A

MATHEMATICAL AND PHILOSOPHICAL

DICTIONARY.

VOL. II.——PART I.

A

MATHEMATICAL AND PHILOSOPHICAL

DICTIONARY:

CONTAINING

AN EXPLANATION OF THE TERMS, AND AN ACCOUNT OF THE SEVERAL SUBJECTS,

COMPRIZED UNDER THE HEADS

MATHEMATICS, ASTRONOMY, AND PHILOSOPHY

BOTH NATURAL AND EXPERIMENTAL:

WITH AN

HISTORICAL ACCOUNT OF THE RISE, PROGRESS, AND PRESENT STATE OF THESE SCIENCES:

ALSO

MEMOIRS OF THE LIVES AND WRITINGS OF THE MOST EMINENT AUTHORS,

BOTH ANCIENT AND MODERN,

WHO BY THEIR DISCOVERIES OR IMPROVEMENTS HAVE CONTRIBUTED TO THE ADVANCEMENT OF THEM.

IN TWO VOLUMES.

WITH MANY CUTS AND COPPER-PLATES.

By CHARLES HUTTON, LL.D. F.R.S.

OF LONDON AND EDINBURGH, AND OF THE PHILOSOPHICAL SOCIETIES OF HAARLEM AND AMERICA; AND PROFESSOR OF MATHEMATICS IN THE ROYAL MILITARY ACADEMY, WOOLWICH.

VOL. II.

LONDON:

PRINTED FOR J. JOHNSON, IN ST. PAUL'S CHURCH-YARD; AND G. G. AND J. ROBINSON, IN PATERNOSTER-ROW.

MDCCXCV.

A
PHILOSOPHICAL and MATHEMATICAL
DICTIONARY.

K.

KALENDAR. See Calendar.

KALENDS. See Calends.

KEILL (Dr. John), an eminent mathematician and philosopher, was born at Edinburgh in 1671, and studied in the university of that city. His genius leading him to the mathematics, he made a great progress under David Gregory the professor there, who was one of the first that had embraced and publicly taught the Newtonian philosophy. In 1694 he followed his tutor to Oxford, where, being admitted of Baliol College, he obtained one of the Scotch exhibitions in that college. It is said he was the first who taught Newton's principles by the experiments on which they are founded: and this it seems he did by an apparatus of instruments of his own providing; by which means he acquired a great reputation in the university. The first public specimen he gave of his skill in mathematical and philosophical knowledge, was his *Examination of Dr. Burnet's Theory of the Earth; with Remarks on Mr. Whiston's New Theory;* which appeared in 1698. These theories were defended by their respective authors; which drew from him, in 1699, *An Examination of the Reflections on the Theory of the Earth*, together with *A Defence of the Remarks on Mr. Whiston's New Theory*. Dr. Burnet was a man of great humanity, moderation, and candour; and it was therefore supposed that Keill had treated him too roughly, considering the great disparity of years between them. Keill however left the doctor in possession of that which has since been thought the great characteristic and excellence of his work; and though he disclaimed him as a philosopher, yet allowed him to be a man of a fine imagination. "Perhaps, says he, many of his readers will be sorry to be undeceived about his theory; for, as I believe never any book was fuller of mistakes and errors in philosophy, so none ever abounded with more beautiful scenes and surprising images of nature. But I write only to those who might expect to find a true philosophy in it: they who read it as an ingenious romance, will still be pleased with their entertainment."

The year following, Dr. Millington, Sedleian professor of natural philosophy in Oxford, who had been appointed physician to king William, substituted Keill as his deputy, to read the lectures in the public school. This office he discharged with great reputation; and, the term of enjoying the Scotch exhibition at Baliol-college now expiring, he accepted an invitation from Dr. Aldrich, dean of Christ-church, to reside there.

In 1701, he published his celebrated treatise, intitled, *Introductio ad Veram Physicam*, which is supposed to be the best and most useful of all his performances. The first edition of this book contained only fourteen lectures; but to the second, in 1705, he added two more. This work was deservedly esteemed, both at home and abroad, as the best introduction to the Principia, or the new mechanical philosophy, and was reprinted in different places; also a new edition in English was printed at London in 1736, at the instance of M. Maupertuis, who was then in England.

Being made Fellow of the Royal Society, he published, in the Philos. Trans. 1708, a paper on the Laws of Attraction, and its physical principles: and being offended at a passage in the *Acta Eruditorum* of Leipsic, where Newton's claim to the first invention of the method

thod of Fluxions was called in queſtion, he warmly vindicated that claim againſt Leibnitz. In 1709 he went to New-England as treaſurer of the Palatines; and ſoon after his return in 1710, he was choſen Savilian profeſſor of aſtronomy at Oxford. In 1711, being attacked by Leibnitz, he entered the liſts with that mathematician, in the diſpute concerning the invention of Fluxions. Leibnitz wrote a letter to Dr. Hans Sloane, then ſecretary to the Royal Society, requiring Keill, in effect, to make him ſatisfaction for the injury he had done him in his paper relating to the paſſage in the *Acta Eruditorum:* he proteſted, that he was far from aſſuming to himſelf Newton's method of Fluxions; and therefore deſired that Keill might be obliged to retract his falſe aſſertion. On the other hand, Keill deſired that he might be permitted to juſtify what he had aſſerted. He made his defence to the approbation of Newton, and other members of the Society. A copy of this was ſent to Leibnitz; who, in a ſecond letter, remonſtrated ſtill more loudly againſt Keill's want of candour and ſincerity; adding, that it was not fit for one of his age and experience to engage in a diſpute with an upſtart, who acted without any authority from Newton, and deſiring that the Royal Society would enjoin him ſilence. Upon this, a ſpecial committee was appointed; who, after examining the facts, concluded their report with "reckoning Mr. Newton the inventor of Fluxions; and that Mr. Keill, in aſſerting the ſame, had been no ways injurious to Mr. Leibnitz." The whole proceedings upon this matter may be ſeen in Collins's *Commercium Epiſtolicum*, with many valuable papers of Newton, Leibnitz, Gregory, and other mathematicians. In the mean time Keill behaved himſelf with great firmneſs and ſpirit; which he alſo ſhewed afterwards in a Latin epiſtle, written in 1720, to Bernoulli, mathematical profeſſor at Baſil, on account of the ſame uſage ſhewn to Newton: in the title-page of which he put the arms of Scotland, viz, a Thiſtle, with this motto, *Nemo me impune laceſſit.*

About the year 1711, ſeveral objections being urged againſt Newton's philoſophy, in ſupport of Des Cartes's notions of a plenum, Keill publiſhed a paper in the Philoſ. Tranſ. on the Rarity of Matter, and the Tenuity of its Compoſition. But while he was engaged in this diſpute, queen Anne was pleaſed to appoint him her Decipherer; and he continued in that place under king George the Firſt till the year 1716. The univerſity of Oxford conferred on him the degree of M. D. in 1713; and, two years after, he publiſhed an edition of Commandine's Euclid, with additions of his own. In 1718 he publiſhed his *Introductio ad Veram Aſtronomiam:* which was afterwards, at the requeſt of the ducheſs of Chandos, tranſlated by himſelf into Engliſh; and, with ſeveral emendations, publiſhed in 1721, under the title of *An Introduction to True Aſtronomy*, &c. This was his laſt gift to the public; being this ſummer ſeized with a violent fever, which terminated his life Sept. 1, in the 50th year of his age.

His papers in the Philoſ. Tranſ. above alluded to, are contained in volumes 26 and 29.

Keill (Dr. *James*), an eminent phyſician and philoſopher, and younger brother of Dr. John Keill above mentioned, was alſo born in Scotland, in 1673. Having travelled abroad, on his return he read lectures on Anatomy with great applauſe in the univerſities of Oxford and Cambridge, by the latter of which he had the degree of M. D. conferred upon him. In 1703 he ſettled at Northampton as a phyſician, where he died of a cancer in the mouth in 1719. His publications are

1. An Engliſh tranſlation of Lemery's Chemiſtry.
2. On Animal Secretion, the quantity of Blood in the Human Body, and on Muſcular Motion.
3. A treatiſe on Anatomy.
4. Several pieces in the Philoſ. Tranſ. volumes 25 and 30.

KEPLER (John), a very eminent aſtronomer and mathematician, was born at Wiel, in the county of Wirtemberg, in 1571. He was the diſciple of Mæſtlinus, a learned mathematician and aſtronomer, of whom he learned thoſe ſciences, and became afterwards profeſſor of them to three ſucceſſive emperors, viz. Matthias, Rudolphus, and Ferdinand the 2d.

To this ſagacious philoſopher we owe the firſt diſcovery of the great laws of the planetary motions, viz. that the planets deſcribe areas that are always proportional to the times; that they move in elliptical orbits, having the ſun in one focus; and that the ſquares of their periodic times, are proportional to the cubes of their mean diſtances; which are now generally known by the name of Kepler's Laws. But as this great man ſtands as it were at the head of the modern reformed aſtronomy, he is highly deſerving of a pretty large account, which we ſhall extract chiefly from the words of that great mathematician Mr. Maclaurin.

Kepler had a particular paſſion for finding analogies and harmonies in nature, after the manner of the Pythagoreans and Platoniſts; and to this diſpoſition we owe ſuch valuable diſcoveries, as are more than ſufficient to excuſe his conceits. Three things, he tells us, he anxiouſly ſought to find out the reaſon of, from his early youth; viz, Why the planets were 6 in number? Why the dimenſions of their orbits were ſuch as Copernicus had deſcribed from obſervations? And what was the analogy or law of their revolutions? He ſought for the reaſons of the two firſt of theſe, in the properties of numbers and plane figures, without ſucceſs. But at length reflecting, that while the plane regular figures may be infinite in number, the regular ſolids are only five, as Euclid had long ago demonſtrated: he imagined, that certain myſteries in nature might correſpond with this remarkable limitation inherent in the eſſences of things; and the rather, as he found that the Pythagoreans had made great uſe of thoſe five regular ſolids in their philoſophy. He therefore endeavoured to find ſome relation between the dimenſions of theſe ſolids and the intervals of the planetary ſpheres; thus, imagining that a cube, inſcribed in the ſphere of Saturn, would touch by its ſix planes the ſphere of Jupiter; and that the other four regular ſolids in like manner fitted the intervals that are between the ſpheres of the other planets: he became perſuaded that this was the true reaſon why the primary planets were preciſely ſix in number, and that the author of the world had determined their diſtances from the ſun, the centre of the ſyſtem, from a regard to this analogy. Being thus poſſeſſed, as he thought, of the grand ſecret of the Pythagoreans, and greatly pleaſed with his diſcovery, he publiſhed it in 1596, under the title of *Myſterium Coſmographicum;* and was for ſome time ſo charmed with it, that he ſaid

he would not give up the honour of having invented what was contained in that book, for the electorate of Saxony.

Kepler fent a copy of this book to Tycho Brahe, who did not approve of thofe abftract fpeculations concerning the fyftem of the world, but wrote to Kepler, firft to lay a folid foundation in obfervations, and then, by afcending from them, to endeavour to come at the caufes of things. Tycho however, pleafed with his genius, was very defirous of having Kepler with him to affift him in his labours: and having fettled, under the protection of the emperor, in Bohemia, where he paffed the laft years of his life, after having left his native country on fome ill-ufage, he prevailed upon Kepler to leave the univerfity of Gratz, and remove into Bohemia, with his family and library, in the year 1600. But Tycho dying the next year, the arranging the obfervations devolved upon Kepler, and from that time he had the title of Mathematician to the Emperor all his life, and gained continually more and more reputation by his works. The emperor Rudolph ordered him to finifh the tables of Tycho Brahe, which were to be called the *Rudolphine Tables*. Kepler applied diligently to the work: but unhappy are thofe learned men who depend upon the good-humour of the intendants of the finances; the treafurers were fo ill-affected towards our author, that he could not publifh thefe tables till 1627. He died at Ratifbon, in 1630, where he was foliciting the payment of the arrears of his penfion.

Kepler made many important difcoveries from Tycho's obfervations, as well as his own. He found, that aftronomers had erred, from the firft rife of the fcience, in afcribing always circular orbits and uniform motions to the planets; that, on the contrary, each of them moves in an ellipfis which has one of its foci in the fun: that the motion of each is really unequable, and varies fo, that a ray fuppofed to be always drawn from the planet to the fun defcribes equal areas in equal times.

It was fome years later before he difcovered the analogy there is between the diftances of the feveral planets from the fun, and the periods in which they complete their revolutions. He eafily faw, that the higher planets not only moved in greater circles, but alfo more flowly than the nearer ones; fo that, on a double account, their periodic times were greater. Saturn, for example, revolves at the diftance from the fun $9\frac{1}{2}$ times greater than the earth's diftance from it; and the circle defcribed by Saturn is in the fame proportion: but as the earth revolves in one year, fo, if their velocities were equal, Saturn ought to revolve in 9 years and a half; whereas the periodic time of Saturn is about 29 years. The periodic times of the planets increafe, therefore, in a greater proportion than their diftances from the fun: but yet not in fo great a proportion as the fquares of thofe diftances; for if that were the law of the motions, (the fquare of $9\frac{1}{2}$ being $90\frac{1}{4}$), the periodic time of Saturn ought to be above 90 years. A mean proportion between that of the diftances of the planets, and that of the fquares of thofe diftances, is the true proportion of the periodic times; as the mean between $9\frac{1}{2}$ and its fquare $90\frac{1}{4}$, gives the periodic time of Saturn in years. Kepler, after having committed feveral miftakes in determining this analogy, hit upon it at laft, May the 15, 1618; for he is fo particular as to mention the precife day when he found that "The fquares of the periodic times were always in the fame proportion as the cubes of their mean diftances from the fun."

When Kepler faw, according to better obfervations, that his difpofition of the five regular folids among the planetary fpheres, was not agreeable to the intervals between their orbits, he endeavoured to difcover other fchemes of harmony. For this purpofe, he compared the motions of the fame planet at its greateft and leaft diftances, and of the different planets in their feveral orbits, as they would appear viewed from the fun; and here he fancied that he found a fimilitude to the divifions of the octave in mufic. Thefe were the dreams of this ingenious man, which he was fo fond of, that, hearing of the difcovery of four new planets (the fatellites of Jupiter) by Galileo, he owns that his firft reflections were from a concern how he could fave his favourite fcheme, which was threatened by this addition to the number of the planets. The fame attachment led him into a wrong judgment concerning the fphere of the fixed ftars: for being obliged, by his doctrine, to allow a vaft fuperiority to the fun in the univerfe, he reftrains the fixed ftars within very narrow limits. Nor did he confider them as funs, placed in the centres of their feveral fyftems, having planets revolving round them; as the other followers of Copernicus have concluded them to be, from their having light in themfelves, from their immenfe diftances, and from the analogy of nature. Not contented with thefe harmonies, which he had learned from the obfervations of Tycho, he gave himfelf the liberty to imagine feveral other analogies, that have no foundation in nature, and are overthrown by the beft obfervations. Thus from the opinions of Kepler, though moft juftly admired, we are taught the danger of efpoufing principles, or hypothefes, borrowed from abftract fciences, and of applying them, with fuch freedom, to natural enquiries.

A more recent inftance of this fondnefs, for difcovering analogies between matters of abftract fpeculation, and the conftitution of nature, we find in Huygens, one of the greateft geometricians and aftronomers any age has produced: when he had difcovered that fatellite of Saturn, which from him is ftill called the Huygenian fatellite, this, with our moon, and the four fatellites of Jupiter, completed the number of fix fecondary planets then difcovered in the fyftem; and becaufe the number of primary planets was alfo fix, and this number is called by mathematicians a perfect number (being equal to the fum of its aliquot parts, 1, 2, 3,) Huygens was hence induced to believe that the number of the planets was complete, and that it was in vain to look for any more. This is not mentioned to leffen the credit of this great man, who never perhaps reafoned in fuch a manner on any other occafion; but only to fhew, by another inftance, how ill-grounded reafonings of this kind have always proved. For, not long after, the celebrated Caffini difcovered four more fatellites about Saturn, not to mention the two more that have lately been difcovered to that planet by Dr. Herfchel, with another new primary planet and its two fatellites, befides many others, of both forts, as yet unknown, which poffibly may belong to our fyftem. The fame Caffini having found that the analogy, difcovered by Kepler, between the periodic times and the diftances from the centre, takes place in the

the leſſer ſyſtems of Jupiter and Saturn, as well as in the great ſolar ſyſtem; his obſervations overturned that groundleſs analogy which had been imagined between the number of the planets, both primary and ſecondary, and the number ſix: but eſtabliſhed, at the ſame time, that harmony in their motions, which will afterwards appear to flow from one real principle extended over the univerſe.

But to return to Kepler; his great ſagacity, and continual meditations on the planetary motions, ſuggeſted to him ſome views of the true principles from which theſe motions flow. In his preface to the Commentaries concerning the planet Mars, he ſpeaks of gravity as of a power that was mutual between bodies, and tells us, that the earth and moon tend towards each other, and would meet in a point, ſo many times nearer to the earth than to the moon, as the earth is greater than the moon, if their motions did not hinder it. He adds, that the tides ariſe from the gravity of the waters towards the moon. But not having notions ſufficiently juſt of the laws of motion, it ſeems he was not able to make the beſt uſe of theſe thoughts; nor does it appear that he adhered to them ſteadily, ſince in his Epitome of Aſtronomy, publiſhed many years after, he propoſes a phyſical account of the planetary motions, derived from different principles.

He ſuppoſes, in that treatiſe, that the motion of the ſun on his axis, is preſerved by ſome inherent vital principle; that a certain virtue, or immaterial image of the ſun, is diffuſed with his rays into the ambient ſpaces, and, revolving with the body of the ſun on his axis, takes hold of the planets, and carries them along with it in the ſame direction; like as a loadſtone turned round near a magnetic needle, makes it turn round at the ſame time. The planet, according to him, by its inertia, endeavours to continue in its place, and the action of the ſun's image and this inertia are in a perpetual ſtruggle. He adds, that this action of the ſun, like his light, decreaſes as the diſtance increaſes; and therefore moves the ſame planet with greater celerity when nearer the ſun, than at a greater diſtance. To account for the planet's approaching towards the ſun as it deſcends from the aphelion to the perihelion, and receding from the ſun while it aſcends to the aphelion again, he ſuppoſes that the ſun attracts one part of each planet, and repels the oppoſite part; and that the part attracted is turned towards the ſun in the deſcent, and the other towards the ſun in the aſcent. By ſuppoſitions of this kind, he endeavoured to account for all the other varieties of the celeſtial motions.

But, now that the laws of motion are better known than in Kepler's time, it is eaſy to ſhew the fallacy of every part of this account of the planetary motions. The planet does not endeavour to ſtop in conſequence of its inertia, but to perſevere in its motion in a right line. An attractive force makes it deſcend from the aphelion to the perihelion in a curve concave towards the ſun: but the repelling force, which he ſuppoſed to begin at the perihelion, would cauſe it to aſcend in a figure convex towards the ſun. There will be occaſion to ſhew afterwards, from Sir Iſaac Newton, how an attraction or gravitation towards the ſun, alone produces the effects, which, according to Kepler, required both an attractive and repelling force; and that the virtue which he aſcribed to the ſun's image, propagated into the planetary regions, is unneceſſary, as it could be of no uſe for this effect, though it were admitted. For now his own prophecy, with which he concludes his book, is verified; where he tells us, that "the diſcovery of ſuch things was reſerved for the ſucceeding ages, when the author of nature would be pleaſed to reveal theſe myſteries."

The works of this celebrated author are many and valuable; as,

1. His *Coſmographical Myſtery*, in 1596.
2. *Optical Aſtronomy*, in 1604.
3. *Account of a New Star in Sagittarius*, 1605.
4. *New Aſtronomy*; or, *Celeſtial Phyſics*, in Commentaries on the planet Mars.
5. *Diſſertations*; with the *Nuncius Siderius* of Galileo, 1610.
6. *New Gauging of Wine Caſks*, 1615. Said to be written on occaſion of an erroneous meaſurement of the wine at his marriage by the revenue officer.
7. *New Ephemerides*, from 1617 to 1620.
8. *Copernican Syſtem*, three firſt books of the, 1618.
9. *Harmony of the World*; and three books of *Comets*, 1619.
10. *Coſmographical Myſtery*, 2d edit. with Notes, 1621.
11. *Copernican Aſtronomy*; the three laſt books, 1622.
12. *Logarithms*, 1624; and the *Supplement*, in 1625.
13. His *Aſtronomical Tables*, called the *Rudolphine Tables*, in honour of the emperor Rudolphus, his great and learned patron, in 1627.
14. *Epitome of the Copernican Aſtronomy*, 1635.

Beſide theſe, he wrote ſeveral pieces on various other branches, as *Chronology*, *Geometry of Solids*, *Trigonometry*, and an excellent treatiſe of *Dioptrics*, for that time.

Kepler's Laws, are thoſe laws of the planetary motions diſcovered by Kepler. Theſe diſcoveries in the mundane ſyſtem, are commonly accounted two, viz. 1ſt, That the planets deſcribe about the ſun, areas that are proportional to the times in which they are deſcribed, namely, by a line connecting the ſun and planet; and 2d, That the ſquares of the times of revolution, are as the cubes of the mean diſtances of the planets from the ſun. Kepler diſcovered alſo that the orbits of the planets are elliptical.

Theſe diſcoveries of Kepler, however, were only found out by many trials, in ſearching among a great number of aſtronomical obſervations and revolutions, what rules and laws were found to obtain. On the other hand, Newton has demonſtrated, *a priori*, all theſe laws, ſhewing that they muſt obtain in the mundane ſyſtem, from the laws of gravitation and centripetal force; viz, the firſt of theſe laws reſulting from a centripetal force urging the planets towards the ſun, and the 2d, from the centripetal force being in an inverſe ratio of the ſquare of the diſtance. And the elliptic form of the orbits, from a projectile force regulated by a centripetal one.

Kepler's *Problem*, is the determining the true from the mean anomaly of a planet, or the determining its place, in its elliptic orbit, anſwering to any given time; and ſo named from the celebrated aſtronomer Kepler, who firſt propoſed it. See Anomaly.

The

The general ſtate of the problem is this: To find the poſition of a right line, which, paſſing through one of the foci of an ellipſis, ſhall cut off an area which ſhall be in any given proportion to the whole area of the ellipſis; which reſults from this property, that ſuch a line ſweeps areas that are proportional to the times.

Many ſolutions have been given of this problem, ſome direct and geometrical, others not: viz, by Kepler, Bullíald, Ward, Newton, Keill, Machin, &c. See Newton's Princip. lib. 1. prop. 31, Keill's Aſtron. Lect. 23, Philoſ. Tranſ. abr. vol. 8. pa. 73, &c.

In the laſt of theſe places, Mr. Machin obſerves, that many attempts have been made at different times, but with no great ſucceſs, towards the ſolution of the problem propoſed by Kepler: To divide the area of a ſemicircle into given parts, by a line drawn from a given point in the diameter, in order to find an univerſal rule for the motion of a body in an elliptic orbit. For among the ſeveral methods offered, ſome are only true in ſpeculation, but are really of no ſervice; others are not different from his own, which he judged improper. And as to the reſt, they are all ſo limited and confined to particular conditions and circumſtances, as ſtill to leave the problem in general untouched. To be more particular; it is evident, that all conſtructions by mechanical curves are ſeeming ſolutions only, but in reality unapplicable; that the roots of infinite ſeries are, on account of their known limitations in all reſpects, ſo far from being ſufficient rules, that they ſerve for little more than exerciſes in a method of calculation. And then, as to the univerſal method, which proceeds by a continued correction of the errors of a falſe poſition, it is no method of ſolution at all in itſelf; becauſe, unleſs there be ſome antecedent rule or hypotheſis to begin the operation (as ſuppoſe that of an uniform motion about the upper focus, for the orbit of a planet; or that of a motion in a parabola for the perihelion part of the orbit of a comet, or ſome other ſuch), it would be impoſſible to proceed one ſtep in it. But as no general rule has ever yet been laid down, to aſſiſt this method, ſo as to make it always operate, it is the ſame in effect as if there were no method at all. And accordingly in experience it is found, that there is no rule now ſubſiſting but what is abſolutely uſeleſs in the elliptic orbits of comets; for in ſuch caſes there is no other way to proceed but that which was uſed by Kepler: to compute a table for ſome part of the orbit, and in it examine if the time to which the place is required, will fall out any where in that part. So that, upon the whole, it appears evident, that this problem, contrary to the received opinion, has never yet been advanced one ſtep towards its true ſolution.

Mr. Machin then proceeds to give his own ſolution of this problem, which is particularly neceſſary in orbits of a great excentricity; and he illuſtrates his method by examples for the orbits of Venus, of Mercury, of the comet of the year 1682, and of the great comet of the year 1680, ſufficiently ſhewing the univerſality of the method.

KEY, in Muſic, is a certain fundamental note, or tone, to which the whole piece, be it concerto, ſonata, cantata, &c, is accommodated; and with which it uſually begins, but always ends.

KEYS denote alſo, in an organ, harpſichord, &c, the pieces of wood or ivory which are ſtruck by the fingers, in playing upon the inſtrument.

KEYSTONE, the middle vouſſoir, or the arch ſtone in the top, or immediately over the centre of an arch.—The length of the keyſtone, or thickneſs of the archivolt at top, is allowed by the beſt architects, to be about the 15th or 16th part of the ſpan.

KILDERKIN, a kind of liquid meaſure, containing two firkins, or 18 gallons, beer-meaſure, or 16 ale-meaſure.

KING-*piece*, or KING-*poſt*, is a piece of timber ſet upright in the middle, between two principal rafters, and having ſtruts or braces going from it to the middle of each rafter.

KIRCH (CHRISTIAN FREDERIC), of Berlin, a celebrated aſtronomer, was born at Guben in 1694. He acquired great reputation in the obſervatories of Dantzic and Berlin. Godfrey Kirch his father, and Mary his mother, alſo acquired conſiderable reputation by their aſtronomical obſervations. This family correſponded with all the learned ſocieties of Europe, and their aſtronomical works are in great repute.

KIRCHER (ATHANASIUS), a famous philoſopher and mathematician, was born at Fulde in 1601. He entered into the ſociety of the Jeſuits in 1618, and taught philoſophy, mathematics, the Hebrew and Syriac Languages, in the univerſity of Wirtſburg, with great applauſe, till the year 1631. He retired to France on account of the ravages committed by the Swedes in Franconia, and lived ſome time at Avignon. He was afterwards called to Rome, where he taught mathematics in the Roman college, collected a rich cabinet of machines and antiquities, and died in 1680, in the 80th year of his age.

The quantity of his works is immenſe, amounting to 22 volumes in folio, 11 in quarto, and three in octavo; enough to employ a man for a great part of his life even to tranſcribe them. Moſt of them are rather curious than uſeful; many of them viſionary and fanciful; and it is not to be wondered at, if they are not always accompanied with the greateſt exactneſs and preciſion. The principal of them are,

1. *Præluſiones Magneticæ.*
2. *Primitiæ Gnomonicæ Catoptricæ.*
3. *Ars magna Lucis et Umbræ.*
4. *Muſurgia Univerſalis.*
5. *Obeliſcus Pamphilius.*
6. *Oedipus Ægyptiacus*; 4 volumes folio.
7. *Itinerarium Extaticum.*
8. *Obeliſcus Ægyptiacus*; 4 volumes folio.
9. *Mundus Subterraneus.*
10. *China Illuſtrata.*

KNOT, a tye, or complication of a rope, cord, or ſtring, or of the ends of two together. There are divers ſorts of knots uſed for different purpoſes, which may be explained by ſhewing the figures of them open, or undrawn, thus. 1. Fig. 1, plate xiii. is a *Thumb knot*. This is the ſimpleſt of all. It is uſed to tye at the end of a rope, to prevent its opening out: it is alſo uſed by taylors &c. at the end of their thread.

Fig. 2, a *Loop knot*. Uſed to join pieces of rope &c. together.

Fig. 3, a *Draw knot*, which is the ſame as the laſt; only one end or both return the ſame way back, as *a b c d*.

a b c d. By drawing at *a*, the part *b c d* comes through, and the knot is loofed.

Fig. 4, a *Ring knot.* This ferves alfo to join pieces of cord &c together.

Fig. 5 is another knot for tying cords together. This is ufed when any cord is often to be loofed.

Fig. 6, a *Running knot,* to draw any thing clofe. By pulling at the end *a*, the cord is drawn through the loop *b*, and the part *c d* is drawn clofe about a beam, &c.

Fig. 7 is another knot, to tye any thing to a poft. And here the end may be put through as often as you pleafe.

Fig. 8, a *Very fmall knot.* A thumb knot is firft made at the end of each piece, and then the end of the other is paffed through it. Thus, the cord *a c* runs through the loop *d*, and *b d* through *c*; and then drawn clofe by pulling at *a* and *b*. If the ends *e* and *f* be drawn, the knot will be loofed again.

Fig. 9, a *Fifher's knot,* or *Water knot.* This is the fame as the 4th, only the ends are to be put twice through the ring, which in the former was but once; and then drawn clofe.

Fig. 10, a *Mefhing knot,* for nets; and is to be drawn clofe.

Fig. 11, a *Barber's knot,* or a knot for cawls of wigs; and is to be drawn clofe.

Fig. 12, a *Bowline knot.* When this is drawn clofe, it makes a loop that will not flip, as fig. 7; and ferves to hitch over any thing.

Fig. 13, a *Wale knot,* which is made with the three ftrands of a rope, fo that it cannot flip. When the rope is put through a hole, this knot keeps it from flipping through. When the three ftrands are wrought round once or twice more, after the fame manner, it is called *crowning.* By this means the knot is made larger and ftronger. A thumb knot, N°. 1, may be applied to the fame ufe as this.

KNOTS mean alfo the divifions of the log line, ufed at fea. Thefe are ufually 7 fathom, or 42 feet afunder; but fhould be $8\frac{1}{3}$ fathom, or 50 feet. And then, as many knots as the log line runs out in half a minute, fo many miles does the fhip fail in an hour; fuppofing her to keep going at an equal rate, and allowing for yaws, leeway, &c.

KOENIG (SAMUEL), a learned philofopher and mathematician, was a Swifs by birth, and came early into eminence by his mathematical abilities. He was profeffor of philofophy and natural law at Franeker, and afterwards at the Hague, where he became alfo librarian to the Stadtholder, and to the Princefs of Orange; and where he died in 1757.

The Academy of Berlin enrolled him among her members; but afterwards expelled him on the following occafion. Maupertuis, the prefident, had inferted in the volume of the Memoirs for 1746, a difcourfe upon the Laws of Motion; which Koenig not only attacked, but alfo attributed the memoir to Leibnitz. Maupertuis, ftung with the imputation of plagiarifm, engaged the Academy of Berlin to call upon him for his proof; which Koenig failing to produce, he was ftruck out of the academy. All Europe was interefted in the quarrel which this occafioned between Koenig and Maupertuis. The former appealed to the public; and his appeal, written with the animation of refentment, procured him many friends. He was author of fome other works, and had the character of being one of the beft mathematicians of the age.

L.

LABEL, a long thin brafs ruler, with a fmall fight at one end, and a central hole at the other; commonly ufed with a tangent-line on the edge of a circumferentor, to take altitudes, and other angles.

LACERTA, *Lizard,* one of the new conftellations of the northern hemifphere, added by Hevelius to the 48 old ones, near Cepheus and Caffiopeia.

This conftellation contains, in Hevelius's catalogue 10 ftars, and in Flamfteed's 16.

LACUNAR, an arched roof or cieling; more efpecially the planking or flooring above the porticos.

LADY-*Day,* the 25th of March, being the Annunciation of the Holy Virgin.

LAGNY (THOMAS FANTET *de*), an eminent French mathematician, was born at Lyons. Fournier's Euclid, and Pelletier's Algebra, by chance falling in his way, developed his genius for the mathematics. It was in vain that his father defigned him for the law; he went to Paris to deliver himfelf wholly up to the ftudy of his favourite fcience. In 1697, the Abbé Bignon, protector-general of letters, got him appointed profeffor-royal of Hydrography at Rochfort. Soon after, the duke of Orleans, then regent of France, fixed him at Paris, and made him fub-director of the General Bank, in which he loft the greateft part of his fortune in the failure of the Bank. He had been received into the ancient academy in 1696; upon the renewal of which he was named Affociate-geometrician in 1699, and penfioner in 1723. After a life fpent in clofe application, he died, April 12, 1734.

In the laft moments of his life, and when he had loft all knowledge of the perfons who furrounded his bed,

bed, one of them, through curiosity, asked him, what is the square of 12? To which he immediately replied, and without seeming to know that he gave any answer, 144.

De Lagny particularly excelled in arithmetic, algebra, and geometry, in which he made many improvements and discoveries. He, as well as Leibnitz, invented a binary arithmetic, in which only two figures are concerned. He rendered much easier the resolution of algebraic equations, especially the irreducible case in cubic equations; and the numeral resolution of the higher powers, by means of short approximating theorems.—He delivered the measures of angles in a new science, called *Goniometry*; in which he measured angles by a pair of compasses, without scales, or tables, to great exactness; and thus gave a new appearance to trigonometry.—*Cyclometry*, or the measure of the circle, was also an object of his attention; and he calculated, by means of infinite series, the ratio of the circumference of a circle to its diameter, to 120 places of figures.—He gave a general theorem for the tangents of multiple arcs. With many other curious or useful improvements, which are found in the great multitude of his papers, that are printed in the different volumes of the Memoirs of the Academy of Sciences, viz, in almost every volume, from the year 1699, to 1729.

LAKE, a collection of water, inclosed in the cavity of some inland place, of a considerable extent and depth. As the Lake of Geneva, &c.

LAMMAS-DAY, the 1st of August; so called, according to some, because lambs then grow out of season, as being too large. Others derive it from a Saxon word, signifying *loaf-mass*, because on that day our forefathers made an offering of bread prepared with new wheat.

It is celebrated by the Romish church in memory of St. Peter's imprisonment.

LAMPÆDIAS, a kind of bearded comet, resembling a burning lamp, being of several shapes; for sometimes its flame or blaze runs tapering upwards like a sword, and sometimes it is double or treble pointed.

LANDEN (JOHN), an eminent mathematician, was born at Peakirk, near Peterborough in Northamptonshire, in January 1719. He became very early a proficient in the mathematics, for we find him a very respectable contributor to the Ladies Diary in 1744; and he was soon among the foremost of those who then contributed to the support of that small but valuable publication, in which almost every English mathematician who has arrived at any degree of eminence for the best part of this century, has contended for fame at one time or other of his life. Mr. Landen continued his contributions to it at times, under various signatures, till within a few years of his death.

It has been frequently observed, that the histories of literary men consist chiefly of the history of their writings; and the observation was never more fully verified, than in the present article concerning Mr. Landen.

In the 48th volume of the Philosophical Transactions, for the year 1754, Mr. Landen gave "An Investigation of some theorems which suggest several very remarkable properties of the Circle, and are at the same time of considerable use in resolving Fractions, the denominators of which are certain Multinomials, into more simple ones, and by that means facilitate the computation of Fluents." This ingenious paper was delivered to the Society by that eminent mathematician Thomas Simpson of Woolwich, a circumstance which will convey to those who are not themselves judges of it, some idea of its merit.

In the year 1755, he published a volume of about 160 pages, intitled *Mathematical Lucubrations*. The title to this publication was made choice of, as a means of informing the world, that the study of the mathematics was at that time rather the pursuit of his leisure hours, than his principal employment: and indeed it continued to be so, during the greatest part of his life; for about the year 1762 he was appointed agent to Earl Fitzwilliam, an employment which he resigned only two years before his death. These Lucubrations contain a variety of tracts relative to the rectification of curve lines, the summation of series, the finding of fluents, and many other points in the higher parts of the mathematics.

About the latter end of the year 1757, or the beginning of 1758, he published proposals for printing by subscription, *The Residual Analysis*, a new Branch of the Algebraic art: and in 1758 he published a small tract, entitled *A Discourse on the Residual Analysis*; in which he resolved a variety of problems, to which the method of fluxions had usually been applied, by a mode of reasoning entirely new: he also compared these solutions with others derived from the fluxionary method; and shewed that the solutions by his new method were commonly more natural and elegant than the fluxionary ones.

In the 51st volume of the Philosophical Transactions, for the year 1760, he gave *A New Method of computing the Sums of a great number of Infinite Series*. This paper was also presented to the Society by his ingenious friend the late Mr. Thomas Simpson.

In 1764, he published the first book of *The Residual Analysis*. In this treatise, besides explaining the principles which his new analysis was founded on, he applied it, in a variety of problems, to drawing tangents, and finding the properties of curve lines; to describing their involutes and evolutes, finding the radius of curvature, their greatest and least ordinates, and points of contrary flexure; to the determination of their cusps, and the drawing of asymptotes: and he proposed, in a second book, to extend the application of this new analysis to a great variety of mechanical and physical subjects. The papers which were to have formed this book lay long by him; but he never found leisure to put them in order for the press.

In the year 1766, Mr. Landen was elected a Fellow of the Royal Society. And in the 58th volume of the Philosophical Transactions, for the year 1768, he gave *A specimen of a New Method of comparing Curvilinear Areas*; by means of which many areas are compared, that did not appear to be comparable by any other method: a circumstance of no small importance in that part of natural philosophy which relates to the doctrine of motion.

In the 60th volume of the same work, for the year 1770, he gave *Some New Theorems* for computing the Whole Areas of Curve Lines, where the Ordinates are expressed

expreffed by Fractions of a certain form, in a more concife and elegant manner than had been done by Cotes, De Moivre, and others who had confidered the fubject before him.

In the 61ft volume, for 1771, he has inveftigated feveral new and ufeful theorems for computing certain fluents, which are affignable by arcs of the conic fections. This fubject had been confidered before, both by Maclaurin and d'Alembert; but fome of the theorems that were given by thefe celebrated mathematicians, being in part expreffed by the difference between an hyperbolic arc and its tangent, and that difference being not directly attainable when the arc and its tangent both become infinite, as they will do when the whole fluent is wanted, although fuch fluent be finite; thefe theorems therefore fail in thefe cafes, and the computation becomes impracticable without farther help. This defect Mr. Landen has removed, by affigning the *limit* of the difference between the hyperbolic arc and its tangent, while the point of contact is fuppofed to be removed to an infinite diftance from the vertex of the curve. And he concludes the paper with a curious and remarkable property relating to pendulous bodies, which is deducible from thofe theorems. In the fame year he publifhed *Animadverfions on Dr. Stewart's Computation of the Sun's Diftance from the Earth.*

In the 65th volume of the Philofophical Tranfactions, for 1775, he gave the inveftigation of a General Theorem, which he had promifed in 1771, for finding the Length of any Curve of a Conic Hyperbola by means of two Elliptic Arcs: and he obferves, that by the theorems there inveftigated, both the elaftic curve and the curve of equable recefs from a given point, may be conftructed in thofe cafes where Maclaurin's elegant method fails.

In the 67th volume, for 1777, he gave "A New Theory of the Motion of bodies revolving about an axis in free fpace, when that motion is difturbed by fome extraneous force, either percuffive or accelerative." At that time he did not know that the fubject had been treated by any perfon before him, and he confidered only the motion of a fphere, fpheroid, and cylinder. After the publication of this paper however he was informed, that the doctrine of rotatory motion had been confidered by d'Alembert; and upon procuring that author's *Opufcules Mathematiques*, he there learned that d'Alembert was not the only one who had confidered the matter before him; for d'Alembert there fpeaks of fome mathematician, though he does not mention his name, who, after reading what had been written on the fubject, doubted whether there be any folid whatever, befide the fphere, in which any line, paffing through the centre of gravity, will be a permanent axis of rotation. In confequence of this, Mr. Landen took up the fubject again; and though he did not then give a folution to the general problem, viz, "to determine the motions of a body of any form whatever, revolving without reftraint about any axis paffing through its centre of gravity," he fully removed every doubt of the kind which had been ftarted by the perfon alluded to by d'Alembert, and pointed out feveral bodies which, under certain dimenfions, have that remarkable property. This paper is given, among many others equally curious, in a volume of *Memoirs*, which he publifhed in the year 1780. That volume is alfo enriched with a very extenfive appendix, containing *Theorems for the Calculation of Fluents*; which are more complete and extenfive than thofe that are found in any author before him.

In 1781, 1782, and 1783, he publifhed three fmall Tracts on the Summation of Converging Series; in which he explained and fhewed the extent of fome theorems which had been given for that purpofe by De Moivre, Stirling, and his old friend Thomas Simpfon, in anfwer to fome things which he thought had been written to the difparagement of thofe excellent mathematicians. It was the opinion of fome, that Mr. Landen did not fhew lefs mathematical fkill in explaining and illuftrating thefe theorems, than he has done in his writings on original fubjects; and that the authors of them were as little aware of the extent of their own theorems, as the reft of the world were before Mr. Landen's ingenuity made it obvious to all.

About the beginning of the year 1782, Mr. Landen had made fuch improvements in his theory of Rotatory Motion, as enabled him, he thought, to give a folution of the general problem mentioned above; but finding the refult of it to differ very materially from the refult of the folution which had been given of it by d'Alembert, and not being able to fee clearly where that gentleman in his opinion had erred, he did not venture to make his own folution public. In the courfe of that year, having procured the Memoirs of the Berlin Academy for 1757, which contain M. Euler's folution of the problem, he found that this gentleman's folution gave the fame refult as had been deduced by d'Alembert; but the perfpicuity of Euler's manner of writing enabled him to difcover where he had differed from his own, which the obfcurity of the other did not do. The agreement, however, of two writers of fuch eftablifhed reputation as Euler and d'Alembert made him long dubious of the truth of his own folution, and induced him to revife the procefs again and again with the utmoft circumfpection; and being every time more convinced that his own folution was right, and theirs wrong, he at length gave it to the public, in the 75th volume of the Philofophical Tranfactions, for 1785.

The extreme difficulty of the fubject, joined to the concife manner in which Mr. Landen had been obliged to give his folution, to confine it within proper limits for the Tranfactions, rendered it too difficult, or at leaft too laborious a tafk for moft mathematicians to read it; and this circumftance, joined to the eftablifhed reputation of Euler and d'Alembert, induced many to think that their folution was right, and Mr. Landen's wrong; and there did not want attempts to prove it; particularly a long and ingenious paper by the learned Mr. Wildbore, a gentleman of very diftinguifhed talents and experience in fuch calculations; this paper is given in the 80th volume of the Philofophical Tranfactions, for the year 1790, in which he agrees with the folutions of Euler and d'Alembert, and againft that of Mr. Landen. This determined the latter to revife and extend his folution, and give it at greater length, to render it more generally underftood. About this time alfo he met by chance with the late Frifi's *Cofmographiæ Phyficæ et Mathematicæ*; in the fecond part of which

which there is a ſolution of this problem, agreeing in the reſult with thoſe of Euler and d'Alembert. Here Mr. Landen learned that Euler had reviſed the ſolution which he had given formerly in the Berlin Memoirs, and given it another form, and at greater length, in a volume publiſhed at Roſtoch and Gryphiſwald in 1765, intitled, *Theoria Motûs Corporum Solidorum ſeu Rigidorum*. Having therefore procured this book, Mr. Landen found the ſame principles employed in it, and of courſe the ſame concluſion reſulting from them, as in M. Euler's former ſolution of the problem. But notwithſtanding that there were thus a coincidence of at leaſt four moſt reſpectable mathematicians againſt him, Mr. Landen was ſtill perſuaded of the truth of his own ſolution, and prepared to defend it. And as he was convinced of the neceſſity of explaining his ideas on the ſubject more fully, ſo he now found it neceſſary to loſe no time in ſetting about it. He had for ſeveral years been ſeverely afflicted with the ſtone in the bladder, and towards the latter part of his life to ſuch a degree as to be confined to his bed for more than a month at a time: yet even this dreadful diſorder did not extinguiſh his ardour for mathematical ſtudies; for the ſecond volume of his *Memoirs*, lately publiſhed, was written and reviſed during the intervals of his diſorder. This volume, beſides a ſolution of the general problem concerning rotatory motion, contains the reſolution of the problem relating to the motion of a Top; with an inveſtigation of the motion of the Equinoxes, in which Mr. Landen has firſt of any one pointed out the cauſe of Sir Iſaac Newton's miſtake in his ſolution of this celebrated problem; and ſome other papers of conſiderable importance. He juſt lived to ſee this work finiſhed, and received a copy of it the day before his death, which happened on the 15th of January 1790, at Milton, near Peterborough, in the 71ſt year of his age.

LARBOARD, the left-hand ſide of a ſhip, when a perſon ſtands with his face towards the head.

LARMIER, in Architecture, a flat ſquare member of the cornice below the cimaſium, and jets out fartheſt; being ſo called from its uſe, which is to diſperſe the water, and cauſe it to fall at a diſtance from the wall, drop by drop, or, as it were, by tears; *larme* in French ſignifying a tear.

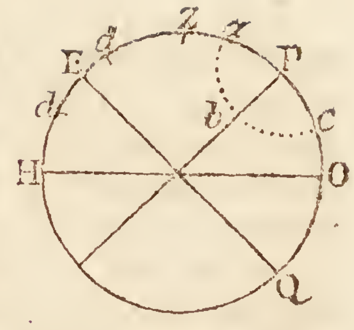
LATERAL Equation, in Algebra, is the ſame with ſimple equation. It has but one root, and may be conſtructed by right lines only.

LATION, is uſed by ſome, for the tranſlation or motion of a body from one place to another.

LATITUDE, in Geography, or Navigation, the diſtance of a place from the equator; or an arch of the meridian, intercepted between its zenith and the equator. Hence the Latitude is either north or ſouth, according as the place is on the north or ſouth ſide of the equator: thus London is ſaid to be in 51° 31′ of north latitude.

Circles parallel to the equator are called *parallels of latitude*, becauſe they ſhew the latitudes of places by their interſections with the meridian.

The Latitude of a place is equal to the elevation of the pole above the horizon of the place: and hence theſe two terms are uſed indifferently for each other.



This will be evident from the figure, where the circle ZHQP is the meridian, Z the zenith of the place, HO the horizon, EQ the equator, and P the pole; then is ZE the latitude, and PO the elevation of the pole above the horizon. And becauſe PE is = ZO, being each a quadrant, if the common part PZ be taken from both, there will remain the latitude ZE = PO the elevation of the pole.—Hence we have a method of meaſuring the circumference of the earth, or of determining the quantity of a degree on its ſurface; for by meaſuring directly northward or ſouthward, till the pole be one degree higher or lower, we ſhall have the number of miles in a degree of a great circle on the ſurface of the earth; and conſequently multiplying that by 360, will give the number of miles round the whole circumference of the earth.

The knowledge of the Latitude of the place, is of the utmoſt conſequence, in geography, navigation, and aſtronomy; it may be proper therefore to lay down ſome of the beſt ways of determining it, both by ſea and land.

1ſt. One method is, to find the Latitude of the pole, to which it is equal, by means of the pole ſtar, or any other circumpolar ſtar, thus: Either draw a true meridian line, or find the times when the ſtar is on the meridian, both above and below the pole; then at theſe times, with a quadrant, or other fit inſtrument, take the altitudes of the ſtar; or take the ſame when the ſtar comes upon your meridian line; which will be the greateſt and leaſt altitude of the ſtar: then ſhall half the ſum of the two be the elevation of the pole, or the latitude ſought.—For, if *abc* be the path of the ſtar about the pole P, Z the zenith, and HO the horizon: then is aO the altitude of the ſtar upon the meridian when above the pole, and cO the ſame when below the pole; hence, becauſe $aP = cP$, therefore $aO + cO = 2OP$, hence the height of the pole OP, or latitude of Z, is equal to half the ſum of aO and cO.

2d. A ſecond method is by means of the declination of the ſun, or a ſtar, and one meridian altitude of the ſame, thus: Having, with a quadrant, or other inſtrument, obſerved the zenith diſtance Zd of the luminary; or elſe its altitude Hd, and taken its complement Zd; then to this zenith diſtance, add the declination dE when the luminary and place are on the ſame ſide of the equator, or ſubtract it when on different ſides, and the ſum or difference will be the latitude EZ ſought. But note, that all altitudes obſerved, muſt be corrected for refraction and the dip of the horizon, and for the ſemidiameter of the ſun, when that is the luminary obſerved.

Many other methods of obſerving and computing the Latitude may be ſeen in Robertſon's Navigation; ſee book 5 and book 9. See alſo the Nautical Almanac for 1771.

Mr. Richard Graham contrived an ingenious inſtrument for taking the latitude of a place at any time of the day. See Philoſ. Tranſ. N°. 435, or Abr. vol. 8. pa. 371.

Latitude, in Aſtronomy, as of a ſtar or planet, is its diſtance from the ecliptic, being an arch of a circle of latitude, reckoned from the ecliptic towards its poles, either north or ſouth. Hence, the aſtronomical latitude is quite different from the geographical, the former meaſuring from the ecliptic, and the latter from the equator, ſo that this latter anſwers to the declination in aſtronomy, which meaſures from the equinoctial.

The ſun has no latitude, being always in the ecliptic; but all the ſtars have their ſeveral latitudes, and the planets are continually changing their latitudes, ſometimes north, and ſometimes ſouth, croſſing the ecliptic from the one ſide to the other; the points in which they croſs the ecliptic being called the *nodes* of the planet, and in theſe points it is that they can paſs over the face of the ſun, or behind his body, viz, when they come both to this point of the ecliptic at the ſame time.

Circle of Latitude, is a great circle paſſing through the poles of the ecliptic, and conſequently perpendicular to it, like as the meridians are perpendicular to the equator, and paſs through its poles.

Latitude, *of the Moon, North aſcending*, is when ſhe proceeds from the aſcending node towards her northern limit, or greateſt elongation.

Latitude, *North deſcending*, is when the moon returns from her northern limit towards the deſcending node.

Latitude, *South deſcending*, is when ſhe proceeds from the deſcending node towards her ſouthern limit.

Latitude, *South aſcending*, is when ſhe returns from her ſouthern limit towards her aſcending node.

And the ſame is to be underſtood of the other planets.

Heliocentric Latitude, of a planet, is its latitude, or diſtance from the ecliptic, ſuch as it would appear from the ſun.—This, when the planet comes to the ſame point of its orbit, is always the ſame, or unchangeable.

Geocentric Latitude, of a planet, is its latitude as ſeen from the earth.—This, though the planet be in the ſame point of its orbit, is not always the ſame, but alters according to the poſition of the earth, in reſpect to the planet.

The latitude of a ſtar is altered only by the aberration of light, and the ſecular variation of latitude.

Difference of Latitude, is an arc of the meridian, or the neareſt diſtance between the parallels of latitude of two places. When the two latitudes are of the ſame name, either both north or both ſouth, ſubtract the leſs latitude from the greater, to give the difference of latitude; but when they are of different names, add them together for the difference of latitude.

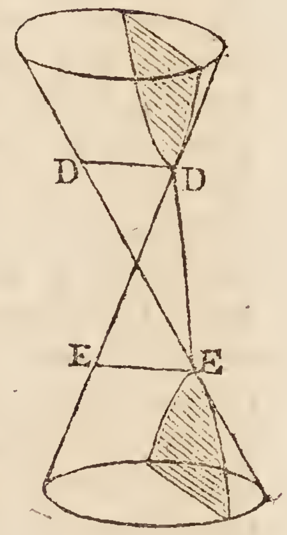
Middle Latitude, is the middle point between two latitudes or places; and is found by taking half the ſum of the two.

Parallax of Latitude. See Parallax.

Refraction of Latitude. See Refraction.

LATUS Rectum, in Conic Sections, the ſame with parameter; which ſee.

Latus *Tranſverſum*, of the hyperbola, is the right line between the vertices of the two oppoſite ſections; or that part of their common axis lying between the two oppoſite cones; as the line DE. It is the ſame as the tranſverſe axis of the hyperbola, or oppoſite hyperbolas.



Latus *Primarium*, a right line, DD, or EE, drawn through the vertex of the ſection of a cone, within the ſame, and parallel to the baſe.

LEAGUE, an extent of three miles in length. A nautical league, or three nautical miles, is the 20th part of a degree of a great circle.

LEAP-Year, the ſame as Bissextile; which ſee. It is ſo called from its leaping a day more that year than in a common year; conſiſting of 366 days, and a common year only of 365. This happens every 4th year, except only ſuch complete centuries as are not exactly diviſible by 4; ſuch as the 17th, 18th, 19th, 21ſt &c. centuries, becauſe 17, 18, 19, 21, &c, cannot be divided by 4 without a remainder.

To find Leap Year, &c. Divide the number of the year by 4; then if 0 remain, it is leap-year; but if 1, 2, or 3 remain, it is ſo many after leap-year.

Or the rule is ſometimes thus expreſſed, in theſe two memorial verſes:

Divide by 4; what's left ſhall be,
For leap-year 0; for paſt, 1, 2, or 3.

Thus if it be required to know what year 1790 is:
then 4) 1790 (447
2 remains:
ſo that 2 remaining, ſhews that 1790 is the 2d year after leap-year. And to find what year 1796 is:
then 4) 1796 (449
here 0 remaining, ſhews that 1796 is a leap-year.

LEAVER. See Lever.

LEE, a term in Navigation, ſignifying that ſide, or quarter, towards which the wind blows.

Lee-Way, of a Ship, is the angle made by the point of the compaſs ſteered upon, and the real line of the ſhip's way, occaſioned by contrary winds and a rough ſea.

All ſhips are apt to make ſome lee-way; ſo that ſomething muſt be allowed for it, in caſting up the log-board. But the lee-way made by different ſhips, under ſimilar circumſtances of wind and ſails, is different; and even the ſame ſhip, with different lading, and having more or leſs ſail ſet, will have more or leſs lee-way. The uſual allowances for it are theſe, as they were given by Mr. John Buckler to the late ingenious Mr. William Jones, who firſt publiſhed them in 1702 in his *Compendium of Practical Navigation*. 1ſt, When a ſhip is cloſe-hauled, has all her ſails ſet, the ſea ſmooth, and a moderate gale of wind, it is then ſuppoſed ſhe makes little or no lee-way. 2d, Allow one point, when it blows ſo freſh that the ſmall ſails are taken in. 3d, Allow two points, when the topſail muſt be cloſe reefed. 4th, Allow two points and a half, when one topſail muſt be handed. 5th, Allow three points and a half, when both topſails muſt be taken in. 6th, Allow four points, when the fore-courſe is handed. 7th, Allow five points, when trying under the mainſail only. 8th, Allow ſix points, when both main and fore-courſes are taken in.

9th,

9th, Allow ſeven points, when the ſhip tries a-hull, or with all ſails handed.

When the wind has blown hard in either quarter, and ſhifts acroſs the meridian into the next quarter, the lee-way will be leſſened. But in all theſe caſes, reſpect muſt be had to the roughneſs of the ſea, and the trim of the ſhip. And hence the mariner will be able to correct his courſe.

LEGS, *of a Triangle*. When one ſide of a triangle is taken as the baſe, the other two are ſometimes called the legs. The term is often uſed too for the baſe and perpendicular of a right-angled triangle, or the two ſides about the right angle.

Hyperbolic LEGS, are the ends of a curve line that partake of the nature of the hyperbola, or having aſymptotes.

LEIBNITZ (GODFREY-WILLIAM), an eminent mathematician and philoſopher, was born at Leipſic in Saxony in 1646. At the age of 15, he applied himſelf to mathematics at Leipſic and Jena; and in 1663, maintained a theſis *de Principiis Individuationis*. The year following he was admitted Maſter of Arts. He read with great attention the Greek philoſophers; and endeavoured to reconcile Plato with Ariſtotle, as he afterwards did Ariſtotle with Des Cartes. But the ſtudy of the law was his principal view; in which faculty he was admitted Bachelor in 1665. The year following he would have taken the degree of Doctor; but was refuſed it on pretence that he was too young, though in reality becauſe he had raiſed himſelf ſeveral enemies by rejecting the principles of Ariſtotle and the Schoolmen.

Upon this he repaired to Altorf, where he maintained a theſis *de Caſibus Perplexis*, with ſuch applauſe, that he had the degree of Doctor conferred on him.

In 1672 he went to Paris, to manage ſome affairs at the French Court for the baron Boinebourg. Here he became acquainted with all the Literati, and made farther and conſiderable progreſs in the ſtudy of mathematics and philoſophy, chiefly, as he ſays, by the works of Paſcal, Gregory St. Vincent, and Huygens. In this courſe, having obſerved the imperfection of Paſcal's arithmetical machine, he invented a new one, as he called it, which was approved of by the miniſter Colbert, and the Academy of Sciences, in which he was offered a ſeat as a member, but refuſed the offers made to him, as it would have been neceſſary to embrace the Catholic religion.

In 1673, he came over to England; where he became acquainted with Mr. Oldenburg, ſecretary of the Royal Society, and Mr. John Collins, a diſtinguiſhed member of the Society; from whom it ſeems he received ſome hints of the method of fluxions, which had been invented, in 1664 or 1665, by the then Mr. Iſaac Newton.

The ſame year he returned to France, where he reſided till 1676, when he again paſſed through England, and Holland, in his journey to Hanover, where he propoſed to ſettle. Upon his arrival there, he applied himſelf to enrich the duke's library with the beſt books of all kinds. The duke dying in 1679, his ſucceſſor Erneſt Auguſtus, then biſhop of Oſnaburgh, ſhewed Mr. Leibnitz the ſame favour as his predeceſſor had done, and engaged him to write the Hiſtory of the Houſe of Brunſwick. To execute this taſk, he travelled over Germany and Italy, to collect materials. While he was in Italy, he met with a pleaſant adventure, which might have proved a more ſerious affair. Paſſing in a ſmall bark from Venice to Meſola, a ſtorm aroſe; during which the pilot, imagining he was not underſtood by a German, whom, being a heretic, he looked on as the cauſe of the tempeſt, propoſed to ſtrip him of his cloaths and money, and throw him overboard. Leibnitz hearing this, without diſcovering the leaſt emotion, drew a ſet of beads from his pocket, and began turning them over with great ſeeming devotion. The artifice ſucceeded; one of the ſailors obſerving to the pilot, that, ſince the man was no heretic, he ought not to be drowned.

In 1700 he was admitted a member of the Royal Academy of Sciences at Paris. The ſame year the elector of Brandenburg, afterwards king of Pruſſia, founded an academy at Berlin by his advice; and he was appointed perpetual Preſident, though his affairs would not permit him to reſide conſtantly at that place. He projected an academy of the ſame kind at Dreſden; and this deſign would have been executed, if it had not been prevented by the confuſions in Poland. He was engaged likewiſe in a ſcheme for an univerſal language, and other literary projects. Indeed his writings had made him long before famous over all Europe, and he had many honours and rewards conferred on him. Beſide the office of Privy Counſellor of Juſtice, which the elector of Hanover had given him, the emperor appointed him, in 1711, Aulic Counſellor; and the czar made him Privy Counſellor of Juſtice, with a penſion of 1000 ducats. Leibnitz undertook at the ſame time to eſtabliſh an academy of ſciences at Vienna; but the plague prevented the execution of it. However, the emperor, as a mark of his favour, ſettled a penſion on him of 2000 florins, and promiſed him one of 4000 if he would come and reſide at Vienna; an offer he was inclined to comply with, but was prevented by his death.

Meanwhile, the Hiſtory of Brunſwick being interrupted by other works which he wrote occaſionally, he found, at his return to Hanover in 1714, that the elector had appointed Mr. Eccard for his colleague in writing that hiſtory. The elector was then raiſed to the throne of Great Britain, which place Leibnitz viſited the latter end of that year, when he received particular marks of friendſhip from the king, and was frequently at court. He now was engaged in a diſpute with Dr. Samuel Clarke, upon the ſubjects of free-will, the reality of ſpace, and other philoſophical ſubjects. This was conducted with great candour and learning; and the papers, which were publiſhed by Clarke, will ever be eſteemed by men of genius and learning. The controverſy ended only with the death of Leibnitz, Nov. 14, 1716, which was occaſioned by the gout and ſtone, in the 70th year of his age.

As to his character and perſon: He was of a middle ſtature, and a thin habit of body. He had a ſtudious air, and a ſweet aſpect, though near-ſighted. He was indefatigably induſtrious to the end of his life. He eat and drank little. Hunger alone marked the time of his meals, and his diet was plain and ſtrong.

He had a very good memory, and it was ſaid could re. peat the Æneid from beginning to end. What he wanted to remember, he wrote down, and never read it afterwards. He always profeſſed the Lutheran religion, but never went to ſermons; and when in his laſt ſickneſs his favourite ſervant deſired to ſend for a miniſter, he would not permit it, ſaying he had no occaſion for one. He was never married, nor ever attempted it but once, when he was about 50 years old; and the lady deſiring time to conſider of it, gave him an opportunity of doing the ſame: he uſed to ſay, "that marriage was a good thing, but a wiſe man ought to conſider of it all his life."

Leibnitz was author of a great multitude of writings; ſeveral of which were publiſhed ſeparately, and many others in the memoirs of different academies. He invented a binary arithmetic, and many other ingenious matters. His claim to the invention of Fluxions, has been ſpoken of under that article. Hanſchius collected, with great care, every thing that Leibnitz had ſaid, in different paſſages of his works, upon the principles of philoſophy; and formed of them a complete ſyſtem, under the title of *G. G. Leibnitzii Principia Philoſophiæ more geometrico demonſtrata* &c, 1728, in 4to. There came out a collection of our author's letters in 1734 and 1735, intitled, *Epiſtolæ ad diverſos theologici, juridici, medici, philoſophici, mathematici, hiſtorici, & philologici argumenti e MSS. auctores: cum annotationibus ſuis primum divulgavit Chriſtian Cortholtus.* But all his works were collected, diſtributed into claſſes by M. Dutens, and publiſhed at Geneva in ſix large volumes 4to, in 1768, intitled, *Gothofredi Guillelmi Leibnitii Opera Omnia &c.*

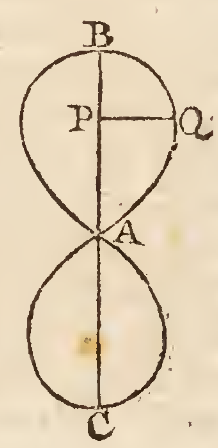
Leibnitzian Philosophy, or the Philoſophy of Leibnitz, is a ſyſtem formed and publiſhed by its author in the laſt century, partly in emendation of the Carteſian, and partly in oppoſition to the Newtonian philoſophy. In this philoſophy, the author retained the Carteſian ſubtile matter, with the vortices and univerſal plenum; and he repreſented the univerſe as a machine that ſhould proceed for ever, by the laws of mechaniſm, in the moſt perfect ſtate, by an abſolute inviolable neceſſity. After Newton's philoſophy was publiſhed, in 1687, Leibnitz printed an Eſſay on the celeſtial motions in the Act. Erud. 1689, where he admits the circulation of the ether with Des Cartes, and of gravity with Newton; though he has not reconciled theſe principles, nor ſhewn how gravity aroſe from the impulſe of this ether, nor how to account for the planetary revolutions in their reſpective orbits. His ſyſtem is alſo defective, as it does not reconcile the circulation of the ether with the free motions of the comets in all directions, or with the obliquity of the planes of the planetary orbits; nor reſolve other objections to which the hypotheſis of the vortices and plenum is liable.

Soon after the period juſt mentioned, the diſpute commenced concerning the invention of the method of Fluxions, which led Mr. Leibnitz to take a very decided part in oppoſition to the philoſophy of Newton. From the goodneſs and wiſdom of the Deity, and his principle of a *ſufficient reaſon*, he concluded, that the univerſe was a perfect work, or the beſt that could poſſibly have been made; and that other things, which are evil or incommodious, were permitted as neceſſary conſequences of what was beſt: that the material ſyſtem, conſidered as a perfect machine, can never fall into diſorder, or require to be ſet right; and to ſuppoſe that God interpoſes in it, is to leſſen the ſkill of the author, and the perfection of his work. He expreſsly charges an impious tendency on the philoſophy of Newton, becauſe he aſſerts, that the fabric of the univerſe and courſe of nature could not continue for ever in its preſent ſtate, but in proceſs of time would require to be re-eſtabliſhed or renewed by the hand of its firſt framer. The perfection of the univerſe, in conſequence of which it is capable of continuing for ever by mechanical laws in its preſent ſtate, led Mr. Leibnitz to diſtinguiſh between the quantity of motion and the force of bodies; and, whilſt he owns in oppoſition to Des Cartes that the former varies, to maintain that the quantity of force is for ever the ſame in the univerſe; and to meaſure the forces of bodies by the ſquares of their velocities.

Mr. Leibnitz propoſes two principles as the foundation of all our knowledge; the firſt, that it is impoſſible for a thing to be, and not to be at the ſame time, which he ſays is the foundation of ſpeculative truth; and ſecondly, that nothing is without a *ſufficient reaſon* why it ſhould be ſo, rather than otherwiſe; and by this principle he ſays we make a tranſition from abſtracted truths to natural philoſophy. Hence he concludes that the mind is naturally determined, in its volitions and elections, by the greateſt apparent good, and that it is impoſſible to make a choice between things perfectly like, which he calls *indiſcernibles*; from whence he infers, that two things perfectly like could not have been produced even by the Deity himſelf: and one reaſon why he rejects a vacuum, is becauſe the parts of it muſt be ſuppoſed perfectly like to each other. For the ſame reaſon too, he rejects atoms, and all ſimilar parts of matter, to each of which, though diviſible *ad infinitum*, he aſcribes a *monad* (Act. Lipſiæ 1698, pa. 435) or active kind of principle, endued with perception and appetite. The eſſence of ſubſtance he places in action or activity, or, as he expreſſes it, in ſomething that is between acting and the faculty of acting. He affirms that abſolute reſt is impoſſible, and holds that motion, or a ſort of *niſus*, is eſſential to all material ſubſtances. Each monad he deſcribes as repreſentative of the whole univerſe from its point of ſight; and yet he tells us, in one of his letters, that matter is not a ſubſtance, but a *ſubſtantiatum*, or *phenomené bien fondé*. See alſo Maclaurin's View of Newton's Philoſophical Diſcoveries, book 1, chap. 4.

LEMMA, is a term chiefly uſed by mathematicians, and ſignifies a propoſition, previouſly laid down to prepare the way for the more eaſy apprehenſion of the demonſtration of ſome theorem, or the conſtruction of ſome problem.

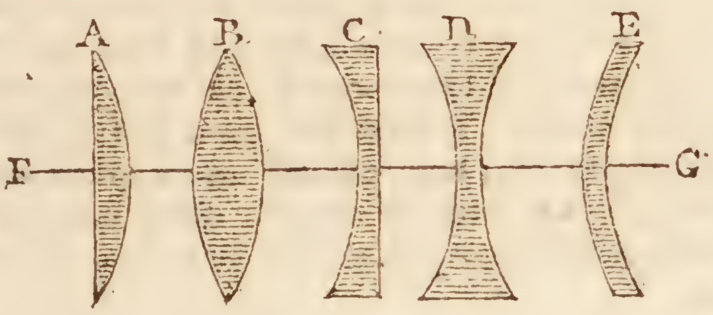
LEMNISCATE, the name of a curve in the form of the figure of 8. If we call A P, x; P Q, y, and the conſtant line A B or A C, a; the equation $ay = x\sqrt{aa - xx}$, or $a^2y^2 = a^2x^2 - x^4$, expreſſing a line of the 4th degree, will denote a lemniſcate, having a double point in the point A. There may be other lemniſcates, as the ellipſe of Caſſini, &c; but that above defined is the ſimpleſt of them.



It easily appears that this curve is quadrable. For since $ay = x\sqrt{a^2 - x^2}$, therefore the fluxion of the curve or $y\dot{x}$ is $= \frac{x}{a}\dot{x}\sqrt{a^2 - x^2}$; the fluent of which is $\frac{1}{3}a^2 - \frac{1}{3a}\cdot\overline{a^2 - x^2}^{\frac{3}{2}}$ for the general area of the curve; which, when x is $= a$, becomes barely $\frac{1}{3}a^2 = AQB$.

LENS, a piece of glafs or other tranfparent fubftance, having its two furfaces fo formed that the rays of light, in paffing through it, have their direction changed, and made to converge and tend to a point beyond the lens, or to become parallel after converging or diverging, or laftly to diverge as if they had proceeded from a point before the lens. Some lenfes are convex, or thicker in the middle; others concave, or thinner in the middle; while others are plano-convex, or plano-concave; and fome again are convex on one fide and concave on the other, which are called menifcufes, the properties of which fee under that word. When the particular figure is not confidered, a lens that is thickeft in the middle is called a convex lens; and that which is thinneft in the middle is called a concave lens, without farther diftinction.

Thefe feveral forms of lenfes are reprefented in the annexed figure:



where A, B are convex lenfes, and C, D, E are concave ones; alfo A is a plano-convex, B is convexo-convex, C is plano-concave, D is concavo-concave, and E is a menifcus.

In every lens, the right line perpendicular to the two furfaces, is called the Axis of the lens, as F G; the points where the axis cuts the furface, are called the Vertices of the lens; alfo the middle point between them is called the Centre; and the diftance between them, the Diameter.

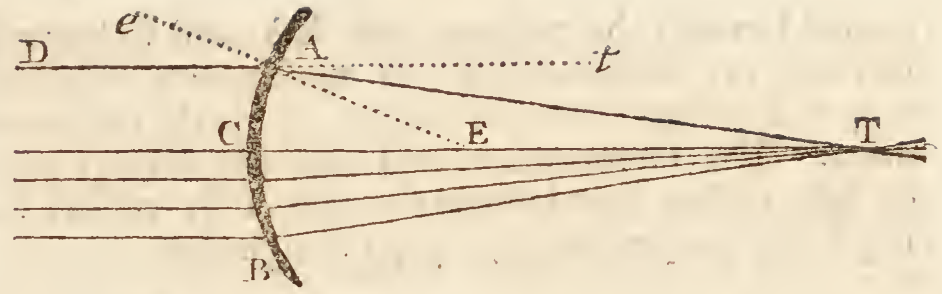
Some confine lenfes within the diameter of half an inch; and fuch as exceed that thicknefs, they call Lenticular Glaffes.

Lenfes are either blown or ground.

Blown LENSES, are fmall globules of glafs, melted in the flame of a lamp or taper. See Microfcope.

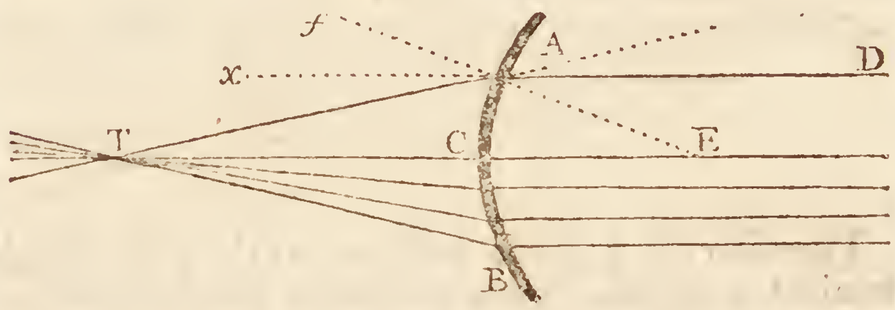
Ground LENSES, are fuch as are ground or rubbed into the defired fhape, and then polifhed. For a method of grinding them, and defcription of a machine for that purpofe, fee Philof. Tranf. vol. xli. pa. 555, or Abr. viii. 281.

Maurolycus firft delivered fomething relative to the nature of lenfes; but we are chiefly indebted to Kepler for explaining the doctrine of refraction through mediums of different forms, the chief fubftance of which may be comprehended in the cafes following.

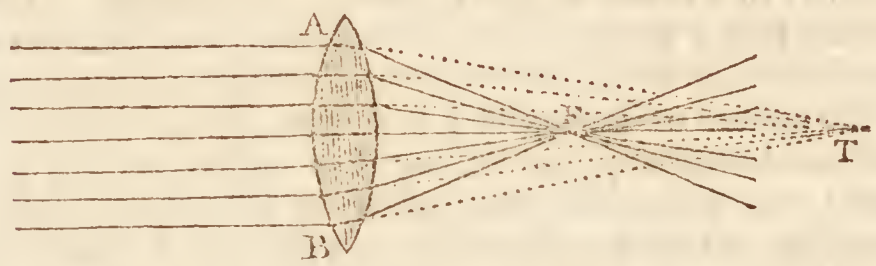


Let DA be a ray of light falling upon a convex denfe medium, having its centre at E. When the ray arrives at A, it will not proceed in the fame direction Af; but it will be there bent, and thrown into a direction AT, nearer the perpendicular AE. In the fame manner, another ray falling on B, at an equal diftance on the other fide of the vertex C, and parallel to the former ray DA, will be refracted into the fame point T. And it will alfo be found, that all the intermediate parallel rays will converge to the fame point, very nearly.

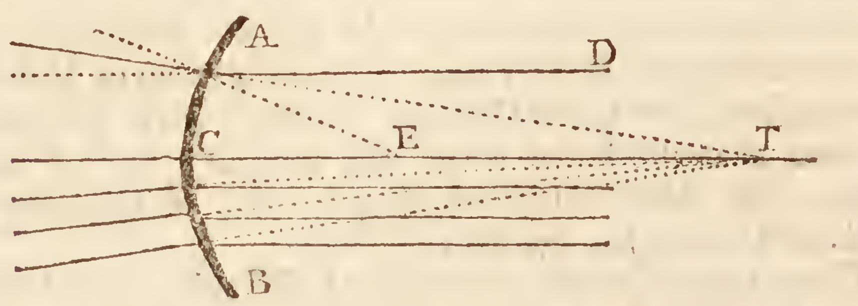
On the other hand, if the rays fall parallel on the infide of this denfer medium, as in the fig. below, they will tend from the perpendicular EAf; and converge to a point T in the air, or any rarer medium. Alfo the ray incident on B, at the fame diftance from the vertex C, will converge to the fame place T, together with all the intermediate parallel rays.



Since therefore rays are made to converge when they pafs either from a rarer or a denfer medium terminated by a convex furface, and converge again when they pafs from the fame medium convex towards the rarer, a lens which is convex on both fides muft, on both accounts, make parallel rays converge to a point beyond it. Thus, the parallel rays between A and B, falling upon

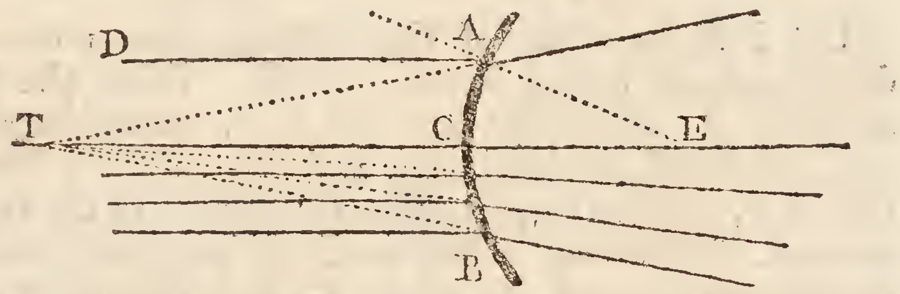


the convex furface of the glafs AB, would in that denfe medium have converged to T; but that medium being terminated by another convex furface, they will be made more converging, and be collected at fome place F, nearer to the lens.

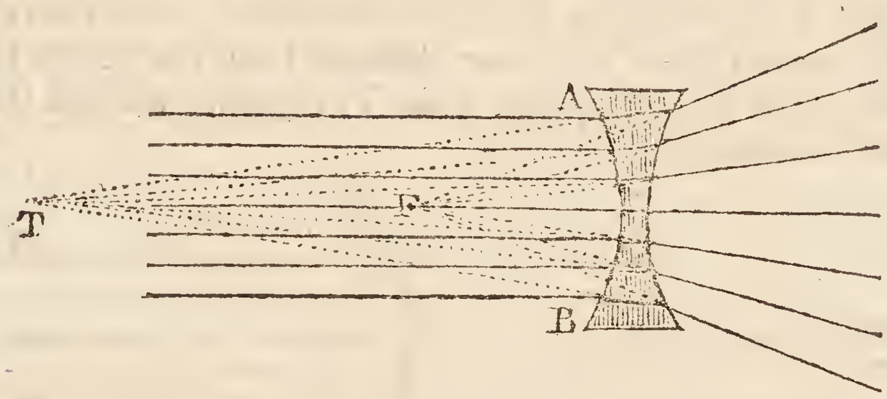


Again, to explain the effects of a concave glafs, let AB be the concave fide of a denfe medium, the centre of concavity being at E. In this cafe, DA will be refracted

fracted towards the perpendicular EA; and ſo likewiſe will the ray incident at B; in conſequence of which they will diverge from one another within the denſe medium. The intermediate rays will alſo diverge more or leſs, as they recede from the axis TC; which, being in the perpendicular, will go ſtraight on.



If the rays be parallel within the denſe medium, they will diverge when they paſs from thence into a rarer medium, through a concave ſurface. For the ray DA will be refracted from the perpendicular AE, as will alſo the ray that is incident at B, together with all the intermediate rays, in proportion to their diſtance from the axis or central ray TC.



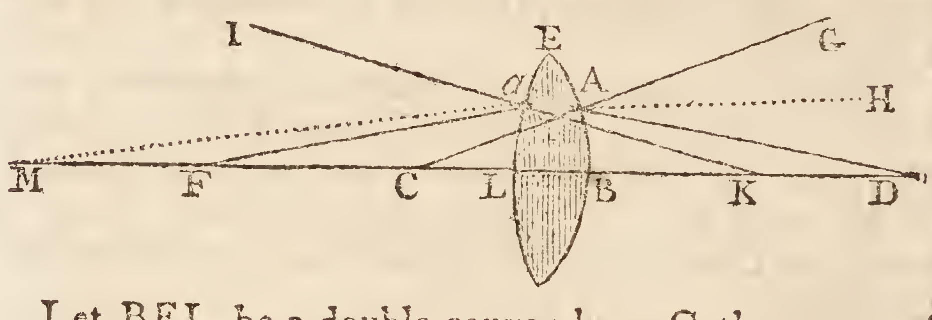
Therefore, if a denſe medium, as the glaſs AB, be terminated by two concave ſurfaces, parallel rays paſſing through it will be made to diverge by both the ſides of it. Thus the firſt ſurface AB will make them diverge as if they had come from the point T; and with the effect of the ſecond ſurface added to this, they will diverge as from a nearer point, F.

It was Kepler, who by theſe inveſtigations firſt gave a clear explanation of the effects of lenſes, in making the rays of a pencil of light converge or diverge. He ſhewed that a plano-convex lens makes rays, that were parallel to its axis, meet at the diſtance of the diameter of the ſphere of convexity; but that if both ſides of the lens be equally convex, the rays will have their focus at the diſtance of the radius of the circle correſponding to that degree of convexity. But he did not inveſtigate any rule for the foci of lenſes unequally convex. He only ſays, in general, that they will fall ſomewhere in the medium, between the foci belonging to the two different degrees of convexity. It is to Cavalerius that we owe this inveſtigation: he laid down this rule, As the ſum of both the diameters is to one of them, ſo is the other to the diſtance of the focus. And it is to be noted that all theſe rules, concerning convex lenſes, are applicable to thoſe that are concave, with this difference, that the focus is on the contrary ſide of the glaſs. See Montucla, vol. 2, pa. 176; or Prieſtley's Hiſt. of Viſion, pa. 65, 4to.

Upon this principle it was not difficult to find the foci of pencils of rays iſſuing from any point in the axis of the lens; ſince thoſe that are parallel will meet in the focus; and if they iſſue from the focus, they will be parallel on the other ſide. If they iſſue from a point between the focus and the glaſs, they will continue to diverge after paſſing the lens, but leſs than before; while thoſe that come from beyond the focus, will converge after paſſing the glaſs, and will meet in a place beyond the oppoſite focus. This philoſopher particularly obſerved, that rays which iſſue from twice the diſtance of the focus, will meet at the ſame diſtance on the other ſide. The moſt important of theſe obſervations have been already illuſtrated by proper figures, and from them the reſt may be eaſily conceived. Later optical writers have aſſigned the diſtances at which rays will meet, that iſſue from any other place in the axis of a lens; but Kepler was too much intent upon his aſtronomical and other purſuits, to give much attention to geometry. But, from the whole, Montucla gives the following rule concerning this ſubject: As the exceſs of the diſtance of the object from the glaſs, above the diſtance of the focus, is to the diſtance of the focus; ſo is this diſtance, to the place of convergency beyond the glaſs. And the ſame rule will find the point of divergency, when the rays iſſue from any place between the lens and the focus: for then the exceſs of the diſtance of the object from the glaſs, above that of the focus, is negative, which is the ſame diſtance taken the contrary way. Montucla, vol. 2, pa. 177.

And from the principle above-mentioned, it will not be difficult to underſtand the application of lenſes, in the rationale of teleſcopes and microſcopes. On theſe principles too is founded the ſtructure of refracting burning glaſſes, by which the ſun's light and heat are exceedingly augmented in the focus of the lens, whether convex or plano-convex; ſince the rays, falling parallel to the axis of the lens, are reduced into a much narrower compaſs; ſo that it is no wonder they burn ſome bodies, melt others, and produce other extraordinary phenomena.

In the Philoſ. Tranſ. vol. xvii. 960, or the Abr. i. 191, Dr. Halley gives an ingenious inveſtigation of the foci of rays refracted through any lenſes, nearly as follows:



Let BEL be a double convex lens, C the centre of the ſegment EB, and K the centre of the ſegment EL; BL the thickneſs or diameter of the lens, and D a point in the axis; it is required to find the point F, or focus, where the rays proceeding from D ſhall be collected, after being refracted through the lens at A and a, points very near to the axis BL. Put the diſtance DA or DB$=d$, the radius CA or CB$=r$, and the radius Ka or KL$=$R; alſo the thickneſs of the lens BL$=t$, and m to n the ratio of the ſine of the angle of incidence DAG to the ſine of the refracted angle HAG or CAM; or m to n will be the ratio of thoſe angles themſelves nearly, ſince very ſmall angles are to each other in the ſame ratio as their ſines. Hence

m is as the angle DAG or DAC,

n is as the angle HAG or MAC,

and becauſe in this caſe the ſides are as their oppo-

ſite

ſite angles, therefore DC : DA : : ∠DAC : ∠C,

or $d+r : d : : m : \frac{dm}{d+r}$ which is as the ∠C;

from this take n or the ∠ MAC,

and there remains $\frac{dm-dn-rn}{d+r}$ as the ∠M;

hence again ∠M : ∠C : : CA : MA or MB,

that is $\frac{dm-dn-rn}{d+r} : \frac{dm}{d+r} : : r : \frac{mdr}{\overline{m-n}\,.\,d-nr} =$ MA or MB; which ſhews in what point the rays would be collected after one refraction, viz, when nr is leſs than $\overline{m-n}\,.\,d$. But when nr is $= \overline{m-n}\,.\,d$, the point would be at an infinite diſtance, or the rays will be parallel to the axis; and when nr is greater than $\overline{m-n}\,.\,d$, then MB is negative, or M falls on the other ſide of the lens beyond D, and the rays ſtill continue to diverge after the firſt refraction.

The point M being now found, to or from which the rays proceed after the firſt refraction, and BM − BL being thus given, which call D, by a proceſs like the former it follows that FL, or the focal diſtance ſought, is equal to $\frac{nDR}{\overline{m-n}\,.\,D+mR} = f$. And here, inſtead of D ſubſtituting MB − LB or $\frac{mdr}{\overline{m-n}\,.\,d-nr} - t$, and putting p for $\frac{n}{m-n}$, the ſame theorem will become

$$\frac{(mpdr - ndt + nprt) \times R}{mdr + mdR - mprR - \overline{m-n}\,.\,dt + nrt} = f,$$

the focal diſtance ſought; in its moſt general form, including the thickneſs of the lens; being the univerſal rule for the foci of double convex glaſſes expoſed to diverging rays.

But if t the thickneſs of the lens be rejected, as not ſenſible, the rule will be much ſhorter,

viz, $\frac{pdrR}{dr+dR-prR} = f$.

If therefore the lens conſiſt of glaſs, whoſe refraction is as 3 to 2, it will be $\frac{2drR}{dr+dR-2rR} = f$. And if it be of water, whoſe refraction is as 4 to 3, it will be $\frac{3drR}{dr+dR-3rR} = f$. But, if the lens could be made of diamond, whoſe refraction is as 5 to 2, it would be $\frac{2drR}{3dr+3dR-2rR} = f$.

If the incident rays, inſtead of diverging, be converging, the diſtance DB or d will be negative, and then the theorem for a double convex glaſs lens will be $\frac{-2drR}{-dr-dR-2rR}$ or $\frac{2drR}{dr+dR+2rR} = f$; in which caſe therefore the focus is always on the other ſide of the glaſs.

And if the rays be parallel, as coming from an infinite diſtance, or nearly ſo, then will d be negative, as well as the terms in the theorem in which it is found; and therefore, the other term prR will be nothing in reſpect of thoſe infinite terms; and by omitting it, the theorem will be $\frac{pdrR}{dr+dR} = \frac{prR}{r+R} = f$,

or for glaſs $\frac{2rR}{r+R} = f$.

And here if $r = R$, or the two ſides of the glaſs be of equal convexity, this laſt will become barely $\frac{2r^2}{2r}$ or barely $r = f$ the focus, which therefore is in the centre of the convexity of the lens.

If the lens be a meniſcus of glaſs; then, making r negative, the theorem is

$\frac{-2drR}{-dr+dR+2rR}$ or $\frac{2drR}{dr-dR-2rR} = f$ for diverging rays,

$\frac{-2drR}{-dr+dR-2rR}$ or $\frac{2drR}{dr-dR+2rR} = f$ for converging rays,

and $\frac{-2rR}{-r+R}$ or $\frac{2rR}{r-R} = f$ for parallel rays.

If the lens be a double concave glaſs, r and R will be both negative, and then the theorem becomes

$\frac{-2drR}{dr+dR+2rR} = f$ for diverging rays, always negative;

$\frac{-2drR}{dr+dR\times 2rR} = f$ for converging rays;

and $\frac{-2rR}{r+R} = f$ for parallel rays.

And here, if the radii of curvature r and R be equal, this laſt will be barely $-r = f$ for parallel rays falling on a double concave glaſs of equal curvature.

Laſtly, when the lens is a plano-convex glaſs; then, r being infinite, the theorem becomes

$\frac{2dR}{d-2R} = f$ for diverging rays,

$\frac{2dR}{d+2R} = f$ for converging rays,

and $2R = f$ for parallel rays.

The theorems for parallel rays, as coming from an infinite diſtance, take place in the common refracting teleſcopes. And thoſe for converging rays are chiefly of uſe to determine the focus reſulting from any ſort of lens placed in a teleſcope, between the focus of the object-glaſs and the glaſs itſelf; the diſtance between the ſaid focus of the object-glaſs and the interpoſed lens being made $= -d$; while thoſe for diverging rays are chiefly of uſe in microſcopes, reading glaſſes, and other caſes in which near objects are viewed.

It is evident that the foregoing general theorem will ſerve to find any of the other circumſtances, as well as the focus, by conſidering this as given. Thus, for inſtance, ſuppoſe it be required to find the diſtance at which an object being placed, it ſhall by a given lens be repreſented as large as the object itſelf; which is of ſingular uſe in viewing and drawing them, by tranſmitting the image through a glaſs in a dark room, as in the camera obſcura, which gives not only the true figure and ſhades, but the colours themſelves as vivid as the life. Now in this caſe d is $= f$, which makes the theorem become $pdrR = d^2r + d^2R - pdrR$, and this

this gives $d = \frac{2prR}{r + R}$. But if the two convexities belong to equal ſpheres, ſo as that $r = R$, then it is $d = pr$, or $= 2r$ when the lens is glaſs. So that if the object be placed at the diameter of the ſphere diſtant from the lens, then the focus will be as far diſtant on the other ſide, and the image as large as the object. But if the glaſs were a plano-convex, the ſame diſtance would be juſt twice as much.

Again, recurring to the firſt general theorem, including t, the thickneſs of the lens; let the lens be a whole ſphere; then $t = 2r$, and $r = R$; and hence the theorem reduces to $\frac{mpdr - 2ndr - 2npr^2}{2nd + 2nr - mpr} = f$.

And here if d be infinite, the theorem contracts to $\frac{mp - 2n}{2n} r$ or $\frac{2n - m}{2m - 2n} r = f$; or for glaſs $\frac{1}{2} r = f$: ſhewing that a ſphere of glaſs collects the ſun's rays at half the radius of the ſphere without it. And for a ſphere of water, the focus is at the diſtance of a whole radius.

For another example; when a hemiſphere is expoſed to parallel rays; then d and R being infinite, and $t = r$, the theorem becomes $\frac{mp - n}{m} r, = \frac{nn}{m^2 - mn} r = f$. That is, in glaſs it is $\frac{4}{3} r$, and in water $\frac{9}{4} r$.

Several other corollaries may be deduced from the foregoing principles. As,

1ſt. That the thickneſs of the lens, being very ſmall, the focus will remain the ſame, whether the one ſide or the other be expoſed to the rays.

2d. If a luminous body be placed in a focus behind a lens, whether plano-convex, or convex on both ſides; or whether equally or unequally ſo; the rays become parallel after refraction, as the refracted rays become what were before the incident rays. And hence, by means of a convex lens, or a little glaſs bubble full of water, a very intenſe light may be projected to a great diſtance. Which furniſhes us with the ſtructure of a lamp or lantern, to throw an intenſe light to an immenſe diſtance: for a lens, convex on both ſides, being placed oppoſite to a concave mirror, if there be placed a lighted candle or wick in the common focus of both, the rays reflected back from the mirror to the lens will be parallel to each other; and after refraction will converge, till they concur at the diſtance of the radius, after which they will again diverge. But the candle being likewiſe in the focus of the lens, the rays it throws on the lens will be parallel; and therefore a very intenſe light meeting with another equally intenſe, at the diſtance of the diameter from the lens, the light will be ſurpriſing: and though it afterwards decreaſe, yet the parallel and diverging rays going a long way together, it will be very great at a great diſtance. Lanterns of this kind are of conſiderable ſervice in the night time, to diſcover remote objects; and are uſed with ſucceſs by fowlers and fiſhermen, to collect their prey together, that ſo it may be taken.

If it be required to have the light, at the ſame time, tranſmitted to ſeveral places, as through ſeveral ſtreets, &c, the number of lenſes and mirrors muſt be increaſed.

3d. The images of objects are ſhewn inverted in the focus of a convex lens: nor is the focus of the ſun's rays any thing elſe, in effect, but the image of the ſun inverted. Hence, in ſolar eclipſes, the ſun's image, eclipſed as it is, may be burnt by a large lens on a board, &c, and exhibit a very entertaining phenomenon.

4th. If a concave mirror be ſo placed, as that an inverted image, formed by refraction through a lens, be found between the centre and the focus, or even beyond the centre, it will again be inverted by reflection, and ſo appear erect; in the firſt caſe beyond the centre, and in the latter between the centre and the focus. And on theſe principles the camera obſcura is conſtructed.

5th. The image of an object, delineated beyond a convex lens, is of ſuch a magnitude, as it would be of, were the object to ſhine into a dark room through a ſmall hole, upon a wall, at the ſame diſtance from the hole, as the focus is from the lens.—When an object is leſs diſtant from a lens than the focus of parallel rays, the diſtance of the image is greater than that of the object; otherwiſe, the diſtance of the image is leſs than that of the object: in the former caſe, therefore, the image is larger than the object; in the latter, it is leſs.

When the images are leſs than the objects, they will appear more diſtinct and vivid; becauſe then more rays are accumulated into a given ſpace. But if the images be made greater than the objects, they will not appear diſtinctly; becauſe in that caſe there are fewer rays which meet after refraction in the ſame point; whence it happens, that rays proceeding from different points of an object, terminate in the ſame point of an image, which is the cauſe of confuſion. Hence it appears, that the ſame aperture of a lens may be admitted in every caſe, if we would keep off the rays which produce confuſion. However, though the image be then more diſtinct, when no rays are admitted but thoſe near the axis, yet for want of rays the image is apt to be dim.

6th. If the eye be placed in the focus of a convex lens, an object viewed through it, appears erect, and enlarged in the ratio of the diſtance of the object from the eye, to that of the eye from the lens, if it be near; but infinitely if remote.

7th. An object viewed through a concave lens, appears erect, and diminiſhed in a ratio compounded of the ratios of the ſpace in the axis between the point of incidence, and the point to which an oblique ray would paſs without refraction, to the ſpace in the axis between the eye and the middle of the object; and the ſpace in the ſame axis between the eye and the point of incidence, to the ſpace between the middle of the object and the point to which the oblique ray would paſs without refraction.

Finally, it may be obſerved, that the very ſmall magnifying glaſſes uſed in microſcopes, moſt properly come under the denomination of lens, as they moſt approach to the figure of the lentil, a ſeed of the vetch or pea kind, from whence the name is derived; but the reading glaſſes, and burning glaſſes, and all that magnify, come under the ſame denomination; for their ſurfaces are convex, although leſs ſo. A drop of water is a lens, and it will ſerve as one; and many have uſed it by way of lens

lens in their microscopes. A drop of any transparent fluid, inclosed between two concave glasses, acquires the shape of a lens, and has all its properties. The crystalline humour of the eye is a lens exactly of this kind; it is a small quantity of a transflucent fluid, contained between two concave and transparent membranes, called the coats of the eye; and it acts as the lens made of water would do, in an equal degree of convexity.

LEO, *the Lion*, a considerable constellation of the northern hemisphere, being one of the 48 old constellations, and the 5th sign of the zodiac. It is marked thus ♌, as a rude sketch of the animal.

The Greeks fabled that this was the Nemæan lion, which had dropped from the moon, but being slain by Hercules, was raised to the heavens by Jupiter, in commemoration of the dreadful conflict, and in honour of that hero. But the hieroglyphical meaning of this sign, so depicted by the Egyptians long before the invention of the fables of Hercules, was probably no more than to signify, by the fury of the lion, the violent heats occasioned by the sun when he entered that part of the ecliptic.

The stars in the constellation Leo, in Ptolomy's catalogue are 27, besides 8 unformed ones, now counted in later times in the constellation Coma Berenices, in Tycho's 30, in that of Hevelius 49, and in Flamsteed's 95; one of them, of the first magnitude, in the breast of the Lion, is called Regulus, and Cor Leonis, or Lion's Heart.

Leo *Minor, the Little Lion*, a constellation of the northern hemisphere, and one of the new ones that were formed out of what were left by the ancients, under the name of Stellæ Informes, or unformed stars, and added to the 48 old ones. It contains 53 stars in Flamsteed's catalogue.

Cor Leonis, *Lion's heart*, a fixed star, of the first magnitude, in the sign Leo; called also Regulus, Basilicus, &c.

LEPUS, *the Hare*, a constellation of the southern hemisphere, and one of the 48 old constellations.

The Greeks fabled, that this animal was placed in the heavens, near Orion, as being one of the animals which he hunted. But it is probable their masters, the Egyptians, had some other meaning in this hieroglyphic.

The stars in the constellation Lepus, in Ptolomy's catalogue are 12, in Tycho's 13, and in Flamsteed's 19.

LEUCIPPUS, a celebrated Greek philosopher and mathematician, who flourished about the 428th year before Christ. He was the first author of the famous system of atoms and vacuums, and of the hypothesis of storms; since attributed to the moderns.

LEVEL, an instrument used to make a line parallel to the horizon, and to continue it out at pleasure; and by this means to find the true level, or the difference of ascent or descent between two or more places, for conveying water, draining fens, &c.

There are several instruments, of different contrivance and matter, invented for the perfection of levelling, as may be seen in De la Hire's and Picard's treatises of Levelling, in Biron's treatise on Mathematical Instruments, also in the Philos. Trans. and the Memoirs de l'Acad. &c. But they may be reduced to the following kinds.

Water-Level, that which shews the horizontal line by means of a surface of water or other fluid; founded on this principle, that water always places itself level or horizontal.

The most simple kind is made of a long wooden trough or canal; which being equally filled with water, its surface shews the line of level. And this is the chorobates of the ancients, described by Vitruvius, lib. viii. cap. 6.

The water-level is also made with two cups fitted to the two ends of a straight pipe, about an inch diameter, and 3 or 4 feet long, by means of which the water communicates from the one cup to the other; and this pipe being moveable on its stand by means of a ball and socket, when the two cups shew equally full of water, their two surfaces mark the line of level.

This instrument, instead of cups, may also be made with two short cylinders of glass three or four inches long, fastened to each extremity of the pipe with wax or mastic. The pipe is filled with common or coloured water, which shews itself through the cylinders, by means of which the line of Level is determined; the height of the water, with respect to the centre of the earth, being always the same in both cylinders. This level, though very simple, is yet very commodious for levelling small distances. See the method of preparing and using a water-level, and a mercurial Level, annexed to Davis's quadrant, for the same purpose, by Mr. Leigh, in Philos. Trans. vol. xl. 417, or Abr. viii. 362.

Air-Level, that which shews the line of Level by means of a bubble of air inclosed with some fluid in a glass tube of an indeterminate length and thickness, and having its two ends hermetically sealed: an invention, it is said, of M. Thevenot. When the bubble fixes itself at a certain mark, made exactly in the middle of the tube, the case or ruler in which it is fixed, is then level. When it is not level, the bubble will rise to one end.—This glass-tube may be set in another of brass, having an aperture in the middle, where the bubble of air may be observed.—The liquor with which the tube is filled, is oil of tartar, or aqua secunda; those not being liable to freeze as common water, nor to rarefaction and condensation as spirit of wine is.

There is one of these instruments with sights, being an improvement upon that last described, which, by the addition of other apparatus, becomes more exact and commodious. It consists of an air-Level, n° 1, (*fig.* 1, *Plate XIV*) about 8 inches long, and about two thirds of an inch in diameter, set in a brass tube, 2, having an aperture in the middle, C. The tubes are carried in a strong straight ruler, of a foot long; at the ends of which are fixed two sights, 3, 3, exactly perpendicular to the tubes, and of an equal height, having a square hole, formed by two fillets of brass crossing each other at right angles; in the middle of which is drilled a very small hole, through which a point on a level with the instrument is seen. The brass tube is fastened to the ruler by means of two screws; the one of which, marked 4, serves to raise or depress the tube at pleasure, for bringing it towards a level. The top of the ball and socket is rivetted to a small ruler that springs, one end of which is fastened with springs to the great ruler, and at the other end is a screw, 5, serving to raise and depress the instrument when nearly level.

But this instrument is still less commodious than the following one: for though the holes be ever so small, yet they will still take in too great a space to determine the point of Level precisely.

Fig.

Fig. 2, is a *Level with Teleſcopic Sights*, firſt invented by Mr. Huygens. It is like the laſt; with this difference, that inſtead of plain ſights, it carries a teleſcope, to determine exactly a point of Level at a conſiderable diſtance. The ſcrew 3, is for raiſing or lowering a little fork, for carrying the hair, and making it agree with the bubble of air when the inſtrument is Level; and the ſcrew 4, is for making the bubble of air, D or E, agree with the teleſcope. The whole is fitted to a ball and ſocket, or otherwiſe moved by joints and ſcrews.—It may be obſerved that a teleſcope may be added to any kind of Level, by applying it upon, or parallel to, the baſe or ruler, when there is occaſion to take the level of remote objects: and it poſſeſſes this advantage, that it may be inverted by turning the ruler and teleſcope half round; and if then the hair cut the ſame point that it did before, the operation is juſt. Many varieties and improvements of this inſtrument have been made by the more modern opticians.

Dr. Deſaguliers propoſed a machine for taking the difference of Level, which contained the principles both of a barometer and thermometer; but it is not accurate in practice: Philoſ. Tranſ. vol. xxxiii. pa. 165, or Abr. vol. vi. 271. *Fig.* 3, 4, 5, 6.

Mr. Hadley too has contrived a Spirit Level to be fixed to a quadrant, for taking a meridian altitude at ſea, when the horizon is not viſible. See the deſcription and figure of it in the Philoſ. Tranſ. vol. xxxviii. 167, or Abr. viii. 357. Various other Spirit Levels, and Mercurial Levels, are alſo invented and uſed upon different occaſions.

Reflecting LEVEL, that made by means of a pretty long ſurface of water, repreſenting the ſame object inverted, which we ſee erect by the eye; ſo that the point where theſe two objects appear to meet, is on a Level with the place where the ſurface of the water is found. This is the invention of M. Mariotte.

There is another reflecting Level, conſiſting of a poliſhed metal mirror, placed a little before the object glaſs of a teleſcope, ſuſpended perpendicularly. This mirror muſt be ſet at an angle of 45 degrees; in which caſe the perpendicular line of the teleſcope becomes a horizontal line, or a line of Level. Which is the invention of M. Caſſini.

Artillery Foot-LEVEL, is in form of a ſquare (fig. 7), having its two legs or branches of an equal length; at the junction of which is a ſmall hole, by which hangs a plummet playing on a perpendicular line in the middle of a quadrant, which is divided both ways from that point into 45 degrees.

This inſtrument may be uſed on other occaſions, by placing the ends of its two branches on a plane; for when the plummet plays perpendicularly over the middle diviſion of the quadrant, the plane is then Level.

To uſe it in Gunnery, place the two ends on the piece of artillery, which may be raiſed to any propoſed height, by means of the plummet, which will cut the degree above the Level. But this ſuppoſes the outſide of the cannon is parallel to its axis, which is not always the caſe; and therefore they uſe another inſtrument now, either to ſet the piece Level, or elevate it at any angle; namely a ſmall quadrant, with one of its radii continued out pretty long, which being put into the inſide of the cylindrical bore, the plummet ſhews the angle of elevation, or the line of Level. See *Gunner's* QUADRANT.

Carpenter's, *Bricklayer's*, or *Pavior's* LEVEL, conſiſts of a long ruler, in the middle of which is fitted at right angles another broader piece, at the top of which is faſtened a plummet, which when it hangs over the middle line of the 2d or upright piece, ſhews that the baſe or long ruler is horizontal or Level. Fig. 8.

Maſon's LEVEL, is compoſed of 3 rules, ſo jointed as to form an iſoſceles triangle, ſomewhat like a Roman A; from the vertex of which is ſuſpended a plummet, which hangs directly over a mark in the middle of the baſe, when this is horizontal or Level. Fig. 8.

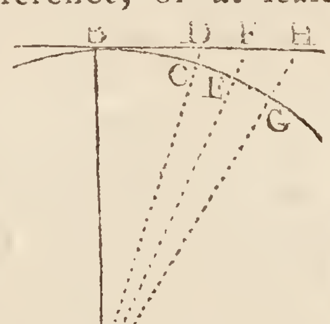
Plumb or Pendulum LEVEL, ſaid to be invented by M. Picard; fig. 10. This ſhews the horizontal line by means of another line perpendicular to that deſcribed by a plummet or pendulum. This Level conſiſts of two legs or branches, joined at right angles, the one of which, of about 18 inches long, carries a thread and plummet; the thread being hung near the top of the branch, at the point 2. The middle of the branch where the thread paſſes is hollow, ſo that it may hang free every where: but towards the bottom, where there is a ſmall blade of ſilver, on which a line is drawn perpendicular to the teleſcope, the ſaid cavity is covered by two pieces of braſs, with a piece of glaſs G, to ſee the plummet through, forming a kind of caſe, to prevent the wind from agitating the thread. The teleſcope, of a proper length, is fixed to the other leg of the inſtrument, at right angles to the perpendicular, and having a hair ſtretched horizontally acroſs the focus of the object-glaſs, which determines the point of Level, when the ſtring of the plummet hangs againſt the line on the ſilver blade. The whole is fixed by a ball and ſocket to its ſtand.

Fig. 12, is a *Balance* LEVEL; which being ſuſpended by the ring, the two ſights, when in equilibrio, will be horizontal, or in a Level.

Some other Levels are alſo repreſented in plate xiv.

LEVELLING, the art or act of finding a line parallel to the horizon at one or more ſtations, to determine the height or depth of one place with reſpect to another; for laying out grounds even, regulating deſcents, draining moraſſes, conducting water, &c.

Two or more places are on a true level when they are equally diſtant from the centre of the earth. Alſo one place is higher than another, or out of level with it, when it is farther from the centre of the earth: and a line equally diſtant from that centre in all its points, is called the *line of true level.* Hence, becauſe the earth is round, that line muſt be a curve, and make a part of the earth's circumference, or at leaſt parallel to it, or concentrical with it; as the line BCFG, which has all its points equally diſtant from A the centre of the earth; conſidering it as a perfect globe.



But the line of ſight BDE &c given by the operations of levels, is a tangent, or a right line perpendicular to the ſemidiameter of the earth at the point of contact B, riſing always higher above the true line of level, the

the farther the diſtance is, is called the *apparent line of level*. Thus, CD is the height of the apparent level above the true level, at the diſtance BC or BD; alſo EF is the exceſs of height at F; and GH at G; &c. The difference, it is evident, is always equal to the exceſs of the ſecant of the arch of diſtance above the radius of the earth.

The common methods of levelling are ſufficient for laying pavements of walks, or for conveying water to ſmall diſtances, &c: but in more extenſive operations, as in levelling the bottoms of canals, which are to convey water to the diſtance of many miles, and ſuch like, the difference between the true and the apparent level muſt be taken into the account.

Now the difference CD between the true and apparent level, at any diſtance BC or BD, may be found thus: By a well known property of the circle $2AC + CD : BD :: BD : CD$; or becauſe the diameter of the earth is ſo great with reſpect to the line CD at all diſtances to which an operation of levelling commonly extends, that 2AC may be ſafely taken for $2AC + CD$ in that proportion without any ſenſible error, it will be $2AC : BD :: BD : CD$ which therefore is $= \frac{BD^2}{2AC}$ or $\frac{BC^2}{2AC}$ nearly; that is, the difference between the true and apparent level, is equal to the ſquare of the diſtance between the places, divided by the diameter of the earth; and conſequently it is always proportional to the ſquare of the diſtance.

Now the diameter of the earth being nearly 7958 miles; if we firſt take BC = 1 mile, then the exceſs $\frac{BC^2}{2AC}$ becomes $\frac{1}{7958}$ of a mile, which is 7·962 inches, or almoſt 8 inches, for the height of the apparent above the true level at the diſtance of one mile. Hence, proportioning the exceſſes in altitude according to the ſquares of the diſtances, the following Table is obtained, ſhewing the height of the apparent above the true level for every 100 yards of diſtance on the one hand, and for every mile on the other.

Diſt. or BC	Dif. of Level, or CD
Yards	Inches
100	0·026
200	0·103
300	0·231
400	0·411
500	0·643
600	0·925
700	1·260
800	1·645
900	2·081
1000	2·570
1100	3·110
1200	3·701
1300	4·344
1400	5·038
1500	5·784
1600	6·580
1700	7·425

Diſt. or BC	Dif. of Level, or CD	
Miles	Feet	Inc.
$\frac{1}{4}$	0	$0\frac{1}{2}$
$\frac{1}{2}$	0	2
$\frac{3}{4}$	0	$4\frac{1}{2}$
1	0	8
2	2	8
3	6	0
4	10	7
5	16	7
6	23	11
7	32	6
8	42	6
9	53	9
10	66	4
11	80	3
12	95	7
13	112	2
14	130	1

By means of theſe Tables of reductions, we can now level to almoſt any diſtance at one operation, which the ancients could not do but by a great multitude; for, being unacquainted with the correction anſwering to any diſtance, they only levelled from one 20 yards to another, when they had occaſion to continue the work to ſome conſiderable extent.

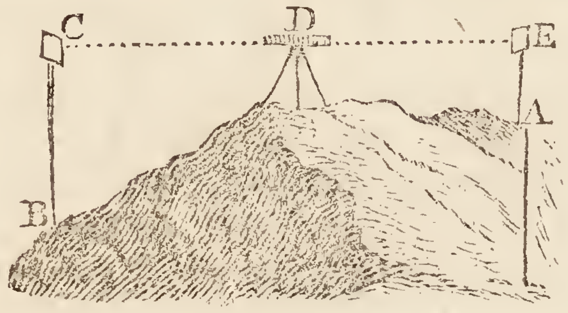
This table will anſwer ſeveral uſeful purpoſes. Thus, firſt, to find the height of the apparent level above the true, at any diſtance. If the given diſtance be contained in the table, the correction of level is found on the ſame line with it: thus at the diſtance of 1000 yards, the correction is 2·57, or two inches and a half nearly; and at the diſtance of 10 miles, it is 66 feet 4 inches. But if the exact diſtance be not found in the table, then multiply the ſquare of the diſtance in yards by 2·57, and divide by 1000000, or cut off 6 places on the right for decimals; the reſt are inches: or multiply the ſquare of the diſtance in miles by 66 feet 4 inches, and divide by 100. 2ndly, To find the extent of the viſible horizon, or how far can be ſeen from any given height, on a horizontal plane, as at ſea, &c. Suppoſe the eye of an obſerver, on the top of a ſhip's maſt at ſea, be at the height of 130 feet above the water, he will then ſee about 14 miles all around. Or from the top of a cliff by the ſea-ſide, the height of which is 66 feet, a perſon may ſee to the diſtance of near 10 miles on the ſurface of the ſea. Alſo, when the top of a hill, or the light in a lighthouſe, or ſuch like, whoſe height is 130 feet, firſt comes into the view of an eye on board a ſhip; the table ſhews that the diſtance of the ſhip from it is 14 miles, if the eye be at the ſurface of the water; but if the height of the eye in the ſhip be 80 feet, then the diſtance will be increaſed by near 11 miles, making in all about 25 miles, diſtance.

3dly, Suppoſe a ſpring to be on one ſide of a hill, and a houſe on an oppoſite hill, with a valley between them; and that the ſpring ſeen from the houſe appears by a levelling inſtrument to be on a level with the foundation of the houſe, which ſuppoſe is at a mile diſtance from it; then is the ſpring 8 inches above the true level of the houſe; and this difference would be barely ſufficient for the water to be brought in pipes from the ſpring to the houſe, the pipes being laid all the way in the ground.

4th, If the height or diſtance exceed the limits of the table: Then, firſt, if the diſtance be given, divide it by 2, or by 3, or by 4, &c, till the quotient come within the diſtances in the table; then take out the height anſwering to the quotient, and multiply it by the ſquare of the diviſor, that is by 4, or 9, or 16, &c, for the height required: So if the top of a hill be juſt ſeen at the diſtance of 40 miles; then 40 divided by 4 gives 10, to which in the table anſwers $66\frac{1}{3}$ feet, which being multiplied by 16, the ſquare of 4, gives $1061\frac{1}{3}$ feet for the height of the hill. But when the height is given, divide it by one of theſe ſquare numbers 4, 9, 16, 25, &c, till the quotient come within the limits of the table, and multiply the quotient by the ſquare root of the diviſor, that is by 2, or 3, or 4, or 5, &c, for the diſtance ſought: So when the top of the pike of Teneriff, ſaid to be almoſt 3 miles or 15840 feet high, juſt comes into view at ſea; divide 15840 by 225, or the ſquare of 15, and the quotient is

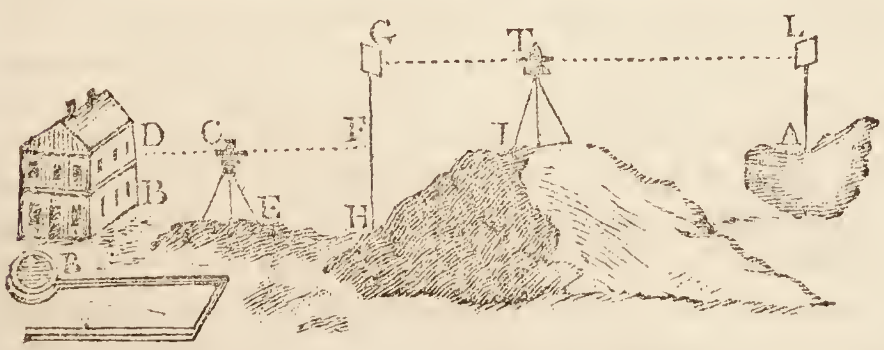
is 70 nearly; to which in the table anſwers, by proportion, nearly 10$\frac{2}{7}$ miles; then multiplying 10$\frac{2}{7}$ by 15, gives 154 miles and $\frac{2}{7}$, for the diſtance of the hill.

Of the Practice of Levelling.



The operation of Levelling is as follows. Suppoſe the height of the point A on the top of a mountain, above that of B, at the foot of it, be required. Place the level about the middle diſtance at D, and ſet up pickets, poles, or ſtaffs, at A and B, where perſons muſt attend with ſignals for raiſing and lowering, on the ſaid poles, little marks of paſteboard or other matter. The level having been placed horizontally by the bubble, &c, look towards the ſtaff AE, and cauſe the perſon there to raiſe or lower the mark, till it appear through the teleſcope, or ſights, &c, at E: then meaſure exactly the perpendicular height of the point E above the point A, which ſuppoſe 5 feet 8 inches, ſet it down in your book. Then turn your view the other way, towards the pole B, and cauſe the perſon there to raiſe or lower his mark, till it appear in the viſual line as before at C; and meaſuring the height of C above B, which ſuppoſe 15 feet 6 inches, ſet this down in your book alſo, immediately above the number of the firſt obſervation. Then ſubtract the one from the other, and the remainder 9 feet 10 inches, will be the difference of level between A and B, or the height of the point A above the point B.

If the point D, where the inſtrument is fixed, be exactly in the middle between the points A and B, there will be no neceſſity for reducing the apparent level to the true one, the viſual ray on both ſides being raiſed equally above the true level. But if not, each height muſt be corrected or reduced according to its diſtance, before the one corrected height is ſubtracted from the other; as in the caſe following.



When the diſtance is very conſiderable, or irregular, ſo that the operation cannot be effected at once placing of the level; or when it is required to know if there be a ſufficient deſcent for conveying water from the ſpring A to the point B; it will be neceſſary to perform this at ſeveral operations. Having choſen a proper place for the firſt ſtation, as at I, fix a pole at the point A near the ſpring, with a proper mark to ſlide up and down it, as L; and meaſure the diſtance from A to I. Then the level being adjuſted in the point I, let the mark L be raiſed or lowered till it is ſeen through the teleſcope or ſights of the level, and meaſure the height AL. Then having fixed another pole at H, direct the level to it, and cauſe the mark G to be moved up or down till it appear through the inſtrument: then meaſure the height GH, and the diſtance from I to H; noting them down in the book. This done, remove the level forwards to ſome other eminence as E, from whence the pole H may be viewed, as alſo another pole at D; then having adjuſted the level in the point E, look back to the pole H; and managing the mark as before, the viſual ray will give the point F; then meaſuring the diſtance HE and the height HF, note them down in the book. Then, turning the level to look at the next pole D, the viſual ray will give the point D; there meaſure the height of D, and the diſtance EB, entering them in the book as before. And thus proceed from one ſtation to another, till the whole is completed.

But all theſe heights muſt be corrected or reduced by the foregoing table, according to their reſpective diſtances; and the whole, both diſtances and heights, with their corrections, entered in the book in the following manner.

Back-ſights.			*Fore-ſights.*		
Diſts.	Hts.	Cors.	Diſts.	Hts.	Cors.
yds	ft in.	inc.	yds	ft in.	inc
IA 1650	AL 11 3	7·0	IH 1265	HG 19 5	4·0
EH 940	HF 10 7	2·2	EB 900	BD 8 1	2·1
2590	21 10	9·2	2165	27 6	6·1
	9·2		2590	6·1	
	21 0·8		Diſt. 4755	26 11·9	
				21 0·8	
			Whole Dif. of level	5 11·1	

Having ſummed up all the columns, add thoſe of the diſtances together, and the whole diſtance from A to B is 4755 yards, or 2 miles and 3 quarters nearly. Then, the ſums of the corrections taken from the ſums of the apparent heights, leave the two corrected heights; the one of which being taken from the other, leaves 5 feet 11·1 inc. for the true difference of level ſought between the two places A and B, which is at the rate of an inch and half nearly to every 100 yards, a quantity more than ſufficient to cauſe the water to run from the ſpring to the houſe.

Or, the operation may be otherwiſe performed, thus: Inſtead of placing the level between every two poles, and taking both back-ſights and fore-ſights; plant it firſt at the ſpring A, and from thence obſerve the level to the firſt pole; then remove it to this pole, and obſerve the 2d pole; next move it to the 2d pole, and obſerve the 3d pole; and ſo on, from one pole to another, always taking foreward ſights or obſervations only. And then at the laſt, add all the corrected heights together,

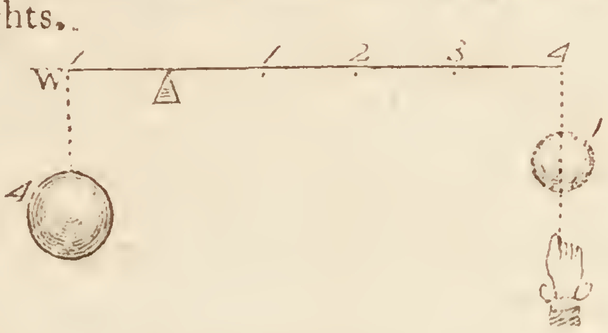
gether, and the ſum will be the whole difference of level ſought.

Dr. Halley ſuggeſted a new method of levelling performed wholly by means of the barometer, in which the mercury is found to be ſuſpended at ſo much the leſs height, as the place is farther remote from the centre of the earth; and hence the different heights of the mercury in two places give the difference of level. This method is, in fact, no other than the method of meaſuring altitudes by the barometer, which has lately been ſo ſucceſsfully practiſed and perfected by M. De Luc and others; but though it ſerves very well for the heights of hills, and other conſiderable altitudes, it is not accurate enough for determining ſmall altitudes, to inches and parts. See the Barometrical Meaſurement of Altitudes.

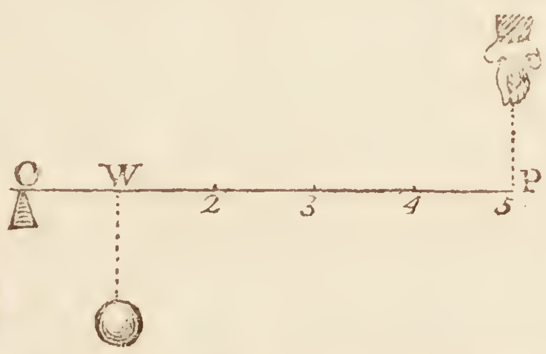
Levelling *Poles*, or *Staves*, are inſtruments uſed in levelling, ſerving to carry the marks to be obſerved, and at the ſame time to meaſure the heights of thoſe marks from the ground. They uſually conſiſt each of two long wooden rulers, made to ſlide over each other, and divided into feet and inches, &c.

LEVER, a ſtraight bar of iron or wood, &c, ſuppoſed to be inflexible, ſupported on a fulcrum or prop by a ſingle point, about which all the parts are moveable.

The Lever is the firſt of thoſe ſimple machines called *mechanical powers*, as being the ſimpleſt of them all; and is chiefly uſed for raiſing great weights to ſmall heights.

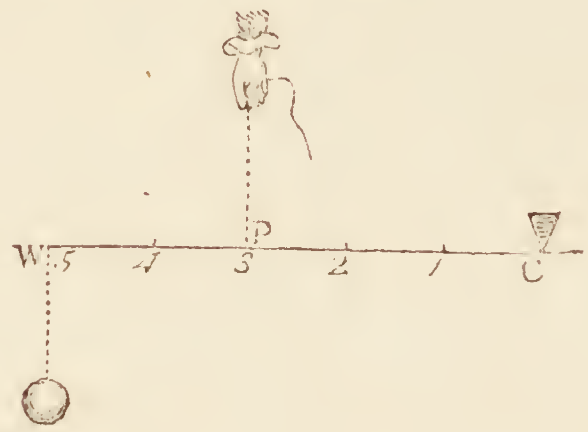


The Lever is of three kinds. Firſt the common ſort, where the weight intended to be raiſed is at one end of it, our ſtrength or another weight called the power is at the other end, and the prop or fulcrum is between them both. In ſtirring up the fire with a poker, we make uſe of this Lever; the poker is the Lever, it reſts upon one of the bars of the grate as a prop, the incumbent fire is the weight to be overcome, and the preſſure of the hand on the other end is the force or power. In this, as in all the other machines, we have only to increaſe the diſtance between the force and the prop, or to decreaſe the diſtance between the weight and the prop, to give the operator the greater power or effect. To this kind of Lever may alſo be referred all ſciſſars, pincers, ſnuffers, &c. The ſteel-yard and the common balance are alſo Levers of this kind.



In the Lever of the 2d kind the prop is at one end, the force or power at the other, and the weight to be raiſed is between them. Thus, in raiſing a water-plug in the ſtreets, the workman puts his iron bar or Lever through the ring or hole of the plug, till the end of it reaches the ground on the other ſide; then making that the prop, he lifts the plug with his force or ſtrength at the other end of the Lever. In this Lever too, the nearer the weight is to the prop, or the farther the power from the prop, the greater is the effect. To this 2d kind of Lever may alſo be referred the oars and rudder of a boat, the maſts of a ſhip, cutting knives fixed at one end, and doors, whoſe hinges ſerve as a fulcrum.

In the Lever of the third kind, the power acts between the weight and the prop; ſuch as a ladder raiſed by a man ſomewhere between the two ends, to rear it againſt a wall, or a pair of tongs, &c.



It is by this kind of Lever too that the muſcular motions of animals are performed, the muſcles being inſerted much nearer to the centre of motion, than the point where is placed the centre of gravity of the weight to be raiſed; ſo that the power of the muſcle is many times greater than the weight it is able to ſuſtain. And in this third kind of Lever, to produce a balance between the power and weight, the power or force muſt *exceed* the weight, in the ſame proportion as it is nearer the prop than the weight is; whereas in the other two kinds, the power is leſs than the weight, in the ſame proportion as its diſtance is greater; that is, univerſally, the power and weight are each of them reciprocally as their diſtance from the prop; as is demonſtrated below.

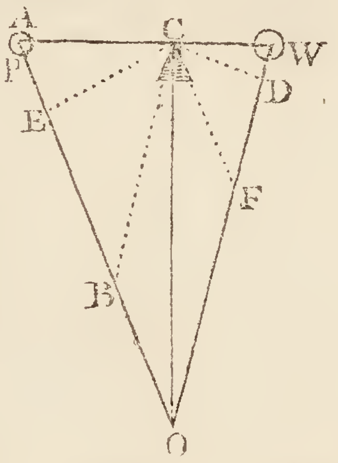
Some authors make a 4th ſort of what is called a bended Lever; ſuch as a hammer in drawing a nail, &c.

In all Levers, the univerſal property is, that the effect of either the weight or the power, to turn the Lever about the fulcrum, is directly as its intenſity and its diſtance from the prop, that is as di, where d denotes the diſtance, and i the intenſity, ſtrength, or weight, &c, of the agent. For it is evident that at a double diſtance it will have a double effect, at a triple diſtance a triple effect, and ſo on; alſo that a double intenſity produces a double effect, a triple a triple, and ſo on: therefore univerſally the effect is as di the product of the two. In like manner, if D be the diſtance of another power or agent, whoſe intenſity is I, then is DI the effect of this alſo to move the Lever. And if theſe two agents act againſt each other on the Lever, and their effects be ſuppoſed equal, or the Lever kept in equilibrio by the equal and contrary effects of theſe two agents; then is $DI = di$, which equation reſolves into this analogy, viz, $D : d :: i : I$; that is, the diſtances of the agents from the prop, are reciprocally

er.

or inverſely as their intenſities, or the power is to the weight, as the diſtance of the latter is to the diſtance of the former.

Writers on mechanics commonly demonſtrate this proportion in a very abſurd manner, viz, by ſuppoſing the Lever put into motion about the prop, and then inferring that, becauſe the momenta of two bodies are equal, when placed upon the Lever at ſuch diſtances, that theſe diſtances are reciprocally proportional to the weights of the bodies, that therefore this is alſo the proportion in caſe of an equilibrium; which is an attempt abſurdly to demonſtrate a thing ſuppoſing the contrary, that a body is at reſt, by ſuppoſing it to be in motion. I ſhall therefore give here a new and univerſal demonſtration of the property, on the pure principles of reſt and preſſure, or force only. Thus, let PW be a lever, C the prop, and P and W any two forces acting on the lever at the points P and W, in the directions PO, WO; then if CE and CD be the perpendicular diſtances of the directions of theſe forces from the prop C, it is to be demonſtrated that P : W : : CD : CE. In order to which join CO, and draw CB parallel to WO, and CF parallel to PO.



Then will CO be the direction of the preſſure on the prop, otherwiſe there could not be an equilibrium, for the directions of three forces that keep each other in equilibrium, muſt neceſſarily meet in the ſame point. And becauſe any three forces that keep each other in equilibrium, are proportional to the three ſides of a triangle formed by drawing lines parallel to the directions of theſe forces; therefore the forces on P, C, and W, are as the three lines BO, CO, CB, which are in the ſame direction, or parallel to them; that is the force P is to the force W, as BO or its equal CF is to CB. But the two triangles CDF, CEB are ſimilar, and have their like ſides proportional,

viz, CF : CB : : CD : CE;

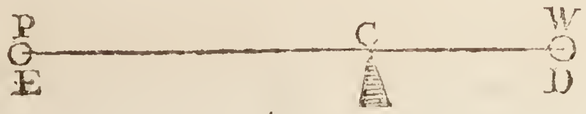
and becauſe it was CF : CB : : P : W;

therefore by equality P : W : : CD : CE;

that is, each force is reciprocally proportional to the diſtance of its direction from the fulcrum. And it will be found that this demonſtration will ſerve alſo for the other kinds of Levers, by drawing the lines as directed. Hence if any given force P be applied to a Lever at A; its effect upon the Lever, to turn it about the centre of motion C, is as the length of the arm CA, and the ſine of the angle of direction CAE. For the perp. CE is as CA × ſin. ∠A.

In any analogy, becauſe the product of the extremes is equal to that of the means; therefore the product of the power by the diſtance of its direction is equal to the product of the weight by the diſtance of its direction. That is, P × CE = W × CD.

If the Lever, with the two weights fixed to it, be made to move about the centre C; the momentum of the power will be equal to that of the weight; and the weights will be reciprocally proportional to their velocities.



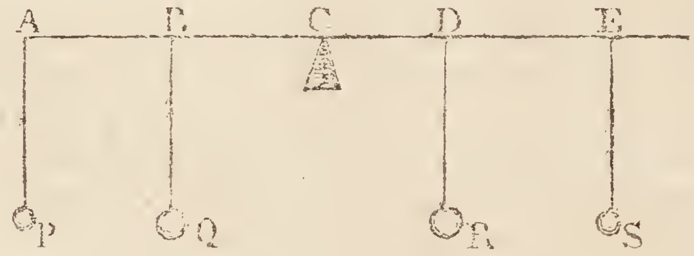
When the two forces act perpendicularly on the Lever, as two weights &c; then, in caſe of an equilibrium, E coincides with P, and D with W; and the diſtances CP, CW, taken on the Lever, or the diſtances of the power and weight, from the fulcrum, are reciprocally proportional to the power and weight.

In a ſtraight Lever, kept in equilibrio by a weight and power acting perpendicularly upon it; then, of theſe three, the power, weight, and preſſure on the prop, any one is as the diſtance of the other two.

And hence too P + W : P : : ED : CD,

and P + W : W : : ED : CP;

that is, the ſum of the weights is to either of them, as the ſum of their diſtances is to the diſtance of the other.



Alſo, if ſeveral weights P, Q, R, S, &c, act on a ſtraight Lever, and keep it in equilibrio; then the ſum of the products on one ſide of the prop, will be equal to the ſum on the other ſide, made by multiplying each weight by its diſtance from the prop; viz, P . AC + Q . BC = R . DC + S . EC + &c.

Hitherto the Lever has been conſidered as a mathematical line void of weight or gravity. But when its weight is conſidered, it is to be done thus: Find the weight and the centre of gravity of the Lever alone, and then conſider it as a mathematical line, but having an equal weight ſuſpended by that centre of gravity; and ſo combine its effect with thoſe of the other weights, as above.

Upon the foregoing principles depends the nature of ſcales and beams for weighing all bodies. For, if the diſtances be equal, then will the weights be equal alſo; which gives the conſtruction of the common ſcales. And the Roman ſtatera, or ſteel-yard, is alſo a Lever, but of unequal arms or diſtances, ſo contrived that one weight only may ſerve to weigh a great many, by ſliding it backwards and forwards to different diſtances upon the longer arm of the Lever. See BALANCE, &c.

Alſo upon the principle of the Lever depends almoſt all other mechanical powers and effects. See WHEEL-AND-AXLE, PULLEY, WEDGE, SCREW, &c.

LEVITY, the privation or want of weight in any body, when compared with another that is heavier; and in this ſenſe it is oppoſed to gravity. Thus cork, and moſt ſorts of wood that float in water, have Levity with reſpect to water, that is, are leſs heavy. The ſchools maintained that there is ſuch a thing as poſitive and abſolute Levity; and to this they imputed the riſe and buoyancy of bodies lighter in ſpecie than the bodies in which they riſe and float. But it is now well known that this happens only in conſequence of the heavier and denſer fluid, which, by its ſuperior gravity, gains the loweſt place, and raiſes up the lighter body by a force which is equal to the difference of their gravities. It was demonſtrated by Archimedes, that a ſolid body will float any where in a fluid of the ſame ſpecific gravity;

gravity; and that a lighter body will always be raifed up in it.

LEUWENHOEK (Antony), a celebrated Dutch philofopher, was born at Delft in 1632; and acquired a great reputation throughout all Europe, by his experiments and difcoveries in Natural Hiftory, by means of the microfcope. He particularly excelled in making glaffes for microfcopes and fpectacles; and he was a member of moft of the literary focieties of Europe; to whom he fent many memoirs. Thofe in the Philofophical Tranfactions, and in the Paris Memoirs, extend through many volumes; the former were extracted, and publifhed at Leyden, in 1722. He died in 1723, at 91 years of age.

LEYDEN Phial, in Electricity, is a glafs phial or jar, coated both within and without with tin foil, or fome other conducting fubftance, that it may be charged, and employed in a variety of ufeful and entertaining experiments. Or even flat glafs, or any other fhape, fo coated and ufed, has alfo received the fame denomination. Alfo a vacuum produced in fuch a jar, &c. has been named the *Leyden Vacuum*.

The Leyden Phial has been fo called, becaufe it is faid that M. Cunæus, a native of Leyden, firft contrived, about the clofe of the year 1745, to accumulate the electrical power in glafs, and ufe it in this way. But Dr. Prieftley afferts that this difcovery was firft made by Von Kleift, dean of the cathedral in Camin; who, on the 4th of November 1745, fent an account of it to Dr. Lieberkuhn at Berlin: however, thofe to whom Kleift's account was communicated, could not fucceed in performing his experiments. The chief circumftances of this difcovery are ftated by Dr. Prieftley in the following manner.

Profeffor Mufchenbroek and his friends, obferving that electrified bodies, when expofed to the common atmofphere, which is always replete with conducting particles of various kinds, foon loft the moft part of their electricity, imagined that if the electrified bodies fhould be terminated on all fides by original electrics, they might be capable of receiving a ftronger power, and retaining it a longer time. Glafs being the moft convenient electric for this purpofe, and water the moft convenient non-electric, they at firft made thefe experiments with water in glafs bottles: but no confiderable difcovery was made, till M. Cunæus, happening to hold his glafs veffel in one hand, containing water, which had a communication with the prime conductor by means of a wire; and with the other hand difengaging it from the conductor, when he fuppofed the water had received as much electricity as the machine could give it, was furprifed by a fudden and unexpected fhock in his arms and breaft. This experiment was repeated, and the firft accounts of it publifhed in Holland by Meffrs. Allamand and Mufchenbroek; by the Abbé Nollet and M. Monnier, in France; and by Meffrs. Gralath and Rugger, in Germany. M. Gralath contrived to increafe the ftrength of the fhock, by altering the fhape and fize of the phial, and alfo by charging feveral phials at the fame time, fo as to form what is now called the *electrical battery*. He likewife made the fhock to pafs through a number of perfons connected in a circuit from the outfide to the infide of the phial. He alfo obferved that a cracked phial would not receive a charge: and he difcovered what is now called the *Refiduum of a charge*.

Dr. Watfon, about this time, obferved a circumftance attending the operation of charging the phial, which, if purfued, might have led him to the difcovery which was afterwards made by Dr. Franklin. He fays, that when the phial is well electrified, and you apply your hand to it, you fee the fire flafh from the outfide of the glafs, wherever you touch it, and it crackles in your hand. He alfo obferved, that when a fingle wire only was faftened about a phial, properly filled with warm water, and charged; upon the inftant of its explofion, the electrical corrufcations were feen to dart from the wire, and to illuminate the water contained in the phial. He likewife found that the ftroke, in the difcharge of the phial, was, *cæteris paribus*, as the points of contact of the non-electrics of the outfide of the glafs; which led to the method of coating glafs: in confequence of which he made experiments, from whence he concluded, that the effect of the Leyden phial was greatly increafed by, if not chiefly owing to, the number of points of non-electric in contact within the glafs, and the denfity of the matter of which thefe points confifted; provided the matter was, in its own nature, a ready conductor of electricity. He farther obferved, that the explofion was greater from hot water inclofed in glaffes, than from cold, and from his coated jars warmed, than when cold.

Mr. Wilfon, in 1746, difcovered a method of giving the fhock to any particular part of the body, without affecting the reft. He alfo increafed the ftrength of the fhock by plunging the phial in water, which gave it a coat of water on the outfide as high as it was filled within. He likewife found, that the law of accumulation of the electric matter in the Leyden phial, was always in proportion to the thinnefs of the glafs, the furface of the glafs, and that of the non-electrics in contact with its outfide and infide. He made alfo a variety of other experiments with the Leyden phial, too long here to be related.

Mr. Canton found, that when a charged phial was placed upon electrics, the wire and coating would give a fpark or two alternately, and that by a continuance of the operation the phial would be difcharged; though he did not obferve that thefe alternate fparks proceeded from the two contrary electricities difcovered by Dr. Franklin.

The Abbé Nollet made feveral experiments with this phial. He received a fhock from one, out of which the air had been exhaufted, and into which the end of his conductor had been inferted. He afcribed the force of the glafs, in giving a fhock, to that property of it, by which it retains it more ftrongly than conductors do, and is not fo eafily divefted of it as they are. It was he alfo who firft tried the effect of the electric fhock on brute animals: and he enlarged the circuit of its conveyance.

M. Monnier, it has been faid, was the firft who difcovered that the Leyden phial would retain its electricity for a confiderable time after it was charged; and that in time of froft he found it continued for 36 hours. It is remarkable too that both the French and Englifh philofophers made feveral experiments, which, with a fmall degree of attention, would have led them to the difcovery of the different qualities of the electricity on the

the contrary ſides of the glaſs. But this diſcovery was reſerved for the ingenious Dr. Franklin; who, in explaining the method of charging the Leyden phial, obſerves, that when one ſide of the glaſs is electrified plus, or poſitively, the other ſide is electrified minus, or negatively: ſo that whatever quantity of fire is thrown upon one ſide of the glaſs, the ſame quantity is drawn out of the other; and in an uncharged phial, none can be thrown into the inſide, when none can be taken from the outſide; and that there is really no more electric fire in the phial after it is charged than before; all that can be done by charging, being only to take from one ſide, and convey to the other. Dr. Franklin alſo obſerved that glaſs was not impervious to electricity, and that as the equilibrium could not be reſtored to the charged phial by any internal communication, it muſt neceſſarily be done by conductors externally joining the inſide and the outſide. Theſe capital diſcoveries he made by obſerving, that when a phial was charged, a cork ball ſuſpended by ſilk, was attracted by the outſide coating, when it was repelled by a wire communicating with the inſide, and *vice verſa*. But the truth of this principle appeared more evident, when he brought the knob of the wire, communicating with the outſide coating, within a few inches of the wire communicating with the inſide coating, and ſuſpended a cork ball between them; for then the ball was attracted by them alternately, till the phial was diſcharged.

Dr. Franklin alſo ſhewed, that when the phial was charged, one ſide loſt exactly as much as the other gained, in reſtoring the equilibrium. Hanging a fine linen thread near the coating of an electrical phial, he obſerved that whenever he brought his finger near the wire, the thread was attracted by the coating; for as the fire was drawn from the inſide by touching the wire, the outſide drew in an equal quantity by the thread. He likewiſe proved, that the coating on one ſide of a phial received juſt as much electricity, as was emitted from the diſcharge of the other, and that in the following manner:—He inſulated his rubber, and then hanging a phial to his conductor, he found it could not be charged, even when his hand was held conſtantly to it; becauſe, though the electric fire might leave the outſide of the phial, there was none collected by the rubber to be conveyed to the inſide. He then took away his hand from the phial, and forming a communication by a wire from the outſide coating to the inſulated rubber, he found that it was charged with eaſe. In this caſe it was plain, that the very ſame fire which left the outſide coating, was conveyed to the inſide by the way of the rubber, the globe, the conductor, and the wire of the phial. This new theory of charging the Leyden phial, led Dr. Franklin to obſerve a greater variety of facts, relating both to the charging and diſcharging it, than other philoſophers had attended to. And this maxim, that it takes in at one ſurface, what it loſes at the other, led Dr. Franklin to think of charging ſeveral phials together with the ſame trouble, by connecting the outſide of one with the inſide of another; by which the fire that was driven out of the firſt would be received by the ſecond, &c. By this means he found, that a great number of jars might be charged with the ſame labour as one only; and that they might be charged equally high, were it not that every one of them receives the new fire, and loſes its old, with ſome reluctance, or rather that it gives ſome ſmall reſiſtance to the charging. And on this principle he firſt conſtructed an electrical battery.

When Dr. Franklin firſt began his experiments on the Leyden phial, he imagined that the electric fire was all crowded into the ſubſtance of the non-electric, in contact with the glaſs. But he afterwards found, that its power of giving a ſhock lay in the glaſs itſelf, and not in the coating, by the following ingenious analyſis of the phial. To find where the ſtrength of the charged bottle lay, having placed it upon a glaſs, he firſt took out the cork and the wire; but not finding the virtue in them, he touched the outſide coating with one hand, and put a finger of the other into the mouth of the bottle; when the ſhock was felt quite as ſtrong as if the cork and wire had been in it. He then charged the phial again, and pouring out the water into an empty bottle which was inſulated, he expected that if the force reſided in the water, it would give the ſhock; but he found it gave none. He therefore concluded that the electric fire muſt either have been loſt in decanting, or muſt remain in the bottle; and the latter he found to be true; for, upon filling the charged bottle with freſh water, he found the ſhock, and was ſatisfied that the power of giving it reſided in the glaſs itſelf. The ſame experiment was made with panes of glaſs, laying the coating on lightly, and charging it, as the water had been before charged in the bottle, when the reſult was preciſely the ſame. He alſo proved in other ways that the electric fire reſided in the glaſs. See Franklin's Letters and Obſervations, &c. Alſo Prieſtley's Hiſt. of Electricity, vol. i, pa. 191, &c.

From this account of Dr. Franklin's method of analyzing the Leyden phial, the manner of charging and diſcharging it, with the reaſon of the proceſs, are eaſily underſtood. Thus, placing a coated phial near the prime conductor, ſo that the knob of its wire may be in contact with it; then upon turning the winch of the machine, the index of the electrometer, E, fixed to the conductor, will gradually riſe as far as 90° nearly, and there reſt; which ſhews that the phial has received its full charge: then holding the diſcharger by its glaſs handle, and applying one of its knobs to the outſide coating of the phial, the other being brought near the knob of the wire, or near the prime conductor which communicates with it, a report will be heard, and luminous ſparks will be ſeen between the diſcharger and the conducting ſubſtances communicating with the ſides of the phial; and by this operation the phial will be diſcharged. But, inſtead of uſing the diſcharger, if a perſon touch the outſide of the phial with one hand, and bring the other hand near the wire of the phial, the ſame ſpark and report will take place, and a ſhock will be felt, affecting the wriſts and elbows, and the breaſt too when the ſhock is ſtrong: a ſhock may alſo be given to any ſingle part of the body, if that part alone be brought into the circuit. If a number of perſons join hands, and the firſt of them touch the outſide of the phial, while the laſt touches the wire communicating with the inſide, they will all feel the ſhock at the ſame time. If the coated phial be held by the wire, and the outſide coating be preſented to the prime conductor,

conductor, it will be charged as readily; but only with this difference, that in this case the outside will be positive, and the inside negative; also if the prime conductor, by being connected with the rubber of the machine, be electrified negatively, the phial will be charged in the same manner; but the side that touches the conductor will be electrified negatively, and the opposite side will be electrified positively. But, by insulating the phial, and repeating the same process, the index of the electrometer will soon rise to 90°, yet the phial will remain uncharged; because the outside, having no communication with the earth, &c, cannot part with its own electricity, and therefore the inside cannot acquire an additional quantity: but when a chain, or any other conductor, connects the outside of the phial with the table, the phial may be charged as before. Moreover, if a phial be insulated, and one side of it, instead of being connected with the earth, be connected with the insulated rubber, whilst the other side communicates with the prime conductor, the phial will be expeditiously charged; because that whilst the rubber exhausts one side, the other side is supplied by the prime conductor; and thus the phial is charged with its own electricity; or the natural electric matter of one of its sides is thus thrown upon the other side. This last experiment may be diversified by insulating the phial, and placing it with its wire at the distance of about half an inch from the prime conductor, and holding the knob of another wire at the same distance from its outside coating; then, upon turning the machine, a spark will be observed to proceed from the prime conductor to the wire of the phial, and another spark will pass at the same time from the outside coating to the knob of the wire presented towards it: and thus it appears that as a quantity of the electric matter is entering the inside of the phial, an equal quantity of it is leaving the outside. If the wire presented to the outside of the phial be pointed, it will be seen illuminated with a star; but if the pointed wire be connected with the coating of the phial, it will appear illuminated with a brush of rays. See *Charge*, *Electrical Shock*, *Experiments*, *&c.*

Mr. Cavallo has described the construction of a phial which, being charged by an electrical kite, in examining the state of the clouds, or in any other way, may be put into the pocket, and which will retain its charge for a considerable time. A phial of this kind has been kept in a charged state for six weeks. See his Electricity, pa. 340. Many other curious experiments with the Leyden phial may be seen in the books above cited, as also in the volumes of the Philos. Trans. and elsewhere. In this last-mentioned work, Mr. Cavallo describes a method of repairing coated phials that have cracked by any means. He first removes the outside coating from the fractured part, and then makes it moderately hot, by holding it to the flame of a candle; and whilst it remains hot, he applies burning sealing-wax to the part, so as to cover the fracture entirely; observing that the thickness of this wax coating may be greater than that of the glass. Lastly, he covers all the sealing-wax, and also part of the surface of the glass beyond it, with a composition made with four parts of bees-wax, one of resin, one of turpentine, and a very little oil of olives; this being spread upon a piece of oiled silk, he applies it in the manner of a plaster. In this way several phials have been so effectually repaired, that after being frequently charged, they were at last broken by a spontaneous discharge, but in a different part of the glass. Philos. Trans. vol. 68, pa. 1011.

LIBRA, *Balance*, one of the mechanical powers. See BALANCE.

LIBRA is also one of the 48 old constellations, and the 7th sign of the zodiac, being opposite to Aries, and marked like a part of a pair of scales, thus ♎. The figure of the balance was probably given to this part of the ecliptic, because when the sun arrives at this part, which is at the time of the autumnal equinox, the days and nights are equal, as if weighed in a balance.

The stars in this constellation are, according to Ptolomy 17, Tycho 10, Hevelius 20, and Flamsteed 51.

LIBRA also denotes the ancient Roman pound, which was divided into 12 unciæ, or ounces, and the ounce into 24 scruples. It seems the mean weight of the scruple was nearly equal to $17\frac{1}{2}$ grains Troy, and consequently the libra, or pound, 5040 grains. It was also the name of a gold coin, equal in value to 20 denarii. See Philos. Trans. vol. 61, pa. 462.

The French livre is derived from the Roman libra, this being used in France for the proportions of their coin till about the year 1100, their sols being so proportioned as that 20 of them were equal to the libra. By degrees it became a term of account, and every thing of the value of 20 sols was called a livre.

LIBRATION, *of the Moon*, is an apparent irregularity in her motion, by which she seems to librate, or waver, about her own axis, one while towards the east, and again another while towards the west. See MOON, and EVECTION. Hence it is that some parts near the moon's western edge at one time recede from the centre of the disc, while those on the other or eastern side approach nearer to it; and, on the contrary, at another time the western parts are seen to be nearer the centre, and the eastern parts farther from it: by which means it happens that some of those parts, which were before visible, set and hide themselves in the hinder or invisible side of the moon, and afterwards return and appear again on the nearer or visible side.

This Libration of the moon was first discovered by Hevelius, in the year 1654; and it is owing to her equable rotation round her own axis, once in a month, in conjunction with her unequal motion in the perimeter of her orbit round the earth. For if the moon moved in a circle, having its centre coinciding with the centre of the earth, whilst it turned on its axis in the precise time of its period round the earth, then the plane of the same lunar meridian would always pass through the earth, and the same face of the moon would be constantly and exactly turned towards us. But since the real motion of the moon is about a point considerably distant from the centre of the earth, that motion is very unequal, as seen from the earth, the plane of no one meridian constantly passing through the earth.

The Libration of the moon is of three kinds.

1st, Her libration in longitude, or a seeming to-and-again motion according to the order of the signs of the zodiac. This libration is nothing twice in each periodical month, viz, when the moon is in her apogeum, and when in her perigeum; for in both these cases the

plane

plane of her meridian, which is turned towards us, is directed alike towards the earth.

2d, Her libration in latitude; which arifes from hence, that her axis not being perpendicular to the plane of her orbit, but inclined to it, fometimes one of her poles, and fometimes the other will nod, as it were, or dip a little towards the earth, and confequently fhe will appear to librate a little, and to fhew fometimes more of her fpots, and fometimes lefs of them, towards each pole. Which libration, depending on the pofition of the moon, in refpect to the nodes of her orbit, and her axis being nearly perpendicular to the plane of the ecliptic, is very properly faid to be in latitude. And this alfo is completed in the fpace of the moon's periodical month, or rather while the moon is returning again to the fame pofition, in refpect of her nodes.

3d, There is alfo a third kind of libration; by which it happens that although another part of the moon be not really turned to the earth, as in the former libration, yet another is illuminated by the fun. For fince the moon's axis is nearly perpendicular to the plane of the ecliptic, when fhe is moft foutherly, in refpect of the north pole of the ecliptic, fome parts near to it will be illuminated by the fun; while, on the contrary, the fouth pole will be in darknefs. In this cafe, therefore, if the fun be in the fame line with the moon's fouthern limit, then, as fhe proceeds from conjunction with the fun towards her afcending node, fhe will appear to dip her northern polar parts a little into the dark hemifphere, and to raife her fouthern polar parts as much into the light one. And the contrary to this will happen two weeks after, while the new moon is defcending from her northern limit; for then her northern polar parts will appear to emerge out of darknefs, and the fouthern polar parts to dip into it. And this feeming libration, or rather thefe effects of the former libration in latitude, depending on the light of the fun, will be completed in the moon's fynodical month. Greg. Aftron. lib. 4, fect. 10.

LIBRATION *of the Earth*, is a term applied by fome aftronomers to that motion, by which the earth is fo retained in its orbit, as that its axis continues conftantly parallel to the axis of the world.

This Copernicus calls the *motion of libration*, which may be thus illuftrated: Suppofe a globe, with its axis parallel to that of the earth, painted on the flag of a maft, moveable on its axis, and conftantly driven by an eaft wind, while it fails round an ifland, it is evident that the painted globe will be fo librated, as that its axis will be parallel to that of the world, in every fituation of the fhip.

LIFE-ANNUITIES, are fuch periodical payments as depend on the continuance of fome particular life or lives. They may be diftinguifhed into Annuities that commence immediately, and fuch as commence at fome future period, called *reverfionary life-annuities*.

The value, or prefent worth, of an annuity for any propofed life or lives, it is evident, depends on two circumftances, the intereft of money, and the chance or expectation of the continuance of life. Upon the former only, it has been fhewn, under the article ANNUITIES, depends the value or prefent worth of an annuity certain, or that is not fubject to the continuance of a life, or other contingency; but the expectation of life being a thing not certain, but only poffeffing a certain chance, it is evident that the value of the certain annuity, as ftated above, muft be diminifhed in proportion as the expectancy is below certainty: thus, if the prefent value of an annuity certain be any fum, as fuppofe 100l. and the value or expectancy of the life be $\frac{1}{2}$, then the value of the life-annuity will be only half of the former, or 50l; and if the value of the life be only $\frac{1}{3}$, the value of the life-annuity will be but $\frac{1}{3}$ of 100l, that is 33l. 6s. 8d; and fo on.

The meafure of the value or expectancy of life, depends on the proportion of the number of perfons that die, out of a given number, in the time propofed; thus, if 50 perfons die, out of 100, in any propofed time, then, half the number only remaining alive, any one perfon has an equal chance to live or die in that time, or the value of his life for that time is $\frac{1}{2}$; but if $\frac{2}{3}$ of the number die in the time propofed, or only $\frac{1}{3}$ remain alive, then the value of any one's life is $\frac{1}{3}$; and if $\frac{3}{4}$ of the number die, or only $\frac{1}{4}$ remain alive, then the value of any life is but $\frac{1}{4}$; and fo on. In thefe proportions then muft the value of the annuity certain be diminifhed, to give the value of the like life annuity.

It is plain therefore that, in this bufinefs, it is neceffary to know the value of life at all the different ages, from fome table of obfervations on the mortality of mankind, which may fhew the proportion of the perfons living, out of a given number, at the end of any propofed time; or from fome certain hypothefis, or affumed principle. Now various tables and hypothefes of this fort were given by the writers on this fubject, as Dr. Halley, Mr. Demoivre, Mr. Thomas Simpfon, Mr. Dodfon, Mr. Kerffeboom, Mr. Parcieux, Dr. Price, Mr. Morgan, Mr. Baron Maferes, and many others. But the fame table of probabilities of life will not fuit all places; for long experience has fhewn that all places are not equally healthy, or that the proportion of the number of perfons that die annually, is different for different places. Dr. Halley computed a table of the annual deaths as drawn from the bills of mortality of the city of Breflaw in Germany, Mr. Smart and Mr. Simpfon from thofe of London, Dr. Price from thofe of Northampton, Mr. Kerffeboom from thofe of the provinces of Holland and Weft-Friefland, and M. Parcieux from the lifts of the French tontines, or long annuities, and all thefe are found to differ from one another. It may not therefore be improper to infert here a comparative view of the principal tables that have been given of this kind, as below, where the firft column fhews the age, and the other columns the number of perfons living at that age, out of 1000 born, or of the age 0, in the firft line of each column.

TABLE.

T A B L E I.

Shewing the Number of Perſons living at all Ages, out of 1000 *that had been born at ſeveral Places, viz.*

Ages.	Vienna.	Berlin.	London.	Norwich.	Northampton.	Breſlaw.	Brandenburg.	Holy-Croſs.	Holland.	France.	Vaud, Switzerland.
0	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
1	542	633	680	798	738	769	775	882	804	805	811
2	471	528	548	651	628	658	718	762	768	777	765
3	430	485	492	595	585	614	687	717	736	750	735
4	400	434	452	566	562	585	664	682	709	727	715
5	377	403	426	544	544	563	642	659	689	711	701
6	357	387	410	526	530	546	622	636	676	697	688
7	344	376	397	511	518	532	607	618	664	686	677
8	337	367	388	500	510	523	595	604	652	676	667
9	331	361	380	490	504	515	585	595	646	667	659
10	326	356	373	481	498	508	577	589	639	660	653
11	322	353	367	474	493	502	570	585	633	654	648
12	318	350	361	469	488	497	564	581	627	649	643
13	314	347	356	464	484	492	559	577	621	644	639
14	310	344	351	460	480	488	554	573	616	639	635
15	306	341	347	455	475	483	549	569	611	635	631
16	302	338	343	451	470	479	544	565	606	631	626
17	299	335	338	446	465	474	539	560	601	626	622
18	295	332	334	442	459	470	535	555	596	621	618
19	291	328	329	437	453	465	531	550	590	616	614
20	287	324	325	432	447	461	527	545	584	610	610
21	284	320	321	426	440	456	522	539	577	604	606
22	280	315	316	421	433	451	517	532	571	598	602
23	276	310	310	415	426	446	512	525	566	592	597
24	273	305	305	409	419	441	507	518	559	586	592
25	269	297	299	404	412	436	502	512	551	580	587
26	265	293	294	398	405	431	498	506	543	574	582
27	261	287	288	392	398	426	495	501	535	568	577
28	256	281	283	385	391	421	492	496	526	562	572
29	251	275	278	378	384	415	489	491	517	556	567
30	247	269	272	372	378	409	486	486	508	550	563
31	243	264	266	365	372	403	482	481	499	544	558
32	239	259	260	361	366	397	477	476	490	438	553
33	235	254	254	355	360	391	472	471	482	532	548
34	231	249	248	350	354	384	467	466	474	526	544
35	226	243	242	344	348	377	462	460	467	520	539
36	221	237	236	338	342	370	456	454	460	514	533
37	216	230	230	333	336	363	450	447	453	508	527
38	211	223	224	327	330	356	444	440	446	503	520
39	205	216	218	322	324	349	438	433	439	497	513
40	199	209	214	317	317	342	432	426	432	492	506
41	194	203	207	311	310	335	427	418	425	487	500
42	189	197	201	306	303	328	422	410	419	482	494
43	185	192	194	300	296	321	417	401	413	476	488
44	181	187	187	294	289	314	412	393	407	471	482
45	176	182	180	287	282	307	407	386	400	466	476
46	171	177	174	281	275	299	400	379	393	460	469
47	165	172	167	274	268	291	394	372	386	455	461
48	159	167	159	268	261	283	388	365	378	449	451
49	153	162	153	261	254	275	381	359	370	443	441
50	147	157	147	255	247	267	374	353	362	436	431
51	142	152	141	248	239	259	367	347	354	429	422
52	137	147	135	242	232	250	359	340	345	422	414
53	133	142	130	235	225	241	351	333	336	414	406

Ages.	Vienna.	Berlin.	London.	Norwich.	North-ampton.	Breslaw.	Branden-burg.	Holy-Crofs.	Holland.	France.	Vaud, Switzer-land.
54	128	137	125	228	218	232	343	326	327	406	397
55	123	132	120	221	211	224	334	318	318	397	388
56	117	127	116	213	204	216	324	310	309	388	377
57	111	121	111	206	197	209	314	301	300	379	364
58	106	115	106	199	190	201	304	292	291	369	348
59	101	109	101	191	183	193	293	283	282	359	331
60	96	103	96	184	176	186	282	273	273	349	314
61	91	97	92	177	169	178	271	263	264	339	299
62	87	92	87	169	162	170	260	253	255	329	286
63	82	88	83	161	155	163	248	243	245	318	274
64	77	84	78	153	148	155	236	233	235	307	262
65	72	80	74	144	141	147	224	223	225	296	250
66	67	75	70	136	134	140	213	213	215	285	236
67	62	70	65	128	127	132	202	203	205	273	220
68	57	65	61	119	120	124	190	193	195	260	202
69	52	60	56	111	113	117	178	182	185	246	184
70	48	55	52	103	106	109	166	171	175	232	168
71	44	51	47	94	99	101	153	161	165	218	153
72	40	47	43	86	92	93	138	151	155	195	140
73	36	43	39	79	85	85	122	142	145	188	129
74	33	39	35	71	78	77	107	134	135	173	119
75	30	35	32	64	71	69	93	126	125	158	109
76	27	32	28	57	64	61	80	119	114	144	98
77	24	29	25	50	58	53	68	112	103	129	85
78	21	26	22	43	52	45	59	105	92	115	71
79	18	23	19	37	46	38	51	98	82	102	58
80	16	20	17	32	40	32	44	90	72	88	46
81	14	18	14	27	34	26	38	81	62	75	36
82	12	16	12	23	28	22	32	71	53	63	29
83	10	14	10	19	23	18	25	61	45	53	24
84	8	12	8	16	19	15	21	51	38	44	20
85	7	10	7	13	16	12	15	41	31	36	17
86	6	8	6	10	13	9	11	32	25	28	14
87	5	7	5	8	11	6	8	24	19	21	11
88	4	6	4	6	8	4	6	17	14	16	9
89	3	5	3	5	6	2	4	11	10	12	7
90	2	4	2	4	4	1	3	7	7	8	5

Thefe tables fhew that the mortality and chance of life are very various in different places; and that therefore, to obtain a fufficient accuracy in this bufinefs, it is neceffary to adapt a table of probabilities or chances of life, to every place for which annuities are to be calculated; or at leaft one fet of tables for large towns, and another for country places, as well as for the fuppofition of different rates of intereft.

Several of the foregoing tables, as they commenced with numbers different from one another, are here reduced to the fame number at the beginning, viz, 1000 perfons, by which means we are enabled by infpection, at any age, to compare the numbers together, and immediately perceive the relative degrees of vitality at the feveral places. The tables are alfo arranged according to the degree of vitality amongft them; the leaft, or that at Vienna, firft; and the reft in their order, to the higheft, which is the province of Vaud in Switzerland. The authorities upon which thefe tables depend, are as they here follow. The firft, taken from Dr. Price's Obfervations on Reverfionary payments, is formed from the bills at Vienna, for 8 years, as given by Mr. Sufmilch, in his *Gottliche* Ordnung; the 2d, for Berlin, from the fame, as formed from the bills there for 4 years, viz, from 1752 to 1755; the 3d, from Dr. Price, fhewing the true probabilities of life in London, formed from the bills for ten years, viz, from 1759 to 1768; the 4th, for Norwich, formed by Dr. Price from the bills for 30 years, viz, from 1740 to 1769; the 5th, by the fame, from the bills for Northampton; the 6th, as deduced by Dr. Halley, from the bills of mortality at Breflaw; the 7th fhews the probabilities of life in a country parifh in Brandenburg, formed from the bills for 50 years, from 1710 to 1759, as given by Mr. Sufmilch; the 8th fhews the probabilities of life in the parifh of Holy-Crofs, near Shrewfbury, formed from a regifter kept by the Rev. Mr. Garfuch, for 20 years, from 1750 to 1770; the 9th, for Holland,

Holland, was formed by M. Kerſſeboom, from the regiſters of certain annuities for lives granted by the government of Holland, which had been kept there for 125 years, in which the ages of the ſeveral annuitants dying during that period had been truly entered; the 10th, for France, were formed by M. Parcieux, from the liſts of the French tontines, or long annuities, and verified by a compariſon with the mortuary regiſters of ſeveral religious houſes for both ſexes; and the 11th, or laſt, for the diſtrict of Vaud in Switzerland, was formed by Dr. Price from the regiſters of 43 pariſhes, given by M. Muret, in the Bern Memoirs for the year 1766.

Now from ſuch liſts as the foregoing, various tables have been formed for the valuation of annuities on ſingle and joint lives, at ſeveral rates of intereſt, in which the value is ſhewn by inſpection. The following are thoſe that are given by Mr. Simpſon, in his Select Exerciſes, as deduced from the London bills of mortality.

TABLE II.

Shewing the Value of an Annuity on One Life, or Number of Years Annuity in the Value, ſuppoſing Money to bear Intereſt at the ſeveral Rates of 3, 4, *and* 5 *per cent.*

Age.	Years value at 3 per cent.	Years value at 4 per cent.	Years value at 5 per cent.	Age.	Years value at 3 per cent.	Years value at 4 per cent.	Years value at 5 per cent.
6	18·8	16·2	14·1	41	13·0	11·4	10·2
7	18·9	16·3	14·2	42	12·8	11·2	10·1
8	19·0	16·4	14·3	43	12·6	11·1	10·0
9	19·0	16·4	14·3	44	12·5	11·0	9·9
10	19·0	16·4	14·3	45	12·3	10·8	9·8
11	19·0	16·4	14·3	46	12·1	10·7	9·7
12	18·9	16·3	14·2	47	11·9	10·5	9·5
13	18·7	16·2	14·1	48	11·8	10·4	9·4
14	18·5	16·0	14·0	49	11·6	10·2	9·3
15	18·3	15·8	13·9	50	11·4	10·1	9·2
16	18·1	15·6	13·7	51	11·2	9·9	9·0
17	17·9	15·4	13·5	52	11·0	9·8	8·9
18	17·6	15·2	13·4	53	10·7	9·6	8·8
19	17·4	15·0	13·2	54	10·5	9·4	8·6
20	17·2	14·8	13·0	55	10·3	9·3	8·5
21	17·0	14·7	12·9	56	10·1	9·1	8·4
22	16·8	14·5	12·7	57	9·9	8·9	8·2
23	16·5	14·3	12·6	58	9·6	8·7	8·1
24	16·3	14·1	12·4	59	9·4	8·6	8·0
25	16·1	14·0	12·3	60	9·2	8·4	7·9
26	15·9	13·8	12·1	61	8·9	8·2	7·7
27	15·6	13·6	12·0	62	8·7	8·1	7·6
28	15·4	13·4	11·8	63	8·5	7·9	7·4
29	15·2	13·2	11·7	64	8·3	7·7	7·3
30	15·0	13·1	11·6	65	8·0	7·5	7·1
31	14·8	12·9	11·4	66	7·8	7·3	6·9
32	14·6	12·7	11·3	67	7·6	7·1	6·7
33	14·4	12·6	11·2	68	7·4	6·9	6·6
34	14·2	12·4	11·0	69	7·1	6·7	6·4
35	14·1	12·3	10·9	70	6·9	6·5	6·2
36	13·9	12·1	10·8	71	6·7	6·3	6·0
37	13·7	11·9	10·6	72	6·5	6·1	5·8
38	13·5	11·8	10·5	73	6·2	5·9	5·6
39	13·3	11·6	10·4	74	5·9	5·6	5·4
40	13·2	11·5	10·3	75	5·6	5·4	5·2

TABLE III.

Shewing the Value of an Annuity for Two Joint Lives, that is, for as long as they exist together.

Age of Younger	Age of Elder	Value at 3 per cent.	Value at 4 per cent.	Value at 5 per cent.
10	10	14·7	13·0	11·6
	15	14·3	12·7	11·3
	20	13·8	12·2	10·8
	25	13·1	11·6	10·2
	30	12·3	10·9	9·7
	35	11·5	10·2	9·1
	40	10·7	9·6	8·6
	45	10·0	9·0	8·1
	50	9·3	8·4	7·6
	55	8·6	7·8	7·1
	60	7·8	7·2	6·6
	65	6·9	6·5	6·1
	70	6·1	5·8	5·5
	75	5·3	5·1	4·9
15	15	13·9	12·3	11·0
	20	13·3	11·8	10·5
	25	12·6	11·2	10·1
	30	11·9	10·6	9·5
	35	11·2	10·0	9·0
	40	10·4	9·4	8·5
	45	9·6	8·8	8·0
	50	8·9	8·2	7·5
	55	8·2	7·6	7·0
	60	7·5	7·0	6·5
	65	6·8	6·4	6·0
	70	6·0	5·7	5·4
	75	5·2	5·0	4·8
20	20	12·8	11·3	10·1
	25	12·2	10·8	9·7
	30	11·6	10·3	9·2
	35	10·9	9·8	8·8
	40	10·2	9·2	8·4
	45	9·5	8·6	7·9
	50	8·8	8·0	7·4
	55	8·1	7·5	6·9
	60	7·4	6·9	6·4
	65	6·7	6·3	5·9
	70	6·0	5·7	5·4
	75	5·2	5·0	4·8
25	25	11·8	10·5	9·4
	30	11·3	10·1	9·0
	35	10·7	9·6	8·6
	40	10·0	9·1	8·2
	45	9·4	8·5	7·8
	50	8·7	7·9	7·3
	55	8·0	7·4	6·8
	60	7·3	6·8	6·3
	65	6·6	6·2	5·8
	70	5·9	5·6	5·3
	75	5·1	4·9	4·7
30	30	10·8	9·6	8·6
	35	10·3	9·2	8·3
	40	9·7	8·8	8·0
	45	9·1	8·3	7·6
	50	8·5	7·8	7·2
	55	7·9	7·3	6·7
	60	7·2	6·7	6·2
	65	6·5	6·1	5·7
	70	5·8	5·5	5·2
	75	5·1	4·9	4·7
35	35	9·9	8·8	8·0
	40	9·4	8·5	7·7
	45	8·9	8·1	7·4
	50	8·3	7·6	7·0
	55	7·7	7·1	6·6
	60	7·1	6·5	6·1
	65	6·4	6·0	5·6
	70	5·7	5·4	5·1
	75	5·0	4·8	4·6
40	40	9·1	8·1	7·3
	45	8·7	7·8	7·1
	50	8·2	7·4	6·8
	55	7·6	6·9	6·4
	60	7·0	6·4	6·0
	65	6·4	5·9	5·5
	70	5·7	5·4	5·1
	75	5·0	4·8	4·6
45	45	8·3	7·4	6·7
	50	7·9	7·1	6·5
	55	7·4	6·7	6·2
	60	6·8	6·3	5·8
	65	6·3	5·8	5·4
	70	5·6	5·3	5·0
	75	4·9	4·7	4·5
50	50	7·6	6·8	6·2
	55	7·2	6·5	6·0
	60	6·7	6·1	5·7
	65	6·2	5·7	5·3
	70	5·5	5·2	4·9
	75	4·8	4·6	4·4
55	55	6·9	6·2	5·7
	60	6·5	5·9	5·5
	65	6·0	5·6	5·2
	70	5·4	5·1	4·8
	75	4·7	4·5	4·3
60	60	6·1	5·6	5·2
	65	5·7	5·3	4·9
	70	5·2	4·9	4·6
	75	4·6	4·4	4·2
65	65	5·4	5·0	4·7
	70	4·9	4·6	4·4
	75	4·4	4·2	4·0
70	70	4·6	4·4	4·2
	75	4·2	4·0	3·9
75	75	3·8	3·7	3·6

TABLE IV.

For the Value of an Annuity upon the Longer of Two Given Lives.

Age of Younger	Age of Elder	Value at 3 per cent.	Value at 4 per cent.	Value at 5 per cent.
10	10	23·4	19·9	17·1
	15	22·9	19·5	16·8
	20	22·5	19·1	16·6
	25	22·2	18·8	16·4
	30	21·9	18·6	16·2
	35	21·6	18·4	16·1
	40	21·4	18·3	16·0
	45	21·2	18·2	15·9
	50	20·9	18·0	15·8
	55	20·7	17·8	15·7
	60	20·4	17·6	15·5
	65	20·1	17·4	15·3
	70	19·8	17·2	15·1
	75	19·5	16·9	14·8
15	15	22·8	19·3	16·7
	20	22·3	18·9	16·4
	25	21·9	18·6	16·2
	30	21·6	18·3	16·0
	35	21·3	18·1	15·9
	40	21·1	17·9	15·7
	45	20·9	17·8	15·6
	50	20·7	17·6	15·4
	55	20·4	17·4	15·3
	60	20·1	17·2	15·2
	65	19·8	16·9	15·0
	70	19·4	16·6	14·7
	75	18·9	16·3	14·4
20	20	21·6	18·3	15·8
	25	21·1	17·9	15·5
	30	20·7	17·6	15·3
	35	20·4	17·4	15·1
	40	20·1	17·2	15·0
	45	19·9	17·0	14·9
	50	19·6	16·8	14·7
	55	19·4	16·6	14·5
	60	19·1	16·3	14·3
	65	18·7	16·0	14·1
	70	18·2	15·7	13·8
	75	17·7	15·3	13·5
25	25	20·3	17·4	15·1
	30	19·8	17·0	14·9
	35	19·4	16·7	14·7
	40	19·2	16·5	14·5
	45	18·9	16·3	14·3
	50	18·7	16·1	14·2
	55	18·4	15·9	14·0
	60	18·0	15·6	13·8
	65	17·6	15·3	13·6
	70	17·2	15·0	13·3
	75	16·7	14·6	12·9
30	30	19·3	16·6	14·5
	35	18·8	16·2	14·2
	40	18·4	15·9	14·0
	45	18·1	15·6	13·8
	50	17·8	15·4	13·6
	55	17·4	15·1	13·4
	60	17·0	14·8	13·2
	65	16·6	14·5	12·9
	70	16·1	14·1	12·6
	75	15·6	13·7	12·2
35	35	18·3	15·8	13·8
	40	17·8	15·4	13·5
	45	17·4	15·1	13·3
	50	17·1	14·8	13·1
	55	16·7	14·5	12·9
	60	16·3	14·2	12·7
	65	15·8	13·8	12·4
	70	15·3	13·4	12·0
	75	14·8	13·0	11·6
40	40	17·3	15·0	13·3
	45	16·8	14·6	13·0
	50	16·3	14·2	12·7
	55	15·9	13·9	12·4
	60	15·4	13·5	12·1
	65	14·9	13·1	11·8
	70	14·5	12·7	11·4
	75	14·0	12·3	11·0
45	45	16·2	14·2	12·8
	50	15·7	13·8	12·5
	55	15·2	13·4	12·1
	60	14·7	12·9	11·7
	65	14·1	12·5	11·4
	70	13·6	12·0	11·0
	75	13·1	11·6	10·6
50	50	15·0	13·3	12·1
	55	14·5	12·9	11·7
	60	13·9	12·4	11·3
	65	13·3	12·0	10·9
	70	12·8	11·5	10·5
	75	12·2	11·0	10·1
55	55	13·6	12·4	11·3
	60	13·0	11·9	10·9
	65	12·4	11·3	10·5
	70	11·8	10·8	10·0
	75	11·3	10·3	9·5
60	60	12·2	11·2	10·5
	65	11·5	10·6	10·0
	70	10·9	10·1	9·5
	75	10·3	9·5	9·0
65	65	10·7	10·0	9·4
	70	10·0	9·4	8·9
	75	9·3	8·7	8·3
70	70	9·2	8·6	8·2
	75	8·4	7·9	7·6
75	75	7·6	7·2	6·9

The uses of these tables may be exemplified in the following problems.

PROB. 1. *To find the Probability or Proportion of Chance, that a person of a Given Age continues in being a proposed number of years.*—Thus, suppose the age be 40, and the number of years proposed 15; then, to calculate by the table of the probabilities for London, in tab. 1. against 40 years stands 214, and against 55 years, the age to which the person must arrive, stands 120, which shews that, of 214 persons who attain to the age of 40, only 120 of them reach the age of 55, and consequently 94 die between the ages of 40 and 55: It is evident therefore that the odds for attaining the proposed age of 55, are as 120 to 94, or as 9 to 7 nearly.

PROB. 2. *To find the Value of an Annuity for a proposed Life.*—This problem is resolved from tab. 2, by looking against the given age, and under the proposed rate of interest; then the corresponding quantity shews the number of years-purchase required. For example, if the given age be 36, the rate of interest 4 per cent, and the proposed annuity L250. Then in the table it appears that the value is 12·1 years purchase, or 12·1 times L250, that is L3025.

After the same manner the answer will be found in any other case falling within the limits of the table. But as there may sometimes be occasion to know the values of lives computed at higher rates of interest than those in the table, the two following practical rules are subjoined; by which the problem is resolved independent of tables.

Rule 1. When the given age is not less than 45 years, nor greater than 85, subtract it from 92; then multiply the remainder by the perpetuity, and divide the product by the said remainder added to $2\frac{1}{2}$ times the perpetuity; so shall the quotient be the number of years purchase required. Where note, that by the *perpetuity* is meant the number of years purchase of the fee-simple; found by dividing 100 by the rate per cent at which interest is reckoned.

Ex. Let the given age be 50 years, and the rate of interest 10 per cent. Then subtracting 50 from 92, there remains 42; which multiplied by 10 the perpetuity, gives 420; and this divided by 67, the remainder increased by $2\frac{1}{2}$ times 10 the perpetuity, quotes 6·3 nearly, for the number of years purchase. Therefore, supposing the annuity to be L100, its value in present money will be L630.

Rule 2. When the age is between 10 and 45 years; take 8 tenths of what it wants of 45, which divide by the rate per cent increased by 1·2; then if the quotient be added to the value of a life of 45 years, found by the preceding rule, there will be obtained the number of years purchase in this case. For example, let the proposed age be 20 years, and the rate of interest 5 per cent. Here taking 20 from 45, there remains 25; $\frac{8}{10}$ of which is 20; which divided by 6·2, quotes 3·2; and this added to 9·8, the value of a life of 45, found by the former rule, gives 13 for the number of years purchase that a life of 20 ought to be valued at.

And the conclusions derived by these rules, Mr. Simpson adds, are so near the true values, computed

from real observations, as seldom to differ from them by more than $\frac{1}{10}$ or $\frac{2}{10}$ of one year's purchase.

The observations here alluded to, are those which are founded on the London bills of mortality. And a similar method of solution, accommodated to the Breslaw observations, will be as follows, viz. "Multiply the difference between the given age and 85 years by the perpetuity, and divide the product by 8 tenths of the said difference increased by double the perpetuity, for the answer." Which, from 8 to 80 years of age, will commonly come within less than $\frac{1}{5}$ of a year's purchase of the truth.

PROB. 3. *To find the Value of an Annuity for Two Joint Lives, that is, for as long as they both continue in being together.*—In table 3, find the younger age, or that nearest to it, in column 1, and the higher age in column 2; then against this last is the number of years purchase in the proper column for the interest. *Ex.* Suppose the two ages be 20 and 35 years; then the value

is 10·9 years purchase at 3 per cent.
or 9·8 - - at 4 per cent.
or 8·8 - - at 5 per cent.

PROB. 4. *To find the Value of the Annuity for the* Longest *of Two Lives, that is, for as long as* either of them *continues in being.*—In table 4, find the age of the youngest life, or the nearest to it, in col. 1, and the age of the elder in col. 2; then against this last is the answer in the proper column of interest.—*Ex.* So, if the two ages be 15 and 40; then the value of the annuity upon the longest of two such lives,

is 21·1 years purchase at 3 per cent.
or 17·9 - - 4 per cent.
or 15·7 - - 5 per cent.

N B. In the last two problems, if the younger age, or the rate of interest, be not exactly found in the tables, the nearest to them may be taken, and then by proportion the value for the true numbers will be nearly found.

Rules and tables for the values of three lives, &c, may also be seen in Simpson, and in Baron Maseres's Annuities, &c. All these calculations have been made from tables of the real mortuary registers, differing unequally at the several ages. But rules have also been given upon other principles, as by De Moivre, upon the supposition that the decrements of life are equal at all ages; an assumption not much differing from the truth, from 7 to 70 years of age.

Life-Annuities, payable half-yearly, &c.—These are worth more than such as are payable yearly, as computed by the foregoing rules and tables, on the two following accounts: First, that parts of the payments are received sooner; and 2dly, there is a chance of receiving some part or parts of a whole year's payment more than when the payments are only made annually. Mr. Simpson, in his Select Exercises, pa. 283, observes, that the value of these two advantages put together, will always amount to $\frac{1}{4}$ of a year's purchase for half-yearly payments, and to $\frac{3}{8}$ of a year's purchase for quarterly payments; and Mr. Maseres, at page 233 &c of his Annuities, by a very elaborate calculation, finds the former difference to be nearly $\frac{1}{4}$ also. But Dr. Price, in an Essay in the Philos. Trans. vol. 66, pa. 109,

pa. 109, ftates the fame differences only

at $\frac{2}{10}$ for half-yearly payments,

and $\frac{3}{10}$ for quarterly payments :

And the Doctor then adds fome algebraical theorems for fuch calculations.

LIFE-ANNUITIES, *fecured by Land.*—Thefe differ from other life-annuities only in this, that the annuity is to be paid up to the very day of the death of the age in queftion, or of the perfon upon whofe life the annuity is granted. To obtain the more exact value therefore of fuch an annuity, a fmall quantity muft be added to the fame as computed by the foregoing rules and obfervations, which is different according as the payments are yearly, half-yearly, or quarterly, &c; and are thus ftated by Dr. Price in his Effay quoted above; viz, the addition

is $\frac{y}{2n}$ for annual payments,

or $\frac{h}{4n}$ for half-yearly payments,

or $\frac{q}{8n}$ for quarterly payments :

where n is the complement of the given age, or what it wants of 86 years; and y, h, q are the refpective values of an annuity *certain* for n years, payable yearly, half-yearly, or quarterly. And, by numeral examples, it is found that the firft of thefe additional quantities is about $\frac{2}{10}$, the fecond $\frac{1}{10}$, and the 3d half a tenth of one year's purchafe.

Complement of LIFE. See COMPLEMENT.

Expectation of LIFE. See EXPECTATION.

Infurance or Affurance on LIVES. See ASSURANCES *on Lives.*

LIGHT, that principle by which objects are made perceptible to our fenfe of feeing; or the fenfation occafioned in the mind by the view of luminous objects.

The nature of Light has been a fubject of fpeculation from the firft dawnings of philofophy. Some of the earlieft philofophers doubted whether objects became vifible by means of any thing proceeding from them, or from the eye of the fpectator. But this opinion was qualified by Empedocles and Plato, who maintained, that vifion was occafioned by particles continually flying off from the furfaces of bodies, which meet with others proceeding from the eye; while the effect was afcribed by Pythagoras folely to the particles proceeding from the external objects, and entering the pupil of the eye. But Ariftotle defines Light to be the act of a tranfparent body, confidered as fuch: and he obferves that Light is not fire, nor yet any matter radiating from the luminous body, and tranfmitted through the tranfparent one.

The Cartefians have refined confiderably on this notion; and hold that Light, as it exifts in the luminous body, is only a power or faculty of exciting in us a very clear and vivid fenfation; or that it is an invifible fluid prefent at all times and in all places, but requiring to be fet in motion, by a body ignited or otherwife properly qualified to make objects vifible to us.

Father Malbranche explains the nature of Light from a fuppofed analogy between it and found.—Thus he fuppofes all the parts of a luminous body are in a rapid motion, which, by very quick pulfes, is conftantly compreffing the fubtle matter between the luminous body and the eye, and excites vibrations of preffion. As thefe vibrations are greater, the body appears more luminous; and as they are quicker or flower, the body is of this or that colour.

But the Newtonians maintain, that Light is not a fluid *per fe*, but confifts of a great number of very fmall particles, thrown off from the luminous body by a repulfive power with an immenfe velocity, and in all directions. And thefe particles, it is alfo held, are emitted in right lines: which rectilinear motion they preferve till they are turned out of their path by fome of the following caufes, viz, by the attraction of fome other body near which they pafs, which is called *inflection*; or by paffing obliquely through a medium of different denfity, which is called *refraction*; or by being turned afide by the oppofition of fome intervening body, which is called *reflection*; or, laftly, by being totally ftopped by fome fubftance into which they penetrate, and which is called their *extinction*. A fucceffion of thefe particles following one another, in an exact ftraight line, is called a *ray of Light*; and this ray, in whatever manner its direction may be changed, whether by refraction, reflection, or inflection, always preferves a rectilinear courfe till it be again changed; neither is it poffible to make it move in the arch of a circle, ellipfis, or other curve. For the above properties of the rays of Light, fee the feveral words, REFRACTION, REFLECTION, &c.

The *velocity* of the particles and rays of Light is truly aftonifhing, amounting to near 2 hundred thoufand miles in a fecond of time, which is near a million times greater than the velocity of a cannon-ball. And this amazing motion of Light has been manifefted in various ways, and firft, from the eclipfes of Jupiter's fatellites. It was firft obferved by Roemer, that the eclipfes of thofe fatellites happen fometimes fooner, and fometimes later, than the times given by the tables of them; and that the obfervation was before or after the computed time, according as the earth was nearer to, or farther from Jupiter, than the mean diftance. Hence Roemer and Caffini both concluded that this circumftance depended on the diftance of Jupiter from the earth; and that, to account for it, they muft fuppofe that the Light was about 14 minutes in croffing the earth's orbit. This conclufion however was afterward abandoned and attacked by Caffini himfelf. But Roemer's opinion found an able advocate in Dr. Halley; who removed Caffini's difficulty, and left Roemer's conclufion in its full force. Yet, in a memoir prefented to the Academy in 1707, M. Maraldi endeavoured to ftrengthen Caffini's arguments; when Roemer's doctrine found a new defender in Mr. Pound. See Philof. Tranf. number 136, alfo Abridg. vol. 1, pa. 409 and 422, and Groves, Phyf. Elem. number 2636. It has fince been found, by repeated experiments, that when the earth is exactly between Jupiter and the fun, his fatellites are feen eclipfed about $8\frac{1}{4}$ minutes *fooner* than they could be according to the tables; but when the earth is nearly in the oppofite point of its orbit, thefe eclipfes happen about $8\frac{1}{4}$ minutes *later* than the tables predict them. Hence

then

then it is certain that the motion of Light is not inftantaneous, but that it takes up about $16\frac{1}{2}$ minutes of time to pafs over a fpace equal to the diameter of the earth's orbit, which is at leaft 190 millions of miles in length, or at the rate of near 200,000 miles per fecond, as above-mentioned. Hence therefore Light takes up about $8\frac{1}{4}$ minutes in paffing from the fun to the earth; fo that, if he fhould be annihilated, we would fee him for $8\frac{1}{4}$ minutes after that event fhould happen; and if he were again created, we fhould not fee him till $8\frac{1}{4}$ minutes afterwards. Hence alfo it is eafy to know the time in which Light travels to the earth, from the moon, or any of the other planets, or even from the fixed ftars when their diftances fhall be known; thefe diftances however are fo immenfely great, that from the neareft of them, fuppofed to be Sirius, the dog-ftar, Light takes up many years to travel to the earth: and it is even fufpected that there are many ftars whofe Light have not yet arrived at us fince their creation. And this, by-the-bye, may perhaps fometimes account for the appearance of new ftars in the heavens.

It may be juft obferved that Galileo firft conceived the notion of meafuring the velocity of Light; and a defcription of his contrivance for this purpofe, is in his Treatife on Mechanics, pa. 39. He had two men with Lights covered; the one was to obferve when the other uncovered his Light, and to exhibit his own the moment he perceived it. This rude experiment was tried at the diftance of a mile, but without fuccefs, as may naturally be imagined: and the members of the Academy Del Cimento repeated the experiment, and placed their obfervers, to as little purpofe, at the diftance of 2 miles.

But our excellent aftronomer, Dr. Bradley, afterwards found nearly the fame velocity of Light as Roemer, from his accurate obfervations, and moft ingenious theory, to account for fome apparent motions in the fixed ftars; for an account of which, fee Aberration of Light. By a long feries of thefe obfervations, he found the difference between the true and apparent place of feveral fixed ftars, for different times of the year; which difference could no otherwife be accounted for, than from the progreffive motion of the rays of Light. From the mean quantity of this difference he ingenioufly found, that the ratio of the velocity of Light to the velocity of the earth in its orbit, was as 10313 to 1, or that Light moves 10313 times fafter than the earth moves in its orbit about the fun; and as this latter motion is at the rate of $18\frac{1}{11}$ miles per fecond nearly, it follows that the former, or the velocity of Light, is at the rate of about 195000 miles in a fecond; a motion according to which it will require juft $8' 7''$ to move from the fun to the earth, or about 95 millions of miles.

It was alfo inferred, from the foregoing principles, that Light proceeds with the fame velocity from all the ftars. And hence it follows, if we fuppofe that all the ftars are not equally diftant from us, as many arguments prove, that the motion of Light, all the way it paffes through the immenfe fpace above our atmofphere, is equable or uniform. And fince the different methods of determining the velocity of Light thus agree in the refult, it is reafonable to conclude that, in the fame medium, Light is propagated with the fame velocity after it has been reflected, as before.

For an account of Mr. Melville's hypothefis of the different velocities of differently coloured rays, fee Colour.

To the doctrine concerning the materiality of Light, and its amazing velocity, feveral objections have been made; of which the moft confiderable is, That as rays of Light are continually paffing in different directions from every vifible point, they muft neceffarily interfere with each other in fuch a manner, as entirely to confound all diftinct perception of objects, if not quite to deftroy the whole fenfe of feeing: not to mention the continual wafte of fubftance which a conftant emiffion of particles muft occafion in the luminous body, and thereby fince the creation muft have greatly diminifhed the matter in the fun and ftars, as well as increafed the bulk of the earth and planets by the vaft quantity of particles of Light abforbed by them in fo long a period of time.

But it has been replied, that if Light were not a body, but confifted in mere preffion or pulfion, it could never be propagated in right lines, but would be continually inflected ad umbram. Thus Sir I. Newton: "A preffure on a fluid medium, i. e. a motion propagated by fuch a medium, beyond any obftacle, which impedes any part of its motion, cannot be propagated in right lines, but will be always inflecting and diffufing itfelf every way, to the quiefcent medium beyond that obftacle. The power of gravity tends downwards; but the preffure of water arifing from it tends every way with an equable force, and is propagated with equal eafe and equal ftrength, in curves, as in ftrait lines. Waves, on the furface of the water, gliding by the extremes of any very large obftacle, inflect and dilate themfelves, ftill diffufing gradually into the quiefcent water beyond that obftacle. The waves, pulfes, or vibrations of the air, wherein found confifts, are manifeftly inflected, though not fo confiderably as the waves of water; and founds are propagated with equal eafe, through crooked tubes, and through ftrait lines; but Light was never known to move in any curve, nor to inflect itfelf ad umbram."

It muft be acknowledged, however, that many philofophers, both Englifh and Foreigners, have recurred to the opinion, that Light confifts of vibrations propagated from the luminous body, through a fubtle etherial medium.

The ingenious Dr. Franklin, in a letter dated April 23, 1752, expreffes his diffatisfaction with the doctrine, that Light confifts of particles of matter continually driven off from the fun's furface, with fo enormous a fwiftnefs. "Muft not, fays he, the fmalleft portion conceivable, have, with fuch a motion, a force exceeding that of a 24 pounder difcharged from a cannon? Muft not the fun diminifh exceedingly by fuch a wafte of matter; and the planets, inftead of drawing nearer to him, as fome have feared, recede to greater diftances through the leffened attraction? Yet thefe particles, with this amazing motion, will not drive before them, or remove, the leaft and flighteft duft they meet with; and the fun appears to continue of his ancient dimenfions, and his attendants move in their ancient orbits." He therefore conjectures that all the phenomena of

Light may be more properly solved, by supposing all space filled with a subtle elastic fluid, which is not visible when at rest, but which, by its vibrations, affects that fine sense in the eye, as those of the air affect the grosser organs of the ear; and even that different degrees of the vibration of this medium may cause the appearances of different colours. Franklin's Exper. and Observ. 1769, pa. 264.

The celebrated Euler has also maintained the same hypothesis, in his Theoria Lucis & Colorum. In the summary of his arguments against the common opinion, recited in Acad. Berl. 1752, pa. 271, besides the objections above-mentioned, he doubts the possibility, that particles of matter, moving with the amazing velocity of Light, should penetrate transparent substances with so much ease. In whatever manner they are transmitted, those bodies must have pores, disposed in right lines, and in all possible directions, to serve as canals for the passage of the rays: but such a structure must take away all solid matter from those bodies, and all coherence among their parts, if they do contain any solid matter.

Doctor Horsley, now Bp. of Rochester, has taken considerable pains to obviate the difficulties started by Dr. Franklin. Supposing that the diameter of each particle of Light does not exceed one millionth of one millionth of an inch, and that the density of each particle is even three times that of iron, that the Light of the sun reaches the earth in 7′, at the distance of 22919 of the earth's semidiameters, he calculates that the momentum or force of motion in each particle of Light coming from the sun, is less than that in an iron ball of a quarter of an inch diameter, moving at the rate of less than an inch in 12 thousand millions of millions of years. And hence he concludes, that a particle of matter, which probably is larger than any particle of Light, moving with the velocity of Light, has a force of motion, which, instead of exceeding the force of a 24 pounder discharged from a cannon, is almost infinitely less than that of the smallest shot discharged from a pocket pistol, or less than any that art can create. He also thinks it possible, that Light may be produced by a continual emission of matter from the sun, without any such waste of his substance as should sensibly contract his dimensions, or alter the motions of the planets, within any moderate length of time. In proof of this, he observes that, for the production of any of the phenomena of Light, it is not necessary that the emanation from the sun should be continual, in a strict mathematical sense, or without any interval; and likewise that part of the Light which issues from the sun, is continually returned to him by reflection from the planets, as well as other Light from the suns of other systems. He proceeds, by calculation, to shew that in 385,130,000 years, the sun would lose but the 13232d part of his matter, and consequently of the gravitation towards him, at any given distance; which is an alteration much too small to discover itself in the motion of the earth, or of any of the planets. He farther computes that the greatest stroke which the retina of a common eye sustains, when turned directly to the sun in a bright day, does not exceed that which would be given by an iron shot, a quarter of an inch diameter, and moving only at the rate of $16\frac{1}{6}$ inches in a year; whereas the ordinary stroke is less than the 2084th part of this. See Philos. Trans. vol. 60 and 61.

In answer to the difficulty respecting the non-interference of the particles of Light with each other, Mr. Melville observes (Edinb. Ess. vol. 2), there is probably no physical point in the visible horizon, that does not send rays to every other point, unless where opaque bodies interpose. Light, in its passage from one system to another, often passes through torrents of Light issuing from other suns and systems, without ever interfering, or being diverted from its course, either by it, or by the particles of that elastic medium, which it has been supposed by some is diffused through all the mundane space. To account for this fact, he supposes that the particles of Light are incomparably rare, even when they are the most dense, or that their diameters are incomparably less than their distance from one another: which obviates the objection urged by Euler and others against the materiality of Light, from its influence in disturbing the freedom and perpetuity of the celestial motions. Boscovich and some others solve the difficulty concerning the non-interference of the particles of Light, by supposing that each particle is endued with an insuperable impulsive force; but in this case, their spheres of impulsion would be more likely to interfere, and on that account they be more liable to disturb one another.

M. Canton shews (Philos. Trans. vol. 58, p. 344), that the difficulty of the interference will vanish, if a very small portion of time be allowed between the emission of every particle and the next that follows in the same direction. Suppose, for instance, that a lucid point in the sun's surface emits 150 particles in a second of time, which, he observes, will be more than sufficient to give continual Light to the eye, without the least appearance of intermission; yet still the particles of such a ray, on account of their great velocity, will be more than 1000 miles behind each other, a space sufficient to allow others to pass in all directions without any perceptible interruption. And if we adopt the conclusions drawn from the experiments on the duration of the sensations excited by Light, by the chevalier D'Arcy, in the Acad. Scienc. 1765, who states it at the 7th part of a second, an interval of more than 20,000 miles may be admitted between every two successive particles.

The doctrine of the materiality of Light is farther confirmed by those experiments, which shew, that the colour and inward texture of some bodies are changed by being exposed to the Light.

Of the Momentum, or Force, of the Particles of Light. Some writers have attempted to prove the materiality of Light, by determining the momentum of their component particles, or by shewing that they have a force so as, by their impulse, to give motion to light bodies. M. Homberg, Ac. Par. 1708, Hist. pa. 25, imagined, that he could not only disperse pieces of amianthus, and other light substances, by the impulse of the solar rays, but also that by throwing them upon the end of a kind of lever, connected with the spring of a watch, he could make it move sensibly quicker; from which, and other experiments, he inferred the weight of the particles of Light. And Hartsoecker made preten-

 sions

ſions of the ſame nature. But M. Du Fay and M. Mairan made other experiments of a more accurate kind, without the effects which the former had imagined, and which even proved that the effects mentioned by them were owing to currents of heated air produced by the burning glaſſes uſed in their experiments, or ſome other cauſes which they had overlooked.

However, Dr. Prieſtley informs us, that Mr. Michell endeavoured to aſcertain the momentum of Light with ſtill greater accuracy, and that his endeavours were not altogether without ſucceſs. Having found that the inſtrument he uſed, acquired, from the impulſe of the rays of light, a velocity of an inch in a ſecond of time, he inferred that the quantity of matter contained in the rays falling upon the inſtrument in that time, amounted to no more than the 12 hundred millionth part of a grain. In the experiment, the Light was collected from a ſurface of about 3 ſquare feet; and as this ſurface reflected only about the half of what fell upon it, the quantity of matter contained in the ſolar rays, incident upon a ſquare foot and a half of ſurface, in a ſecond of time, ought to be no more than the 12 hundred millionth part of a grain, or upon one ſquare foot only, the 18 hundred millionth part of a grain. But as the denſity of the rays of Light at the ſurface of the ſun, is 45000 times greater than at the earth, there ought to iſſue from a ſquare foot of the ſun's ſurface, in one ſecond of time, the 40 thouſandth part of a grain of matter; that is, a little more than 2 grains a day, or about 4,752,000 grains, which is about 670 pounds avoirdupois, in 6000 years, the time ſince the creation; a quantity which would have ſhortened the ſun's ſemidiameter by no more than about 10 feet, if it be ſuppoſed of no greater denſity than water only.

The *Expanſion* or *Extenſion* of any portion of Light, is inconceivable. Dr. Hook ſhews that it is as unlimited as the univerſe; which he proves from the immenſe diſtance of many of the fixed ſtars, which only become viſible to the eye by the beſt teleſcopes. Nor, adds he, are they only the great bodies of the ſun or ſtars that are thus liable to diſperſe their Light through the vaſt expanſe of the univerſe, but the ſmalleſt ſpark of a lucid body muſt do the ſame, even the ſmalleſt globule ſtruck from a ſteel by a flint.

The *Intenſity* of different Lights, or of the ſame Light in different circumſtances, affords a curious ſubject of ſpeculation. M. Bouguer, Traité de Optique, found that when one Light is from 60 to 80 times leſs than another, its preſence or abſence will not be perceived by an ordinary eye; that the moon's Light, when ſhe is 19° 16′ high above the horizon, is but about $\frac{1}{3}$ of her Light at 66° 11′ high; and when one limb juſt touched the horizon, her Light was but the 2000th part of her Light at 66° 11′ high; and that hence Light is diminiſhed in the proportion of 3 to 1 by traverſing 7469 toiſes of denſe air. He found alſo, that the centre of the ſun's diſc is conſiderably more luminous than the edges of it; whereas both the primary and ſecondary planets are more luminous at their edges than near their centres: That, farther, the Light of the ſun is about 300,000 times greater than that of the moon; and therefore it is no wonder that philoſophers have had ſo little ſucceſs in their attempts to collect the Light of the moon with burning-glaſſes; for, ſhould one of the largeſt of them even increaſe the Light 1000 times, it will ſtill leave the Light of the moon in the focus of the glaſs, 300 times leſs than the intenſity of the common Light of the ſun.

Dr. Smith, in his Optics, vol. 1, pa. 29, thought he had proved that the Light of the full moon would be only the 90,900th part of the full day Light, if no rays were loſt at the moon. But Mr. Robins, in his Tracts, vol. 2, pa. 225, ſhews that this is too great by one half. And Mr. Michell, by a more eaſy and accurate mode of computation, found that the denſity of the ſun's Light on the ſurface of the moon is but the 45,000th part of the denſity at the ſun; and that therefore, as the moon is nearly of the ſame apparent magnitude as the ſun, if ſhe reflected to us all the Light received on her ſurface, it would be only the 45,000th part of our day Light, or that which we receive from the ſun. Admitting therefore, with M. Bouguer, that the moon Light is only the 300,000th part of the day or ſun's Light, Mr. Michell concludes that the moon reflects no more than between the 6th and 7th part of what ſhe receives.

Dr. Graveſande ſays, a lucid body is that which emits or gives fire a motion in right lines, and makes the difference between Light and heat to conſiſt in this, that to produce the former, the fiery particles muſt enter the eye in a rectilinear motion, which is not required in the latter: on the contrary, an irregular motion ſeems more proper for it, as appears from the rays coming directly from the ſun to the tops of mountains, which have not near that effect with thoſe in the valley, agitated with an irregular motion, by ſeveral reflections.

Sir I. Newton obſerves, that bodies and Light act mutually on one another; bodies on Light, in emitting, reflecting, refracting, and inflecting it; and Light on bodies, by heating them, and putting their parts into a vibrating motion, in which heat principally conſiſts. For all fixed bodies, he obſerves, when heated beyond a certain degree, do emit Light, and ſhine; which ſhining &c appears to be owing to the vibrating motion of their parts; and all bodies, abounding in earthy and ſulphureous particles, if ſufficiently agitated, emit Light, which way ſoever that agitation be effected. Thus, ſea water ſhines in a ſtorm; quickſilver, when ſhaken in vacuo; cats or horſes, when rubbed in the dark; and wood, fiſh, and fleſh, when putrefied.

Light proceeding from putreſcent animal and vegetable ſubſtances, as well as from glow-worms, is mentioned by Ariſtotle. And Bartholin mentions four kinds of luminous inſects, two of which have wings: but in hot climates it is ſaid they are found in much greater numbers, and of different ſpecies. Columna obſerves, that their Light is not extinguiſhed immediately on the death of the animal. The firſt diſtinct account that occurs of Light proceeding from putreſcent animal fleſh, is that which is given by Fabricius ab Aquapendente in 1592, de Viſione &c, pa. 45. And Bartholin gives an account of a ſimilar appearance, which happened at Montpelier in 1641, in his treatiſe De Luce Animalium.

Mr. Boyle ſpeaks of a piece of ſhining rotten wood, which was extinguiſhed in vacuo; but upon re-admitting the air, it revived again, and ſhone as before; though

though he could not perceive that it was increafed in condenfed air. But in Birch's Hiftory of the Royal Soc. vol. 2, pa. 254, there is an account of the Light of a fhining fifh, which was rendered more vivid by putting the fifh into a condenfing engine. The fifh called Whitings were thofe commonly ufed by Mr. Boyle in his experiments: though in a difcourfe read before the R. Soc. in 1681, it was afferted that, of all fifhy fubftances, the eggs of lobfters, after they had been boiled, fhone the brighteft. Birch's Hift. vol. 2, pa. 70. In 1672 Mr. Boyle accidentally obferved Light iffuing from flefh meat; and, among other remarks on this fubject, he obferves that extreme cold extinguifhes the Light of fhining wood; probably becaufe extreme cold checks the putrefaction, which is the caufe of the Light. The fhell fifh called Pholas, is remarkable for its luminous quality. The *luminoufnefs of the Sea* has been alfo a fubject of frequent obfervation. See *Ignis fatuus*, *Phofphorus*, and *Putrefaction*, &c.

Mr. Hawkfbee, and many writers on the fubject of electricity fince his time, have produced a great variety of inftances of the artificial production of Light, by the attrition of bodies naturally not luminous; as of amber rubbed on woollen cloth in vacuo; of glafs on woollen, of glafs on glafs, of oyfter fhells on woollen, and of woollen on woollen, all in vacuo. On the feveral experiments of this kind, he makes thefe following reflections: that different forts of bodies afford Light of various kinds, different both in colour and in force; that the effects of an attrition are various, according to the different preparations and treatment of the bodies that are to endure it; and that bodies which have yielded a particular Light, may be brought by friction to yield no more of that Light.

M. Bernoulli found by experiment, that mercury amalgamated with tin, and rubbed on glafs, produced a confiderable Light in the air; that gold rubbed on glafs, exhibited the fame in a greater degree; but that the moft exquifite Light of all was produced by the attrition of a diamond, this being equally vivid with that of a burning coal brifkly agitated with the bellows. See Electricity, &c.

Of the Attraction of Light. That the particles of Light are attracted by thofe of other bodies, is evident from numerous experiments. This phenomenon was obferved by Sir I. Newton, who found, by repeated trials, that the rays of Light, in their paffage near the edges of bodies, are diverted out of the right lines, and always inflected or bent towards thofe bodies, whether they be opaque or tranfparent, as pieces of metals, the edges of knives, broken glaffes, &c. See Inflection and Rays. The curious obfervations that had been made on this fubject by Dr. Hook and Grimaldi, led Sir I. Newton to repeat and diverfify their experiments, and to purfue them much farther than they had done. For a particular account of his experiment and obfervations, fee his treatife on Optics, pa. 293 &c.

This action of bodies on Light is found to exert itfelf at a fenfible diftance, though it always increafes as the diftance is diminifhed; as appears very fenfibly in the paffage of a ray between the edges of two thin planes at different apertures; which is attended with this peculiar circumftance, that the attraction of one edge is increafed as the other is brought nearer it. The rays of Light, in their paffage out of glafs into a vacuum, are not only inflected towards the glafs, but if they fall too obliquely, they will revert back again to the glafs, and be totally reflected. Now the caufe of this reflection cannot be attributed to any refiftance of the vacuum, but muft be entirely owing to fome force or power in the glafs, which attracts or draws back the rays as they were paffing into the vacuum. And this appears farther from hence, that if you wet the back furface of the glafs with water, oil, honey, or a folution of quickfilver, then the rays which would otherwife have been reflected, will pervade and pafs through that liquor; which fhews that the rays are not reflected till they come to that back furface of the glafs, nor even till they begin to go out of it; for if, at their going out, they fall into any of the aforefaid mediums, they will not then be reflected, but will perfift in their former courfe, the attraction of the glafs being in this cafe counterbalanced by that of the liquor.

M. Maraldi profecuted experiments fimilar to thofe of Sir I. Newton on inflected Light. And his obfervations chiefly refpect the inflection of Light towards other bodies, by which their fhadows are partially illuminated. Acad. Paris 1723, Mem. p. 159. See alfo Prieftley's Hift. pa. 521 &c.

M. Mairan, without attempting the difcovery of new facts, endeavoured to explain the old ones, by the hypothefis of an atmofphere furrounding all bodies; and confequently two reflections and refractions of Light that impinges upon them, one at the furface of the atmofphere, and the other at the furface of the body itfelf. This atmofphere he fuppofed to be of a variable denfity and refractive power, like the air.

M. Du Tour fucceeded Mairan, and imagined that he could account for all the phenomena by the help of an atmofphere of an uniform denfity, but of a lefs refractive power than the air furrounding all bodies. Du Tour alfo varied the Newtonian experiments, and difcovered more than three fringes in the colours produced by the inflection of light. He farther concludes that the refracting atmofpheres, furrounding all kinds of bodies, are of the fame fize; for when he ufed a great variety of fubftances, and of different fizes too, he always found coloured ftreaks of the fame dimenfions. He alfo obferves, that his hypothefis contradicts an obfervation of Sir I. Newton, viz, that thofe rays are the moft inflected which pafs the neareft to any body. Mem. de Math. & de Phyf. vol. 5, pa. 650, or Prieftley's Hift. pa. 531.

M. Le Cat found that objects fometimes appear magnified by means of the inflection of Light. Looking at a diftant fteeple, when a wire, of a lefs diameter than the pupil of his eye, was held pretty near to it, and drawing it feveral times between that object and his eye, he was furprifed to find that every time the wire paffed before his eye, the fteeple feemed to change its place, and fome hills beyond the fteeple feemed to have the fame motion, juft as if a lens had been drawn between them and his eye. This difcovery led him to feveral others depending on the inflection of the rays of Light. Thus, he magnified fmall objects, as the head of a pin, by viewing them through a fmall hole in a card; fo that the rays which formed the image muft neceffarily

neceffarily pafs fo near the circumference of the hole, as to be attracted by it. He exhibited alfo other appearances of a fimilar nature. Traité des Sens, pa. 299. Prieftley, ubi fupra, pa. 537.

Reflection and Refraction of Light. From the mutual attraction between the particles of Light and other bodies, arife two other grand phenomena, befides the inflection of Light, which are called the reflection and refraction of Light. It is well known that the determination of bodies in motion, efpecially elaftic ones, is changed by the interpofition of other bodies in their way: thus alfo Light, impinging on the furfaces of bodies, fhould be turned out of its courfe, and beaten back or reflected, fo as, like other ftriking bodies, to make the angle of its reflection equal to the angle of incidence. This, it is found by experience, Light does; and yet the caufe of this effect is different from that juft now affigned: for the rays of Light are not reflected by ftriking on the very parts of the reflecting bodies, but by fome power equally diffufed over the whole furface of the body, by which it acts on the Light, either attracting or repelling it, without contact: by which fame power, in other circumftances, the rays are refracted; and by which alfo the rays are firft emitted from the luminous body; as Newton abundantly proves by a great variety of arguments. See Reflection and Refraction.

That great author puts it paft doubt, that all thofe rays which are reflected, do not really touch the body, though they approach it infinitely near; and that thofe which ftrike on the parts of folid bodies, adhere to them, and are as it were extinguifhed and loft. Since the reflection of the rays is afcribed to the action of the whole furface of the body without contact, if it be afked, how it happens that all the rays are not reflected from every furface; but that, while fome are reflected, others pafs through, and are refracted? the anfwer given by Newton is as follows:—Every ray of Light, in its paffage through any refracting furface, is put into a certain tranfient conftitution or ftate, which in the progrefs of the ray returns at equal intervals, and difpofes the ray at every return to be eafily tranfmitted through the next refracting furface, and between the returns to be eafily reflected by it: which alteration of reflection and tranfmiffion it appears is propagated from every furface, and to all diftances. What kind of action or difpofition this is, and whether it confifts in a circulating or vibrating motion of the ray, or the medium, or fomething elfe, he does not enquire; but allows thofe who are fond of hypothefes to fuppofe, that the rays of Light, by impinging on any reflecting or refracting furface, excite vibrations in the reflecting or refracting medium, and by that means agitate the folid parts of the body. Thefe vibrations, thus produced in the medium, move fafter than the rays, fo as to overtake them; and when any ray is in that part of the vibration which confpires with its motion, its velocity is increafed, and fo it eafily breaks through a refracting furface; but when it is in a contrary part of the vibration, which impedes its motion, it is eafily reflected; and thus every ray is fucceffively difpofed to be eafily reflected or tranfmitted by every vibration which meets it. Thefe returns in the difpofition of any ray to be reflected, he calls *fits of eafy reflection;* and the returns in the difpofition to be tranfmitted, he calls *fits of eafy tranfmiffion;* alfo the fpace between the returns, *the interval of the fits.* Hence then the reafon why the furfaces of all thick tranfparent bodies reflect part of the Light incident upon them, and refract the reft, is that fome rays at their incidence are in fits of eafy reflection, and others of eafy tranfmiffion. For *the properties of reflected Light,* fee Reflection, Mirror, &c.

Again, a ray of Light, paffing out of one medium into another of different denfity, and in its paffage making an oblique angle with the furface that feparates the mediums, will be refracted, or turned out of its direction; becaufe the rays are more ftrongly attracted by a denfer than by a rarer medium. That thefe rays are not refracted by ftriking on the folid parts of bodies, but that this is effected without a real contact, and by the fame force by which they are emitted and reflected, only exerting itfelf differently in different circumftances, is proved in a great meafure by the fame arguments by which it is demonftrated that reflection is performed without contact. See Refraction, Lens, Colour, Vision, &c.

LIGHTNING, a large bright flame, fhooting fwiftly through the atmofphere, of momentary or very fhort duration, and commonly attended with thunder.

Some philofophers accounted for this awful natural phenomenon in this manner, viz, that an inflammable fubftance is formed of the particles of fulphur, nitre, and other combuftible matter, which are exhaled from the earth, and carried into the higher regions of the atmofphere, and that by the collifion of two clouds, or otherwife, this fubftance takes fire, and darts out into a train of Light, larger or fmaller according to the ftrength and quantity of the materials. And others have explained the phenomenon of Lightning by the fermentation of fulphureous fubftances with nitrous acids. See Thunder.

But it is now univerfally allowed, that Lightning is really an electrical explofion or phenomenon. Philofophers had not proceeded far in their experiments and enquiries on this fubject, before they perceived the obvious analogy between Lightning and electricity, and they produced many arguments to evince their fimilarity. But the method of proving this hypothefis beyond a doubt, was firft propofed by Dr. Franklin, who, about the clofe of the year 1749, conceived the practicability of drawing Lightning down from the clouds. Various circumftances of refemblance between Lightning and electricity were remarked by this ingenious philofopher, and have been abundantly confirmed by later difcoveries, fuch as the following: Flafhes of Lightning are ufually feen crooked and waving in the air; fo the electric fpark drawn from an irregular body at fome diftance, and when it is drawn by an irregular body, or through a fpace in which the beft conductors are difpofed in an irregular manner, always exhibits the fame appearance: Lightning ftrikes the higheft and moft pointed objects in its courfe, in preference to others, as hills, trees, fpires, mafts of fhips, &c; fo all pointed conductors receive and throw off the electric fluid more readily than thofe that are terminated by flat furfaces: Lightning is obferved to take and follow the readieft and beft conductor; and the fame is the cafe with electricity in the difcharge of the Leyden

8 phial;

phial; from whence the doctor infers, that in a thunder-ſtorm, it would be ſafer to have one's cloaths wet than dry: Lightning burns, diſſolves metals, rends ſome bodies, ſometimes ſtrikes perſons blind, deſtroys animal life, deprives magnets of their virtue, or reverſes their poles; and all theſe are well-known properties of electricity.

But Lightning alſo gives polarity to the magnetic needle, as well as to all bodies that have any thing of iron in them, as bricks &c; and by obſerving afterwards which way the magnetic poles of theſe bodies lie, it may thence be known in what direction the ſtroke paſſed. Perſons are ſometimes killed by Lightning, without exhibiting any viſible marks of injury; and in this caſe Sig. Beccaria ſuppoſes that the Lightning does not really touch them, but only produces a ſudden vacuum near them, and the air ruſhing violently out of their lungs to ſupply it, they cannot recover their breath again: and in proof of this opinion he alleges, that the lungs of ſuch perſons are found flaccid; whereas theſe are found inflated when the perſons are really killed by the electric ſhock. Though this hypotheſis is controverted by Dr. Prieſtley.

To demonſtrate however, by actual experiment, the identity of the electric fluid with the matter of Lightning, Dr. Franklin contrived to bring Lightning from the heavens, by means of a paper kite, properly fitted up for the purpoſe, with a long fine wire ſtring, and called an electrical kite, which he raiſed when a thunder-ſtorm was perceived to be coming on: and with the electricity thus obtained, he charged phials, kindled ſpirits, and performed all other ſuch electrical experiments as are uſually exhibited by an excited glaſs globe or cylinder. This happened in June 1752, a month after the electricians in France, in purſuance of the method which he had before propoſed, had verified the ſame theory, but without any knowledge of what they had done. The moſt active of theſe were Meſſrs. Dalibard and Delor, followed by M. Mazeas and M. Monnier.

In April and June 1753, Dr. Franklin diſcovered that the air is ſometimes electrified negatively, as well as ſometimes poſitively; and he even found that the clouds would change from poſitive to negative electricity ſeveral times in the courſe of one thunder-guſt. This curious and important diſcovery he ſoon perceived was capable of being applied to practical uſe in life, and in conſequence propoſed a method, which he ſoon accompliſhed, of ſecuring buildings from being damaged by Lightning, by means of CONDUCTORS. See the word.

Nor had the Engliſh philoſophers been inattentive to this ſubject: but, for want of proper opportunities of trying the neceſſary experiments, and from ſome other unfavourable circumſtances, they had failed of ſucceſs. Mr. Canton, however, ſucceeded in July 1752; and in the following month Dr. Bevis and Mr. Wilſon obſerved near the ſame appearances as Mr. Canton had done before. By a number of experiments Mr. Canton alſo ſoon after obſerved that ſome clouds were in a poſitive, while ſome were in a negative ſtate of electricity; and that the electricity of his conductor would ſometimes change, from one ſtate to the other, five or ſix times in leſs than half an hour.

But Sig. Beccaria diſcovered this variable ſtate of thunder clouds, before he knew that it had been obſerved by Dr. Franklin or any other perſon; and he has given a very exact and particular account of the external appearances of theſe clouds. From the obſervations of his apparatus within doors, and of the Lightning abroad, he inferred, that the quantity of electric matter in a common thunder ſtorm, is inconceivably great, conſidering how many pointed bodies, as ſpires, trees, &c, are continually drawing it off, and what a prodigious quantity is repeatedly diſcharged to or from the earth. This matter is in ſuch abundance, that he thinks it impoſſible for any cloud or number of clouds to contain it all, ſo as either to receive or diſcharge it. He obſerves alſo, that during the progreſs and increaſe of the ſtorm, though the lightning frequently ſtruck to the earth, the ſame clouds were the next moment ready to make a ſtill greater diſcharge, and his apparatus continued to be as much affected as ever; ſo that the clouds muſt have received at one part, in the ſame moment when a diſcharge was made from them in another. And from the whole he concludes, that the clouds ſerve as conductors to convey the electric fluid from thoſe parts of the earth that are overloaded with it, to thoſe that are exhauſted of it. The ſame cauſe by which a cloud is firſt raiſed, from vapours diſperſed in the atmoſphere, draws to it thoſe that are already formed, and ſtill continues to form new ones, till the whole collected maſs extends ſo far as to reach a part of the earth where there is a deficiency of the electric fluid, and where the electric matter will diſcharge itſelf on the earth. A channel of communication being thus formed, a freſh ſupply of electric matter is raiſed from the overloaded part, which continues to be conveyed by the medium of the clouds, till the equilibrium of the fluid is reſtored between the two places of the earth. Sig. Beccaria obſerves, that a wind always blows from the place from which the thunder-cloud proceeds; and it is plain that the ſudden accumulation of ſuch a prodigious quantity of vapours muſt diſplace the air, and repel it on all ſides. Indeed many obſervations of the deſcent of Lightning, confirm his theory of the manner of its aſcent; for it often throws before it the parts of conducting bodies, and diſtributes them along the reſiſting medium, through which it muſt force its paſſage; and upon this principle the longeſt flaſhes of Lightning ſeem to be made, by forcing into its way part of the vapours in the air. One of the chief reaſons why theſe flaſhes make ſo long a rumbling, is that they are occaſioned by the vaſt length of a vacuum made by the paſſage of the electric matter: for although the air collapſes the moment after it has paſſed, and that the vibration, on which the ſound depends, commences at the ſame moment; yet when the flaſh is directed towards the perſon who hears the report, the vibrations excited at the nearer end of the track, will reach his ear much ſooner than thoſe from the more remote end; and the ſound will, without any echo or repercuſſion, continue till all the vibrations have ſucceſſively reached him.

How it happens that particular parts of the earth, or the clouds, come into the oppoſite ſtates of poſitive and negative electricity, is a queſtion not abſolutely determined: though it is eaſy to conceive that when particular clouds, or different parts of the earth, poſſeſs oppoſite

posite electricities, a discharge will take place within a certain distance; or the one will strike into the other, and in the discharge a flash of Lightning will be seen. Mr. Canton queries whether the clouds do not become possessed of electricity by the gradual heating and cooling of the air; and whether air suddenly rarefied, may not give electric fire to clouds and vapours passing through it, and air suddenly condensed receive electric fire from them.——Mr. Wilcke supposes, that the air contracts its electricity in the same manner that sulphur and other substances do, when they are heated and cooled in contact with various bodies. Thus, the air being heated or cooled near the earth, gives electricity to the earth, or receives it from it; and the electrified air, being conveyed upwards by various means, communicates its electricity to the clouds.—Others have queried, whether, since thunder commonly happens in a sultry state of the air, when it seems charged with sulphureous vapours, the electric matter then in the clouds may not be generated by the fermentation of sulphureous vapours with mineral or acid vapours in the air.

With regard to places of safety in times of thunder and Lightning, Dr. Franklin's advice is, to sit in the middle of a room, provided it be not under a metal lustre suspended by a chain, sitting on one chair, and laying the feet on another. It is still better, he says, to bring two or three mattresses or beds into the middle of the room, and folding them double, to place the chairs upon them; for as they are not so good conductors as the walls, the Lightning will not be so likely to pass through them: but the safest place of all, is in a hammock hung by silken cords, at an equal distance from all the sides of a room. Dr. Priestley observes, that the place of most perfect safety must be the cellar, and especially the middle of it; for when a person is lower than the surface of the earth, the Lightning must strike it before it can possibly reach him. In the fields, the place of safety is within a few yards of a tree, but not quite near it. Beccaria cautions persons not always to trust too much to the neighbourhood of a higher or better conductor than their own body; since he has repeatedly found that the Lightning by no means descends in one undivided track, but that bodies of various kinds conduct their share of it at the same time, in proportion to their quantity and conducting power. See Franklin's Letters, Beccaria's Lettre dell' Ellettricessimo, Priestley's Hist. of Electric., and Lord Mahon's Principles of Electricity.

Lord Mahon observes that damage may be done by Lightning, not only by the main stroke and lateral explosion, but also by what he calls the returning stroke; by which is meant the sudden violent return of that part of the natural share of electricity which had been gradually expelled from some body or bodies, by the superinduced elastic electrical pressure of the electrical atmosphere of a thunder cloud.

Artificial LIGHTNING, an imitation of real or natural Lightning by gunpowder, aurum fulminans, phosphorus, &c, but especially the last, between which and Lightning there is much more resemblance than the others.

Phosphorus, when newly made, gives a sort of artificial Lightning visible in the dark, which would surprise those not used to such a phenomenon. It is usual to keep this preparation under water; and if it is desired to see the corruscations to the greatest advantage, it should be kept in a deep cylindrical glass, not more than three quarters filled with water. At times the phosphorus will send up corruscations, which will pierce through the incumbent water, and expand themselves with great brightness in the upper or empty part of the glass, and much resembling Lightning. The season of the year, as well as the newness of the phosphorus, must concur to produce these flashes; for they are as common in winter as Lightning is, though both are very frequent in warm weather. The phosphorus, while burning, acts the part of a corrosive, and when it goes out resolves into a menstruum, which dissolves gold, iron, and other metals; and Lightning, in like manner, melts the same substances.

LIKE QUANTITIES, or *Similar Quantities*, in Algebra, are such as are expressed by the same letters, to the same power, or equally repeated in each quantity; though the numeral coefficients may be different.

Thus $4a$ and $5a$ are Like quantities,
as are also $3a^2$ and $12a^2$,
and also $6bxy^2$ and $10bxy^2$.

But $4a$ and $5b$, or $3a^2b$ and $10a^2b^2$, &c, are unlike quantities; because they have not every where the same dimensions, nor are the letters equally repeated. —Like quantities can be united into one quantity, by addition or subtraction; but unlike quantities can only be added or subtracted by placing the signs of these operations between them.

LIKE *Signs*, in Algebra, are the same signs, either both positive or both negative. But when one is positive and the other negative, they are unlike signs.

So, $+ 3ab$ and $+ 5cd$ have Like signs,
as have also $- 2a^2c$ and $- 2ax^2$;
but $+ 3ab$ and $- 5cd$ have unlike signs,
as also $- 2ax$ and $3ax$.

LIKE *Figures*, or *Arches*, &c, are the same as *Similar* figures, arches, &c. See SIMILAR.

All Like figures have their homologous lines in the same ratio. Also Like plane figures are in the duplicate ratio, or as the squares of their homologous lines or sides; and Like solid figures are in the triplicate ratio, or as the cubes of their homologous lines or sides.

LILLY (WILLIAM), a noted English astrologer, born in Leicestershire in 1602. His father was not able to give him farther education than common reading and writing; but young Lilly being of a forward temper, and endued with shrewd wit, he resolved to push his fortune in London; where he arrived in 1620, and, for a present support, articled himself as a servant to a mantua-maker in the parish of St. Clement Danes. But in 1624 he moved a step higher, by entering into the service of Mr. Wright in the Strand, master of the Salters company, who not being able to write, Lilly among other offices kept his books. On the death of his master, in 1627, Lilly paid his addresses to the widow, whom he married with a fortune of 1000l. Being now his own master, he followed the bent of his inclinations, which led him to follow the puritanical preachers. Afterwards, turning his mind to judicial astrology, in 1632 he became pupil, in that art, to one Evans, a

profligate

profligate Welsh parson; and the next year gave the public a specimen of his skill, by an intimation that the king had chosen an unlucky horoscope for the coronation in Scotland. In 1634, getting a manuscript copy of the *Ars Noticia* of Cornelius Agrippa, with alterations, he drank in the doctrine of the magic circle, and the invocation of spirits, with great eagerness, and practised it for some time; after which he treated the mystery of recovering stolen goods, &c, with great contempt, claiming a supernatural sight, and the gift of prophetical predictions; all which he well knew how to turn to good advantage.

Mean while, he had buried his first wife, purchased a moiety of 13 houses in the Strand, and married a second wife, who, joining to an extravagant temper a termagant spirit, which he could not lay, made him unhappy, and greatly reduced his circumstances. With this uncomfortable yokemate he removed, in 1636, to Hersham in Surrey, where he staid till 1641; when, seeing a prospect of fishing in troubled waters, he returned to London. Here having purchased several curious books in this art, which were found on pulling down the house of another astrologer, he studied them incessantly, finding out secrets contained in them, which were written in an imperfect Greek character; and, in 1644, published his *Merlinus Anglicus*, an almanac, which he continued annually till his death, and several other astrological works; devoting his pen, and other labours, sometimes to the king's party, and sometimes to that of the parliament, but mostly to the latter, raising his fortune by favourable predictions to both parties, sometimes by presents, and sometimes by pensions: thus, in 1648, the council of state gave him in money 50l. and a pension of 100l. per annum, which he received for two years, and then resigned it on some disgust. By his advice and contrivance, the king attempted several times to make his escape from his confinement: he procured and sent the aqua-fortis and files to cut the iron bars of his prison windows at Carisbrook castle; but still advising and writing for the other party at the same time. Mean while he read public lectures on astrology, in 1648 and 1649, for the improvement of young students in that art; and in short, plied his business so well, that in 1651 and 1652 he laid out near 2000l. for lands and a house at Hersham.

During the siege of Colchester, he and Booker were sent for thither, to encourage the soldiers; which they did by assuring them that the town would soon be taken; which proved true in the event.—Having, in 1650, written publicly that the parliament should not continue, but a new government arise; agreeably to which, in his almanac for 1653, he asserted that the parliament stood upon a ticklish foundation, and that the commonalty and soldiery would join together against them. Upon which he was summoned before the committee of plundered ministers; but, receiving notice of it before the arrival of the messenger, he applied to his friend Lenthal the speaker, who pointed out the offensive passages. He immediately altered them; attended the committee next morning, with 6 copies printed, which six alone he acknowledged to be his; and by that means came off with only 13 days custody by the serjeant at arms. This year he was engaged in a dispute with Mr. Thomas Gataker.—In 1655 he was indicted at Hicks's-hall, for giving judgment upon stolen goods; but was acquitted. And in 1659, he received, from the king of Sweden, a present of a gold chain and medal, worth about 50l. on account of his having mentioned that monarch with great respect in his almanacs of 1657 and 1658.—After the Restoration, in 1660, being taken into custody, and examined by a committee of the house of commons, touching the execution of Charles the 1st, he declared, that Robert Spavin, then Secretary to Cromwell, dining with him soon after the fact, assured him it was done by cornet Joyce. The same year he sued out his pardon under the broad seal of England; and afterwards continued in London till 1665; when, upon the raging of the plague there, he retired to his estate at Hersham. Here he applied himself to the study of physic, having, by means of his friend Elias Ashmole, procured from archbishop Sheldon a licence to practise it, which he did, as well as astrology, from thence till the time of his death.—In October 1666 he was examined before a committee of the house of commons concerning the fire of London, which happened in September that year. A little before his death, he adopted for his son, by the name of *Merlin junior*, one Henry Coley, a taylor by trade; and at the same time gave him the impression of his almanac, which had been printed for 36 years successively. This Coley became afterwards a celebrated astrologer, publishing in his own name, almanacs, and books of astrology, particularly one intitled *A Key to Astrology*.

Lilly died of a palsy 1681, at 79 years of age; and his friend Mr. Ashmole placed a monument over his grave in the church of Walton upon Thames.

Lilly was author of many works. His *Observations on the Life and Death of Charles late King of England*, if we overlook the astrological nonsense, may be read with as much satisfaction as more celebrated histories; Lilly being not only very well informed, but strictly impartial. This work, with the Lives of Lilly and Ashmole, written by themselves, were published in one volume, 8vo, in 1774, by Mr. Burman. His other works were principally as follow:

1. Merlinus Anglicus junior.—2. Supernatural Sight.—3. The White King's Prophecy.—4. England's Prophetical Merlin: all printed in 1644.—5. The Starry Messenger, 1645.—6. Collection of Prophecies, 1646.—7. A Comment on the White King's Prophecy, 1646.—8. The Nativities of Archbishop Laud and Thomas earl of Strafford, 1646.—9. Christian Astrology, 1647: upon this piece he read his lectures in 1648, mentioned above.—10. The third book of Nativities, 1647.—11. The World's Catastrophe, 1647.—12. The Prophecies of Ambrose Merlin, with a Key, 1647.—13. Trithemius, or the Government of the World by Presiding Angels, 1647.—14. A treatise of the Three Suns seen in the winter of 1647, printed in 1648.—15. Monarchy or no Monarchy, 1651.—16. Observations on the Life and Death of Charles, late king of England, 1651; and again in 1651, with the title of Mr. William Lilly's True History of king James and king Charles the 1st, &c.—17. Annus Tenebrosus; or, the Black Year. This drew him into the dispute with Gataker, which Lilly carried on in his Almanac in 1654.

LIMB, the outermost border, or graduated edge, of a quadrant, astrolabe, or such like mathematical instrument.

The word is also used for the arch of the primitive circle, in any projection of the sphere in plano.

Limb also signifies the outermost border or edge of the sun or moon; as the upper Limb, or edge; the lower Limb; the preceding Limb, or side; the following Limb.—Astronomers observe the upper or lower Limb of the sun or moon, to find their true height, or that of the centre, which differs from the others by the semidiameter of the disc.

LIMBERS, in Artillery, a sort of advanced train, joined to the carriage of a cannon on a march. It is composed of two shafts, wide enough to receive a horse between them, called the *fillet horse*: these shafts are joined by two bars of wood, and a bolt of iron at one end, and mounted on a pair of rather small wheels. Upon the axle-tree rises a strong iron spike, which is put into a hole in the hinder part of the train of the gun-carriage, to draw it by. But when a gun is in action, the Limbers are taken off, and run out behind it.—See the dimensions and figure of it in Müller's Treatise of Artillery, pa. 187.

LIMIT, is a term used by mathematicians, for some determinate quantity, to which a variable one continually approaches, and may come nearer to it than by any given difference, but can never go beyond it; in which sense a circle may be said to be the Limit of all its inscribed and circumscribed polygons: because these, by increasing the number of their sides, can be made to be nearer equal to the circle than by any space that can be proposed, how small soever it may be.

In Algebra, the term *Limit* is applied to two quantities, of which the one is greater and the other less than some middle quantity, as the root of an equation, &c. And in this sense it is used when speaking of the Limits of equations, a method by which their solution is greatly facilitated.

Limit *of Distinct Vision*, in Optics. See *Distinct* Vision.

Limit *of a Planet*, has been sometimes used for its greatest heliocentric latitude.

Limited *Problem*, denotes a problem that has but one solution, or some determinate number of solutions: as to describe a circle through three given points that do not lie in a right line, which is limited to one solution only; to divide a parallelogram into two equal parts by a line parallel to one side, which admits of two solutions, according as the line is parallel to the length or breadth of the parallelogram; or to divide a triangle in any ratio by a line parallel to one side, which is limited to three solutions, as the line may be parallel to any of the three sides.

LINE, in Geometry, a quantity extended in length only, without either breadth or thickness.

A Line is sometimes considered as generated by the flux or motion of a point; and sometimes as the limit or termination of a superficies, but not as any part of that surface, however small.

Lines are either *right* or *curved*. A *right*, or straight Line, is the nearest distance between two points, which are its extremes or ends; or it is a Line which has in every part of it the same direction or position. But a *curve Line* has in every part of it a different direction, and is not the shortest distance between its extremes or ends.

Right Lines are all of the same species; but curves are of an infinite number of different sorts. As many may be conceived as there are different compound motions, or as many as there may be different relations between their ordinates and abscisses. See Curves.

Again, *Curve* Lines are usually divided into *geometrical* and *mechanical*.

Geometrical Lines, are those which may be found exactly in all their parts. See Geometrical Line.

Mechanical Lines are such as are not determined exactly in all their parts, but only nearly, or tentatively. But

Des Cartes, and his followers, define geometrical Lines to be those which may be expressed by an algebraical equation of a determinate or finite degree; called its *locus*. And mechanical Lines, such as cannot be expressed by such an equation.

But others distinguish the same Lines by the name *algebraical* and *transcendental*.

Lines are also divided into orders, by Newton, according to the number of intersections which may be made by them and a right Line, viz, the 1st, 2d, 3d, 4th, &c, order, according as they may be cut by a right Line, in 1, or 2, or 3, or 4, &c, points. In this way of considering them, the right Line only is of the 1st order, being but one in number; the 2d order contains 4 curves only, being such as may be cut from a cone by a plane, viz, the circle, the ellipse, the hyperbola, and the parabola; the lines of the 3d order have been enumerated by Newton, in a particular treatise, who makes their number amount to 72; but Mr. Stirling found 4 others, and Mr. Stone 2 more; though it is disputed by some whether these 2 last ought to be accounted different from some of Newton's, or not. See Newton's Enumer. Lin. Tertii Ordin. also Stirling's Lineæ Tert. Ordin. Newtonianæ Oxon. 1717, 8vo. and Philos. Trans. number 456, &c. Again,

Algebraical Lines are divided into different orders according to the power or degree of their equations. So, the simple equation $a + by + cx = 0$ or equation of the 1st degree, denotes the 1st order or right line; the equation $a + by + cx + dyy + exy + fxx = 0$, of the 2d degree, denotes the Lines of the 2d order; and the equation

$$a + by + cx + dyy + exy + fxx + gy^3 + hxy^2 + ix^2y + kx^3 = 0$$

of the 3d degree, expresses the Lines of the 3d order; and so on. See Cramer's Introd. à l'Analyse des Lignes Courbes.

Lines, considered as to their positions, are either *parallel*, *perpendicular*, or *oblique*. And the construction and properties of each of these, see under the respective terms.

Line also denotes a French measure of length, being the 12th part of an inch, or the 144th part of a foot.

In *Astronomy*,

Line *of the Apses*, or *Apsides*, the Line joining the two apses, or the longer axis of the orbit of a planet.

Fiducial Line, the index line or edge of the ruler, which passes through the middle of an astrolabe, or other instrument, on which the sights are fitted, and marking the divisions.

Horizontal

Horizontal Line, a Line parallel to the horizon.

Line *of the Nodes*, that which joins the nodes of the orbit of a planet, being the common ſection of the plane of the orbit with the plane of the ecliptic.

In *Dialling*,

Horizontal Line, is the common ſection of the horizon and the dial-plate.

Horary, or *Hour Lines*, are the common interſections of the hour-circles of the ſphere with the plane of the dial.

Equinoctial Line is the common interſection of the equinoctial and the plane of the dial.

In *Fortification*, *Line* is ſometimes uſed for a ditch, bordered with its parapet: and ſometimes for a row of gabions, or ſacks of earth, extended lengthwiſe on the ground, to ſerve as a ſhelter againſt the enemy's fire.

When the trenches were carried on within 30 paces of the glacis, they drew two Lines, one on the right, and the other on the left, for a place of arms.

Lines are commonly made to ſhut up an avenue or entrance to ſome place; the ſides of the entrance being covered by rivers, woods, mountains, moraſſes, or other obſtructions, not eaſy to be paſſed over by an army. When they are conſtructed in an open country, they are carried round the place to be defended, and reſemble the Lines ſurrounding a camp, called Lines of circumvallation. Lines are alſo thrown up to ſtop the progreſs of an army; but the term is moſt uſed for the Line which covers a paſs that can only be attacked in front.

When lines are made to cover a camp, or a large tract of land, where a conſiderable body of troops is poſted, the work is not made in one ſtraight, or uniformly bending Line; but, at certain diſtances, the Lines project in ſaliant angles, called redents, redans, or flankers, towards the enemy. The diſtance between theſe angles is commonly between the limits of 200 and 260 yards; the ordinary flight of a muſket ball, point blank, being commonly within thoſe limits; though muſkets a little elevated will do effectual ſervice at the diſtance of 360 yards.

Fundamental Line, is the firſt Line drawn for the plan of a place, and which ſhews its area.

Central Line, is the Line drawn from the angle of the centre to the angle of the baſtion.

Line of Defence, &c. See Defence &c.

Line of Approach, or *Attack*, ſignifies the work which the beſiegers carry on under cover, to gain the moat, and the body of the place.

Line of Circumvallation, is a Line or trench cut by the beſiegers, within cannon-ſhot of the place, which ranges round the camp, and ſecures its quarters againſt any relief to be brought to the beſieged.

Line of Contravallation, is a ditch bordered with a parapet, ſerving to cover the beſiegers on the ſide next the place, and to ſtop the ſallies of the garriſon.

Lines of Communication are thoſe which run from one work to another.

Line of the Baſe, is that which joins the points of the two neareſt baſtions.

To *Line* a work, ſignifies to face it, as with brick or ſtone; for example, to ſtrengthen a rampart with a firm wall, or to encompaſs a parapet or moat with good turf, &c.

Line, in Geography and Navigation, is emphatically uſed for the Equator or Equinoctial Line.

The ſeamen uſe to baptize their freſh men, and paſſengers, the firſt time they croſs the Line: that is, to dip them in the ſea, ſuſpended by a rope from the yard-arm, unleſs they compound for it, by giving ſomething to drink.

In *Perſpective*,

The *Geometrical Line*, is a right Line drawn in any manner on the geometrical plane.

Terreſtrial or *Fundamental Line*, is the common interſection of the geometrical plane and plane of the picture.

Line of the Front, is any Line parallel to the terreſtrial Line.

Vertical Line, is the ſection of the vertical and draft planes.

Viſual Line, is the Line or ray conceived to paſs from the object to the eye.

Objective Line, is any Line drawn on the geometrical plane, whoſe repreſentation is ſought for in the draught or picture.

Line of Meaſures, is uſed by Oughtred, and others, to denote the diameter of the primitive circle, in the projection of the ſphere in plano, or that Line in which falls the diameter of any circle to be projected.

Linear Numbers, are ſuch as have relation to length only; ſuch, for example, as expreſs one ſide of a plane figure; and when the plane figure is a ſquare, the linear number is called a root.

Linear Problem, is one that can be ſolved geometrically by the interſection of two right lines. This is called a ſimple problem, and is capable of only one ſolution.

LIQUID, a fluid which wets or ſmears ſuch bodies as are immerſed in it, ariſing from ſome configuration of its particles, which diſpoſes them to adhere to the ſurfaces of bodies contiguous to them. Thus water, oil, milk, &c, are Liquids, as well as fluids; but quickſilver is not a Liquid, but ſimply a fluid.

LISLE (William de), a very learned French geographer, was born at Paris in 1675. His father being much occupied in the ſame way, young Liſle began at 9 years of age to draw maps, and ſoon made a great progreſs in this art. In 1699 he firſt diſtinguiſhed himſelf to the public, by giving a map of the world, and other pieces, which procured him a place in the Academy of Sciences, 1702. He was afterwards appointed geographer to the king, with a penſion, and had the honour of inſtructing the king himſelf in geography, for whoſe particular uſe he drew up ſeveral works. De Liſle's reputation was ſo great, that ſcarcely any hiſtory or travels came out without the embelliſhment of his maps. Nor was his name leſs celebrated abroad than in his own country. Many ſovereigns in vain attempted to draw him out of France. The Czar Peter, when at Paris on his travels, paid him a viſit, to communicate to him ſome remarks upon Muſcovy; but more eſpecially, ſays Fontenelle, to learn from him, better than he could any where elſe, the extent and ſituation of his own dominions. De Liſle died of an apoplexy in 1726, at 51 years of age. Beſide the excellent maps he publiſhed, he wrote

many pieces in the Memoirs of the Academy of Sciences.

LIST, or LISTEL, a ſmall ſquare moulding, ſerving to crown or accompany larger mouldings; or on occaſion to ſeparate the flutings of columns.

LITERAL ALGEBRA. See ALGEBRA.

LIZARD, in Aſtronomy. See LACERTA.

LOADSTONE, or MAGNET; which ſee.

LOCAL *Problem*, is one that is capable of an infinite number of different ſolutions; becauſe the point, which is to ſolve the problem, may be indifferently taken within a certain extent; as ſuppoſe any where in ſuch a line, within ſuch a plane figure, &c, which is called a *geometrical Locus*.

A Local problem is *ſimple*, when the point ſought is in a right line; *plane*, when the point ſought is in the circumference of a circle; *ſolid*, when it is in the circumference of a conic ſection; or *ſurſolid*, when the point is in the perimeter of a line of a higher kind.

LOCAL MOTION, or *Loco-Motion*, the change of place: See MOTION.

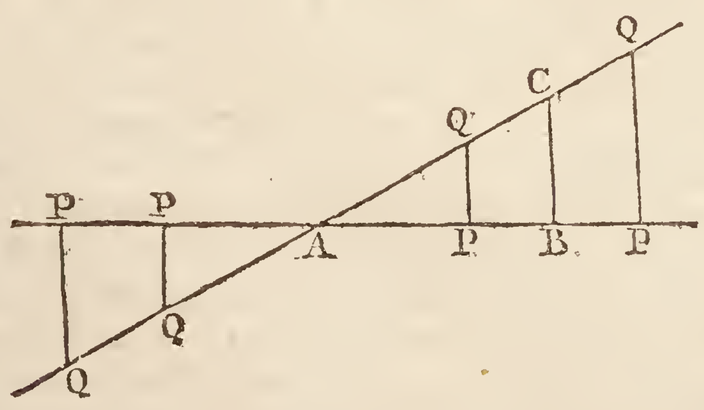
LOCI, the plural of LOCUS, which ſee.

LOCUS, is ſome line by which a local or indeterminate problem is ſolved; or a line of which any point may equally ſolve an indeterminate problem.

Loci are expreſſed by algebraic equations of different orders according to the nature of the Locus. If the equation is conſtructed by a right line, it is called *Locus ad rectum*; if by a circle, *Locus ad circulum*; if by a parabola, *Locus ad parabolam*; if by an ellipſis, *Locus ad ellipſim*; and ſo on.

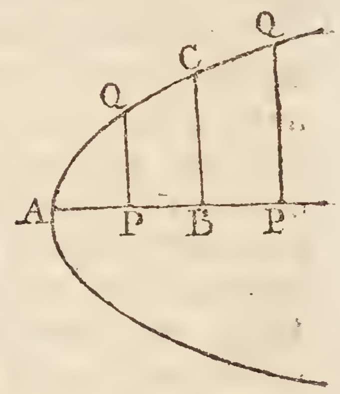
The Loci of ſuch equations as are right lines or circles, the ancients called *plane loci*; and of thoſe that are conic ſections, *ſolid loci*; but ſuch as are curves of a higher order, *ſurſolid loci*. But the moderns diſtinguiſh the Loci into orders according to the dimenſions of the equations by which they are expreſſed, or the number of the powers of indeterminate or unknown quantities in any one term: thus, the equation $ay = bx + c$ denotes a Locus of the 1ſt order, but $y^2 = ax$, or $= ax - x^2$, &c, a Locus of the 2d order, and $y^3 = a^2x$, or $= ax^2 - x^3$, &c, a Locus of the 3d order, and ſo on; where x and y are unknown or indeterminate quantities, and the others known or determinate ones; alſo x denotes the abſciſs, and y the ordinate of the curve or line which is the Locus of the equation.

For inſtance, ſuppoſe two variable or indeterminate right lines AP, AQ, making any given angle PAQ between

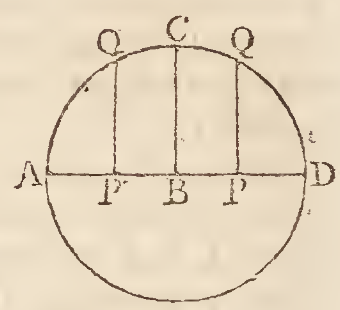


them, where they are ſuppoſed to commence, and to extend indefinitely both ways from the point A: then calling any AP, x, and its correſponding ordinate PQ, y, continually changing its poſition by moving parallel to itſelf along the indefinite line AP; alſo in the line AP aſſume AB $= a$, and from B draw BC parallel to PQ and $= b$: then the indefinite line AQ is called in general a geometrical Locus, and in particular the Locus of the equation $y = \frac{bx}{a}$; for whatever point Q is, the triangles ABC, APQ are always ſimilar, and therefore AB : BC :: AP : PQ, that is $a : b :: x : y$, and therefore $y = \frac{bx}{a}$ is the equation to the right line AQ, or AQ is the Locus of the equation $y = \frac{bx}{a}$.

Again, if AQ be a parabola, the nature of which is ſuch, that AB : AP :: BC² : PQ², or $a : x :: b^2 : y^2$, and therefore $y^2 = \frac{b^2x}{a}$ is the equation which has the parabola for its Locus, or the parabola is the Locus to every equation of this form $y^2 = \frac{b^2x}{a}$.



Or if AQ be a circle, having its radius AB $= a$, the nature of which is this, that $PQ^2 = AP \cdot PD$, or $y^2 = x \cdot \overline{2a - x}$ or $2ax - x^2$; therefore the Locus of the equation of this form $y^2 = 2ax - x^2$, is always a circle.



In like manner it will appear, that the ellipſe is the Locus to the equation $y^2 = \frac{c^2}{t^2} \times \overline{tx - x^2}$, and the hyperbola the Locus to the equation $y^2 = \frac{c^2}{t^2} \times \overline{tx + x^2}$; where t is the tranſverſe, and c the conjugate axis of the ellipſe or hyperbola.

All equations, whoſe Loci are of the firſt order, may be reduced to one of the 4 following forms:

1ſt $y = \frac{bx}{a}$; 2d $y = \frac{bx}{a} + c$; 3d $y = \frac{bx}{a} - c$; 4th $y = c - \frac{bx}{a}$;

where the letter c denotes the diſtance that the ordinates commence from the line AP, either on the one ſide or the other of it, according as the ſign of that quantity is + or —.

All Loci of the 2d degree are conic ſections, viz, either the parabola, the circle, ellipſis, or hyperbola. Therefore when an equation is given, whoſe Locus is of the 2d degree, and it is required to draw that Locus, or, which is the ſame thing, to conſtruct the equation generally; bring over all the terms of the equation to one ſide, ſo that the other ſide be 0; then to know which of the conic ſections it denotes, there will be two general caſes, viz, either when the rectangle xy is in the equation, or when it is not in it.

Caſe 1. When the term xy is not in the propoſed equation. Then, 1ſt, if only one of the ſquares

x^2, y^2

x^2, y^2 be found in it, the Locus will be a parabola. 2d, If both the squares be in it, and if they have the same sign, the Locus will be a circle or an ellipse. 3d, But if the signs of the squares x^2, y^2 be different, the Locus will be an hyperbola, or the opposite hyperbolas.

Case 2. When the rectangle xy is in the proposed equation; then 1st, If neither of the squares x^2, y^2, or only one of them be in the equation, the Locus will be an hyperbola between the asymptotes. 2d, If both x^2 and y^2 be in it, having different signs, the Locus will be an hyperbola, having the abscisses on its diameter. 3d, If both the squares be in it, and with the same sign, then if the coefficient of x^2 be greater than the square of half the coefficient of xy, the Locus will be an ellipse; if equal, a parabola; and if less, an hyperbola.

This method of determining geometric Loci, by reducing them to the most compound or general equations, was first published by Mr. Craig, in his Treatise on the Quadrature of Curves, in 1693. It is explained at large in the 7th and 8th books of l'Hospital's Conic Sections. See this subject particularly illustrated in Maclaurin's Algebra. The method of Des Cartes, of finding the Loci of equations of the 2d order, is a good one, viz, by extracting the root of the equation. See his Geometry; as also Stirling's Illustratio Linearum Tertii Ordinis. The doctrine of these Loci is likewise well treated by De Witt in his Elementa Curvarum. And Bartholomæus Intieri, in his Aditus ad Nova Arcana Geometrica delegenda, has shewn how to find the Loci of equations of the higher orders. Mr. Stirling too, in his treatise above-mentioned, has given an example or two of finding the Loci of equations of 3 dimensions. Euclid, Apollonius, Aristæus, Fermat, Viviani, have also written on the subject of Loci.

LOG, in Navigation, is a piece of thin board, of a sectoral or quadrantal form, loaded in the circular side with lead sufficient to make it swim upright in the water; to which is fastened a line of about 150 fathoms, or 300 yards long, called the Log-line, which is divided into certain spaces, called Knots, and wound on a reel which turns very freely, for the line to wind easily off.

The use of the Log, or Log-line, is to measure the velocity of the ship, or rate at which she runs, which is done from time to time, as the foundation upon which the ship's reckoning, or finding her place, is kept; and the practice is to heave the Log into the sea, with the line tied to it, and observe how much of the line is run off the reel, while the ship sails, during the space of half a minute, which time is measured by a sand-glass made to run that time very exactly. About 10 fathoms of stray or waste line is left next the Log before the knotting or counting commence, that space being usually allowed to carry the Log out of the eddy of the ship's wake.

The using of the Log for finding the velocity of the ship, is called *Heaving the Log*, and is thus performed: One man holds the reel, and another the half-minute glass; an officer of the watch throws the Log over the ship's stern, on the lee-side, and when he observes the stray line, and the first mark is going off, he cries *turn!* when the glass-holder instantly turns the glass crying out *done!* then watching the glass, the moment it is run out he says *stop!* upon which the reel being quickly stopt, the last mark run off shews the number of knots, and the distance of that mark from the reel is estimated in fathoms: then the knots and fathoms together shew the distance run in half a minute, or the distance per hour nearly, by considering the knots as miles, and the fathoms as decimals of a mile: thus if 7 knots and 4 fathoms be observed, then the ship runs at the rate of 7·4 miles an hour.

It follows, therefore, that the length of each knot, or division of the line, ought to be the same part of a sea mile, as half a minute is of an hour, that is $\frac{1}{120}$th part. Now it is found that a degree of the meridian contains nearly 366,000 feet, therefore $\frac{1}{60}$ of this, or a nautical mile, will be 6100 feet; the $\frac{1}{120}$th of which, or 51 feet nearly, should be the length of each knot, or division of the Log-line. But because it is safer to have the reckoning rather before the ship than after it, therefore it is usual now to make each knot equal to 8 fathoms or 48 feet. But the knots are made sometimes to contain only 42 feet; and this method of dividing the Log-line was founded on the supposition, that 60 miles, of 5000 feet each, made a degree; for $\frac{1}{120}$th of 5000 is $41\frac{2}{3}$, or in round numbers 42 feet. And although many mariners find by experience that this length of the knot is too short, yet rather than quit the old way, they use sand-glasses for half-minute ones that run only 24 or 25 seconds. The sand, or half-minute glass, may be tried by a pendulum vibrating seconds, in the following manner: On a round nail or peg, hang a thread or fine string that has a musket ball fixed to one end, carefully measuring between the centre of the ball and the string's loop over the nail $39\frac{1}{8}$ inches, being the length of a second pendulum; then make it swing or vibrate very small arches, and count one for every time it passes under the nail, beginning at the second time it passes; and the number of swings made during the time the glass is running out, shews the seconds in the glass.

It is not known who was the inventor of this method of measuring the ship's way, or her rate of sailing; but no mention of it occurs till the year 1607, in an East-India voyage, published by Purchas; and from that time its name occurs in other voyages in his collections; after which it became famous, being noticed both by our own authors, and by foreigners; as by Gunter in 1623; Snellius, in 1624; Metius, in 1631; Oughtred, in 1633; Herigone, in 1634; Saltonstall, in 1636; Norwood, in 1637; Fournier, in 1643; and almost all the succeeding writers on navigation of every country. Various improvements have lately been made of this instrument by different persons.

LOGARITHM, from the Greek λογος *ratio*, and αριθμος *number*; q. d. *ratio of numbers*, or perhaps rather *number of ratios*; the indices of the ratios of numbers to one another; or a series of numbers in arithmetical proportion, corresponding to as many others in geometrical proportion, in such sort that 0 corresponds to, or is the index of 1, in the geometricals. They have been devised for the ease of large arithmetical calculations.

Thus

Thus,

	0,	1,	2,	3,	4,	&c, indices or Logarithms,
	1,	2,	4,	8,	16,	&c, the geometrical progressions, or common numbers.
or	2^0,	2^1,	2^2,	2^3,	2^4,	&c,
	1,	3,	9,	27,	81,	&c,
or	3^0,	3^1,	3^2,	3^3,	3^4,	&c,
	1,	10,	100,	1000,	10000,	&c,
or	10^0,	10^1,	10^2,	10^3,	10^4,	&c,

Where the ſame indices, or Logarithms, ſerve equally for any geometric ſeries; and from which it is evident, that there may be an endleſs variety of ſets of Logarithms to the ſame common numbers, by varying the 2d term 2, or 3, or 10, &c of the geometric ſeries; as this will change the original ſeries of terms whoſe indices are the numbers 1, 2, 3, &c; and by interpolation the whole ſyſtem of numbers may be made to enter the geometrical ſeries, and receive their proportional Logarithms, whether integers or decimals.

Or the Logarithm of any given number, is the index of ſuch a power of ſome other number, as is equal to the given one. So if N be $= r^n$, then the Logarithm of N is n, which may be either poſitive or negative, and r any number whatever, according to the different ſyſtems of Logarithms. When N is 1, then n is $= 0$, whatever the value of r is; and conſequently the Logarithm of 1 is always 0 in every ſyſtem of Logarithms. When n is $= 1$, then N is $= r$; conſequently the root r is always the number whoſe Logarithm is 1, in every ſyſtem. When r is $= 2{\cdot}718281828459$ &c, the indices are the hyperbolic Logarithms; ſo that n is always the hyperbolic Logarithm of $\overline{2{\cdot}718 \text{ \&c}}^n$. But in the common Logarithms, r is $= 10$; ſo that the common Logarithm of any number, is the index of that power of 10 which is equal to the ſaid number; ſo the common Logarithm of N $= 10^n$, is n the index of the power of 10; for example, 1000, being the 3d power of 10, has 3 for its Logarithm; and if 50 be $= 10^{1{\cdot}69897}$, then is $1{\cdot}69897$ the common Logarithm of 50. And hence it follows that this decimal ſeries of terms

1000, 100, 10, 1, ·1, ·01, ·001,
or 10^3, 10^2, 10^1, 10^0, 10^{-1}, 10^{-2}, 10^{-3},
have 3, 2, 1, 0, -1, -2, -3,

reſpectively for the Logarithms of thoſe terms.

The Logarithm of a number contained between any two terms of the firſt ſeries, is included between the two correſponding terms of the latter; and therefore that Logarithm will conſiſt of the ſame index, whether poſitive or negative, as the ſmaller of thoſe two terms, together with a decimal fraction, which will always be poſitive. So the number 50 falling between 10 and 100, its Logarithm will fall between 1 and 2, being indeed equal to 1·69897 nearly: alſo the number ·05 falling between the terms ·1 and ·01, its Logarithm will fall between -1 and -2, and is indeed $= -2 + {\cdot}69897$, the index of the leſs term together with the decimal ·69897. The index is alſo called the Characteriſtic of the Logarithms, and is always an integer, either poſitive or negative, or elſe $= 0$; and it ſhews what place is occupied by the firſt ſignificant figure of the given number, either above or below the place of units, being in the former caſe + or poſitive; in the latter − or negative.

When the characteriſtic of a Logarithm is negative, the ſign − is commonly ſet over it, to diſtinguiſh it from the decimal part, which, being the Logarithm found in the tables, is always poſitive: ſo $-2 + {\cdot}69897$, or the Logarithm of ·05, is written thus $\bar{2}{\cdot}69897$. But on ſome occaſions it is convenient to reduce the whole expreſſion to a negative form; which is done by making the characteriſtic leſs by 1, and taking the *arithmetical complement* of the decimal, that is, beginning at the left hand, ſubtract each figure from 9, except the laſt ſignificant figure, which is ſubtracted from 10; ſo ſhall the remainders form the Logarithm wholly negative: thus the Logarithm of ·05, which is $\bar{2}{\cdot}69897$ or $-2 + {\cdot}69897$, is alſo expreſſed by $-1{\cdot}30103$, which is all negative. It is alſo ſometimes thought more convenient to expreſs ſuch Logarithms entirely as poſitive, namely by only joining to the tabular decimal the complement of the index to 10; and in this way the above Logarithm is expreſſed by 8·69897; which is only increaſing the indices in the ſcale by 10.

The Properties of Logarithms.—From the definition of Logarithms, either as being the indices of a ſeries of geometricals, or as the indices of the powers of the ſame root, it follows that the multiplication of the numbers will anſwer to the addition of their Logarithms; the diviſion of numbers, to the ſubtraction of their Logarithms; the raiſing of powers, to the multiplying the Logarithm of the root by the index of the power; and the extracting of roots, to the dividing the Logarithm of the given number by the index of the root required to be extracted.

So, 1ſt,

Log. ab or of $a \times b$ is $=$ log. a + log. b,
Log. 18 or of 3×6 is $=$ log. 3 + log. 6,
Log. $5 \times 9 \times 73$ is $=$ log. 5 + log. 9 + log. 73.

Secondly,

Log. $a \div b$ is $=$ log. a − log. b,
Log. $18 \div 6$ is $=$ log. 18 − log. 6,
Log. $79 \times 5 \div 9$ is $=$ log. 79 + log. 5 − log. 9,
Log. $\frac{1}{2}$ or $1 \div 2$ is $=$ l. 1 − l. 2 $= 0 -$ l. 2 $= -$ l. 2,
Log. $\frac{1}{n}$ or $1 \div n$ is $=$ l. 1 − l. $n = -$ l. n.

Thirdly,

Log. r^n is $= n$ l. r; Log. $r^{\frac{1}{n}}$ or of $\sqrt[n]{r}$ is $= \frac{1}{n}$ l. r;

Log. $r^{\frac{m}{n}}$ is $= \frac{m}{n}$ l. r; Log. 2^6 is $= 6$ l. 2; log. $2^{\frac{1}{3}}$ or of $\sqrt[3]{2}$ is $= \frac{1}{3}$ l. 2; and Log. $2^{\frac{3}{4}}$ is $= \frac{3}{4}$ l. 2.

So that any number and its reciprocal have the ſame Logarithm, but with contrary ſigns; and the ſum of the Logarithms of any number and its reciprocal, or complement, is equal to 0.

Hiſtory and Conſtruction of Logarithms.—The properties of Logarithms hitherto mentioned, or of arithmetical indices to powers or geometricals, with their various uſes and properties, as above-mentioned, are taken notice of by Stifelius, in his Arithmetic; and indeed they were not unknown to the ancients; but they come all far ſhort of the uſe of Logarithms in Trigo-

Trigonometry, as first discovered by John Napier, baron of Merchiston in Scotland, and published at Edinburgh in 1614, in his Mirifici Logarithmorum Canonis Descriptio; which contained a large canon of Logarithms, with the description and uses of them; but their construction was reserved till the sense of the Learned concerning his invention should be known. This work was translated into English by the celebrated Mr. Edward Wright, and published by his son in 1616. In the year 1619, Robert Napier, son of the inventor of Logarithms, published a new edition of his late father's work, together with the promised Construction of the Logarithms, with other miscellaneous pieces written by his father and Mr. Briggs. And in the same year, 1619, Mr John Speidell published his New Logarithms, being an improved form of Napier's.

All these tables were of the kind that have since been called hyperbolical, because the numbers express the areas between the asymptote and curve of the hyperbola. And Logarithms of this kind were also soon after published by several other persons; as by Ursinus in 1619, Kepler in 1624, and some others.

On the first publication of Napier's Logarithms, Henry Briggs, then professor of Geometry in Gresham College in London, immediately applied himself to the study and improvement of them, and soon published the Logarithms of the first 1000 numbers, but on a new scale, which he had invented, viz, in which the Logarithm of the ratio of 10 to 1 is 1, the Logarithm of the same ratio in Napier's system being 2·30258 &c; and in 1624, Briggs published his Arithmetica Logarithmica, containing the Logarithms of 30,000 natural numbers, to 14 places of figures besides the index, in a form which Napier and he had agreed upon together, which is the present form of Logarithms; also in 1633 was published, to the same extent of figures, his Trigonometria Britannica, containing the natural and logarithmic sines, tangents, &c.

With various and gradual improvements, Logarithms were also published successively, by Gunter in 1620, Wingate in 1624, Henrion in 1626, Miller and Norwood in 1631, Cavalerius in 1632 and 1643, Vlacq and Rowe in 1633, Frobenius in 1634, Newton in 1658, Caramuel in 1670, Sherwin in 1706, Gardiner in 1742, and Dodson's Antilogarithmic Canon in the same year; besides many others of lesser note; not to mention the accurate and comprehensive tables in the Tables Portative, and in my own Logarithms lately published, where a complete history of this science may be seen, with the various ways of constructing them that have been invented by different authors.

In Napier's construction of Logarithms, the natural numbers, and their Logarithms, as he sometimes called them, or at other times the artificial numbers, are supposed to arise, or to be generated, by the motions of points, describing two lines, of which the one is the natural number, and the other its Logarithm, or artificial. Thus, he conceived the line or length of the radius to be described, or run over, by a point moving along it in such a manner, that in equal portions of time it generated, or cut off, parts in a decreasing geometrical progression, leaving the several remainders, or sines, in geometrical progression also; whilst another point described equal parts of an indefinite line, in the same equal portions of time; so that the respective sums of these, or the whole line generated, were always the arithmeticals or Logarithms of the aforesaid natural sines. In this idea of the generation of the Logarithms and numbers, Napier assumed 0 as the Logarithm of the greatest sine or radius; and next he limited his system, not by assuming a particular value to some assigned number, or part of the radius, but by supposing that the two generating points, which, by their motions along the two lines, described the natural numbers and Logarithms, should have their velocities equal at the beginning of those lines. And this is the reason that, in his table, the natural sines and their Logarithms, at the complete quadrant, have equal differences or increments; and this is also the reason why his scale of Logarithms happens accidentally to agree with what have since been called the hyperbolical Logarithms, which have likewise numeral differences equal to those of their natural numbers at the beginning; except only that these latter increase with the natural numbers, while his on the contrary decrease; the Logarithm of the ratio of 10 to 1 being the same in both, namely 2·30258509 &c.

Having thus limited his system, Napier proceeds, in the posthumous work of 1619, to explain his construction of the Logarithmic canon. This he effects in various ways, but chiefly by generating, in a very easy manner, a series of proportional numbers, and their arithmeticals or Logarithms; and then finding, by proportion, the Logarithms to the natural sines from those of the natural numbers, among the original proportionals; a particular account of which may be seen in my book of Logarithms above mentioned.

The methods above alluded to, relate to Napier's or the hyperbolical system of Logarithms, and indeed are in a manner peculiar to that sort of them. But in an appendix to the posthumous work, mention is made of other methods, by which the common Logarithms, agreed upon by him and Briggs, may be constructed, and which it appears were written after that agreement. One of these methods is as follows: Having assumed 0 for the Logarithm of 1, and 1000 &c for the Logarithm of 10; this Logarithm of 10, and the successive quotients, are to be divided ten times by 5, by which divisions there will be obtained these other ten Logarithms, namely 2000000000, 400000000, 80000000, 16000000, 3200000, 640000, 128000, 25600, 5120, 1024; then this last Logarithm, and its quotients, being divided ten times by 2, will give these other ten Logarithms,

viz, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1.

And the numbers answering to these twenty Logarithms are to be found in this manner, viz, Extract the 5th root of 10 (with ciphers), then the 5th root of that root, and so on for ten continual extractions of the 5th root: so shall these ten roots be the natural numbers belonging to the first ten Logarithms above found, in dividing continually by 5. Next, out of the last 5th root is to be extracted the square root, then the square root of this last root, and so on for ten successive extractions of the square root: so shall these last ten roots be the natural numbers corresponding to the Logarithms or quotients arising from the

the laſt ten diviſions by the number 2. And from theſe twenty Logarithms, 1, 2, 4, 8, &c, and their natural numbers, the author obſerves that other Logarithms and their numbers may be formed, namely by adding the Logarithms, and multiplying their correſponding numbers. But, beſides the immenſe labour of this method, it is evident that this proceſs would generate rather an antilogarithmic canon, ſuch as Dodſon's, than the table of Briggs.

Napier next mentions another method of deriving a few of the primitive numbers and their Logarithms, namely, by taking continually geometrical means, firſt between 10 and 1, then between 10 and this mean, and again between 10 and the laſt mean, and ſo on; and then taking the arithmetical means between their correſponding Logarithms.

He then lays down various relations between numbers and their Logarithms, ſuch as, that the products and quotients of numbers, anſwer to the ſums and differences of their Logarithms; and that the powers and roots of numbers, anſwer to the products and quotients of the Logarithms when multiplied or divided by the index of the power or root, &c; as alſo that, of any two numbers, whoſe Logarithms are given, if each number be raiſed to the power denoted by the Logarithm of the other, the two reſults will be equal; thus, if x be the Logarithm of any number X, and y the Logarithm of Y, then is $X^y = Y^x$. Napier then adverts to another method of making the Logarithms to a few of the prime integer numbers, which is well adapted to the conſtruction of the common table of Logarithms: this method eaſily follows from what has been ſaid above, and it depends on this property, that the Logarithm of any number in this ſcale, is one leſs than the number of places or figures contained in that power of the given number whoſe exponent is 10000000000, or the Logarithm of 10, at leaſt as to integer numbers, for they really differ by a fraction, as is ſhewn by Mr. Briggs in his illuſtrations of theſe properties; printed at the end of this Appendix to the Conſtruction of Logarithms.

Kepler gave a conſtruction of Logarithms ſomewhat varied from Napier's. His work is divided into two parts: In the firſt, he raiſes a regular and purely mathematical ſyſtem of proportions, and the meaſures of them, demonſtrating both the nature and principles of the conſtruction of Logarithms, which he calls the *meaſures of ratios:* and in the ſecond part, he applies thoſe principles in the actual conſtruction of his table, which contains only 1000 numbers and their Logarithms. The fundamental principles are briefly theſe: That at the beginning of the Logarithms, their increments or differences are equal to thoſe of the natural numbers: that the natural numbers may be conſidered as the decreaſing coſines of increaſing arcs: and that the ſecants of thoſe arcs at the beginning have the ſame differences as the coſines, and therefore the ſame differences as the Logarithms. Then, ſince the ſecants are the reciprocals of the coſines of the ſame arcs, from the foregoing principles, he eſtabliſhes the following method of raiſing the firſt 100 Logarithms, to the numbers 1000, 999, 998, &c, to 900; viz, in this manner: Divide the radius 1000, increaſed with ſeven ciphers, by each of theſe numbers ſeparately, and the quotients will be the ſecants of thoſe arcs which have the diviſors for their coſines; continuing the diviſion to the 8th figure, as it is in that place only that the arithmetical and geometrical means differ. Then by adding continually the arithmetical means between every two ſucceſſive ſecants, the ſums will be the the ſeries of Logarithms. Or by adding continually every two ſecants, the ſucceſſive ſums will be the ſeries of the double Logarithms. He then derives all the other Logarithms from theſe firſt 100, by common principles.

Briggs firſt adverts to the methods mentioned above, in the Appendix to Napier's Conſtruction, which methods were common to both theſe authors, and had doubtleſs been jointly agreed upon by them. He firſt gives an example of computing a Logarithm by the property, that the Logarithm is one leſs than the number of places or figures contained in that power of the given number whoſe exponent is the Logarithm of 10 with ciphers. Briggs next treats of the other general method of finding the Logarithms of prime numbers, which he thinks is an eaſier way than the former, at leaſt when many figures are required. This method conſiſts in taking a great number of continued geometrical means between 1 and the given number whoſe Logarithm is required; that is, firſt extracting the ſquare root of the given number, then the root of the firſt root, the root of the 2d root, the root of the 3d root, and ſo on, till the laſt root ſhall exceed 1 by a very ſmall decimal, greater or leſs according to the intended number of places to be in the Logarithm ſought: then finding the Logarithm of this ſmall number, by eaſy methods deſcribed afterwards, he doubles it as often as he made extractions of the ſquare root, or, which is the ſame thing, he multiplies it by ſuch power of 2 as is denoted by the ſaid number of extractions, and the reſult is the required Logarithm of the given number; as is evident from the nature of Logarithms.

But as the extraction of ſo many roots is a very troubleſome operation, our author deviſes ſome ingenious contrivances to abridge that labour, chiefly by a proper application of the ſeveral orders of the differences of numbers, forming the firſt inſtance of what may called *the differential method;* but for a particular deſcription of theſe methods, ſee my Treatiſe of Logarithms, above quoted, pag. 65 &c.

Mr. James Gregory, in his Vera Circuli Hyperbolæ Quadratura, printed at Padua in 1667, having approximated to the hyperbolic aſymptotic ſpaces by means of a ſeries of inſcribed and circumſcribed polygons, from thence ſhews how to compute the Logarithms, which are analogous to the areas of thoſe ſpaces: and thus the quadrature of the hyperbolic ſpaces became the ſame thing as the computation of the Logarithms. He here alſo lays down various methods to abridge the computation, with the aſſiſtance of ſome properties of numbers themſelves, by which the Logarithms of all prime numbers under 1000 may be computed, each by one multiplication, two diviſions, and the extraction of the ſquare root. And the ſame ſubject is farther purſued in his Exercitationes Geometricæ. In this latter place, he firſt finds an algebraic expreſſion, in an infinite ſeries, for the Logarithm of $\frac{1+a}{1}$, and then the like for the

Logarithm of $\frac{1}{1-a}$; and as the one ſeries has all its terms poſitive, while thoſe of the other are alternately poſitive and negative, by adding the two together, every 2d term is cancelled, and the double of the other terms gives the Logarithm of the product of

$\frac{1+a}{1}$ and $\frac{1}{1-a}$, or the Logarithm of the $\frac{1+a}{1-a}$, that is of the ratio of $1-a$ to $1+a$: thus, he finds,

$$\text{firſt } a - \tfrac{1}{2}a^2 + \tfrac{1}{3}a^3 - \tfrac{1}{4}a^4 \text{ \&c} = \text{log. of } \frac{1+a}{1},$$

$$\text{and } a + \tfrac{1}{2}a^2 + \tfrac{1}{3}a^3 + \tfrac{1}{4}a^4 \text{ \&c} = \text{log. of } \frac{1}{1-a},$$

$$\text{theref. } 2a + \tfrac{2}{3}a^3 + \tfrac{2}{5}a^5 + \tfrac{2}{7}a^7 \text{ \&c} = \text{l. of } \frac{1+a}{1-a},$$

Which may be accounted Mr. James Gregory's method of making Logarithms.

In 1668, Nicholas Mercator publiſhed his Logarithmotechnia, ſive Methodus Conſtruendi Logarithmos, nova, accurata, & facilis; in which he delivers a new and ingenious method for computing the Logarithms upon principles purely arithmetical; and here, in his modes of thinking and expreſſion, he cloſely follows the celebrated Kepler, in his writings on the ſame ſubject; accounting Logarithms as the meaſures of ratios, or as the number of ratiunculæ contained in the ratio which any number bears to unity. Purely from theſe principles, then, the number of the equal ratiunculæ contained in ſome one ratio, as of 10 to 1, being ſuppoſed given, our author ſhews how the Logarithm, or meaſure, of any other ratio may be found. But this, however, only by-the-bye, as not being the principal method he intends to teach, as his laſt and beſt. Having ſhewn, then, that theſe Logarithms, or numbers of ſmall ratios, or meaſures of ratios, may be all properly repreſented by numbers, and that of 1, or the ratio of equality, the Logarithm or meaſure being always 0, the Logarithm of 10, or the meaſure of the ratio of 10 to 1, is moſt conveniently repreſented by 1 with any number of ciphers; he then proceeds to ſhew how the meaſures of all other ratios may be found from this laſt ſuppoſition: and he explains theſe principles by ſome examples in numbers.

In the latter part of the work, Mercator treats of his other method, given by an infinite ſeries of algebraic terms, which are collected in numbers by common addition only. He here ſquares the hyperbola, and finally finds that the hyperbolic Logarithm of $1+a$, is equal to the infinite ſeries $a - \frac{1}{2}a^2 + \frac{1}{3}a^3 - \frac{1}{4}a^4$ &c; which may be conſidered as Mercator's quadrature of the hyperbola, or his general expreſſion of an hyperbolic Logarithm, in an infinite ſeries.

And this method was farther improved by Dr. Wallis, in the Philoſ. Tranſ. for the year 1668. The celebrated Newton invented alſo the ſame ſeries for the quadrature of the hyperbola, and the conſtruction of Logarithms, and that before the ſame were given by Gregory and Mercator, though unknown to one another, as appears by his letter to Mr. Oldenburg, dated October 24, 1676. The explanation and conſtruction of the Logarithms are alſo farther purſued in his Fluxions, publiſhed in 1736 by Mr. Colſon.

Dr. Halley, in the Philoſ. Tranſ. for the year 1695, gave a very ingenious eſſay on the conſtruction of Logarithms, intitled, "A moſt compendious and facile method for conſtructing the Logarithms, and exemplified and demonſtrated from the nature of numbers, without any regard to the hyperbola, with a ſpeedy method for finding the number from the given Logarithm."

Inſtead of the more ordinary definition of Logarithms, viz, 'numerorum proportionalium æquidifferentes comites,' the learned author adopts this other, 'numeri rationum exponentes,' as better adapted to the principle on which Logarithms are here conſtructed, conſidering them as the number of ratiunculæ contained in the given ratios whoſe Logarithms are in queſtion. In this way he firſt arrives at the Logarithmic ſeries before given by Newton and others, and afterwards, by various combinations and ſections of the ratios, he derives others, converging ſtill faſter than the former. Thus he found the Logarithms of ſeveral ratios, as below, viz, when multiplied by the modulus peculiar to the ſcale of Logarithms,

$$q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4 \text{ \&c, the Log. of 1 to } 1+q,$$

$$q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4 \text{ \&c, the Log. of 1 to } 1-q,$$

$$\frac{x}{a} - \frac{x^2}{2a^2} + \frac{x^3}{3a^3} - \frac{x^4}{4a^4} \text{ \&c, the Log. of } a \text{ to } b, \text{ or}$$

$$\frac{x}{b} + \frac{x^2}{2b^2} + \frac{x^3}{3b^3} + \frac{x^4}{4b^4} \text{ \&c, the ſame Log. of } a \text{ to } b, \text{ or}$$

$$\frac{2x}{z} + \frac{2x^3}{3z^3} + \frac{2x^5}{5z^5} + \frac{2x^7}{7z^7} \text{ \&c, the ſame Log. of } a \text{ to } b,$$

$$\frac{x^2}{2z^2} + \frac{x^4}{4z^4} + \frac{x^6}{6z^6} + \frac{x^8}{8z^8} \text{ \&c, the Log. of } \sqrt{ab} \text{ to } \tfrac{1}{2}z, \text{ or}$$

$$\frac{1}{y^2} + \frac{1}{3y^6} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} \text{ \&c, the ſame Log. of } \sqrt{ab} \text{ to } \tfrac{1}{2}z;$$

where a, b, q, are any quantities, and the values of x, y, z, are thus, viz, $x = b - a$, $z = b + a$, $y = ab + \frac{1}{4}z^2$.

Dr. Halley alſo, firſt of any, performed the reverſe of the problem, by aſſigning the number to a given Logarithm; viz,

$$\frac{b}{a} = 1 + l + \tfrac{1}{2}l^2 + \frac{1}{2 \cdot 3}l^3 + \frac{1}{2 \cdot 3 \cdot 4}l^4 \text{ \&c, or}$$

$$\frac{a}{b} = 1 - l + \tfrac{1}{2}l^2 - \frac{1}{2 \cdot 3}l^3 + \frac{1}{2 \cdot 3 \cdot 4}l^4 \text{ \&c.}$$

where l is the Logarithm of the ratio of a the leſs, to b the greater of any two terms.

Mr. Abraham Sharp of Yorkſhire made many calculations and improvements in Logarithms, &c. The moſt remarkable of theſe were, his quadrature of the circle to 72 places of figures, and his computation of Logarithms to 61 figures, viz, for all numbers to 100, and for all prime numbers to 1100.

The celebrated Mr. Roger Cotes gave to the world a learned tract on the nature and conſtruction of Logarithms: this was firſt printed in the Philoſ. Tranſ. No 338, and afterwards with his Harmonia Menſurarum in 1722, under the title Logometria. This tract has juſtly been complained of, as very obſcure and intricate, and the principle is ſomething between that of Kepler and the method of Fluxions. He invented the terms Modulus and Modular ratio, this being the ratio

$$\text{of } 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} \text{ \&c to 1 or}$$

$$\text{of 1 to } 1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} - \frac{1}{2 \cdot 3 \cdot 4 \cdot 5} \text{ \&c;}$$

&c; that is the ratio of 2·718281828459 &c to 1, or the ratio of 1 to 0·367879441171 &c; the modulus of any syftem being the meafure or Logarithm of that ratio, which in the hyp. Logarithms is 1, and in Briggs's or the common Logarithms is 0·434294481903 &c.

The learned Dr. Brook Taylor gave another method of computing Logarithms in the Philof. Tranf. No. 352, which is founded on thefe three principles, viz, 1ft, That the fum of the Logarithms of any two numbers is the Logarithm of the product of thofe numbers; 2d, That the Logarithm of 1 is 0, and confequently that the nearer any number is to 1, the nearer will its Logarithm be to 0; 3d, That the product of two numbers or factors, of which the one is greater and the other lefs than 1, is nearer to 1, than that factor is which is on the fame fide of 1 with itfelf; fo of the two numbers $\frac{2}{3}$ and $\frac{4}{3}$, the product $\frac{8}{9}$ is lefs than 1, but yet nearer to it than $\frac{2}{3}$ is, which is alfo lefs than 1.—And on thefe principles he founds an ingenious, though not very obvious, approximation to the Logarithms of given numbers.

In the Philof. Tranf. a Mr. John Long gave a method of conftructing Logarithms, by means of a fmall table, fomething in the manner of one of Briggs's methods for the fame purpofe.

Alfo in the Philof. Tranf. vol. 61, a tract on the conftruction of Logarithms is given by the ingenious Mr. William Jones. In this method, all numbers are confidered as fome certain powers of a conftant determined root: thus, any number x is confidered as the z power of any root r, or $x = r^z$ is taken as a general expreffion for all numbers in terms of the conftant root r and a variable exponent z. Now the index z being the Logarithm of the number x, therefore to find this Logarithm, is the fame thing as to find what power of the radix r is equal to the number x.

An elegant tract on Logarithms, as a comment on Dr. Halley's method, was alfo given by Mr. Jones in his Synopfis Palmariorum Mathefeos, publifhed in the year 1706.

In the year 1742, Mr. James Dodfon publifhed his Anti-logarithmic Canon, containing all Logarithms under 100,000, and their correfponding natural numbers to eleven places of figures, with all their differences and the proportional parts; the whole arranged in the order contrary to that ufed in the common tables of numbers and Logarithms, the exact Logarithms being here placed firft, and their correfponding neareft numbers in the columns oppofite to them.

And in 1767, Mr. Andrew Reid publifhed an "Effay on Logarithms," in which he fhews the computation of Logarithms from principles depending on the binomial theorem, and on the nature of the exponents of powers, the Logarithms of numbers being here confidered as the exponents of the powers of 10. In this way he brings out the ufual feries for Logarithms, and exemplifies Dr. Halley's conftruction of them. But for the particulars of this, and the methods given by the other authors, we muft refer to the hiftorical preface to my treatife on Logarithms.

Befides the authors above-mentioned, many others have treated on the fubject of Logarithms; among the principal of whom are Leibnitz, Euler, Maclaurin, Wolfius, Keill, and profeffor Simfon in an ingenious geometrical tract on Logarithms, contained in his pofthumous works, elegantly printed at Glafgow in the year 1776, at the expence of the learned Earl Stanhope, and by his lordfhip difpofed of in prefents among gentlemen moft eminent for mathematical learning.

For the defcription and ufes of Logarithms in numeral calculations, with the fhorteft method of conftructing them, fee the Hiftorical Introduction to my Logarithms, pa. 124 & feq.

Briggs's or *Common* LOGARITHMS, are thofe that have 1 for the Logarithm of 10, or which have 0·4342944819 &c for the modulus; as has been explained above.

Hyperbolic LOGARITHMS, are thofe that were computed by the inventor Napier, and called alfo fometimes *Natural Logarithms*, having 1 for their modulus, or 2·302585092994 &c for the Logarithm of 10. Thefe have fince been called Hyperbolical Logarithms, becaufe they are analogous to the areas of a right-angled hyperbola between the afymptotes and the curve. See LOGARITHMS, alfo HYPERBOLA and ASYMPTOTIC SPACE.

Logiftic LOGARITHMS, are certain Logarithms of fexagefimal numbers or fractions, ufeful in aftronomical calculations. The Logiftic Logarithm of any number of feconds, is the difference between the common Logarithm of that number and the Logarithm of 3600, the feconds in 1 degree.

The chief ufe of the table of Logiftic Logarithms, is for the ready computing a proportional part in minutes and feconds, when two terms of the proportion are minutes and feconds, or hours and minutes, or other fuch fexagefimal numbers. See the Introd. to my Logarithms, pa. 144.

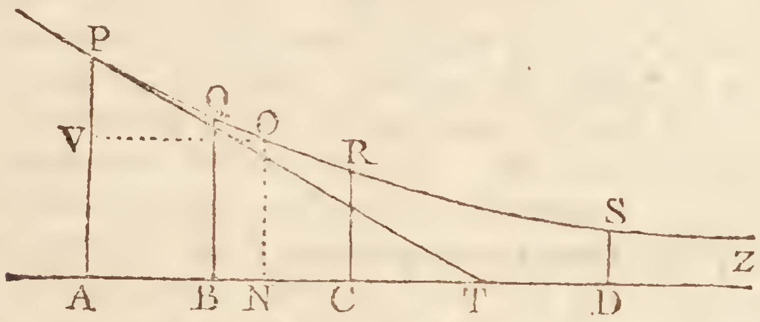
Imaginary LOGARITHM, a term ufed in the Log. of imaginary and negative quantities; fuch as $-a$, or $\sqrt{-a^2}$ or $a\sqrt{-1}$. The fluents of certain imaginary expreffions are alfo Imaginary Logarithms; as of $\frac{\dot{x}}{x\sqrt{-1}}$, or of $\frac{a\dot{x}}{cx\sqrt{-1}}$, &c. See Euler Analyf. Infin. vol. i. pa. 72, 74.

It is well known that the expreffion $\frac{\dot{x}}{x}$ reprefents the fluxion of the Logarithm of x, and therefore the fluent of $\frac{\dot{x}}{x}$ is the Logarithm of x; and hence the fluent of $\frac{\dot{x}}{x\sqrt{-1}}$ is the Imaginary Logarithm of x.

However, when thefe Imaginary Logarithms occur in the folutions of problems, they may be transformed into circular arcs or fectors; that is, the Imaginary Logarithm, or imaginary hyperbolic fector, becomes a real circular fector. See Bernoulli Oper. tom. i, pa. 400, and pa. 512. Maclaurin's Fluxions, art. 762. Cotes's Harmon. Menf. pa. 45. Walmefley, Anal. des Mef. pa. 63.

LOGARITHMIC, or LOGISTIC CURVE, a curve fo called from its properties and ufes, in explaining and conftructing the Logarithms, becaufe its ordinates are in geometrical progreffion, while the abfciffes are in arithmetical progreffion; fo that the abfciffes are as the Logarithms of the correfponding ordinates. And hence the

the curve will be constructed in this manner: Upon any right line, as an axis, take the equal parts AB, BC, CD, &c, or the arithmetical progression AB, AC, AD, &c; and at the points A, B, C, D, &c, erect the perpendicular ordinates AP, BQ, CR, DS, &c, in a geometrical progression; so is the curve line drawn through all the points P, Q, R, S, &c, the Logarithmic, or Logistic Curve; so called, because any abscis AB, is as the Logarithm of its ordinate BQ. So that the axis ABC &c is an asymptote to the curve.



Hence, if any abscis AN $= x$, its ordinate NO $= y$, AP $= 1$, and $a =$ a certain constant quantity, or the modulus of the Logarithms; then the equation of the curve is $x = a \times \log.$ of $y = \log. y^a$.

And if the fluxion of this equation be taken, it will be $\dot{x} = \frac{a\dot{y}}{y}$; which gives this proportion,

$$\dot{y} : \dot{x} :: y : a$$

but in any curve $\dot{y} : \dot{x} :: y :$ the subtangent AT; and therefore the subtangent of this curve is everywhere equal to the constant quantity a, or the modulus of the Logarithms.

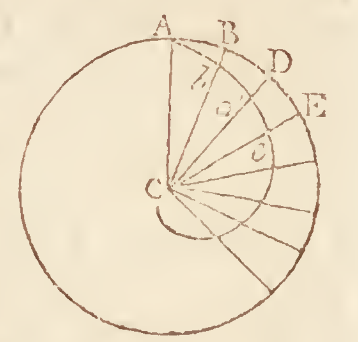
To find the Area contained between two ordinates. Here the fluxion of the area $\dot{A}$ or $y\dot{x}$ is $y \times \frac{a\dot{y}}{y} = a\dot{y}$; and the correct fluent is $A = a \times \overline{AP - y} = a \times \overline{AP - NO} = a \times PV = AT \times PV$. That is, the area APON between any two ordinates, is equal to the rectangle of the constant subtangent and the difference of the ordinates. And hence, when the abscis is infinitely long, or the farther ordinate equal to nothing, then the infinitely long area APZ is equal AT × AP, or double the triangle APT.

For the Solid formed by the curve revolved about its axis AZ. The fluxion of the solid is $\dot{s} = py^2\dot{x} = py^2 \times \frac{a\dot{y}}{y} = pay\dot{y}$, where p is $= 3\cdot1416$; and the correct fluent is $s = \frac{1}{2}pa \times \overline{AP^2 - y^2} = \frac{1}{2}p \times AT \times \overline{AP^2 - NO^2}$, which is half the difference between two cylinders of the common altitude a or AT, and the radii of their bases AP, NO. And hence supposing the solid infinitely long towards Z, where y or the ordinate is nothing, the infinitely long solid will be equal to $\frac{1}{2}pa \times AP^2 = \frac{1}{2}p \times AT \times AP^2$, or half the cylinder on the same base and its altitude AT.

It has been said that Gunter gave the first idea of a curve whose abscisses are in arithmetical progression, while the corresponding ordinates are in geometrical progression, or whose abscis are the Logarithms of their ordinates; but I do not find it noticed in any part of his writings. This curve was afterwards considered by others, and named the Logarithmic or Logistic Curve by Huygens in his Dissertatio de Causa Gravitatis, where he enumerates all the principal properties of it, shewing its analogy to Logarithms. Many other learned men have also treated of its properties; particularly Le Seur and Jacquier, in their Comment on Newton's Principia; Dr. John Keill, in the elegant little Tract on Logarithms subjoined to his edition of Euclid's Elements; and Francis Maseres Esq. Cursitor Baron of the Exchequer, in his ingenious Treatise on Trigonometry: see also Bernoulli's Discourse in the Acta Eruditorum for the year 1696, pa. 216; Guido Grando's Demonstratio Theorematum Huygeneanorum circa Logisticam seu Logarithmicam Lineam; and Emerson on Curve Lines, pa. 19.—It is indeed rather extraordinary that this curve was not sooner announced to the public, since it results immediately from Napier's manner of conceiving the generation of Logarithms, by only supposing the lines which represent the natural numbers as placed at right angles to that upon which the Logarithms are taken.

This curve greatly facilitates the conception of Logarithms to the imagination, and affords an almost intuitive proof of the very important property of their fluxions, or very small increments, namely, that the fluxion of the number is to the fluxion of the Logarithm, as the number is to the subtangent: as also of this property, that if three numbers be taken very nearly equal, so that their ratios may differ but a little from a ratio of equality, as the three numbers 10000000, 10000001, 10000002, their differences will be very nearly proportional to the Logarithms of the ratios of those numbers to each other: all which follows from the Logarithmic arcs being very little different from their chords, when they are taken very small. And the constant subtangent of this curve is what was afterwards by Cotes called the Modulus of the System of Logarithms.

Logarithmic, or *Logistic, Spiral*, a curve constructed as follows. Divide the arch of a circle into any equal parts AB, BD, DE, &c; and upon the radii drawn to the points of division take C*b*, C*d*, C*e*, &c, in a geometrical progression; so is the curve A*bde* &c the Logarithmic Spiral; so called, because it is evident that AB, AD, AE, &c, being arithmeticals, are as the the Logarithms of CA, C*b*, C*d*, C*e*, &c, which are geometricals; and a Spiral, because it winds continually about the centre C, coming continually nearer, but without ever really falling into it.



In the Philos. Trans. Dr. Halley has happily applied this curve to the division of the meridian line in Mercator's chart. See also Cotes's Harmonia Mens., Guido Grando's Demonst. Theor. Huygen., the Acta Erudit. 1691, and Emerson's Curves, &c.

LOGISTICS, or LOGISTICAL Arithmetic, a name sometimes employed for the arithmetic of sexagesimal fractions, used in astronomical computations.

This name was perhaps taken from a Greek treatise of Barlæmus, a Monk, who wrote a book of Sexagesimal Multiplication, which he called Logistic. Vossius places this author about the year 1350, but he mistakes the work for a Treatise on Algebra.

The same term however has been used for the rules of

of computations in Algebra, and in other ſpecies of Arithmetic: witneſs the Logiſtics of Vieta and other writers.

Shakerly, in his Tabulæ Britannicæ, has a Table of Logarithms adapted to ſexageſimal fractions, and which he calls Logiſtical Logarithms; and the expeditious arithmetic, obtained by means of them, he calls Logiſtical Arithmetic.

Logistical *Curve*, *Line*, or *Spiral*, the ſame as the Logarithmic, which ſee.

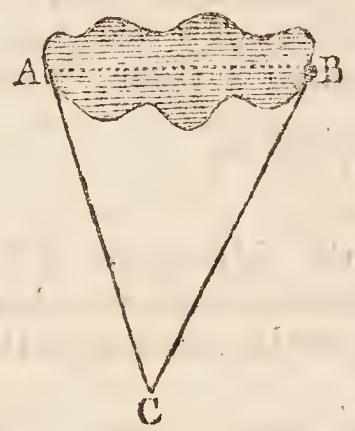
LONG (Roger), D.D. maſter of Pembroke-hall in Cambridge, Lowndes's profeſſor of aſtronomy in that univerſity, &c, was author of a well-known and much approved treatiſe of aſtronomy, and the inventor of a remarkably curious aſtronomical machine. This was a hollow ſphere, of 18 feet diameter, in which more than 30 perſons might ſit conveniently. Within ſide the ſurface, which repreſented the heavens, was painted the ſtars and conſtellations, with the zodiac, meridians, and axis parallel to the axis of the world, upon which it was eaſily turned round by a winch. He died, December 16, 1770, at 91 years of age.

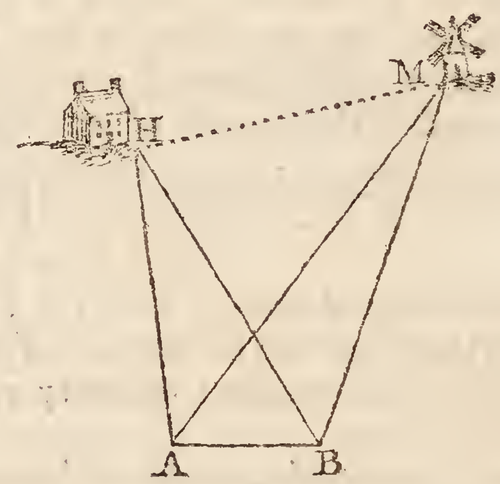
A few years before his death, Mr. Jones gave ſome anecdotes of Dr. Long, as follows: "He is now in the 88th year of his age, and for his years vegete and active. He was lately put in nomination for the office of vice-chancellor: he executed that truſt once before, I think in the year 1737. He is a very ingenious perſon, and ſometimes very facetious. At the public Commencement, in the year 1713, Dr. Greene (maſter of Bennet college, and afterwards biſhop of Ely) being then vice-chancellor, Mr. Long was pitched upon for the tripos performance; it was witty and humorous, and has paſſed through divers editions. Some that remembered the delivery of it, told me, that in addreſſing the vice-chancellor (whom the univerſity wags uſually ſtyled *Miſs Greene)*, the tripos-orator, being a native of Norfolk, and aſſuming the Norfolk dialect, inſtead of ſaying, *Domine Vice-Cancellarie*, archly pronounced the words thus, *Domina Vice-Cancellaria*; which occaſioned a general ſmile in that great auditory. His friend the late Mr. Bonfoy of Ripton told me this little incident: 'That he and Dr. Long walking together in Cambridge in a duſky evening, and coming to a ſhort *poſt* fixed in the pavement, which Mr. Bonfoy in the midſt of chat and inattention, took to be a *boy* ſtanding in his way, he ſaid in a hurry, 'Get out of my way, boy!' '*That boy, Sir*, ſaid the Doctor very calmly and ſlily, *is a* poſt-boy, *who turns out of his way for nobody*.' I could recollect ſeveral other ingenious repartees if there were occaſion. One thing is remarkable, he never was a hale and hearty man, always of a tender and delicate conſtitution, yet took great care of it: his common drink water; he always dines with the Fellows in the Hall Of late years he has left off eating fleſh-meats; in the room thereof, puddings; vegetables, &c; ſometimes a glaſs or two of wine."

LONGIMETRY, the art of meaſuring lengths or diſtances, both acceſſible and inacceſſible, forming a part of what is called Heights and Diſtances, being an application of geometry and trigonometry to ſuch meaſurements.

As to acceſſible lengths, they are eaſily meaſured by the actual application of a rod, a chain, or wheel, or ſome other meaſure of length.

But inacceſſible lengths require the practice and properties of geometry and trigonometry, either in the meaſurement and conſtruction, or in the computation. For example, Suppoſe it were required to know the length or diſtance between the two places A and B, to which places there is free acceſs, but not to the intermediate parts, on account of water or ſome other impediment; meaſure therefore, from A and B, the diſtances to any convenient place C, which ſuppoſe to be thus, viz, AC = 735, and BC = 840 links; and let the angle at C, taken with a theodolite or other inſtrument, be 55° 40′. From theſe meaſures the length or diſtance AB may be determined, either by geometrical meaſurement, or by trigonometrical computation. Thus, firſt, lay down an angle C = 55° 40′, and upon its legs ſet off, from any convenient ſcale of equal parts, CA = 735, and CB = 840; then meaſure the diſtance between the points A and B by the ſame ſcale of equal parts, which will be found to be 740 nearly.





Or this by calculation,

840 180° − 55° 40′ = 124° 20′, its half 62° 10′,
735

Sum 1575	- - -	1·1972806
Dif. 105	- - -	0·0211893
Tang. 62° 10′	- -	10·2773793
Tang. 7 11$\frac{1}{4}$	-	9·1012880
ſ. Sum or ∠ A = 99° 21′$\frac{1}{4}$		9·9711092
to ſ. ∠ C = 55 40		9·9168593
So BC = 840		0·9242793
To AB = 741·2		0·8699404

For a 2d Example—Suppoſe it were required to find the diſtance between two inacceſſible objects, as between the houſe and mill, H and M; firſt meaſure any convenient line on the ground, as AB, 300 yards; then at the ſtation A take the angles BAM = 58° 20′; and MAH = 37°; alſo at the ſtation B take the angles ABH = 53° 30′, and HBM = 45° 15′; from hence the diſtance or length MH may be found, either by geometrical conſtruction, or by trigonometrical calculation, thus:

Firſt draw a line AB of the given length of 300, by a convenient ſcale of equal parts; then at the point A lay down the angles BAM and MAH of the magnitudes

nitudes above given ; and alſo at the point B the given angles ABH and HBM : then by applying the length HM to the ſame ſcale of equal parts, it is found to be nearly 480 yards.

Otherwiſe, by calculation. Firſt, by adding and ſubtracting the angles, there is found as below :

	37° 00′	58° 20′		53° 30′
	58 20	53 30		45 15
	53 30	45 15	ſum	98 45 ∠ ABM
ſums	148 50	157 05		
from	180 00	180 00		
∠ AHB	31 10	22 55 ∠ AMB		

Then,

as ſin. AHB : ſin. ABH : : AB : AH $= 465\cdot9776$,
and, as ſin. AMB : ſin. ABM : : AB : AM $= 761\cdot4655$;

their ſum is $1227\cdot4431$
and their diff. $295\cdot4879$

Then as ſum AM + AH : to dif. AM − AH : :
tang. $\frac{1}{2}$ AHM + $\frac{1}{2}$ AMH $= 71° 30'$,
to tang. $\frac{1}{2}$ AHM − $\frac{1}{2}$ AMH $= 35\ 44$

the dif. of which is AMH $= 35\ 46$.

Laſtly,

as ſ. AMH : ſ. MAH : : AH : HM $= 479\cdot7933$, the diſtance ſought.

LONGITUDE *of the Earth*, is ſometimes uſed to denote its extent from weſt to eaſt, according to the direction of the equator. By which it ſtands contradiſtinguiſhed from the Latitude of the earth, which denotes its extent from one pole to the other.

Longitude *of a Place*, in Geography, is its longitudinal diſtance from ſome firſt meridian, or an arch of the equator intercepted between the meridian of that place and the firſt meridian.

Longitude *in the Heavens*, as of a ſtar, &c, is an arch of the ecliptic, counted from the beginning of Aries, to the place where it is cut by a circle perpendicular to it, and paſſing through the place of the ſtar.

Longitude *of the Sun or Star from the next equinoctial point*, is the degrees they are diſtant from the beginning of Aries or Libra, either before or after them ; which can never exceed 180 degrees.

Longitude, *Geocentric*, *Heliocentric*, &c, the Longitude of a planet as ſeen from the earth, or from the ſun. See the reſpective terms.

Longitude, *in Navigation*, is the diſtance of a ſhip, or place, eaſt or weſt, from ſome other place or meridian, counted in degrees of the equator. When this diſtance is counted in leagues, or miles, or in degrees of the meridian, and not in thoſe proper to the parallel of Latitude, it is uſually called Departure.

An eaſy practicable method of finding the Longitude at ſea, is the only thing wanted to render the Art of Navigation perfect, and is a problem that has greatly perplexed mathematicians for the laſt two centuries : accordingly moſt of the commercial nations of Europe have offered great rewards for the diſcovery of it ; and in conſequence very conſiderable advances have been made towards a perfect ſolution of the problem, eſpecially by the Engliſh.

In the year 1598, the government of Spain offered a reward of 1000 crowns for the ſolution of this problem ; and ſoon after the States of Holland offered 10 thouſand florins for the ſame. Encouraged by ſuch offers, in 1635, M. John Morin, profeſſor of mathematics at Paris, propoſed to cardinal Richlieu, a method of reſolving it ; and though the commiſſioners, who were appointed to examine this method, on account of the imperfect ſtate of the lunar tables, judged it inſufficient, cardinal Mazarin, in 1645, procured for the author a penſion of 2000 livres.

In 1714 an act was paſſed in the Britiſh parliament, allowing 2000l. towards making experiments ; and alſo offering a reward to the perſon who ſhould diſcover the Longitude at ſea, proportioned to the degree of accuracy that might be attained by ſuch diſcovery ; viz, a reward of 10,000l. if it determines the Longitude to one degree of a great circle, or 60 geographical miles ; 15,000l. if it determines the ſame to two-thirds of that diſtance ; and 20,000l. if it determines it to half that diſtance ; with other regulations and encouragements. 12 Ann. cap. 15. See alſo ſtat. 14 Geo. II, cap. 39, and 26 Geo. II, cap. 25. But, by ſtat. Geo. III, all former acts concerning the Longitude at ſea are repealed, except ſo much of them as relates to the appointment and authority of the commiſſioners, and ſuch clauſes as relate to the publiſhing of nautical almanacs, and other uſeful tables ; and it enacts, that any perſon who ſhall diſcover a method for finding the Longitude by means of a time-keeper, the principles of which have not hitherto been made public, ſhall be entitled to the reward of 5000l. if it ſhall enable a ſhip to keep her Longitude, during a voyage of 6 months, within 60 geographical miles, or one degree of a great circle ; to 7500l. if within 40 geographical miles, or two-thirds of a degree of a great circle ; or to a reward of 10,000l. if within 30 geographical miles, or half a degree of a great circle. But if the method ſhall be by means of improved ſolar and lunar tables, the author of them ſhall be entitled to a reward of 5000l. if they ſhew the diſtance of the moon from the ſun and ſtars within 15″ of a degree, anſwering to about 7′ of Longitude, after making an allowance of half a degree for the errors of obſervation, and after compariſon with aſtronomical obſervations for a period of $18\frac{1}{2}$ years, or during the period of the irregularities of the lunar motions. Or that in caſe any other method ſhall be propoſed for finding the Longitude at ſea, beſides thoſe before-mentioned, the author ſhall be entitled to 5000l. if it ſhall determine the Longitude within one degree of a great circle, or 60 geographical miles ; to 7500l. if within two-thirds of that diſtance ; and to 10,000l. if within half the ſaid diſtance.

Accordingly, many attempts have been made for ſuch diſcovery, and ſeveral ways propoſed, with various degrees of ſucceſs. Theſe however have been chiefly directed to methods of determining the difference of time between any two points on the earth ; for the Longitude of any place being an arch of the equator intercepted between two meridians, and this arc being proportional to the time required by the ſun to move from the one meridian to the other, at the rate of 4 minutes of time to one degree of the arch, it follows that the difference of time being known, and turned into

into degrees according to that proportion, it will give the Longitude.

This meaſurement of time has been attempted by ſome perſons by means of clocks, watches, and other automata: for if a clock or watch were contrived to go uniformly at all ſeaſons, and in all places and ſituations; ſuch a machine being regulated, for inſtance, to London or Greenwich time, would always ſhew the time of the day at London or Greenwich, wherever it ſhould be carried to; then the time of the day at this place being found by obſervations, the difference between theſe two times would give the difference of Longitude, according to the proportion of one degree to 4 minutes of time.

Gemma Friſius, in his tract De Principiis Aſtronomiæ et Geographiæ, printed at Antwerp in 1530, it ſeems firſt ſuggeſted the method of finding the Longitude at ſea by means of watches, or time-keepers; which machines, he ſays, were then but lately invented. And ſoon after, the ſame was attempted by Metius, and ſome others; but the ſtate of watch-making was then too imperfect for that purpoſe. Dr. Hooke and Mr. Huygens alſo, about the year 1664, applied the invention of the pendulum-ſpring to watches; and employed it for the purpoſe of diſcovering the Longitude at ſea. Some diſputes however between Dr. Hooke and the Engliſh Miniſtry prevented any experiments from being made with watches conſtructed by him; but many experiments were made with ſome conſtructed by Huygens; particularly Major Holmes, in a voyage from the coaſt of Guinea in 1665, by one of theſe watches predicted the Longitude of the iſland of Fuego to a great degree of accuracy. This ſucceſs encouraged Huygens to improve the ſtructure of his watches, (ſee Philoſ. Tranſ. for May 1669); but experience ſoon convinced him, that unleſs methods could be diſcovered for preſerving the regular motion of ſuch machines, and preventing the effects of heat and cold, and other diſturbing cauſes, they could never anſwer the intention of diſcovering the Longitude, and on this account his attempts failed.

The firſt perſon who turned his thoughts this way, after the public encouragement held out by the act of 1714, was Henry Sully, an Engliſhman; who, in the ſame year, printed at Vienna, a ſmall tract on the ſubject of watch-making; and afterwards removing to Paris, he employed himſelf there in improving time-keepers for the diſcovery of the Longitude. It is ſaid he greatly diminiſhed the friction in the machine, and rendered uniform that which remained: and to him is principally to be attributed what is yet known of watch-making in France: for the celebrated Julien le Roy was his pupil, and to him owed moſt of his inventions, which he afterwards perfected and executed: and this gentleman, with his ſon, and M. Berthoud, are the principal perſons in France who have turned their thoughts this way ſince the time of Sully. Several watches made by theſe laſt two artiſts, have been tried at ſea, it is ſaid with good ſucceſs, and large accounts have been publiſhed of theſe trials.

In the year 1726 our countryman, Mr. John Harriſon, produced a time-keeper of his own conſtruction, which did not err above one ſecond in a month, for 10 years together: and in the year 1736 he had a machine tried in a voyage to and from Liſbon; which was the means of correcting an error of almoſt a degree and a half in the computation of the ſhip's reckoning. In conſequence of this ſucceſs, Mr. Harriſon received public encouragement to proceed, and he made three other time-keepers, each more accurate than the former, which were finiſhed ſucceſſively in the years 1739, 1758, and 1761; the laſt of which proved ſo much to his own ſatisfaction, that he applied to the commiſſioners of the Longitude to have this inſtrument tried in a voyage to ſome port in the Weſt Indies, according to the directions of the ſtatute of the 12th of Anne above cited. Accordingly, Mr. William Harriſon, ſon of the inventor, embarked in November 1761, on a voyage for Jamaica, with this 4th time-keeper or watch; and on his arrival there, the Longitude, as ſhewn by the time-keeper, differed but one geographical mile and a quarter from the true Longitude, deduced from aſtronomical obſervations. The ſame gentleman returned to England, with the time-keeper, in March 1762; when he found that it had erred, in the 4 months, no more than $1' 54''\frac{1}{2}$ in time, or $28\frac{5}{8}$ minutes of Longitude; whereas the act requires no greater exactneſs than 30 geographical miles, or minutes of a great circle, in ſuch a voyage. Mr. Harriſon now claimed the whole reward of 20,000l, offered by the ſaid act: but ſome doubts ariſing in the minds of the commiſſioners, concerning the true ſituation of the iſland of Jamaica, and the manner in which the time at that place had been found, as well as at Portſmouth; and it being farther ſuggeſted by ſome, that although the time-keeper happened to be right at Jamaica, and after its return to England, it was by no means a proof that it had been always ſo in the intermediate times; another trial was therefore propoſed, in a voyage to the iſland of Barbadoes, in which precautions were taken to obviate as many of theſe objections as poſſible. Accordingly, the commiſſioners previouſly ſent out proper perſons to make aſtronomical obſervations at that iſland, which, when compared with other correſponding ones made in England, would determine, beyond a doubt, its true ſituation: and Mr. William Harriſon again ſet out with his father's time-keeper, in March 1764, the watch having been compared with equal altitudes at Portſmouth, before he ſet out, and he arrived at Barbadoes about the middle of May; where, on comparing it again by equal altitudes of the ſun, it was found to ſhew the difference of Longitude, between Portſmouth and Barbadoes, to be $3^h\ 55^m\ 3^s$; the true difference of Longitude between theſe places, by aſtronomical obſervations, being $3^h\ 54^m\ 20^s$; ſo that the error of the watch was 43^s, or $10'\ 45''$ of Longitude. In conſequence of this, and the former trials, Mr. Harriſon received one moiety of the reward offered by the 12th of Queen Anne, after explaining the principles on which his watch was conſtructed, and delivering this as well as the three former to the Commiſſioners of the Longitude, for the uſe of the public: and he was promiſed the other moiety of the reward, when other time-keepers ſhould be made, on the ſame principles, either by himſelf or others, performing equally well with that which he had laſt made. In the mean time, this laſt time-keeper was ſent down to the Royal Obſervatory at Greenwich, to be tried there under the direction of the Rev. Dr. Maſkelyne, the Aſtronomer Royal. But it did

did not appear, during this trial, that the watch went with the regularity that was expected; from which it was apprehended, that the performance even of the same watch, was not at all times equal; and consequently that little certainty could be expected in the performance of different ones. Moreover, the watch was now found to go faster than during the voyage to and from Barbadoes, by 18 or 19 seconds in 24 hours: but this circumstance was accounted for by Mr. Harrison; who informs us that he had altered the rate of its going by trying some experiments, which he had not time to finish before he was ordered to deliver up the watch to the Board. Soon after this trial, the Commissioners of Longitude agreed with Mr. Kendal, one of the watch-makers appointed by them to receive Mr. Harrison's discoveries, to make another watch on the same construction with this, to determine whether such watches could be made from the account which Mr. Harrison had given, by other persons, as well as himself. The event proved the affirmative; for the watch produced by Mr. Kendal, in consequence of this agreement, went considerably better than Mr. Harrison's did. Mr. Kendal's watch was sent out with Capt. Cook, in his 2d voyage towards the south pole and round the globe, in the year 1772, 1773, 1774, and 1775; when the only fault found in the watch was, that its rate of going was continually accelerated; though in this trial, of 3 years and a half, it never amounted to $14''\frac{1}{2}$ a day. The consequence was, that the House of Commons in 1774, to whom an appeal had been made, were pleased to order the 2d moiety of the reward to be given to Mr. Harrison, and to pass the act above mentioned. Mr. Harrison had also at different times received some other sums of money, as encouragements to him to continue his endeavours, from the Board of Longitude, and from the India Company, as well as from many individuals. Mr. Arnold and some other persons have since also made several very good watches for the same purpose.

Others have proposed various astronomical methods for finding the Longitude. These methods chiefly depend on having an ephemeris or almanac suited to the meridian of some place, as Greenwich for instance, to which the Nautical Almanac is adapted, which shall contain for every day computations of the times of all remarkable celestial motions and appearances, as adapted to that meridian. So that, if the hour and minute be known when any of the same phenomena are observed in any other place, whose Longitude is desired, the difference between this time and that to which the time of the said phenomenon was calculated and set down in the almanac, will be known, and consequently the difference of Longitude also becomes known, between that place and Greenwich, allowing at the rate of 15 degrees to an hour.

Now it is easy to find the time at any place, by means of the altitude or azimuth of the sun or stars; which time it is necessary to find by such means, both in these astronomical modes of determining the Longitude, and in the former by a time-keeper; and it is the difference between that time, so determined, and the time at Greenwich, known either by the time-keeper or by the astronomical observations of celestial phenomena, which gives the difference of Longitude, at the rate above-mentioned. Now the difficulty in these methods lies in the fewness of proper phenomena, capable of being thus observed; for all slow motions, such as belong to the planet Saturn for instance, are quite excluded, as affording too small a difference, in a considerable space of time, to be properly observed; and it appears that there are no phenomena in the heavens proper for this purpose, except the eclipses or motions of Jupiter's satellites, and the eclipses or motions of the moon, viz, such as her distance from the sun or certain fixed stars lying near her path, or her Longitude or place in the zodiac, &c. Now of these methods,

1st, That by the eclipses of the moon is very easy, and sufficiently accurate, if they did but happen often, as every night. For at the moment when the beginning, or middle, or end of an eclipse is observed by a telescope, there is no more to be done but to determine the time by observing the altitude or azimuth of some known star; which time being compared with that in the tables, set down for the happening of the same phenomenon at Greenwich, gives the difference in time, and consequently of Longitude sought. But as the beginning or end of an eclipse of the moon cannot generally be observed nearer than one minute, and sometimes 2 or 3 minutes of time, the Longitude cannot certainly be determined by this method, from a single observation, nearer than one degree of Longitude. However, by two or more observations, as of the beginning and end &c, a much greater degree of exactness may be attained.

2d, The moon's place in the zodiac is a phenomenon more frequent than that of her eclipses; but then the observation of it is difficult, and the calculus perplexed and intricate, by reason of two parallaxes; so that it is hardly practicable, to any tolerable degree of accuracy.

3d, But the moon's distances from the sun, or certain fixed stars, are phenomena to be observed many times in almost every night, and afford a good practical method of determining the Longitude of a ship at almost any time; either by computing, from thence, the moon's true place, to compare with the same in the almanac; or by comparing her observed distance itself with the same as there set down.

It is said that the first person who recommended the finding the Longitude from this observed distance between the moon and some star, was John Werner, of Nuremberg, who printed his annotations on the first book of Ptolomy's Geography in 1514. And the same thing was recommended in 1524, by Peter Apian, professor of mathematics at Ingolstadt; also about 1530, by Oronce Finé, of Briançon; and the same year by the celebrated Kepler, and by Gemma Frisius, at Antwerp; and in 1560, by Nonius or Pedro Nunez.

Nor were the English mathematicians behind hand on this head. In 1665 Sir Jonas Moore prevailed on king Charles the 2d to erect the Royal Observatory at Greenwich, and to appoint Mr. Flamsteed his astronomical observer, with this express command, that he should apply himself with the utmost care and diligence to the rectifying the table of the motions of the heavens, and the places of the fixed stars, in order to find out the so much desired Longitude at sea, for perfecting the Art of Navigation. And to the fidelity and industry with

with which Mr. Flamsteed executed his commission, it is that we are chiefly indebted for that curious theory of the moon, which was afterwards formed by the immortal Newton. This incomparable philosopher made the best possible use of the observations with which he was furnished; but as these were interrupted and imperfect, his theory would sometimes differ from the heavens by 5 minutes or more.

Dr. Halley bestowed much time on the same object; and a Starry Zodiac was published under his direction, containing all the stars to which the moon's appulse can be observed; but for want of correct tables, and proper instruments, he could not proceed in making the necessary observations. In a paper on this subject, in the Philos. Trans. number 421, he expresses his hope, that the instrument just invented by Mr. Hadley might be applied to taking angles at sea with the desired accuracy. This great astronomer, and after him the Abbé de la Caille, and others, have reckoned the best astronomical method for finding the Longitude at sea, to be that in which the distance of the moon from the sun or from a star is used; for the moon's daily motion being about 13 degrees, her hourly mean motion is above half a degree, or one minute of a degree in two minutes of time; so that an error of one minute of a degree in position will produce an error of 2 minutes in time, or half a degree in Longitude. Now from the great improvements made by Newton in the theory of the moon, and more lately by Euler and others on his principles, professor Mayer, of Gottengen, was enabled to calculate lunar tables more correct than any former ones; having so far succeeded as to give the moon's place within one minute of the truth, as has been proved by a comparison of the tables with the observations made at the Greenwich observatory by the late Dr. Bradley, and by Dr. Maskeline, the present Astronomer Royal; and the same have been still farther improved under his direction, by the late Mr. Charles Mason, by several new equations, and the whole computed to tenths of a second. These new tables, when compared with the above-mentioned series of observations, a proper allowance being made for the unavoidable error of observation, seem to give always the moon's Longitude in the heavens correctly within 30 seconds of a degree; which greatest error, added to a possible error of one minute in taking the moon's distance from the sun or a star at sea, will at a medium only produce an error of 42 minutes of Longitude. To facilitate the use of the tables, Dr. Maskelyne proposed a nautical ephemeris, the scheme of which was adopted by the Commissioners of Longitude, and first executed in the year 1767, since which time it has been regularly continued, and published as far as for the year 1800. But as the rules that were given in the appendix to one of those publications, for correcting the effects of refraction and parallax, were thought too difficult for general use, they have been reduced to tables. So that, by the help of the ephemeris, these tables, and others that are also provided by the Board of Longitude, the calculations relating to the Longitude, which could not be performed by the most expert mathematician in less than four hours, may now be completed with great ease and accuracy in half an hour.

As this method of determining the Longitude depends on the use of the tables annually published for this purpose, those who wish for farther information are referred to the instructions that accompany them, and particularly to those that are annexed to the *Tables requisite to be used with the Astronomical and Nautical Ephemeris*, 2d edit. 1781.

4th. The phenomena of Jupiter's satellites have commonly been preferred to those of the moon, for finding the Longitude; because they are less liable to parallaxes than these are, and besides they afford a very commodious observation whenever the planet is above the horizon. Their motion is very swift, and must be calculated for every hour. These satellites of Jupiter were no sooner announced by Galileo, in his Syderius Nuncius, first printed at Venice in 1610, than the frequency of their eclipses recommended them for this purpose; and among those who treated on this subject, none was more successful than Cassini. This great astronomer published, at Bologna, in 1688, tables for calculating the appearances of their eclipses, with directions for finding the Longitudes of places by them; and being invited to France by Louis the 14th, he there, in the year 1693, published more correct tables of the same. But the mutual attractions of the satellites rendering their motions very irregular, those tables soon became useless for this purpose; insomuch that they require to be renewed from time to time; a service which has been performed by several ingenious astronomers, as Dr. Pound, Dr. Bradley, M. Cassini the son, and more especially by Mr. Wargentin, whose tables are much esteemed, which have been published in several places, as also in the Nautical Almanacs for 1771 and 1779.

Now, to find the Longitude by these satellites; with a good telescope observe some of their phenomena, as the conjunction of two of them, or of one of them with Jupiter, &c; and at the same time find the hour and minute, from the altitudes of the stars, or by means of a clock or watch, previously regulated for the place of observation; then, consulting tables of the satellites, observe the time when the same appearance happens in the meridian of the place for which the tables are calculated; and the difference of time, as before, will give the Longitude.

The eclipses of the first and second of Jupiter's satellites are the most proper for this purpose; and as they happen almost daily, they afford a ready means of determining the Longitude of places at land, having indeed contributed much to the modern improvements in geography; and if it were possible to observe them with proper telescopes, in a ship under sail, they would be of great service in ascertaining its Longitude from time to time. To obviate the inconvenience to which these observations are liable from the motions of the ship, a Mr. Irwin invented what he called a marine chair; this was tried by Dr. Maskelyne, in his voyage to Barbadoes, when it was not found that any benefit could be derived from the use of it. And indeed, considering the great power requisite in a telescope proper for these observations, and the violence, as well as irregularities in the motion of a ship, it is to be feared that the complete management of a telescope on ship-board, will always remain among the desiderata in this part of nautical science. And farther, since all methods that depend on the phenomena of the heavens have also this other defect, that they cannot be observed at all times, this renders

renders the improvement of time-keepers an object of the greater importance.

Many other schemes and proposals have been made by different persons, but most of them of very little or no use; such as by the space between the flash and report of a great gun, proposed by Messrs Whiston and Ditton; and another proposed by Mr. Whiston, by means of the inclinatory or dipping needle; besides a method by the variation of the magnetic needle, &c, &c.

LONGITUDE *of Motion*, is a term used by Dr. Wallis for the measure of motion, estimated according to its line of direction; or it is the distance or length gone through by the centre of any moving body, as it moves on in a right line.

The same author calls the measure of any motion, estimated according to the line of direction of the vis motrix, the *Altitude* of it.

LONGOMONTANUS (CHRISTIAN), a learned astronomer, born in Denmark in 1562, in the village of Longomontum, whence he took his name. Vossius, by mistake, calls him Christopher. Being the son of a poor man, a plowman, he was obliged to suffer, during his studies, all the hardships to which he could be exposed, dividing his time, like the philosopher Cleanthes, between the cultivation of the earth and the lessons he received from the minister of the place. At length, at 15 years old, he stole away from his family, and went to Wiburg, where there was a college, in which he spent 11 years; and though he was obliged to earn his livelihood as he could, his close application to study enabled him to make a great progress in learning, particularly in the mathematical sciences.

From hence he went to Copenhagen; where the professors of that university soon conceived a very high opinion of him, and recommended him to the celebrated Tycho Brahe; with whom Longomontanus lived 8 years, and was of great service to him in his observations and calculations. At length, being very desirous of obtaining a professor's chair in Denmark, Tycho Brahe consented, with some difficulty, to his leaving him; giving him a discharge filled with the highest testimonies of his esteem, and furnishing him with money for the expence of his long journey from Germany, whither Tycho had retired.

He accordingly obtained a professorship of mathematics in the university of Copenhagen in 1605; the duty of which he discharged very worthily till his death, which happened in 1647, at 85 years of age.

Longomontanus was author of several works, which shew great talents in mathematics and astronomy. The most distinguished of them, is his *Astronomica Danica*, first printed in 4to, 1621, and afterwards in folio in 1640, with augmentations. He amused himself with endeavouring to square the circle, and pretended that he had made the discovery of it; but our countryman Dr. John Pell attacked him warmly on that subject, and proved that he was mistaken.——It is remarkable that, obscure as his village and father were, he contrived to dignify and eternize them both; for he took his name from his village, and in the title page to some of his works he wrote himself *Christianus Longomontanus Severini filius*, his father's name being Severin or Severinus.

LOXODROMIC CURVE, or SPIRAL, is the same as the Rhumb line, or path of a ship sailing always on the same course in an oblique direction, or making always the same angle with every meridian. It is a species of logarithmic spiral, described on the surface of the sphere, having the meridians for its radii.

LOXODROMICS, the art or method of oblique sailing, by the loxodromic or rhumb line.

LOZENGE, an oblique-angled parallelogram; being otherwise called a rhombus, or a rhomboides.

LUBIENIETSKI (STANISLAUS), a Polish gentleman, born at Cracow, in 1623, and educated with great care by his father. He was learned in astronomy, and became a celebrated Socinian minister. He took great pains to obtain a toleration from the German princes for his Socinian brethren. His endeavours however were all in vain; being himself persecuted by the Lutheran ministers, and banished from place to place; till at length he was banished out of the world, with his two daughters, by poison, in 1675, his wife narrowly escaping.

We have, of his writing, *A History of the Reformation in Poland*; and a Treatise on Comets, intitled *Theatrum Cometicum*, printed at Amsterdam in 2 volumes folio; which is a most elaborate work, containing a minute historical account of every single comet that had been seen or recorded.

LUCIDA CORONÆ, a fixed star of the 2d magnitude, in the northern crown. See CORONA *Borealis*.

LUCIDA HYDRÆ. See COR *Hydræ*.

LUCIDA LYRÆ, a bright star of the first magnitude in the constellation Lyra.

LUCIFER, a name given to the planet Venus, when she appears in the morning before sunrise.

LUMINARIES, a term used for the sun and moon, by way of eminence, for their extraordinary lustre, and the great quantity of light they give us.

LUNA, the Moon; which see.

LUNAR, something relating to the moon.

LUNAR *Cycle*, or *Cycle of the Moon*. See CYCLE.

LUNAR *Method for the Longitude*, a method of keeping or finding the Longitude by means of the moon's motions, particularly by her observed distances from the sun and stars; for which, see the article LONGITUDE.

LUNAR *Month*, is either Periodical, Synodical, or Illuminative. Which see; also MONTH.

LUNAR *Year*, consists of 354 days, or 12 synodical months, of $29\frac{1}{2}$ days each. See YEAR.

In the early ages, the lunar year was used by all nations; the variety of course being more frequent and conspicuous in this planet, and consequently better known to men, than those of any other. The Romans regulated their year, in part, by the moon, even till the time of Julius Cæsar. The Jews too had their lunar month and year.

LUNAR *Dial*, *Eclipse*, *Horoscope*, and *Rainbow*. See the several substantives.

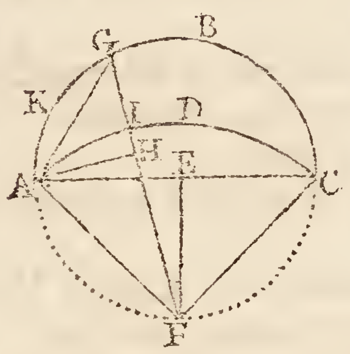
LUNATION, the period or time between one new moon and another; it is also called the synodical month, consisting of 29 days 12 hrs. 44 m. 3 sec. 11 thirds; exceeding the periodical month by 2 ds. 5 hrs. 0 m. 55 sec.

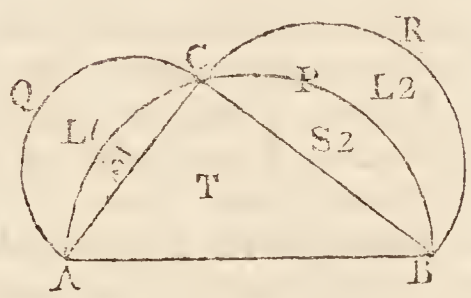
LUNE, or LUNULA, or little moon, is a geometrical

cal figure, in form of a creſcent, terminated by the arcs of two circles that interſect each other within.

Though the quadrature of the whole circle has never been effected, yet many of its parts have been ſquared. The firſt of theſe partial quadratures was that of the Lunula, given by Hippocrates of Scio, or Chios; who, from being a ſhipwrecked merchant, commenced geometrician. But although the quadrature of the Lune be generally aſcribed to Hippocrates, yet Proclus expreſsly ſays it was found out by Oenopidas of the ſame place. See Heinius in Mem. de l'Acad. de Berlin, tom. ii. pa. 410, where he gives a diſſertation concerning this Oenopidas. See alſo CIRCLE, and QUADRATURE.

The Lune of Hippocrates is this: Let ABC be a ſemicircle, having its centre E, and ADC a quadrant, having its centre F; then the Figure ABCDA, contained between the arcs of the ſemicircle and quadrant, is his Lune; and it is equal to the right-angled triangle ACF, as is thus eaſily proved. Since $AF^2 = 2AE^2$, that is, the ſquare of the radius of the quadrant equal to double the ſquare of the radius of the ſemicircle; therefore the quadrantal area ADCFA is = the ſemicircle ABCEA; from each of theſe take away the common ſpace ADCEA, and there remains the triangle ACF = the Lune ABCDA.





Another property of this Lune, which is the more general one of the former, is, that if FG be any line drawn from the point F, and AH perpendicular to it; then is the intercepted part of the Lune AGIA = the triangle AGH cut off by the chord line AG; or in general, that the ſmall ſegment AKGA is equal to the trilineal AIHA. For, the angle AFG being at the centre of the one circle, and at the circumference of the other, the arcs cut off AG, AI are ſimilar to the wholes ABC, ADC, therefore the ſmall ſeg. AKGA is to the ſemiſegment AIH, as the whole ſemicircle ABCA to the ſemiſegment or quadrant ADCF, that is in a ratio of equality.

Again, if ABC (fig. 2) be a triangle, right angled at C, and if ſemicircles be deſcribed on the three ſides as diameters; then the triangle T (ABC) is equal to the ſum of the two Lunes L1, L2. For, the greateſt ſemicircle is equal to the ſum of both the other two; from the greateſt ſemicircle take away the ſegments S1 and S2, and there remains the triangle T; alſo from the two leſs ſemicircles take away the ſame two ſegments S1 and S2, and there remains the two Lunes L1 and L2; therefore the triangle T = L1 + L2 the two Lunes.

LUNETTE, in Fortification, an inveloped counterguard, or mound of earth, made beyond the ſecond ditch, oppoſite to the place of arms; differing from the ravelins only in their ſituation. Lunettes are uſually made in wet ditches, and ſerve the ſame purpoſe as fauſſe-brays, to defend the paſſage of the ditch.

LUPUS, the *Wolf*, a ſouthern conſtellation, joined to the Centaur, containing together 19 ſtars in Ptolomy's catalogue, but 24 in the Britannic catalogue.

LYNX, a conſtellation of the northern hemiſphere, compoſed by Hevelius out of the unformed ſtars. In his catalogue it conſiſts of 19 ſtars, but in the Britannic 44.

LYONS (ISRAEL), a good mathematician and botaniſt, was the ſon of a Poliſh Jew ſilverſmith, and teacher of Hebrew at Cambridge in England, where he was come to ſettle, and where young Lyons was born, 1739. He was a very extraordinary young man for parts and ingenuity; and ſhewed very early in life a great inclination to learning, particularly mathematics, on which account he was much patronized by Dr. Smith, maſter of Trinity college. About 1755 he began to ſtudy botany, which he continued occaſionally till his death; in which he made a conſiderable progreſs, and could remember not only the Linnæan names of almoſt all the Engliſh plants, but even the ſynonyma of the old botaniſts; and he had prepared large materials for a *Flora Cantabrigienſis*, deſcribing fully every part of each plant from the ſpecimen, without being obliged to conſult, or being liable to be miſled by, former authors.

In 1758, he obtained much celebrity by publiſhing *A Treatiſe on Fluxions*, dedicated to his patron, Dr. Smith; and in 1763, *Faſciculus Plantarum circa Cantabrigiam, &c.* In the ſame year, or the year before, he read Lectures on Botany at Oxford with great applauſe, to at leaſt 60 pupils; but he could not be prevailed on to make a long abſence from Cambridge.

Mr. Lyons was ſome time employed as one of the computers of the Nautical Almanac; and beſides he received frequent other preſents from the Board of Longitude for his own inventions.——He had ſtudied the Engliſh hiſtory; and could quote whole paſſages from the Monkiſh writers verbatim. He could read Latin and French with eaſe, but wrote the former ill. He was appointed by the Board of Longitude to ſail with Capt. Phipps, in his voyage towards the North Pole, in 1773, as aſtronomical obſervator; and he diſcharged that office to the ſatisfaction of his employers. After his return from this voyage, he married, and ſettled in London, where he died of the meazles in about two years.

At the time of his death he was engaged in preparing for the preſs, a complete edition of all the works of the late learned Dr. Halley; a work very much wanted. —His *Calculations in Spherical Trigonometry abridged*, were printed in the Philoſ. Tranſ. vol. 65, for the year 1775, pa. 470.—After his death, his name appeared in the title-page of *A Geographical Dictionary*, the aſtronomical parts of which were ſaid to be "taken from the papers of the late Mr. Iſrael Lyons of Cambridge, author of ſeveral valuable mathematical productions, and aſtronomer in lord Mulgrave's voyage to the northern hemiſphere."—The aſtronomical and other mathematical calculations, printed in the account of captain Phipps's voyage towards the north pole, mentioned above, were made by Mr. Lyons. This appeared afterwards, by the acknowledgment of captain Phipps, when Dr. Horſley detected a material error in ſome part of

of them, in his *Remarks on the Obſervations made in the late Voyage, &c*, 1774.

"The Scholar's Inſtructor, or Hebrew Grammar, by Iſrael Lyons, Teacher of the Hebrew Tongue in the Univerſity of Cambridge," the 2d edit. &c, 1757, 8vo, was the production of his father; as was alſo another Treatiſe printed at the Cambridge preſs, under the title of "Obſervations and Enquiries relating to various parts of Scripture Hiſtory, 1761."

LYRA, the *Harp*, a conſtellation in the northern hemiſphere, containing 10 ſtars in Ptolomy's catalogue, 11 in Tycho's, 17 in Hevelius's, and 21 in the Britannic catalogue.

M.

M, In *Aſtronomical Tables*, &c, is uſed for *Meridional* or ſouthern; and ſometimes for *Meridian*, or mid-day.—In the Roman numeration, it denotes 1000, one thouſand.

MACHINE, denotes any thing that ſerves to augment, or to regulate moving powers: or it is any body deſtined to produce motion, ſo as to ſave either time or force. The word, in Greek, ſignifies an *Invention*, or *Art*: and hence, in ſtrictneſs, a machine is ſomething that conſiſts more in art and invention, than in the ſtrength and ſolidity of the materials; for which reaſon it is that the inventors of machines are called *Ingenieurs*, or *engineers*.

Machines are either ſimple or compound. The ſimple machines are the ſeven mechanical powers, viz, the lever, balance, pulley, wheel-and-axle, wedge, ſcrew, and inclined plane; which are otherwiſe called the ſimple mechanic powers.

Theſe ſimple machines ſerve for different purpoſes, according to the different ſtructures of them; and it is the buſineſs of the ſkilful mechaniſt to chooſe them, and combine them, in the manner that may be beſt adapted to produce the deſired effect. The lever is a very handy machine for many purpoſes, and its power immediately varied as the occaſion may require; when weights are to be raiſed only a little way, ſuch as ſtones out of quarries, &c. On the other hand, the wheel-and-axle ſerves to raiſe weights from the greateſt depth, or to the greateſt height. Pulleys, being eaſily carried, are therefore much employed in ſhips. The balance is uſeful for aſcertaining an equality of weight. The wedge is excellent for ſeparating the parts of bodies; and being impelled by the force of percuſſion, it is incomparably greater than the other powers. The ſcrew is uſeful for compreſſing or ſqueezing bodies together, and alſo for raiſing very heavy weights to a ſmall height; its great friction is even of conſiderable uſe, to preſerve the effect already produced by the machine.

Compound MACHINE, is formed from theſe ſimple machines, combined together for different purpoſes. The number of compound machines is almoſt infinite; and yet it would ſeem that the Ancients went far beyond the Moderns in the powers and effects of them; eſpecially their machines of war and architecture.

Accurate deſcriptions and drawings of machines would be a very curious and uſeful work. But to make a collection of this kind as beneficial as poſſible, it ſhould contain alſo an analyſis of them; pointing out their advantages and diſadvantages, with the reaſons of the conſtructions; alſo the general problems implied in theſe conſtructions, with their ſolutions, ſhould be noticed. Though a complete work of this kind be ſtill wanting, yet many curious and uſeful particulars may be gathered from Strada, Beſſon, Beroaldus, Auguſtinus de Ramellis, Bockler, Leupold, Beyer, Limpergh, Van Zyl, Perault, and others; a ſhort account of whoſe works may be found in Wolfii Commentatio de Præcipuis Scriptis Mathematicis; Elem. Matheſ. Univ. tom. 5, pa. 84. To theſe may be added, Belidor's Architecture Hydraulique, Deſaguliers's Courſe of Experimental Philoſophy, and Emerſon's Mechanics. The Royal Academy of Sciences at Paris have alſo given a collection of machines and inventions approved of by them. This work, publiſhed by M. Gallon, conſiſts of 6 volumes in quarto, containing engraved draughts of the machines, with their deſcriptions annexed.

MACHINE, *Architectonical*, is an aſſemblage of pieces of wood ſo diſpoſed as that, by means of ropes and pulleys, a ſmall number of men may raiſe great loads, and lay them in their places: ſuch as cranes, &c.—— It is hard to conceive what ſort of machines the Ancients muſt have uſed to raiſe thoſe immenſe ſtones found in ſome of the antique buildings; as ſome of thoſe ſtill found in the walls of Balbeck in Turkey, the ancient Heliopolis, which are 63 feet long, 12 feet broad, and 12 feet thick, and which muſt weigh 6 or 7 hundred tons a piece.

Blowing MACHINE. See BELLOWS.

Boyleian MACHINE. Mr. Boyle's Air-Pump.

Electrical MACHINE. See ELECTRICAL *Machine*.

Wind MACHINE. See ANEMOMETER, and WIND *Machine*.

Hydraulic, or *Water* MACHINE, is uſed either to ſignify a ſimple Machine, ſerving to conduct or raiſe water; as a ſluice, pump, and the like, or ſeveral of theſe acting together, to produce ſome extraordinary effect; as the

MACHINE *of Marli*. See MARLI. See alſo FIRE-*engine*, STEAM-*engine*, and WATER-*works*.

Military MACHINES, among the Ancients, were of

 three

three kinds: the firft ferving to launch arrows, as the fcorpion; or javelins, as the catapult; or ftones, as the balifta; or fiery darts, as the pyrabolus: the 2d fort ferving to beat down walls, as the battering ram and terebra: and the 3d fort to fhelter thofe who approach the enemy's wall, as the tortoife or teftudo, the vinea, and the towers of wood. See the refpective articles.

The Machines of war now in ufe, confift in artillery, including cannon, mortars, petards, &c.

MACLAURIN, (Colin), a moft eminent mathematician and philofopher, was the fon of a clergyman, and born at Kilmoddan in Scotland, in the year 1698. He was fent to the univerfity of Glafgow in 1709; where he continued five years, and applied to his ftudies in a very intenfe manner, and particularly to the mathematics. His great genius for mathematical learning difcovered itfelf fo early as at 12 years of age; when, having accidentally met with a copy of Euclid's Elements in a friend's chamber, he became in a few days mafter of the firft 6 books without any affiftance: and it is certain, that in his 16th year he had invented many of the propofitions which were afterwards publifhed as part of his work intitled *Geometria Organica*. In his 15th year he took the degree of Mafter of Arts; on which occafion he compofed and publicly defended a thefis on the power of gravity, with great applaufe. After this he quitted the univerfity, and retired to a country feat of his uncle, who had the care of his education; his parents being dead fome time. Here he fpent two or three years in purfuing his favourite ftudies; but, in 1717, at 19 years of age only, he offered himfelf a candidate for the profefforfhip of mathematics in the Marifchal College of Aberdeen, and obtained it after a ten days trial, againft a very able competitor.

In 1719, Mr. Maclaurin vifited London, where he left his *Geometria Organica* to print, and where he became acquainted with Dr. Hoadley then bifhop of Bangor, Dr. Clarke, Sir Ifaac Newton, and other eminent men; at which time alfo he was admitted a member of the Royal Society: and in another journey, in 1721, he contracted an intimacy with Martin Folkes, Efq. the prefident of it, which continued during his whole life.

In 1722, lord Polwarth, plenipotentiary of the king of Great Britain at the congrefs of Cambray, engaged Maclaurin to go as a tutor and companion to his eldeft fon, who was then to fet out on his travels. After a fhort ftay at Paris, and vifiting other towns in France, they fixed in Lorrain; where he wrote his piece, On the Percuffion of Bodies, which gained him the prize of the Royal Academy of Sciences for the year 1724. But his pupil dying foon after at Montpelier, he returned immediately to his profeffion at Aberdeen. He was hardly fettled here, when he received an invitation to Edinburgh; the curators of that univerfity being defirous that he fhould fupply the place of Mr. James Gregory, whofe great age and infirmities had rendered him incapable of teaching. He had here fome difficulties to encounter, arifing from competitors, who had good intereft with the patrons of the univerfity, and alfo from the want of an additional fund for the new profeffor; which however at length were all furmounted, principally by the means of Sir Ifaac Newton. Accordingly, in Nov. 1725, he was introduced into the univerfity; as was at the fame time his learned colleague and intimate friend, Dr. Alexander Monro, profeffor of anatomy. After this, the Mathematical claffes foon became very numerous, there being generally upwards of 100 ftudents attending his Lectures every year; who being of different ftandings and proficiency, he was obliged to divide them into four or five claffes, in each of which he employed a full hour every day from the firft of November to the firft of June. In the firft clafs he taught the firft 6 books of Euclid's Elements, Plane Trigonometry, Practical Geometry, the Elements of Fortification, and an Introduction to Algebra. The fecond clafs ftudied Algebra, with the 11th and 12th books of Euclid, Spherical Trigonometry, Conic Sections, and the general Principles of Aftronomy. The third went on in Aftronomy and Perfpective, read a part of Newton's Principia, and had performed a courfe of experiments for illuftrating them: he afterwards read and demonftrated the Elements of Fluxions. Thofe in the fourth clafs read a Syftem of Fluxions, the Doctrine of Chances, and the remainder of Newton's Principia.

In 1734, Dr. Berkley, bifhop of Cloyne, publifhed a piece called The Analift; in which he took occafion, from fome difputes that had arifen concerning the grounds of the fluxionary method, to explode the method itfelf; and alfo to charge mathematicians in general with infidelity in religion. Maclaurin thought himfelf included in this charge, and began an anfwer to Berkley's book: but other anfwers coming out, and as he proceeded, fo many difcoveries, fo many new theories and problems occurred to him, that inftead of a vindicatory pamphlet, he produced a Complete Syftem of Fluxions, with their application to the moft confiderable problems in Geometry and Natural Philofophy. This work was publifhed at Edinburgh in 1742, 2 vols 4to; and as it coft him infinite pains, fo it is the moft confiderable of all his works, and will do him immortal honour, being indeed the moft complete treatife on that fcience that has yet appeared.

In the mean time, he was continually obliging the public with fome obfervation or performance of his own, feveral of which were publifhed in the 5th and 6th volumes of the Medical Effays at Edinburgh. Many of them were likewife publifhed in the Philofophical Tranfactions; as the following: 1. On the Conftruction and Meafure of Curves, vol. 30.—2. A New Method of defcribing all kinds of Curves, vol. 30.—3. On Equations with Impoffible Roots, vol. 34.—4. On the Roots of Equations, &c. vol. 34.—5. On the Defcription of Curve Lines, vol. 39.—6. Continuation of the fame, vol. 39.—7. Obfervations on a Solar Eclipfe, vol. 40.—8. A Rule for finding the Meridional Parts of a Spheroid with the fame Exactnefs as in a Sphere, vol. 41.—9. An Account of the Treatife of Fluxions, vol. 42.—10. On the Bafes of the Cells where the Bees depofit their Honey, vol. 42.

In the midft of thefe ftudies, he was always ready to lend his affiftance in contriving and promoting any fcheme which might contribute to the public fervice. When the earl of Morton went, in 1739, to vifit his eftates in Orkney and Shetland, he requefted Mr. Maclaurin to affift him in fettling the geography of thofe countries, which is very erroneous in all our maps; to examine their natural hiftory, to furvey the coafts, and to take the meafure of a degree of the meridian. Maclaurin's

laurin's family affairs would not permit him to comply with this requeſt: he drew up however a memorial of what he thought neceſſary to be obſerved, and furniſhed proper inſtruments for the work, recommending Mr. Short, the noted optician, as a fit operator for the management of them.

Mr. Maclaurin had ſtill another ſcheme for the improvement of geography and navigation, of a more extenſive nature; which was the opening a paſſage from Greenland to the South Sea by the North Pole. That ſuch a paſſage might be found, he was ſo fully perſuaded, that he uſed to ſay, if his ſituation could admit of ſuch adventures, he would undertake the voyage, even at his own charge. But when ſchemes for finding it were laid before the parliament in 1741, and he was conſulted by ſeveral perſons of high rank concerning them, and before he could finiſh the memorials he propoſed to ſend, the premium was limited to the diſcovery of a north-weſt paſſage: and he uſed to regret that the word Weſt was inſerted, becauſe he thought that paſſage, if at all to be found, muſt lie not far from the pole.

In 1745, having been very active in fortifying the city of Edinburgh againſt the rebel army, he was obliged to fly from thence into England, where he was invited by Dr. Herring, archbiſhop of York, to reſide with him during his ſtay in this country. In this expedition however, being expoſed to cold and hardſhips, and naturally of a weak and tender conſtitution, which had been much more enfeebled by cloſe application to ſtudy, he laid the foundation of an illneſs which put an end to his life, in June 1746, at 48 years of age, leaving his widow with two ſons and three daughters.

Mr. Maclaurin was a very good, as well as a very great man, and worthy of love as well as admiration. His peculiar merit as a philoſopher was, that all his ſtudies were accommodated to general utility; and we find, in many places of his works, an application even of the moſt abſtruſe theories, to the perfecting of mechanical arts. For the ſame purpoſe, he had reſolved to compoſe a courſe of Practical Mathematics, and to reſcue ſeveral uſeful branches of the ſcience from the ill treatment they often met with in leſs ſkilful hands. Theſe intentions however were prevented by his death; unleſs we may reckon, as a part of his intended work, the tranſlation of Dr. David Gregory's Practical Geometry, which he reviſed, and publiſhed with additions, in 1745.

In his lifetime, however, he had frequent opportunities of ſerving his friends and his country by his great ſkill. Whatever difficulty occurred concerning the conſtructing or perfecting of machines, the working of mines, the improving of manufactures, the conveying of water, or the execution of any public work, he was always ready to reſolve it. He was employed to terminate ſome diſputes of conſequence that had ariſen at Glaſgow concerning the ganging of veſſels; and for that purpoſe preſented to the commiſſioners of the exciſe two elaborate memorials, with their demonſtrations, containing rules by which the officers now act. He made alſo calculations relating to the proviſion, now eſtabliſhed by law, for the children and widows of the Scotch clergy, and of the profeſſors in the univerſities, entitling them to certain annuities and ſums, upon the voluntary annual payment of a certain ſum by the incumbent. In contriving and adjuſting this wiſe and uſeful ſcheme, he beſtowed a great deal of labour, and contributed not a little towards bringing it to perfection.

Of his works, we have mentioned his *Geometria Organica*, in which he treats of the deſcription of curve lines by continued motion; as alſo of his piece which gained the prize of the Royal Academy of Sciences in 1724. In 1740, he likewiſe ſhared the prize of the ſame Academy, with the celebrated D. Bernoulli and Euler, for reſolving the problem relating to the motion of the tides from the theory of gravity: a queſtion which had been given out the former year, without receiving any ſolution. He had only ten days to draw this paper up in, and could not find leiſure to tranſcribe a fair copy; ſo that the Paris edition of it is incorrect. He afterwards reviſed the whole, and inſerted it in his Treatiſe of Fluxions; as he did alſo the ſubſtance of the former piece. Theſe, with the Treatiſe of Fluxions, and the pieces printed in the Medical Eſſays and the Philoſophical Tranſactions, a liſt of which is given above, are all the writings which our author lived to publiſh. Since his death, however, two more volumes have appeared; his *Algebra*, and his *Account of Sir Iſaac Newton's Philoſophical Diſcoveries*. The Algebra, though not finiſhed by himſelf, is yet allowed to be excellent in its kind; containing, in no large volume, a complete elementary treatiſe of that ſcience, as far as it has hitherto been carried; beſides ſome neat analytical papers on curve lines. His Account of Newton's Philoſophy was occaſioned in the following manner:—Sir Iſaac dying in the beginning of 1728, his nephew, Mr. Conduitt, propoſed to publiſh an account of his life, and deſired Mr. Maclaurin's aſſiſtance. The latter, out of gratitude to his great benefactor, cheerfully undertook, and ſoon finiſhed, the Hiſtory of the Progreſs which Philoſophy had made before Newton's time; and this was the firſt draught of the work in hand; which not going forward, on account of Mr. Conduitt's death, was returned to Mr. Maclaurin. To this he afterwards made great additions, and left it in the ſtate in which it now appears. His main deſign ſeems to have been, to explain only thoſe parts of Newton's philoſophy, which have been controverted: and this is ſuppoſed to be the reaſon why his grand diſcoveries concerning light and colours are but tranſiently and generally touched upon; for it is known, that whenever the experiments, on which his doctrine of light and colours is founded, had been repeated with due care, this doctrine had not been conteſted; while his accounting for the celeſtial motions, and the other great appearances of nature, from gravity, had been miſunderſtood, and even attempted to be ridiculed.

MACULÆ, in Aſtronomy, are dark ſpots appearing on the luminous ſurfaces of the ſun and moon, and even ſome of the planets.

The Solar Maculæ are dark ſpots of an irregular and changeable figure, obſerved in the face of the ſun. Theſe were firſt obſerved in November and December of the year 1610, by Galileo in Italy, and Harriot in England, unknown to, and independent of each other, ſoon after they had made or procured teleſcopes. They were afterwards alſo obſerved by Scheiner, Hevelius, Flamſteed, Caſſini, Kirch, and others. See Philoſ. Tranſ. vol. 1, pa. 274, and vol. 64, pa. 194.

There

There have been various obſervations made of the phenomena of the ſolar maculæ, and hypotheſes invented for explaining them. Many of theſe maculæ appear to conſiſt of heterogeneous parts; the darker and denſer being called, by Hevelius, nuclei, which are encompaſſed as it were with atmoſpheres, ſomewhat rarer and leſs obſcure; but the figure, both of the nuclei and entire maculæ, is variable. Theſe maculæ are often ſubject to ſudden mutations: In 1644 Hevelius obſerved a ſmall thin macula, which in two days time grew to ten times its bulk, appearing alſo much darker, and having a larger nucleus: the nucleus began to fail ſenſibly before the ſpot diſappeared; and before it quite vaniſhed, it broke into four, which re-united again two days after. Some maculæ have laſted 2, 3, 10, 15, 20, 30, but ſeldom 40 days; though Kirchius obſerved one in 1681, that was viſible from April 26th to the 17th of July. It is found that the ſpots move over the ſun's diſc with a motion ſomewhat ſlacker near the edge than in the middle parts; that they contract themſelves near the limb, and in the middle appear larger; that they often run into one in the diſc, though ſeparated near the centre; that many of them firſt appear in the middle, and many diſappear there; but that none of them deviate from their path near the horizon; whereas Hevelius, obſerving Mercury in the ſun near the horizon, found him too low, being depreſſed 27″ beneath his former path.

From theſe phenomena are collected the following conſequences. 1. That ſince Mercury's depreſſion below his path ariſes from his parallax, the maculæ, having no parallax from the ſun, are much nearer him than that planet.

2. That, ſince they riſe and diſappear again in the middle of the ſun's diſc, and undergo various alterations with regard both to bulk, figure, and denſity, they muſt be formed *de novo*, and again diſſolved about the ſun; and hence ſome have inferred, that they are a kind of ſolar clouds, formed out of his exhalations; and if ſo, the ſun muſt have an atmoſphere.

3. Since the ſpots appear to move very regularly about the ſun, it is hence inferred, that it is not that they really move, but that the ſun revolves round his axis, and the ſpots accompany him, in the ſpace of 27 days 12 hours 20 minutes.

4. Since the ſun appears with a circular diſc in every ſituation, his figure, as to ſenſe, muſt be ſpherical.

The magnitude of the ſurface of a ſpot may be eſtimated by the time of its tranſit over a hair in a fixed teleſcope. Galileo eſtimates ſome ſpots as larger than both Aſia and Africa put together: but if he had known more exactly the ſun's parallax and diſtance, as they are known now, he would have found ſome of thoſe ſpots much larger than the whole ſurface of the earth. For, in 1612, he obſerved a ſpot ſo large as to be plainly viſible to the naked eye; and therefore it ſubtended an angle of about a minute. But the earth, ſeen at the diſtance of the ſun, would ſubtend an angle of only about 17″: therefore the diameter of the ſpot was to the diameter of the earth, as 60 to 17, or $3\frac{1}{2}$ to 1 nearly; and conſequently the ſurface of the ſpot, if circular, to a great circle of the earth, as $12\frac{1}{4}$ to 1, and to the whole ſurface of the earth, as $12\frac{1}{4}$ to 4, or nearly 3 to 1. Gaſſendus obſerved a ſpot whoſe breadth was $\frac{1}{20}$ of the ſun's diameter, and which therefore ſubtended an angle at the eye of above a minute and a half; and conſequently its ſurface was above ſeven times larger than the ſurface of the whole earth. He ſays he obſerved above 40 ſpots at once, though without ſenſibly diminiſhing the light of the ſun.

Various opinions have been formed concerning the nature, origin, and ſituation of the ſolar ſpots; but the moſt probable ſeems to be that of Dr. Wilſon, profeſſor of practical aſtronomy in the univerſity of Glaſgow. By attending particularly to the different phaſes preſented by the umbra, or ſhady zone, of a ſpot of an extraordinary ſize that appeared on the ſun, in the month of November 1769, during its progreſs over the ſolar diſc, Dr. Wilſon was led to form a new and ſingular conjecture on the nature of theſe appearances; which he afterwards greatly ſtrengthened by repeated obſervations. The reſults of theſe obſervations are, that the ſolar maculæ are cavities in the body of the ſun; that the nucleus, as the middle or dark part has uſually been called, is the bottom of the excavations; and that the umbra, or ſhady zone ſurrounding it, is the ſhelving ſides of the cavity. Dr. Wilſon, beſides having ſatisfactorily aſcertained the reality of theſe immenſe excavations in the body of the ſun, has alſo pointed out a method of meaſuring the depth of them. He eſtimates, in particular, that the nucleus, or bottom of the large ſpot above-mentioned, was not leſs than a ſemidiameter of the earth, or about 4000 miles below the level of the ſun's ſurface; while its other dimenſions were of a much larger extent. He obſerved that a ſpot near the middle of the ſun's diſc, is ſurrounded equally on all ſides with its umbra; but that when, by its apparent motion over the ſun's diſc, it comes near the weſtern limb, that part of the umbra which is next the ſun's centre gradually diminiſhes in breadth, till near the edge of the limb it totally diſappears; whilſt the umbra on the other ſide of it is little or nothing altered. After a ſemirevolution of the ſun on his axis, if the the ſpot appear again, it will be on the oppoſite ſide of the diſc, or on the left hand, and the part of the umbra which had before diſappeared, is now plainly to be ſeen; while the umbra on the other ſide of the ſpot, ſeems to have vaniſhed in its turn; being hid from the view by the upper edge of the excavation, from the oblique poſition of its ſloping ſides with reſpect to the eye. But as the ſpot advances on the ſun's diſc, this umbra, or ſide of the cavity, comes in ſight; at firſt appearing narrow, but afterwards gradually increaſing in breadth, as the ſpot moves towards the middle of the diſc. Which appearances perfectly agree with the phaſes that are exhibited by an excavation in a ſpherical body, revolving on its axis; the bottom of the cavity being painted black, and the ſides lightly ſhaded.

From theſe, and other obſervations, it is inferred, that the body of the ſun, at the depth of the nucleus, emits little or no light, when ſeen at the ſame time, and compared with that reſplendent, and probably, in ſome degree, fluid ſubſtance, that covers his ſurface.

This manner of conſidering theſe phenomena, naturally gives riſe to many curious ſpeculations and inquiries. It is natural, for inſtance, to inquire, by what great commotion this refulgent matter is thrown up on all ſides, ſo as to expoſe to our view the darker part of

the

the ſun's body, which was before covered by it? what is the nature of this ſhining matter? and why, when an excavation is made in it, is the luſtre of this ſhining ſubſtance, which forms the ſhelving ſides of the cavity, ſo far diminiſhed, as to give the whole the appearanee of a ſhady zone, or darkiſh atmoſphere, ſurrounding the denuded part of the ſun's body? On theſe, and many other ſubjects, Dr. Wilſon has advanced ſome ingenious conjectures; for which ſee the Philoſ. Tranſ. vol. 64, art. 1. See alſo ſome remarks on this theory, by Mr. Woolaſton, in the ſame vol. pa. 337, &c.

MADRIER, in Artillery, is a thick plank, armed with plates of iron, and having a cavity ſufficient to receive the mouth of a petard, with which it is applied againſt a gate, or any thing elſe intended to be broken down.

This term is alſo applied to certain flat beams, fixed to the bottom of a moat, to ſupport a wall.

There are alſo Madriers lined with tin, and covered with earth; ſerving as defences againſt artificial fires, in lodgments, &c, where there is need of being covered overhead.

MÆSTLIN (Michael), in Latin Mæſtlinus, a noted aſtronomer of Germany, was born in the duchy of Wittemberg; but ſpent his youth in Italy, where he made a ſpeech in favour of Copernicus's ſyſtem, which brought Galileo over from Ariſtotle and Ptolomy, to whom he was before wholly devoted. He afterwards returned to Germany, and became profeſſor of mathematicsat Tubingen; where, among his other ſcholars, he taught the celebrated Kepler, who has commended ſeveral of his ingenious inventions, in his Aſtronomia Optica.

Mæſtlin publiſhed many mathematical and aſtronomical works; and died in 1590.—Though Tycho Brahe did not aſſent to Mæſtlin's opinion, yet he allowed him to be an extraordinary perſon, and deeply ſkilled in the ſcience of aſtronomy.

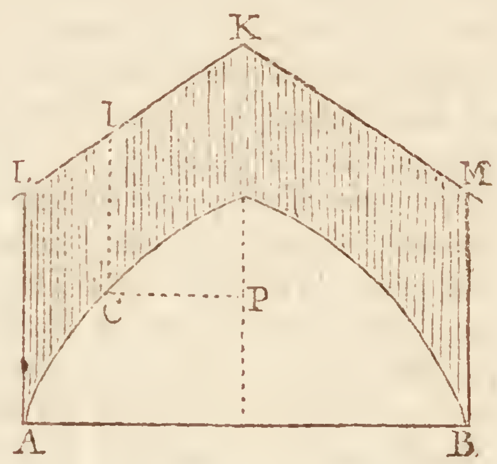
MAGAZINE, a place in which ſtores are kept, of arms, ammunition, proviſions, &c.

Artillery Magazine, or the Magazine to a field battery, is made about 25 or 30 yards behind the battery, towards the parallels, and at leaſt 3 feet under ground, to receive the powder, loaded ſhells, port-fires, &c.—Its roof and ſides ſhould be well ſecured with boards, to prevent the earth from falling in: it has a door, and a double trench or paſſage ſunk from the magazine to the battery, the one to enter, and the other to go out at, to prevent confuſion. Sometimes traverſes are made in the paſſages, to prevent ricochet ſhot from entering the magazine.

Powder-Magazine, is the place where powder is kept in large quantities. Authors differ very much with regard to the ſituation and conſtruction of theſe magazines; but all agree, that they ought to be arched and bomb-proof. In fortifications, they were formerly placed in the rampart; but of late they have been built in different parts of the town. The firſt powder-magazines were made with Gothic arches: but M. Vauban finding theſe too weak, conſtructed them of a ſemicircular form, the dimenſions being 60 feet long within, and 25 feet broad; the foundations are 8 or 9 feet thick, and 8 feet high from the foundation to the ſpring of the arch; alſo the floor 2 feet from the ground, to keep it from dampneſs.

It is a conſtant obſervation, that after the centering of ſemicircular arches is ſtruck, they ſettle at the crown, and riſe up at the hances, even with a ſtraight horizontal extrados; and ſtill much more ſo in powder-magazines, where the outſide at top is formed, like the roof of a houſe, by inclined planes joining in an angle over the top of the arch, to give a proper deſcent to the rain; which effects are exactly what might be expected from the true theory of arches. Now, this ſhrinking of the arches, as it muſt be attended with very bad conſequences, by breaking the texture of the cement after it has in ſome degree been dried, and alſo by opening the joints of the vouſoirs at one end, ſo a remedy is provided for this inconvenience, with regard to bridges, by the arch of equilibration, in my book on the Principles of Bridges: but as the ill conſequences of it are much greater in powder-magazines, in queſtion 96 of my Mathematical Miſcellany, I propoſed to find an arch of equilibration for them alſo; which queſtion was there reſolved both by Mr. Wildbore and myſelf, both upon general principles, and which I illuſtrated by an application to a particular caſe, which is there conſtructed, and accompanied with a table of numbers for that purpoſe. Thus, if ALKMB repreſent a vertical tranſverſe ſection of the arch, the roof forming an angle LKM of 112° 37′, alſo PC an ordinate parallel to the horizon taken in any part, and IC perpendicular to the ſame; then for properly conſtructing the curve ſo as to be the ſtrongeſt, or an arch of equilibration in all its parts, the correſponding values of PC and CI will be as in the following table, where thoſe numbers may denote any lengths whatever, either inches, or feet, or half-yards.

Value of PC	Value of IC
1	7·031
2	7·125
3	7·264
4	7·501
5	7·789
6	8·164
7	8·574
8	9·078
9	9·663
10	10·333



Magazine, or *Powder-Room*, on ſhip-board, is a cloſe room or ſtore-houſe, built in the fore or after part of the hold, in which to preſerve the gunpowder for the uſe of the ſhip. This apartment is ſtrongly ſecured againſt fire, and no perſon is allowed to enter it with a lamp or candle. it is therefore lighted, as occaſion requires, by means of the candles or lamps in the light-room contiguous to it.

MAGELLANIC-Clouds, whitiſh appearances like clouds, ſeen in the heavens towards the ſouth pole, and having the ſame apparent motion as the ſtars. They are three in number, two of them near each other.—The largeſt lies far from the ſouth pole; but the other two are not many degrees more remote from it than the neareſt conſpicuous ſtar, that is, about 11 degrees.

Mr. Boyle conjectures that if these clouds were seen through a good telescope, they would appear to be multitudes of small stars, like the milky way.

MAGIC Lantern, an optical machine, by means of which small painted images are represented on the wall of a dark room, magnified to any size at pleasure. This machine was contrived by Kircher, (see his Ars Magna Lucis and Umbræ, pa. 768); and it was so called, because the images were made to represent strange phantasms, and terrible apparitions, which have been taken for the effect of magic, by such as were ignorant of the secret.

This machine is composed of a concave speculum, from 4 to 12 inches diameter, reflecting the light of a candle through the small hole of a tube, at the end of which is fixed a double convex lens of about 3 inches focus. Between the two are successively placed, many small plain glasses, painted with various figures, usually such as are the most formidable and terrifying to the spectators, when represented at large on the opposite wall.

Thus, (Pl. 13, fig. 14) ABCD is a common tin lantern, to which is added a tube FG to draw out. In H is fixed the metallic concave speculum, from 4 to 12 inches diameter; or else, instead of it, near the extremity of the tube, there must be placed a convex lens, consisting of a segment of a small sphere, of but a few inches in diameter. The use of this lens is to throw a strong light upon the image; and sometimes a concave speculum is used with the lens, to render the image still more vivid. In the focus of the concave speculum or lens, is placed the lamp L; and within the tube, where it is soldered to the side of the lantern, is placed a small lens, convex on both sides, being a portion of a small sphere, having its focus about the distance of 3 inches. The extreme part of the tube FM is square, and has an aperture quite through, so as to receive an oblong frame NO passing into it; in which frame there are round holes, of an inch or two in diameter. Answering to the magnitude of these holes there are drawn circles on a plain thin glass; and in these circles are painted any figures, or images, at pleasure, with transparent water colours. These images fitted into the frame, in an inverted position, at a small distance from the focus of the lens I, will be projected on an opposite white wall of a dark room, in all their colours, greatly magnified, and in an erect position. By having the instrument so contrived, as that the lens I may move on a slide, the focus may be made, and consequently the image appear distinct, at almost any distance.

Or thus: Every thing being managed as in the former case, into the sliding tube FG, insert another convex lens K, the segment of a sphere rather larger than I. Now, if the picture be brought nearer to I than the distance of the focus, diverging rays will be propagated as if they proceeded from the object; wherefore, if the lens K be so placed, as that the object be very near its focus, the image will be exhibited on the wall, greatly magnified.

Magic Square, is a square figure, formed of a series of numbers in arithmetical progression, so disposed in parallel and equal ranks, as that the sums of each row, taken either perpendicularly, horizontally, or diagonally, are equal to one another. As the annexed square, formed of these nine numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, where the sum of the three figures in every row, in all directions, is always the same number, viz 15. But if the same numbers be placed in this natural order, the first being 1, and the last of them a square number, they will form what is called a natural square. As in the first 25 numbers, viz, 1, 2, 3, 4, 5, &c to 25.

4	9	2
3	5	7
8	1	6

Natural Square.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Magic Square.

16	14	8	2	25
3	22	20	11	9
15	6	4	23	17
24	18	12	10	1
7	5	21	19	13

where every row and diagonal in the magic square makes just the sum 65, being the same as the two diagonals of the natural square.

It is probable that these magic squares were so called, both because of this property in them, viz, that the ranks in every direction make the same sum, appeared extremely surprising, especially in the more ignorant ages, when mathematics passed for magic, and because also of the superstitious operations they were employed in, as the construction of talismans, &c; for, according to the childish philosophy of those days, which ascribed virtues to numbers, what might not be expected from numbers so seemingly wonderful!

The Magic Square was held in great veneration among the Egyptians, and the Pythagoreans their disciples, who, to add more efficacy and virtue to this square, dedicated it to the then known seven planets divers ways, and engraved it upon a plate of the metal that was esteemed in sympathy with the planet. The square thus dedicated, was inclosed by a regular polygon, inscribed in a circle, which was divided into as many equal parts as there were units in the side of the square; with the names of the angels of the planet, and the signs of the zodiac written upon the void spaces between the polygon and the circumference of the circumscribed circle. Such a talisman or metal they vainly imagined would, upon occasion, befriend the person who carried it about him.

To Saturn they attributed the square of 9 places or cells, the side being 3, and the sum of the numbers in every row 15: to Jupiter the square of 16 places, the side being 4, and the amount of each row 34: to Mars the square of 25 places, the side being 5, and the amount of each row 65: to the Sun the square with 36 places, the side being 6, and the sum of each row 111; to Venus the square of 49 places, the side being 7, and the amount of each row 175: to Mercury the square with 64 places, the side being 8, and the sum of each

each row 260: and to the Moon the ſquare of 81 places, the ſide being 9, and the amount of each row 369. Finally, they attributed to imperfect matter, the ſquare with 4 diviſions, having 2 for its ſide; and to God the ſquare of only one cell, the ſide of which is alſo an unit, which multiplied by itſelf, undergoes no change.

However, what was at firſt the vain practice of conjurers and makers of taliſmans, has ſince become the ſubject of a ſerious reſearch among mathematicians. Not that they imagine it will lead them to any thing of ſolid uſe or advantage; but rather as it is a kind of play, in which the difficulty makes the merit, and it may chance to produce ſome new views of numbers, which mathematicians will not loſe the occaſion of.

It would ſeem that Eman. Moſchopulus, a Greek author of no high antiquity, is the firſt now known of, who has ſpoken of magic ſquares: he has left ſome rules for their conſtruction; though, by the age in which he lived, there is reaſon to imagine he did not look upon them merely as a mathematician.

In the treatiſe of Cornelius Agrippa, ſo much accuſed of magic, are found the ſquares of ſeven numbers, viz, from 3 to 9 incluſive, diſpoſed magically; and it is not to be ſuppoſed that thoſe ſeven numbers were preferred to all others without ſome good reaſon: indeed it is becauſe their ſquares, according to the ſyſtem of Agrippa and his followers, are planetary. The ſquare of 3, for inſtance, belongs to Saturn; that of 4 to Jupiter; that of 5 to Mars; that of 6 to the Sun; that of 7 to Venus: that of 8 to Mercury; and that of 9 to the Moon.

M. Bachet applied himſelf to the ſtudy of magic ſquares, on the hint he had taken from the planetary ſquares of Agrippa, as being unacquainted with Moſchopulus's work, which is only in manuſcript in the French king's library; and, without the aſſiſtance of any author, he found out a new method for the ſquares of uneven numbers; for inſtance, 25, or 49, &c; but he could not ſucceed with thoſe that have even roots.

M. Frenicle next engaged in this ſubject. It was the opinion of ſome, that although the firſt 16 numbers might be diſpoſed 20922789888000 different ways in a natural ſquare, yet they could not be diſpoſed more than 16 ways in a magic ſquare; but M. Frenicle ſhewed, that they might be thus diſpoſed in 878 different ways.

To this buſineſs he thought fit to add a difficulty that had not yet been conſidered; which was, to take away the marginal numbers quite around, or any other circumference at pleaſure, or even ſeveral of ſuch circumferences, and yet that the remainder ſhould ſtill be magical.

Again he inverted that condition, and required that any circumference taken at pleaſure, or even ſeveral circumferences, ſhould be inſeparable from the ſquare; that is, that it ſhould ceaſe to be magical when they were removed, and yet continue magical after the removal of any of the reſt. M. Frenicle however gives no general demonſtration of his methods, and it often ſeems that he has no other guide but chance. It is true, his book was not publiſhed by himſelf, nor did it appear till after his death, viz, in 1693.

In 1703 M. Poignard, canon of Bruſſels, publiſhed a treatiſe on ſublime magic ſquares. Before his time there had been no magic ſquares made, but for ſerieſes of natural numbers that formed a ſquare; but M. Poignard made two very conſiderable improvements. 1ſt, Inſtead of taking all the numbers that fill a ſquare, for inſtance, the 36 ſucceſſive numbers, which would fill all the cells of a natural ſquare whoſe ſide is 6, he only takes as many ſucceſſive numbers as there are units in the ſide of the ſquare, which in this caſe are 6; and theſe ſix numbers alone he diſpoſes in ſuch manner, in the 36 cells, that none of them occur twice in the ſame rank, whether it be horizontal, vertical, or diagonal; whence it follows, that all the ranks, taken all the ways poſſible, muſt always make the ſame ſum; and this method M. Poignard calls repeated progreſſions. 2d, Inſtead of being confined to take theſe numbers according to the ſeries and ſucceſſion of the natural numbers, that is in arithmetical progreſſion, he takes them likewiſe in a geometrical progreſſion; and even in an harmonical progreſſion, the numbers of all the ranks always following the ſame kind of progreſſion: he makes ſquares of each of theſe three progreſſions repeated.

M. Poignard's book gave occaſion to M. de la Hire to turn his thoughts to the ſame ſubject, which he did with ſuch ſucceſs, that he greatly extended the theory of magic ſquares, as well for even numbers as thoſe that are uneven; as may be ſeen at large in the Memoirs of the Royal Academy of Sciences, for the years 1705 and 1710. See alſo Saunderſon's Algebra, vol. 1, pa. 354, &c; as alſo Ozanam's Mathematical Recreations, who lays down the following eaſy method of filling up a magic ſquare.

To form a magic ſquare of an odd number of terms in the arithmetic progreſſion 1, 2, 3, 4, &c. Place the leaſt term 1 in the cell immediately under the middle, or central one, and the reſt of the terms, in their natural order, in a deſcending diagonal direction, till they run off either at the bottom, or on the ſide: when the number runs off at the bottom, carry it to the uppermoſt cell, that is not occupied, of the ſame column that it would have fallen in below, and then proceed deſcending diagonalwiſe again as far as you can, or till the numbers either run off at bottom or ſide, or are interrupted by coming at a cell already filled: now when any number runs off at the right-hand ſide, then bring it to the fartheſt cell on the left-hand of the ſame row or line it would have fallen in towards the right-hand: and when the progreſs diagonalwiſe is interrupted by meeting with a cell already occupied by ſome other number, then deſcend diagonally to the left from this cell till an empty one is met with, where enter it; and thence proceed as before.

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Thus, to make a magic ſquare of the 49 numbers 1, 2, 3, 4, &c. Firſt place the 1 next below the centre cell, and thence deſcend to the right till the 4 runs off at the bottom, which therefore carry to the top corner on the ſame column as it would have fallen in; but as runs off at the ſide, bring it to the beginning of the ſecond line, and

and thence descend to the right till they arrive at the cell occupied by 1; carry the 8 therefore to the next diagonal cell to the left, and so proceed till 10 run off at the bottom, which carry therefore to the top of its column, and so proceed till 13 runs off at the side, which therefore bring to the beginning of the same line, and thence proceed till 15 arrives at the cell occupied by 8; from this therefore descend diagonally to the left; but as 16 runs off at the bottom, carry it to the top of its proper column, and thence descend till 21 run off at the side, which is therefore brought to the beginning of its proper line; but as 22 arrives at the cell occupied by 15, descend diagonally to the left, which brings it into the 1st column, but off at the bottom, and therefore it is carried to the top of that column; thence descending till 29 runs off both at bottom and side, which therefore carry to the highest unoccupied cell in the last column; and here, as 30 runs off at the side, bring it to the beginning of its proper column, and thence descend till 35 runs off at the bottom, which therefore carry to the beginning or top of its own column; and here, as 36 meets with the cell occupied by 29, it is brought from thence diagonally to the left; thence descending, 38 runs off at the side, and therefore it is brought to the beginning of its proper line; thence descending, 41 runs off at the bottom, which therefore is carried to the beginning or top of its column; from whence descending, 43 arrives at the cell occupied by 36, and therefore it is brought down from thence to the left; thence descending, 46 runs off at the side, which therefore is brought to the beginning of its line; but here, as 47 runs off at the bottom, it is carried to the beginning or top of its column, from whence descending with 48 and 49, the square is completed, the sum of every row and column and diagonal making just 175.

There are many other ways of filling up such squares, but none that are easier than the above one.

It was observed before, that the sum of the numbers in the rows, columns and diagonals, was 15 in the square of 9 numbers, 34 in a square of 16, 65 in a square of 25, &c; hence then is derived a method of finding the sums of the numbers in any other square, viz, by taking the successive differences till they become equal, and then adding them successively to produce or find out the amount of the following sums. Thus,

Side	Cells	Sums	Diffs.		
0	0	0		0	
1	1	1	1	3	3
2	4	5	4	6	3
3	9	15	10	9	3
4	16	34	19	12	3
5	25	65	31	15	3
6	36	111	46	18	3
7	49	175	64	21	3
8	64	260	85	24	3
9	81	369	109	27	3
10	100	505	136	30	

having ranged the sides and cells in two columns, and a few of the first sums in a third column, take the first differences of these, which will be 1, 4, 10, 19, &c, as in the 4th column; and of these take the differences 0, 3, 6, 9, 12, &c, as in the 5th column; and again, of these the differences 3, 3, 3 &c, as in the 6th or last column. Then, returning back again, add always 3, the constant last or 3d difference, to the last found of the 2d differences, which will complete the remainder of the column of these, viz, 15, 18, 21, 24, &c: then add these 2d differences to the last found of the 1st differences, which will complete the column of these, viz, giving 31, 46, 64, &c: lastly, add always these corresponding 1st differences to the last found number or amount of the sums, and the column of sums will thus be completed.

Again, like as the terms of an arithmetical progression arranged magically, give the same sum in every row &c, so the terms of a geometrical series arranged magically give the same product in every row &c, by multiplying the numbers continually together; so this progression 1, 2, 4, 8, 16, &c, arranged as in the margin, gives, for each continual product, 4096 in every row &c, which is just the cube of the middle term, 16.

8	256	2
4	16	64
128	1	32

Also, the terms of an harmonical progression being ranged magically, as in the margin, have the terms in each row &c in harmonical progression.

1260	840	630
504	420	360
315	280	252

The ingenious Dr. Franklin, it seems, carried this curious speculation farther than any of his predecessors in the same way. He constructed both a *magic square of squares*, and a *magic circle of circles*, the description of which is as follows. The magic square of squares is formed by dividing the great square as in fig. 1, Pl. 15. The great square is divided into 256 little squares, in which all the numbers from 1 to 256, or the square of 16, are placed, in 16 columns, which may be taken either horizontally or vertically. Their chief properties are as follow:

1. The sum of the 16 numbers in each column or row, vertical or horizontal, is 2056.

2. Every half column, vertical and horizontal, makes 1028, or just one half of the same sum 2056.

3. Half a diagonal ascending, added to half a diagonal descending, makes also the same sum 2056; taking these half diagonals from the ends of any side of the square to the middle of it; and so reckoning them either upward or downward; or sideways from right to left, or from left to right.

4. The same with all the parallels to the half diagonals, as many as can be drawn in the great square: for any two of them being directed upward and downward, from the place where they begin, to that where they end, their sums still make the same 2056. Also the same holds true downward and upward; as well as if taken sideways to the middle, and back to the same side again. Only one set of these half diagonals and their parallels, is drawn in the same square upward and downward; but another set may be drawn from any of the other three sides.

5. The four corner numbers in the great square added to the four central numbers in it, make 1028, the half

half fum of any vertical or horizontal column, which contains 16 numbers; and alfo equal to half a diagonal or its parallel.

6. If a fquare hole, equal in breadth to four of the little fquares or cells, be cut in a paper, through which any of the 16 little cells in the great fquare may be feen, and the paper be laid upon the great fquare; the fum of all the 16 numbers, feen through the hole, is always equal to 2056, the fum of the 16 numbers in any horizontal or vertical column.

The *Magic Circle of Circles*, fig. 2, pl. 15, by the fame author, is compofed of a feries of numbers, from 12 to 75 inclufive, divided into 8 concentric circular fpaces, and ranged in 8 radii of numbers, with the number 12 in the centre; which number, like the centre, is common to all thefe circular fpaces, and to all the radii.

The numbers are fo placed, that 1ft, the fum of all thofe in either of the concentric circular fpaces above mentioned, together with the central number 12, amount to 360, the fame as the number of degrees in a circle.

2. The numbers in each radius alfo, together with the central number 12, make juft 360.

3. The numbers in half of any of the above circular fpaces, taken either above or below the double horizontal line, with half the central number 12, make juft 180, or half the degrees in a circle.

4. If any four adjoining numbers be taken, as if in a fquare, in the radial divifions of thefe circular fpaces; the fum of thefe, with half the central number, make alfo the fame 180.

5. There are alfo included four fets of other circular fpaces, bounded by circles that are excentric with regard to the common centre; each of thefe fets containing five fpaces; and the centres of them being at A, B, C, D. For diftinction, thefe circles are drawn with different marks, fome dotted, others by fhort unconnected lines, &c; or ftill better with inks of divers colours, as blue, red, green, yellow.

Thefe fets of excentric circular fpaces interfect thofe of the concentric, and each other; and yet, the numbers contained in each of the excentric fpaces, taken all around through any of the 20, which are excentric, make the fame fum as thofe in the concentric, namely 360, when the central number 12 is added. Their halves alfo, taken above or below the double horizontal line, with half the central number, make up 180.

It is obfervable, that there is not one of the numbers but what belongs at leaft to two of the circular fpaces; fome to three, fome to four, fome to five: and yet they are all fo placed, as never to break the required number 360, in any of the 28 circular fpaces within the primitive circle. They have alfo other properties. See Franklin's Exp. and Obf. pa. 350, edit. 4to, 1769; or Ferguson's Tables and Tracts, 1771, pa. 318.

MAGICAL *Picture*, in Electricity, was firft contrived by Mr. Kinnerfley, and is thus made: Having a large mezzotinto with a frame and glafs, as of the king for inftance, take out the print, and cut a pannel out of it, near two inches diftant from the frame all around; then with thin pafte or gum-water, fix the border that is cut off on the infide of the glafs, preffing it fmooth and clofe; then fill up the vacancy by gilding the glafs well with leaf gold or brafs. Gild likewife the inner edge of the back of the frame all round, except the top part; and form a communication between that gilding and the gilding behind the glafs; then put in the board, and that fide is finifhed. Next turn up the glafs, and gild the forefide exactly over the back gilding, and when it is dry, cover it by pafting on the pannel of the picture that has been cut out, obferving to bring the correfponding parts of the border and picture together, by which means the picture will appear entire, as at firft, only part behind the glafs, and part before.

Hold the picture horizontally by the top, and place a fmall moveable gilt crown on the king's head. If now the picture be moderately electrified, and another perfon take hold of the frame with one hand, fo that his fingers touch its infide gilding, and with the other hand endeavour to take off the crown, he will receive a violent blow, and fail in the attempt. If the picture were highly charged, the confequence might be as fatal as that of high treafon. The operator, who holds the picture by the upper end, where the infide of the frame is not gilt, to prevent its falling, feels nothing of the fhock, and may touch the face of the picture without danger. And if a ring of perfons take the fhock among them, the experiment is called the confpirators. See Franklin's Exper. and Obferv. pa. 30.

MAGINI (John-Anthony), or Maginus, profeffor of mathematics in the univerfity of Bologna, was born at Padua in the year 1536. Magini was remarkable for his great affiduity in acquiring and improving the knowledge of the mathematical fciences, with feveral new inventions for thefe purpofes, and for the extraordinary favour he obtained from moft princes of his time. This doubtlefs arofe partly from the celebrity he had in matters of aftrology, to which he was greatly addicted, making horofcopes, and foretelling events, both relating to perfons and things. He was invited by the emperor Rodolphus to come to Vienna, where he promifed him a profeffor's chair, about the year 1597; but not being able to prevail on him to fettle there, he neverthelefs gave him a handfome penfion.

It is faid, he was fo much addicted to aftrological predictions, that he not only foretold many good and evil events relative to others with fuccefs; but even foretold his own death, which came to pafs the fame year: all which he reprefented as under the influence of the ftars. Tomafini fays, that Magini, being advanced to his 61ft year, was ftruck with an apoplexy, which ended his days; and that a long while before, he had told him and others, that he was afraid of that year. And Roffeni, his pupil, fays, that Magini died under an afpect of the planets, which, according to his own prediction, would prove fatal to him; and he mentions Riccioli as affirming that he faid, the figure of his nativity, and his climacteric year, doomed him to die about that time; which happened in 1618, in the 62d year of his age.

His writings do honour to his memory, as they were very confiderable, and upon learned fubjects. The principal were the following: 1. His Ephemeris, in 3 volumes, from the year 1580 to 1630.—2. Tables of Secondary Motions.—3. Aftronomical, Gnomonical, and Geographical Problems.—4. Theory of the Planets, according to Copernicus.—5. A Confutation of Scaliger's Differtation concerning the Preceffion of the Equinox.

Equinox.—6. A Primum Mobile, in 12 books.—7. A Treatife of Plane and Spherical Trigonometry.—8. A Commentary on Ptolomy's Geography.—9. A Chorographical Defcription of the Regions and Cities of Italy, illuftrated with 60 maps; with fome other papers on Aftrological fubjects.

MAGNET, Magnes, the *Loadftone;* a kind of ferruginous ftone, refembling iron ore in weight and colour, though rather harder and heavier; and is endued with divers extraordinary properties, attractive, directive, inclinatory, &c. See Magnetism.

The Magnet is alfo called *Lapis Heracleus*, from Heraclea, a city of Magnefia, a port of the ancient Lydia, where it was faid it was firft found, and from which it is ufually fuppofed that it took its name. Though fome derive the word from a fhepherd named *Magnes*, who firft difcovered it on Mount Ida with the iron of his crook. It is alfo called *Lapis Nauticus*, from its ufe in navigation; alfo *Siderites*, from its virtue in attracting iron, which the Greeks call σιδηρος.

The Magnet is ufually found in iron mines, and fometimes in very large pieces, half magnet, half iron. Its colour is different, as found in different countries. Norman obferves, that the beft are thofe brought from China and Bengal, which are of an irony or fanguine colour; thofe of Arabia are reddifh; thofe of Macedonia, blackifh; and thofe of Hungary, Germany, England, &c, the colour of unwrought iron. Neither its figure nor bulk are conftant or determined; being found of all fhapes and fizes.

The Ancients reckoned five kinds of Magnets, different in colour and virtue: the Ethiopic, Magnefian, Bœotic, Alexandrian, and Natolian. They alfo took it to be male and female: but the chief ufe they made of it was in medicine; efpecially for the cure of burns and defluxions of the eyes.—The Moderns, more happy, take it to conduct them in their voyages.

The moft diftinguifhing properties of the Magnet are, That it attracts iron, and that it points towards the poles of the world; and in other circumftances alfo dips or inclines to a point beneath the horizon, directly under the pole; it alfo communicates thefe properties, by touch, to iron. By means of which, are obtained the mariner's needles, both horizontal, and inclinatory or dipping needles.

The Attractive Power of the Magnet, was known to the Ancients, and is mentioned even by Plato and Euripides, who call it the *Herculean ftone*, becaufe it commands iron, which fubdues every thing elfe: but the knowledge of its directive power, by which it difpofes its poles along the meridian of every place, or nearly fo, and caufes needles, pieces of iron, &c, touched with it, to point nearly north and fouth, is of a much later date; though the difcoverer himfelf, and the exact time of the difcovery, be not now known. The firft mention of it is about 1260, when it has been faid that Marco Polo, a Venetian, introduced the mariner's compafs; though not as an invention of his own, but as derived from the Chinefe, who it feems had the ufe of it long before; though fome imagine that the Chinefe rather borrowed it from the Europeans.

But Flavio de Gira, a Neapolitan, who lived in the 13th century, is the perfon ufually fuppofed to have the beft title to the difcovery; and yet Sir G. Wheeler mentions, that he had feen a book of aftronomy much older, which fuppofed the ufe of the needle; though not as applied to the purpofes of navigation, but of aftronomy. And in Guiot de Provins, an old French poet, who wrote about the year 1180, there is an exprefs mention made of the loadftone and the compafs; and their ufe in navigation obliquely hinted at.

The Variation of the Magnet, or needle, or its deviation from the pole, was firft difcovered by Sebaftian Cabot, a Venetian, in 1500; and the variation of that variation, or change in its direction, by Mr. Henry Gellibrand, profeffor of aftronomy in Grefham college, about the year 1625.

Laftly, the Dip or inclination of the needle, when at liberty to play vertically, to a point beneath the horizon, was firft difcovered by another of our countrymen, Mr. Robert Norman, about the year 1576.

The Phenomena of the Magnet, are as follow: 1, In every Magnet there are two poles, of which the one points northwards, the other fouthwards; and if the Magnet be divided into ever fo many pieces, the two poles will be found in each piece. The poles of a Magnet may be found by holding a very fine fhort needle over it; for where the poles are, the needle will ftand upright, but no where elfe.—2, Thefe poles, in different parts of the globe, are differently inclined towards a point under the horizon.—3, Thefe poles, though contrary to each other, do help mutually towards the Magnet's attraction, and fufpenfion of iron.—4, If two Magnets be fpherical, one will turn or conform itfelf to the other, fo as either of them would do to the earth; and after they have fo conformed or turned themfelves, they endeavour to approach or join each other; but if placed in a contrary pofition, they avoid each other.—5, If a Magnet be cut through the axis, the fegments or parts of the ftone, which before were joined, will now avoid and fly each other.—6, If the Magnet be cut perpendicular to its axis, the two points, which before were conjoined, will become contrary poles; one in the one, and one in the other fegment.—7, Iron receives virtue from the Magnet by application to it, or barely from an approach near it, though it do not touch it; and the iron receives this virtue varioufly, according to the parts of the ftone it is made to touch, or even approach to.—8, If an oblong piece of iron be anyhow applied to the ftone, it receives virtue from it only lengthways.—9, The Magnet lofes none of its own virtue by communicating any to the iron; and this virtue it can communicate to the iron very fpeedily: though the longer the iron joins or touches the ftone, the longer will its communicated virtue hold; and a better Magnet will communicate more of it, and fooner, than one not fo good.—10, Steel receives virtue from the Magnet better than iron.—11, A needle touched by a Magnet will turn its ends the fame way towards the poles of the world, as the Magnet itfelf does.—12, Neither loadftone nor needles touched by it do conform their poles exactly to thofe of the world, but have ufually fome variation from them: and this variation is different in divers places, and at divers times in the fame places.—13, A loadftone will take up much more iron when armed, or capped, than it can alone. (A loadftone is faid to be armed, when its poles are furrounded with plates

plates of ſteel: and to determine the quantity of ſteel to be applied, try the Magnet with ſeveral ſteel bars; and the greateſt weight it takes up, with a bar on, is to be the weight of its armour.) And though an iron ring or key be ſuſpended by the loadſtone, yet this does not hinder the ring or key from turning round any way, either to the right or left.—14, The force of a loadſtone may be variouſly increaſed or leſſened by variouſly applying to it, either iron, or another loadſtone.—15, A ſtrong Magnet at the leaſt diſtance from a ſmaller or a weaker, cannot draw to it a piece of iron adhering actually to ſuch ſmaller or weaker ſtone; but if it come to touch it, it can draw it from the other: but a weaker Magnet, or even a ſmall piece of iron, can draw away or ſeparate a piece of iron contiguous to a larger or ſtronger Magnet.—16, In theſe northern parts of the world, the ſouth pole of a Magnet will raiſe up more iron than its north pole.—17, A plate of iron only, but no other body interpoſed, can impede the operation of the loadſtone, either as to its attractive or directive quality.—18, The power or virtue of a loadſtone may be impaired by lying long in a wrong poſition, as alſo by ruſt, wet, &c; and may be quite deſtroyed by fire, lightning, &c.—19, A piece of iron wire well touched, upon being bent round in a ring, or coiled round on a ſtick, &c, will always have its directive virtue diminiſhed, and often quite deſtroyed. And yet if the whole length of the wire were not entirely bent, ſo that the ends of it, though but for the length of one-tenth of an inch, were left ſtraight, the virtue will not be deſtroyed in thoſe parts; though it will in all the reſt.—20, The ſphere of activity of Magnets is greater and leſs at different times. Alſo, the variation of the needle from the meridian, is various at different times of the day.—21, By twiſting a piece of wire touched with a Magnet, its virtue is greatly diminiſhed; and ſometimes ſo diſordered and confuſed, that in ſome parts it will attract, and in others repel; and even, in ſome places, one ſide of the wire ſeems to be attracted, and the other ſide repelled, by one and the ſame pole of the ſtone.—22, A piece of wire that has been touched, on being ſplit, or cleft in two, the poles are ſometimes changed, as in a cleft Magnet; the north pole becoming the ſouth, and the ſouth the north: and yet ſometimes one half of the wire will retain its former poles, and the other half will have them changed. —23, A wire being touched from end to end with one pole of a Magnet, the end at which you begin will always turn contrary to the pole that touched it: and if it be again touched the ſame way with the other pole of the Magnet, it will then be turned the contrary way.—24, If a piece of wire be touched in the middle with only one pole of the Magnet, without moving it backwards or forwards; in that place will be the pole of the wire, and the two ends will be the other pole.—25, If a Magnet be heated red hot, and again cooled either with its ſouth pole towards the north in a horizontal poſition, or with its ſouth pole downwards in a perpendicular poſition, its poles will be changed. —26, Mr. Boyle (to whom we are indebted for the following magnetical phenomena) found he could preſently change the poles of a ſmall fragment of a loadſtone, by applying them to the oppoſite vigorous poles of a large one.—27, Hard iron tools well tempered, when heated by a briſk attrition, as filing, turning, &c, will attract thin filings or chips of iron, ſteel, &c; and hence we obſerve that files, punches, augres, &c, have a ſmall degree of magnetic virtue.—28, The iron bars of windows, &c, which have ſtood a long time in an erect poſition, grow permanently magnetical; the lower ends of ſuch bars being the north pole, and the upper end the ſouth pole.—29, A bar of iron that has not ſtood long in an erect poſture, if it be only held perpendicularly, will become magnetical, and its lower end the north pole, as appears from its attracting the ſouth pole of a needle: but then this virtue is tranſient, and by inverting the bar, the poles change their places. In order therefore to render the quality permanent in an iron bar, it muſt continue a long time in a proper poſition. But fire will produce the effect in a ſhort time: for as it will immediately deprive a loadſtone of its attractive virtue; ſo it ſoon gives a verticity to a bar of iron, if, being heated red hot, it be cooled in an erect poſture, or directly north and ſouth. Even tongs and fireforks, by being often heated, and ſet to cool again in a poſture nearly erect, have gained this magnetic property. Sometimes iron bars, by long ſtanding in a perpendicular poſition, have acquired the magnetic virtue in a ſurpriſing degree. A bar about 10 feet long, and three inches thick, ſupporting the ſummer beam of a room, was able to turn the needle at 8 or 10 feet diſtance, and exceeded a loadſtone of $3\frac{1}{2}$ pounds weight: from the middle point upwards it was a north pole, and downwards a ſouth pole. And Mr. Martin mentions a bar, which had been the beam of a large ſteel-yard that had ſeveral poles in it.—30, Mr. Boyle found, that by heating a piece of Engliſh oker red-hot, and placing it to cool in a proper poſture, it manifeſtly acquired a magnetic virtue. And an excellent Magnet, belonging to the ſame ingenious gentleman, having lain near a year in an inconvenient poſture, had its virtue greatly impaired, as if it had been by fire.—31, A needle well touched, it is known, will point north and ſouth: if it have one contrary touch of the ſame ſtone, it will be deprived of its faculty; and by another ſuch touch, it will have its poles interchanged.—32, If an iron bar have gained a verticity by being heated red-hot and cooled again, north and ſouth, and then hammered at the two ends; its virtue will be deſtroyed by two or three ſmart blows on the middle.—33, By drawing the back of a knife, or a long piece of ſteel-wire, &c, leiſurely over the pole of a loadſtone, carrying the motion from the middle of the ſtone to the pole; the knife or wire will attract one end of a needle; but if the knife or wire be paſſed from the ſaid pole to the middle of the ſtone, it will repel the ſame end of the needle.—34, Either a Magnet or a piece of iron being laid on a piece of cork, ſo as to float freely on water; it will be found, that, whichſoever of the two is held in the hand, the other will be drawn to it: ſo that iron attracts the Magnet as much as it is attracted by it; action and re-action being always equal. In this experiment, if the Magnet be ſet afloat, it will direct its two poles to the poles of the world nearly.—35, A knife &c touched with a Magnet, acquires a greater or leſs degree of virtue, according to the part it is touched on. It receives the ſtrongeſt virtue, when it is drawn leiſurely from the handle

handle towards the point over one of the poles. And if the ſame knife thus touched, and thus poſſeſſed of a ſtrong attractive power, be retouched in a contrary direction, viz, by drawing it from the point towards the handle over the ſame pole, it immediately loſes all its virtue.—36, A Magnet acts with equal force in vacuo as in the open air.—37, The ſmalleſt Magnets have uſually the greateſt power in proportion to their bulk. A large Magnet will ſeldom take up above 3 or 4 times its own weight, while a ſmall one will often take up more than ten times its weight. A Magnet worn by Sir Iſaac Newton in a ring, and which weighed only 3 grains, would take up 746 grains, or almoſt 250 times its own weight. A magnetic bar made by Mr. Canton, weighing 10 oz. 12 dwts, took up more than 79 ounces; and a flat ſemicircular ſteel Magnet, weighing 1 oz. 13 dwts, took up an iron wedge of 90 ounces.

Armed Magnet, denotes one that is capped, caſed, or ſet in iron or ſteel, to make it take up a greater weight, and alſo more readily to diſtinguiſh its poles. For the methods of doing this, ſee Mr. Michell's book on this ſubject.

Artificial Magnet, is a bar of iron or ſteel, impregnated with the magnetic virtue, ſo as to poſſeſs all the properties of the natural loadſtone, and be uſed inſtead of it. How to make Magnets of this kind, by means of a natural Magnet, and even without the aſſiſtance of any Magnet, was ſuggeſted many years ſince by Mr. Savary, and particularly deſcribed in the Philoſ. Tranſ. number 414. See alſo Abridgment, vol. 6, pa. 260. But as his method was tedious and operoſe, though capable of communicating a very conſiderable virtue, it was little practiſed. Dr. Gowin Knight firſt brought this kind of Magnets to their preſent ſtate of perfection, ſo as to be even of much greater efficacy than the natural ones. But as he refuſed to diſcover his methods upon any terms whatever (even, as he ſaid, though he ſhould receive in return as many guineas as he could carry), theſe curious and valuable ſecrets in a great meaſure died with him. The reſult of his method however was firſt publiſhed in the Philoſ. Tranſ. for 1744, art. 8, and for 1745, art. 3. See alſo the vol. for 1747, art. 2. And in the 69th vol. Mr. Benjamin Wilſon has given a proceſs, which at leaſt diſcovers one of the leading principles of Dr. Knight's art. The method, according to Mr. Wilſon, was as follows. Having provided a great quantity of clean iron filings, he put them into a large tub that was more than one-third filled with clean water; he then, with great labour, ſhook the tub to and fro for many hours together, that the friction between the grains of iron, by this treatment, might break or rub off ſuch ſmall parts as would remain ſuſpended in the water for ſome time. The water being thus rendered very muddy, he poured it into a clean iron veſſel, leaving the filings behind; and when the water had ſtood long enough to become clear, he poured it out carefully, without diſturbing ſuch of the ſediment as ſtill remained, which now appeared reduced almoſt to impalpable powder. This powder was afterwards removed into another veſſel, to dry it. Having, by ſeveral repetitions of this proceſs, procured a ſufficient quantity of this very fine powder, the next thing was to make a paſte of it, and that with ſome vehicle containing a good quantity of the phlogiſtic principle; for this purpoſe, he had recourſe to linſeed oil, in preference to all other fluids. With theſe two ingredients only, he made a ſtiff paſte, and took great care to knead it well before he moulded it into convenient ſhapes. Sometimes, while the paſte continued in its ſoft ſtate, he would put the impreſſion of a ſeal; one of which is in the Britiſh Muſeum. This paſte ſo moulded was then ſet upon wood, or a tile, to dry or bake it before a moderate fire, being placed at about one foot diſtance. He found that a moderate fire was moſt proper, becauſe a greater degree of heat would make the compoſition crack in many places. The time requiſite for the baking or drying of this paſte, was uſually about 5 or 6 hours, before it attained a ſufficient degree of hardneſs. When that was done, and the ſeveral baked pieces were become cold, he gave them their magnetic virtue in any direction he pleaſed, by placing them between the extreme ends of his large magazine of artificial magnets, for a few ſeconds. The virtue they acquired by this method was ſuch, that, when any of thoſe pieces were held between two of his beſt ten-guinea bars, with its poles purpoſely inverted, it immediately of itſelf turned about to recover its natural direction, which the force of thoſe very powerful bars was not ſufficient to counteract. Philoſ. Tranſ. vol. 65, for 1779.

Methods for artificial Magnets were alſo diſcovered and publiſhed by the Rev. Mr. John Michell, in a Treatiſe on Artificial Magnets, printed in 1750, and by Mr. John Canton, in the Philoſ. Tranſ. for 1751. The proceſs for the ſame purpoſe was alſo found out by other perſons, particularly by Du Hamel, Hiſt. Acad. Roy. 1745 and 1750, and by Marul Uitgeleeze Natuurkund. Verhand. tom. 2, p. 261.

Mr. Canton's method is as follows: Procure a dozen of bars; 6 of ſoft ſteel, and 6 of hard; the former to be each 3 inches long, a quarter of an inch broad, and 1-20th of an inch thick; with two pieces of iron, each half the length of one of the bars, but of the ſame breadth and thickneſs; and the 6 hard bars to be each $5\frac{1}{2}$ inches long, half an inch broad, and 3-20ths of an inch thick, with two pieces of iron of half the length, but the whole breadth and thickneſs of one of the hard bars; and let all the bars be marked with a line quite around them at one end. Then take an iron poker and tongs (fig. 1, plate 16), or two bars of iron, the larger they are, and the longer they have been uſed, the better; and fixing the poker upright between the knees, hold to it, near the top, one of the ſoft bars, having its marked end downwards by a piece of ſewing ſilk, which muſt be pulled tight by the left hand, that the bar may not ſlide: then graſping the tongs with the right hand, a little below the middle, and holding them nearly in a vertical poſition, let the bar be ſtroked by the lower end, from the bottom to the top, about ten times on each ſide, which will give it a magnetic power ſufficient to lift a ſmall key at the marked end: which end, if the bar were ſuſpended on a point, would turn towards the north, and is therefore called the north pole; and the unmarked end is, for the ſame reaſon, called

called the ſouth pole. Four of the ſoft bars being impregnated after this manner, lay the two (fig. 2) parallel to each other, at a quarter of an inch diſtance, between the two pieces of iron belonging to them, a north and a ſouth pole againſt each piece of iron: then take two of the four bars already made magnetical, and place them together ſo as to make a double bar in thickneſs, the north pole of one even with the ſouth pole of the other; and the remaining two being put to theſe, one on each ſide, ſo as to have two north and two ſouth poles together, ſeparate the north from the ſouth poles at one end by a large pin, and place them perpendicularly with that end downward on the middle of one of the parallel bars, the two north poles towards its ſouth end, and the two ſouth poles towards its north end: ſlide them three or four times backward and forward the whole length of the bar; then removing them from the middle of this bar, place them on the middle of the other bar as before directed, and go over that in the ſame manner; then turn both the bars the other ſide upwards, and repeat the former operation: this being done, take the two from between the pieces of iron; and, placing the two outermoſt of the touching bars in their ſtead, let the other two be the outermoſt of the four to touch theſe with; and this proceſs being repeated till each pair of bars have been touched three or four times over, which will give them a conſiderable magnetic power. Put the half-dozen together after the manner of the four (fig. 3), and touch them with two pair of the hard bars placed between their irons, at the diſtance of about half an inch from each other; then lay the ſoft bars aſide, and with the four hard ones let the other two be impregnated (fig. 4), holding the touching bars apart at the lower end near two-tenths of an inch; to which diſtance let them be ſeparated after they are ſet on the parallel bar, and brought together again before they are taken off: this being obſerved, proceed according to the method deſcribed above, till each pair have been touched two or three times over. But as this vertical way of touching a bar, will not give it quite ſo much of the magnetic virtue as it will receive, let each pair be now touched once or twice over in their parallel poſition between the irons (fig. 5), with two of the bars held horizontally, or nearly ſo, by drawing at the ſame time the north end of one from the middle over the ſouth end, and the ſouth of the other from the middle over the north end of a parallel bar; then bringing them to the middle again, without touching the parallel bar, give three or four of theſe horizontal ſtrokes to each ſide. The horizontal touch, after the vertical, will make the bars as ſtrong as they poſſibly can be made, as appears by their not receiving any additional ſtrength, when the vertical touch is given by a great number of bars, and the horizontal by thoſe of a ſuperior magnetic power.

This whole proceſs may be gone through in about half an hour; and each of the large bars, if well hardened, may be made to lift 28 Troy ounces, and ſometimes more. And when theſe bars are thus impregnated, they will give to a hard bar of the ſame ſize its full virtue in leſs than two minutes; and therefore will anſwer all the purpoſes of Magnetiſm in navigation and experimental philoſophy, much better than the loadſtone, which has not a power ſufficient to impregnate hard bars. The half dozen being put into a caſe (fig. 6), in ſuch a manner as that no two poles of the ſame name may be together, and their irons with them as one bar, they will retain the virtues they have received; but if their power ſhould, by making experiments, be ever ſo far impaired, it may be reſtored without any foreign aſſiſtance in a few minutes. And if, perchance, a much larger ſet of bars ſhould be required, theſe will communicate to them a ſufficient power to proceed with; and they may, in a ſhort time, by the ſame method, be brought to their full ſtrength.

MAGNETISM, the quality or conſtitution of a body, by which it is rendered magnetical, or a magnet, ſenſibly attracting iron, and giving it a meridional direction.

This is a tranſient power, capable of being produced, deſtroyed, or reſtored.

The Laws of MAGNETISM.

Theſe laws are laid down by Mr. Whiſton in the following propoſitions.——1, The Loadſtone has both an attractive and a directive power united together, while iron touched by it has only the former; i. e. the magnet not only attracts needles, or ſteel filings, but alſo directs them to certain different angles, with reſpect to its own ſurface and axis; whereas iron, touched with it, does little or nothing more than attract them; ſtill ſuffering them to lie along or ſtand perpendicular to its ſurface and edges in all places, without any ſuch ſpecial direction.

2. Neither the ſtrongeſt nor the largeſt magnets give a better directive touch to needles, than thoſe of a leſs ſize or virtue: to which may be added, that whereas there are two qualities in all magnets, an attractive and a directive one; neither of them depend on, or are any argument of the ſtrength of the other.

3. The attractive power of magnets, and of iron, will greatly increaſe or diminiſh the weight of needles on the balance; nay, it will overcome that weight, and even ſuſtain ſome other additional alſo: while the directive power has a much ſmaller effect. Gaſſendus indeed, as well as Merſennus and Gilbert, aſſert that it has none at all: but by miſtake; for Whiſton found, from repeated trials on large needles, that after the touch they weighed leſs than before. One of $4584\frac{1}{8}$ grains, loſt $2\frac{5}{8}$ grains by the touch; and another of 65726 grains weight, no leſs than 14 grains.

4. It is probable that iron conſiſts almoſt wholly of the attractive particles; and the magnet, of the attractive and directive together; mixed, probably, with other heterogeneous matter; as having never been purged by the fire, which iron has; and hence may ariſe the reaſon why iron, after it has been touched, will lift up a much greater weight than the loadſtone that touched it.

5. The quantity and direction of magnetic powers, communicated to needles, are not properly, after ſuch communication, owing to the magnet which gave the touch; but to the goodneſs of the ſteel that receives it, and to the ſtrength and poſition of the terreſtrial loadſtone, whoſe influence alone thoſe needles are afterwards ſubject to, and directed by: ſo that all ſuch needles, if good, move with the ſame ſtrength, and point to the ſame angle, whatever loadſtone they may have been excited by, provided it be but a good one. Nor does it ſeem

feem that the touch does much more in magnetical cafes, than attrition does in electrical ones; i. e. ferving to rub off fome obftructing particles, that adhere to the furface of the fteel, and opening the pores of the body touched, and fo make way for the entrance and exit of fuch effluvia as occafion or affift the powers we are fpeaking of. Hence Mr. Whifton takes occafion to obferve, that the directive power of the loadftone feems to be mechanical, and to be derived from magnetic effluvia, circulating continually round it.

6. The abfolute attractive power of different armed loadftones, is, *cæteris paribus*, not according to either the diameters or folidities of the loadftones, but according to the quantity of their furfaces, or in the duplicate proportion of their diameters.

7. The power of good magnets unarmed, fenfibly equal in ftrength, fimilar in figure and pofition, but unequal in magnitude, is fometimes a little greater, fometimes a little lefs, than in the proportion of their fimilar diameters.

8. The loadftone attracts needles that have been touched, and others that have not been touched, with equal force at diftances unequal, viz. when the diftance of the former is to the diftance of the latter, as 5 to 2.

9. Both poles of a magnet equally attract needles, till they are touched; then it is, and then only, that one pole begins to attract one end, and repel the other: though the repelling pole will ftill attract upon contact, and even at very fmall diftances.

10. The attractive power of loadftones, in their fimilar pofition to, but different diftances from, magnetic needles, is in the fefquiduplicate proportion of the diftances of their furfaces from their needles reciprocally; or as the mean proportionals between the fquares and the cubes of thofe diftances reciprocally; or as the fquare roots of the 5th powers of thofe diftances reciprocally. Thus, the magnetic force of attraction, at twice the diftance from the furface of the loadftone, is between a 5th and 6th part of the force at the firft diftance; at thrice the diftance, the force is between the 15th and 16th part; at four times the diftance, the power is the 32d part of the firft; and at fix times the diftance, it is the 88th part. Where it is to be noted, that the diftances are not counted from the centre, as in the laws of gravity, but from the furface: all experience affuring us, that the magnetic power refides chiefly, if not wholly, in the furfaces of the loadftone and iron; without any particular relation to any centre at all. The proportion here laid down was determined by Mr. Whifton, from a great number of experiments by Mr. Hawkfbee, Dr. Brook Taylor, and himfelf; meafuring the force by the chords of thofe arcs by which the magnet at feveral diftances draws the needle out of its natural direction, to which chords, as he demonftrates, it is always proportional. The numbers in fome of their moft accurate trials, he gives in the following Table, fetting down the half chords, or the fines of half thofe arcs of declination, as the true meafures of the force of magnetic attraction.

Diftances in inches.	*Degrees of inclination.*	*Sines of* $\frac{1}{2}$ *arcs.*	*Sefquiduplicate ratio.*
20	2	175	466
$14\frac{8}{9}$	4	349	216
$13\frac{5}{8}$	6	523	170
$12\frac{2}{8}$	8	697	138
$11\frac{1}{8}$	10	871	105
$10\frac{1}{4}$	12	1045	87
$9\frac{1}{4}$	14	1219	70

Other perfons however have found fome variations in the proportions of magnetic force with refpect to diftance: Thus, Newton fuppofes it to decreafe nearly in the triplicate ratio of the diftance: Mr. Martin obferves, that the power of his loadftone decreafes in the fefquiduplicate ratio of the diftances inverfely: but Dr. Helfham and Mr. Michell found it to be as the fquare of the diftance inverfely: while others, as Dr. Brook Taylor and M. Mufchenbroek, are of opinion, that this power follows no certain ratio at all, and that the variation is different in different ftones.

11. An inclinatory, or dipping-needle, of 6 inches radius, and of a prifmatic or cylindric figure, when it ofcillates along the magnetic meridian, performs there every mean vibration in about 6″ or 360‴, and every fmall ofcillation in about $5''\frac{1}{2}$, or 330‴: and the fame kind of needle, 4 feet long, makes every mean ofcillation in about 24″, and every fmall one in about 22″.

12. The whole power of Magnetifm in this country, as it affects needles a foot long, is to that of gravity nearly as 1 to 300; and as it affects needles 4 feet long, as 1 to 600.

13. The quantity of magnetic power accelerating the fame dipping-needle, as it ofcillates in different vertical planes, is always as the cofines of the angles made by thofe planes with the magnetic meridian, taken on the horizon.

Thus, in eftimating the quantity of force in the horizontal and in the vertical fituations of needles at London, it is found that the latter, in needles of a foot long, is to the whole force along the magnetic meridian, as 96 to 100; and in needles 4 feet long, as 9667 to 10000: whereas, in the former, the whole force in needles of a foot long, is as 28 to 100; and in thofe of 4 feet long, as 256 to 1000. Whence it follows, that the power by which horizontal needles are governed in thefe parts of the world, is but the quarter of the power by which the dipping-needle is moved.

Hence alfo, as the horizontal needle is moved only by a part of the power that moves the dipping-needle; and as it only points to a certain place in the horizon, becaufe that place is the neareft to its original tendency of any that its fituation will allow it to tend to; whenever the dipping-needle ftands exactly perpendicular to the horizon, the horizontal needle will not refpect one point of the compafs more than another, but will wheel about any way uncertainly.

14. The time of ofcillation and vibration, both in dipping and horizontal needles, that are equally good, is as their length directly; and the actual velocities of their points along their arcs, are always unequal. And hence, magnetical needles are, *cæteris paribus*, ftill better, the longer they are; and that in the fame proportion with their lengths.

Of the Causes of MAGNETISM. Though many authors have proposed hypotheses, or written concerning the cause of Magnetism, as Plutarch, Descartes, Boyle, Newton, Gilbert, Hartsoeker, Halley, Whiston, Knight, Beccaria, &c; nothing however has yet appeared that can be called a satisfactory solution of its phenomena. It is certain indeed, that both natural and artificial electricity will give polarity to needles, and even reverse their poles; but though from this it may appear probable that the electric fluid is also the cause of Magnetism, yet in what manner the fluid acts while producing the magnetical phenomena, seems to be quite unknown.

Dr. Knight indeed deduces from several experiments the following propositions, which he offers, not so much to explain the nature of the cause of Magnetism, as the manner in which it acts: the magnetic matter of a loadstone, he says, moves in a stream from one pole to the other internally, and is then carried back in a curve line externally, till it arrive again at the pole where it first entered, to be again admitted: the immediate cause why two or more magnetical bodies attract each other, is the flux of one and the same stream of magnetical matter through them; and the immediate cause of magnetic repulsion, is the conflux and accumulation of the magnetic matter. Philos. Trans. vol. 44, pa. 665.

Mr. Michell rejects the motion of a subtle fluid; but though he proposed to publish a theory of Magnetism established by experiments, no such theory has appeared.

Signor Beccaria, from observing that a sudden stroke of lightning gives polarity to Magnets, conjectures, that a regular and constant circulation of the whole mass of the electric fluid from north to south may be the original cause of Magnetism in general. This current he would not suppose to arise from one source, but from several, in the northern hemisphere of the earth: the aberration of the common centre of all the currents from the north point, may be the cause of the variation of the needle; the period of this declination of the centre of the currents, may be the period of the variation; and the obliquity with which the currents strike into the earth, may be the cause of the dipping of the needle, and also why bars of iron more easily receive the magnetic virtue in one particular direction. Lettre dell' Elettricismo, pa. 269; or Priestley's Hist. Elec. vol. 1, pa. 409. See also Cavallo's Treatise on Magnetism.

MAGNIFYING, is the making of objects appear larger than they usually and naturally appear to the eye; whence convex lenses, which have the power of doing this, are called Magnifying Glasses.

The Magnifying power of dense mediums of certain figures, was known to the Ancients; though they were far from understanding the cause of this effect. Seneca says, that small and obscure letters appear larger and brighter through a glass globe filled with water; and he absurdly accounts for it by saying, that the eye slides in the water, and cannot lay hold of its object. And Alexander Aphrodisiensis, about two centuries after Seneca, says, that the reason why apples appear large when immersed in water, is, that the water which is contiguous to any body is affected with the same quality and colour; so that the eye is deceived in imagining the body itself larger. But the first distinct account we have of the Magnifying power of glasses, is in the 12th century, in the writings of Roger Bacon, and Alhazen; and it is not improbable that from their observations the construction of spectacles was derived. In the Opus Majus of Bacon, it is demonstrated, that if a transparent body, interspersed between the eye and an object, be convex towards the eye, the object will appear magnified.

MAGNIFYING *Glass*, in Optics, is a small spherical convex lens; which, in transmitting the rays of light, inflects them more towards the axis, and so exhibits objects viewed through them larger than when viewed by the naked eye. See MICROSCOPE.

MAGNITUDE, any thing made up of parts locally extended, or continued; or that has several dimensions; as a line, surface, solid, &c. Quantity is often used as synonymous with Magnitude. See QUANTITY.

Geometrical MAGNITUDES, are usually, and most properly, considered as generated or produced by motion; as lines by the motion of points, surfaces by the motion of lines, and solids by the motion of surfaces.

Apparent MAGNITUDE, is that which is measured by the optic or visual angle, intercepted between rays drawn from its extremes to the centre of the pupil of the eye. It is a fundamental maxim in optics, that whatever things are seen under the same or equal angles, appear equal; and vice versa.—The apparent Magnitudes of an object at different distances, are in a ratio less than that of their distances reciprocally.

The apparent Magnitudes of the two great luminaries, the sun and moon, at rising and setting, are a phenomenon that has greatly embarrassed the modern philosophers. According to the ordinary laws of vision, they should appear the least when nearest the horizon, being then farthest from the eye; and yet it is found that the contrary is true in fact. Thus, it is well known that the mean apparent diameter of the moon, at her greatest height in the meridian, is nearly 31′ in round numbers, subtending then an angle of that quantity as measured by any instrument. But, being viewed when she rises or sets, she seems to the eye as two or three times as large as before; and yet when measured by the instrument, her diameter is not found increased at all.

Ptolomy, in his Almagest, lib. 1, cap. 3, taking for granted, that the angle subtended by the moon was really increased, ascribed the increase to a refraction of the rays by vapours, which actually enlarge the angle under which the moon appears; just as the angle is enlarged by which an object is seen from under water: and his commentator Theon explains distinctly how the dilatation of the angle in the object immersed in water is caused. But it being afterwards discovered, that there is no alteration in the angle, another solution was started by the Arab Alhazen, which was followed and improved by Bacon, Vitello, Kepler, Peckham, and others. According to Alhazen, the sight apprehends the surface of the heavens as flat, and judges of the stars as it would of ordinary visible objects extended upon a wide plain; the eye sees then under equal angles indeed, but withal perceives a difference in their distances,

diftances, and (on account of the femidiameter of the earth, which is interpofed in one cafe, and not in the other) it is hence induced to judge thofe that appear more remote to be greater." Some farther improvement was made in this explanation by Mr. Hobbes, though he fell into fome miftakes in his application of geometry to this fubject: for he obferves, that this deception operates gradually from the zenith to the horizon; and that if the apparent arch of the fky be divided into any number of equal parts, thofe parts, in defcending towards the horizon, will fubtend an angle that is gradually lefs and lefs. And he was the firft who expreffly confidered the vaulted appearance of the fky as a real portion of a circle.

Des Cartes, and from him Dr. Wallis, and moft other authors, account for the appearance of a different diftance under the fame angle, from the long feries of objects interpofed between the eye and the extremity of the fenfible horizon; which makes us imagine it more remote than when in the meridian, where the eye fees nothing in the way between the object and itfelf. This idea of a great diftance makes us imagine the luminary the larger; for an object being feen under any certain angle, and believed at the fame time very remote, we naturally judge it muft be very large, to appear under fuch an angle at fuch a diftance. And thus a pure judgment of the mind makes us fee the fun, or the moon, larger in the horizon than in the meridian; notwithftanding their diameters meafured by any inftrument are really lefs in the former fituation than the latter.

James Gregory, in his Geom. Pars Univerfalis, pa. 141, fubfcribes to this opinion: Father Mallebranche alfo, in the firft book of his Recherche de la Verité, has explained this phenomenon almoft in the expreffion of Des Cartes: and Huygens, in his Treatife on the Parhelia, tranflated by Dr. Smith, Optics, art. 536, has approved, and very clearly illuftrated, the received opinion. The caufe of this fallacy, fays he, in fhort, is this; that we think the fun, or any thing elfe in the heavens, farther from us when it is near the horizon, than when it approaches towards the vertex, becaufe we imagine every thing in the air that appears near the vertex to be farther from us than the clouds that fly over our heads; whereas, on the other hand, we are ufed to obferve a large extent of land lying between us and the objects near the horizon, at the farther end of which the convexity of the fky begins to appear; which therefore, with the objects that appear in it, are ufually imagined to be much farther from us. Now when two objects of equal magnitude appear under the fame angle, we always judge that object to be larger which we think is remoter. And this, according to them, is the true caufe of the deception in queftion. It is really aftonifhing that an hypothefis fo palpably falfe fhould ever be held and maintained by fuch eminent men; for it is daily feen that the moon or fun, when near the horizon, very fuddenly change their magnitude, as they afcend or defcend, though all the intervening objects are feen juft as before; and that the luminary appears largeft of all when feweft objects appear on the earth, as in a thick fog or mift. It is no wonder therefore that other reafons have been affigned for this remarkable phenomenon.

Accordingly Gaffendus was of opinion, that this effect arifes from hence; that the pupil of the eye, being always more open as the place is more dark, as in the morning and evening, when the light is lefs, and befides the earth being then covered with grofs vapours, through a longer column of which the rays muft pafs to reach the horizon; the image of the luminary enters the eye at a greater angle, and is really painted there larger than when the luminary is higher. See APPARENT *Diameter* and *Magnitude*.

F. Gouge advances another hypothefis, which is, that when the luminaries are in the horizon, the proximity of the earth, and the grofs vapours with which they then appear enveloped, have the fame effect with regard to us, as a wall, or other denfe body, placed behind a column; which in that cafe appears larger than when infulated, and encompaffed on all fides with an illuminated air.

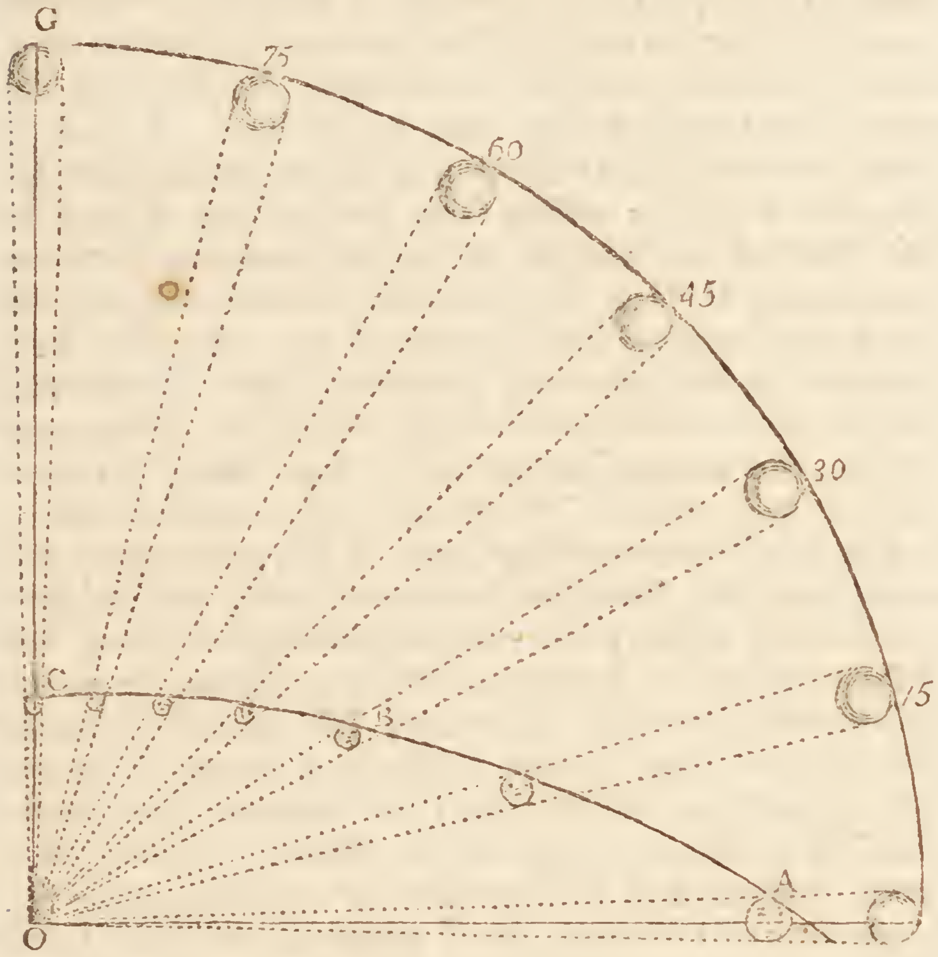
The commonly received opinion has been difputed, not only by F. Gouge, who obferves, Acad. Sci. 1700, pa. 11, that the horizontal moon appears equally large acrofs the fea, where there are no objects to produce the effect afcribed to them; but alfo by Mr. Molyneux, who fays, Philof. Tranf. abr. vol. 1, pa. 221, that if this hypothefis be true, we may at any time increafe the apparent magnitude of the moon, even in the meridian; for, in order to divide the fpace between it and the eye, we need only to look at it behind a clufter of chimneys, the ridge of a hill, or the top of a houfe, &c. He makes alfo the fame obfervation with F. Gouge, above mentioned, and farther obferves, that when the height of all the intermediate objects is cut off; by looking through a tube, the imagination is not helped, and yet the moon feems ftill as large as before. However, Mr. Molyneux advances no hypothefis of his own.

Bifhop Berkley fuppofed, that the moon appears larger near the horizon, becaufe fhe then appears fainter, and her beams affect the eye lefs. And Mr. Robins has recited fome other opinions on this fubject, Math. Tracts, vol. 2, pa. 242.

Dr. Defaguliers has illuftrated the doctrine of the horizontal moon, Philof. Tranf. abr. vol. 8, pa. 130, upon the fuppofition of our imagining the vifible heavens to be only a fmall portion of a fpherical furface, and confequently fuppofing the moon to be farther from us in the horizon than near the zenith; and by feveral ingenious contrivances he demonftrated how liable we are to fuch deceptions. The fame idea is purfued ftill farther by Dr. Smith, in his Optics, where he determines that, the centre of the apparent fpherical fegment of the fky lying much below the eye, or the horizon, the apparent diftance of its parts near the horizon was about 3 or 4 times greater than the apparent diftance of its parts over head; from which reafon it is, he infers, that the moon always appears the larger as fhe is lower, and alfo that we always think the height of a celeftial object to be more than it really is. Thus, he determined, by meafuring the actual height of fome of the heavenly bodies, when to his eye they feemed to be half way between the horizon and the zenith; that their real altitude was then only 23°: when the fun was about 30° high, the upper always appeared lefs than the under; and he thought that it was conftantly greater when the fun was 18° or 20° high. Mr. Robins, in his

his Tracts, vol. 2, pa. 245, shews how to determine the apparent concavity of the sky in a more accurate and geometrical manner; by which it appears, that if the altitude of any of the heavenly bodies be 20°, at the time when it seems to be half way between the horizon and the zenith, the horizontal distance will be hardly less than 4 times the perpendicular distance; but if that altitude be 28°, it will be little more than 2 and a half.

Dr. Smith, having determined the apparent figure of the sky, thus applies it to explain the phenomenon of the horizontal moon, and other similar appearances in the heavens. Suppose the arc ABC to re-



present that apparent concavity; then the diameter of the sun and moon would seem to be greater in the horizon than at any altitude, measured by the angle AOB, in the ratio of its apparent distances, AO, BO. The numbers that express these proportions he reduced into the annexed table, answering to the corresponding altitudes of the sun or moon, which are also exactly represented to the eye in the figure, in which the moon, placed in the quadrantal arc FG described about the centre O, are all equal to each other, and represent the body of the moon in the heights there noted, and the unequal moons in the concavity ABC

The alt. of the sun or moon in degrees.	Apparent diameters or distances.
00	100
15	68
30	50
45	40
60	34
75	31
90	30

are terminated by the visual rays coming from the circumference of the real moon, at those heights to the eye, at O. Dr. Smith also observes, that the apparent concave of the sky, being less than a hemisphere, is the cause that the breadths of the colours in the inward and outward rainbows, and the interval between the bows, appear least at the top, and greater at the bottom. This theory of the horizontal moon is also confirmed by the appearances of the tails of comets, which, whatever be their real figure, magnitude, and situation in absolute space, do always appear to be an arc of the concave sky. Dr. Smith however justly acknowledges that, at different times, the moon appears of very different magnitudes, even in the same horizon, and occasionally of an extraordinary large size; which he is not able to give a satisfactory explanation of. Smith's Optics, vol. 1, pa. 63, &c, Remarks, pa. 53.

MAIGNAN (Emanuel), a religious minim, and one of the greatest philosophers of his age, was born at Thoulouse in 1601. Like the famous Pascal, he became a complete mathematician without the assistance of a teacher; and filled the professor's chair at Rome in 1636, where, at the expence of Cardinal Spada, he published his book *De Perspectiva Horaria*, in 1648. Upon this book, Baillet, in his Life of Des Cartes, has the following passage: "M. Carcavi acquainted Des Cartes, that there was at Rome one father Maignan, a minim, of greater learning and more depth than father Mersenne, who made him expect some objections against his principles. This father's proper name was Emanuel, and his native place Thoulouse: but he lived at that time at Rome, where he taught divinity in the convent of the Trinity upon Mount Pincio, which they otherwise call the convent of the French minims." Maignan returned to Thoulouse in 1650, and was created Provincial. His knowledge in mathematics, and physical experiments, were very early known; especially from a dispute which arose between him and father Kircher, about the invention of a catoptrical work.

The king, who in 1660 amused himself with the machines and curiosities in the father's cell, made him offers by Cardinal Mazarin, to draw him to Paris; but he humbly desired to spend the remainder of his days in a cloyster.—He published a Course of Philosophy, in 4 volumes 8vo, at Thoulouse, in 1652; to the second edition of which, in folio, 1673, he added two Treatises; the one against the vortices of Des Cartes, the other upon the speaking trumpet invented by Sir Samuel Morland.—He formed a machine, which shewed, by its movements, that Des Cartes's supposition concerning the manner in which the universe was formed, or might have been formed, and concerning the centrifugal force, was entirely without foundation.

Thus this great philosopher and divine passed a life of tranquillity, in writing books, making experiments, and reading lectures. He was frequently consulted by the most eminent philosophers; and has had a thousand answers to make, either by writing or otherwise. Never was mortal less inclined to idleness. It is said that he even studied in his sleep; for his very dreams employed him in problems, which he pursued sometimes till he came to a solution or demonstration; and he has frequently been awaked out of his sleep of a sudden, by the exquisite pleasure which he felt upon discovery of it. The excellence of his manners, and his unspotted virtues, rendered him no less worthy of esteem, than his genius and learning.—It is said that he composed with great ease, and without any alterations at all.—He died at Thoulouse in 1676, at 75 years of age.

MALLEABLE, the property of a solid ductile body, from which it may be beaten, forged, and extended

tended under the hammer, without breaking, which is a property of all metals.

MANFREDI (Eustachio), a celebrated aſtronomer and mathematician, born at Bologna in 1674. His genius was always above his age. He was a tolerable poet, and wrote ingenious verſes while he was but a child. And while very young he formed in his father's houſe an academy of youth of his own age, who became the Academy of Sciences, or the Inſtitute, there. He became Profeſſor of Mathematics at Bologna in 1698, and Superintendant of the waters there in 1704. The ſame year he was placed at the head of the College of Montalte, founded at Bologna for young men intended for the church. In 1711 he obtained the office of Aſtronomer to the Inſtitute of Bologna. He became member of the Academy of Sciences of Paris in 1726, and of the Royal Society of London in 1729; and died the 15th of February 1739.—His works are:

1. *Ephemerides Motuum Cœleſtium ab anno* 1715 *ad annum* 1750; 4 volumes in 4to.—The firſt volume is an excellent introduction to aſtronomy; and the other three contain numerous calculations. His two ſiſters were greatly aſſiſting to him in compoſing this work.

2. *De Tranſitu Mercurii per Solem, anno* 1723. Bologna 1724, in 4to.

3. *De Annuis Inerrantium Stellarum Aberrationibus*, Bologna 1729, in 4to.—Beſides a number of papers in the Memoirs of the Academy of Sciences, and in other places.

MANILIUS (Marcus), a Latin aſtronomical poet, who lived in the reign of Auguſtus Cæſar. He wrote an ingenious poem concerning the ſtars and the ſphere, called *Aſtronomicon;* which, not being mentioned by any of the ancient poets, was unknown, till about two centuries ſince, when it was found buried in ſome German library, and publiſhed by Poggius. There is no account to be found of this author, but what can be drawn from his poem; which contains a ſyſtem of the ancient aſtronomy and aſtrology, together with the philoſophy of the Stoics. It conſiſts of five books; though there was a ſixth, which has not been recovered. In this work, Manilius hints at ſome opinions, which later ages have been ready to glory in as their own diſcoveries. Thus, he defends the fluidity of the heavens, againſt the hypotheſis of Ariſtotle: he aſſerts that the fixed ſtars are not at all in the ſame concave ſuperficies of the heavens, and equally diſtant from the centre of the world: he maintains that they are all of the ſame nature and ſubſtance with the ſun, and that each of them has a particular vortex of its own: and laſtly, he ſays, that the milky way is only the united luſtre of a great many ſmall imperceptible ſtars; which indeed the Moderns now ſee to be ſuch through their teleſcopes.

The beſt editions of Manilius are, that of Joſeph Scaliger, in 4to, 1600; that of Bentley, in 4to, 1738, and that of Edmund Burton, Eſq. in 8vo, 1783.

MANOMETER, or Manoscope, an inſtrument to ſhew or meaſure the alterations in the rarity or denſity of the air.

The Manometer differs from the barometer in this; That the latter only ſerves to meaſure the *weight* of the atmoſphere, or of the column of air over it; but the former, the denſity of the air in which it is found; which denſity depends not only on the weight of the atmoſphere, but alſo on the action of heat and cold, &c. Authors however often confound the two together; and Mr. Boyle himſelf has given a very good Manometer of his contrivance, under the name of a Statical Barometer, conſiſting of a bubble of thin glaſs, about the ſize of an orange, which being counterpoiſed when the air was in a mean ſtate of denſity, by means of a nice pair of ſcales, ſunk when the atmoſphere became lighter, and roſe as it grew heavier.

The Manometer uſed by captain Phipps, in his voyage towards the North Pole, conſiſted of a tube of a ſmall bore, with a ball at the end. The barometer being at 29·7, a ſmall quantity of quickſilver was put into the tube, to take off the communication between the external air, and that confined in the ball and the part of the tube below this quickſilver. A ſcale is placed on the ſide of the tube, which marks the degrees of dilatation ariſing from the increaſe of heat in this ſtate of the weight of the air, and has the ſame graduation as that of Fahrenheit's thermometer, the point of freezing being marked 32. In this ſtate therefore it will ſhew the degrees of heat in the ſame manner as a thermometer. But when the air becomes lighter, the bubble incloſed in the ball, being leſs compreſſed, will dilate itſelf, and occupy a ſpace as much larger as the compreſſing force is leſs; therefore the changes ariſing from the increaſe of heat, will be proportionably larger; and the inſtrument will ſhew the differences in the denſity of the air, ariſing from the changes in its weight and heat. Mr. Ramſden found, that a heat equal to that of boiling water, increaſed the magnitude of the air, from what it was at the freezing point, by $\frac{414}{1000}$ of the whole. Hence it follows, that the ball and the part of the tube below the beginning of the ſcale, is of a magnitude equal to almoſt 414 degrees of the ſcale. If the height of both the Manometer and thermometer be given, the height of the barometer may be thence deduced, by this rule;

as the height of the Manometer increaſed by 414,
to the height of the thermometer increaſed by 414,
ſo is 29·7, to the height of the barometer;

or if m denote the height of the Manometer, and t the height of the thermometer; then

$$m + 414 : t + 414 :: 29{\cdot}7 : \frac{t + 414}{m + 414} \times 29{\cdot}7,$$

which is the height of the barometer.

Another kind of Manometer was made uſe of by colonel Roy, in his attempts to correct the errors of the barometer; which is deſcribed in the Philoſ. Tranſ. vol. 67, pa. 689.

MANTELETS, a kind of moveable parapet, or ſcreen, of about 6 feet high, ſet upon trucks or little wheels, and guided by a long pole; ſo that in a ſiege it may be driven before the pioneers, and ſerve as blinds, or ſcreens, to ſhelter them from the enemy's ſmall ſhot. Mantelets are made of different materials, ſo as to render them muſket proof; as of ſtrong boards nailed together, and covered with tin; or of thick leather, or of layers of rope, &c, firmly bound together.

There are alſo other ſorts of Mantelets, covered on the top, uſed by the miners in approaching the walls or works of an enemy. The double Mantelets form an angle,

angle, and stand square, making two fronts. It appears from Vegetius, that Mantelets were in use among the Ancients, under the name of Vineæ.

MANTLE, or MANTLE-*tree*, is the lower part of the breast or front of a chimney. It was formerly a piece of timber that lay across the jambs, supporting the breastwork; but by a late act of parliament, chimney-breasts are not to be supported by a wooden mantle-tree, or turning piece, but by an iron bar, or by an arch of brick or stone.

MAP, a plane figure representing the surface of the earth, or some part of it; being a projection of the globular surface of the earth, exhibiting countries, seas, rivers, mountains, cities, &c, in their due positions, or nearly so.

Maps are either Universal or Particular, that is Partial.

Universal MAPS are such as exhibit the whole surface of the earth, or the two hemispheres.

Particular, or *Partial* MAPS, are those that exhibit some particular region, or part of the earth.

Both kinds are usually called Geographical, or Land-Maps, as distinguished from Hydrographical, or Sea-Maps, which represent only the seas and sea coasts, and are properly called *Charts*.

Anaximander, the scholar of Thales, it is said, about 400 years before Christ, first invented geographical tables, or Maps. The Pentingerian Tables, published by Cornelius Pentinger of Ausburgh, contain an itinerary of the whole Roman Empire; all places, except seas, woods, and desarts, being laid down according to their measured distances, but without any mention of latitude, longitude, or bearing.

The Maps published by Ptolomy of Alexandria, about the 144th year of Christ, have meridians and parallels, the better to define and determine the situation of places, and are great improvements on the construction of Maps. Though Ptolomy himself owns that his Maps were copied from some that were made by Marinus, Tirus, &c, with the addition of some improvements of his own. But from his time till about the 14th century, during which, geography and most sciences were neglected, no new Maps were published. Mercator was the first of note among the Moderns, and next to him Ortelius, who undertook to make a new set of Maps, with the modern divisions of countries and names of places; for want of which, those of Ptolomy were become almost useless. After Mercator, many others published Maps, but for the most part they were mere copies of his. Towards the middle of the 17th century, Bleau in Holland, and Sanson in France, published new sets of Maps, with many improvements from the travellers of those times, which were afterwards copied, with little variation, by the English, French, and Dutch; the best of these being those of Vischer and De Witt. And later observations have furnished us with still more accurate and copious sets of Maps, by De Lisle, Robert, Wells, &c, &c. Concerning Maps, see Varenius's Geog. lib. 3, cap. 3, prop. 4; Fournier's Hydrog. lib. 4, c. 24; Wolfius's Elem. Hydrog. c. 9; John Newton's Idea of Navigation; Mead's Construction of Globes and Maps; Wright's Constructions of Maps, &c, &c.

Construction of MAPS. Maps are constructed by making a projection of the globe, either on the plane of some particular circle, or by the eye placed in some particular point, according to the rules of Perspective, &c; of which there are several methods.

First, to construct a Map of the World, or a general Map.

1st *Method.*—A map of the world must represent two hemispheres; and they must both be drawn upon the plane of that circle which divides the two hemispheres. The first way is to project each hemisphere upon the plane of some particular circle, by the rules of Orthographic projection, forming two hemispheres, upon one common base or circle. When the plane of projection is that of a meridian, the maps will be the east and west hemispheres, the other meridians will be ellipses, and the parallel circles will be right lines. Upon the plane of the equinoctial, the meridians will be right lines crossing in the centre, which will represent the pole, and the parallels of latitude will be circles having that common centre, and the Maps will be the northern and southern hemispheres. The fault of this way of drawing Maps, is, that near the outside the circles are too near one another; and therefore equal spaces on the earth are represented by very unequal spaces upon the Map.

2d *Method.*—Another way is to project the same hemispheres by the rules of Stereographic projection; in which way, all the parallels will be represented by circles, and the meridians by circles or right lines. And here the contrary fault happens, viz, the circles towards the outsides are too far asunder, and about the middle they are too near together.

3d *Method.*—To remedy the faults of the two former methods, proceed as follows. First, for the east and west hemispheres, describe the circle PENQ for the meridian (pl. xvii, fig. 1), or plane of projection; through the centre of which draw the equinoctial EQ, and axis PN perpendicular to it, making P and N the north and south pole. Divide the quadrants PE, EN, NQ, and QP into 9 equal parts, each representing 10 degrees, beginning at the equinoctial EQ: divide also CP and CN into 9 equal parts; beginning at EQ; and through the corresponding points draw the parallels of latitude. Again, divide CE and CQ into 9 equal parts; and through the points of division, and the two poles P and N, draw circles, or rather ellipses, for the meridians. So shall the Map be prepared to receive the several places and countries of the earth.

Secondly, for the north or south hemisphere, draw AQBE, for the equinoctial (fig. 2), dividing it into the four quadrants EA, AQ, QB, and BE; and each quadrant into 9 equal parts, representing each 10 degrees of longitude; and then, from the points of division, draw lines to the centre C, for the circles of longitude. Divide any circle of longitude, as the first meridian EC, into 9 equal parts, and through these points describe circles from the centre C, for the parallels of latitude; numbering them as in the figure.

In this 3d method, equal spaces on the earth are represented by equal spaces on the Map, as near as any projection will bear; for a spherical surface can no way be represented exactly upon a plane. Then the several countries of the world, seas, islands, sea-coasts, towns, &c,

&c, are to be entered in the Map, according to their latitudes and longitudes.

In filling up the Map, all places repreſenting land are filled with ſuch things as the countries contain; but the ſeas are left white; the ſhores adjoining to the ſea being ſhaded. Rivers are marked by ſtrong lines, or by double lines, drawn winding in form of the rivers they repreſent; and ſmall rivers are expreſſed by ſmall lines. Different countries are beſt diſtinguiſhed by different colours, or at leaſt the borders of them. Foreſts are repreſented by trees; and mountains ſhaded to make them appear. Sands are denoted by ſmall points or ſpecks; and rocks under water by a ſmall croſs. In any void ſpace, draw the mariner's compaſs, with the 32 points or winds.

II. *To draw a Map of any particular Country.*

1ſt *Method.*—For this purpoſe its extent muſt be known, as to latitude and longitude; as ſuppoſe Spain, lying between the north latitudes 36 and 44, and extending from 10 to 23 degrees of longitude; ſo that its extent from north to ſouth is 8 degrees, and from eaſt to weſt 13 degrees.

Draw the line AB for a meridian paſſing through the middle of the country (fig. 3), on which ſet off 8 degrees from B to A, taken from any convenient ſcale; A being the north, and B the ſouth point. Through A and B draw the perpendiculars CD, EF, for the extreme parallels of latitude. Divide AB into 8 parts, or degrees, through which draw the other parallels of latitude, parallel to the former.

For the meridians; divide any degree in AB into 60 equal parts, or geographical miles. Then, becauſe the length of a degree in each parallel decreaſes towards the pole, from the table ſhewing this decreaſe, under the article DEGREE, take the number of miles anſwering to the latitude of B, which is $48\frac{1}{2}$ nearly, and ſet it from B, 7 times to E, and 6 times to F; ſo is EF divided into degrees. Again, from the ſame table take the number of miles of a degree in the latitude A, viz $43\frac{1}{6}$ nearly; which ſet off, from A, 7 times to C, and 6 times to D. Then from the points of diviſion in the line CD, to the correſponding points in the line EF, draw ſo many right lines, for the meridians. Number the degrees of latitude up both ſides of the Map, and the degrees of longitude on the top and bottom. Alſo, in ſome vacant place make a ſcale of miles; or of degrees, if the Map repreſent a large part of the earth; to ſerve for finding the diſtances of places upon the Map.

Then make the proper diviſions and ſubdiviſions of the country: and having the latitudes and longitudes of the principal places, it will be eaſy to ſet them down in the Map: for any town, &c, muſt be placed where the circles of its latitude and longitude interſect. For inſtance, Gibraltar, whoſe latitude is 36° 11′, and longitude 12° 27′, will be at G: and Madrid, whoſe lat. is 40° 10′, and long. 14° 44′, will be at M. In like manner the mouth of a river muſt be ſet down; but to deſcribe the whole river, the latitude and longitude of every turning muſt be marked down, and the towns and bridges by which it paſſes. And ſo for woods, foreſts, mountains, lakes, caſtles, &c. The boundaries will be deſcribed, by ſetting down the remarkable places on the ſea-coaſt, and drawing a continued line through them all. And this way is very proper for ſmall countries.

2d *Method.*—Maps of particular places are but portions of the globe, and therefore may be drawn after the ſame manner as the whole is drawn. That is, ſuch a Map may be drawn either by the orthographic or ſtereographic projection of the ſphere, as in the laſt prob. But in partial Maps, an eaſier way is as follows. Having drawn the meridian AB (fig. 3), and divided it into equal parts as in the laſt method, through all the points of diviſion draw lines perpendicular to AB, for the parallels of latitude; CD, EF being the extreme parallel. Then to divide theſe, ſet off the degrees in each parallel, diminiſhed after the manner directed for the two extreme parallels CD, EF, in the laſt method: and through all the correſponding points draw the meridians, which will be curve lines; which were right lines in the laſt method; becauſe only the extreme parallels were divided by the table. This method is proper for a large tract, as Europe, &c: in which caſe the parallels and meridians need only be drawn to every 5 or 10 degrees. This method is much uſed in drawing Maps; as all the parts are nearly of their due magnitude, but a little diſtorted towards the outſide, from the oblique interſections of the meridians and parallels.

3d *Method.*—Draw PB of a convenient length, for a meridian; divide it into 9 equal parts, and through the points of diviſion, deſcribe as many circles for the parallels of latitude, from the centre P, which repreſents the pole. Suppoſe AB (fig. 4) the height of the Map; then CD will be the parallel paſſing through the greateſt latitude, and EF will repreſent the equator. Divide the equator EF into equal parts, of the ſame ſize as thoſe in AB, both ways, beginning at B. Divide alſo all the parallels into the ſame number of equal parts, but leſſer, in proportion to the numbers for the ſeveral latitudes, as directed in the laſt method for the rectilineal parallels. Then through all the correſponding diviſions, draw curve lines, which will repreſent the meridians, the extreme ones being EC and FD. Laſtly, number the degrees of latitude and longitude, and place a ſcale of equal parts, either of miles or degrees, for meaſuring diſtances.—This is a very good way of drawing large Maps, and is called the globular projection; all the parts of the earth being repreſented nearly of their due magnitude, excepting that they are a little diſtorted on the outſides.

When the place is but ſmall that a Map is to be made of, as if a county was to be exhibited; the meridians, as to ſenſe, will be parallel to one another, and the whole will differ very little from a plane. Such a Map will be made more eaſily than by the preceding rules. It will here be ſufficient to meaſure the diſtances of places in miles, and ſo lay them down in a plane rectangular map. But this belongs more properly to Surveying.

The Uſe of MAPS is obvious from their conſtruction. The degrees of the meridians and parallels ſhew the latitudes and longitudes of places, and the ſcale of miles annexed, their diſtances; the ſituation of places, with regard to each other, as well as to the cardinal points, appears by inſpection; the top of the map being always the north, the bottom the ſouth, the right hand the eaſt,

east, and the left hand the west; unless the compass, usually annexed, shew the contrary.

MARALDI (James Philip), a learned astronomer and mathematician, was born in 1665 at Perinaldo in the county of Nice, a place already honoured by the birth of his maternal uncle the celebrated Cassini. Having made a considerable progress in mathematics, at the age of 22 his uncle, who had been a long time settled in France, invited him there, that he might himself cultivate the promising genius of his nephew. Maraldi no sooner applied himself to the contemplation of the heavens, than he conceived the design of forming a catalogue of the fixed stars, the foundation of all the astronomical edifice. In consequence of this design, he applied himself to observe them with the most constant attention; and he became by this means so intimate with them, that on being shewn any one of them, however small, he could immediately tell what constellation it belonged to, and its place in that constellation. He has been known to discover those small comets, which astronomers often take for the stars of the constellation in which they are seen, for want of knowing precisely what stars the constellation consists of, when others, on the spot, and with eyes directed equally to the same part of the heavens, could not for a long time see any thing of them.

In 1700 he was employed under Cassini in prolonging the French meridian to the northern extremity of France, and had no small share in completing it. He then set out for Italy, where Clement the 11th invited him to assist at the assemblies of the Congregation then sitting in Rome to reform the calendar. Bianchini also availed himself of his assistance to construct the great meridian of the Carthusian church in that city. In 1718 Maraldi, with three other academicians, prolonged the French meridian to the southern extremity of that country. He was admitted a member of the Academy of Sciences of Paris in 1699, in the department of Astronomy, and communicated a great multitude of papers, which are printed in their memoirs, in almost every year from 1699 to 1729, and usually several papers in each of the years; for he was indefatigable in his observations of every thing that was curious and useful in the motions and phenomena of the heavenly bodies. As to the catalogue of the fixed stars, it was not quite completed: just as he had placed a mural quadrant on the terras of the observatory, to observe some stars towards the north and the zenith, he fell sick, and died the 1st of December 1729.

MARCH, the 3d month of the year, according to the common way of computing, and consists of 31 days. The sun enters the sign Aries about the 20th or 21st day of this month.

Among the Romans, March was the first month; and in some ecclesiastical computations, that order is still preserved. In England, before the alteration of the stile, March was the 1st month in order, the year always commencing with the 25th day of the month.

It has been said it was Romulus who first divided the year into months; to the first of which he gave the name of his supposed father Mars. It is observed by Ovid, however, that the people of Italy had the month of March before the time of Romulus; but that they placed it differently; some making it the third, some the 4th, some the 5th, and others the 10th month of the year.

MARINE Barometer. See Barometer.

MARINERS-Compass. See Compass.

MARIOTTE (Edme), an eminent French philosopher and mathematician, was born at Dijon, and admitted a member of the Academy of Sciences of Paris in 1666. His works however are better known than his life. He was a good mathematician, and the first French philosopher who applied much to experimental physics. The law of the shock or collision of bodies, the theory of the pressure and motion of fluids, the nature of vision, and of the air, particularly engaged his attention. He carried into his philosophical researches, that spirit of scrutiny and investigation so necessary to those who would make any considerable progress in it. He died in 1684.

He communicated a number of curious and valuable papers to the Academy of Sciences, which were printed in the collection of their Memoirs dated 1666, viz, from volume 1 to volume 10. And all his works were collected into 2 volumes in 4to, and printed at Leyden in 1717.

MARS, one of the seven primary planets now known, and the first of the four superior ones, being placed immediately next above the earth. It is usually denoted by this character ♂, being a mark rudely formed from a man holding a spear protruded, representing the god of war of the same name.

The mean distance of Mars from the sun, is 1524 of those parts, of which the distance of the earth from the sun is 1000; his excentricity 141; and his real distance 145 millions of miles. The inclination of his orbit to the plane of the ecliptic, is $1° 52'$; the length of his year, or the period of one revolution about the sun, is $686\frac{23}{24}$ of our days, or $667\frac{3}{4}$ of his own days, which are 40 minutes longer than ours, the revolution on his axis being performed in 24 hours 40 minutes. His mean diameter is 4444 miles; and the same seen from the sun is $11''$: the inclination of the axis to his orbit $0° 0'$; the inclination of his orbit to the ecliptic $1° 52'$; place of the aphelion ♍ $0° 32'$; place of his ascending node ♉ $17° 17'$; and his parallax, according to Dr. Hook and Mr. Flamsteed, is scarce 30 seconds.

Dr. Hook, in 1665, observed several spots in Mars; which having a motion, he concluded the planet turned round its centre. In 1666, M. Cassini observed several spots in the two faces or hemispheres of Mars, which he found made one revolution in 24hours 40minutes. These observations were repeated in 1670, and confirmed by Maraldi in 1704, and 1719: whence both the motion and period, or natural day, of that planet, were determined.

In the Philos. Trans. for 1781, Mr. Herschel gave a series of observations on the rotation of this planet about its axis, from which he concluded that one mean sidereal rotation was between 24 h. 39 m. 5 sec. and 24 h. 39 m. 22 sec.; and in the Philos. Trans. for 1784, is given a paper by the same gentleman, on the remarkable appearances at the polar regions of the planet Mars, the inclination of its axis, the position of its poles, and its spheroidical figure; with a few hints relating to its real diameter and atmosphere, deduced from his

his obfervations taken from the year 1777 to 1783 inclufively. He obferved feveral remarkable bright fpots near both poles, which had fome fmall motion; and the refults of his obfervations are as follow; viz,

"Inclination of axis to the ecliptic, 59° 22′.

The node of the axis is in ♓ 17° 47′.

Obliquity of the planet's ecliptic 28° 42′.

The point Aries on Mars's ecliptic anfwers to our ♐ 19° 28′.

The figure of Mars is that of an oblate fpheroid, whofe equatorial diameter is to the polar one, as 1355 to 1272, or as 16 to 15 nearly.

The equatorial diameter of Mars, reduced to the mean diftance of the earth from the fun, is 9″ 8‴.

And the planet has a confiderable, but moderate atmofphere, fo that its inhabitants probably enjoy a fituation in many refpects fimilar to ours."

Mars always appears with a ruddy troubled light; owing, it is fuppofed, to the nature of his atmofphere, through which the light paffes.

In the acronical rifing of this planet, or when in oppofition to the fun, it is five times nearer to us than when in conjunction with him; and fo appears much larger and brighter than at other times.

Mars, having his light from the fun, and revolving round it, has an increafe and decreafe like the moon: it may alfo be obferved almoft bifected, when in the quadratures, or in perigæon; but is never feen cornicular, as the inferior planets. All which fhews both that his orbit includes that of the earth within it, and that he fhines not by his own light.

MARTIN (Benjamin), was born in 1704, and became one of the moft celebrated mathematicians and opticians of his time. He firft taught a fchool in the country; but afterwards came up to London, where he read lectures on experimental philofophy for many years, and carried on a very extenfive trade as an optician and globe-maker in Fleet-ftreet, till the growing infirmities of old age compelled him to withdraw from the active part of bufinefs. Trufting too fatally to what he thought the integrity of others, he unfortunately, though with a capital more than fufficient to pay all his debts, became a bankrupt. The unhappy old man, in a moment of defperation from this unexpected ftroke, attempted to deftroy himfelf; and the wound, though not immediately mortal, haftened his death, which happened the 9th of February 1782, at 78 years of age.

He had a valuable collection of foffils and curiofities of almoft every fpecies; which after his death were almoft given away by public auction. He was indefatigable as an artift, and as a writer he had a very happy method of explaining his fubject, and wrote with clearnefs, and even confiderable elegance. He was chiefly eminent in the fcience of optics; but he was well fkilled in the whole circle of the mathematical and philofophical fciences, and wrote ufeful books on every one of them; though he was not diftinguifhed by any remarkable inventions or difcoveries of his own. His publications were very numerous, and generally ufeful: fome of the principal of them were as follow:

The Philofophical Grammar; being a View of the prefent State of Experimental Phyfiology, or Natural Philofophy, 1735, 8vo.—A new, complete, and univerfal Syftem or Body of Decimal Arithmetic, 1735, 8vo.—The Young Student's Memorial Book, or Pocket Library, 1735, 8vo.—Defcription and Ufe of both the Globes, the Armillary Sphere and Orrery, Trigonometry, 1736, 2 vols. 8vo.—Syftem of the Newtonian Philofophy, 1759, 3 vols.—New Elements of Optics, 1759.—Mathematical Inftitutions, 1764, 2 vols.—Philologic and Philofophical Geography, 1759.—Lives of Philofophers, their inventions, &c. 1764.—Young Gentleman and Lady's Philofophy, 1764, 3 vols.—Mifcellaneous Correfpondence, 1764, 4 vols.—Inftitutions of Aftronomical Calculations, 3 parts, 1765.—Introduction to the Newtonian Philofophy, 1765.—Treatife of Logarithms.—Treatife on Navigation.—Defcription and Ufe of the Air-pump.—Defcription of the Torricellian Barometer.—Appendix to the Ufe of the Globes.—Philofophia Britannica, 3 vols.—Principles of Pump-work.—Theory of the Hydrometer.—Defcription and Ufe of a Cafe of Mathematical Inftruments.—Ditto of a Univerfal Sliding Rule.—Micrographia, on the Microfcope.—Principles of Perfpective.—Courfe of Lectures.—Optical Effays.—Effay on Electricity.—Effay on Vifual Glaffes or Spectacles.—Horologia Nova, or New Art of Dialling.—Theory of Comets.—Nature and Conftruction of Solar Eclipfes.—Venus in the Sun.—The Mariner's Mirror.—Thermometrum Magnum.—Survey of the Solar Syftem.—Effay on Ifland Chryftal.—Logarithmologia Nova, &c. &c.

MASCULINE *Signs*. Aftrologers divide the Signs, &c, into Mafculine and Feminine; by reafon of their qualities, which are either active, and hot, or cold, accounted Mafculine; or paffive, dry, and moift, which are feminine. On this principle they call the Sun, Jupiter, Saturn, and Mars, Mafculine; and the Moon and Venus, feminine. Mercury, they fuppofe, partakes of the two. Among the Signs, they account Aries, Libra, Gemini, Leo, Sagittarius, and Aquarius, Mafculine; but Cancer, Capricornus, Taurus, Virgo, Scorpio, and Pifces are feminine.

MASS, the quantity of matter in any body. This is rightly eftimated by its weight; whatever be its figure, or whether its bulk or magnitude be large or fmall.

MATERIAL, relating to Matter.

MATHEMATICAL, relating to Mathematics.

Mathematical *Sect*, is one of the two leading philofophical fects, which arofe about the beginning of the 17th century; the other being the Metaphyfical fect. The former directed its refearches by the principles of Gaffendi, and fought after truth by obfervation and experience. The difciples of this fect denied the poffibility of erecting on the bafis of metaphyfical and abftract truths, a regular and folid fyftem of philofophy, without the aid of affiduous obfervation and repeated experiments, which are the moft natural and effectual means of philofophical progrefs and improvement. The advancement and reputation of this fect, and of natural knowledge in general, were much owing to the plan of philofophizing propofed by lord Bacon, to the eftablifhment of the Royal Society in London,

London, to the genius and induſtry of Mr. Boyle, and to the unparaleled reſearches and diſcoveries of Sir Iſaac Newton. Barrow, Wallis, Locke, and many other great luminaries in learning, adorned this ſect.

MATHEMATICS, the ſcience of quantity; or a ſcience that conſiders magnitudes either as computable or meaſurable.

The word in its original, μαθησις, *matheſis*, ſignifies *diſcipline* or *ſcience* in general; and, it ſeems, has been applied to the doctrine of quantity, either by way of eminence, or becauſe, this having the ſtart of all other ſciences, the reſt took their common name from it.

As to the origin of the Mathematics, Joſephus dates it before the flood, and makes the ſons of Seth obſervers of the courſe and order of the heavenly bodies: he adds, that to perpetuate their diſcoveries, and ſecure them from the injuries either of a deluge or a conflagration, they had them engraven on two pillars, the one of ſtone, the other of brick; the former of which, he ſays, was yet ſtanding in Syria in his time.

Indeed it is pretty generally agreed that the firſt cultivators of Mathematics, after the flood, were the Aſſyrians and Chaldeans; from whom, Joſephus adds, the ſcience was carried by Abraham to the Egyptians; who proved ſuch notable proficients, that Ariſtotle even fixes the firſt riſe of Mathematics among them. From Egypt, 584 years before Chriſt, Mathematics paſſed into Greece, being carried thither by Thales; who having learned geometry of the Egyptian prieſts, taught it in his own country. After Thales, came Pythagoras; who, among other Mathematical arts, paid a particular regard to arithmetic; drawing the greateſt part of his philoſophy from numbers. He was the firſt, according to Laertius, who abſtracted geometry from matter; and to him we owe the doctrine of incommenſurable magnitude, and the five regular bodies, beſides the firſt principles of muſic and aſtronomy. To Pythagoras ſucceeded Anaxagoras, Oenopides, Briſo, Antipho, and Hippocrates of Scio; all of whom particularly applied themſelves to the quadrature of the circle, the duplicature of the cube, &c; but the laſt with moſt ſucceſs of any: he is alſo mentioned by Proclus, as the firſt who compiled elements of Mathematics.

Democritus excelled in Mathematics as well as phyſics; though none of his works in either kind are extant; the deſtruction of which is by ſome authors aſcribed to Ariſtotle. The next in order is Plato, who not only improved geometry, but introduced it into phyſics, and ſo laid the foundation of a ſolid philoſophy. From his ſchool aroſe a crowd of mathematicians. Proclus mentions 13 of note; among whom was Leodamus, who improved the analyſis firſt invented by Plato; Theætetus, who wrote Elements; and Archytas, who has the credit of being the firſt that applied Mathematics to uſe in life. Theſe were ſucceeded by Neocles and Theon, the laſt of whom contributed to the elements. Eudoxus excelled in arithmetic and geometry, and was the firſt founder of a ſyſtem of aſtronomy. Menechmus invented the conic ſections, and Theudius and Hermotimus improved the elements.

For Ariſtotle, his works are ſo ſtored with Mathematics, that Blancanus compiled a whole book of them: out of his ſchool came Eudemus and Theophraſtus; the firſt of whom wrote upon numbers, geometry, and inviſible lines; and the latter compoſed a mathematical hiſtory. To Ariſteus, Iſidorus, and Hypſicles, we owe the books of Solids; which, with the other books of Elements, were improved, collected, and methodiſed by Euclid, who died 284 years before the birth of Chriſt.

A hundred years after Euclid, came Eratoſthenes and Archimedes: and contemporary with the latter was Conon, a geometrician and aſtronomer. Soon after came Apollonius Pergæus; whoſe excellent conics are ſtill extant. To him are alſo aſcribed the 14th and 15th books of Euclid, and which, it is ſaid, were contracted by Hypſicles. Hipparchus and Menelaus wrote on the ſubtenſes of the arcs in a circle; and the latter alſo on ſpherical triangles. Theodoſius's 3 books of Spherics are ſtill extant. And all theſe, Menelaus excepted, lived before Chriſt.

Seventy years after Chriſt, was born Ptolomy of Alexandria; a good geometrician, and the prince of aſtronomers: to him ſucceeded the philoſopher Plutarch, ſome of whoſe Mathematical problems are ſtill extant. After him came Eutocius, who commented on Archimedes, and occaſionally mentions the inventions of Philo, Diocles, Nicomedes, Sporus, and Heron, on the duplicature of the cube. To Cteſebes of Alexandria we are indebted for pumps; and Geminus, who lived ſoon after, is preferred by Proclus to Euclid himſelf.

Diophantus of Alexandria was a great maſter of numbers, and the firſt Greek writer on Algebra. Among others of the Ancients, Nicomachus is celebrated for his arithmetical, geometrical, and muſical works: Serenus, for his books on the ſection of the cylinder; Proclus, for his commentaries on Euclid; and Theon has the credit among ſome, of being author of the books of elements aſcribed to Euclid. The laſt to be named among the Ancients, is Pappus of Alexandria, who flouriſhed about the year of Chriſt 400, and is juſtly celebrated for his books of Mathematical collections, ſtill extant.

Mathematics are commonly diſtinguiſhed into *Speculative* and *Practical*, *Pure* and *Mixed*.

Speculative MATHEMATICS, is that which barely contemplates the properties of things: and

Practical MATHEMATICS, that which applies the knowledge of thoſe properties to ſome uſes in life.

Pure MATHEMATICS is that branch which conſiders quantity abſtractedly, and without any relation to matter or bodies.

Mixed MATHEMATICS conſiders quantity as ſubſiſting in material being; for inſtance, length in a pole, depth in a river, height in a tower, &c.

Pure Mathematics, again, either conſiders quantity as diſcrete, and ſo computable, as arithmetic; or as concrete, and ſo meaſureable, as geometry.

Mixed Mathematics are very extenſive, and are diſtinguiſhed by various names, according to the different ſubjects it conſiders, and the different views in which it is taken; ſuch as Aſtronomy, Geography, Optics, Hydroſtatics, Navigation, &c, &c.

Pure Mathematics has one peculiar advantage, that it occaſions no conteſts among wrangling diſputants, as happens in other branches of knowledge: and the

reaſon is, becauſe the definitions of the terms are premiſed, and every perſon that reads a propoſition has the ſame idea of every part of it. Hence it is eaſy to put an end to all mathematical controverſies, by ſhewing, either that our adverſary has not ſtuck to his definitions, or has not laid down true premiſes, or elſe that he has drawn falſe concluſions from true principles; and in caſe we are not able to do either of theſe, we muſt acknowledge the truth of what he has proved.

It is true, that in mixed Mathematics, where we reaſon mathematically upon phyſical ſubjects, ſuch juſt definitions cannot be given as in geometry: we muſt therefore be content with deſcriptions; which will be of the ſame uſe as definitions, provided we be conſiſtent with ourſelves, and always mean the ſame thing by thoſe terms we have once explained.

Dr. Barrow gives a very elegant deſcription of the excellence and uſefulneſs of mathematical knowledge, in his inaugural oration, upon being appointed Profeſſor of Mathematics at Cambridge. The Mathematics, he obſerves, effectually exerciſe, not vainly delude, nor vexatiouſly torment ſtudious minds with obſcure ſubtilties, but plainly demonſtrate every thing within their reach, draw certain concluſions, inſtruct by profitable rules, and unfold pleaſant queſtions. Theſe diſciplines likewiſe enure and corroborate the mind to a conſtant diligence in ſtudy; they wholly deliver us from a credulous ſimplicity, moſt ſtrongly fortify us againſt the vanity of ſcepticiſm, effectually reſtrain us from a raſh preſumption, moſt eaſily incline us to a due aſſent, and perfectly ſubject us to the government of right reaſon. While the mind is abſtracted and elevated from ſenſible matter, diſtinctly views pure forms, conceives the beauty of ideas, and inveſtigates the harmony of proportions; the manners themſelves are ſenſibly corrected and improved, the affections compoſed and rectified, the fancy calmed and ſettled, and the underſtanding raiſed and excited to more divine contemplations.

MATTER, an extended ſubſtance. Other properties of Matter are, that it reſiſts, is ſolid, diviſible, moveable, paſſive, &c; and it forms the principles of which all bodies are compoſed.

Matter and form, the two ſimple and original principles of all things, according to the Ancients, compoſing ſome ſimple natures, which they called Elements; from the various combinations of which all natural things were afterwards compoſed.

Dr. Woodward was of opinion, that Matter is originally and really various, being at firſt creation divided into ſeveral ranks, ſets, or kinds of corpuſcles, differing in ſubſtance, gravity, hardneſs, flexibility, figure, ſize, &c; from the various compoſitions and combinations of which, he thinks, ariſe all the varieties in bodies as to colour, hardneſs, gravity, taſtes, &c. But it is Sir Iſaac Newton's opinion, that all thoſe differences reſult from the various arrangements of the ſame Matter; which he accounts homogeneous and uniform in all bodies.

The quantity of Matter in any body, is its meaſure ariſing from the joint conſideration of the magnitude and denſity of the body: as if one body be twice as denſe as another, and alſo occupy twice the ſpace, then will it contain 4 times the Matter of the other. This quantity of Matter is beſt diſcovered by the weight or gravity of the body, to which it is always proportional.

Newton obſerves, that "it ſeems probable, God, in the beginning, formed Matter in ſolid, maſſy, hard, impenetrable, moveable particles, of ſuch ſizes, figures, and with ſuch other properties, and in ſuch proportion to ſpace, as moſt conduced to the end for which he formed them; and that theſe primitive particles, being ſolid, are incomparably harder than any porous bodies compounded of them; even ſo very hard, as never to wear, and break in pieces: no ordinary power being able to divide what God himſelf made one in the firſt creation. While the particles continue entire, they may compoſe bodies of one and the ſame nature and texture in all ages; but ſhould they wear away, or break in pieces, the nature of things depending on them would be changed. Water and earth, compoſed of old worn particles, would not be of the ſame nature and texture now with water and earth compoſed of entire particles in the beginning. And therefore, that nature may be laſting, the changes of corporeal things are to be placed only in the various ſeparations and new aſſociations and motions of theſe permanent particles; compound bodies being apt to break, not in the midſt of ſolid particles, but where thoſe particles are laid together, and touch in a few points. It ſeems farther, he continues, that theſe particles have not only a vis inertiæ, accompanied with ſuch paſſive laws of motion as naturally reſult from that force, but alſo, that they are moved by certain active principles, ſuch as is that of gravity, and that which cauſeth fermentation, and the coheſion of bodies. Theſe principles are to be conſidered not as occult qualities, ſuppoſed to reſult from the ſpecific forms of things, but as general laws of nature, by which the things themſelves are formed; their truth appearing to us by phenomena, though their cauſes are not yet diſcovered."

Hobbes, Spinoza, &c, maintain that all the beings in the univerſe are material, and that their differences ariſe from their different modifications, motions, &c. Thus they conceive that Matter extremely ſubtile, and in a briſk motion, may think; and ſo they exclude ſpirit out of the world.

Dr. Berkley, on the contrary, argues againſt the exiſtence of Matter itſelf; and endeavours to prove that it is a mere *ens rationis*, and has no exiſtence out of the mind.

Some late philoſophers have advanced a new hypotheſis concerning the nature and eſſential properties of Matter. The firſt of theſe who ſuggeſted, or at leaſt publiſhed an account of this hypotheſis, was M. Boſcovich, in his Theoria Philoſophiæ Naturalis. He ſuppoſes that Matter is not impenetrable, but that it conſiſts of phyſical points only, endued with powers of attraction and repulſion, taking place at different diſtances, that is, ſurrounded with various ſpheres of attraction and repulſion; in the ſame manner as ſolid Matter is generally ſuppoſed to be. Provided therefore that any body move with a ſufficient degree of velocity, or have ſufficient momentum to overcome any power of repulſion that it may meet with, it will find no difficulty in making its way through any body whatever. If the velocity of ſuch a body in motion be ſufficiently great, Boſcovich contends, that the particles of any body through which it paſſes, will not even be moved

moved out of their place by it. With a degree of velocity fomething lefs than this, they will be confiderably agitated, and ignition might perhaps be the confequence, though the progrefs of the body in motion would not be fenfibly interrupted; and with a ftill lefs momentum it might not pafs at all.

Mr. Michell, Dr. Prieftley, and fome others of our own country, are of the fame opinion. See Prieftley's Hiftory of Difcoveries relating to Light, pa. 390.—In conformity to this hypothefis, this author maintains, that Matter is not that inert fubftance that it has been fuppofed to be; that powers of attraction or repulfion are neceffary to its very being, and that no part of it appears to be impenetrable to other parts. Accordingly, he defines Matter to be a fubftance, poffeffed of the property of extenfion, and of powers of attraction or repulfion, which are not diftinct from Matter, and foreign to it, as it has been generally imagined, but abfolutely effential to its very nature and being: fo that when bodies are divefted of thefe powers, they become nothing at all. In another place, Dr. Prieftley has given a fomewhat different account of Matter; according to which it is only a number of centres of attraction and repulfion; or more properly of centres, not divifible, to which divine agency is directed; and as fenfation and thought are not incompatible with thefe powers, folidity, or impenetrability, and confequently a vis inertiæ only having been thought repugnant to them, he maintains, that we have no reafon to fuppofe that there are in man two fubftances abfolutely diftinct from each other. See Difquifitions on Matter and Spirit.

But Dr. Price, in a correfpondence with Dr. Prieftley, publifhed under the title of A Free Difcuffion of the Doctrines of Materialifm and Philofophical Neceffity, 1778, has fuggefted a variety of unanfwerable objections againft this hypothefis of the penetrability of Matter, and againft the conclufions that are drawn from it. The vis inertiæ of Matter, he fays, is the foundation of all that is demonftrated by natural philofophers concerning the laws of the collifion of bodies. This, in particular, is the foundation of Newton's philofophy, and efpecially of his three laws of motion. Solid Matter has the power of acting on other Matter by impulfe; but unfolid Matter cannot act at all by impulfe; and this is the only way in which it is capable of acting, by any action that is properly its own. If it be faid, that one particle of Matter can act upon another without contact and impulfe, or that Matter can, by its own proper agency, attract or repel other Matter which is at a diftance from it, then a maxim hitherto univerfally received muft be falfe, that "nothing can act where it is not." Newton, in his letters to Bentley, calls the notion, that Matter poffeffes an innate power of attraction, or that it can act upon Matter at a diftance, and attract and repel by its own agency, an abfurdity into which he thought no one could poffibly fall. And in another place he expreffly difclaims the notion of innate gravity, and has taken pains to fhew that he did not take it to be an effential property of bodies. By the fame kind of reafoning purfued, it muft appear, that Matter has not the power of attracting and repelling; that this power is the power of fome foreign caufe, acting upon Matter according to ftated laws; and confequently that attraction and repulfion, not being actions, much lefs inherent qualities of Matter, as fuch, it ought not to be defined by them. And if Matter has no other property, as Dr. Prieftley afferts, than the power of attracting and repelling, it muft be a non-entity; becaufe this is a property that cannot belong to it. Befides, all power is the power of fomething; and yet if Matter is nothing but this power, it muft be the power of nothing; and the very idea of it is a contradiction. If Matter be not folid extenfion, what can it be more than mere extenfion?

Farther, Matter that is not folid, is the fame with pore; and therefore it cannot poffefs what philofophers mean by the momentum or force of bodies, which is always in proportion to the quantity of Matter in bodies, void of pore.

MAUNDY THURSDAY, is the Thurfday in Paffion week; which was called *Maundy* or *Mandate Thurfday*, from the command which Chrift gave his apoftles to commemorate him in the Lord's Supper, which he this day inftituted; or from the new commandment which he gave them to love one another, after he had wafhed their feet as a token of his love to them.

MAUPERTUIS (PETER LOUIS MORCEAU DE), a celebrated French mathematician and philofopher, was born at St Malo in 1698, and was there privately educated till he attained his 16th year, when he was placed under the celebrated profeffor of philofophy, M. le Blond, in the college of la Marche, at Paris; while M. Guifnée, of the Academy of Sciences, was his inftructor in mathematics. For this fcience he foon difcovered a ftrong inclination, and particularly for geometry. He likewife practifed inftrumental mufic in his early years with great fuccefs; but fixed on no profeffion till he was 20, when he entered into the army; in which he remained about 5 years, during which time he purfued his mathematical ftudies with great vigour; and it was foon remarked by M. Freret and other academicians, that nothing but mathematics could fatisfy his active foul and unbounded thirft for knowledge.

In the year 1723, he was received into the Royal Academy of Sciences, and read his firft performance, which was a memoir upon the conftruction and form of mufical inftruments. During the firft years of his admiffion, he did not wholly confine his attention to mathematics; he dipt into natural philofophy, and difcovered great knowledge and dexterity in obfervations and experiments upon animals.

If the cuftom of travelling into remote countries, like the fages of antiquity, in order to be initiated into the learned myfteries of thofe times, had ftill fubfifted, no one would have conformed to it with more eagernefs than Maupertuis. His firft gratification of this paffion was to vifit the country which had given birth to Newton; and during his refidence at London he became as zealous an admirer and follower of that philofopher as any one of his own countrymen. His next excurfion was to Bafil in Switzerland, where he formed a friendfhip with the celebrated John Bernoulli and his family, which continued till his death. At his return to Paris, he applied himfelf to his favourite ftudies with greater zeal than ever. And how well he ful-

filled the duties of an academician, may be feen by running over the Memoirs of the Academy from the year 1724 to 1744; where it appears that he was neither idle, nor occupied by objects of fmall importance. The moft fublime queftions in the mathematical fciences, received from his hand that elegance, clearnefs, and precifion, fo remarkable in all his writings.

In the year 1736, he was fent to the polar circle, to meafure a degree of the meridian, in order to afcertain the figure of the earth; in which expedition he was accompanied by Meff. Clairault, Camus, Monnier, Outhier, and Celfus the celebrated profeffor of aftronomy at Upfal. This bufinefs rendered him fo famous, that on his return he was admitted a member of almoft every academy in Europe.

In the year 1740, Maupertuis had an invitation from the king of Pruffia to go to Berlin; which was too flattering to be refufed. His rank among men of letters had not wholly effaced his love for his firft profeffion, that of arms. He followed the king to the field, but at the battle of Molwitz was deprived of the pleafure of being prefent, when victory declared in favour of his royal patron, by a fingular kind of adventure. His horfe, during the heat of the action, running away with him, he fell into the hands of the enemy; and was at firft but roughly treated by the Auftrian Huffars, to whom he could not make himfelf known for want of language; but being carried prifoner to Vienna, he received fuch honours from the emperor as never were effaced from his memory. Maupertuis lamented very much the lofs of a watch of Mr. Graham's, the celebrated Englifh artift, which they had taken from him; the emperor, who happened to have another by the fame artift, but enriched with diamonds, prefented it to him, faying, "the Huffars meant only to jeft with you, they have fent me your watch, and I return it to you."

He went foon after to Berlin; but as the reform of the academy which the king of Pruffia then meditated was not yet mature, he repaired to Paris, where his affairs called him, and was chofen in 1742 director of the Academy of Sciences. In 1743 he was received into the French Academy; which was the firft inftance of the fame perfon being a member of both the academies at Paris at the fame time. Maupertuis again affumed the foldier at the fiege of Fribourg, and was pitched upon by marfhal Coigny and the count d'Argenfon to carry the news to the French king of the furrender of that citadel.

Maupertuis returned to Berlin in the year 1744, when a marriage was negotiated and brought about, by the good offices of the queen mother, between our author and madamoifelle de Borck, a lady of great beauty and merit, and nearly related to M. de Borck at that time minifter of ftate. This determined him to fettle at Berlin, as he was extremely attached to his new fpoufe, and regarded this alliance as the moft fortunate circumftance of his life.

In the year 1746, Maupertuis was declared, by the king of Pruffia, Prefident of the Royal Academy of Sciences at Berlin, and foon after by the fame prince was honoured with the Order of Merit. However, all thefe accumulated honours and advantages, fo far from leffening his ardour for the fciences, feemed to furnifh new allurements to labour and application. Not a day paffed but he produced fome new project or effay for the advancement of knowledge. Nor did he confine himfelf to mathematical ftudies only: metaphyfics, chemiftry, botany, polite literature, all fhared his attention, and contributed to his fame. At the fame time he had, it feems, a ftrange inquietude of fpirit, with a dark atrabilaire humour, which rendered him miferable amidft honours and pleafures. Such a temperament did not promife a pacific life; and he was in fact engaged in feveral quarrels. One of thefe was with Koenig the profeffor of philofophy at Franeker, and another more terrible with Voltaire. Maupertuis had inferted in the volume of Memoirs of the Academy of Berlin for 1746, a difcourfe upon the laws of motion; which Koenig was not content with attacking, but attributed to Leibnitz. Maupertuis, ftung with the imputation of plagiarifm, engaged the academy of Berlin to call upon him for his proof; which Koenig failing to produce, his name was ftruck out of the academy, of which he was a member. Several pamphlets were the confequence of this meafure; and Voltaire, for fome reafon or other, engaged in the quarrel againft Maupertuis. We fay, for fome reafon or other; becaufe Maupertuis and Voltaire were apparently upon the moft amicable terms; and the latter refpected the former as his mafter in the mathematics. Voltaire upon this occafion exerted all his wit and fatire againft him; and upon the whole was fo much tranfported beyond what was thought right, that he found it expedient in 1753 to quit the court of Pruffia.

Our philofopher's conftitution had long been confiderably impaired by the great fatigues of various kinds in which his active mind had involved him; though from the amazing hardfhips he had undergone, in his northern expedition, moft of his bodily fufferings may be traced. The intenfe fharpnefs of the air could only be fupported by means of ftrong liquors; which helped but to lacerate his lungs, and bring on a fpitting of blood, which began at leaft 12 years before he died. Yet ftill his mind feemed to enjoy the greateft vigour; for the beft of his writings were produced, and moft fublime ideas developed, during the time of his confinement by ficknefs, when he was unable to occupy his prefidial chair at the academy. He took feveral journeys to St. Malo, during the laft years of his life, for the recovery of his health: and though he always received benefit by breathing his native air, yet ftill, upon his return to Berlin, his diforder likewife returned with greater violence. His laft journey into France was undertaken in the year 1757; when he was obliged, foon after his arrival there, to quit his favourite retreat at St. Malo, on account of the danger and confufion which that town was thrown into by the arrival of the Englifh in its neighbourhood. From thence he went to Bourdeaux, hoping there to meet with a neutral fhip to carry him to Hamburgh, in his way back to Berlin; but being difappointed in that hope, he went to Touloufe, where he remained feven months. He had then thoughts of going to Italy, in hopes a milder climate would reftore him to health; but finding himfelf grow worfe, he rather inclined towards Germany, and went no Neufchatel, where for three months

months he enjoyed the converſation of lord Mariſchal, with whom he had formerly been much connected. At length he arrived at Baſil, October 16, 1758, where he was received by his friend Bernoulli and his family with the utmoſt tenderneſs and affection. He at firſt found himſelf much better here than he had been at Neufchatel: but this amendment was of ſhort duration; for as the winter approached, his diſorder returned, accompanied by new and more alarming ſymptoms. He languiſhed here many months, during which he was attended by M. de la Condamine; and died in 1759, at 61 years of age.

The works which he publiſhed were collected into 4 volumes 8vo, publiſhed at Lyons in 1756, where alſo a new and elegant edition was printed in 1768. Theſe contain the following works:

1. Eſſay on Coſmology.—2. Diſcourſe on the different Figures of the Stars.—3. Eſſay on Moral Philoſophy.—4. Philoſophical Reflections upon the Origin of Languages, and the Signification of Words—5. Animal Phyſics, concerning Generation &c.—6. Syſtem of Nature, or the Formation of bodies—7. Letters on various ſubjects.—8. On the Progreſs of the Sciences.—9. Elements of Geography.—10. Account of the Expedition to the Polar Circle, for determining the Figure of the Earth; or the Meaſure of the Earth at the Polar Circle.—11. Account of a Journey into the Heart of Lapland, to ſearch for an Ancient Monument.—12. On the Comet of 1742.—13. Various Academical Diſcourſes, pronounced in the French and Pruſſian Academies.—14. Diſſertation upon Languages.—15. Agreement of the Different Laws of Nature, which have hitherto appeared incompatible.—16. Upon the Laws of Motion.—17. Upon the Laws of Reſt.—18. Nautical Aſtronomy.—19. On the Parallax of the Moon.—20. Operations for determining the Figure of the Earth, and the Variations of Gravity.—21. Meaſure of a Degree of the Meridian at the Polar Circle.

Beſide theſe works, Maupertuis was author of a great multitude of intereſting papers, particularly thoſe printed in the Memoirs of the Paris and Berlin Academies, far too numerous here to mention; viz, in the Memoirs of the Academy at Paris, from the year 1724, to 1749; and in thoſe of the Academy of Berlin, from the year 1746, to 1756.

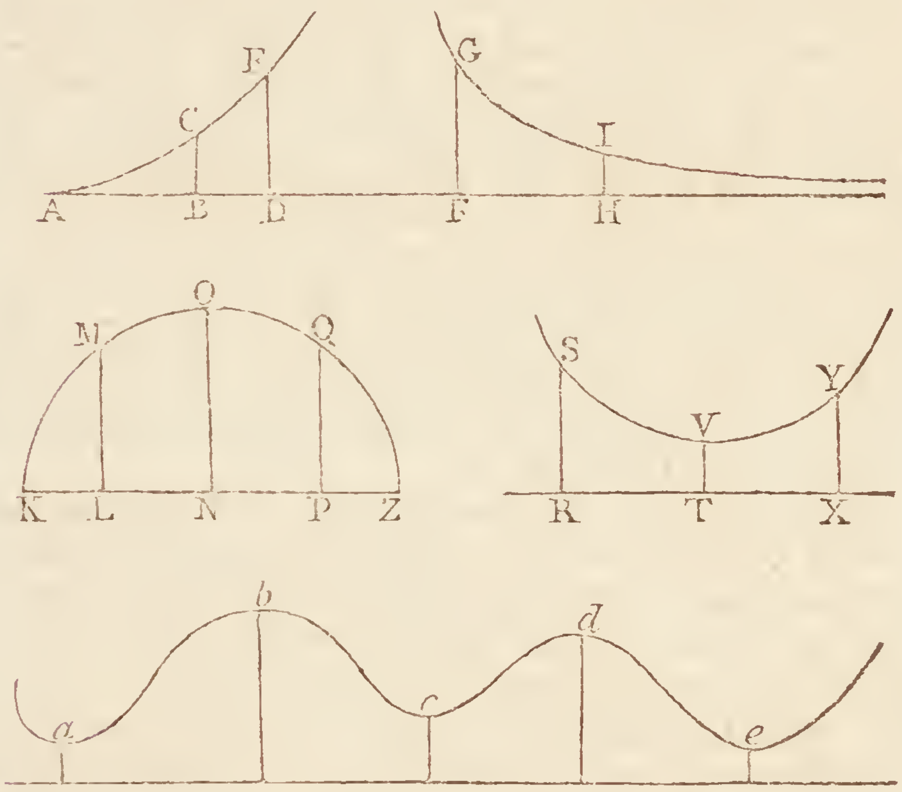
MAXIMUM, denotes the greateſt ſtate or quantity attainable in any given caſe, or the greateſt value of a variable quantity. By which it ſtands oppoſed to Minimum, which is the leaſt poſſible quantity in any caſe.

As in the algebraical expreſſion $a^2 - bx$, where a and b are conſtant or invariable quantities, and x a variable one. Now it is evident that the value of this remainder or difference, $a^2 - bx$, will increaſe as the term bx, or x, decreaſes; and therefore that will be the greateſt when this is the ſmalleſt; that is, $a^2 - bx$ is a maximum, when x is the leaſt, or nothing at all.

Again, the expreſſion or difference $a^2 - \frac{b}{x}$, evidently increaſes as the fraction $\frac{b}{x}$ diminiſhes; and this diminiſhes as x increaſes; therefore the given expreſſion will be the greateſt, or a maximum, when x is the greateſt, or infinite.

Alſo, if along the diameter KZ *(the 3d fig. below)* of a circle, a perpendicular ordinate LM be conceived to move, from K towards Z; it is evident that, from K it increaſes continually till it arrive at the centre, in the poſition NO, where it is at the greateſt ſtate; and from thence it continually decreaſes again, as it moves along from N to Z, and quite vaniſhes at the point Z. So that the maximum ſtate of the ordinate is NO, equal to the radius of the circle.

Methodus de MAXIMIS *et* MINIMIS, a method of finding the greateſt or leaſt ſtate or value of a variable quantity.



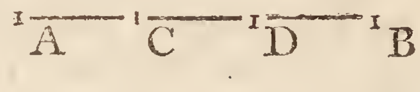
Some quantities continually increaſe, and ſo have no maximum but what is infinite; as the ordinates BC, DE of the parabola ACE: Some continually decreaſe, and ſo their leaſt or minimum ſtate is nothing; as the ordinates FG, HI, to the aſymptotes of the hyperbola. Others increaſe to a certain magnitude, which is their maximum, and then decreaſe again; as the ordinates LM &c of the circle. And others again decreaſe to a certain magnitude TV, which is their minimum, and then increaſe again; as the ordinates of the curve SVY. While others admit of ſeveral maxima and minima; as the ordinates of the curve *abcde*, where at *b* and *d* they are maxima, and *a*, *c*, *e*, minima. And thus the maxima and minima of all other variable quantities may be conceived; expreſſing thoſe quantities by the ordinates of ſome curves.

The firſt maxima and minima are found in the Elements of Euclid, or flow immediately from them: thus, it appears, by the 5th prop. of book 2, that the greateſt rectangle that can be made of the two parts of a given line, any how divided, is when the line is divided equally in the middle; prob. 7, book 3, ſhews that the greateſt line that can be drawn from a given point within a circle, is that which paſſes through the centre; and that the leaſt line that can be ſo drawn, is the continuation of the ſame to the other ſide of the circle: prop. 8 ib. ſhews the ſame for lines drawn from a point without the circle: and thus other inſtances might be pointed out in the Elements.—Other writers on the Maxima and Minima, are, Apollonius, in the whole 5th book of his Conic Sections; and

and in the Preface or Dedication to that book, he ſays others had then alſo treated the ſubject, though in a ſlighter manner.—Archimedes; as in prop. 9 of his Treatiſe on the Sphere and Cylinder, where he demonſtrates that, of all ſpherical ſegments under equal ſuperficies, the hemiſphere is the greateſt.—Serenus, in his 2d book, or that on the Conic Sections.—Pappus, in many parts of his Mathematical Collections; as in lib. 3, prop. 28 &c, lib. 6, prop. 31 &c, where he treats of ſome curious caſes of variable geometrical quantities, ſhewing how ſome increaſe and decreaſe both ways to infinity; while others proceed only one way, by increaſe or decreaſe, to infinity, and the other way to a certain magnitude; and others again both ways to a certain magnitude, giving a maximum and minimum; alſo lib. 7, prop. 13, 14, 165, 166, &c. And all theſe are the geometrical Maxima and Minima of the Ancients; to which may be added ſome others of the ſame kind, viz. Viviani De Maximis & Minimis Geometrica Divinatio in quintum Conicorum Apollonii Pergæi, in fol. at Flor. 1659; alſo an ingenious little tract in Thomas Simpſon's Geometry, on the Maxima and Minima of Geometrical Quantities.

Other writings on the Maxima and Minima are chiefly treated in a more general way by the modern analyſis; and firſt among theſe perhaps may be placed that of Fermat. This, and other methods, are beſt referred to, and explained by the ordinates of curves. For when the ordinate of a curve increaſes to a certain magnitude, where it is greateſt, and afterwards decreaſes again, it is evident that two ordinates on the contrary ſides of the greateſt ordinate may be equal to each other; and the ordinates decreaſe to a certain point, where they are at the leaſt, and afterwards increaſe again; there may alſo be two equal ordinates, one on each ſide of the leaſt ordinate. Hence then an equal ordinate correſponds to two different abſciſſes, or for every value of an ordinate there are two values of abſciſſes. Now as the difference between the two abſciſſes is conceived to become leſs and leſs, it is evident that the two equal ordinates, correſponding to them, approach nearer and nearer together; and when the differences of the abſciſſes are infinitely little, or nothing, then the equal ordinates unite in one, which is either the maximum or minimum. The method hence derived then, is this: Find two values of an ordinate, expreſſed in terms of the abſciſſes: put thoſe two values equal to each other, cancelling the parts that are common to both, and dividing all the remaining terms by the difference between the abſciſſes, which will be a common factor in them: next, ſuppoſing the abſciſſes to become equal, that the equal ordinates may concur in the maximum or minimum, that difference will vaniſh, as well as all the terms of the equation that include it; and therefore, ſtriking thoſe terms out of the equation, the remaining terms will give the value of the abſciſs correſponding to the maximum or minimum.

For example, ſuppoſe it were required to find the greateſt ordinate in a circle KMQ. Put the diameter $KZ = a$, the abſciſs $KL = x$, the ordinate $LM = y$; hence the other part of the diameter is $LZ = a - x$, and conſequently, by the nature of the circle $KL \times LZ$ being equal LM^2, $x \times \overline{a - x}$ or $ax - x^2 = y^2$. Again, put another abſciſs $KP = x + d$, where d is the difference LP, the ordinate PQ, being equal to LM or y; here then again $KP \times PZ = PQ^2$, or $\overline{x + d} \times \overline{a - x - d} = ax - x^2 - 2dx + ad - d^2 = y^2$: put now theſe two values of y^2 equal to each other, ſo ſhall $ax - x^2 = ax - x^2 - 2dx + ad - d^2$; cancel the common terms ax and x^2, then $0 = -2dx + ad - d^2$, or $2dx + d^2 = ad$; divide all by d, ſo ſhall $2x + d = a$, a general equation derived from the equality of the two ordinates. Now, bringing the two equal ordinates together, or making the two abſciſſes equal, their difference d vaniſhes, and the laſt equation becomes barely $2x = a$, or $x = \frac{1}{2}a$, $=$ KN, the value of the abſciſs KN when the ordinate NO is a maximum, viz, the greateſt ordinate biſects the diameter. And the operation and concluſion it is evident will be the ſame, to divide a given line into two parts, ſo that their rectangle ſhall be the greateſt poſſible.



For a ſecond example, let it be required to divide the given line AB into two ſuch parts, that the one line drawn into the ſquare of the other may be the greateſt poſſible. Putting the given line $AB = a$, and one part $AC = x$; then the other part CB will be $a - x$, and therefore $x^2 \times \overline{a - x} = ax^2 - x^3$ is the product of one part by the ſquare of the other. Again, let one part be $AD = x + d$, then the other part is $a - x - d$, and $\overline{x + d}|^2 \times \overline{a - x - d} = ax^2 - x^3 - 3dx^2 + \overline{2ad - 3d^2} \cdot x + ad^2 - d^3$. Then, putting theſe two products equal to each other, cancelling the common terms $ax^2 - x^3$, and dividing the remainder by d, there reſults

$0 = -3x^2 + \overline{2a - 3d} \cdot x + ad - d^2$; hence, cancelling all the terms that contain d, there remains $0 = -3x^2 + 2ax$, or $3x = 2a$, and, $x = \frac{2}{3}a$; that is, the given line muſt be divided into two parts in the ratio of 3 to 2. See Fermat's Opera Varia, pa. 63, and his Letters to F. Merſenne.

The next method was that of John Hudde, given by Schooten among the additions to Des Cartes's Geometry, near the end of the 1ſt vol. of his edition. This method is alſo drawn from the property of an equation that has two equal roots. He there demonſtrates that, having ranged the terms of an equation, that has two roots equal, according to the order of the exponents of the unknown quantity, taking all the terms over to one ſide, and ſo making them equal to nothing on the other ſide; if then the terms in that order be multiplied by the terms of any arithmetical progreſſion, the reſulting equation will ſtill have one of its roots equal to one of the two equal roots of the former equation. Now ſince, by what has been ſaid of the foregoing method, when the ordinate of a curve, admitting of a maximum or minimum, is expreſſed in terms of the abſciſſa, that abſciſſa, or the value of x, will be two-fold, becauſe there are two ordinates of the ſame value; that is, the equation has at leaſt two unequal roots or values of x: but when the ordinate becomes a maximum or minimum, the two abſciſſes unite in one, and the two roots, or values of x, are equal; therefore, from the above ſaid property, the terms of this equation for the maximum or minimum being multiplied by the terms of any arithmetical progreſſion, the root of the reſulting equation

tion will be one of the ſaid equal roots, or the value of the abſciſs x when the ordinate is a maximum.

Although the terms of any arithmetic progreſſion may be uſed for this purpoſe, ſome are more convenient than others; and Mr. Hudde directs to make uſe of that progreſſion which is formed by the exponents of x, viz, to multiply each term by the exponent of its power, and putting all the reſulting products equal to nothing; which, it is evident, is exactly the ſame proceſs as taking the fluxions of all the terms, and putting them equal to nothing; being the common proceſs now uſed for the ſame purpoſe.

Thus, in the former of the two foregoing examples, where $ax - x^2$, or y^2, is to be a maximum;
mult. by 1 2
gives $ax - 2x^2 = 0$; hence $2x = a$, and $x = \frac{1}{2}a$, as before.

And in the 2d example, where $ax^2 - x^3$, is to be a maximum; mult. by - - 2 3
gives - - - - - $2ax^2 - 3x^3 = 0$;
hence $2a - 3x = 0$, or $3x = 2a$, and $x = \frac{2}{3}a$, as before.

The next general method, and which is now uſually practiſed, is that of Newton, or the method of Fluxions, which proceeds upon a principle different from that of the two former methods of Fermat and Hudde. Theſe proceed upon the idea of the two equal ordinates of a curve uniting into one, at the place of the maximum and minimum; but Newton's upon the principle, that the fluxion or increment of an ordinate is nothing, at the point of the maximum or minimum; a circumſtance which immediately follows from the nature of that doctrine: for, ſince a quantity ceaſes to increaſe at the maximum, and to decreaſe at the minimum, at thoſe points it neither increaſes nor decreaſes; and ſince the fluxion of a quantity is proportional to its increaſe or decreaſe, therefore the fluxion is nothing at the maximum or minimum. Hence this rule: Take the fluxion of the algebraical expreſſion denoting the maximum or minimum, and put it equal to nothing; and that equation will determine the value of the unknown letter or quantity in queſtion.

So in the firſt of the two foregoing examples, where it is required to determine x when $ax - x^2$ is a maximum: the fluxion of this is $a\dot{x} - 2x\dot{x} = 0$; divide by $\dot{x}$, ſo ſhall $a - 2x = 0$, or $a = 2x$, and $x = \frac{1}{2}a$.

Alſo, in the 2d example, where $ax^2 - x^3$ muſt be a maximum: here the fluxion is $2ax\dot{x} - 3x^2\dot{x} = 0$; hence $2a - 3x = 0$, or $2a = 3x$, and $x = \frac{2}{3}a$.

When a quantity becomes a maximum or minimum, and is expreſſed by two or more affirmative and negative terms, in which only one variable letter is contained; it is evident that the fluxion of the affirmative terms will be equal to the fluxion of the negative ones; ſince their difference is equal to nothing.

And when, in the expreſſion for the fluxion of a maximum or minimum, there are two or more fluxionary letters, each contained in both affirmative and negative terms; the ſum of the terms containing the fluxion of each letter, will be equal to nothing: For, in order that any expreſſion be a maximum or minimum, which contains two or more variable quantities, it muſt produce a maximum or minimum, if but one of thoſe quantities be ſuppoſed variable. So if $ax - 2xy + by$ denote a minimum; its fluxion is $a\dot{x} - 2y\dot{x} - 2x\dot{y} + b\dot{y}$; hence $a\dot{x} - 2y\dot{x} = 0$, and $b\dot{y} - 2x\dot{y} = 0$; from the former of theſe $y = \frac{1}{2}a$, and from the latter $x = \frac{1}{2}b$. Or, in ſuch a caſe, take the fluxion of the whole expreſſion, ſuppoſing only one quantity variable; then take the fluxion again, ſuppoſing another quantity only variable: and ſo on, for all the ſeveral variable quantities; which will give the ſame number of equations for determining thoſe quantities. So, in the above example, $ax - 2xy + by$, the fluxion is $a\dot{x} - 2y\dot{x} = 0$, ſuppoſing only x variable; which gives $y = \frac{1}{2}a$: and the fluxion is $- 2x\dot{y} + b\dot{y} = 0$, when y only is variable; which gives $x = \frac{1}{2}b$; the ſame as before.

Farther, when any quantity is a maximum or minimum, all the powers or roots of it will be ſo too; as will alſo the reſult be, when it is increaſed or decreaſed, or multiplied, or divided by a given or conſtant quantity; and the logarithm of the ſame will be alſo a maximum or minimum.

To find whether a propoſed algebraic quantity admits of a maximum or minimum.—Every algebraic expreſſion does not admit of a maximum or minimum, properly ſo called; for it may either increaſe continually to infinity, or decreaſe continually to nothing; in both which caſes there is neither a proper maximum nor minimum; for the true maximum is that value to which an expreſſion increaſes, and after which it decreaſes again; and the minimum is that value to which the expreſſion decreaſes, and after that it increaſes again. Therefore when the expreſſion admits of a maximum, its fluxion is poſitive before that point, and negative after it; but when it admits of a minimum, its fluxion is negative before, and poſitive after it. Hence, take the fluxion of the expreſſion a little before the fluxion is equal to nothing, and a little after it; if the firſt fluxion be poſitive, and the laſt negative, the middle ſtate is a maximum; but if the firſt fluxion be negative, and the laſt poſitive, the middle ſtate is a minimum. See Maclaurin's Fluxions, book 1, chap. 9, and book 2, chap. 5, art. 859.

MAY, *Maius*, the fifth month in the year, reckoning from our firſt or January; but the third, counting the year to begin with March, as the Romans did anciently. It was called Maius by Romulus, in reſpect to the ſenators and nobles of his city, who were named *majores*; as the following month was called *Junius*, in honour of the youth of Rome, *in honorem juniorum*, who ſerved him in the war. Though ſome ſay it has been thus called from *Maia*, the mother of Mercury, to whom they offered ſacrifice on the firſt day of this month: and Papias derives the name from *Madius, eo quod tunc terra madeat*.

In this month the ſun enters the ſign Gemini, and the plants of our hemiſphere begin moſtly to flower.

MAYER (Tobias), one of the greateſt aſtronomers and mechaniſts of the 18th century, was born at Maſpach, in the duchy of Wirtemberg, 1723. He taught himſelf mathematics, and at 14 years of age deſigned machines and inſtruments with the greateſt dexterity and juſtneſs. Theſe purſuits did not hinder him from cultivating the Belles Lettres. He acquired the Latin tongue, and wrote it with elegance. In 1750, the univerſity of Gottingen choſe him for their mathematical profeſſor; and every year of his ſhort life was

was thenceforward marked with ſome conſiderable diſcoveries in geometry and aſtronomy. He publiſhed ſeveral works in this way, which are all accounted excellent of their kind; and ſome papers are inſerted in the ſecond volume of the Memoirs of the Univerſity of Gottingen. He was very accurate and indefatigable in his aſtronomical obſervations; indeed his labours ſeem to have very early exhauſted him; for he died worn out in 1762, at no more than 39 years of age.

His Table of Refractions, deduced from his aſtronomical obſervations, very nicely agrees with that of Doctor Bradley; and his Theory of the Moon, and Aſtronomical Tables and Precepts, were ſo well eſteemed, that they were rewarded by the Engliſh Board of Longitude, with the premium of three thouſand pounds, which ſum was paid to his widow after his death. Theſe tables and precepts were publiſhed by the Board of Longitude in 1770.

MEAN, a middle ſtate between two extremes: as a mean motion, mean diſtance, arithmetical mean, geometrical mean, &c.

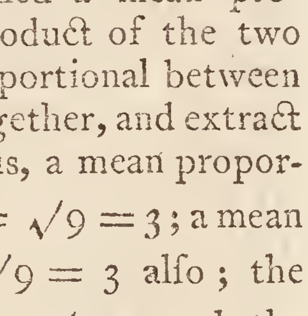
Arithmetical MEAN, is half the ſum of the extremes. So, 4 is an arithmetical mean between 2 and 6, or between 3 and 5, or between 1 and 7; alſo an arithmetical mean between a and b is $\frac{a+b}{2}$ or $\frac{1}{2}a + \frac{1}{2}b$.

Geometrical MEAN, commonly called a mean proportional, is the ſquare root of the product of the two extremes; ſo that, to find a mean proportional between two given extremes, multiply theſe together, and extract the ſquare root of the product. Thus, a mean proportional between 1 and 9, is $\sqrt{1 \times 9} = \sqrt{9} = 3$; a mean between 2 and $4\frac{1}{2}$ is $\sqrt{2 \times 4\frac{1}{2}} = \sqrt{9} = 3$ alſo; the mean between 4 and 6 is $\sqrt{4 \times 6} = \sqrt{24}$; and the mean between a and b is $\sqrt{ab}$.

The geometrical mean is always leſs than the arithmetical mean, between the ſame two extremes. So the arithmetical mean between 2 and $4\frac{1}{2}$ is $3\frac{1}{4}$, but the geometrical mean is only 3. To prove this generally; let a and b be any two terms, a the greater, and b the leſs; then, univerſally, the arithmetical mean $\frac{a+b}{2}$ ſhall be greater than the geometrical mean $\sqrt{ab}$, or $a+b$ greater than $2\sqrt{ab}$. For, by

ſquaring both, they are $a^2 + 2ab + b^2 > 4ab$;
ſubtr. $4ab$ from each, then $a^2 - 2ab + b^2 > 0$,
that is - - - $(a - b)^2 > 0$.

To find a Mean Proportional Geometrically, between two given lines M and N. Join the two given lines together at C in one continued line AB; upon the diameter AB deſcribe a ſemicircle, and erect the perpendicular CD; which will be the mean proportional between AC and CB, or M and N.



To find two Mean Proportionals between two given extremes. Multiply each extreme by the ſquare of the other, viz, the greater extreme by the ſquare of the leſs, and the leſs extreme by the ſquare of the greater; then extract the cube root out of each product, and the two roots will be the two mean proportionals ſought. That is, $\sqrt[3]{a^2b}$ and $\sqrt[3]{ab^2}$ are the two means between a and b. So, between 2 and 16, the two mean proportionals are 4 and 8; for $\sqrt[3]{2^2 \times 16} = \sqrt[3]{64} = 4$, and $\sqrt[3]{2 \times 16^2} = \sqrt[3]{512} = 8$.

In a ſimilar manner we proceed for three means, or four means, or five means, &c. From all which it appears that the ſeries of the ſeveral numbers of mean proportionals between a and b will be as follows: viz,

one mean, $\sqrt{ab}$;
two means, $\sqrt[3]{a^2b}$, $\sqrt[3]{ab^2}$;
three means, $\sqrt[4]{a^3b}$, $\sqrt[4]{a^2b^2}$, $\sqrt[4]{ab^3}$;
four means, $\sqrt[5]{a^4b}$, $\sqrt[5]{a^3b^2}$, $\sqrt[5]{a^2b^3}$; $\sqrt[5]{ab^4}$;
five means, $\sqrt[6]{a^5b}$, $\sqrt[6]{a^4b^2}$, $\sqrt[6]{a^3b^3}$, $\sqrt[6]{a^2b^4}$, $\sqrt[6]{ab^5}$;
&c, &c.

Harmonical MEAN, is double a fourth proportional to the ſum of the extremes, and the two extremes themſelves a and b: thus, as $a+b : a :: 2b : \frac{2ab}{a+b} = m$ the harmonical mean between a and b. Or it is the reciprocal of the arithmetical mean between the reciprocals of the given extremes; that is, take the reciprocals of the extremes a and b, which will be $\frac{1}{a}$ and $\frac{1}{b}$; then take the arithmetical mean between theſe reciprocals, or half their ſum, which will be $\frac{1}{2a} + \frac{1}{2b}$ or $\frac{a+b}{2ab}$; laſtly, the reciprocal of this is $\frac{2ab}{a+b} = m$ the harmonical mean: for, arithmeticals and harmonicals are mutually reciprocals of each other;

ſo that if a, m, b, &c be arithmeticals,
then ſhall $\frac{1}{a}$, $\frac{1}{m}$, $\frac{1}{b}$, &c be harmonicals;
or if the former be harmonicals, the latter will be arithmeticals.

For example, to find a harmonical mean between 2 and 6; here $a = 2$, and $b = 6$; therefore $\frac{2ab}{a+b} = \frac{2 \times 2 \times 6}{2+6} = \frac{24}{8} = 3 = m$ the harmonical mean ſought between 2 and 6.

In the 3d book of Pappus's Mathematical Collections we have a very good tract on all the three ſorts of mean proportionals, beginning at the 5th propoſition. He obſerves, that the Ancients could not reſolve, in a geometrical way, the problem of finding two mean proportionals; and becauſe it is not eaſy to deſcribe the conic ſections in plano, for that purpoſe, they contrived eaſy and convenient inſtruments, by which they obtained good mechanical conſtructions of that problem; as appears by their writings; as in the Meſolabe of Eratoſthenes, of Philo, with the Mechanics and Catapultics of Hero. For theſe, rightly deeming the problem a ſolid one, effected the conſtruction only by inſtruments, and Apollonius Pergæus by means of the conic ſections; which others again performed by the *loci ſolidi* of Ariſtæus; alſo Nicomedes ſolved it by the conchoid, by means of which

which likewise he trisected an angle: and Pappus himself gave another solution of the same problem.

Pappus adds definitions of the three foregoing different sorts of means, with many problems and properties concerning them, and, among others, this curious similarity of them, viz, a, m, b, being three continued terms, either arithmeticals, geometricals, or harmonicals; then in the

Arithmeticals, $a : a :: a - m : m - b$;
Geometricals, $a : m :: a - m : m - b$;
Harmonicals, $a : b :: a - m : m - b$.

MEAN-*and-Extreme Proportion*, or *Extreme-and-Mean Proportion*, is when a line, or any quantity is so divided, that the less part is to the greater, as the greater is to the whole.

MEAN *Anomaly*, of a planet, is an angle which is always proportional to the time of the planet's motion from the aphelion, or perihelion, or proportional to the area described by the radius vector; that is, as the whole periodic time in one revolution of the planet, is to the time past the aphelion or perihelion, so is 360° to the Mean anomaly. See Anomaly.

MEAN *Axis*, in Optics. See AXIS.

MEAN *Conjunction* or *Opposition*, is when the mean place of the sun is in conjunction, or opposition, with the mean place of the moon in the ecliptic.

MEAN *Diameter*, in Gauging, is a Mean between the diameters at the head and bung of a cask.

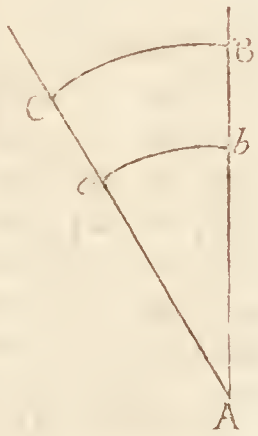
MEAN *Distance*, of a Planet from the Sun, is an arithmetical mean between the planet's greatest and least distances; and this is equal to the semitransverse axis of the elliptic orbit in which it moves, or to the right line drawn from the sun or focus to the extremity of the conjugate axis of the same.

MEAN *Motion*, is that by which a planet is supposed to move equably in its orbit; and it is always proportional to the time.

MEAN *Time*, or Equal time, is that which is measured by an equable motion, as a clock; as distinguished from apparent time, arising from the unequal motion of the earth or sun.

MEASURE, denotes any quantity, assumed as unity, or one, to which the ratio of other homogeneous or like quantities may be expressed.

MEASURE *of an Angle*, is an arc of a circle described from the angular point as a centre, and intercepted between the legs or sides of the angle: and it is usual to estimate and express the Measure of the angle by the number of degrees and parts contained in that arc, of which 360 make up the whole circumference. So, the measure of the angle BAC, is the arc BC to the radius AB, or the arc *bc* to the radius A*b*.



Hence, a right angle is measured by a quadrant, or 90 degrees; and any angle, as BAC, is in proportion to a right angle, as the arc BC is to a quadrant, or as the degrees in BC are to 90 degrees.

Common MEASURE. See COMMON *Measure*.

MEASURE *of a Figure*, or Plane Surface, is a square inch, or square foot, or square yard, &c, that is, a square whose side is an inch, or a foot, or a yard, or some other determinate length; and this square is called the *measuring unit*.

MEASURE *of a Line*, is any right line taken at pleasure, and considered as unity; as an inch, or a foot, or a yard, &c.

Line of MEASURES. See LINE *of Measures*.

MEASURE *of a Mass*, or *Quantity of Matter*, is its weight.

MEASURE *of a Number*, is any number that divides it, without leaving a remainder. So, 2 is a Measure of 4, of 8, or of any even number; and 3 is a Measure of 6, or of 9, or of 12, &c.

MEASURE *of a Ratio*, is its logarithm, in any system of logarithms; or it is the exponent of the power to which the ratio is equal, the exponent of some given ratio being assumed as unity. So, if the logarithm or Measure of the ratio of 10 to 1, be assumed equal to 1; then the Measure of the ratio of 100 to 1, will be 2, because 100 is $= 10^2$, or because 100 to 1 is in the duplicate ratio of 10 to 1; and the Measure of the ratio of 1000 to 1, will be 3, because 1000 is $= 10^3$, or because 1000 to 1 is triplicate of the ratio of 10 to 1.

MEASURE *of a Solid*, is a cubic inch, or cubic foot, or cubic yard, &c; that is, a cube whose side is an inch, or a foot, or a yard, &c.

MEASURE *of a Superficies*, the same as the Measure of a figure.

MEASURE *of Velocity*, is the space uniformly passed over by a moving body in a given time.

Universal or *Perpetual* MEASURE, is a kind of Measure unalterable by time or place, to which the Measures of different ages and nations might be reduced, and by which they may be compared and estimated. Such a Measure would be very useful, if it could be attained; since, being used at all times, and in all places, a great deal of confusion and error would be avoided.

Huygens, in his Horol. Oscil. proposes, for this purpose, the length of a pendulum that should vibrate seconds, measured from the point of suspension to the point of oscillation: the 3d part of such a pendulum to be called horary foot, and to serve as a standard to which the Measure of all other feet might be referred. Thus, for instance, the proportion of the Paris foot to the horary foot, would be that of 864 to 881; because the length of 3 Paris feet is 864 half lines, and the length of a pendulum, vibrating seconds, contains 881 half lines. But this Measure, in order to its being universal, supposes that the action of gravity is the same on every part of the earth's surface, which is contrary to fact; for which reason it would really serve only for places under the same parallel of latitude: so that, if every different latitude were to have its foot equal to the 3d part of the pendulum vibrating seconds there, any latitude would still have a different length of foot. And besides, the difficulty of measuring exactly the distance between the centres of motion and oscillation are such, that hardly any two measurers would make it the same quantity.

M. Mouton, canon of Lyons, has also a treatise *De Mensura posteris transmittenda*.

Since that time various other expedients have been proposed for establishing an universal Measure, but hitherto

hitherto without the perfect effect. In 1779, a method was propofed to the Society of Arts, &c, by a Mr. Hatton, in confequence of a premium, which had been 4 years advertifed by that inftitution, of a gold medal, or 100 guineas, ‘ for obtaining invariable ftandards for weights and Meafures, communicable at all times and to all nations.’ Mr. Hatton’s plan confifted in the application of a moveable point of fufpenfion to one and the fame pendulum, in order to produce the full and abfolute effect of two pendulums, the difference of whofe lengths was the intended Meafure. Mr. Whitehurft much improved upon this idea, by very curious and accurate machinery, in his tract publifhed 1787, intitled “An Attempt towards obtaining invariable Meafures of Length, Capacity, and Weight, from the Menfuration of time, &c. Mr. Whitehurft’s plan is, to obtain a Meafure of the greateft length that conveniency will permit, from two pendulums whofe vibrations are in the ratio of 2 to 1, and whofe lengths coincide with the Englifh ftandard in whole numbers. The numbers he has chofen fhew great ingenuity. On a fuppofition that the length of a feconds pendulum, in the latitude of London, is 39·2 inches, the length of one vibrating 42 times in a minute, muft be 80 inches; and of another vibrating 84 times in a minute, muft be 20 inches; their difference, 60 inches or 5 feet, is his ftandard Meafure. By his experiments, however, the difference in the lengths of the two pendulums was found to be 59·892 inches inftead of 60, owing to the error in the affumed length of the feconds pendulum, 39·2 inches being greater than the truth. Mr. Whitehurft has fully accomplifhed his defign, and fhewn how an invariable ftandard may, at all times, be found for the fame latitude. He has alfo afcertained a fact, as accurately as human powers feem capable of afcertaining it, of great confequence in natural philofophy. The difference between the lengths of the rods of two pendulums whofe vibrations are known, is a datum from which may be derived the true length of pendulums, the fpaces through which heavy bodies fall in a given time, with many other particulars relative to the doctrine of gravitation, the figure of the earth, &c, &c. The refult deduced from this experiment is, that the length of a feconds pendulum, vibrating in a circular arc of 3° 20′, is 39·119 inches very nearly; but vibrating in the arc of a cycloid it would be 39·136 inches; and hence, heavy bodies will fall, in the firft fecond of their defcent, 16·094 feet, or 16 feet $1\frac{1}{8}$ inch, very nearly.

It is faid, the French philofophers have a plan in contemplation, to take for a univerfal Meafure, the length of a whole meridian circle of the earth, and take all other Meafures from fub-divifions of that; which will be a very good way.—Other projects have alfo been devifed, but of little or no confideration.

Measure, in a legal, commercial, and popular fenfe, denotes a certain quantity or proportion of any thing, bought, fold, valued, or the like.

The regulation of weights and Meafures ought to be univerfally the fame throughout the nation, and indeed all nations; and they fhould therefore be reduced to fome fixed rule or ftandard.

Meafures are various, according to the various kinds or dimenfions of the things meafured. Hence arife *Lineal* or *Longitudinal* Measures, for lines or lengths:

Square Measures, for areas or fuperficies: and

Solid or *Cubic* Measures, for the folid contents and capacities of bodies.

The feveral Meafures ufed in England, are as in the following Tables:

1. *Englifh Long Meafure.*

Barley Corns						
3 =	1 Inch					
36 =	12 =	1 Foot				
108 =	36 =	3 =	1 Yard			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole		
23760 =	7920 =	660 =	220 =	40 =	1 Furlong	
190082 =	63360 =	5280 =	1760 =	320 =	8 =	1 Mile

Alfo, 4 Inches = 1 Hand
6 Feet, or 2 yds = 1 Fathom
3 Miles = 1 League
60 Nautical or Geograph. Miles = 1 Degree
or $69\frac{1}{3}$ Statute Miles = 1 Degree nearly
360 Degrees, or 25000 Miles nearly = the Circumference of the Earth.

2. *Cloth Meafure.*

Inches				
$2\frac{1}{4}$ =	1 Nail			
9 =	4 =	1 Quarter		
36 =	16 =	4 =	1 Yard	
27 =	12 =	3 =	1 Ell Flemifh	
45 =	20 =	5 =	1 Ell Englifh	
54 =	24 =	6 =	1 Ell French.	

3. *Square Meafure.*

Inches					
144 =	1 Foot				
1296 =	9 =	1 Yard			
39204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1 Pole		
1568160 =	10890 =	1210 =	40 =	1 Rood	
6272640 =	43560 =	4840 =	160 =	4 =	1 Acre.

4. *Solid, or Cubical Meafure.*

Inches		
1728 =	1 Foot	
46656 =	27 =	1 Yard.

5. *Wine Meafure.*

Pints						
2 =	1 Quart					
8 =	4 =	1 Gallon = 231 Cubic Inches.				
336 =	168 =	42 =	1 Tierce			
504 =	252 =	63 =	$1\frac{1}{2}$ =	1 Hogfhead		
672 =	336 =	84 =	2 =	$1\frac{1}{3}$ =	1 Puncheon	
1008 =	504 =	126 =	3 =	2 =	$1\frac{1}{2}$ =	1 Pipe
2016 =	1008 =	252 =	6 =	4 =	3 =	2 = 1 Tun.

Alfo, 231 Cubic Inches = 1 Gallon
10 Gallons = 1 Anker
18 Gallons = 1 Runlet
$31\frac{1}{2}$ Gallons = 1 Barrel.

6. *Ale*

6. *Ale and Beer Measure.*

Pints.

2 = 1 Quart.
8 = 4 = 1 Gallon = 282 Cubic Inches.
72 = 36 = 9 = 1 Firkin.
144 = 72 = 18 = 2 = 1 Kilderkin.
288 = 144 = 36 = 4 = 2 = 1 Barrel.
432 = 216 = 54 = 6 = 3 = $1\frac{1}{2}$ = 1 Hogshead.
576 = 288 = 72 = 8 = 4 = 2 = $1\frac{1}{3}$ = 1 Puncheon.
864 = 432 = 108 = 12 = 6 = 3 = 2 = $1\frac{1}{2}$ = 1 Butt.

Note, The Ale gallon contains 282 cubic inches.

7. *Dry Measure.*

Pints.

8 = 1 Gallon = $268\frac{4}{5}$ Cubic Inches.
16 = 2 = 1 Peck.
64 = 8 = 4 = 1 Bushel.
256 = 32 = 16 = 4 = 1 Coom.
512 = 64 = 32 = 8 = 2 = 1 Quarter.
2560 = 320 = 160 = 40 = 10 = 5 = 1 Wey.
5120 = 640 = 320 = 80 = 20 = 10 = 2 = 1 Last.

Also, $268\frac{4}{5}$ Cubic Inches = 1 Gallon.
and 36 Bushels of Coals = 1 Chaldron.

8. *Proportions of the Long Measures of several Nations to the English Foot.*

		Thousandth Parts.	Inches.
English - - -	foot	1000	12·000
Paris - - -	foot	$1065\frac{3}{4}$	12·792
Rynland, or Leyden -	foot	1033	12·396
Amsterdam - -	foot	942	11·304
Brill - - -	foot	1103	13·236
Antwerp - - -	foot	946	11·352
Dort - - - -	foot	1184	14·208
Lorrain - - -	foot	958	11·496
Mechlin - - -	foot	919	11·028
Middleburgh - -	foot	991	11·892
Strasburgh - - -	foot	920	11·040
Bremen - - -	foot	964	11·568
Cologn - - -	foot	954	11·448
Frankfort ad Mœnum -	foot	948	11·376
Spanish - - -	foot	1001	12·012
Toledo - - -	foot	899	10·788
Roman - - -	foot	967	11·604
On the monument of Cestius Statilius -	foot	972	11·664
Bononia - - -	foot	1204	14·448
Mantua - - -	foot	1569	18·838
Venice - - -	foot	1162	13·944
Dantzic - - -	foot	944	11·328
Copenhagen - -	foot	965	11·580
Prague - - -	foot	1026	12·312
Riga - - -	foot	1831	21·972
Turin - - -	foot	1062	12·744
The Greek - -	foot	1007	12·084
Old Roman - -	foot	970	11·640
Lyons - - -	ell	3967	47·604
Bologna - -	ell	2076	24·912

		Thousandth Parts.	Inches.
Amsterdam - -	ell	2269	27·228
Antwerp - - -	ell	2273	27·276
Rynland, or Leyden -	ell	2260	27·120
Frankfort - - -	ell	1826	21·912
Hamburgh - - -	ell	1905	22·860
Leipsic - - -	ell	2260	27·120
Lubeck - - -	ell	1908	22·896
Noremburgh - -	ell	2227	26·724
Bavaria - - -	ell	954	11·448
Vienna - - -	ell	1053	12·636
Bononia - -	ell	2147	25·764
Dantzic - - -	ell	1903	22·836
Florence - ·	Brace or ell	1913	22·956
Spanish, or Castile -	palm	751	9·012
Spanish - - -	vare	3004	36·040
Lisbon - - -	vare	2750	33·000
Gibraltar - - -	vare	2760	33·120
Toledo - - -	vare	2685	32·220
Naples - -	palm	861	10·332
	brace	2100	25·200
	canna	6880	82·560
Genoa - - -	palm	830	9·960
Milan - -	calamus	6544	78·528
Parma - - -	cubit	1866	22·392
China - - -	cubit	1016	12·192
Cairo - -	cubit	1824	21·888
Old Babylonian - -	cubit	1520	18·240
Old Greek - -	cubit	1511	18·132
Old Roman - -	cubit	1458	17·496
Turkish - -	pike	2200	26·400
Persian - - -	arash	3197	38·364

MEASURING, the same as MENSURATION, which see.

MECHANICS, a mixed mathematical science, that treats of forces, motion, and moving powers, with their effects in machines, &c. The science of Mechanics is distinguished, by Sir Isaac Newton, into Practical and Rational: the former treats of the Mechanical Powers, and of their various combinations; the latter, or Rational Mechanics, comprehends the whole theory and doctrine of forces, with the motions and effects produced by them.

That part of Mechanics, which treats of the weight, gravity,

gravity, and equilibrium of bodies and powers, is called Statics; as diſtinguiſhed from that part which conſiders the Mechanical powers, and their application, which is properly called Mechanics.

Some of the principles of Statics were eſtabliſhed by Archimedes, in his Treatiſe on the Centre of Gravity of Plane Figures: beſides which, little more upon Mechanics is to be found in the writings of the Ancients, except what is contained in the 8th book of Pappus's Mathematical Collections, concerning the five Mechanical Powers. Galileo laid the beſt foundation of Mechanics, when he inveſtigated the deſcent of heavy bodies; and ſince his time, by the aſſiſtance of the new methods of computation, a great progreſs has been made, eſpecially by Newton, in his Principia, which is a general treatiſe on Rational and Phyſical Mechanics, in its largeſt extent. Other writers on this ſcience, or ſome branch of it, are, Guido Ubaldus, in his Liber Mechanicorum; Torricelli, Libri de Motu Gravium naturaliter Deſcendentium & Projectorum; Balianus, Tractatus de Motu naturali Gravium; Huygens, Horologium Oſcillatorium, and Tractatus de Motu Corporum ex Percuſſione; Leibnitz, Reſiſtentia Solidorum in Acta Eruditor. an. 1684; Guldinus, De Centro Gravitatis; Wallis, Tractatus de Mechanica; Varignon, Projet d'une Nouvelle Mechanique, and his papers in the Memoir. Acad. an. 1702; Borelli, Tractatus De Vi Percuſſionis, De Motionibus Naturalibus a Gravitate pendentibus, and De Motu Animalium; De Chales, Treatiſe on Motion; Pardies, Diſcourſe of Local Motion; Parent, Elements of Mechanics and Phyſics; Caſatus, Mechanica; Oughtred, Mechanical Inſtitutions; Rohault, Tractatus de Mechanica; Lamy, Mechanique; Keill, Introduction to true Philoſophy; De la Hire, Mechanique; Mariotte, Traité du Choc des Corps; Ditton, Laws of Motion; Herman, Phoronomia; Graveſande, Phyſics: Euler, Tractatus de Motu; Muſſchenbroek, Phyſics; Boſſu, Mechanique; Deſaguliers, Mechanics; Rowning, Natural Philoſophy; Emerſon, Mechanics; Parkinſon, Mechanics; La Grange, Mechanique Analytique; Nicholſon, Introduction to Natural Philoſophy; Enfield, Inſtitutes of Natural Philoſophy, &c, &c. As to the Deſcription of Machines, ſee Strada, Zeiſingius, Beſſon, Auguſtine de Ramellis, Boetler, Leopold, Sturmy, Perrault, Limberg, Emerſon, Royal Academy of Sciences, &c.

In treating of machines, we ſhould conſider the weight that is to be raiſed, the power by which it is to be raiſed, and the inſtrument or engine by which this effect is to be produced. And, in treating of theſe, there are two principal problems that preſent themſelves: the firſt is, to determine the proportion which the power and weight ought to have to each other, that they may juſt be in equilibrio; the ſecond is, to determine what ought to be the proportion between the power and weight, that a machine may produce the greateſt effect in a given time. All writers on Mechanics treat on the firſt of theſe problems, but few have conſidered the ſecond, though not leſs uſeful than the other.

As to the firſt problem, this general rule holds in all powers; namely, that when the power and weight are reciprocally proportional to the diſtances of the directions in which they act, from the centre of motion; or when the product of the power by the diſtance of its direction, is equal to the product of the weight by the diſtance of its direction; this is the caſe in which the power and weight ſuſtain each other, and are in equilibrio; ſo that the one would not prevail over the other, if the engine were at reſt; and if it were in motion, it would continue to proceed uniformly, if it were not for the friction of its parts, and other reſiſtances. And, in general, the effect of any power, or force, is as the product of that force multiplied by the diſtance of its direction from the centre of motion, or the product of the power and its velocity when in motion, ſince this velocity is proportional to the diſtance from that centre.

The ſecond general problem in Mechanics, is, to determine the proportion between the power and weight, ſo that when the power prevails, and the machine is in motion, the greateſt effect poſſible may be produced by it in a given time. It is manifeſt, that this is an enquiry of the greateſt importance, though few have treated of it. When the power is only a little greater than what is ſufficient to ſuſtain the weight, the motion uſually is too ſlow; and though a greater weight be raiſed in this caſe, it is not ſufficient to compenſate for the loſs of time. On the other hand, when the power is much greater than what is ſufficient to ſuſtain the weight, this is raiſed in leſs time; but it may happen that this is not ſufficient to compenſate for the loſs ariſing from the ſmallneſs of the load. It ought therefore to be determined when the product of the weight multiplied by its velocity, is the greateſt poſſible; for this product meaſures the effect of the engine in a given time, which is always the greater in proportion both as the weight is greater, and as its velocity is greater. For ſome calculations on this problem, ſee Maclaurin's Account of Newton's Diſcoveries, p. 171, &c; alſo his Fluxions, art. 908 &c. And, for the various properties in Mechanics, ſee the ſeveral terms Motion, Force, Mechanical Powers, Lever, &c.

MECHANIC, or Mechanical, ſomething relating to Mechanics, or regulated by the nature and laws of motion.

Mechanical is alſo uſed in Mathematics, to ſignify a conſtruction or proof of ſome problem, not done in an accurate and geometrical manner, but coarſely and unartfully, or by the aſſiſtance of inſtruments; as are moſt problems relating to the duplicature of the cube, and the quadrature of the circle.

Mechanical *Affections*, ſuch properties in matter, as reſult from their figure, bulk, and motion.

Mechanical *Cauſes*, are ſuch as are founded on Mechanical Affections.

Mechanical *Curve*, called alſo *Tranſcendental*, is one whoſe nature cannot be expreſſed by a finite Algebraical equation.

Mechanical *Philoſophy*, alſo called the *Corpuſcular Philoſophy*, is that which explains the phenomena of nature, and the operations of corporeal things, on the principles of Mechanics; viz, the motion, gravity, figure, arrangement, diſpoſition, greatneſs, or

or ſmallneſs of the parts which compoſe natural bodies.

Mechanical *Solution*, of a Problem, is either when the thing is done by repeated trials, or when the lines uſed in the ſolution are not truly geometrical, or by organical conſtruction.

Mechanical *Powers*, are certain ſimple machines which are uſed for raiſing greater weights, or overcoming greater reſiſtances than could be effected by the natural ſtrength without them.

Theſe ſimple machines are uſually accounted ſix in number, viz, the Lever, the Wheel and Axle, or Axis in Peritrochio, the Pulley, the Inclined Plane, the Wedge, and the Screw. Of the various combinations of theſe ſimple powers do all engines, or compound machines, conſiſt: and in treating of them, ſo as to ſettle their theory and properties, they are conſidered as mathematically exact, or void of weight and thickneſs, and moving without friction. See the properties and demonſtrations of each of theſe under the ſeveral words Lever, &c. To which may be added the following general obſervations on them all, in a connective way.

1. A *Lever*, the moſt ſimple of all the mechanic powers, is an engine chiefly uſed to raiſe large weights to ſmall heights; ſuch as a handſpike, when of wood; and a crow, when of iron. In theory, a lever is conſidered as an inflexible line, like the beam of a balance, and ſubject to the ſame proportions; only that the power applied to it, is commonly an animal power; and from the different ways of uſing it, or applying it, it is called a lever of the firſt, ſecond, or third kind: viz, of the 1ſt kind, when the weight is on one ſide of the prop, and the power on the other; of the 2d kind, when the weight is between the prop and the power; and of the 3d kind, when the power is between the prop and the weight.

Many of the inſtruments in common uſe, are levers of one of the three kinds; thus, pincers, ſheers, forceps, ſnuffers, and ſuch like, are compounded of two levers of the firſt kind; for the joint about which they move, is the fulcrum, or centre of motion; the power is applied to the handles, to preſs them together; and the weight is the body which they pinch or cut. The cutting knives uſed by druggiſts, patten-makers, blockmakers, and ſome other trades, are levers of the 2d kind: for the knife is fixed by a ring at one end, which makes the fulcrum, or fixed point; the other end is moved by the hand, or power; and the body to be cut, or the reſiſtance to be overcome, is the weight. Doors are levers of the 2d kind; the hinges being the centre of motion; the hand applied to the lock is the power; while the door or weight lies between them. A pair of bellows conſiſts of two levers of the 2d kind; the centre of motion is where the ends of the boards are fixed near the pipe; the power is applied at the handles; and the air preſſed out from between the boards, by its reſiſtance, acts againſt the middle of the boards like a weight. The oars of a boat are levers of the 2d kind: the fixed point is the blade of the oar in the water; the power is the hand acting at the other end; and the weight to be moved is the boat. And the ſame of the rudder of a veſſel. Spring ſheers and tongs are levers of the 3d kind; where the centre of motion is at the bow-ſpring at one end; the weight or reſiſtance is acted on by the other end; and the hand or power is applied between the ends. A ladder reared by a man againſt a wall, is a lever of the 3d kind: and ſo are alſo almoſt all the bones and muſcles of animals.

In all levers, the effect of any power or weight, is both proportional to that power or weight, and alſo to its diſtance from the centre of motion. And hence it is that, in raiſing great weights by a lever, we chuſe the longeſt levers; and alſo reſt it upon a point as far from the hand or power, and as near to the weight, as poſſible. Hence alſo there will be an equilibrium between the power and weight, when thoſe two products are equal, viz, the power multiplied by its diſtance, equal to the weight multiplied by its diſtance; when, alſo, the weight and power are to each other reciprocally as their diſtances from the prop or fixed point.

2. The Axis in Peritrochio, or Wheel and Axle, is a ſimple engine conſiſting of a wheel fixed upon the end of an axle, ſo that they both turn round together in the ſame time. This engine may be referred to the lever: for the centre of the axis, or wheel, is the fixed point; the radius of the wheel is the diſtance of the power, acting at the circumference of the wheel, from that point; and the radius of the axle is the diſtance of the weight from the ſame point. Hence the effect of the power, independent of its own natural intenſity, is as the radius of the wheel; and the effect of the weight is as the radius of the axle: ſo that the two will be in equilibrio, when the two products are equal, which are made by multiplying each of theſe, the weight and power, by the radius, or diſtance at which it acts; and then alſo, the weight and power are reciprocally proportional to thoſe radii.

In practice, the thickneſs of the rope, that winds upon the axle, and to which the weight is faſtened, is to be conſidered: which is done, by adding half its thickneſs to the radius of the axis, for its diſtance from the fixed point, when there is only one fold of rope upon the axle; or as many times the thickneſs as there are folds, wanting only one half when there are ſeveral folds of the rope, one over another: which is the reaſon that more power muſt be applied when the axis is thus thickened; as often happens in drawing water from a deep and narrow well, over which a long axle cannot be placed.

If the rope to which the power is faſtened, be ſucceſſively applied to different wheels, whoſe diameters are larger and larger; the axis will be turned with ſtill more and more eaſe, unleſs the intenſity of the power be diminiſhed in the ſame proportion; and if ſo, the axis will always be drawn with the ſame ſtrength by a power continually diminiſhing. This is practiſed in ſpring clocks and watches; where the ſpiral ſpring, which is ſtrongeſt in its action when firſt wound up, draws the fuzee, or continued axis in peritrochio, firſt by the ſmaller wheels, and as it unbends and becomes weak, draws at the larger wheels, in ſuch manner that the watch work is always carried round with the ſame force.

As a very ſmall axis would be too weak for very great weights, or a large wheel would be expenſive as well

well as cumberſome, and take more room than perhaps can be ſpared for it; therefore, that the action of the power may be increaſed, without incurring either of thoſe inconveniences, a compound Axis in Peritrochio is uſed, which is effected by combining wheels and axles by means of pinions, or ſmall wheels, upon the axles, the teeth of which take hold of teeth made in the large wheels; as is ſeen in clocks, jacks, and other compound machines. And in ſuch a combination of wheels and axles, the effect of the power is increaſed in the ratio of the continual product of all the axles, or ſmall wheels, to that of all the large ones. Thus, if there be two ſmall wheels and an axle, turning three large wheels; the axle being 2 inches diameter, and each of the ſmall wheels 4 inches, while the large ones are 2 feet or 24 inches diameter; then $2 \times 4 \times 4 = 32$ is the continual product of the ſmall diameters, and $24 \times 24 \times 24 = 13824$ is that of the large ones; therefore 13824 to 32, or 432 to 1, is the ratio in which the power is increaſed: and if the power be a man, whoſe natural ſtrength is equal, ſuppoſe, to 150 pounds weight, then $432 \times 150 = 64800$lb, or 28 ton 18 cwt 64lb, is the weight he would be able to balance, ſuſpended about the axle.

3. *A Single Pulley*, is a ſmall wheel, moveable round an axis, called its centre pin; which of itſelf is not properly one of the mechanical powers, becauſe it produces no gain of power; for, as the weight hangs by one end of the cord that paſſes over the pulley, and the power acts at the other end of the ſame, theſe act at equal diſtances from the centre or axis of motion, and conſequently the power is equal to the weight when in equilibrio. So that the chief uſe of the ſingle pulley is to change the direction of the power from upwards to downwards, &c, and to convey bodies to a great height or diſtance, without a perſon moving from his place.

But by combining ſeveral ſingle pulleys together, a conſiderable gain of power is made, and that in proportion to the additional number of ropes made to paſs over them; and yet it enjoys at the ſame time the properties of a ſingle pulley, by changing the direction of the action in any manner.

4. *The Inclined Plane*, is made by planks, bars, or beams, laid aſlope; by which, large and heavy bodies may be more eaſily raiſed or lowered, by ſliding them up or down the plane; and the gain in power is in proportion as the length of the plane to its height, or as radius to the ſine of the angle of inclination of the plane with the horizon.

In drawing a weight up an inclined plane, the power acts to the greateſt advantage, when its direction is parallel to the plane.

5. *The Wedge*, which reſembles a double inclined plane, is very uſeful to drive in below very heavy weights to raiſe them but a ſmall height, alſo in cleaving and ſplitting blocks of wood, and ſtone &c; and the power gained, is in proportion of the ſlant ſide to half the thickneſs of the back. So that, if the back of a wedge be 2 inches thick, and the ſide 20 inches long, any weight preſſing on the back will balance 20 times as much acting on the ſide. But the great advantage of a wedge lies in its being urged, not by preſſure, but uſually by percuſſion, as the blow of a hammer or mallet; by which means a wedge may be driven in below, and ſo be made to lift, almoſt any the greateſt weight, as the largeſt ſhip, by a man ſtriking the back of a wedge with a mallet.

To the wedge may be referred the axe or hatchet, the teeth of ſaws, the chiſel, the augur, the ſpade and ſhovel, knives and ſwords of all kinds, as alſo the bodkin and needle, and in a word all ſorts of inſtruments which, beginning from edges or points, become gradually thicker as they lengthen; the manner in which the power is applied to ſuch inſtruments, being different according to their different ſhapes, and the various uſes for which they have been contrived.

6. *The Screw*, is a kind of perpetual or endleſs Inclined Plane; the power of which is ſtill farther aſſiſted by the addition of a handle or lever, where the power acts; ſo that the gain in power, is in the proportion of the circumference deſcribed or paſſed through by the power, to the diſtance between thread and thread in the ſcrew.

The uſes to which the ſcrew is applied, are various; as, the preſſing of bodies cloſe together; ſuch as the preſs for napkins, for bookbinders, for packers, hotpreſſers, &c.

In the ſcrew, and the wedge, the power has to overcome both the weight, and alſo a very great friction in thoſe machines; ſuch indeed as amounts ſometimes to as much as the weight to be raiſed, or more. But then this friction is of uſe in retaining the weight and machine in its place, even after the power is taken off.

If machines or engines could be made without friction, the leaſt degree of power added to that which balances the weight, would be ſufficient to raiſe it. In the lever, the friction is little or nothing; in the wheel and axle, it is but ſmall; in pulleys, it is very conſiderable; and in the inclined plane, wedge, and ſcrew, it is very great.

It is a general property in all the Mechanic powers, that when the weight and power are regulated ſo as to balance each other, in every one of theſe machines, if they be then put in motion, the power and weight will be to each other reciprocally as the velocities of their motion, or the power is to the weight as the velocity of the weight is to the velocity of the power; ſo that their two momenta are equal, viz, the product of the power multiplied by its velocity, equal to the product of the weight multiplied by its velocity. And hence too, univerſally, what is gained in power, is loſt in time; for the weight moves as much ſlower as the power is ſmaller.

Hence alſo it is plain, that the force of the power is not at all increaſed by engines; only the velocity of the weight, either in lifting or drawing, is ſo diminiſhed by the application of the inſtrument, as that the momentum of the weight is not greater than the force of the power. Thus, for inſtance, if any force can raiſe a pound weight with a given velocity, it is impoſſible by any engine to raiſe 2 pound weight with the ſame velocity: but by an engine it may be made to raiſe 2 pound weight with half the velocity, or even 1000 times the weight with the 1000th part of the velocity.

See

See Maclaurin's Account of Newton's Philof. Difcov. book 2, chap. 3; Hamilton's Philof. Eff. 1; Philof. Tranf. 53, pa. 116; or Landen's Memoirs, vol. 1, pa. 1.

MECHANISM, either the conftruction or the machinery employed in any thing; as the Mechanifm of the barometer, of the microfcope, &c.

MEDIUM, the fame as mean, either arithmetical, geometrical, or harmonical.

MEDIUM denotes alfo that fpace, or region, or fluid, &c, through which a body paffes in its motion towards any point. Thus, the air, or atmofphere, is the medium in which birds and beafts live and move, and in which a projectile moves; water is the medium in which fifhes move; and æther is a fuppofed fubtile Medium in which the planets move. Glafs is alfo called a Medium, being that through which the rays of light move and pafs.

Mediums refift the motion of bodies moving through them, in proportion to their denfity or fpecific gravity.

Subtile or *Ætherial* MEDIUM, is an univerfal one whofe exiftence is by Newton rendered probable. He makes it univerfal; and vaftly more rare, fubtile, elaftic, and active than air; and by that means freely permeating the pores and interftices of all other Mediums, and diffufing itfelf through the whole creation. By the intervention of this fubtile Medium he thinks it is that moft of the great phenomena of nature are effected. See ÆTHER.

This Medium it would feem he has recourfe to, as the firft and moft remote phyfical fpring, and the ultimate of all natural caufes. By the vibrations of this Medium, he fuppofes that heat is propagated from lucid bodies; as alfo the intenfenefs of heat increafed and preferved in hot bodies, and from them communicated to cold ones.

By this Medium, he fuppofes that light is reflected, inflected, refracted, and put alternately into fits of eafy reflection and tranfmiffion; which effects he alfo elfewhere afcribes to the power of attraction; fo that it would feem, this Medium is the fource and caufe even of attraction itfelf.

Again, this Medium being much rarer within the heavenly bodies, than in the heavenly fpaces, and growing denfer as it recedes farther from them, he fuppofes this is the caufe of the gravitation of thefe bodies towards each other, and of the parts towards the bodies.

Again, from the vibrations of this fame Medium, excited in the bottom of the eye by the rays of light, and thence propagated through the capillaments of the optic nerves into the fenforium, he fuppofes that vifion is performed: and fo likewife hearing, from the vibrations of this or fome other Medium, excited in the auditory nerves by the tremors of the air, and propagated through the capillaments of thofe nerves into the fenforium: and fo of the other fenfes.

And again, he conceives that mufcular motion is performed by the vibrations of the fame Medium, excited in the brain at the command of the will, and thence propagated through the capillaments of the nerves into the mufcles; and thus contracting and dilating them.

The elaftic force of this Medium, he fhews, muft be prodigioufly great. Light moves at the rate of confiderably more than 10 millions of miles in a minute; yet the vibrations and pulfations of this Medium, to caufe the fits of eafy reflection and tranfmiffion, muft be fwifter than light, which is yet 7 hundred thoufand times fwifter than found. The elaftic force of this Medium, therefore, in proportion to its denfity, muft be above 490000 million of times greater than the elaftic force of the air, in proportion to its denfity; the velocities and pulfes of the elaftic Mediums being in a fubduplicate ratio of the elafticities, and the rarities of the Mediums, taken together. And thus may it be conceived that the vibration of this Medium is the caufe alfo of the elafticity of bodies.

Farther, the particles of this Medium being fuppofed indefinitely fmall, even fmaller than thofe of light; if they be likewife fuppofed, like our air, endued with a repelling power, by which they recede from each other, the fmallnefs of the particles may exceedingly contribute to the increafe of the repelling power, and confequently to that of the elafticity and rarity of the Medium; by that means fitting it for the free tranfmiffion of light, and the free motions of the heavenly bodies. In this Medium may the planets and comets roll without any confiderable refiftance. If it be 700,000 times more elaftic, and as many times rarer, than air, its refiftance will be above 600 million times lefs than that of water; a refiftance that would caufe no fenfible alteration in the motion of the planets in ten thoufand years.

MEGAMETER. See MICROMETER.

MEIBOMIUS (MARCUS), a very learned perfon of the 17th century, of a family in Germany which had long been famous for learned men. He devoted himfelf to literature and criticifm, but particularly to the learning of the Ancients; as their mufic, the ftructure of their galleys, &c. In 1652 he publifhed a collection of feven Greek authors, who had written upon Ancient Mufic, to which he added a Latin verfion by himfelf. This work he dedicated to queen Chriftina of Sweden; in confequence of which he received an invitation to that Princefs's court, like feveral other learned men, which he accepted. The queen engaged him one day to fing an air of ancient mufic, while a perfon danced the Greek dances to the found of his voice; and the immoderate mirth which this occafioned in the fpectators, fo covered him with ridicule, and difgufted him fo vehemently, that he abruptly left the court of Sweden immediately, after heartily battering with his fifts the face of Bourdelot, the favourite phyfician and buffon to the queen, who had perfuaded her to exhibit that fpectacle.

Meibomius pretended that the Hebrew copy of the Bible was full of errors, and undertook to correct them by means of a metre, which he fancied he had difcovered in thofe ancient writings; but this it feems drew upon him no fmall raillery from the Learned. Neverthelefs, befides the work above mentioned, he produced feveral others, which fhewed him to be a good fcholar; witnefs his Notes upon Diogenes Laertius in Menage's edition; his *Liber de Fabrica Triremium*, 1671, in which he thinks he difcovered the

method

method in which the Ancients difpofed their bancs of oars; his edition of the Ancient Greek Mythologifts; and his Dialogues on Proportions, a curious work, in which the interlocutors, or perfons reprefented as fpeaking, are Euclid, Archimedes, Apollonius, Pappus, Eutocius, Theo, and Hermotimus. This laft work was oppofed by Langius, and by Dr. Wallis, in a confiderable Tract, printed in the firft volume of his works.

MELODY, is the agreeable effect of different mufical founds, ranged or difpofed in a proper fucceffion, being the effect only of one fingle part, voice, or inftrument; by which it is diftinguifhed from harmony, which properly refults from the union of two or more mufical founds heard together.

MENISCUS, a lens or glafs, convex on one fide, and concave on the other. Sometimes alfo called a Lune or Lunula. See its figure under the article LENS.

To find the Focus of a Menifcus, the rule is, as the difference between the diameters of the convexity and concavity, is to either of them, fo is the other diameter, to the focal length, or diftance of the focus from the Menifcus. So that, having given the diameter of the convexity, it is eafy to find that of the concavity, fo as to remove the focus to any propofed diftance from the Menifcus. For, if D and d be the diameters of the two fides, and f the focal diftance; then fince,

by the rule $D - d : D :: d : f$,
therefore $d : D :: f - d : f$,
or $f - d : f :: d : D$.

Hence, if D the diameter of the concavity be double to d that of the convexity, f will be equal to D, or the focal diftance equal to the diameter; and therefore the Menifcus will be equivalent to a plano-convex lens.

Again, if $D = 3d$, or the diameter of the concavity triple to that of the convexity, then will $f = \frac{1}{2}D$, or the focal diftance equal to the radius of concavity; and therefore the Menifcus will be equivalent to a lens equally convex on either fide.

But if $D = 5d$, then will $f = \frac{1}{4}D$; and therefore the Menifcus will be equivalent to a fphere.

Laftly, if $D = d$, then will f be infinite; and therefore a ray falling parallel to the axis, will ftill continue parallel to it after refraction.

MENSTRUUM, SOLVENT, or DISSOLVENT, any fluid that will diffolve hard bodies, or feparate their parts. Sir Ifaac Newton accounts for the action of Menftruums from the acids with which they are impregnated; the particles of acids being endued with a ftrong attractive force, in which their activity confifts, and by virtue of which they diffolve bodies. By this attraction they gather together about the particles of bodies, whether metallic, ftony, or the like, and adhere very clofely to them, fo as fcarce to be feparated from them by diftillation, or fublimation. Thus ftrongly attracting, and gathering together on all fides, they raife, disjoin, and fhake afunder the particles of bodies, i. e. they diffolve them; and by the attractive power with which they rufh againft the particles of the bodies, they move the fluid, and fo excite heat, fhaking fome of the particles to that degree, as to convert them into air, and fo generating bubbles.

Dr. Keill has given the theory or foundation of the action of Menftruums, in feveral propofitions. See ATTRACTION. From thofe propofitions are perceived the reafons of the different effects of different Menftruums; why fome bodies, as metals, diffolve in a faline Menftruum; others again, as refins, in a fulphureous one; &c: particularly why filver diffolves in aqua fortis, and gold only in aqua regis; all the varieties of which are accountable for, from the different degrees of cohefion, or attraction in the parts of the body to be diffolved, the different diameters and figures of its pores, the different degrees of attraction in the Menftruum, and the different diameters and figures of its parts.

MENSURABILITY, the fitnefs of a body for being applied, or conformable to a certain meafure.

MENSURATION, the act, or art, of meafuring figured extenfion and bodies; or of finding the dimenfions, and contents of bodies, both fuperficial and folid.

Every different fpecies of Menfuration is eftimated and meafured by others of the fame kind; fo, the folid contents of bodies are meafured by cubes, as cubic inches, or cubic feet, &c; furfaces by fquares, as fquare inches, feet, &c; and lengths or diftances by other lines, as inches, feet, &c.

The contents of rectilinear figures, whether plane or folid, can be accurately determined, or expreffed; but of many curved ones, not. So the quadrature of the circle, and cubature of the fphere, are problems that have never yet been accurately folved. See the various kinds of Menfuration, as well as that of the different figures, under their refpective terms.

The firft writers on Geometry were chiefly writers on Menfuration; as Euclid, Archimedes, &c. See QUADRATURE; alfo the Preface to my Menfuration, for the moft ample information.

MERCATOR (GERARD), an eminent geographer and mathematician, was born in 1512, at Ruremonde in the Low Countries. He applied himfelf with fuch induftry to the fciences of geography and mathematics, that it has been faid he often forgot to eat and fleep. The emperor Charles the 5th encouraged him much in his labours; and the duke of Juliers made him his cofmographer. He compofed and publifhed a Chronology; a larger and fmaller Atlas; and fome Geographical Tables; befide other books in Philofophy and Divinity. He was alfo fo curious, as well as ingenious, that he engraved and coloured his maps himfelf. He made various maps, globes, and other mathematical inftruments for the ufe of the emperor; and gave the moft ample proofs of his uncommon fkill in what he profeffed. His method of laying down charts is ftill ufed, which bear the name of *Mercator's Charts*; alfo a part of navigation is from him called *Mercator's Sailing*.—He died at Duifbourg in 1594, at 82 years of age.—See MERCATOR'S *Chart*, below.

MERCATOR *(Nicholas)*, an eminent mathematician and aftronomer, whofe name in High-Dutch was *Hauffman*, was born, about the year 1640, at Holftein in Denmark. From his works we learn, that he had an early and liberal education, fuitable to his diftinguifhed genius, by which he was enabled to extend his

researches into the mathematical sciences, and to make very considerable improvements: for it appears from his writings, as well as from the character given of him by other mathematicians, that his talent rather lay in improving, and adapting any discoveries and improvements to use, than invention. However, his genius for the mathematical sciences was very conspicuous, and introduced him to public regard and esteem in his own country, and facilitated a correspondence with such as were eminent in those sciences, in Denmark, Italy, and England. In consequence, some of his correspondents gave him an invitation to this country, which he some time after accepted, and he afterwards continued in England till his death. He had not been long here before he was admitted F. R. S. and gave frequent proofs of his close application to study, as well as of his eminent abilities in improving some branch or other of the sciences. But he is charged sometimes with borrowing the inventions of others, and adopting them as his own. And it appeared upon some occasions that he was not of an over liberal mind in scientific communications. Thus, it had some time before him been observed, that there was an analogy between a scale of logarithmic tangents and Wright's protraction of the nautical meridian line, which consisted of the sums of the secants; though it does not appear by whom this analogy was first discovered. It appears however to have been first published, and introduced into the practice of navigation, by Henry Bond, who mentions this property in an edition of Norwood's Epitome of Navigation, printed about 1645; and he again treats of it more fully in an edition of Gunter's Works, printed in 1653, where he teaches, from this property, to resolve all the cases of Mercator's Sailing by the logarithmic tangents, independent of the table of meridional parts. This analogy had only been found to be nearly true by trials, but not demonstrated to be a mathematical property. Such demonstration seems to have been first discovered by Mercator, who, desirous of making the most advantage of this and another concealed invention of his in navigation, by a paper in the Philosophical Transactions for June 4, 1666, invites the public to enter into a wager with him on his ability to prove the truth or falsehood of the supposed analogy. This mercenary proposal it seems was not taken up by any one, and Mercator reserved his demonstration. Our author however distinguished himself by many valuable pieces on philosophical and mathematical subjects. His first attempt was, to reduce Astrology to rational principles, which proved a vain attempt. But his writings of more particular note, are as follow:

1. *Cosmographia, sive Descriptio Cœli & Terræ in Circulos, qua fundamentum sterniter sequentibus ordine Trigonometriæ Sphericorum Logarithmicæ, &c, a Nicolao Hauffman Holsato*; printed at Dantzick, 1651, 12mo.

2. *Rationes Mathematicæ subductæ anno* 1653; Copenhagen, in 4to.

3. *De Emendatione annua Diatribæ duæ, quibus exponuntur & demonstrantur Cycli Solis & Lunæ, &c*; in 4to.

4. *Hypothesis Astronomica nova, et Consensus ejus cum Observationibus*; Lond. 1664, in folio.

5. *Logarithmotechnia, sive Methodus Construendi Logarithmos nova, accurata, et facilis; scripto antehac communicata anno sc.* 1667 *nonis Augusti; cui nunc accedit, Vera Quadratura Hyperbolæ, & Inventio summæ Logarithmorum. Auctore Nicolao Mercatore Holsato è Societate Regia. Huic etiam jungitur Michaelis Angeli Riccii Exercitatio Geometrica de Maximis et Minimis, hic ob argumenti præstantiam & exemplarium raritatem recusa*: Lond. 1668, in 4to.

6. *Institutionum Astronomicarum libri duo, de Motu Astrorum communi & proprio, secundum hypotheses veterum & recentiorum præcipuas; deque Hypotheseon ex observatis constructione, cum tabulis Tychonianis, Solaribus, Lunaribus, Lunæ-solaribus, & Rudolphinis Solis, Fixarum & quinque Errantium, earumque usu præceptis et exemplis commonstrato. Quibus accedit Appendix de iis, quæ novissimis temporibus cœlitus innotuerunt*: Lond. 1676, 8vo.

7. *Euclidis Elementa Geometrica, novo ordine ac methodo fere, demonstrata. Una cum Nic. Mercatoris in Geometriam Introductione brevi, qua Magnitudinum Ortus ex genuinis Principiis, & Ortarum Affectiones ex ipsa Genesi derivantur.* Lond. 1678, 12mo.

His papers in the Philosophical Transactions, are,

1. A Problem on some Points in Navigation: vol. 1, pa. 215.

2. Illustrations of the Logarithmo-technia: vol. 3, pa. 759.

3. Considerations concerning his Geometrical and Direct Method for finding the Apogees, Excentricities, and Anomalies of the Planets: vol. 5, pa. 1168.

Mercator died in 1594, about 54 years of age.

MERCATOR's *Chart*, or *Projection*, is a projection of the surface of the earth in plano, so called from Gerrard Mercator, a Flemish Geographer, who first published maps of this sort in the year 1556; though it was Edward Wright who first gave the true principles of such charts, with their application to Navigation, in 1599.

In this chart or projection, the meridians, parallels, and rhumbs, are all straight lines, the degrees of longitude being every where increased so as to be equal to one another, and having the degrees of latitude also increased in the same proportion; namely, at every latitude or point on the globe, the degrees of latitude, and of longitude, or the parallels, are increased in the proportion of radius to the sine of the polar distance, or cosine of the latitude; or, which is the same thing, in the proportion of the secant of the latitude to radius; a proportion which has the effect of making all the parallel circles be represented by parallel and equal right lines, and all the meridians by parallel lines also, but increasing infinitely towards the poles.

From this proportion of the increase of the degrees of the meridian, viz, that they increase as the secant of the latitude, it is very evident that the length of an arch of the meridian, beginning at the equator, is proportional to the sum of all the secants of the latitude, i. e. that the increased meridian, is to the true arch of it, as the sum of all those secants, to as many times the radius. But it is not so evident that the same increased meridian is also analogous to a scale of the logarithmic tangents, which however it is. "It does not appear by whom, nor by what accident, was discovered the analogy

analogy between a ſcale of logarithmic tangents and Wright's protraction of the nautical meridian line, which conſiſted of the ſums of the ſecants. It appears however to have been firſt publiſhed, and introduced into the practice of navigation, by Mr. Henry Bond, who mentions this property in an edition of Norwood's Epitome of Navigation, printed about 1645; and he again treats of it more fully in an edition of Gunter's Works, printed in 1653, where he teaches, from this property, to reſolve all the caſes of Mercator's Sailing by the logarithmic tangents, independent of the table of meridional parts. This analogy had only been found however to be nearly true by trials, but not demonſtrated to be a mathematical property. Such demonſtration, it ſeems, was firſt diſcovered by Mr. Nicholas Mercator, which he offered a wager to diſcloſe, but this not being accepted; Mercator reſerved his demonſtration; as mentioned in the account of his life in the foregoing page. The propoſal however excited the attention of mathematicians to the ſubject, and demonſtrations were not long wanting. The firſt was publiſhed about two years after, by James Gregory, in his Exercitationes Geometricæ; from hence, and other ſimilar properties there demonſtrated, he ſhews how the tables of logarithmic tangents and ſecants may eaſily be computed from the natural tangents and ſecants.

"The ſame analogy between the logarithmic tangents and the meridian line, as alſo other ſimilar properties, were afterwards more elegantly demonſtrated by Dr. Halley, in the Philoſ. Tranſ. for Feb. 1696, and various methods given for computing the ſame, by examining the nature of the ſpirals into which the rhumbs are transformed in the ſtereographic projection of the ſphere on the plane of the equator: the doctrine of which was rendered ſtill more eaſy and elegant by the ingenious Mr. Cotes, in his Logometria, firſt printed in the Philoſ. Tranſ. for 1714, and afterwards in the collection of his works publiſhed 1732, by his couſin Dr. Robert Smith, who ſucceeded him as Plumian profeſſor of philoſophy in the Univerſity of Cambridge."

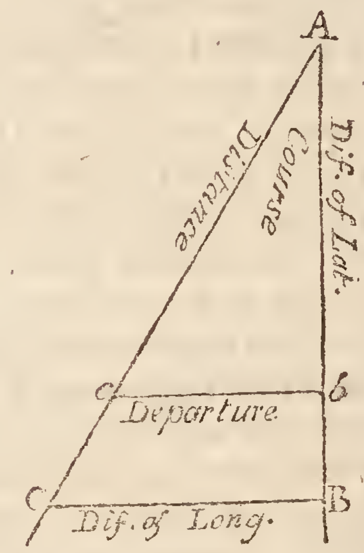
The learned Dr. Iſaac Barrow alſo, in his Lectiones Geometricæ, Lect. xi, Append. firſt publiſhed in 1672, delivers a ſimilar property, namely, "that the ſum of all the ſecants of any arc, is analogous to the logarithm of the ratio of $r + s$ to $r - s$, viz, radius plus ſine to radius minus ſine; or, which is the ſame thing, that the meridional parts anſwering to any degree of latitude, are as the logarithms of the ratios of the verſed ſines of the diſtances from the two poles." Preface to my Logarithms, pa. 100.

The meridian line in Mercator's Chart, is a ſcale of logarithmic tangents of the half colatitudes. The differences of longitude on any rhumb, are the logarithms of the ſame tangents, but of a different ſpecies; thoſe ſpecies being to each other, as the tangents of the angles made with the meridian. Hence any ſcale of logarithmic tangents is a table of the differences of longitude, to ſeveral latitudes, upon ſome one determinate rhumb; and therefore, as the tangent of the angle of ſuch a rhumb, is to the tangent of any other rhumb, ſo is the difference of the logarithms of any two tangents, to the difference of longitude on the propoſed rhumb, intercepted between the two latitudes, of whoſe half complements the logarithmic tangents were taken.

It was the great ſtudy of our predeceſſors to contrive ſuch a chart in plano, with ſtraight lines, on which all, or any parts of the world, might be truly ſet down, according to their longitudes and latitudes, bearings and diſtances. A method for this purpoſe was hinted by Ptolomy, near 2000 years ſince; and a general map, on ſuch an idea, was made by Mercator; but the principles were not demonſtrated, and a ready way ſhewn of deſcribing the chart, till Wright explained how to enlarge the meridian line by the continual addition of ſecants; ſo that all degrees of longitude might be proportional to thoſe of latitude, as on the globe: which renders this chart, in ſeveral reſpects, far more convenient for the navigator's uſe, than the globe itſelf; and which will truly ſhew the courſe and diſtance from place to place, in all caſes of ſailing.

Mercator's *Sailing*, or more properly *Wright's* Sailing, is the method of computing the caſes of ſailing on the principles of Mercator's chart, which principles were laid down by Edward Wright in the beginning of the laſt century; or the art of finding on a plane the motion of a ſhip upon any aſſigned courſe, that ſhall be true as well in longitude and latitude, as diſtance; the meridians being all parallel, and the parallels of latitude ſtraight lines.

In the right-angled triangle A*bc*, let A*b* be the true difference of latitude between two places, the angle *b*A*c* the angle of the courſe ſailed, and A*c* the true diſtance ſailed; then will *bc* be what is called the departure, as in plane ſailing: produce A*b* till AB be equal to the meridional difference of latitude, and draw BC parallel to *bc*; ſo ſhall BC be the difference of longitude.



Now from the ſimilarity of the two triangles A*bc*, ABC, when three of the parts are given, the reſt may be found; as in the following analogies: As

Radius : ſin. courſe : : diſtance : departure;
Radius : cos. courſe : : diſtance : dif. lat.;
Radius : tan. courſe : : merid. dif. lat : dif. longitude.

And by means of theſe analogies may all the caſes of Mercator's Sailing be reſolved.

MERCURY, the ſmalleſt of the inferior planets, and the neareſt to the ſun, about which it is carried with a very rapid motion. Hence it was, that the Greeks called this planet after the name of the nimble meſſenger of the Gods, and repreſented it by the figure of a youth with wings at his head and feet; from whence is derived ☿, the character in preſent uſe for this planet.

The mean diſtance of Mercury from the ſun, is to that of the earth from the ſun, as 387 to 1000, and therefore his diſtance is about 36 millions of miles, or little more than one-third of the earth's diſtance from the

the ſun. Hence the ſun's diameter will appear at Mercury, near 3 times as large as at the earth; and hence alſo the ſun's light and heat received there is about 7 times thoſe at the earth; a degree of heat ſufficient to make water boil. Such a degree of heat therefore muſt render Mercury not habitable to creatures of our conſtitution: and if bodies on its ſurface be not inflamed, and ſet on fire, it muſt be becauſe their degree of denſity is proportionably greater than that of ſuch bodies is with us.

The diameter of Mercury is alſo nearly one-third of the diameter of the earth, or about 2600 miles. Hence the ſurface of Mercury is nearly 1-9th, and his magnitude or bulk 1-27th of that of the earth.

The inclination of his orbit to the plane of the ecliptic, is 6° 54′; his period of revolution round the ſun, 87days 23hours; his greateſt elongation from the ſun 28°; the excentricity of his orbit $\frac{1}{5}$ of his mean diſtance, which is far greater than that of any of the other planets; and he moves in his orbit about the ſun at the amazing rate of 95000 miles an hour.

The place of his aphelion is ♐ 23° 8′; place of aſcending node ♉ 14° 43′, and conſequently that of the deſcending node ♏ 14° 43′.

His Length of day, or rotation on his axis, Inclination of axis to his orbit, Gravity on his ſurface, Denſity, and Quantity of matter, are all unknown.

Mercury changes his phaſes, like the moon, according to his various poſitions with regard to the earth and ſun; except only, that he never appears quite full, becauſe his enlightened ſide is never turned directly towards us, unleſs when he is ſo near the ſun as to be loſt to our ſight in his beams. And as his enlightened ſide is always towards the ſun, it is plain that he ſhines not by any light of his own; for if he did, he would conſtantly appear round.

The beſt obſervations of this planet are thoſe made when it is ſeen on the ſun's diſc, called its tranſit; for in its lower conjunction, it ſometimes paſſes before the ſun like a little ſpot, eclipſing a ſmall part of the ſun's body, only obſervable with a teleſcope. That node from which Mercury aſcends northward above the ecliptic, is in the 15th degree of Taurus, and the oppoſite in the 15th degree of Scorpio. The earth is in thoſe parts on the 6th of November, and 4th of May, new ſtyle; and when Mercury comes to either of his nodes at his inferior conjunction about theſe times, he will appear in this manner to paſs over the diſc of the ſun. But in all other parts of his orbit, his conjunctions are inviſible, becauſe he goes either above or below the ſun. The firſt obſervation of this kind was made by Gaſſendi, in November 1631. Several following obſervations of the like tranſits are collected in Du Hamel's Hiſt. of the Royal Acad. of Sciences, pa. 470, ed. 2. And Mr. Whiſton has given a liſt of ſeveral periods at which Mercury may be ſeen on the ſun's diſc, viz, in 1782, Nov. 12, at 3h 44m afternoon; in 1786, May 4th, at 6h 57m in the forenoon; in 1789, Dec. 6th, at 3h 55m afternoon; and in 1799, May 7th, at 2h 34m afternoon. There are alſo ſeveral intermediate tranſits, but none of them viſible at London. See Dr. Halley's account of the Tranſits of Mercury and Venus, in the Philoſ. Tranſ. nº. 193.

MERIDIAN, in Aſtronomy, is a great circle of the celeſtial ſphere, paſſing through the poles of the world, and both the zenith and nadir, croſſing the equinoctial at right angles, and dividing the ſphere into two equal parts, or hemiſpheres, the one eaſtern, and the other weſtern. Or, the Meridian is a vertical circle paſſing through the poles of the world.

It is called Meridian, from the Latin *meridies*, midday or noon, becauſe when the ſun comes to the ſouth part of this circle, it is noon to all thoſe places ſituated under it.

MERIDIAN, in Geography, is a great circle paſſing through the poles of the earth, and any given place whoſe Meridian it is; and it lies exactly under, or in the plane of, the celeſtial Meridian.

Theſe Meridians are various, and change according to the longitude of places; ſo that their number may be ſaid to be infinite, for that all places from eaſt to weſt have their ſeveral Meridians. Farther, as the Meridian inveſts the whole earth, there are many places ſituated under the ſame Meridian. Alſo, as it is noon whenever the centre of the ſun is in the celeſtial Meridian; and as the Meridian of the earth is in the plane of the former; it follows, that it is noon at the ſame time, in all places ſituated under the ſame Meridian.

Firſt MERIDIAN, is that from which the reſt are counted, reckoning both eaſt and weſt; and is the beginning of longitude.

The fixing of the Firſt Meridian is a matter merely arbitrary; and hence different perſons, nations, and ages, have fixed it differently: from which circumſtance ſome confuſion has ariſen in geography. The rule among the Ancients was, to make it paſs through the place fartheſt to the weſt that was known. But the Moderns knowing that there is no ſuch place on the earth as can be eſteemed the moſt weſterly, the way of computing the longitudes of places from one fixed point is much laid aſide.

Ptolomy aſſumed the Meridian that paſſes through the fartheſt of the Canary Iſlands, as his firſt Meridian; that being the moſt weſtern place of the world then known. After him, as more countries were diſcovered in that quarter, the Firſt Meridian was remov farther off. The Arabian geographers choſe to the Firſt Meridian upon the utmoſt ſhore of the weſtern ocean. Some fixed it to the iſland of St. Nicholas near the Cape Verd; Hondius to the iſle of St. James; others to the iſland of Del Corvo, one of the Azores; becauſe on that iſland the magnetic needle at that time pointed directly north, without any variation: and it was not then known that the variation of the needle is itſelf ſubject to variation. The lateſt geographers, particularly the Dutch, have pitched on the Pike of Teneriffe; others on the Iſle of Palm, another of the Canaries; and laſtly, the French, by order of the king, on the iſland of Fero, another of the Canaries.

But, without much regard to any of theſe rules, geographers and map-makers often aſſume the Meridian of the place where they live, or the capital of their country, or its chief obſervatory, for a Firſt Meridian; and from thence reckon the longitudes of places, eaſt and weſt.

Aſtronomers, in their calculations, uſually chooſe

the Meridian of the place where their obſervations are made, for their Firſt Meridian; as Ptolomy at Alexandria; Tycho Brahe at Uranibourg; Riccioli at Bologna; Flamſteed at the Royal Obſervatory at Greenwich; and the French at the Obſervatory at Paris.

There is a ſuggeſtion in the Philoſ. Tranſ. that the Meridians vary in time. And it has been ſaid that this is rendered probable, from the old Meridian line in the church of St. Petronio at Bologna, which is ſaid to vary no leſs than 8 degrees from the true Meridian of the place at this time; and from the Meridian of Tycho at Uranibourg, which M. Picart obſerves, varies 18 minutes from the modern Meridian. If there be any thing of truth in this hint, Dr. Wallis ſays, the alteration muſt ariſe from a change of the terreſtrial poles (here on earth, of the earth's diurnal motion), not of their pointing to this or that of the fixed ſtars: for if the poles of the diurnal motion remain fixed to the ſame place on the earth, the Meridians, which paſs through theſe poles, muſt remain the ſame.

But the notion of the changes of the Meridian ſeems overthrown by an obſervation of M. Chazelles, of the French Academy of Sciences, who, when in Egypt, found that the four ſides of a pyramid, built 3000 years ago, ſtill looked very exactly to the four cardinal points. A poſition which cannot be conſidered as merely fortuitous.

Meridian *of a Globe*, or *Sphere*, is the brazen circle, in which the globe hangs and turns.

It is divided into four 90's, or 360 degrees, beginning at the equinoctial: on it, each way, from the equinoctial, on the celeſtial globes, is counted the north and ſouth declination of the ſun, moon, or ſtars; and on the terreſtrial globe, the latitude of places, north and ſouth. There are two points on this circle called the poles; and a diameter, continued from thence through the centre of either globe, is called the axis of the earth, or heavens, on which it is ſuppoſed they turn round.

On the terreſtrial globes there are uſually drawn 36 Meridians, one through every 10th degree of the equator, or through every 10th degree of longitude.

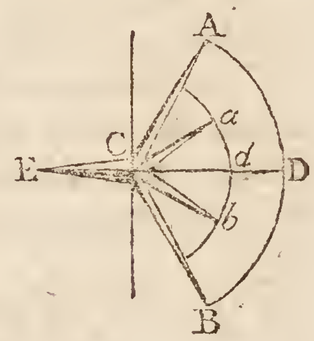
The uſes of this circle are, to ſet the globes in any particular latitude, to ſhew the ſun's or a ſtar's declination, right aſcenſion, greateſt altitude, &c.

Meridian *Line*, an arch, or part, of the Meridian of the place, terminated each way by the horizon. Or, a Meridian line is the interſection of the plane of the Meridian of the place with the plane of the horizon, often called a north-and-ſouth line, becauſe its direction is from north to ſouth.

The Meridian line is of moſt eſſential uſe in aſtronomy, geography, dialling, &c; and the greateſt pains are taken by aſtronomers to fix it at their obſervatories to the utmoſt preciſion. M. Caſſini has diſtinguiſhed himſelf by a Meridian line drawn on the pavement of the church of St. Petronio, at Bologna; being extended to 120 feet in length. In the roof of this church, 1000 inches above the pavement, is a ſmall hole, through which the ſun's image, when in the meridian, falling upon the line, marks his progreſs all the year. When finiſhed, M. Caſſini, by a public writing, quaintly informed the mathematicians of Europe, of a new oracle of Apollo, or the ſun, eſtabliſhed in a temple, which might be conſulted, with entire confidence, as to all difficulties in aſtronomy. See Gnomon.

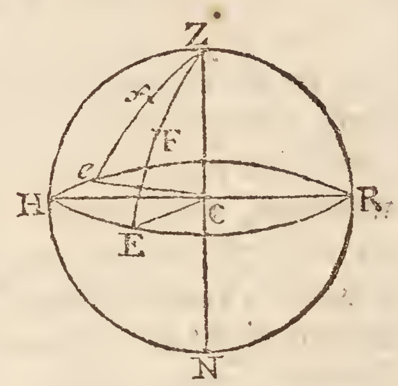
To draw a Meridian Line.—There are many ways of doing this; but ſome of the eaſieſt and ſimpleſt are as follow:

1. On an horizontal plane deſcribe ſeveral concentric circles AB, *ab*, &c, and on the common centre C erect a ſtile, or gnomon, perpendicular to the horizontal plane, of about a foot in length. About the 21ſt of June, between the hours of 9 and 11 in the morning, and between 1 and 3 in the afternoon, obſerve the points A, *a*, B, *b*, &c, in the circles, where the ſhadow of the ſtile terminates. Biſect the arches AB, *ab*, &c, in D, *d*, &c. If then the ſame right line DE biſect all theſe arches, it will be the Meridian line ſought.



As it is not eaſy to determine preciſely the extremity of the ſhadow, it will be beſt to make the ſtile flat at top, and to drill a ſmall hole through it, noting the lucid point projected by it on the arches AB and *ab*, inſtead of marking the extremity of the ſhadow itſelf.

2. Another method is thus: Knowing the ſouth quarter pretty nearly, obſerve the altitude FE of ſome ſtar on the eaſt ſide of it, and not far from the Meridian HZRN: then, keeping the quadrant firm on its axis, ſo as the plummet may ſtill cut the ſame degree, direct it to the weſtern ſide of the Meridian, and wait till you find the ſtar has the ſame altitude as before, as *fe*. Laſtly, biſect the angle EC*e*, formed by the interſection of the two planes in which the quadrant has been placed at the time of the two obſervations, by the right line HR, which will be the Meridian ſought.



Many other methods are given by authors, of deſcribing a Meridian line; as by the pole ſtar, or by equal altitudes of the ſun, &c; by Schooten in his Exercitationes Geometriæ; Grey, Derham, &c, in the Philoſ. Tranſ. and by Ferguſon in his Lectures on Select Subjects.

From what has been ſaid it is evident that whenever the ſhadow of the ſtile covers the Meridian line, the centre of the ſun is in the Meridian, and therefore it is then noon. And hence the uſe of a Meridian line in adjuſting the motion of clocks to the ſun.

If another ſtile be erected perpendicularly on any other horizontal plane, and a ſignal be given when the ſhadow of the former ſtile covers the Meridian line drawn on another plane, noting the apex or extremity of the ſhadow projected by the ſecond ſtile, a line drawn through that point and the foot of the ſtile will be a Meridian line at the 2d place.

Or, inſtead of the 2d ſtile, a plumb line may be hung up, and its ſhadow noted on a plane, upon a ſignal given that the ſhadow of another plummet, or of

of a ſtile, falls exactly in another Meridian line, at a little diſtance; which ſhadow will give the other Meridian line parallel to the former.

MERIDIAN *Line*, on a Dial, is a right line ariſing from the interſection of the Meridian of the place with the plane of the dial. This is the line of noon, or 12 o'clock, and from hence the diviſion of the hour-line begins.

MERIDIAN *Line*, on Gunter's ſcale, is divided unequally towards 87 degrees, in ſuch manner as the Meridian in Mercator's chart is divided and numbered.

This line is very uſeful in navigation. For, 1ſt, It ſerves to graduate a ſea-chart according to the true projection. 2d, Being joined with a line of chords, it ſerves for the protraction and reſolution of ſuch rectilineal triangles as are concerned in latitude, longitude, courſe, and diſtance, in the practice of ſailing; as alſo in pricking the chart truly at ſea.

Magnetical MERIDIAN, is a great circle paſſing through or by the magnetical poles; to which Meridians the magnetical needle conforms itſelf.

Meridian Altitude, of the ſun or ſtars, is their altitude when in the meridian of the place where they are obſerved.

MERIDIONAL *Diſtance*, in Navigation, is the ſame with the Departure, or eaſting and weſting, or diſtance between two meridians.

MERIDIONAL *Parts*, *Miles*, or *Minutes*, in Navigation, are the parts of the increaſed or enlarged meridian, in the Mercator's chart. Tables of theſe parts are in moſt books of navigation; and they ſerve both for conſtructing that ſort of charts, and for working that kind of navigation.

Under the article MERCATOR's *Chart*, it is ſhewn that the parts of the enlarged Meridian increaſe in proportion as the coſine of the latitude to radius, or, which is the ſame thing, as radius to the ſecant of the latitude; and therefore it follows, that the whole length of the enlarged nautical Meridian, from the equator to any point, or latitude, will be proportional to the ſum of all the ſecants of the ſeveral latitudes up to that point of the Meridian. And on this principle was the firſt Table of Meridional Parts conſtructed, by the inventor of it, Mr. Edward Wright, and publiſhed in 1599; viz, he took the Meridional parts

of $1' =$ the ſec. of $1'$;
of $2' =$ ſec. of $1'$ + ſec. of $2'$;
of $3' =$ ſecants of 1, 2, and 3 min.
of $4' =$ ſecants of 1, 2, 3, and 4 min.

and ſo on by a conſtant addition of the ſecants.

The Tables of Meridional Parts, ſo conſtructed, are perhaps exact enough for ordinary practice in navigation; but they would be more accurate if the Meridian were divided into more or ſmaller parts than ſingle minutes; and the ſmaller the parts, ſo much the greater the accuracy. But, as a continual ſubdiviſion would greatly augment the labour of calculation, other ways of computing ſuch a table have been deviſed, and treated of, by Bond, Gregory, Oughtred, Sir Jonas Moor, Dr. Wallis, Dr. Halley, and others. See MERCATOR's *Chart*, and Robertſon's Navigation, vol. 2, book 8. The beſt of theſe methods was derived from this property, viz, that the Meridian line, in a Mercator's chart, is analogous to a ſcale of logarithmic tangents of half the complements of the latitudes; from which property alſo a method of computing the caſes of Mercator's Sailing has been deduced, by Dr. Halley. Vide ut ſupra, alſo the Philoſ. Tranſ. vol. 46, pa. 559.

To find the MERIDIONAL PARTS *to any Spheroid, with the ſame exactneſs as in a Sphere.*

Let the ſemidiameter of the equator be to the diſtance of the centre from the focus of the generating ellipſe, as m to 1. Let A repreſent the latitude for which the meridional parts are required, s the ſine of the latitude, to the radius 1: Find the arc B, whoſe ſine is $\frac{s}{m}$; take the logarithmic tangent of half the complement of B, from the common tables; ſubtract the log. tangent from 10·0000000, or the log. tangent of 45°; multiply the remainder by the number 7915·7044679, and divide the product by m; then the quotient ſubtracted from the Meridional parts in the ſphere, computed in the uſual manner for the latitude A, will give the Meridional parts, expreſſed in minutes, for the ſame latitude in the ſpheroid, when it is the oblate one.

Example. If $mm : 1 :: 1000 : 22$, then the greateſt difference of the Meridional parts in the ſphere and ſpheroid is 76·0929 minutes. In other caſes it is found by multiplying the remainder above mentioned by the number 1174·078.

When the ſpheroid is oblong, the difference in the Meridional parts between the ſphere and ſpheroid, for the ſame latitude, is then determined by a circular arc. See Philoſ. Tranſ. no. 461, ſect. 14. Alſo Maclaurin's Fluxions, art. 895, 899. And Murdoch's Mercator's Sailing &c.

MERLON, in Fortification, that part of the Parapet, which lies between two embraſures.

MERSENNE (MARTIN), a learned French author, was born at Bourg of Oyſe, in the province of Maine, 1588. He ſtudied at La Fleche at the ſame time with Des Cartes; with whom he contracted a ſtrict friendſhip, which continued till death. He afterwards went to Paris, and ſtudied at the Sorbonne; and in 1611 entered himſelf among the Minims. He became well ſkilled in Hebrew, philoſophy, and mathematics. From 1615 to 1619, he taught philoſophy and theology in the convent of Nevers; and became the Superior of that convent. But being deſirous of applying himſelf more freely and cloſely to ſtudy, he reſigned all the poſts he enjoyed in his order, and retired to Paris, where he ſpent the remainder of his life; excepting ſome ſhort excurſions which he occaſionally made into Italy, Germany, and the Netherlands.

Study and literary converſation were afterwards his whole employment. He held a correſpondence with moſt of the learned men of his time; being as it were the very centre of communication between literary men of all countries, by the mutual correſpondence which he managed between them; being in France what Mr. Collins was in England. He omitted no opportunity to engage them to publiſh their works; and the world is obliged to him for ſeveral excellent diſcoveries, which would probably have been loſt, but for his encouragement; and on all accounts he had the reputation of being one of the beſt men, as well as philoſophers, of

of his time. No perfon was more curious in penetrating into the fecrets of nature, and carrying all the arts and fciences to perfection. He was the chief friend and literary agent of Des Cartes at Paris; giving him advice and affiftance upon all occafions, and informing him of all that paffed at Paris and elfewhere. For, being a perfon of univerfal learning, but particularly excelling in phyfical and mathematical knowledge, Des Cartes fcarcely ever did any thing, or at leaft was not perfectly fatisfied with any thing he had done, without firft knowing what Merfenne thought of it. It is even faid, that when Merfenne gave out in Paris, that Des Cartes was erecting a new fyftem of phyfics upon the foundation of a vacuum, and found the public very indifferent to it on that very account, he immediately fent notice to Des Cartes, that a vacuum was not then the fafhion at Paris; upon which, that philofopher changed his fyftem, and adopted the old doctrine of a plenum.

Merfenne was a man of good invention alfo himfelf; and he had a peculiar talent in forming curious queftions, though he did not always fucceed in refolving them; however, he at leaft gave occafion to others to do it. It is faid he invented the Cycloid, otherwife called the Roulette. Prefently the chief geometricians of the age engaged in the contemplation of this new curve, among whom Merfenne himfelf held a diftinguifhed rank. After a very ftudious and ufeful life, he died at Paris in 1648, at 60 years of age.

Merfenne was author of many ufeful works, particularly the following:

1. *Quæftiones celeberrimæ in Genefim.*
2. *Harmonicorum Libri.*
3. *De Sonorum Natura, Caufis, et Effectibus.*
4. *Cogitata Phyfico-Mathematica*; 2 vols. 4to.
5. *La Verité des Sciences.*
6. *Les Queftions inouies.*

Befides many letters in the works of Des Cartes, and other authors.

MESOLABE, or Mesolabium, a mathematical inftrument invented by the Ancients, for finding two mean proportionals mechanically, which they could not perform geometrically. It confifts of three parallelograms, moving in a groove to certain interfections. Its figure is defcribed by Eutocius, in his Commentary on Archimedes. See alfo Pappus, lib. 3.

MESO-Logarithm, a term ufed by Kepler to fignify the logarithms of the cofines and cotangents.

METO, or Meton, the fon of Paufanias, a famous mathematician of Athens, who flourifhed 432 years before Chrift. In the firft year of the 87th Olympiad, he obferved the folftice at Athens: and publifhed his *Anneadecateride*, that is, his *Cycle of* 19 *Years*; by which he endeavoured to adjuft the courfe of the fun to that of the moon, and to make the folar and lunar years begin at the fame point of time. See Cycle.

Metonic Cycle, called alfo the *Golden Number*, and *Lunar Cycle*, or *Cycle of the Moon*, that which was invented by Meton the Athenian; being a period of 19 years. See Cycle.

METOPE, or Metopa, in Architecture, the fquare fpace between the triglyphs of the Doric Freeze; which among the Ancients ufed to be adorned with the heads of beafts, bafons, vafes, and other inftruments ufed in facrificing.

A *Demi-Metope* is a fpace fomewhat lefs than half a Metope, at the corner of the Doric Freeze.

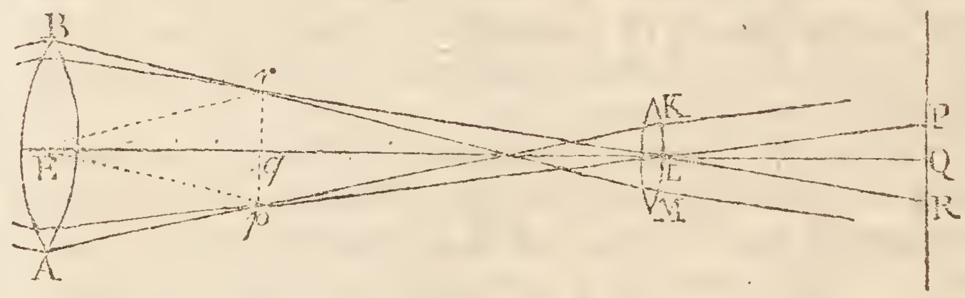
MICHAELMAS, the feaft of St. Michael the archangel; held on the 29th of September.

MICROCOUSTICS, the fame with Microphones.

MICROMETER, is an inftrument ufually fitted to a telefcope, in the focus of the object-glafs, for meafuring fmall angles or diftances; as the apparent diameters of the planets, &c.

There are feveral forts of thefe inftruments, upon different principles; the origin of which has been difputed. The general principle is, that the inftrument moves a fine wire parallel to itfelf, in the plane of the picture of an object, formed in the focus of a telefcope, and fo with great exactnefs to meafure its perpendicular diftance from a fixed wire in the fame plane: and thus are meafured fmall angles, fubtended by remote objects at the naked eye.

For example, Let a planet be viewed through the telefcope; and when the parallel wires are opened to fuch a diftance as to appear exactly to touch two oppofite points in the circumference of the planet, it is evident that the perpendicular diftance between the wires is then equal to the diameter of the picture of the planet, formed in the focus of the object-glafs. Let this diftance, whofe meafure is given by the mechanifm of the micrometer, be reprefented by the line



pq; then, fince the meafure of the focal diftance qL may be alfo known, the ratio of qL to qp, that is, of radius to the tangent of the angle qLp, will give the angle itfelf, by a table of fines and tangents; and this angle is equal to the oppofite angle PLQ, which the real diameter of the planet fubtends at L, or at the naked eye.

With refpect to the invention of the Micrometer; Meff. Azout and Picard have the credit of it in common fame, as being the firft who publifhed it, in the year 1666; but Mr. Townley, in the Philof. Tranf. reclaims it for one of our own countrymen, Mr. Gafcoigne. He relates that, from fome fcattered papers and letters of this gentleman, he had learnt that before our civil wars he had invented a Micrometer, of as much effect as that fince made by M. Azout, and had made ufe of it for fome years, not only in taking the diameters of the planets, and diftances upon land, but in determining other matters of nice importance in the heavens; as the moon's diftance, &c. Mr. Gafcoigne's inftrument alfo fell into the hands of Mr. Townley, who fays farther, that by the help of it he could make above 40,000 divifions in a foot. This inftrument being fhewn to Dr. Hook, he gave a drawing and defcription of it, and propofed feveral improvements in it; which may be feen in the Philof. Tranf. vol. 1, pa. 63, and Abr. vol. 1, pa. 217. Mr. Gafcoigne divided the image of an object, in the focus of the object-glafs, by the approach of two

two pieces of metal, ground to a very fine edge; inſtead of which, Dr. Hook would ſubſtitute two fine hairs, ſtretched parallel to each other: and two other methods of Dr. Hook, different from this, are deſcribed in his poſthumous works, pa. 497 &c. An account of ſeveral curious obſervations which Mr. Gaſcoigne made by the help of his Micrometer, particularly in meaſuring the diameter of the moon and other planets, may be ſeen in the Philoſ. Tranſ. vol. 48, pa. 190; where Dr. Bevis refers to an original letter of Mr. Gaſcoigne, to Mr. Oughtred, written in 1641, for an account given by the author of his own invention, &c.

Monſ. De la Hire, in a diſcourſe on the æra of the inventions of the Micrometer, pendulum clock, and teleſcope, read before the Royal Academy of Sciences in 1717, makes M. Huygens the inventor of the Micrometer. That author, he obſerves, in his Obſervations on Saturn's Ring, &c, publiſhed in 1659, gives a method of finding the diameters of the planets by means of a teleſcope, viz, by putting an object, which he calls a virgula, of a ſize proper to take in the diſtance to be meaſured, in the focus of the convex object-glaſs: in this caſe, ſays he, the ſmalleſt object will be ſeen very diſtinctly in that place of the glaſs. By ſuch means, he adds, he meaſured the diameter of the planets, as he there delivers them. See Huygens's Syſtem of Saturn.

This Micrometer, M. De la Hire obſerves, is ſo very little different from that publiſhed by the marquis De Malvaſia, in his Ephemerides, three years after, that they ought to be eſteemed the ſame: and the Micrometer of the marquis differed yet leſs from that publiſhed four years after his, by Azout and Picard. Hence, De la Hire concludes, that it is to Huygens the world is indebted for the invention of the Micrometer; without taking any notice of the claim of our countryman Gaſcoigne, which however is many years prior to any of them.

De la Hire ſays, that there is no method more ſimple or commodious for obſerving the digits of an eclipſe, than a net in the focus of the teleſcope. Theſe, he ſays, were uſually made of ſilken threads; and for this particular purpoſe ſix concentric circles had alſo been uſed, drawn upon oiled paper; but he adviſes to draw the circles on very thin pieces of glaſs, with the point of a diamond. He alſo gives ſome particular directions to aſſiſt perſons in uſing them. In another memoir, he ſhews a method of making uſe of the ſame net for all eclipſes, by uſing a teleſcope with two object-glaſſes, and placing them at different diſtances from each other. Mem. 1701 and 1717.

M. Caſſini invented a very ingenious method of aſcertaining the right aſcenſions and declinations of ſtars, by fixing four croſs hairs in the focus of the teleſcope, and turning it about its axis, ſo as to make them move in a line parallel to one of them. But the later improved Micrometers will anſwer this purpoſe with greater exactneſs. Dr. Maſkelyne has publiſhed directions for the uſe of it, extracted from Dr. Bradley's papers, in the Philoſ. Tranſ. vol. 62. See alſo Smith's Optics, vol. 2, pa. 343.

Wolfius deſcribes a Micrometer of a very eaſy and ſimple ſtructure, firſt contrived by Kirchius.

Dr. Derham tells us, that his Micrometer is not put into a tube, as is uſual, but is contrived to meaſure the ſpectres of the ſun on paper, of any radius, or to meaſure any part of them. By this means he can eaſily, and very exactly, with the help of a fine thread, take the declination of a ſolar ſpot at any time of the day; and, by his half-ſeconds watch, meaſure the diſtance of the ſpot from either limb of the ſun.

J. And. Segner propoſed to enlarge the field of view in theſe Micrometers, by making them of a conſiderable extent, and having a moveable eye-glaſs, or ſeveral eye-glaſſes, placed oppoſite to different parts of it. He thought however, that two would be quite ſufficient, and he gives particular directions how to make uſe of ſuch Micrometers in aſtronomical obſervations. See Comm. Gotting. vol. 1, pa. 27.

A conſiderable improvement in the Micrometer was communicated to the Royal Society, in 1743, by Mr. S. Savary; an account of which, extracted from the minutes by Mr. Short, was publiſhed in the Philoſ. Tranſ. for 1753. The firſt hint of ſuch a Micrometer was ſuggeſted by M. Roemer, in 1675: and M. Bouguer propoſed a conſtruction ſimilar to that of M. Savary, in 1748; for which ſee HELIOMETER. The late Mr. Dollond made a farther improvement in this kind of Micrometer, an account of which was given to the Royal Society by Mr. Short, and publiſhed in the Philoſ. Tranſ. vol. 48. Inſtead of two object-glaſſes, he uſed only one, which he neatly cut into two ſemicircles, and fitted each ſemicircle in a metal frame, ſo that their diameters ſliding in one another, by means of a ſcrew, may have their centres ſo brought together as to appear like one glaſs, and ſo form one image; or by their centres receding, may form two images of the ſame object: it being a property of ſuch glaſſes, for any ſegment to exhibit a perfect image of an object, although not ſo bright as the whole glaſs would give it. If proper ſcales are fitted to this inſtrument, ſhewing how far the centres recede, relative to the focal length of the glaſs, they will alſo ſhew how far the two parts of the ſame object are aſunder, relative to its diſtance from the object-glaſs; and conſequently give the angle under which the diſtance of the parts of that object are ſeen. This divided object-glaſs Micrometer, which was applied by the late Mr. Dollond to the object end of a reflecting teleſcope, and has been with equal advantage adapted by his ſon to the end of an achromatic teleſcope, is of ſo eaſy uſe, and affords ſo large a ſcale, that it is generally looked upon by aſtronomers as the moſt convenient and exact inſtrument for meaſuring ſmall diſtances in the heavens. However, the common Micrometer is peculiarly adapted for meaſuring differences of right aſcenſion, and declination, of celeſtial objects, but leſs convenient and exact for meaſuring their abſolute diſtances; whereas the object-glaſs Micrometer is peculiarly fitted for meaſuring diſtances, though generally ſuppoſed improper for the former purpoſe. But Dr. Maſkelyne has found that this may be applied with very little trouble to that purpoſe alſo; and he has furniſhed the directions neceſſary to be followed when it is uſed in this manner. The addition requiſite for this purpoſe, is a cell, containing two wires, interſecting each other at right angles, placed in the focus of the eye-glaſs of the teleſcope, and moveable round about, by the turning of a button. For the deſcription of this apparatus, with the method of applying and uſing it,

it, see Dr. Maskelyne's paper on the subject, in the Philos. Trans. vol. 61, pa. 536 &c.

After all, the use of the object-glass Micrometer is attended with difficulties, arising from the alterations in the focus of the eye, which are apt to cause it to give different measures of the same angle at different times. To obviate these difficulties, Dr. Maskelyne, in 1776, contrived a prismatic Micrometer, or a Micrometer consisting of two achromatic prisms, or wedges, applied between the object-glass and eye-glass of an achromatic telescope, by moving of which wedges nearer to or farther from the object-glass, the two images of an object produced by them appeared to approach to, or recede from, each other, so that the focal length of the object-glass becomes a scale for measuring the angular distance of the two images. The rationale and use of this Micrometer are explained in the Philos. Trans. vol. 67, pa. 799, &c. And a similar invention by the abbé Rochon, and improved by the abbé Boscovich, was also communicated to the Royal Society, and published in the same volume of the Transactions, pa. 789 &c.

Mr. Ramsden has lately described two new Micrometers, which he has contrived for remedying the defects of the object-glass Micrometer. One of these is a catoptric Micrometer, which, besides the advantage it derives from the principle of reflection, of not being disturbed by the heterogeneity of light, avoids every defect of other Micrometers, and can have no aberration, nor any defect arising from the imperfection of materials, or of execution; as the great simplicity of its construction requires no additional mirrors or glasses, to those required for the telescope; and the separation of the image being effected by the inclination of the two specula, and not depending on the focus of lens or mirror, any alteration in the eye of an observer cannot affect the angle measured. It has peculiar to itself the advantages of an adjustment, to make the images coincide in a direction perpendicular to that of their motion; and also of measuring the diameter of a planet on both sides of the zero; which will appear no inconsiderable advantage to observers who know how much easier it is to ascertain the contact of the external edges of two images than their perfect coincidence.

The other Micrometer invented and described by Mr. Ramsden, is suited to the principle of refraction. This Micrometer is applied to the erect eye-tube of a refracting telescope, and is placed in the conjugate focus of the first eye-glass, as the image is considerably magnified before it comes to the Micrometer, any imperfection in its glass will be magnified only by the remaining eye-glasses, which in any telescope seldom exceeds 5 or 6 times; and besides, the size of the Micrometer glass will not be the 100th part of the area which would be required, if it were placed at the object-glass; and yet the same extent of scale is preserved, and the images are uniformly bright in every part of the field of the telescope. See the description and construction of these two Micrometers in the Philos. Trans. vol. 69, part 2, art. 27.

In vol. 72 of the Philos. Trans. for the year 1782, Dr. Herschel, after explaining the defects and imperfections of the parallel-wire Micrometer, especially for measuring the apparent diameter of stars, and the distances between double and multiple stars, describes one, for these purposes, which he calls a lamp Micrometer; one that is free from such defects, and has the advantage of a very enlarged scale. In speaking of the application of this instrument, he says, "It is well known to opticians and others, who have been in the habit of using optical instruments, that we can with one eye look into a microscope or telescope, and see an object much magnified, while the naked eye may see a scale upon which the magnified picture is thrown. In this manner I have generally determined the power of my telescopes; and any one who has acquired a facility of taking such observations, will very seldom mistake so much as one in 50 in determining the power of an instrument, and that degree of exactness is fully sufficient for the purpose.

"The Newtonian form is admirably adapted to the use of this Micrometer; for the observer stands always erect, and looks in a horizontal direction, notwithstanding the telescope should be elevated to the zenith. —The scale of the Micrometer at the convenient distance of 10 feet from the eye, with the power of 460, is above a quarter of an inch to a second; and by putting on my power of 932, I obtain a scale of more than half an inch to a second, without increasing the distance of the Micrometer; whereas the most perfect of my former Micrometers, with the same instrument, had a scale of less than the 2000th part of an inch to a second.

"The measures of this Micrometer are not confined to double stars only, but may be applied to any other objects that require the utmost accuracy, such as the diameters of the planets or their satellites, the mountains of the moon, the diameters of the fixed stars, &c."

The Micrometer has not only been applied to telescopes, and employed for astronomical purposes; but there have been various contrivances for adapting it to microscopical observations. Mr. Leeuwenhoek's method of estimating the size of small objects, was by comparing them with grains of sand, of which 100 in a line took up an inch. These grains he laid upon the same plate with his objects, and viewed them at the same time. Dr. Jurin's method was similar to this; for he found the diameter of a piece of fine silver wire, by wrapping it very close upon a pin, and observing how many rings made an inch: and he used this wire in the same manner as Leeuwenhoek used his sand. Dr. Hook used to look upon the magnified object with one eye, while at the same time he viewed other objects, placed at the same distance, with the other eye. In this manner he was able, by the help of a ruler, divided into inches and small parts, and laid on the pedestal of the microscope, as it were to cast the magnified appearance of the object upon the ruler, and thus exactly to measure the diameter which it appeared to have through the glass; which being compared with the diameter as it appeared to the naked eye, easily shewed the degree in which it was magnified. A little practice, says Mr. Baker, will render this method exceedingly easy and pleasant.

Mr. Martin, in his Optics, recommends such a Micrometer for a microscope as had been applied to telescopes; for he advises to draw a number of parallel lines on a piece of glass, with the fine point of a diamond, at the distance of one 40th of an inch from one another, and to place it in the focus of the eye-glass.

By this method, Dr. Smith contrived to take the exact draught of objects viewed by a double microſcope; for he adviſes to get a lattice, made with ſmall ſilver wires or ſquares, drawn upon a plain glaſs by the ſtrokes of a diamond, and to put it into the place of the image formed by the object-glaſs. Then, by transferring the parts of the object, ſeen in the ſquares of the glaſs or lattice, upon ſimilar correſponding ſquares drawn on paper, the picture may be exactly taken. Mr. Martin alſo introduced into compound microſcopes another Micrometer, conſiſting of a ſcrew. See both theſe methods deſcribed in his Optics, pa. 277.

A very accurate diviſion of a ſcale is performed by Mr. Coventry, of Southwark. The Micrometers of his conſtruction are parallel lines drawn on glaſs, ivory, or metal, from the 10th to the 10,000th part of an inch. Theſe may be applied to microſcopes, for meaſuring the ſize of minute objects, and the magnifying power of the glaſſes; and to teleſcopes, for meaſuring the ſize and diſtance of objects, and the magnifying power of the inſtrument. To meaſure the ſize of an object in a ſingle microſcope; lay it on a Micrometer, whoſe lines are ſeen magnified in the ſame proportion with it, and they give at one view the real ſize of the object. For meaſuring the magnifying power of the compound microſcope, the beſt and readieſt method is the following: On the ſtage in the focus of the object-glaſs, lay a Micrometer, conſiſting of an inch divided into 100 equal parts; count how many diviſions of the Micrometer are taken into the field of view; then lay a two-foot rule parallel to the Micrometer: fix one eye on the edge of the field of light, and the other eye on the end of the rule, which move, till the edge of the field of light and the end of the rule correſpond; then the diſtance from the end of the rule to the middle of the ſtage, will be half the diameter of the field: ex. gr. If the diſtance be 10 inches, the whole diameter will be 20, and the number of the diviſions of the Micrometer contained in the diameter of the field, is the magnifying power of the microſcope. For meaſuring the height and diſtance of objects by a Micrometer in the teleſcope, ſee TELESCOPE.

Mr. Adams has applied a Micrometer, that inſtantly ſhews the magnifying power of any teleſcope.

In the Philoſ. Tranſ. for 1791, a very ſimple ſcale Micrometer for meaſuring ſmall angles with the teleſcope is deſcribed by Mr. Cavallo. This Micrometer conſiſts of a thin and narrow ſlip of mother-of-pearl finely divided, and placed in the focus of the eye-glaſs of a teleſcope, juſt where the image of the object is formed; whether the teleſcope is a reflector or a refractor, provided the eye-glaſs be a convex lens. This ſubſtance Mr. Cavallo, after many trials, found much more convenient than either glaſs, ivory, horn, or wood, as it is a very ſteady ſubſtance, the diviſions very eaſy marked upon it, and when made as thin as common writing paper it has a very uſeful degree of tranſparency.

Upon this ſubject, ſee M. Azout's Tract on it, contained in *Divers Ouvrages de Mathematique & de Phiſique; par Meſſieurs de l'Academie Royal des Sciences; M. de la Hire's Aſtronomicæ Tabulæ*; Mr. *Townley*, in the *Philoſ. Tranſ.* n°. 21; *Wolfius*, in his *Elem. Aſtron.* § 508; Dr. *Hook*, and many others, in the *Philoſ. Tranſ.* n°. 29 &c; *Hevelius*, in the *Acta Eruditorum, ann.* 1708; Mr. *Balſhaſer*, in his *Micrometria*; alſo ſeveral volumes of the *Paris Memoirs*, &c.

MICROPHONES, inſtruments contrived to magnify ſmall ſounds, as microſcopes do ſmall objects.

MICROSCOPE, an optical inſtrument, compoſed of lenſes or mirrors, by means of which ſmall objects are made to appear larger than they do to the naked eye.

MICROSCOPES are diſtinguiſhed into ſimple and compound, or ſingle and double.

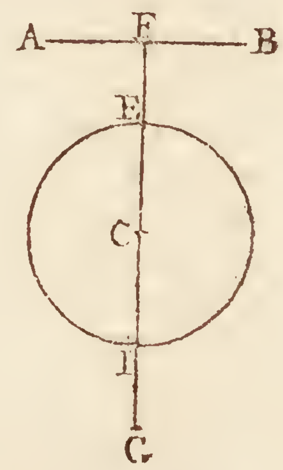
Simple, or *Single* MICROSCOPES, are ſuch as conſiſt of a ſingle lens, or a ſingle ſpherule. And a

Compound MICROSCOPE conſiſts of ſeveral lenſes duly combined.—As optics have been improved, other varieties have been contrived in this inſtrument: Hence reflecting Microſcopes, water Microſcopes, &c.

It is not certainly known when, or by whom, Microſcopes were firſt invented; although it is probable they would ſoon follow upon the uſe of teleſcopes, ſince a Microſcope is like a teleſcope inverted. We are informed by Huygens, that one Drebell, a Dutchman, had the firſt Microſcope, in the year 1621, and that he was reputed the inventor of it: though F. Fontana, a Neapolitan, in 1646, claims the invention to himſelf, and dates it from the year 1618. Be this as it may, it ſeems they were firſt uſed in Germany about 1621. According to Borelli, they were invented by Zacharias Janſen and his ſon, who preſented the firſt Microſcopes they had conſtructed to prince Maurice, and Albert arch-duke of Auſtria. William Borelli, who gives this account in a letter to his brother Peter, ſays, that when he was ambaſſador in England, in 1619, Cornelius Drebell ſhewed him a Microſcope, which he ſaid was the ſame that the arch-duke had given him, and had been made by Janſen himſelf. Borelli De vero Teleſcopii inventore, pa. 35. See LENS.

Theory and *Foundation* of MICROSCOPES.

If an object be placed in the focus of the convex lens of a ſingle Microſcope, and the eye be very near on the other ſide, the object will appear diſtinct in an erect ſituation, and magnified in the ratio of the focal diſtance of the lens, to the ordinary diſtance of diſtinct viſion, viz, about 8 inches.



So, if the object AB be placed in the focus F, of a ſmall glaſs ſphere, and the eye behind it, as in the focus G, the object will appear diſtinct, and in an erect poſture, increaſed as to diameter in the ratio of $\frac{3}{4}$ of the diameter EI to 8 inches. If, ex. gr. the diameter EI of the ſmall ſphere be $\frac{1}{10}$ of an inch; then CE $= \frac{1}{20}$, and FE $= \frac{1}{2}$CE $= \frac{1}{40}$, ſo that CF $= \frac{3}{40}$; then as $\frac{3}{40} : 8$, or as $3 : 320$, or as $1 : 106\frac{2}{3}$: : the natural ſize to the magnified appearance; that is, the object is magnified about 107 times.

Hence the ſmaller the ſpherule or the lens is, ſo much the more is the object magnified. But then, ſo much

much the lefs part is comprehended at one view, and fo much the lefs diftinct is the appearance of the object.

Equal appearances of the fame object, formed by different combinations, become obfcure in proportion as the number of rays conftituting each pencil decreafes, that is, in proportion to the fmallnefs of the object-glafs.

Wherefore, if the diameter of the object-glafs exceeds the diameter of the pupil, as many times as the diameter of the appearance exceeds the diameter of the object; the appearance fhall be as clear and bright as the object itfelf.

The diameter of the object-glafs cannot be fo much increafed, without increafing at the fame time the focal-diftances of all the glaffes, and confequently the length of the inftrument: Otherwife the rays would fall too obliquely upon the eye-glafs, and the appearance become confufed and irregular.

There are feveral kinds of fingle Microfcopes; of which the following is the moft fimple.

AB (Plate xviii, fig. 1) is a little tube, to one end of which BC, is fitted a plain glafs; to which any object, as a gnat, the wing of an infect, or the like, is applied; to the other end AD, at a proper diftance from the object, is applied a lens, convex on both fides, of about an inch in diameter: the plane glafs is turned to the fun, or the light of a candle, and the object is feen magnified. And if the tube be made to draw out, lenfes or fegments of different fpheres may be ufed.

Again, a lens, convex on both fides, is inclofed in a cell AC (fig. 2), and held there by the fcrew H. Through the ftem or pedeftal CD paffes a long fcrew EF, carrying a ftile or needle EG: In E is a fmall tube; on which, and on the point G, the various objects are to be difpofed. Thus, lenfes of various fpheres may be applied.

A good fimple inftrument of this kind is Mr. Wilfon's pocket Microfcope, which has 9 different magnifying glaffes, 8 of which may be ufed with two different inftruments, for the better applying them to various objects. One of thefe inftruments is reprefented at AABB (fig. 3), which is made either of brafs or ivory. There are three thin brafs plates at E, and a fpiral fpring H of fteel wire within it: to one of the thin plates of brafs is fixed a piece of leather F, with a fmall furrow G, both in the leather, and brafs to which it is fixed: in one end of this inftrument there is a long fcrew D, with a convex glafs C, placed in the end of it: in the other end of the inftrument there is a hollow fcrew *oo*, in which any of the magnifying glaffes, M, are fcrewed, when they are to be made ufe of. The 9 different magnifying glaffes are all fet in ivory, 8 of which are fet in the manner expreffed at M. The greateft magnifier is marked upon the ivory, in which it is fet, number 1, the next number 2, and fo on to number 8; the 9th glafs is not marked, but is fet in the manner of a little barrel box of ivory, as at *b*. At *ee* is a flat piece of ivory, of which there are 8 belonging to this fort of Microfcopes (though any one who has a mind to keep a regifter of objects may have as many of them as he pleafes); in each of them there are 3 holes *fff*, in which 3 or more objects are placed between two thin glaffes, or talcs, when they are to be ufed with the greater magnifiers.

The ufe of this inftrument AABB is this. A handle W, from fig. 4, being fcrewed upon the button S, take one of the flat pieces of ivory or fliders *ee*, and flide it between the two thin plates of brafs at E, through the body of the Microfcope, fo that the object to be viewed be juft in the middle; remarking to put that fide of the plate *ee*, where the brafs rings are, fartheft from the end AA: then fcrew into the hollow fcrew, *oo*, the 3d, 4th, 5th, 6th, or 7th magnifying glafs M; which being done, put the end AA clofe to your eye, and while looking at the object through the magnifying glafs, fcrew in or out the long fcrew D, which moving round upon the leather F, held tight to it by the fpiral wire H, will bring your object to the true diftance; which may be known by feeing it clearly and diftinctly.

Thus may be viewed all tranfparent objects, dufts, liquids, cryftals of falts, fmall infects, fuch as fleas, mites, &c. If they be infects that will creep away, or fuch objects as are to be kept, they may be placed between the two regifter glaffes *ff*. For, by taking out the ring that keeps in the glaffes *ff*, where the object lies, they will fall out of themfelves; fo the object may be laid between the two hollow fides of them, and the ring put in again as before; but if the objects be dufts or liquids, a fmall drop of the liquid, or a little of the duft laid on the outfide of the glafs *ff*, and applied as before, will be feen very eafily.

As to the 1ft, 2d, and 3d magnifying glaffes, being marked with a + upon the ivory in which they are fet, they are only to be ufed with thofe plates or fliders that are alfo marked with a +, in which the objects are placed between two thin talcs; becaufe the thicknefs of the glaffes in the other plates or fliders, hinders the object from approaching to the true diftance from thefe greater magnifiers. But the manner of ufing them is the fame with the former.

For viewing the circulation of the blood at the extremities of the arteries and veins, in the tranfparent parts of fifhes tails, &c, there are two glafs tubes, a larger and a fmaller, as expreffed at *gg*, into which the animal is put. When thefe tubes are to be ufed, unfcrew the end fcrew D in the body of the Microfcope, until the tube *gg* can be eafily received into that little cavity G of the brafs plate faftened to the leather F under the other two thin plates of brafs at E. When the tail of the fifh lies flat on the glafs tube, fet it oppofite to the magnifying glafs, and bringing it to the proper diftance by fcrewing in or out the end fcrew D, when the blood will be feen clearly circulating.

To view the blood circulating in the foot of a frog; choofe fuch a frog as will juft go into the tube; then with a little ftick expand its hinder foot, which apply clofe to the fide of the tube, obferving that no part of the frog hinders the light from coming on its foot; and when it is brought to the proper diftance, by means of the fcrew D, the rapid motion of the blood will be feen in its veffels, which are very numerous, in the tranfparent thin membrane or web between the toes. For this object, the 4th and 5th magnifiers will do very

well; but the circulation may be feen in the tails of water-newts in the 6th and 7th glaffes, becaufe the globules of the blood of thofe newts are as large again as the globules of the blood of frogs or fmall fifh, as has been remarked in number 280 of the Philof. Tranf. pa. 1184.

The circulation cannot fo well be feen by the 1ft, 2d, and 3d magnifiers, becaufe the thicknefs of the glafs tube, containing the fifh, hinders the approach of the object to the focus of the magnifying glafs. Fig. 4 is another inftrument for this purpofe.

In viewing objects, one ought to be careful not to hinder the light from falling upon them by the hat, hair, or any other thing, efpecially in looking at opaque objects; for nothing can be feen with the beft of glaffes, unlefs the object be at a due diftance, with a fufficient light. The beft lights for the plates or fliders, when the object lies between the two glaffes, is a clear fky-light, or where the fun fhines on fomething white, or the reflection of the light from a looking-glafs. The light of a candle is alfo good for viewing very fmall objects, though it be a little uneafy to thofe who are not practifed in the ufe of Microfcopes.

To caft fmall Glafs Spherules for MICROSCOPES.— There are feveral methods for this purpofe. Hartfocker firft improved fingle Microfcopes by ufing fmall globules of glafs, melted in the flame of a candle; by which he difcovered the animalculæ in femine mafculino, and thereby laid the foundation of a new fyftem of generation. Wolfius defcribes the following method of making fuch globules: A fmall piece of very fine glafs, fticking to the wet point of a fteel needle, is to be applied to the extreme bluifh part of the flame of a lamp, or rather of fpirits of wine, which will not black it; being there melted, and run into a fmall round drop, it is to be removed from the flame, on which it inftantly ceafes to be fluid. Then folding a thin plate of brafs, and making very fmall fmooth perforations, fo as not to leave any roughnefs on the furfaces, and alfo fmoothing them over to prevent any glaring, fit the fpherule between the plates againft the apertures, and put the whole in a frame, with objects convenient for obfervation.

Mr. Adams gives another method, thus: Take a piece of fine window-glafs, and rafe it, with a diamond, into as many lengths as you think needful, not more than 1-8th of an inch in breadth; then holding one of thofe lengths between the fore finger and thumb of each hand, over a very fine flame, till the glafs begins to foften, draw it out till it be as fine as a hair, and break; then applying each of the ends into the pureft part of the flame, you prefently have two fpheres, which may be made greater or lefs at pleafure: if they remain long in the flame, they will have fpots; fo they muft be drawn out immediately after they are turned round. Break the ftem off as near the globule as poffible; and, lodging the remainder of the ftem between the plates, by drilling the hole exactly round, all the protuberances are buried between the plates; and the Microfcope performs to admiration.

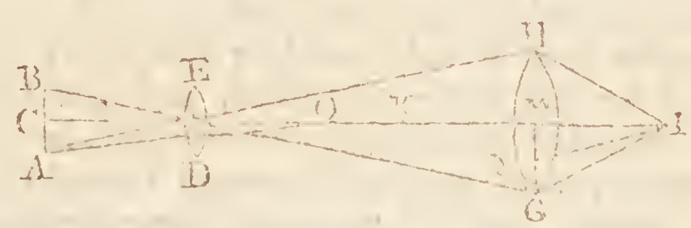
Mr. Butterfield gave another manner of making thefe globules, in number 141 Philof. Tranf.

In any of thefe ways may the fpherules be made much fmaller than any lens; fo that the beft fingle Microfcopes, or fuch as magnify the moft, are made of them. Leeuwenhoeck and Muffchenbroek have fucceeded very well in fpherical Microfcopes, and their greateft magnifiers enlarged the diameter of an object about 160 times; Philof. Tranf. vol. 7, pa. 129, and vol. 8, pa. 121. But the fmalleft globules, and confequently the higheft magnifiers for Microfcopes, were made by F. de Torre of Naples, who, in 1765, fent four of them to the Royal Society. The largeft of them was only two Paris points in diameter, and magnified a line 640 times; the fecond was the fize of one Paris point, and magnified 1280 times; and the 3d no more than half a Paris point, or the 144th part of an inch in diameter, and magnified 2560 times. But fince the focus of a glafs globule is at the diftance of one-4th of its diameter, and therefore that of the 3d globule of de Torre, above mentioned, only the 576th part of an inch diftant from the object, it muft be with the utmoft difficulty that globules fo minute as thofe can be employed to any purpofe; and Mr. Baker, to whofe examination they were referred, confiders them as matters of curiofity rather than of real ufe. Philof. Tranf. vol. 55, pa. 246, vol. 56, pa. 67.

Water MICROSCOPE. Mr. S. Gray, and, after him, Wolfius and others, have contrived water Microfcopes, confifting of fpherules or lenfes of water, inftead of glafs. But fince the diftance of the focus of a lens or fphere of water is greater than that in one of glafs, the fpheres of which they are fegments being the fame, confequently water Microfcopes magnify lefs than thofe of glafs, and therefore are lefs efteemed. Mr. Gray firft obferved, that a fmall drop or fpherule of water, held to the eye by candle light or moon light, without any other apparatus, magnified the animalcules contained in it, vaftly more than any other Microfcope. The reafon is, that the rays coming from the interior furface of the firft hemifphere, are reflected fo as to fall under the fame angle on the furface of the hinder hemifphere, to which the eye is applied, as if they came from the focus of the fpherule; whence they are propagated to the eye in the fame manner as if the objects were placed without the fpherule in its focus.

Hollow glafs fpheres of about half an inch diameter, filled with fpirit of wine, are often ufed for Microfcopes; but they do not magnify near fo much.

Theory of Compound or Double MICROSCOPES.— Suppofe an object-glafs ED, the fegment of a very fmall



fphere, and the object AB placed without the focus F. Suppofe an eye-glafs GH, convex on both fides, and the fegment of a fphere greater than that of DE, though not too great; and, the focus being at K, let it be fo difpofed behind the object,

that CF : CL :: CL : CK.

Laftly fuppofe LK : LM :: LM : LI.

 If

If then O be the place where an object is seen distinct with the naked eye; the eye in this case, being placed in I, will see the object AB distinctly, in an inverted position, and magnified in the compound ratio of MK × LC to LK × CO ; as is proved by the laws of dioptrics; that is, the image is larger than the object, and we are able to view it distinctly at a less distance. For Examp.—If the image be 20 times larger than the object, and by the help of the eye-glass we are able to view it 5 times nearer than we could have done with the naked eye, it will, on both these accounts, be magnified 5 times 20, or 100 times.

Laws of Double MICROSCOPES.

1. The more an object is magnified by the Microscope, the less is its field, i. e. the less of it is taken in at one view.

2. To the same eye-glass may be successively applied object-glasses of various spheres, so as that both the entire objects, but less magnified, and their several parts, much more magnified, may be viewed through the same Microscope. In which case, on account of the different distance of the image, the tube in which the lenses are fitted, should be made to draw out.

3. Since it is proved, that the distance of the image LK, from the object-glass DE, will be greater, if another lens, concave on both sides, be placed before its focus; it follows, that the object will be magnified the more, if such a lens be here placed between the object-glass DE, and the eye-glass GH. Such a Microscope is much commended by Conradi, who used an object-lens, convex on both sides, whose radius was 2 digits, its aperture equal to a mustard seed; a lens, concave on both sides, from 12 to 16 digits; and an eye-glass, convex on both sides, of 6 digits.

4. Since the image is projected to the greater distance, the nearer another lens, of a segment of a larger sphere, is brought to the object-glass; a Microscope may be composed of three lenses, which will magnify prodigiously.

5. From these considerations it follows, that the object will be magnified the more, as the eye-glass is the segment of a smaller sphere; but the field of vision will be the greater, as the same is a segment of a larger sphere. Therefore if two eye-glasses, the one a segment of a larger sphere, the other of a smaller one, be so combined, as that the object appearing very near through them, i. e. not farther distant than the focus of the first, be yet distinct; the object, at the same time, will be vastly magnified, and the field of vision much greater than if only one lens was used; and the object will be still more magnified, and the field enlarged, if both the object-glass and eye-glass be double. But because an object appears dim when viewed through so many glasses, part of the rays being reflected in passing through each, it is not adviseable greatly to multiply glasses; so that, among compound Microscopes, the best are those which consist of one object-glass, and two eye-glasses.

Dr. Hook, in the preface to his Micrography, tells us, that in most of his observations he used a Microscope of this kind, with a middle eye-glass of a considerable diameter, when he wanted to see much of the object at one view, and took it out when he would examine the small parts of an object more accurately: for the fewer refractions there are, the more light and clear the object appears.

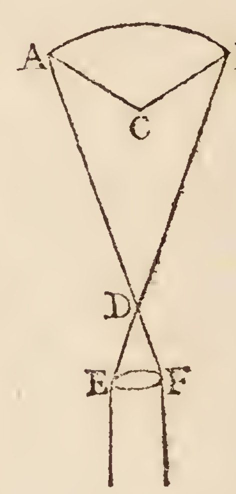
For a Microscope of three lenses De Chales recommends an object glass of $\frac{1}{3}$ or $\frac{1}{4}$ of a digit; and the first eye-glass he makes 2 or $2\frac{1}{2}$ digits; and the distance between the object-glass and eye-glass about 20 lines. Conradi had an excellent Microscope, whose object-glass was half a digit, and the two eye-glasses (which were placed very near) 4 digits; but it answered best when, instead of the object-glass, he used two glasses, convex on both sides, their sphere about a digit and a half, and at most 2, and their convexities touching each other within the space of half a line. Eustachius de Divinis, instead of an object-glass convex on both sides, used two plano-convex lenses, whose convexities touched. Grindelius did the same; only that the convexities did not quite touch. Zahnius made a binocular Microscope, with which both eyes were used. But the most commodious double Microscope, it is said, is that of our countryman Mr. Marshal; though some improvement was made in it by Mr. Culpepper and Mr. Scarlet. These are exhibited in figures 5 and 6.

It is observed, that compound Microscopes sometimes exhibit a fallacious appearance, by representing convex objects concave, and vice versa. Philos. Trans. numb. 476, pa. 387.

To fit Microscopes, as well as Telescopes, to short-sighted eyes, the object-glass and the eye-glass must be placed a little nearer together, so that the rays of each pencil may not emerge parallel, but may fall diverging upon the eye.

Reflecting MICROSCOPE, is that which magnifies by reflection, as the foregoing ones do by refraction. The inventor of this Microscope was Sir Isaac Newton.

The structure of such a Microscope may be conceived thus: near the focus of a concave speculum AB, place a minute object C, that its image may be formed larger than itself in D; to the speculum join a lens, convex on both sides, EF, so as the image D may be in its focus.



The eye will here see the image inverted, but distinct, and enlarged; consequently the object will be larger than if viewed through the lens alone.

Any telescope is changed into a Microscope, by removing the object-glass to a greater distance from the eye-glass. And since the distance of the image is various, according to the distance of the object from the focus; and it is magnified the more, as its distance from the object-glass is greater; the same telescope may be successively changed into Microscopes which magnify the object in different degrees. See some instruments of this sort described in Smith's Optics, Remarks, pa. 94.

Solar MICROSCOPE, called also the Camera-Obscura Microscope, was invented by Mr. Lieberkuhn in 1738 or 1739, and consists of a tube, a looking-glass, a convex lens, and a Wilson's Microscope. The tube (fig. 7) is brass, near 2 inches in diameter, fixed in a circular collar of mahogany, with a groove on the outside

fide of its periphery, denoted by 2, 3, and connected by a cat-gut to the pulley 4 on the upper part; which turning round at pleafure, by the pin 5 within, in a fquare frame, may be eafily adjufted to a hole in the fhutter of a window, by the fcrews 1, 1, fo clofely that no light can enter the room but through the tube of the inftrument. The mirror G is faftened to the frame by hinges, on the fide that goes without the window: this glafs, by means of a jointed brafs wire, 6, 7, and the fcrew H 8, coming through the frame, may be moved either vertically or horizontally, to throw the fun's rays through the brafs tube into the darkened room. The end of the brafs tube without the fhutter has a convex lens, 5, to collect the rays thrown on it by the glafs G, and bring them to a focus in the other part, where D is a tube fliding in and out, to adjuft the object to a due diftance from the focus. And to the end G of another tube F, is fcrewed one of Wilfon's fimple pocket Microfcopes, containing the object to be magnified in a flider; and by tube F, fliding on the fmall end E, of the other tube D, it is brought to a true focal diftance.

The Solar Microfcope has been introduced into the fmall and portable Camera Obfcura, as well as the large one: and if the image be received upon a piece of half-ground glafs, fhaded from the light of the fun, it will be fufficiently vifible. Mr. Lieberkuhn made confiderable improvements in his Solar Microfcope, particularly in adapting it to the viewing of opaque objects; and M. Aepinus, Nov. Com. Petrop. vol. 9, pa. 326, has contrived, by throwing the light upon the forefide of any object, before it is tranfmitted through the object lens, to reprefent all kinds of objects by it with equal advantage. In this improvement, the body of the common Solar Microfcope is retained, and only an addition made of two brafs plates, AB, AC, (fig 8), joined by a hinge, and held at a proper diftance by a fcrew. A fection of thefe plates, and of all the neceffary parts of the inftrument, may be feen in fig. 9, where *a c* reprefent rays of the fun converging from the illuminating lens, and falling upon the mirror *bd*, which is fixed to the nearer of the brafs plates. From this they are thrown upon the object at *ef*, and are thence tranfmitted through the object lens at K, and a perforation in the farther plate, upon a fcreen, as ufual. The ufe of the fcreen *n* is to vary the diftance of the two plates, and thereby to adjuft the mirror to the object with the greateft exactnefs. M. Euler alfo contrived a method of introducing vifion by reflected light into this Microfcope.

The MICROSCOPE *for Opaque Objects* was alfo invented by M. Lieberkuhn, about the fame time with the former, and remedies the inconvenience of having the dark fide of an object next the eye; for by means of a concave fpeculum of filver, highly polifhed, having a magnifying lens placed in its centre, the objectj is fo ftrongly illuminated, that it may be examined with eafe. A convenient apparatus of this kind, with 4 different fpeculums and magnifiers of different powers, was brought to perfection by Mr. Cuff. Philof. Tranf. number 458, § 9.

MICROSCOPIC *Objects*. All things too minute to be viewed diftinctly by the naked eye, are proper objects for the Microfcope. Dr. Hook has diftinguifhed them into thefe three general kinds; viz, exceeding fmall bodies, exceeding fmall pores, or exceeding fmall motions. The fmall bodies may be feeds, infects, animalcules, fands, falts, &c: the pores may be the interftices between the folid parts of bodies, as in ftones, minerals, fhells, &c. or the mouths of minute veffels in vegetables, or the pores of the fkin, bones, and other parts of animals: the fmall motions, may be the movements of the feveral parts or members of minute animals, or the motion of the fluids, contained either in animal or vegetable bodies. Under one or other of thefe three general heads, almoft every thing about us affords matter of obfervation, and may conduce both to our amufement and inftruction.

Great caution is to be ufed in forming a judgment on what is feen by the Microfcope, if the objects are extended or contracted by force or drynefs.

Nothing can be determined about them, without making the proper allowances; and different lights and pofitions will often fhew the fame object as very different from itfelf. There is no advantage in any greater magnifier than fuch as is capable of fhewing the object in view diftinctly; and the lefs the glafs magnifies, the more pleafantly the object is always feen.

The colours of objects are very little to be depended on, as feen by the Microfcope; for their feveral component particles, being thus removed to great diftances from one another, may give reflections very different from what they would, if feen by the naked eye.

The motions of living creatures too, or of the fluids contained in their bodies, are by no means to be haftily judged of, from what we fee by the Microfcope, without due confideration; for as the moving body, and the fpace in which it moves, are magnified, the motion muft alfo be magnified; and therefore that rapidity with which the blood feems to pafs through the veffels of fmall animals, muft be judged of accordingly. Baker on the Microfcope, pa. 52, 62, &c. See alfo an elegant work on this fubject, lately publifhed by that ingenious optician Mr. George Adams.

MIDDLE *Latitude*, is half the fum of two given latitudes; or the arithmetical mean, or the middle between two parallels of latitude. Therefore,

If the latitudes be of the fame name, either both north or both fouth, add the one number to the other, and divide the fum by 2; the quotient is the middle latitudes, which is of the fame name with the two given latitudes. But

If the latitudes be of different names, the one north and the other fouth; fubtract the lefs from the greater, and divide the remainder by 2, fo fhall the quotient be the middle latitude, of the fame name with the greater of the two.

	Ex. 1.		*Ex.* 2.
One lat.	35° 27′ N.		35° 27′ S.
the other	21 13 N.		21 13 N.
	2) 56 40		2) 14 14
Mid. lat.	28 20 N.	Mid. lat.	7 7 S.

MIDDLE *Latitude Sailing*, is a method of refolving the cafes of globular failing, by means of the Middle Latitude, on the principles of plane and parallel failing jointly.

This

This method is not quite accurate, yet often agrees pretty nearly with Mercator's Sailing, and is founded on the following principle, viz, That the departure is accounted a meridional diſtance in the middle latitude between the latitude ſailed from and the latitude arrived at.

This artifice ſeems to have been invented, on account of the eaſy manner in which the ſeveral caſes may be reſolved by the Traverſe Table, and to ſerve where a table of meridional parts is wanting. It is ſufficiently near the truth either when the two parallels are near the equator, or not far diſtant from one another, in any latitude. It is performed by theſe two rules:

1. As the coſine of the middle latitude :
   Is to radius : :
   So is the departure :
   To the difference of longitude .
2. As the coſine of the middle latitude :
   Is to the tangent of the courſe : :
   So is the difference of latitude :
   To the difference of longitude .

Ex. A ſhip ſails from latitude 37° north, ſteering conſtantly N. 33° 19′ eaſt, for 8 days, when ſhe was found in latitude 51° 18′ north; required her difference of longitude.

	51° 18′			51° 18′	
	37 00			37 00	
2)	88 18		Diff. lat.	14 18 = 858 m.	
As cos. mid. l.	44 09	-		0·14417	
To tang. cour.	33 19	-		9·81776	
So diff. lat.	858	-		2·93349	
To diff. long.	786	-		2·89542	

or 13° 6′ diff. of long. ſought.

Middle *Region*. See Region.

Mid-Heaven, *Medium Cœli*, is that point of the ecliptic which culminates, or is higheſt, or is in the meridian at any time.

MIDSUMMER-*Day*, is held on the 24th of June, the ſame day as the Nativity of St. John the Baptiſt is held.

MILE, a long meaſure, by which the Engliſh, Italians, and ſome other nations, uſe to expreſs the diſtance between places: the ſame as the French uſe the word *League*.

The Mile is of different lengths in different countries. The geographical, or Italian Mile, contains 1000 geometrical paces, *mille paſſus*, whence the term Mile is derived. The Engliſh Mile conſiſts of 8 furlongs, each furlong of 40 poles, and each pole of $16\frac{1}{2}$ feet: ſo that the Mile is = 8 furlongs = 320 poles = 1760 yards = 5280 feet.

The following table ſhews the length of the Mile, or league, in the principal nations of Europe, expreſſed in geometrical paces:

			Geomet. Paces.
Mile of Ruſſia	-	-	750
of Italy	-	-	1000
of England	,	-	1200
of Scotland and Ireland			1500
Old League of France	-	-	1500
Small League, ibid.	-	-	2000
Mean League of France	-	-	2500
Great League, ibid.	-	-	3000
Mile of Poland	-	-	3000
of Spain	-	-	3428
of Germany	-	-	4000
of Sweden	-	-	5000
of Denmark	-	-	5000
of Hungary	-	-	6000

MILITARY *Architecture*. The ſame with Fortification.

Milky Way, *Via Lactea*, or *Galaxy*, a broad track or path, encompaſſing the whole heavens, diſtinguiſhable by its white appearance, whence it obtains the name. It extends itſelf in ſome parts by a double path, but for the moſt part it is ſingle. Its courſe lies through the conſtellations Caſſiopeia, Cygnus, Aquila, Perſeus, Andromeda, part of Ophiucus and Gemini, in the northern hemiſphere; and in the ſouthern, it takes in part of Scorpio, Sagittarius, Centaurus, the Argonavis, and the Ara. There are ſome traces of the ſame kind of light about the ſouth pole, but they are ſmall in compariſon of this: theſe are called by ſome, luminous ſpaces, and Magellanic clouds; but they ſeem to be of the ſame kind with the Milky way.

The Milky way has been aſcribed to various cauſes. The Ancients fabled, that it proceeded from a ſtream of milk, ſpilt from the breaſt of Juno, when ſhe puſhed away the infant Hercules, whom Jupiter laid to her breaſt to render him immortal. Some again, as Ariſtotle, &c, imagined that this path conſiſted only of a certain exhalation hanging in the air; while Metrodorus, and ſome Pythagoreans, thought the ſun had once gone in this track, inſtead of the ecliptic; and conſequently that its whiteneſs proceeds from the remains of his light. But it is now well known, by the help of teleſcopes, that this track in the heavens conſiſts of an immenſe multitude of ſtars, ſeemingly very cloſe together, whoſe mingled light gives this appearance of whiteneſs; by Milton beautifully deſcribed as a path "powdered with ſtars."

MILL, properly denotes a machine for grinding corn, &c; but in a more general ſignification, is applied to all machines whoſe action depends on a circular motion. Of theſe there are ſeveral kinds, according to the various methods of applying the moving power; as water-mills, wind-mills, horſe-mills, hand-mills, &c, and even ſteam-mills, or ſuch as are worked by the force of ſteam; as that noble ſtructure that was erected near Blackfriars Bridge, called the Albion Mills, but lately deſtroyed by fire.

The water acts both by its impulſe and weight in an overſhot water-mill, but only by its impulſe in an underſhot one; but here the velocity is greater, becauſe the water is ſuffered to deſcend to a greater depth before it ſtrikes the wheel. Mr. Ferguſon obſerves, that where there is but a ſmall quantity of water, and a fall great enough for the wheel to lie under it, the bucket or overſhot wheel is always uſed: but where there is a large body of water, with a little fall, the breaſt or float-board wheel muſt take place: and where there is a large ſupply of water, as a river, or large ſtream or brook, with very little fall, then the underſhot wheel is the eaſieſt, cheapeſt, and moſt ſimple ſtructure.

Dr.

Dr. Desaguliers, having had occasion to examine many undershot and overshot Mills, generally found that a well made overshot Mill ground as much corn, in the same time, as an undershot Mill does with ten times as much water; supposing the fall of water at the overshot to be 20 feet, and at the undershot about 6 or 7 feet: and he generally observed that the wheel of the overshot Mill was of 15 or 16 feet diameter, with a head of water of 4 or 5 feet, to drive the water into the buckets with some momentum.

In Water-mills, some few have given the preference to the undershot wheel, but most writers prefer the overshot one. M. Belidor greatly preferred the undershot to any other construction. He had even concluded, that water applied in this way will do more than six times the work of an overshot wheel; while Dr. Desaguliers, in overthrowing Belidor's position, determined that an overshot wheel would do ten times the work of an undershot wheel with an equal quantity of water. So that between these two celebrated authors, there is a difference of no less than 60 to 1. In consequence of such monstrous disagreement, Mr. Smeaton began the course of experiments mentioned below.

In the Philos. Trans. vol. 51, for the year 1759, we have a large paper with experiments on Mills turned both by water and wind, by that ingenious and experienced engineer Mr. Smeaton. From those experiments it appears, pa. 129, that the effects obtained by the overshot wheel are generally 4 or 5 times as great as those with the undershot wheel, in the same time, with the same expence of water, descending from the same height above the bottom of the wheels; or that the former performs the same effect as the latter, in the same time, with an expence of only one-4th or one-5th of the water, from the same head or height. And this advantage seems to arise from the water lodging in the buckets, and so carrying the wheel about by their weight. But, in pa. 130, Mr. Smeaton reckons the effect of overshot only double to that of the undershot wheel. And hence he infers, in general, "that the higher the wheel is in proportion to the whole descent, the greater will be the effect; because it depends less upon the impulse of the head, and more upon the gravity of the water in the buckets. However, as every thing has its limits, so has this; for thus much is desirable, that the water should have somewhat greater velocity, than the circumference of the wheel, in coming thereon; otherwise the wheel will not only be retarded, by the buckets striking the water, but thereby dashing a part of it over, so much of the power is lost." He is farther of opinion, that the best velocity for an overshot wheel is when its circumference moves at the rate of about 3 feet in a second of time. See Wind Mill.

Considerable differences have also arisen as to the mathematical theory of the force of water striking the floats of a wheel in motion. M. Parent, Maclaurin, Desaguliers, &c, have determined, by calculation, that a wheel works to the greatest effect, when its velocity is equal to one-third of the velocity of the water which strikes it; or that the greatest velocity that the wheel acquires, is one-third of that of the water. And this determination, which has been followed by all mathematicians till very lately, necessarily results from a position which they assume, viz, that the force of the water against the wheel, is proportional to the square of its relative velocity, or of the difference between the absolute velocity of the water and that of the wheel. And this position is itself an inference which they make from the force of water striking a body at rest, being as the square of the velocity, because the force of each particle is as the velocity it strikes with, and the number of particles or the whole quantity that strikes is also as the same velocity. But when the water strikes a body in motion, the quantity of it that strikes is still as the absolute velocity of the water, though the force of each particle be only as the relative velocity, or that with which it strikes. Hence it follows, that the whole force or effect is in the compound ratio of the absolute and relative velocities of the water; and therefore is greater than the before mentioned effect or force, in the ratio of the absolute to the relative velocity. The effect of this correction is, that the maximum velocity of the wheel becomes one-half the velocity of the water, instead of one-third of it only: a determination which nearly agrees with the best experiments, as those of Mr. Smeaton.

This correction has been lately made by Mr. W. Waring, in the 3d volume of the Transactions of the American Philosophical Society, pa. 144. This ingenious writer says, 'Being lately requested to make some calculations relative to Mills, particularly Dr. Barker's construction as improved by James Rumsey, I found more difficulty in the attempt than I at first expected. It appeared necessary to investigate new theorems for the purpose, as there are circumstances peculiar to this construction, which are not noticed, I believe, by any author; and the theory of Mills, as hitherto published, is very imperfect, which I take to be the reason it has been of so little use to practical mechanics.

'The first step, then, toward calculating the power of any water-mill (or wind-mill) or proportioning their parts and velocities to the greatest advantage, seems to be,

'*The Correction of an Essential Mistake adopted by Writers on the Theory of Mills.*

'This is attempted with all the deference due to eminent authors, whose ingenious labours have justly raised their reputation and advanced the sciences; but when any wrong principles are successively published by a series of such pens, they are the more implicitly received, and more particularly claim a public rectification; which must be pleasing, even to these candid writers themselves.'

A very ingenious writer in England, 'in his masterly treatise on the rectilinear motion and rotation of bodies, published so lately as 1784, continues this oversight, with its pernicious consequences, through his propositions and corollaries (pa. 275 to 284), although he knew the theory was suspected: for he observes (pa. 382) "Mr. Smeaton in his paper on mechanic "power (published in the Philosophical Transactions "for the year 1776) allows, that the theory usually "given will not correspond with matter of fact, when "compared with the motion of machines; and seems "to attribute this disagreement, rather to deficiency "in the theory, than to the obstacles which have pre-

" vented the application of it to the complicated mo-" tion of engines, &c. In order to satisfy himself con-" cerning the reason of this disagreement, he construct-" ed a set of experiments, which, from the known" abilities and ingenuity of the author, certainly de-" serve great consideration and attention from every" one who is interested in these inquiries." 'And notwithstanding the same learned author says, "The evidence upon which the theory rests is scarcely less than mathematical;" I am sorry to find, in the present state of the sciences, one of his abilities concluding (pa. 380) "It is not probable that the theory of motion, however incontestible its principles may be, can afford much assistance to the practical mechanic," although indeed his theory, compared with the above cited experiments, might suggest such an inference. But to come to the point, I would just premise these

Definitions.

'If a stream of water impinge against a wheel in motion, there are three different velocities to be considered, appertaining thereto, viz,

First, the absolute velocity of the water;
Second, the absolute velocity of the wheel;
Third, the relative velocity of the water to that of the wheel,

i. e. the difference of the absolute velocities, or the velocity with which the water overtakes or strikes the wheel.'

'Now the mistake consists in supposing the momentum or force of the water against the wheel, to be in the *duplicate ratio of the relative velocity:* Whereas,

PROP. I.

'The force of an Invariable Stream, impinging against a Mill-wheel in Motion, is in the *Simple Direct Proportion of the Relative Velocity.*'

'For, if the relative velocity of a fluid against a single plane be varied, either by the motion of the plane, or of the fluid from a given aperture, or both, then, the number of particles acting on the plane in a given time, and likewise the momentum of each particle, being respectively as the relative velocity, the force on both these accounts, must be in the *duplicate* ratio of the relative velocity, agreeably to the common theory, with respect to this *single plane:* but, the number of these planes, or parts of the wheel acted on in a given time, will be as the velocity of the wheel, or *inversely as the relative velocity;* therefore, the moving force of the wheel must be in the simple direct ratio of the relative velocity. Q. E. D.

'Or the proposition is manifest from this consideration; that, while the stream is invariable, whatever be the velocity of the wheel, the same number of particles or quantity of the fluid, must strike it somewhere or other in a given time; consequently the variation of force is *only* on account of the varied impingent velocity of the same body, occasioned by a change of motion in the wheel; that is, the momentum is as the relative velocity.'

'Now, this true principle substituted for the erroneous one in use, will bring the theory to agree remarkably with the notable experiments of the ingenious Smeaton, before mentioned, published in the Philosophical Transactions of the Royal Society of London for the year 1751, vol. 51, for which the honorary annual medal was adjudged by the society, and presented to the author by their president. An instance or two of the importance of this correction may be adduced as below.'

PROP. II.

'The velocity of a wheel, moved by the impact of a stream, must be half the velocity of the fluid, to produce the greatest possible effect.—For let

$V =$ the velocity, $m =$ the momentum of the fluid;
$v =$ the velocity, $p =$ the power of the wheel.

Then $V - v =$ the relative velocity, by def. 3d;

and as $V : V - v :: m : \frac{m}{V} \times \overline{V - v} = p$ (prop. 1);

this multiplied by v, gives $pv = \frac{m}{V} \times \overline{Vv - v^2} =$ a maximum; hence $Vv - v^2 =$ a maximum, and its fluxion (v being the variable quantity) is $V\dot{v} - 2v\dot{v} = 0$; therefore $v = \frac{1}{2}V$, that is, the velocity of the wheel = half that of the fluid, at the place of impact, when the effect is a maximum. Q. E. D.'

'The usual theory gives $v = \frac{1}{3}V$; where the error is not less than one third of the true velocity of the wheel.'

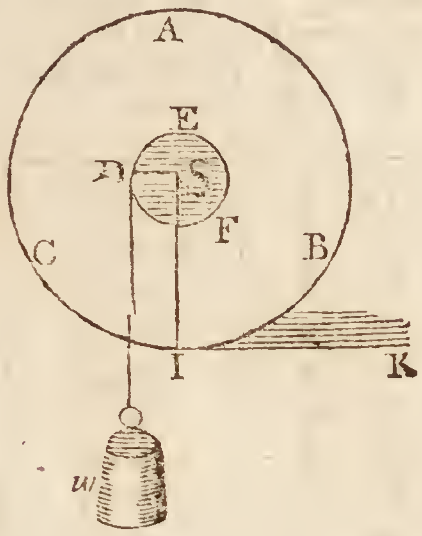
'This proposition is applicable to undershot wheels, and corresponds with the accurate experiments before cited, as appears from the author's conclusion (Philos. Trans. for 1776, pa. 457), viz, "The velocity of the" wheel, which according to M. Parent's determina-" tion, adopted by Desaguliers and Maclaurin, ought to" be no more than one third of that of the water, varies" at the maximum in the experiments of table 1, be-" tween one third and one half; but in all the cases" there related, in which the most work is performed" in proportion to the water expended, and which ap-" proach the nearest to the circumstances of great" works when properly executed, the maximum lies" much nearer one half than one third, *one half seeming*" *to be the true maximum,* if nothing were lost by the" resistance of the air, the scattering of the water car-" ried up by the wheel, &c." Thus he fully shews the common theory to have been very defective; but, I believe, none have since pointed out wherein the deficiency lay, nor how to correct it; and now we see the agreement of the true theory with the result of his experiments.' For another problem,

PROB. III.

'Given, the momentum (m) and velocity (V) of the fluid at I, the place of impact; the radius ($R =$ IS) of the wheel ABC; the radius ($r =$ DS) of the small wheel DEF on the same axle or shaft; the weight (w) or resistance to be overcome at D, and the friction (f) or force necessary to move the wheel without the weight; required the velocity (v) of the wheel &c."

'Here we have $V : V - v :: m : m \times \frac{V - v}{V} =$ the acting force at I in the direction KI, as before (prop. 2). Now $R : r :: w : \frac{rw}{R} =$ the power at

at I neceffary to counterpoife the weight w; hence $\frac{rw}{R} + f =$ the whole refiftance oppofed to the action



of the fluid at I; which deducted from the moving force, leaves $m \times \frac{V - v}{V} - \frac{rw}{R} - f =$ the accelerating force of the machine; which, when the motion becomes uniform, will be evanefcent or $= 0$; therefore $m \times \frac{V - v}{V} = \frac{rw}{R} + f$, which gives

$v = V \times \overline{1 - \frac{rw}{mR} - \frac{f}{m}} =$ the true velocity required; or, if we reject the friction, then

$v = V \times \overline{1 - \frac{rw}{mR}}$ is the theorem for the velocity of the wheel. This, by the common theory, would be $v = V \times 1 - \sqrt{\frac{rw}{mR}}$, which is too little by

$V\sqrt{\frac{rw}{mR}} - V\frac{rw}{mR}$. No wonder why we have hitherto derived fo little advantage from the theory.'

'Corol. 1. If the weight (w) or refiftance be required, fuch as juft to admit of that velocity which would produce the greateft effect; then, by fubftituting $\frac{1}{2}V$ for its equivalent v (by prop. 2), we have $\frac{1}{2}V = V \times \overline{1 - \frac{rw}{mR} - \frac{f}{m}}$; hence $w = \frac{\frac{1}{2}m - f}{r} \times R$; or, if $f = 0$, $w = \frac{mR}{2r}$; but theorifts make this $\frac{4mR}{9r}$, where the error is $\frac{mR}{18r}$.'

'Corol. 2. We have alfo $r = \frac{\frac{1}{2}m - f}{w} \times R$; or, rejecting friction, $r = \frac{mR}{2w}$, when the greateft effect is produced, inftead of $r = \frac{4mR}{9w}$, as has been fuppofed: this is an important theorem in the conftruction of mills.'

In the fame volume of the American Tranfactions, pa. 185, is another ingenious paper, by the fame author, on the power and machinery of Dr. Barker's Mill, as improved by Mr. James Rumfey, with a defcription of it. This is a Mill turned by the refifting force of a ftream of water that iffues from an orifice, the rotatory part, in which that orifice is, being impelled the contrary way by its reaction againft the ftream that iffues from it.

Mr. Ferguson has given the following directions for conftructing water mills in the beft manner; with a table of the feveral correfponding dimenfions proper to a great variety of perpendicular falls of the water.

When the float-boards of the water-wheel move with a 3d part of the velocity of the water that acts upon them, the water has the greateft power to turn the Mill: and when the millftone makes about 60 turns in a minute, it is found to perform its work the beft: for, when it makes but about 40 or 50, it grinds too flowly; and when it makes more than 70, it heats the meal too much, and cuts the bran fo fmall that a great part of it mixes with the meal, and cannot be feparated from it by fifting or boulting. Confequently the utmoft perfection of mill-work lies in making the train fo as that the millftone fhall make about 60 turns in a minute when the water wheel moves with a 3d part of the velocity of the water. To have it fo, obferve the following rules:

1. Meafure the perpendicular height of the fall of water, in feet, above the middle of the aperture, where it is let out to act by impulfe againft the float-boards on the loweft fide of the underfhot wheel.

2. Multiply that height of the fall in feet by the conftant number $64\frac{1}{3}$, and extract the fquare root of the product, which will be the velocity of the water at the bottom of the fall, or the number of feet the water moves per fecond.

3. Divide the velocity of the water by 3; and the quotient will be the velocity of the floats of the wheel in feet per fecond.

4. Divide the circumference of the wheel in feet, by the velocity of its floats; and the quotient will be the number of feconds in one turn or revolution of the great water-wheel, on the axis of which is fixed the cog-wheel that turns the trundle.

5. Divide 60 by the number of feconds in one turn of the water-wheel or cog-wheel; and the quotient will be the number of turns of either of thefe wheels in a minute.

6. Divide 60 (the number of turns the millftone ought to have in a minute) by the abovefaid number of turns; and the quotient will be the number of turns the millftone ought to have for one turn of the water or cog-wheel. Then,

7. As the required number of turns of the millftone in a minute is to the number of turns of the cog-wheel in a minute, fo muft the number of cogs in the wheel be to the number of ftaves or rounds in the trundle on the axis of the millftone, in the neareft whole number that can be found.

By thefe rules the following table is calculated; in which, the diameter of the water-wheel is fuppofed 18 feet, and confequently its circumference $56\frac{4}{7}$ feet, and the diameter of the millftone is 5 feet.

The Mill-Wright's Table.

Perpendicular height of the fall of water.	Velocity of the water in feet per second.	Velocity of the wheel in feet per second.	Number of turns of the wheel in a minute.	Required n°. of turns of the millstone for each turn of the wheel.	Nearest number of cogs and staves for that purpose. Cogs.	Staves.	Number of turns of the millstone for one turn of the wheel by these cogs and staves.	Number of turns of the millstone in a minute by these cogs and staves.
1	8·02	2·67	2·83	21·20	127	6	21·17	59·91
2	11·40	3·78	4·00	15·00	105	7	15·00	60·00
3	13·89	4·63	4·91	12·22	98	8	12·25	60·14
4	16·04	5·35	5·67	10·58	95	9	10·56	59·87
5	17·93	5·98	6·34	9·46	85	9	9·44	59·84
6	19·64	6·55	6·94	8·64	78	9	8·66	60·10
7	21·21	7·07	7·50	8·00	72	9	8·00	60·00
8	22·68	7·56	8·02	7·48	67	9	7·44	59·67
9	24·05	8·02	8·51	7·05	70	10	7·00	59·57
10	25·35	8·45	8·97	6·69	67	10	6·70	60·09
11	26·59	8·86	9·40	6·38	64	10	6·40	60·16
12	27·77	9·26	9·82	6·11	61	10	6·10	59·90
13	28·91	9·64	10·22	5·87	59	10	5·80	60·18
14	30·00	10·00	10·60	5·66	56	10	5·60	59·36
15	31·05	10·35	10·99	5·46	55	10	5·40	60·48
16	32·07	10·69	11·34	5·29	53	10	5·30	60·10
17	33·06	11·02	11·70	5·13	51	10	5·10	59·67
18	34·02	11·34	12·02	4·99	50	10	5·00	60·10
19	34·95	11·65	12·37	4·85	49	10	4·80	60·61
20	35·86	11·92	12·68	4·73	47	10	4·70	59·59

For the theory and construction of Wind-mills, see Wind-*mill*.

MILLION, the number of ten hundred thousand, or a thousand times a thousand.

MINE, in Fortification &c, is a subterraneous canal or passage, dug under any place or work intended to be blown up by gunpowder. The passage of a mine leading to the powder is called the *Gallery*; and the extremity, or place where the powder is placed, is called the *Chamber*. The line drawn from the centre of the chamber perpendicular to the nearest surface, is called the *Line of least Resistance*; and the pit or hole, made by the mine when sprung, or blown up, is called the *Excavation*.

The Mines made by the besiegers in the attack of a place, are called simply *Mines*; and those made by the besieged, *Counter-mines*.

The fire is conveyed to the Mine by a pipe or hose, made of coarse cloth, of about an inch and half in diameter, called *Saucisson*, extending from the powder in the chamber to the beginning or entrance of the gallery, to the end of which is fixed a match, that the miner who sets fire to it may have time to retire before it reaches the chamber.

It is found by experiments, that the figure of the excavation made by the explosion of the powder, is nearly a paraboloid, having its focus in the centre of the powder, and its axis the line of least resistance; its diameter being more or less according to the quantity of the powder, to the same axis, or line of least resistance. Thus, M. Belidor lodged seven different quantities of powder in as many different mines, of the same depth, or line of least resistance 10 feet; the charges and greatest diameters of the excavation, measured after the explosion, were as follow:

	Powder.	Diam.
1st	120lb	$22\frac{2}{3}$ feet
2d	160	26
3d	200	29
4th	240	$31\frac{1}{4}$
5th	280	$33\frac{1}{2}$
6th	320	36
7th	360	38

From which experiments it appears that the excavation, or quantity of earth blown up, is in the same proportion with the quantity of powder; whence the charge of powder necessary to produce any other proposed effect, will be had by the rule of Proportion.

Mine-*Dial*, is a box and needle, with a brass ring divided into 360 degrees, with several dials graduated upon it, commonly made for the use of miners.

MINUTE, is the 60th part of a degree, or of an hour. The minutes of a degree are marked with the acute accent, thus ′; the seconds by two, ″; the thirds by three, ‴. The minutes, seconds, thirds, &c, in time, are sometimes marked the same way; but, to avoid confusion, the better way is, by the initials of the words; as minutes ^{m}, seconds ^{s}, thirds ^{t}, &c.

Minute, in Architecture, usually denotes the 60th part of a module, but sometimes only the 30th part.

MIRROR, a ſpeculum, looking-glaſs, or any poliſhed body, whoſe uſe is to form the images of diſtinct objects by reflexion of the rays of light.

Mirrors are either plane, convex, or concave. The firſt ſort reflects the rays of light in a direction exactly ſimilar to that in which they fall upon it, and therefore repreſents bodies of their natural magnitude. But the convex ones make the rays diverge much more than before reflexion, and therefore greatly diminiſh the images of thoſe objects which they exhibit: while the concave ones, by collecting the rays into a focus, not only magnify the objects they ſhew, but will alſo burn very fiercely when expoſed to the rays of the ſun; and hence they are commonly known by the name of *burning Mirrors.*

In ancient times the Mirrors were made of ſome kind of metal; and from a paſſage in the Moſaic writings we learn, that the Mirrors uſed by the Jewiſh women, were made of braſs; a practice doubtleſs learned from the Egyptians.

Any kind of metal, when well poliſhed, will reflect very powerfully; but of all others, ſilver reflects the moſt, though it has always been too expenſive a material for common uſe. Gold is alſo very powerful; and all metals, or even wood, gilt and poliſhed, will act very powerfully as burning Mirrors. Even poliſhed ivory, or ſtraw nicely plaited together, will form Mirrors capable of burning, if on a large ſcale.

Since the invention of glaſs, and the application of quickſilver to it, have become generally known, it has been univerſally employed for thoſe plane Mirrors uſed as ornaments to houſes; but in making reflecting teleſcopes they have been found much inferior to metallic ones. It does not appear however that the ſame ſuperiority belongs to the metallic burning Mirrors, conſidered merely as burning ſpeculums; ſince the Mirror with which Mr. Macquer melted platina, though only 22 inches diameter, and made of quickſilvered glaſs, produced much greater effects than M. Villette's metal ſpeculum, which was of a much larger ſize. It is very probable, however, that M. Villette's Mirror was not ſo well poliſhed as it ought to have been; as the art of preparing the metal for taking the fineſt poliſh, has but lately been diſcovered, and publiſhed in the Philoſ. Tranſactions, by Dr. Mudge of Plymouth, and, after him, by Mr. Edwards, Dr. Herſchel, &c.

Some of the more remarkable laws and phenomena of plane Mirrors, are as follow:

1. A ſpectator will ſee his image of the ſame ſize, and erect, but reverſed as to right and left, and as far beyond the ſpeculum as he is before it. As he moves to or from the ſpeculum, his image will, at the ſame time, move towards or from the ſpeculum alſo on the other ſide. In like manner if, while the ſpectator is at reſt, an object be in motion, its image behind the ſpeculum will be ſeen to move at the ſame rate. Alſo when the ſpectator moves, the images of objects that are at reſt will appear to approach or recede from him, after the ſame manner as when he moves towards real objects.

2. If ſeveral Mirrors, or ſeveral fragments or pieces of Mirrors, be all diſpoſed in the ſame plane, they will only exhibit an object once.

3. If two plane Mirrors, or ſpeculums, meet in any angle, the eye, placed within that angle, will ſee the image of an object placed within the ſame, as often repeated as there may be perpendiculars drawn determining the places of the images, and terminated without the angle. Hence, as the more perpendiculars, terminated without the angle, may be drawn as the angle is more acute; the acuter the angle, the more numerous the images. Thus, Z. Traber found, at an angle of one-3d of a circle, the image was repreſented twice, at $\frac{1}{4}$th thrice, at $\frac{1}{6}$th five times, and at $\frac{1}{12}$th eleven times.

Farther, if the Mirrors be placed upright, and ſo contracted; or if you retire from them, or approach to them, till the images reflected by them coaleſce, or run into one, they will appear monſtrouſly diſtorted. Thus, if they be at an angle ſomewhat greater than a right one, the image of one's face will appear with only one eye; if the angle be leſs than a right one, you will ſee 3 eyes, 2 noſes, 2 mouths, &c. At an angle ſtill leſs, the body will have two heads. At an angle ſomewhat greater than a right one, at the diſtance of 4 feet, the body will be headleſs, &c. Again, if the Mirrors be placed, the one parallel to the horizon, the other inclined to it, or declined from it, it is eaſy to perceive that the images will be ſtill more romantic. Thus, one being declined from the horizon to an angle of 144 degrees, and the other inclined to it, a man ſees himſelf ſtanding with his head to another's feet.

Hence it appears how Mirrors may be managed in gardens, &c, ſo as to convert the images of thoſe near them into monſters of various kinds; and ſince glaſs Mirrors will reflect the image of a lucid object twice or thrice, if a candle, &c, be placed in the angle between two Mirrors, it will be multiplied a great number of times.

Laws of Convex Mirrors.

1. In a ſpherical convex Mirror, the image is leſs than the object. And hence the uſe of ſuch Mirrors in the art of painting, where objects are to be repreſented leſs than the life.

2. In a convex Mirror, the more remote the object, the leſs its image; alſo the ſmaller the Mirror, the leſs the image.

3. In a convex Mirror, the right hand is turned to the left, and the left to the right; and magnitudes perpendicular to the Mirror appear inverted.

4. The image of a right line, perpendicular to the Mirror, is a right line; but that of a right line oblique or parallel to the Mirror, is convex.

5. Rays reflected from a convex Mirror, diverge more than if reflected from a plane Mirror; and the ſmaller the ſphere, the more the rays diverge.

Laws of Concave Mirrors.

The effects of concave Mirrors are, in general, the reverſe of thoſe of convex ones; rays being made to converge more, or diverge leſs than in plane Mirrors; the image is magnified, and the more ſo as the ſphere is ſmaller; &c, &c.

MITRE, in Architecture, is the workmen's term for an angle that is juſt 45 degrees, or half a right angle. And if the angle be the half of this, or a quarter of a right angle, they call it a *half-mitre.*

Mixt *Angle*, or *Figure*, is one contained by both right and curved lines.

Mixt *Number*, is one that is partly an integer, and partly a fraction; as $3\frac{1}{2}$.

Mixt *Ratio*, or *Proportion*, is when the sum of the antecedent and consequent is compared with the difference of the antecedent and consequent;

$$\text{as if} \left\{ \begin{array}{l} 4 : 3 :: 12 : 9 \\ a : b :: c : d \end{array} \right.$$

$$\text{then} \left\{ \begin{array}{l} 7 \quad : \quad 1 \quad :: \quad 21 \quad : \quad 3 \\ a+b : a-b :: c+d : c-d. \end{array} \right.$$

MOAT, in Fortification, a deep trench dug round a town or fortress, to be defended, on the outside of the wall, or rampart.

The breadth and depth of a Moat often depend on the nature of the soil; according as it is marshy, rocky, or the like. The brink of the Moat next the rampart, is called the Scarp; and the opposite side, the Counterscarp.

Dry Moat, is one that is without water; which ought to be deeper than one that has water, called a Wet Moat. A Dry Moat, or one that has a little water, has often a small notch or ditch run all along the middle of its bottom, called a Cuvette.

Flat-bottomed Moat, is that which has no sloping, its corners being somewhat rounded.

Lined Moat, is that whose scarp and counterscarp are cased with a wall of mason's work lying aslope.

MOBILE, *Primum*, in the Ancient Astronomy, was a 9th heaven, or sphere, conceived above those of the planets and fixed stars. It was supposed that this was the first mover, and carried all the lower spheres about with it; by its rapidity communicating to them a motion carrying them round in 24 hours. But the diurnal apparent revolution of the heavens is now better accounted for, by the rotation of the earth on its axis, without the assistance of any such Primum Mobile.

MOBILITY, an aptitude or facility to be moved.

The Mobility of Mercury is owing to the smallness and sphericity of its particles; and these also render its fixation so difficult.

The hypothesis of the Mobility of the earth is the most plausible, and is universally admitted by the later astronomers.

Pope Paul V. appointed commissioners to examine the opinion of Copernicus touching the Mobility of the earth. The result of their enquiry was, a prohibition to assert, not that the Mobility was possible, but that it was really true: that is, they allowed the Mobility of the earth to be held as an hypothesis, which gives an easy and sensible solution of the phenomena of the heavenly motions; but forbade the Mobility of the earth to be maintained as a thesis, or real effective thing; because they conceived it contrary to Scripture.

MODILLIONS, small inverted consoles under the soffit or bottom of the drip, or of the corniche, seeming to support the projecture of the larmier, in the Ionic, Composite, and Corinthian orders.

MODULE, or Little Measure, in Architecture, a certain measure, taken at pleasure, for regulating the proportions of columns, and the symmetry or distribution of the whole building. Architects usually choose the diameter, or the semidiameter, of the bottom of the column, for their Module; which they subdivide into minutes; for estimating all the other parts of the building by.

MOINEAU, a flat bastion raised before a curtin, when it is too long, and the bastions of the angles too remote to be able to defend one another. Sometimes the Moineau is joined to the curtin, and sometimes it is divided from it by a moat. Here musquetry are placed to fire each way.

MOLYNEUX (William), an excellent mathematician and astronomer, was born at Dublin in 1656. After the usual grammar education, which he had at home, he was entered of the university of that city. Here he distinguished himself by the probity of his manners, as well as by the strength of his parts; and having made a remarkable progress in academical learning, and particularly in the new philosophy, as it was then called, after four years spent in this university, he was sent over to London, where he was admitted into the Middle Temple in 1675. Here he spent three years, in the study of the laws of his country. But the bent of his genius lay strongly toward mathematical and philosophical studies; and even at the university he conceived a dislike to scholastic learning, and fell into the methods of lord Bacon.

Returning to Ireland in 1678, he shortly after married Lucy the daughter of Sir William Domville, the king's attorney-general. Being master of an easy fortune, he continued to indulge himself in prosecuting such branches of natural and experimental philosophy as were most agreeable to his fancy; in which astronomy having the greatest share, he began, about 1681, a literary correspondence with Mr. Flamsteed, the king's astronomer, which he kept up for several years. In 1683 he formed a design of erecting a Philosophical Society at Dublin, in imitation of the Royal Society at London; and, by the countenance and encouragement of Sir William Petty, who accepted the office of president, began a weekly meeting that year, when our author was appointed their first secretary.

Mr. Molyneux's reputation for learning recommended him, in 1684, to the notice and favour of the first great duke of Ormond, then lord-lieutenant of Ireland; by whose influence chiefly he was appointed that year, jointly with sir William Robinson, surveyor-general of the king's buildings and works, and chief engineer.

In 1685, he was chosen fellow of the Royal Society at London; and that year he was sent by the government to view the most considerable fortresses in Flanders. Accordingly he travelled through that country and Holland, with part of Germany and France; and carrying with him letters of recommendation from Flamsteed to Cassini, he was introduced to him, and others, the most eminent astronomers in the several places through which he passed.

Soon after his return from abroad, he printed at Dublin, in 1686, his *Sciothericum Telescopium*, containing a Description of the Structure and Use of a Telescopic Dial, invented by him: another edition of which was published at London in 1700.

In 1688 the Philosophical Society of Dublin was broken up and dispersed by the confusion of the times.

Mr.

Mr. Molyneux had diſtinguiſhed himſelf as a Member of it from the beginning, and preſented ſeveral diſcourſes upon curious ſubjects; ſome of which were tranſmitted to the Royal Society at London, and afterwards printed in the Philoſophical Tranſactions. In 1689, among great numbers of other Proteſtants, he withdrew from the diſturbances in Ireland, occaſioned by the ſeverities of Tyrconnel's government; and after a ſhort ſtay at London, he fixed himſelf with his family at Cheſter. In this retirement, he employed himſelf in putting together the materials he had ſome time before prepared for his *Dioptrics*, in which he was much aſſiſted by Mr. Flamſteed; and in Auguſt 1690, he went to London to put it to the preſs, where the ſheets were reviſed by Dr. Halley, who, at our author's requeſt, gave leave for printing, in the appendix, his celebrated Theorem for finding the Foci of Optic Glaſſes. Accordingly the book came out, 1692, in 4to, under the title of "*Dioptrica Nova*: a Treatiſe of Dioptrics, in two parts; wherein the various effects and appearances of ſpherical glaſſes, both convex and concave, ſingle and combined, in teleſcopes and microſcopes, together with their uſefulneſs in many concerns of human life, are explained." He gave it the title of *Dioptrica Nova*, both becauſe it was almoſt wholly new, very little being borrowed from other writers, and becauſe it was the firſt book that appeared in Engliſh upon the ſubject. The work contains ſeveral of the moſt generally uſeful propoſitions for practice, demonſtrated in a clear and eaſy manner, for which reaſon it was for many years uſed by the artificers: and the ſecond part is very entertaining, eſpecially in the hiſtory which he gives of the ſeveral optical inſtruments, and of the diſcoveries made by them.

Before he left Cheſter he loſt his lady, who died ſoon after ſhe had brought him a ſon. Illneſs had deprived her of her eye-ſight 12 years before, that is, ſoon after her marriage; from which time ſhe had been very ſickly, and afflicted with great pains in her head.

As ſoon as the public tranquillity was ſettled in his native country, he returned home; and, upon the convening of a new parliament in 1692, was choſen one of the repreſentatives for the city of Dublin. In the next parliament, in 1695, he was choſen to repreſent the univerſity there, and continued to do ſo to the end of his life; that learned body having lately conferred on him the degree of doctor of laws. He was likewiſe nominated by the lord-lieutenant one of the commiſſioners for the forfeited eſtates, to which employment was annexed a ſalary of 500l. a year; but looking upon it as an invidious office, he declined it.

In 1698, he publiſhed "The Caſe of Ireland ſtated, in regard to its being bound by Acts of Parliament made in England:" in which it is ſuppoſed he has delivered all, or moſt, that can be ſaid upon this ſubject, with great clearneſs and ſtrength of reaſoning.

Among many learned perſons with whom he maintained correſpondence and friendſhip, Mr. Locke was in a particular manner dear to him, as appears from their letters. In the above mentioned year, which was the laſt of our author's life, he made a journey to England, on purpoſe to pay a viſit to that great man; and not long after his return to Ireland, he was ſeized with a fit of the ſtone, which terminated his exiſtence.

Beſides the three works already mentioned, viz, the *Sciothericum Teleſcopium*, the *Dioptrica Nova*, and the *Caſe of Ireland ſtated*; he publiſhed a great number of pieces in the Philoſophical Tranſactions, which are contained in the volumes 14, 15, 16, 18, 19, 20, 21, 22, 23, 26, 29, ſeveral papers commonly in each volume.

Molyneux *(Samuel)*, ſon of the former, was born at Cheſter in July 1689; and educated with great care by his father, according to the plan laid down by Locke on that ſubject. When his father died, he fell under the management of his uncle, Dr. Thomas Molyneux, an excellent ſcholar and phyſician at Dublin, and alſo an intimate friend of Mr. Locke, who executed his truſt ſo well, that Mr. Molyneux became afterwards a moſt polite and accompliſhed gentleman, and was made ſecretary to George the 3d when prince of Wales. Aſtronomy and Optics being his favourite ſtudies, as they had been his father's, he projected many ſchemes for the advancement of them, and was particularly employed in the years 1723, 1724, and 1725, in perfecting the method of making teleſcopes; one of which inſtruments, of his own making, he had preſented to John the 5th, king of Portugal.

Being ſoon after appointed a commiſſioner of the admiralty, he became ſo engaged in public affairs, that he had not leiſure to purſue thoſe enquiries any farther, as he intended. He therefore gave his papers to Dr. Robert Smith, profeſſor of aſtronomy at Cambridge, whom he invited to make uſe of his houſe and apparatus of inſtruments, in order to finiſh what he had left imperfect. But Mr. Molyneux dying ſoon after, Dr. Smith loſt the opportunity; he however ſupplied what was wanting from M. Huygens and others, and publiſhed the whole in his "Complete Treatiſe of Optics."

MOMENT, in Time, is ſometimes taken for an extremely ſmall part of duration; but, more properly, it is only an inſtant or termination or limit in time, like a point in geometry. Maclaurin's Fluxions, vol. 1, pa. 245.

Moments, in the new Doctrine of Infinites, denote the indefinitely ſmall parts of quantity; or they are the ſame with what are otherwiſe called infiniteſimals, and differences, or increments and decrements; being the momentary increments or decrements of quantity conſidered as in a continual flux.

Moments are the generative principles of magnitude: they have no determined magnitude of their own; but are only inceptive of magnitude.

Hence, as it is the ſame thing, if, inſtead of theſe Moments, the velocities of their increaſes and decreaſes be made uſe of, or the finite quantities that are proportional to ſuch velocities; the method of proceeding which conſiders the motions, changes, or fluxions of quantities, is denominated, by Sir Iſaac Newton, the Method of Fluxions.

Leibnitz, and moſt foreigners, conſidering theſe infinitely ſmall parts, or infiniteſimals, as the differences of two quantities; and thence endeavouring to find the differences of quantities, i. e. ſome Moments, or quantities indefinitely ſmall, which taken an infinite number of times ſhall equal given quantities; call theſe Moments

ments, Differences; and the method of procedure, the Differential Calculus.

Moment, or *Momentum*, in Mechanics, is the same thing with Impetus, or the quantity of motion in a moving body.

In comparing the motions of bodies, the ratio of their Momenta is always compounded of the quantity of matter and the celerity of the moving body: so that the momentum of any such body, may be considered as the rectangle or product of the quantity of matter and the velocity of the motion. As, if b denote any body, or the quantity or mass of matter, and v the velocity of its motion; then bv will express, or be proportional to, its Momentum m. Also if B be another body, and V its velocity; then its Momentum M, is as BV. So that, in general, $M : m :: BV : bv$, i. e. the Momenta are as the products of the mass and velocity. Hence, if the Momenta M and m be equal, then shall the two products BV and bv be equal also; and consequently $B : b :: v : V$, or the bodies will be to each other in the inverse or reciprocal ratio of their velocities; that is, either body is so much the greater as its velocity is less. And this force of Momentum is of a different kind from, and incomparably greater than, any mere dead weight, or pressure, whatever.

The Momentum also of any moving body, may be considered as the aggregate or sum of all the Momenta of the parts of that body; and therefore when the magnitudes and number of particles are the same, and also moved with the same celerity, then will the Momenta of the wholes be the same also.

MONADES. Digits.

MONOCEROS, the *Unicorn*, one of the new constellations of the northern hemisphere, or one of those which Hevelius has added to the 48 old asterisms, and formed out of the stellæ informes, or those which were not comprized within the outlines of any of the others. In Hevelius's catalogue, the Unicorn contains 19 stars, but in the Britannic catalogue 31.

MONOCHORD, a musical instrument with only one string, used by the Ancients to try the variety and proportion of sounds. It was formed of a rule, divided and subdivided into several parts, on which there is a moveable string stretched over two bridges at the extremes of it. In the interval between these is a sliding or moveable bridge, by means of which, in applying it to the different divisions of the line, the sounds are found to bear the same proportion to each other, as the division of the line cut by the bridge. This instrument is also called the *harmonical canon*, or the *canonical rule*, because it serves to measure the degrees of gravity or acuteness. Ptolomy examines his harmonical intervals by the Monochord. When the chord was divided into two equal parts, so that the parts were as 1 to 1, they called them *unisons*; but if they were as 2 to 1, they called them *octaves* or *diapasons*; when they were as 3 to 2, they called them *diapentes*, or *fifths*; if they were as 4 to 3, they called them *diatessarons*, or *fourths*; if the parts were as 5 to 4, they called them *diton*, or *major-third*; but if they were as 6 to 5, they were called a *demi-diton*, or *minor-third*; and lastly, if the parts were as 24 to 25, a *demitone*, or *dieze*.

The Monochord, being thus divided, was properly what they called a system, of which there were many kinds, according to the different divisions of the Monochord.

Monochord is also used for any musical instrument consisting of only one chord or string. Such is the Trump-marine.

MONOMIAL, in Algebra, is a simple or single nomial, consisting of only one term; as a or ax, or a^2bx^3, &c.

MONOTRIGLYPH, a term in Architecture, denoting the space of one triglyph between two pilasters, or two columns.

MONSOON, a regular or periodical wind, that blows one way for 6 months together, and the contrary way the other 6 months of the year. These prevail in several parts of the eastern and southern oceans.

MONTH, the 12th part of the year, and is so called from the Moon, by whose motions it was regulated; being properly the time in which the moon runs through the zodiac. The lunar Month is either *illuminative*, *periodical*, or *synodical*.

Illuminative Month, is the interval between the first appearance of one new moon and that of the next following. As the moon appears sometimes sooner after one change than after another, the quantity of the Illuminative Month is not always the same. The Turks and Arabs reckon by this Month.

Lunar Periodical Month, is the time in which the moon runs through the zodiac, or returns to the same point again; the quantity of which is 27days 7hrs 43m. 8 sec.

Lunar Synodical Month, called also a Lunation, is the time between two conjunctions of the moon with the sun, or between two new moons; the quantity of which is 29 days, 12 hours, 44m. 3 sec. 11 thirds.

The ancient Romans used Lunar Months, and made them alternately of 29 and 30 days: They marked the days of each Month by three terms, viz, Calends, Nones, and Ides.

Solar Month, is the time in which the sun runs through one entire sign of the ecliptic, the mean quantity of which is 30 days 10 hours 29 min. 5 sec. being the 12th part of 365 ds. 5 hrs. 49 min. the mean solar year.

Astronomical or *Natural* Month, is that measured by some exact interval corresponding to the motion of the sun or moon. Such are the lunar and solar months above-mentioned.

Civil or *Common* Month, is an interval of a certain number of whole days, approaching nearly to the quantity of some astronomical month. These may be either lunar or solar. The

Civil Lunar Month, consists alternately of 29 and 30 days. Thus will two Civil Months be equal to astronomical ones, abating for the odd minutes; and so the new moon will be kept to the first day of such Civil Months for a long time together. This was the Month in Civil or common use among the Jews, Greeks, and Romans, till the time of Julius Cæsar. The

Civil Solar Month, consisted alternately of 30 and 31 days, excepting one Month of the twelve, which consisted only of 29 days, but every 4th year of 30 days. And this form of Civil Months was introduced by Julius Cæsar. Under Augustus, the 6th Month, till

till then from its place called Sextilis, received the name Auguſtus, now Auguſt, in honour of that prince; and, to make the compliment ſtill the greater, a day was added to it; which made it conſiſt of 31 days, though till then it had only contained 30 days; to compenſate for which, a day was taken from February, making it conſiſt of 28 days, and 29 every 4th year. And ſuch are the Civil or Calendar Months now uſed through Europe.

MOON, *Luna*, ☾, one of the heavenly bodies, being a ſatellite, or ſecondary planet to the earth, conſidered as a primary planet, about which ſhe revolves in an elliptic orbit, or rather the earth and Moon revolve about a common centre of gravity, which is as much nearer to the earth's centre than to the Moon's, as the maſs of the former exceeds that of the latter.

The mean time of a revolution of the Moon about the earth, from one new moon to another, when ſhe overtakes the ſun again, is 29d. 12h. 44m. 3s. 11th.; but ſhe moves once round her own orbit in 27d. 7h. 43m. 8s. moving about 2290 miles every hour; and turns once round her axis exactly in the time that ſhe goes round the earth, which is the reaſon that ſhe ſhews always the ſame ſide towards us; and that her day and night taken together are juſt as long as our lunar month.

The mean diſtance of the Moon from the earth is $60\frac{1}{2}$ radii, or $30\frac{1}{4}$ diameters, of the earth; which is about 240,000 miles. The mean excentricity of her orbit is $\frac{55}{1000}$, or $\frac{1}{18}$th nearly of her mean diſtance, amounting to about 13,000 miles.

The Moon's diameter is to that of the earth, as 20 to 73, or nearly as 3 to 11, or 1 to $3\frac{2}{3}$; and therefore it is equal to 2180 miles: her mean apparent diameter is $31' \, 16''\frac{1}{2}$, that of the ſun being $32' \, 12''$. The ſurface of the Moon is to the ſurface of the earth, as 1 to $13\frac{1}{3}$, or as 3 to 40; ſo that the earth reflects 13 times as much light upon the Moon, as ſhe does upon the earth; and the ſolid content to that of the earth, as 3 to 146, or as 1 to $48\frac{2}{3}$. The denſity of the Moon's body is to that of the earth, as 5 to 4; and therefore her quantity of matter to that of the earth, as 1 to 39 very nearly: the force of gravity on her ſurface, is to that on the earth, as 100 to 293. The Moon has little or no difference of ſeaſons; becauſe her axis is almoſt perpendicular to the ecliptic.

Phenomena and Phaſes of the MOON. The Moon being a dark, opaque, ſpherical body, only ſhining with the light ſhe receives from the ſun, hence only that half turned towards him, at any inſtant, can be illuminated, the oppoſite half remaining in its native darkneſs: then as the face of the Moon viſible on our earth, is that part of her body turned towards us; whence, according to the various poſitions of the Moon, with reſpect to the earth and ſun, we perceive different degrees of illumination; ſometimes a large and ſometimes a leſs portion of the enlightened ſurface being viſible: And hence the Moon appears ſometimes increaſing, then waning; ſometimes horned, then half round; ſometimes gibbous, then full and round. This may be eaſily illuſtrated by means of an ivory ball, which being before a candle in various poſitions, will preſent a greater or leſs portion of its illuminated hemiſphere to the view of the obſerver, according to its ſituation in moving it round the candle.

The ſame phaſes may be otherwiſe exhibited thus: Let S repreſent the ſun, T the earth, and ABCD &c the Moon's orbit. (Plate xv, fig. 3.) Now, when the Moon is at A, in conjunction with the ſun S, her dark ſide being entirely turned towards the earth, ſhe will be inviſible, as at *a*, and is then called the new Moon. When ſhe comes to her firſt octant at B, or has run through the 8th part of her orbit, a quarter of her enlightened hemiſphere will be turned towards the earth, and ſhe will then appear horned, as at *b*. When ſhe has run through the quarter of her orbit, and arrived at C, ſhe ſhews us the half of her enlightened hemiſphere, as at *c*, when it is ſaid ſhe is one half full. At D ſhe is in her 2d octant, and by ſhewing us more of her enlightened hemiſphere than at C, ſhe appears gibbous, as at *d*. At her oppoſition at E her whole enlightened ſide is turned towards the earth, when ſhe appears round, as at *e*, and ſhe is ſaid to be full; having increaſed all the way round from A to E. On the other ſide ſhe decreaſes again all the way from E to A: thus, in her 3d octant at F, part of her dark ſide being turned towards the earth, ſhe again appears gibbous, as at *f*. At G ſhe appears ſtill farther decreaſed, ſhewing again juſt one half of her illuminated ſide, as at *g*. But when ſhe comes to her 4th octant at H, ſhe preſents only a quarter of her enlightened hemiſphere, and ſhe again appears horned, as at *h*. And at A, having now completed her courſe, ſhe again diſappears, or becomes a new moon again, as at firſt. And the earth preſents all the very ſame phaſes to a ſpectator in the Moon, as ſhe does to us, but only in a contrary order, the one being full when the other changes, &c.

The Motions of the MOON are moſt of them very irregular, and very conſiderably ſo. The only equable motion ſhe has, is her revolution on her own axis, in the ſpace of a month, or time in which ſhe moves round the earth; which is the reaſon that ſhe always turns the ſame face towards us.

This expoſure of the ſame face is not ſo uniformly ſo however, but that ſhe turns ſometimes a little more of the one ſide, and ſometimes of the other, called the Moon's Libration; and alſo ſhews ſometimes a little more towards one pole, and ſometimes towards the other, by a motion like a kind of Wavering, or Vacillation. The former of theſe motions happens from this: the Moon's rotation on her axis is equable or uniform; while her motion in her orbit is unequal, being quickeſt when the Moon is in her perigee, and ſloweſt when in the apogee, like all other planetary motions; which cauſes that ſometimes more of one ſide is turned to the earth, and ſometimes of the other. And the other irregularity ariſes from this: that the axis of the Moon is not perpendicular, but a little inclined to the plane of her orbit: and as this axis maintains its paralleliſm, in the Moon's motion round the earth; it muſt neceſſarily change its ſituation, in reſpect of an obſerver on the earth; whence it happens that ſometimes the one, and ſometimes the other pole of the Moon becomes viſible.

The very orbit of the Moon is changeable, and does not always perſevere in the ſame figure: for though her orbit be elliptical, or nearly ſo, having the earth in one focus, the excentricity of the ellipſe is varied, being ſometimes increaſed, and ſometimes diminiſhed; viz, being

being greateſt when the line of the apſes coincides with that of the ſyzygies, and leaſt when theſe lines are at right angles to each other.

Nor is the apogee of the Moon without an irregularity; being found to move forward, when it coincides with the line of the ſyzygies; and backward, when it cuts that line at right angles. Neither is this progreſs or regreſs uniform; for in the conjunction or oppoſition, it goes briſkly forward; and in the quadratures, it either moves ſlowly forward, ſtands ſtill, or goes backward.

The motion of the nodes is alſo variable; being quicker and ſlower in different poſitions.

The Phyſical Cauſe of the MOON'S *Motion*, about the earth, is the ſame as that of all the primary planets about the ſun, and of the ſatellites about their primaries, viz, the mutual attraction between the earth and Moon.

As for the particular irregularities in the Moon's motion, to which the earth and other planets are not ſubject, they ariſe from the ſun which acts on, and diſturbs her in her ordinary courſe through her orbit; and are all mechanically deducible from the ſame great law by which her general motion is directed, viz, the law of gravitation and attraction. The other ſecondary planets, as thoſe of Jupiter, Saturn, &c, are alſo ſubject to the like irregularities with the Moon; as they are expoſed to the ſame perturbating or diſturbing force of the ſun; but their diſtance ſecures them from being ſo greatly affected as the Moon is, and alſo from being ſo well obſerved by us.

For a familar idea of this matter, it muſt firſt be conſidered, that if the ſun acted equally on the earth and Moon, and always in parallel lines, this action would ſerve only to reſtrain them in their annual motions round the ſun, and no way affect their actions on each other, or their motions about their common centre of gravity. But becauſe the Moon is nearer the ſun, in one half of her orbit, than the earth is, but farther off in the other half of her orbit; and becauſe the power of gravity is always leſs at a greater diſtance; it follows, that in one half of her orbit the Moon is more attracted than the earth towards the ſun, and leſs attracted than the earth in the other half: and hence irregularities neceſſarily ariſe in the motions of the Moon; the exceſs of attraction in the firſt caſe, and the defect in the ſecond, becoming a force that diſturbs her motion: and beſides, the action of the ſun, on the earth and Moon, is not directed in parallel lines, but in lines that meet in the centre of the ſun; which makes the effect of the diſturbing force ſtill the more complex and embarraſſing. And hence, as well as from the various ſituations of the Moon, ariſe the numerous irregularities in her motions, and the equations, or corrections, employed in calculating her places, &c.

Newton, as well as others, has computed the quantities of theſe irregularities, from their cauſes. He finds that the force added to the gravity of the Moon in her quadratures, is to the gravity with which ſhe would revolve in a circle about the earth, at her preſent mean diſtance, if the ſun had no effect on her, as 1 to $178\frac{29}{40}$: he finds that the force ſubducted from her gravity in the conjunctions and oppoſitions, is double of this quantity; and that the area deſcribed in a given time in the quarters, is to the area deſcribed in the ſame time in the conjunctions and oppoſitions, as 10973 to 11073: and he finds that, in ſuch an orbit, her diſtance from the earth in her quarters, would be to her diſtance in the conjunctions and oppoſitions, as 70 to 69. Upon theſe irregularities, ſee Maclaurin's Account of Newton's Diſcoveries, book 4, chap. 4; as alſo moſt books of aſtronomy. Other particulars relating to the Moon's motions, &c, have been ſtated as follow: The power of the Moon's influence, as to the tides, is to that of the ſun, as $6\frac{1}{3}$ to 1, according to Sir I. Newton; but different according to others.

As to the figure of the Moon, ſuppoſing her at firſt to have been a fluid, like the ſea, Newton calculates, that the earth's attraction would raiſe the water there near 90 feet high, as the attraction of the Moon raiſes our ſea 12 feet: whence the figure of the Moon muſt be a ſpheroid, whoſe greateſt diameter extended, will paſs through the centre of the earth; and will be longer than the other diameter, perpendicular to it, by 180 feet; and hence it comes to paſs, that we always ſee the ſame face of the Moon; for ſhe cannot reſt in any other poſition, but always endeavours to conform herſelf to this ſituation: Princip. lib. 3, prop. 38.

Newton eſtimates the mean apparent diameter of the Moon at 32′ 12″; as the ſun is 31′ 27″.

The denſity of the Moon he concludes is to that of the earth, as 9 to 5 nearly; and that the maſs, or quantity of matter, in the Moon, is to that of the earth, as 1 to 26 nearly.

The plane of the Moon's orbit is inclined to that of the ecliptic, and makes with it an angle of about 5 degrees: but this inclination varies, being greateſt when ſhe is in the quarters, and leaſt when in her ſyzygies.

As to the inequality of the Moon's motion, ſhe moves ſwifter, and by the radius drawn from her to the earth deſcribes a greater area in proportion to the time, alſo has an orbit leſs curved, and by that means comes nearer to the earth, in her ſyzygies or conjunctions, than in the quadratures, unleſs the motion of her eccentricity hinders it: which eccentricity is the greateſt when the Moon's apogee falls in the conjunction, but leaſt when this falls in the quadratures: her motion is alſo ſwifter in the earth's aphelion, than in its perihelion. The apogee alſo goes forward ſwifter in the conjunction, and goes ſlower at the quadratures: but her nodes are at reſt in the conjunctions, and recede ſwifteſt of all in the quadratures.

The Moon alſo perpetually changes the figure of her orbit, or the ſpecies of the ellipſe ſhe moves in.

There are alſo ſome other inequalities in the motion of this planet, which it is very difficult to reduce to any certain rule: as the velocities or horary motions of the apogee and nodes, and their equations, with the difference between the greateſt eccentricity in the conjunctions, and the leaſt in the quadratures; and that inequality which is called the Variation of the Moon. All theſe do increaſe and decreaſe annually, in a triplicate ratio of the apparent diameter of the ſun: and this variation is increaſed and diminiſhed in a duplicate ratio of the time between the quadratures; as is proved by Newton in many parts of his Principia.

He

He also found that the apogees in the Moon's syzygies, go forward in respect of the fixed stars, at the rate of 23′ each day; and backwards in the quadratures 16′½ per day: and therefore the mean annual motions he estimates at 40 degrees.

The gravity of the Moon towards the earth, is increased by the action of the sun, when the Moon is in the quadratures, and diminished in the syzygies: and, from the syzygies to the quadrature, the gravity of the Moon towards the earth is continually increased, and she is continually retarded in her motion: but from the quadrature to the syzygy, the Moon's motion is perpetually diminished, and the motion in her orbit is accelerated.

The Moon is less distant from the earth at the syzygies, and more at the quadratures.

As radius is to $\frac{3}{2}$ of the sine of double the Moon's distance from the syzygy, so is the addition of gravity in the quadratures, to the force which accelerates or retards the Moon in her orbit.

And as radius is to the sum or difference of $\frac{1}{2}$ the radius and $\frac{3}{2}$ the cosine of double the distance of the Moon from the syzygy, so is the addition of gravity in the quadratures, to the decrease or increase of the gravity of the Moon at that distance.

The apses of the Moon go forward when she is in the syzygies, and backward in the quadratures. But, in a whole revolution of the Moon, the progress exceeds the regress.

In a whole revolution, the apses go forward the fastest of all when the line of the apses is in the nodes; and in the same case they go back the slowest of all in the same revolution.

When the line of the apses is in the quadratures, the apses are carried in consequentia, the least of all in the syzygies; but they return the swiftest in the quadratures; and in this case the regress exceeds the progress, in one entire revolution of the Moon.

The eccentricity of the orbit undergoes various changes every revolution. It is the greatest of all when the line of the apses is in the syzygies, and the least when that line is in the quadratures.

Considering one entire revolution of the Moon, cæteris paribus, the nodes move in antecedentia swiftest of all when she is in the syzygies; then slower and slower, till they are at rest, when she is in the quadratures.

The line of nodes acquires successively all possible situations in respect of the sun; and every year it goes twice through the syzygies, and twice through the quadratures.

In one whole revolution of the Moon, the nodes go back very fast when they are in the quadratures; then slower till they come to rest, when the line of nodes is in the syzygies.

The inclination of the plane of the orbit is changed by the same force with which the nodes are moved; being increased as the Moon recedes from the node, and diminished as she approaches it.

The inclination of the orbit is the least of all when the nodes are come to the syzygies. For in the motion of the nodes from the syzygies to the quadratures, and in one entire revolution of the Moon, the force which increases the inclination exceeds that which diminishes it; therefore the inclination is increased; and it is the greatest of all when the nodes are in the quadratures.

The Moon's motion being considered in general: her gravity towards the earth is diminished coming near the sun, and the periodical time is the greatest; as also the distance of the Moon, cæteris paribus, the greatest when the earth is in the perihelion.

All the errors in the Moon's motion are something greater in the conjunction than in the opposition.

All the disturbing forces are inversely as the cube of the distance of the sun from the earth; which when it remains the same, they are as the distance of the Moon from the earth. Considering all the disturbing forces together, the diminution of gravity prevails.

The figure of the Moon's *path*, about the earth, is, as has been said, nearly an ellipse; but her path, in moving, together with the earth about the sun, is made up of a series or repetition of epicycloids, and is in every point concave towards the earth. See Maclaurin's Account of Newton's Discov. pa. 336, 4to. Ferguson's Astron. pa. 129, &c; and Rowe's Flux. pa. 225, edit. 2.

Astronomy of the Moon.

To determine the Periodical and Synodical Months; or, the period of the Moon's revolution about the earth, and the period between one opposition or conjunction and another.

In the middle of a lunar eclipse, the Moon is in opposition to the sun: compute therefore the time between two such eclipses, at some considerable distance of time from each other; and divide this by the number of lunations that have passed in the mean time; so shall the quotient be the quantity of the synodical month. Compute also the sun's mean motion during the time of this synodical month, which add to 360°. Then, as the sum is to 360°, so is the synodical to the periodical month.

For example, Copernicus observed two eclipses of the Moon, the one at Rome on November 6, 1500, at 12 at night, and the other at Cracow on August 1, 1523, at 4 h. 25 min. the dif. of meridians being 0 h. 29 min.: hence the quantity of the synodical month is thus determined:

2d Observ.	1523^y	237^d	4^h	25^m
1st Observ.	1500	310	0	29
Difference	22	292	3	56
Add intercalary days		5		
Exact interval - -	22	297	3	56

which divided by 282, the number of lunations in that time, gives the synodical month $29^d\ 12^h\ 41^m$.

From two other observations of eclipses, the one at Cracow, the other at Babylon, the same author determines more accurately the quantity of the synodical month to be $29^d\ 12^h\ 43^m$ &c; and from other observations, probably more accurate still, the same is fixed at $29^d\ 12^h\ 44^m$.

The sun's mean motion in that time 29° 6′ 24″ 18‴, added to 360°, gives the Moon's motion 389 6 24 18;

Therefore the periodical month is $27^d\ 7^h\ 43^m\ 5^s$.

According

According to the observations of Kepler, the mean synodical month is $29^d\ 12^h\ 44^m\ 3^s\ 2^{th}$, and the mean periodical month $27\ \ 7\ \ 43\ \ 8$

Hence, 1, the quantity of the periodical month being given, by the rule of three are found the Moon's diurnal or horary motion, &c: and thus may tables of the mean motion of the Moon be constructed.

2. If the mean diurnal motion of the sun be subtracted from that of the Moon, the remainder will give the Moon's diurnal motion from the sun: and thus may a table of this motion be constructed.

3. Since the Moon is in the node at the time of a total eclipse, if the sun's place be found for that time, and 6 signs be added to the same, the sum will give the place of that node.

4. By comparing the ancient observations with the modern, it appears, that the nodes have a motion, and that they proceed in antecedentia, or backwards from Taurus to Aries, from Aries to Pisces, &c. Therefore if the diurnal motion of the nodes be added to the Moon's diurnal motion, the sum will be the motion of the Moon from the node; and thence by the rule of three, may be found in what time the Moon goes 360° from the dragon's head, or ascending node, or in what time she goes from, and returns to it; that is, the quantity of the Dracontic Month.

5. If the motion of the apogee be subtracted from the mean motion of the Moon, the remainder will be the Moon's mean motion from the apogee; and hence, by the rule of three, the quantity of the Anomalistic Month is determined.

Thus, according to Kepler's observations,

The mean synodical month is	29^d	12^h	44^m	3^s	2^{th}
The periodical month	27	7	43	8	
The place of the apogee for the year 1700 Jan. 1 old style, was	11^s	8°	57′	1″	
The place of the ascending node	4	27	39	17	
Mean diurnal motion of the Moon		13	10	35	
Diurnal motion of the apogee			6	41	
Diurnal motion of the nodes			3	11	
Theref. diurnal mot. from the latter		13	13	46	
And the diurnal motion from the apogee		13	3	54	

Lastly, the eccentricity is 4362, of such parts as the semidiameter of the eccentric is 100,000.

To find nearly the MOON's *Age or Change.*

To the epact add the number and day of the month; their sum, abating 30 if it be above, is the Moon's age; and her age taken from 30, shews the day of the change.

The numbers of the months, or monthly epacts, are the Moon's age at the beginning of each month, when the solar and lunar years begin together; and are thus:

0	2	1	2	3	4	5	6	8	8	10	10
Jan.	Feb.	Mar.	Ap.	Ma.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.

For Ex. To find the Moon's age the 14th of Oct. 1783.

Here, the epact is	26
Number of the month	8
Day of the month	14
The sum is	48
Subtract or abate	30
Leaves Moon's age	18
Taken from	30
Days till the change	12
Answering to Oct.	26

To find nearly the MOON's *Southing*, or coming to the Meridian.

Take $\frac{4}{5}$ or $\frac{8}{10}$ of her age, for her southing nearly; after noon, if it be less than 12 hours; but if greater, the excess is the time after last midnight.

For Ex. Oct. 14, 1783;
The Moon's age is 18 days
$\frac{8}{10}$ of which is 14·4 or $14^h\ 24^m$
Subtract - 12 00
Rem. Moon's southing 2 24 in the morning.

Mr. Ferguson, in his Select Exercises, pa. 135 &c, has given very easy tables and rules for finding the new and full Moons near enough the truth for any common almanac. But the Nautical Almanac, which is now always published for several years before hand, in a great measure supersedes the necessity of these and other such contrivances.

Of the Spots and Mountains &c in the MOON.

The face of the Moon is greatly diversified with inequalities, and parts of different colours, some brighter and some darker than the other parts of her disc. When viewed through a telescope, her face is evidently diversified with hills and valleys; and the same is also shewn by the edge or border of the Moon appearing jagged, when so viewed, especially about the confines of the illuminated part when the Moon is either horned or gibbous.

The astronomers Florenti, Langreni, Hevelius, Grimaldi, Riccioli, Cassini, and De la Hire, &c, have drawn the face of the Moon as viewed through telescopes; noting all the more shining parts, and, for the better distinction, marking them with some proper name; some of these authors calling them after the names of philosophers, astronomers, and other eminent men; while others denominate them from the known names of the different countries, islands, and seas on the earth. The names adopted by Riccioli however are mostly followed, as the names of Hipparchus, Tycho, Copernicus, &c. Fig. 4, plate xv, is a pretty exact representation of the full Moon in her mean libration, with the numbers to the principal spots according to Riccioli, Cassini, Mayer, &c, which denote the names as in the following List of them: also the asterisk refers to one of the volcanoes observed by Herschel.

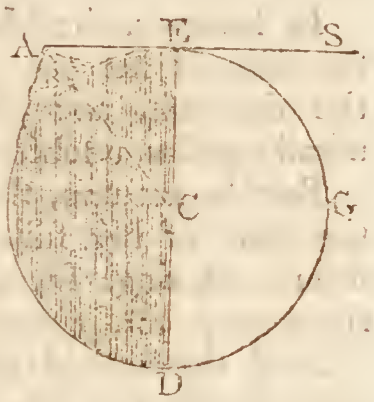
* Herschel's Volcano	12 Helicon
1 Grimaldi	13 Capuanus
2 Galileo	14 Bulliald
3 Aristarchus	15 Eratosthenes
4 Kepler	16 Timocharis
5 Gassendi	17 Plato
6 Schikard	18 Archimedes
7 Harpalus	19 Insula Sinus Medii
8 Heraclides	20 Pitatus
9 Lansberg	21 Tycho
10 Reinhold	22 Eudoxus
11 Copernicus	23 Aristotle

24. Ma-

24	Manilius	36	Cleomedes
25	Menelaus	37	Snell and Furner
26	Hermes	38	Petavius
27	Poſſidonius	39	Langrenus
28	Dionyſius	40	Taruntius
29	Pliny	A	Mare Humorum
30	Catharina Cyrillus, Theophilus	B	Mare Nubium
		C	Mare Imbrium
31	Fracaſtor	D	Mare Nectaris
32	Promontorium acutum, Cenſorinus	E	Mare Tranquilitatis
		F	Mare Serenitatis
33	Meſſala	G	Mare Fœcunditatis
34	Promontorium Somnii	H	Mare Criſium
35	Proclus		

That the ſpots in the Moon, which are taken for mountains and valleys, are really ſuch, is evident from their ſhadows. For in all ſituations of the Moon, the elevated parts are conſtantly found to caſt a triangular ſhadow in a direction from the ſun; and, on the contrary, the cavities are always dark on the ſide next the ſun, and illuminated on the oppoſite one; which is exactly conformable to what we obſerve of hills and valleys on the earth. And as the tops of theſe mountains are conſiderably elevated above the other parts of the ſurface; they are often illuminated when they are at a conſiderable diſtance from the confines of the enlightened hemiſphere, and by this means afford us a method of determining their heights.

Thus, let ED be the Moon's diameter, ECD the boundary of light and darkneſs; and A the top of a hill in the dark part beginning to be illuminated; with a teleſcope take the proportion of AE to the diameter ED; then there are given the two ſides AE, EC of a right-angled triangle ACE, the ſquares of which being added together give the ſquare of the third ſide AC, and the root extracted is that ſide itſelf; from which ſubtracting the radius BC, leaves AB the height of the mountain. In this way, Riccioli obſerved the top of the hill called St. Catherine, on the 4th day after the new moon, to be illuminated when it was diſtant from the confines of the enlightened hemiſphere about one 16th part of the Moon's diameter; and thence found its height muſt be near 9 miles.



It is probable however that this determination is too much. Indeed, Galileo makes AE to be only one 20th of ED, and Hevelius makes it only one 26th of ED; the former of theſe would give $5\frac{1}{2}$ miles, and the latter only $3\frac{1}{4}$ miles, for AB, the height of the mountain: and probably it ſhould be ſtill leſs than either of theſe.

Accordingly, they are greatly reduced by the obſervations of Herſchel, whoſe method of meaſuring them may be ſeen in the Philoſ. Tranſ. an. 1780, pa. 507. This gentleman meaſured the height of many of the lunar prominences, and draws at laſt the following concluſions:—"From theſe obſervations I believe it is evident, that the height of the lunar mountains in general is greatly over-rated; and that, when we have excepted a few, the generality do not exceed half a mile in their perpendicular elevation." And this is confirmed by the meaſurement of ſeveral mountains, as may be ſeen in the place above quoted.

As the Moon has on her ſurface mountains and valleys in common with the earth, ſome modern aſtronomers have diſcovered a ſtill greater ſimilarity, viz, that ſome of theſe are really volcanoes, emitting fire as thoſe on the earth do. An appearance of this kind was diſcovered ſome few years ago by Don Ulloa in an eclipſe of the ſun. It was a ſmall bright ſpot like a ſtar near the margin of the Moon, and which he at that time ſuppoſed to be a hole or valley with the ſun's light ſhining through it. Succeeding obſervations, however, have induced aſtronomers to attribute appearances of this kind to the eruption of volcanic fire; and Mr. Herſchel has particularly obſerved ſeveral eruptions of the lunar volcanos, the laſt of which he gives an account of in the Philoſ. Tranſ. for 1787. April 19, 10h. 36m. ſidereal time, I perceived, ſays he, three volcanos in different places of the dark part of the new Moon. Two of them are either already nearly extinct, or otherwiſe in a ſtate of going to break out; which perhaps may be decided next lunation. The third ſhews an actual eruption of fire or luminous matter: its light is much brighter than the nucleus of the comet which M. Mechain diſcovered at Paris the 10th of this month." The following night he found it burnt with greater violence; and by meaſurement he found that the ſhining or burning matter muſt be more than 3 miles in diameter; being of an irregular round figure, and very ſharply defined on the edges. The other two volcanos reſembled large faint nebulæ, that are gradually much brighter in the middle; but no well-defined luminous ſpot was diſcovered in them. He adds, "the appearance of what I have called the *actual fire*, or eruption of a volcano, exactly reſembled a ſmall piece of burning charcoal when it is covered by a very thin coat of white aſhes, which frequently adhere to it when it has been ſome time ignited; and it had a degree of brightneſs about as ſtrong as that with which a coal would be ſeen to glow in faint day-light."

It has been diſputed whether the Moon has any atmoſphere or not. The following arguments have been urged by thoſe who deny it.

1. The Moon, ſay they, conſtantly appears with the ſame brightneſs when our atmoſphere is clear; which could not be the caſe if ſhe were ſurrounded with an atmoſphere like ours, ſo variable in its denſity, and ſo often obſcured by clouds and vapours. 2. In an appulſe of the Moon to a ſtar, when ſhe comes ſo near it that a part of her atmoſphere comes between our eye and the ſtar, refraction would cauſe the latter to ſeem to change its place, ſo that the Moon would appear to touch it later than by her own motion ſhe would do. 3. Some philoſophers are of opinion, that becauſe there are no ſeas or lakes in the Moon, there is therefore no atmoſphere, as there is no water to be raiſed up in vapours.

But all theſe arguments have been anſwered by other aſtronomers in the following manner. It is denied that the Moon appears always with the ſame brightneſs, even when our atmoſphere appears equally clear. Hevelius relates, that he has ſeveral times found in

R 2 ſkies

ſkies perfectly clear, when even ſtars of the 6th and 7th magnitude were viſible, that at the ſame altitude of the Moon with the ſame elongation from the ſun, and with the ſame teleſcope, the Moon and her maculæ do not appear equally lucid, clear, and conſpicuous at all times; but are much brighter and more diſtinct at ſome times than at others. And hence it is inferred that the cauſe of this phenomenon is neither in our air, in the tube, in the Moon, nor in the ſpectator's eye; but muſt be looked for in ſomething exiſting about the Moon. An additional argument is drawn from the different appearances of the Moon in total eclipſes, which it is ſuppoſed are owing to the different conſtitutions of the lunar atmoſphere.

To the 2d argument Dr. Long replies, that Newton has ſhewn (Princip. prop. 37, cor. 5), that the weight of any body upon the Moon is but a third part of what the weight of the ſame would be upon the earth: now the expanſion of the air is reciprocally as the weight that compreſſes it; therefore the air ſurrounding the Moon, being preſſed together by a weight of one-third, or being attracted towards the centre of the Moon by a force equal only to one-third of that which attracts our air towards the centre of the earth, it thence follows, that the lunar atmoſphere is only one-third as denſe as that of the earth, which is too little to produce any ſenſible refraction of the ſtar's light. Other aſtronomers have contended, that ſuch refraction was ſometimes very apparent. Mr. Caſſini ſays, that he often obſerved that Saturn, Jupiter, and the fixed ſtars, had their circular figures changed into an elliptical one, when they approached either to the Moon's dark or illuminated limb, though they own that, in other occultations, no ſuch change could be obſerved. And, with regard to the fixed ſtars, it has been urged that, granting the Moon to have an atmoſphere of the ſame nature and quantity as ours, no ſuch effect as a gradual diminution of light ought to take place; at leaſt none that we could be capable of perceiving. At the height of 44 miles, our atmoſphere is ſo rare as to be incapable of refracting the rays of light: this height is the 180th part of the earth's diameter; but ſince clouds are never obſerved higher than 4 miles, it appears that the vapourous or obſcure part is only the 1980th part. The mean apparent diameter of the Moon is 31′ 29″, or 1889″: therefore the obſcure parts of her atmoſphere, when viewed from the earth, muſt ſubtend an angle of leſs than one ſecond; which ſpace is paſſed over by the Moon in leſs than two ſeconds of time. It can therefore hardly be expected that obſervation ſhould generally determine whether the ſuppoſed obſcuration takes place or not.

As to the 3d argument, it concludes nothing, becauſe it is not known that there is no water in the Moon; nor, though this could be proved, would it follow that the lunar atmoſphere anſwers no other purpoſe than the raiſing of water into vapour. There is however a ſtrong argument in favour of the exiſtence of a lunar atmoſphere, taken from the appearance of a luminous circle round the Moon in the time of total ſolar eclipſes; a circumſtance that has been obſerved by many aſtronomers; eſpecially in the total eclipſe of the ſun which happened May 1, 1706.

Of the Harveſt Moon. It is remarkable that the Moon, during the week in which ſhe is full about the time of harveſt, riſes ſooner after ſun-ſetting, than ſhe does in any other full-moon week in the year. By this means ſhe affords an immediate ſupply of light after ſun-ſet, which is very beneficial for the harveſt and gathering in the fruits of the earth: and hence this full Moon is diſtinguiſhed from all the others in the year, by calling it the Harveſt-Moon.

To conceive the reaſon of this phenomenon; it may firſt be conſidered, that the Moon is always oppoſite to the ſun when ſhe is full; that ſhe is full in the ſigns Piſces and Aries in our harveſt months, thoſe being the ſigns oppoſite to Virgo and Libra, the ſigns occupied by the ſun about the ſame ſeaſon; and becauſe thoſe parts of the ecliptic riſe in a ſhorter ſpace of time than others, as may eaſily be ſhewn and illuſtrated by the celeſtial globe: conſequently, when the Moon is about her full in harveſt, ſhe riſes with leſs difference of time, or more immediately after ſun-ſet, than when ſhe is full at other ſeaſons of the year.

In our winter, the Moon is in Piſces and Aries about the time of her firſt quarter, when ſhe riſes about noon; but her riſing is not then noticed, becauſe the ſun is above the horizon.

In ſpring, the Moon is in Piſces and Aries about the time of her change; at which time, as ſhe gives no light, and riſes with the ſun, her riſing cannot be perceived.

In ſummer, the Moon is in Piſces and Aries about the time of her laſt quarter; and then, as ſhe is on the decreaſe, and riſes not till midnight, her riſing uſually paſſes unobſerved.

But in autumn, the Moon is in Piſces and Aries at the time of her full, and riſes ſoon after ſun-ſet for ſeveral evenings ſucceſſively; which makes her regular riſing very conſpicuous at that time of the year.

And this would always be the caſe, if the Moon's orbit lay in the plane of the ecliptic. But as her orbit makes an angle of 5° 18′ with the ecliptic, and croſſes it only in the two oppoſite points called the nodes, her riſing when in Piſces and Aries will ſometimes not differ above 1 h. and 40 min. through the whole of 7 days; and at other times, in the ſame two ſigns ſhe will differ 3 hours and a half in the time of her riſing in a week, according to the different poſitions of the nodes with reſpect to theſe ſigns; which poſitions are conſtantly changing, becauſe the nodes go backward through the whole ecliptic in 18 years 225 days.

This revolution of the nodes will cauſe the Harveſt Moons to go through a whole courſe of the moſt and leaſt beneficial ſtates, with reſpect to the harveſt, every 19 years. The following Table ſhews in what years the Harveſt Moons are leaſt beneficial as to the times of their riſing, and in what years they are moſt beneficial, from the year 1790 to 1861; the column of years under the letter L, are thoſe in which the Harveſt-Moons are leaſt of all beneficial, becauſe they fall about the deſcending node; and thoſe under the letter M are the moſt of all beneficial, becauſe they fall about the aſcending node.

Harveſt

Harvest Moons.

L	M	L	M	L	M	L	M
1790	1798	1807	1816	1826	1835	1844	1843
1791	1799	1808	1817	1827	1836	1845	1854
1792	1800	1809	1818	1828	1837	1846	1855
1793	1801	1810	1819	1829	1838	1847	1856
1794	1802	1811	1820	1830	1839	1848	1857
1795	1803	1812	1821	1831	1840	1849	1858
1796	1804	1813	1822	1832	1841	1850	1859
1797	1805	1814	1823	1833	1842	1851	1860
	1806	1815	1824	1834	1843	1852	1861
			1825				

As to the Influence of the Moon, on the changes of the weather, and the constitution of the human body, it may be observed, that the vulgar doctrine concerning it is very ancient, and has also gained much credit among the Learned, though perhaps without sufficient examination. The common opinion is, that the Lunar Influence is chiefly exerted about the time of the full and change, but more especially the latter; and it would seem that long experience has in some degree established the fact: hence, persons observed at those times to be a little deranged in their intellects, are called Lunatics; and hence many persons anxiously look for the new Moon to bring a change in the weather. The Moon's Influence on the sea, in producing tides, being agreed upon on all hands, it is argued that she must also produce similar changes in the atmosphere, but in a much higher degree; which changes and commotions there, must, it is inferred, have a considerable influence on the weather, and on the human body.

Beside the observations of the Ancients, which tend to establish this doctrine, several among the Modern Philosophers have defended the same opinion, and that upon the strength of experience and observation; while others as strenuously deny the fact. The celebrated Dr. Mead was a believer in the Influence of the Sun and Moon on the human body, and published a book to this purpose, intitled, De Imperio Solis ac Lunæ in Corpore Humano. The existence of such influence is however opposed by Dr. Horsley, the present bishop of Rochester, in a learned paper upon this subject in the Philos. Trans. for the year 1775; where he gives a specimen of arranging tables of meteorological observations, so as to deduce from them facts, that may either confirm or refute this popular opinion; recommending it to the Learned, to collect a large series of such observations, as no conclusions can be drawn from one or two only. On the other hand professor Toaldo, and some French philosophers, take the opposite side of the question; and, from the authority of a long series of observations, pronounce decidedly in favour of the Lunar Influence.

Acceleration of the Moon. See Acceleration.

Moon-*Dial*. See Dial.

Horizontal Moon. See *Apparent* Magnitude.

MOORE (Sir Jonas), a very respectable mathematician, Fellow of the Royal Society, and Surveyor-general of the Ordnance, was born at Whitby in Yorkshire about the year 1620. After enjoying the advantages of a liberal education, he bent his studies principally to the mathematics, to which he had always a strong inclination. In the expeditions of King Charles the 1st into the northern parts of England, our author was introduced to him, as a person studious and learned in those sciences; when the king expressed much approbation of him, and promised him encouragement; which indeed laid the foundation of his fortune. He was afterwards appointed mathematical master to the king's second son James, to instruct him in arithmetic, geography, the use of the globes, &c. During Cromwell's government it seems he followed the profession of a public teacher of mathematics; for I find him styled, in the title-page of some of his publications, "professor of the mathematics." After the return of Charles the 2d, he found great favour and promotion, becoming at length surveyor-general of the king's ordnance. He was it seems a great favourite both with the king and the duke of York, who often consulted him, and were advised by him upon many occasions. And it must be owned that he often employed his interest with the court to the advancement of learning and the encouragement of merit. Thus, he got Flamsteed house built in 1675, as a public observatory, recommending Mr. Flamsteed to be the king's astronomer, to make the observations there: and being surveyor-general of the ordnance himself, this was the reason why the salary of the astronomer royal was made payable out of the office of ordnance. Being a governor of Christ's hospital, it seems that by his interest the king founded the mathematical school there, allowing a handsome salary for a master to instruct a certain number of the boys in mathematics and navigation, to qualify them for the sea service. Here he soon found an opportunity of exerting his abilities in a manner somewhat answerable to his wishes, namely, that of serving the rising generation. And considering with himself the benefit the nation might receive from a mathematical school, if rightly conducted, he made it his utmost care to promote the improvement of it. The school was settled; but there still wanted a methodical institution from which the youths might receive such necessary helps as their studies required: a laborious work, from which his other great and assiduous employments might very well have exempted him, had not a predominant regard to a more general usefulness engaged him to devote all the leisure hours of his declining years to the improvement of so useful and important a seminary of learning.

Having thus engaged himself in the prosecution of this general design, he next sketched out the plan of a course or system of mathematics for the use of the school, and then drew up and printed several parts of it himself, when death put an end to his labours, before the work was completed. I have not found in what year this happened; but it must have been but little before 1681, the year in which the work was published by his sons-in-law, Mr. Hanway and Mr. Potinger. Of this work, the Arithmetic, Practical Geometry, Trigonometry, and Cosmography, were written by Sir Jonas himself, and printed before his death. The Algebra, Navigation, and the books of Euclid were supplied by Mr. Perkins, the then master of the mathematical school. And the Astronomy, or Doctrine of the Sphere, was written by Mr. Flamsteed, the astronomer royal.

The

The list of Sir Jonas's works, as far as I have seen them, are the following:

1. The New System of Mathematics; above mentioned, in 2 vols 4to, 1681.

2. Arithmetic in two books, viz, Vulgar Arithmetic and Algebra. To which are added two Treatises, the one A new Contemplation Geometrical, upon the Oval Figure called the Ellipsis; the other, The two first books of Mydorgius, his Conical Sections analized &c. 8vo, 1660.

3. A Mathematical Compendium; or Useful Practices in Arithmetic, Geometry, and Astronomy, Geography and Navigation, &c, &c. 12mo, 4th edition in 1705.

4. A General Treatise of Artillery: or, Great Ordnance, Written in Italian by Tomaso Moretii of Brescia. Translated into English, with notes thereupon, and some additions out of French for Sea-Gunners. By Sir Jonas Moore, Kt. 8vo, 1683.

MORTALITY. *Bills of Mortality*, are accounts or registers specifying the numbers born, and buried, and sometimes married, in any town, parish, or district. These are of great use, not only in the doctrine of Life Annuities, but in shewing the degrees of healthiness and prolificness, with the progress of population in the places where they are kept. It is therefore much to be wished that such accounts had always been correctly kept in every kingdom, and regularly published at the end of every year. We should then have had under inspection the comparative strength of every kingdom, as far as it depends on the number of inhabitants, and its increase or decrease at different periods.

Such accounts are rendered still more useful, when they include the ages of the dead, and the distempers of which they have died. In this case they convey some of the most important instructions, by furnishing the means of ascertaining the law which governs the waste of human life, the values of annuities dependent on the continuance of any lives, or any survivorships between them, and the favourableness or unfavourableness of different situations to the duration of human life.

There are but few registers of this kind; nor has this subject, though so interesting to mankind, ever engaged much attention till lately. Indeed, bills of Mortality for the several parishes of the city of London have been kept from the year 1592, with little interruption; and a very ample account of them has been published down to the year 1759, by Dr. Birch, in a large 4to vol. which is perhaps the fullest work of the kind extant; containing besides the bills of Mortality, with the diseases and casualties, several other valuable tracts on the subject of them, and on political arithmetic, by several other authors, as Capt. John Graunt, F. R. S.; Sir William Petty, F. R. S.; Corbyn Morris, Esq. F. R. S.; and J. P. Esq. F. R. S.; the whole forming a valuable repository of materials; and it would be well if a continuation were published down to the present time, and so continued from time to time.

Bills containing the ages of the dead, were long since published for the town of Breslaw in Silesia. It is well known what use has been made of these by Dr. Halley, and after him by Mr. De Moivre. A table of the probabilities of the duration of human life at every age, deduced from them by Dr. Halley, was published in the Philos. Trans. vol. 17, and has been inserted in this work under the article LIFE-*Annuities*; which is the first table of this kind that has been published. Since the publication of this table, similar bills have been established in many other places, in England, Germany, Switzerland, France, Holland, &c, but most especially in Sweden; the results of some of which may be seen in the large comparative table of the duration of life, under the article LIFE-*Annuities*, in this work.

MORTAR, or MORTAR-PIECE, a short piece of ordnance, thick and wide, proper for throwing bomb-shells, carcases, stones, grape-shot, &c.

It is thought that the use of Mortars is older than that of cannon: for they were employed in the wars of Italy, to throw balls of red-hot iron, and stones, long before the invention of shells: and it is generally believed that the Germans were the first inventors. The practice of throwing red-hot balls out of Mortars, was first practised at the siege of Stralsund in 1675, by the elector of Brandenburg; though some say, in 1653, at the siege of Bremen.

Mortars are made either of brass or iron, and it is usual to distinguish them by the diameter of the bore; as, the 13 inch, the 10 inch, or the 8 inch Mortar: there are some of a smaller sort, as Coehorns of 4·6 inches, and Royals of 5·8 inches in diameter. As to the larger sizes, as 18 inches, &c, they are now disused by the English, as well as most other European nations. For the circumstances relating to Mortars, see Muller's Artillery.

Coehorn MORTAR, a small kind of one, invented by the celebrated engineer baron Coehorn, to throw small shells or grenades. These Mortars are often fixed, to the number of a dozen, on a block of oak, at the elevation of 45°.

MOTION, or *Local* MOTION, is a continued and successive change of place. Borelli defines it, the successive passage of a body from one place to another, in a determinate time, by becoming successively contiguous to all the parts of the intermediate space.

Motion is considered as of various kinds; as Natural, Violent, Absolute and Relative, &c, &c.

Natural MOTION, is that which has its principle, or actuating force, within the moving body. Such is that of a stone falling towards the earth. And

Violent MOTION, is that whose principle is without, and against which the moving body makes a resistance. Such is that of a stone thrown upwards, or of a ball shot off from a gun, &c.

Motion is again divided into Absolute and Relative.

Absolute MOTION, is the change of absolute place, in any moving body, considered independently of any other motion; whose celerity therefore will be measured by the quantity of absolute space which the moveable body runs through. And

Relative MOTION, is the change of the relative place of a moving body, or considered with respect to the motion of some other body; and has its celerity estimated by the quantity of relative space run through.

As to the Continuation of MOTION, or the cause why a body once in Motion comes to persevere in it: this has been

been much controverted among physical writers; and yet it follows very evidently from one of the grand Laws of Nature; viz, that all bodies persevere in their present state, whether of rest or motion, unless disturbed by some foreign powers. Motion therefore, once begun, would be continued in infinitum, were it to meet with no interruption from external causes; as the power of gravity, the resistance of the medium, &c.

Nor has the communication of motion, or how a moving body comes to affect another at rest, or how much of its motion is communicated by the first to the last, been less disputed. See the Laws of it under the word PERCUSSION.

Motion is the proper subject of mechanics; and mechanics is the basis of all natural philosophy; which hence becomes denominated Mechanical.

In effect, all the phenomena of nature, all the changes that happen in the system of bodies, are owing to Motion; and are directed according to the laws of it. Hence the modern philosophers have applied themselves with peculiar ardour to consider the doctrine of Motion; to investigate the properties and laws of it; by observation and experiment, joined to the use of geometry. And to this is owing the great advantage of the modern philosophy above that of the Ancients; who were extremely disregardful of the effects of Motion.

Among all the Ancients, there is nothing extant on Motion, excepting some things in Archimedes's books, De Æquiponderantibus. To Galileo is owing a great part of the doctrine of Motion: he first discovered the general laws of it, and particularly of the descent of heavy bodies, both perpendicularly and on inclined planes; the laws of the Motion of projectiles; the vibration of pendulums, and of stretched cords, with the theory of resistances, &c: things which the Ancients had little notion of.

Torricelli polished and improved the discoveries of his master, Galileo; and added many experiments concerning the force of percussion, and the equilibrium of fluids. Huygens improved very considerably on the doctrine of the pendulum; and both he and Borelli on the force of percussion. Lastly, Newton, Leibnitz, Varignon, Mariotte, &c, have brought the doctrine of Motion still much nearer to perfection.

The general laws of Motion were first brought into a system, and analytically demonstrated together, by Dr. Wallis, Sir Christopher Wren, and M. Huygens, all much about the same time; the first in bodies not elastic, and the two latter in elastic bodies. Lastly, the whole doctrine of Motion, including all the discoveries both of the Ancients and Moderns on that head, was given by Dr. Wallis in his Mechanica, sive De Motu, published in 1670.

Quantity of MOTION, is the same as MOMENTUM, which see. It is a principle maintained by the Cartesians, and some others, that the Creator at the beginning impressed a certain Quantity of Motion on bodies; and that under such laws, as that no part of it should be lost, but the same portion of Motion should be constantly preserved in matter: and hence they conclude, that if any moving body strike another body, the former loses no more of its Motion than it communicates to the latter. This position however has been opposed by other philosophers, and perhaps justly, unless the preservation of Motion be understood only of the quantity of it as estimated always in the same direction; for then it seems the principle will hold good. However, the reasoning ought to have proceeded in the contrary order; by first observing from experiment, or otherwise, that when two bodies act upon each other, the one gains exactly the Motion which is lost by the other, in the same direction; and from hence made the inference, that there is therefore the same Quantity of Motion preserved in the universe, as was created by God in the beginning; since no body can act upon another, without being itself equally acted upon in the opposite or contrary direction.

The Continuation of MOTION, or the cause why a body once in Motion comes to persevere in it, has been much controverted among physical writers; and yet it follows very evidently from one of the grand Laws of Nature; viz, that all bodies persevere in their present state, whether of Motion or rest, unless they are disturbed by some foreign powers. Motion therefore, once begun, would be continued for ever, were it to meet with no interruption from external causes; as the power of gravity, the resistance of the medium, &c.

The Communication of MOTION, or the manner in which a moving body comes to affect another at rest, or how much of its Motion is communicated by the first to the last, has also been the subject of much discussion and controversy. See the Laws of it under the word PERCUSSION.

MOTION may be considered either as Equable, and Uniform; or as Accelerated, and Retarded. Equable Motion, again, may be considered either as Simple, or as Compound; and Compound Motion either as Rectilinear, or as Curvilinear.

And all these again may be considered either with regard to themselves, or with regard to the manner of their production, and communication, by percussion, &c.

Equable MOTION, is that by which the moving body proceeds with exactly the same velocity or celerity; passing always over equal spaces in equal times.

The Laws of Uniform Motion, are these: 1. The spaces described, or passed over, are in the compound ratio of the velocities, and the times of describing those spaces. So that, if V and v be any two uniform velocities, S and s the spaces described or passed over by them, in the respective times T and t:

$$\text{then is } S : s :: TV : tv,$$
$$\text{or } 20 : 12 :: 4 \times 5 : 3 \times 4;$$
$$\text{taking } T = 4,\ t = 3,\ V = 5, \text{ and } v = 4.$$

2. In Uniform Motions, the time is as the space directly, and as the velocity reciprocally; or as the space divided by the velocity. So that

$$T : t :: \frac{S}{V} : \frac{s}{v} \text{ or } :: Sv : sV.$$

3. The velocity is as the space directly, and the time reciprocally; or as the space divided by the time.

$$\text{That is, } V : v :: \frac{S}{T} : \frac{s}{t} \text{ or } :: St : sT.$$

Accelerated MOTION, is that which continually receives fresh accessions of velocity. And it is said to be uniformly

uniformly accelerated, when its acceſſions of velocity are equal in equal times; ſuch as that which is produced by the continual action of one and the ſame force, like the force of gravity, &c.

Retarded MOTION, is that whoſe velocity continually decreaſes. And it is ſaid to be uniformly Retarded, when its decreaſe is continually proportional to the time, or by equal quantities in equal times; like that which is produced by the continual oppoſition of one and the ſame force; ſuch as the force of gravity, in uniformly retarding the Motion of a body that is thrown upwards.

The Laws of Motion, uniformly accelerated or retarded, are theſe:

1. In uniformly varied motions, the ſpace, S or s, is as the ſquare of the time, or as the ſquare of the greateſt velocity, or as the rectangle or product of the time and velocity.

That is, $S : s :: T^2 : t^2 :: V^2 : v^2 :: TV : tv$.

2. The velocity is the time, or as the ſpace divided by the the time, or as the ſquare root of the ſpace.

That is, $V : v :: T : t :: \frac{S}{T} : \frac{s}{t} :: \sqrt{S} : \sqrt{s}$.

3. The time is as the velocity, or as the ſpace divided by the velocity, or as the ſquare root of the ſpace.

That is, $T : t :: V : v :: \frac{S}{V} : \frac{s}{v} :: \sqrt{S} : \sqrt{s}$.

4. When a ſpace is deſcribed, or paſſed over, by an uniformly varied Motion, the velocity either beginning at nothing, and continually accelerated; or elſe beginning at ſome determinate velocity, and continually retarded till the velocity be reduced to nothing; then the ſpace, ſo run over by the variable Motion, will be exactly equal to half the ſpace that would be run over in the ſame time by the greateſt velocity if uniformly continued for that time. So, for inſtance, if g denote the ſpace run over in one ſecond, or any other time, by ſuch a variable Motion; then $2g$ would be the ſpace that would be run over in one ſecond, or the ſame time, by the greateſt velocity uniformly continued for the ſame time; or $2g$ would be the greateſt velocity per ſecond which the moving body had. Conſequently, if t be any other time, s the ſpace run over in that time, and v the greateſt velocity attained in it; then, from the foregoing articles, it will be

$$1'' : t'' :: 2g : 2gt = v \text{ the velocity,}$$
$$\text{and } 1^2 : t^2 :: g : gt^2 = s \text{ the ſpace.}$$

And hence, for any ſuch uniformly varied Motions, the relations among the ſeveral quantities concerned, will be expreſſed by the following equations: viz,

$$s = gt^2 = \tfrac{1}{2}tv = \frac{v^2}{4g},$$

$$v = 2gt = \frac{2s}{t} = 2\sqrt{gs},$$

$$t = \frac{v}{2g} = \frac{2s}{v} = \sqrt{\frac{s}{g}},$$

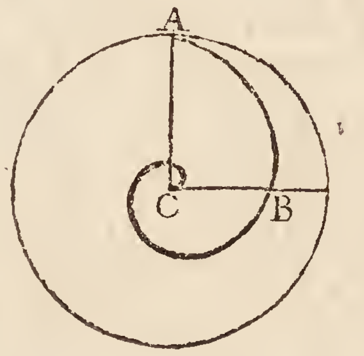
$$g = \frac{v}{2t} = \frac{s}{t^2} = \frac{v^2}{4s}.$$

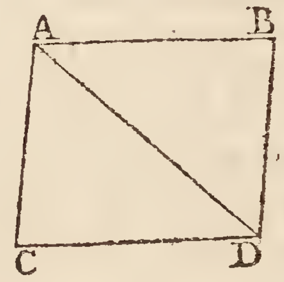
And theſe equations will hold good in the Motion either generated or deſtroyed by the force of gravity, or by any other uniform force whatever. See alſo the articles GRAVITY, ACCELERATION, RETARDATION, &c. Again,

Simple MOTION, is that which is produced by ſome one power or force only, and is always rectilinear, or in one direction, whether the force be only momentary or continued. And

Compound MOTION, is that which is produced by two or more powers acting in different directions. See COMPOUND, and COMPOSITION *of Motion*.

If a moving body be acted on by a double power; the one according to the direction AB, the other according to AC; with the Compound Motion, or that which is compounded of theſe two together, it will deſcribe the diagonal AD of the parallelogram, whoſe ſides AB and AC it would have deſcribed in the ſame time with each of the reſpective powers apart.





And if the radius of a circle be carried round upon the centre C, while a point in the radius ſets off from A, and keeps moving along the radius towards the centre; then, by this Compound Motion, the path of the point will be a kind of a ſpiral ABC.

For the Particular Laws of MOTION, *ariſing from the Colliſion of bodies, both Elaſtic and Non-elaſtic, and that where the directions are both Perpendicular and Oblique*, ſee PERCUSSION.

For CIRCULAR *Motion*, and *the Laws of* PROJECTILES, ſee the reſpective words.

For the Motion of Pendulums, and the Laws of Oſcillation, ſee PENDULUM.

Perpetual MOTION, is a Motion which is ſupplied and renewed from itſelf, without the intervention of any external cauſe.

The celebrated problem of a Perpetual Motion, conſiſts in the inventing a machine, which has the principle of its Motion within itſelf; and is a problem that has employed the mathematicians for 2000 years; though none perhaps have proſecuted it with attention and earneſtneſs equal to thoſe of the preſent age. Infinite are the ſchemes, deſigns, plans, engines, wheels, &c, to which this long-deſired Perpetual Motion has given birth.

But M. De la Hire has proved the impoſſibility of any ſuch machine, and finds that it amounts to this; viz, to find a body which is both heavier and lighter at the ſame time; or to find a body which is heavier than itſelf. Indeed there ſeems but little in nature to countenance all this aſſiduity and expectation: among all the laws of matter and Motion, we know of none yet that ſeem likely to furniſh any principle or foundation for ſuch an effect.

Action and reaction it is allowed are always equal; and a body that gives any quantity of Motion to another, always loses just so much of its own; but under the present state of things, the resistance of the air, the friction of the parts of machines, &c, do necessarily retard every Motion.

To continue the Motion therefore either, first, there must be a supply from some foreign cause; which in a Perpetual Motion is excluded.

Or, 2dly, all resistance from the friction of the parts of matter must be removed; which necessarily implies a change in the nature of things.

Or, 3dly and lastly, there must be some method of gaining a force equivalent to what is lost, by the artful disposition and combination of mechanic powers; to which last point then all endeavours are to be directed: but how, or by what means, such force should be gained, is still a mystery.

The multiplication of powers or forces, it is certain, avails nothing; for what is gained in power is lost in time, so that the quantity of Motion still remains the same. This is an inviolable law of nature; by which nothing is left to art, but the choice of the several combinations that may produce the same effect.

There are various ways by which absolute force may be gained; but since there is always an equal gain in opposite directions, and no increase obtained in the same direction; in the circle of actions necessary to make a perpetual movement, this gain must be presently lost, and will not serve for the necessary expence of force employed in overcoming friction, and the resistance of the medium. And therefore, though it could be shewn, that in an infinite number of bodies, or in an infinite machine, there could be a gain of force for ever, and a Motion continued to infinity, it does not follow that a perpetual movement can be made. That which was proposed by M. Leibnitz in the Leipsic Acts of 1690, as a consequence of the common estimation of the forces of bodies in Motion, is of this kind, and for this and other reasons ought to be rejected. See PERPETUAL *Motion*; also ORFFYREUS's *Wheel*, &c.

Animal MOTION, is that by which the situation, figure, magnitude, &c, of the parts and members of animals are changed. Under these Motions, come all the animal functions; as respiration, circulation of the blood, excretion, walking, running, &c.

Animal Motions are usually divided into two species; viz, Natural and Spontaneous.

Natural MOTION, is that involuntary one which is effected without the command of the will, by the mere mechanism of the parts. Such as the Motion of the heart and pulse; the Peristaltic Motion of the intestines, &c. But

Spontaneous, or *Muscular* MOTION, is that which is performed by means of the muscles, at the command of the will; which is hence called Voluntary Motion. Borelli has a celebrated treatise on this subject, entitled De Motu Animalium.

Intestine MOTION, denotes an agitation of the particles of which a body consists.—Some philosophers will have every body, and every particle of a body, in continual Motion. As for fluids, it is the definition they give of them, that their parts are in continual Motion. And as to solids, they infer the like Motion from the effluvia continually emitted through their pores. Hence Intestine Motion is represented to be a Motion of the internal and smaller parts of matter, continually excited by some external, latent agent, which of itself is insensible, and only discovers itself by its effects; appointed by Nature to be the great instrument of the changes in bodies.

MOTION, in Astronomy, is peculiarly applied to the orderly courses of the heavenly bodies.

Mean MOTION. See MEAN.

The Motions of the celestial luminaries are of two kinds: Diurnal, or Common; and Secondary, or Proper.

Diurnal, or *Primary* MOTION, is that with which all the heavenly bodies, and the whole mundane sphere, appear to revolve every day round the earth, from east to west. This is also called the Motion of the Primum Mobile, and the Common Motion, to distinguish it from that rotation which is peculiar to each planet, &c.

Secondary, or *Proper* MOTION, is that with which a star, planet, or the like, advances a certain space every day from the west towards the east. See the several Motions of each luminary, with the irregularities, &c, of them, under the proper articles, EARTH, MOON, STAR, &c.

Angular MOTION, is that by which the angular position of any thing varies. See ANGULAR.

Horary MOTION, is the Motion during each hour. See HORARY.

Paracentric MOTION *of Impetus*. See PARACENTRIC.

MOTION *of Trepidation*, &c. See TREPIDATION and LIBRATION.

MOTIVE *Power* or *Force*, is the whole power or force acting upon any body, or quantity of matter, to move it; and is proportional to the momentum or quantity of motion it can produce in a given time. To distinguish it from the Accelerative force, which is considered as affecting the celerity only.

MOTRIX, something that has the power or faculty of moving. See *Vis Motrix*, and MOTION.

MOVEABLE, something susceptible of motion, or that is disposed to be moved. A sphere is the most Moveable of all bodies, or is the easiest to be moved on a plane. A door is Moveable on its hinges; the magnetic needle on a pin or pivot, &c. Moveable is often used in contradistinction to Fixed or Fixt.

MOVEABLE *Feasts*, are such as are not always held on the same day of the year or month; though they may be on the same day of the week. Thus, Easter is a Moveable Feast; being always held on the Sunday which falls upon or next after the first full moon following the 21st of March. See Philos. Trans. numb. 240, pa. 185. All the other Moveable Feasts follow Easter, keeping their constant distance from it; so that they are fixed with respect to it, though Moveable through the course of the year. Such are Septuagesima, Sexagesima, Ash-Wednesday, Ascension-Day, Pentecost, Trinity-Sunday, &c.

MOVEMENT, a term often used in the same sense with Automaton. The most usual Movements for keeping time, are Clocks and Watches: the latter are such as shew the parts of time by inspection, and are portable in the pocket; the former such as publish it by sounds, and are fixed as furniture.

Movement, in its popular uſe, ſignifies all the inner works of a clock, watch, or other machine, that move, and by that motion carry on the deſign of the inſtrument. The Movement of a clock, or watch, is the inſide; or that part which meaſures the time, and ſtrikes, &c; excluſive of the frame, caſe, dial-plate, &c.

The parts common to both of theſe Movements are, the Main-ſpring with its appurtenances, lying in the ſpring box, and in the middle of it lapping about the ſpring-arbor, to which one end of it is faſtened. A-top of the ſpring-arbor is the Endleſs ſcrew, and its wheel; but in ſpring clocks this is a ratchet-wheel with its click, that ſtops it. That which the main-ſpring draws, and round which the chain or ſtring is wrapped, is called the fuſee: this is moſtly taper; in large works, going with weights, it is cylindrical, and is called the barrel. The ſmall teeth at the bottom of the fuſee or barrel, which ſtop it in winding up, is called the Ratchet; and that which ſtops it when wound up, and is for that end driven up by the ſpring, the Gardegut. The Wheels are various: the parts of a wheel are, the Hoop or Rim; the Teeth, the Croſs, and the Collet, or piece of braſs ſoldered on the arbor or ſpindle on which the wheel is riveted. The little wheels, playing in the teeth of the larger, are called Pinions; and their teeth, which are 4, 5, 6, 8, &c, are called Leves; the ends of the ſpindle are called Pivots; and the guttured wheel, with iron ſpikes at bottom, in which the line of common clocks runs, the Pulley.

Theory of Calculating the Numbers for Movements.

1. It is firſt to be obſerved, that a wheel, divided by its pinion, ſhews how many turns the pinion has to one turn of the wheel.

2. That from the fuſee to the balance the wheels drive the pinions, conſequently the pinions run faſter, or make more revolutions, than the wheel; but it is the contrary from the great wheel to the dial-wheel.

3. That the wheels and pinions are written down either as vulgar fractions, or in the way of diviſion in common arithmetic: for example, a wheel of 60, moving a pinion of 5, is ſet down either thus $\frac{60}{5}$, or thus 5)60, which is better. And the number of turns the pinion has in one turn of the wheel, as a quotient, thus 5) 60 (12. A whole Movement may be written as follows:

4)	36	(9
5)	55	(11
5)	45	(9
5)	40	(8
		17

where the uppermoſt number expreſſes the pinion of report 4, the dial-wheel 36, and the turns of the pinion 9; the ſecond, the pinion and great wheel; the third, the ſecond wheel &c; the fourth, the contrate wheel; and the laſt, 17, the crown-wheel.

4. Hence, from the number of turns any pinion makes, in one turn of the wheel it works in, may be determined the number of turns a wheel or pinion has at any greater diſtance, viz, by multiplying the quotients together; the product being the number of turns. Thus, ſuppoſe the wheels and pinions as in the caſe above; the quotient 11 multiplied by 9, gives 99, the number of turns in the ſecond pinion 5 to one turn of the wheel 55, which runs concentrical, or on the ſame ſpindle, with the pinion 5. Again, 99 multiplied by 8, gives 792, the number of turns the laſt pinion has to one turn of the firſt wheel 5. Hence we proceed to find, not only the turns, but the number of beats of the balance, in the time of thoſe turns. For, having found the number of turns the crown-wheel has in one turn of the wheel propoſed, thoſe turns multiplied by its notches, give half the number of beats in that one turn of the wheel. Suppoſe, for example, the crown-wheel to have 720 turns, to one of the firſt wheel; this number multiplied by 15, the notches in the crown-wheel, produces 10800, half the number of ſtrokes of the balance in one turn of the firſt wheel of 80 teeth.

The general diviſion of a Movement is, into the clock, and watch parts.

MOULDINGS, in Architecture, are certain projections beyond the naked of a wall, column, wainſcot &c, the aſſemblage of which forms cornices, door-caſes, and other decorations of architecture.

Mouldings, are annexed to great guns by way of ornament, and perhaps in ſome parts for ſtrength; and probably are derived from the hoops or rings which bound the long iron bars together, anciently uſed in making cannon.

MOYNEAU. See Moineau.

MULLER (John), commonly called Regiomontanus, from Mons Regius, or Koningſberg, a town in Franconia, where he was born in 1436, and became the greateſt aſtronomer and mathematician of his time. He was indeed a very prodigy for genius and learning. Having firſt acquired grammatical learning in his own country, he was admitted, while yet a boy, into the academy at Leipſic, where he formed a ſtrong attachment to the mathematical ſciences, arithmetic, geometry, aſtronomy, &c. But not finding proper aſſiſtance in theſe ſtudies at this place, he removed, at only 15 years of age, to Vienna, to ſtudy under the famous Purbach, the profeſſor there, who read lectures in thoſe ſciences with the higheſt reputation. A ſtrong and affectionate friendſhip ſoon took place between theſe two, and our author made ſuch rapid improvement in the ſciences, that he was able to be aſſiſting to his maſter, and to become his companion in all his labours. In this manner they ſpent about ten years together; elucidating obſcurities, obſerving the motions of the heavenly bodies, and comparing and correcting the tables of them; particularly thoſe of Mars, which they found to diſagree with the motions, ſometimes as much as two degrees.

About this time there arrived at Vienna the cardinal Beſſarion, who came to negociate ſome affairs for the pope; who, being a lover of aſtronomy, ſoon formed an acquaintance with Purbach and Regiomontanus. He had begun to form a Latin Verſion of Ptolomy's Almageſt, or an Epitome of it; but not having time to go on with it himſelf, he requeſted Purbach to complete the work, and for that purpoſe to return with him into Italy, to make himſelf maſter of the Greek tongue, which he was as yet unacquainted with. To theſe propoſals Purbach only aſſented, on condition that Regiomontanus would accompany him, and ſhare in all the labours. They firſt however, by

means of an Arabic Verſion of Ptolomy, made ſome progreſs in the work; but this was ſoon interrupted by the death of Purbach, which happened in 1461, in the 39th year of his age. The whole taſk then devolved upon Regiomontanus, who finiſhed the work, at the requeſt of Purbach, made to him when on his death-bed. This work our author afterwards reviſed and perfected at Rome, when he had learned the Greek language, and conſulted the commentator Theon, &c.

Regiomontanus accompanied the cardinal Beſſarion in his return to Rome, being then near 30 years of age. Here he applied himſelf diligently to the ſtudy of the Greek language; not neglecting however to make aſtronomical obſervations and compoſe various works in that ſcience; as his Dialogue againſt the Theories of Cremonenſis. The cardinal going to Greece ſoon after, Regiomontanus went to Ferrara, where he continued the ſtudy of the Greek language under Theodore Gaza; who explained to him the text of Ptolomy, with the commentaries of Theon; till at length he became ſo perfect in it, that he could compoſe verſes, and read it like a critic.—In 1463 he went to Padua, where he became a member of the univerſity; and, at the requeſt of the ſtudents, explained Alfraganus, an Arabian philoſopher.—In 1464 he removed to Venice, to meet and attend his patron Beſſarion. Here he wrote, with great accuracy, his Treatiſe of Triangles, and a Refutation of the Quadrature of the Circle, which Cardinal Cuſan pretended he had demonſtrated. The ſame year he returned with Beſſarion to Rome; where he made ſome ſtay, to procure the moſt curious books: thoſe he could not purchaſe, he took the pains to tranſcribe, for he wrote with great facility and elegance; and others he got copied at a great expence. For as he was certain that none of theſe books could be had in Germany, he thought on his return thither, he would at his leiſure tranſlate and publiſh ſome of the beſt of them. During this time too he had a fierce conteſt with George Trabezonde, whom he had greatly offended by animadverting on ſome paſſages in his tranſlation of Theon's Commentary.

Being now weary of rambling about, and having procured a great number of manuſcripts, which was one great object of his travels, he returned to Vienna, and performed for ſome time the offices of his profeſſorſhip, by reading of lectures &c. After being a while thus employed, he went to Buda, on the invitation of Matthias king of Hungary, who was a great lover of letters and the ſciences, and had founded a rich and noble library there: for he had bought up all the Greek books that could be found on the ſacking of Conſtantinople; alſo thoſe that were brought from Athens, or wherever elſe they could be met with through the whole Turkiſh dominions, collecting them all together into a library at Buda. But a war breaking out in this country, he looked out for ſome other place to ſettle in, where he might purſue his ſtudies, and for this purpoſe he retired to Noremberg. He tells us, that the reaſons which induced him to deſire to reſide in this city the remainder of his life were, that the artiſts there were dextrous in fabricating his aſtronomical machines; and beſides, he could from thence eaſily tranſmit his letters by the merchants into foreign countries. Being now well verſed in all parts of learning, and made the utmoſt proficiency in mathematics, he determined to occupy himſelf in publiſhing the beſt of the ancient authors, as well as his own lucubrations. For this purpoſe he ſet up a printing-houſe, and formed a nomenclature of the books he intended to publiſh, which ſtill remains.

Here that excellent man, Bernard Walther, one of the principal citizens, who was well ſkilled in the ſciences, eſpecially aſtronomy, cultivated an intimacy with Regiomontanus; and as ſoon as he underſtood thoſe laudable deſigns of his, he took upon himſelf the expence of conſtructing the aſtronomical inſtruments, and of erecting a printing-houſe. And firſt he ordered aſtronomical rules to be made of tin, for obſerving the altitudes of the ſun, moon and planets. He next conſtructed a rectangular, or aſtronomical radius, for taking the diſtances of thoſe luminaries. Then an armillary aſtrolabe, ſuch as was uſed by Ptolomy and Hipparchus, for obſerving the places and motions of the ſtars. Laſtly, he made other ſmaller inſtruments, as the torquet, and Ptolomy's meteoroſcope, with ſome others which had more of curioſity than utility in them. From this apparatus it evidently appears, that Regiomontanus was a moſt diligent obſerver of the laws and motions of the celeſtial bodies, if there were not ſtill ſtronger evidences of it in the accounts of the obſervations themſelves which he made with them.

With regard to the printing-houſe, which was the other part of his deſign in ſettling at Noremberg, as ſoon as he had completed it, he put to preſs two works of his own, and two others. The latter were, The *New Theories* of his maſter Purbach, and the *Aſtronomicon* of Manilius. And his own were, the *New Calendar*, in which were given (as he ſays in the Index of the books which he intended to publiſh) the true conjunctions and oppoſitions of the luminaries, their eclipſes, their true places every day, &c. His other work was his *Ephemerides*, of which he thus ſpeaks in the ſaid index: "The Ephemerides, which they vulgarly call an Almanac, for 30 years: where you may every day ſee the true motion of all the planets, of the moon's nodes, with the aſpects of the moon to the ſun and planets, the eclipſes of the luminaries; and in the fronts of the pages are marked the latitudes." He publiſhed alſo moſt acute commentaries on Ptolomy's Almageſt; a work which cardinal Beſſarion ſo highly valued, that he ſcrupled not to eſteem it worth a whole province. He prepared alſo new verſions of Ptolomy's Coſmography; and at his leiſure hours examined and explained works of another nature. He enquired how high the vapours are carried above the earth, which he fixed to be not more than 12 German miles. He ſet down obſervations of two comets that appeared in the years 1471 and 1472.

In 1474, pope Sixtus the 4th conceived a deſign of reforming the calendar; and ſent for Regiomontanus to Rome, as the propereſt and ableſt perſon to accompliſh his purpoſe. Regiomontanus was very unwilling to interrupt the ſtudies, and printing of books, he was engaged in at Noremberg; but receiving great promiſes from the pope, who alſo for the preſent named him biſhop of Ratiſbon, he at length conſented to go. He arrived at Rome in 1475, but died there the year after, at only 40 years of age; not without a

 ſuſpicion

fuspicion of being poisoned by the sons of George Trabezonde, in revenge for the death of their father, which was said to have been caused by the grief he felt on account of the criticisms made by Regiomontanus on his translation of Ptolomy's Almagest.

Purbach first of any reduced the trigonometrical tables of sines, from the old sexagesimal division of the radius, to the decimal scale. He supposed the radius to be divided into 600000 equal parts, and computed the sines of the arcs to every ten minutes, in such equal parts of the radius, by the decimal notation. This project of Purbach was perfected by Regiomontanus; who not only extended the sines to every minute, the radius being 600000, as designed by Purbach, but afterwards, disliking that scheme, as evidently imperfect, he computed them likewise to the radius 1000000, for every minute of the quadrant. Regiomontanus also introduced the tangents into trigonometry, the canon of which he called *fœcundus*, because of the many great advantages arising from them. Beside these things, he enriched trigonometry with many theorems and precepts. Indeed, excepting for the use of logarithms, the trigonometry of Regiomontanus is but little inferior to that of our own time. His Treatise, on both Plane and Spherical Trigonometry, is in 5 books; it was written about the year 1464, and printed in folio at Noremberg in 1533. In the 5th book are various problems concerning rectilinear triangles, some of which are resolved by means of algebra: a proof that this science was not wholly unknown in Europe before the treatise of Lucas de Burgo.

Regiomontanus was author of some other works beside those before mentioned. Peter Ramus, in the account he gives of the admirable works attempted and performed by Regiomontanus, tells us, that in his workshop at Noremberg there was an automaton in perpetual motion: that he made an artificial fly, which taking its flight from his hand, would fly round the room, and at last, as if weary, would return to his master's hand: that he fabricated an eagle, which, on the emperor's approach to the city, he sent out, high in the air, a great way to meet him, and that it kept him company to the gates of the city. Let us no more wonder, adds Ramus, at the dove of Archytas, since Noremberg can shew a fly, and an eagle, armed with geometrical wings. Nor are those famous artificers, who were formerly in Greece, and Egypt, any longer of such account, since Noremberg can boast of her Regiomontanuses. For Wernerus first, and then the Schoneri, father and son, afterwards, revived the spirit of Regiomontanus.

MULTANGULAR Figure, is one that has many angles, and consequently many sides also. These are otherwise called polygons.

MULTILATERAL Figures, are such as have many sides, or more than four sides.

MULTINOMIAL, or Multinomial *Roots*, are such as are composed of many names, parts, or members; as, $a + b + c + d$ &c.

For the raising an infinite Multinomial to any proposed power, or extracting any root out of such power, see a method by Mr. De Moivre, in the Philos. Trans. numb. 230. See also Polynomial.

MULTIPLE, Multiplex, a number which comprehends some other number several times. Thus, 6 is a Multiple of 2, this being contained in 6 just 3 times. Also 12 is a common Multiple of 6, 4, and 3; comprehending the first twice, the second thrice, and the third four times.

Multiple *Ratio* or *Proportion*, is that which is between Multiple numbers &c. If the less term of a ratio be an aliquot part of the greater, the ratio of the greater to the less is called Multiple; and that of the less to the greater Submultiple.

A Submultiple number, is that which is contained in the Multiple. Thus, the numbers 2, 3, and 4 are Submultiples of 12 and 24.

Duple, triple, &c ratios; as also subduples, subtriples, &c, are so many species of Multiple and Submultiple ratios.

Multiple *Superparticular Proportion*, is when one number or quantity contains another more than once, and a certain aliquot part; as 10 to 3, or $3\frac{1}{3}$ to 1.

Multiple *Superpartient Proportion*, is when one number or quantity contains another several times, and some parts besides; as 29 to 6, or $4\frac{5}{6}$ to 1.

MULTIPLICAND, is one of the two factors in the rule of multiplication, being that number given to be multiplied by the other, called the multiplicator, or multiplier.

MULTIPLICATION, is, in general, the taking or repeating of one number or quantity, called the Multiplicand, as often as there are units in another number, called the Multiplier, or Multiplicator; and the number or quantity resulting from the Multiplication, is called the Product of the two foregoing numbers or factors.

Multiplication is a compendious addition; performing at once, what in the usual way of addition would require many operations: for the multiplicand is only added to itself, or repeated, as often as is expressed by the units in the multiplier. Thus, if 6 were to be multiplied by 5, the product is 30, which is the sum arising from the addition of the number 6 five times to itself.

In every Multiplication, 1 is in proportion to the multiplier, as the multiplicand is to the product.

Multiplication is of various kinds, in whole numbers, in fractions, decimals, algebra, &c.

1. Multiplication *of Whole Numbers*, is performed by the following rules: When the multiplier consists of only one figure, set it under the first, or right-hand figure, of the multiplicand; then, drawing a line underneath, and beginning at the said first figure, multiply every figure of the multiplicand by the multiplier; setting down the several products below the line, proceeding orderly from right to left. But if any of these products amount to 10, or several 10's, either with or without some overplus, then set down only the overplus, or set down 0 if there be no overplus; and carry, to the next product, as many units as the former contained of tens. Thus, to multiply 35092 by 4.

Multiplicand	35092
Multiplier	4
Product	140368

When

When the multiplier conſiſts of ſeveral figures; multiply the multiplicand by each figure of it, as before, and place the ſeveral lines of products underneath each other in ſuch order, that the firſt figure or cipher of each line may fall ſtraight under its reſpective multiplier, or multiplying figure; then add theſe ſeveral lines of products together, as they ſtand, and the ſum of them all will be the product of the whole multiplication. Thus, to multiply 63017 by 236:

Multiplicand - -	63017
Multiplier - -	236
Product of 63017 by 6 -	378102
Product of 63017 by 30	189051
Product of 63017 by 200	126034
Whole product	14872012

The ſeveral lines of products may be ſet down in any order, or any of them firſt, and any other of them ſecond, &c; for the order of placing them can make no difference in the ſum total. There are many abbreviations, and peculiar caſes, according to circumſtances, which may be ſeen in moſt books of arithmetic.

The mark or character now uſed for Multiplication, is either the × croſs or a ſingle point · ; the former being introduced by Oughtred, and the latter I think by Leibnitz.

To Prove Multiplication. This may be done various ways; either by dividing the product by the multiplier, then the quotient will be equal to the multiplicand; or divide the ſame product by the multiplicand, and the quotient will come out equal to the multiplier; or in general divide the product by either of the two factors, and the quotient will come out equal to the other factor, when the operations are all right. But the more uſual, and compendious way of proving Multiplication, is by what is called caſting out the nines; which is thus performed: Add the figures of the multiplicand all together, and as often as the ſum amounts to 9, reject it always, and ſet down the laſt overplus as in the margin; this in the foregoing example is 8. Then do the ſame by the multiplier, ſetting down the laſt overplus, which is 2, on the right of the former remainder 8. Next multiply theſe two remainders, 2 and 8, together, and from their product 16, caſt out the 9, and there remains 7, which ſet down over the two former. Laſtly, add up, in the ſame manner, all the figures of the whole product of the multiplication, viz 14872012, caſting out the 9's, and then there remains 7, to be ſet down under the two firſt remains. Then when the figure at top, is the ſame as that at bottom, as they are here both 7's, the work it may be preſumed is right; but if theſe two figures ſhould not be the ſame, it is certainly wrong.

	7	
8		2
	7	

2. *To Multiply Money, or any other thing, conſiſting of different Denominations together, by any number, uſually called Compound Multiplication.* Beginning at the loweſt, multiply the number of each denomination ſeparately by the multiplier, ſetting down the products below them. But if any of theſe products amount to as much as 1 or more of the next higher denominations, carry ſo many to the next product, and ſet down only the overplus. *For Ex.* To find the amount of 9 things at 1 l 12 s $4\frac{1}{2}$d. each; or to multiply 1 l 12 s $4\frac{1}{2}$d by 9: ſet the multiplier 9 under the given ſum as in the margin, and multiply thus: 9 halfpence make 4d halfpenny, ſet down $\frac{1}{2}$ penny, and carry 4; then 9 times 4 are 36, and 4 to carry make 40 pence, which are 3 s and 4d, ſet down 4 and carry 3; next 9 times 12 are 108, and 3 to carry, make 111 ſhillings, or 5 l 11 s, ſet down 11, and carry 5; laſtly 9 times 1 are 9, and 5 to carry, make 14, which ſet down; and then the whole amount, or product, comes to 14 l 11 s $4\frac{1}{2}$d.

l	s	d
1	12	$4\frac{1}{2}$
		9
14	11	$4\frac{1}{2}$

3. *To Multiply Vulgar Fractions.*—Multiply all the given numerators together for the numerator of the product, and all the denominators together for the denominator of the product ſought.

Thus, $\frac{2}{3}$ multiplied by $\frac{4}{5}$, or $\frac{2}{3} \times \frac{4}{5}$ make $\frac{8}{15}$.

And $\frac{3}{5} \times \frac{2}{5} \times \frac{3}{7}$ make $\frac{18}{175}$.

And here it may be noted that, when there are any common numbers in the numerators and denominators, theſe may be omitted from both, which will make the operation ſhorter, and bring out the whole product in a fraction much ſimpler and in lower terms. Thus, $\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}$, by leaving out the two 3's, become $\frac{2 \times 5}{4 \times 6} = \frac{10}{24}$ or $\frac{5}{12}$

Alſo, when any numerators and denominators will both abbreviate or divide by one and the ſame number, let them be divided, and the quotients uſed inſtead of them. So, in the above example, after omitting the two 3's, let the 2 and 6 be both divided by 2, and uſe the quotients 1 and 3 inſtead of them, ſo ſhall the expreſſion become $\frac{1 \times 5}{4 \times 3} = \frac{5}{12}$, as before.

4. *To Multiply Decimals.*—Multiply the given numbers together the ſame as if they were whole numbers, and point off as many decimals in the whole product as there are in both factors together; as in the annexed example, where the number of decimals is five, becauſe there are three in the multiplicand, and two in the multiplier.—When it happens that there are not ſo many figures in the product as there muſt be decimals, then prefix as many ciphers as will ſupply the defect.

2·305
21·86
13830
18440
2305
4610
50·38730

5. *Croſs* Multiplication, otherwiſe called *Duodecimal Arithmetic*, is the multiplying of numbers together whoſe ſubdiviſions proceed by 12's; as feet, inches, and parts, that is 12th parts, &c; a thing of very frequent uſe in ſquaring, or multiplying toge-

ther the dimenſions of the works of bricklayers, carpenters, and other artificers. *For Example.* To multiply 5 feet 3 inches by 2 feet 4 inches. Set them down as in the margin, and multiply all the parts of the multiplicand by each part of the multiplier; thus, 2 times 3 make 6 inches, and 2 times 5 make 10 feet; then 4 times 3 make 12 parts, or 1 inch to carry; and 4 times 5 make 20, and 1 to carry makes 21 inches, or 1f. 9inc. to ſet down below the former line: Laſtly adding the two lines together, the whole ſum or product amounts to 12f. 3inc.

F	I
5	3
2	4
10	6
1	9
12	3

6. Multiplication *in Algebra.* This is performed, 1. When the quantities are ſimple, by only joining the letters together like a word; and if the ſimple quantities have any coefficients or numbers joined with them, multiply the numbers together, and prefix the product of them to the letters ſo joined together. But, in algebra, we have not only to attend to the quantities themſelves, but alſo to the ſigns of them; and the general rule for the ſigns is this: When the ſigns are alike, or the ſame, either both + or both −, then the ſign of the product will always be +; but when the ſigns are different, or unlike, the one +, and the other −, then the ſign of the product will be −. Hence theſe

Examples.

Mult.	$+a$	$-2a$	$+6x$	$-8x$	$-3ab$
By	$+b$	$-4b$	$-3a$	$+5a$	$-5ac$
Products	$+ab$	$+8ab$	$-18ax$	$-40ax$	$+15a^2bc$

2. In Compound quantities, multiply every term or part of the multiplicand by each term ſeparately of the multiplier, and ſet down all the products with their ſigns, collecting always into one ſum as many terms as are ſimilar or like to one another.

Examples.

$$\begin{array}{lll}
a+b & a-b & a+b \\
a+b & a-b & a-b \\
\hline
a^2+ab & a^2-ab & a^2+ab \\
\quad +ab+b^2 & \quad -ab+b^2 & \quad -ab-b^2 \\
\hline
a^2+2ab+b^2 & a^2-2ab+b^2 & a^2 \quad \cdot \quad -b^2
\end{array}$$

$$\begin{array}{lll}
2a-3b & 2a+4x & a^2-ax \\
4a+5b & 2a-4x & 2a+2x \\
\hline
8a^2-12ab & 4a^2+8ax & 2a^3-2a^2x \\
\quad +10ab-15b^2 & \quad -8ax-16x^2 & \quad +2a^2x-2ax^2 \\
\hline
8a^2-\ 2ab-15b^2 & 4a^2 \quad \cdot \quad -16x^2 & 2a^3 \quad \cdot \quad -2ax^3
\end{array}$$

3. In Surd quantities, if the terms can be reduced to a common ſurd, the quantities under each may be multiplied together, and the mark of the ſame ſurd prefixed to the product; but if not, then the different ſurds may be ſet down with ſome mark of multiplication between them, to denote their product.

Examples.

$7\sqrt{ax}$	$\sqrt{7}$	$\sqrt[3]{7ab}$	$\sqrt{12a}$	$6a\sqrt{2cx}$
$5\sqrt{cx}$	$\sqrt{5}$	$\sqrt[3]{4ac}$	$\sqrt{3a}$	$2b\sqrt{3ax}$
$35\sqrt{acx^2}$	$\sqrt{35}$	$\sqrt[3]{28a^2bc}$	$\sqrt{36a^2} = 6a$	$12ab\sqrt{6acx^2}$

4. Powers or Roots of the ſame quantity are multiplied together, by adding their exponents.

Thus, $a^2 \times a^3 = a^5$; and $\overline{a+x}|^3 \times \overline{a+x}|^5 = \overline{a+x}|^8$: alſo $x^2 \times x^{\frac{1}{2}} = x^{\frac{5}{2}}$; and $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^1$, or a.

To Multiply Numbers together by Logarithms.—This is performed by adding together the logarithms of the given numbers, and taking the number anſwering to that ſum, which will be the product ſought.

Des Cartes, at the beginning of his Geometry, performs Multiplication (and indeed all the other common arithmetical rules) in geometry, or by lines; but this is no more than taking a 4th proportional to three given lines, of which the firſt repreſents unity, and the 2d and 3d the two factors or terms to be multiplied, the product being expreſſed by the 4th proportional; becauſe, in every multiplication, unity or 1 is to either of the two factors, as the other factor is to the product.

MULTIPLICATOR, is the number or quantity by which another is multiplied; and is otherwiſe called the multiplier.

MULTIPLIER, or Multiplicator, is the number or quantity which multiplies another, called the multiplicand, in any operation of multiplication.

MUNSTER (Sebastian), an eminent German divine and mathematician, was born at Ingelheim in 1489. At the age of 14 he was ſent to Heidelberg to ſtudy. Two years after, he entered the convent of the Cordeliers; where he aſſiduouſly ſtudied divinity, mathematics, and geography. He was the firſt who publiſhed a Chaldee Grammar and Lexicon; and he ſhortly after gave the world a Talmudic Dictionary. He afterwards became profeſſor of the Hebrew language at Baſil. He was one of the firſt who attached himſelf to Luther, and embraced Proteſtantiſm: yet behaved himſelf with great moderation; never concerning himſelf with their diſputes; but ſhut himſelf up at home and purſued his favourite ſtudies, which were mathematics, natural philoſophy, with the Hebrew and other Oriental languages. He publiſhed a great number of books on theſe ſubjects; particularly, a Latin verſion, from the Hebrew, of all the books of the Old Teſtament, with learned notes, printed at Baſil in 1534 and 1546; Joſephus's Hiſtory of the Jews in Latin; a Treatiſe of Dialling, in folio, 1536; Univerſal Coſmography, in 6 books folio, Baſil 1550. For theſe works he was ſtyled the German Strabo; as he was the German Eſdras, for his Oriental writings.

Munſter was a meek-tempered, pacific, ſtudious, retired man, who wrote a great number of books, but never

never meddled in controverſy.—He died of the plague at Baſil, in 1552, at 63 years of age.

MURDERERS, a ſmall ſpecies of ordnance once uſed on ſhipboard; but now out of uſe.

MUSIC, the ſcience of ſound, conſidered as capable of producing melody, or harmony.

Among the Ancients, Muſic was taken in a much more extenſive ſenſe than among the Moderns: what we call the ſcience of Muſic, was by the Ancients rather called Harmonica.

Muſic is one of the ſeven ſciences called liberal, and comprehended alſo among the mathematical ſciences, as having for its object diſcrete quantity or number; not however conſidering it in the abſtract, like arithmetic; but in relation to time and ſound, with intent to conſtitute a delightful harmony.

This ſcience is alſo Theoretical and Practical. Theoretical, which examines the nature and properties of concords and diſcords, explaining the proportions between them by numbers. And Practical, which teaches not only compoſition, or the manner of compoſing tunes, or airs; but alſo the art of ſinging with the voice, and playing on muſical inſtruments.

It appears that Muſic was one of the moſt ancient of the arts; and, of all others, Vocal Muſic muſt doubtleſs have been the firſt kind. For man had not only the various tones of his own voice to make his obſervations on, before any other art or inſtrument was found out, but had the various natural ſtrains of birds to give him occaſion to improve his own voice, and the modulations of ſounds it was capable of. The firſt invention of wind inſtruments Lucretius aſcribes to the obſervation of the winds whiſtling in the hollow reeds. As for other kinds of inſtruments, there were ſo many occaſions for cords or ſtrings, that men could not be long in obſerving their various ſounds; which might give riſe to ſtringed inſtruments. And for the pulſative inſtruments, as drums and cymbals, they might ariſe from the obſervation of the naturally hollow noiſe of concave bodies.

As to the inventors and improvers of Muſic, Plutarch, in one place, aſcribes the firſt invention of it to Apollo; and in another place to Amphion, the ſon of Jupiter and Antiope. The latter indeed, it is pretty generally allowed, firſt brought Muſic into Greece, and invented the lyre.

To him ſucceeded Chiron, the demigod; then Demodocus; Hermes Triſmegiſtus: Olympus; and Orpheus, whom ſome make the firſt introducer of Muſic into Greece, and the inventor of the lyre: to whom add Phemius, and Terpander, who was contemporary with Lycurgus, and ſet his laws to Muſic; to whom alſo ſome attribute the firſt inſtitution of muſical modes, and the invention of the lyre: laſtly, Thales; and Thamyris, who, it has been ſaid, was the firſt inventor of inſtrumental Muſic without ſinging.

Theſe were the eminent muſicians before Homer's time: others of a later date were, Laſus Hermionenſis, Melanippides, Philoxenus, Timotheus, Phrynnis, Epigonius, Lyſander, Simmicus, and Diodorus; who were all of them conſiderable improvers of Muſic. Laſus, it is ſaid, was the firſt author who wrote upon Muſic, in the time of Darius Hyſtaſpis; Epigonius invented an inſtrument of 40 ſtrings, called the Epigonium. Simmicus alſo invented an inſtrument of 35 ſtrings, called a Simmicium; Diodorus improved the Tibia, by adding new holes; and Timotheus the Lyre, by adding a new ſtring; for which he was fined by the Lacedemonians.

As the accounts we have of the inventors of muſical inſtruments among the Ancients are very obſcure, ſo alſo are the accounts of thoſe inſtruments themſelves; of moſt of them indeed we know little more than the bare names.

The general diviſion of inſtruments is, into ſtringed inſtruments, wind inſtruments, and thoſe of the pulſatile kind. Of ſtringed inſtruments, mention is made of the ly raor cithara, the pſalterium, trigonum, ſambuca, pectis, magas, barbiton, teſtudo, epigonium, ſimmicium, and panderon; which were all ſtruck with the hand, or a plectrum. Of wind inſtruments, were the tibia, fiſtula, hydraulic organs, tubæ, cornua, and lituus. And the pulſatile inſtruments were the tympanum, cymbalum, creptaculum, tintinnabulum, crotalum, and ſiſtrum.

Muſic has ever been in the higheſt eſteem in all ages, and among all people; nor could authors expreſs their opinion of it ſtrongly enough, but by inculcating that it was uſed in heaven, and as one of the principal entertainments of the gods, and the ſouls of the bleſſed. The effects aſcribed to it by the Ancients are almoſt miraculous: by its means, it has been ſaid, diſeaſes have been cured, unchaſtity corrected, ſeditions quelled, paſſions raiſed and calmed, and even madneſs occaſioned. Athenæus aſſures us, that anciently all laws, divine and civil, exhortations to virtue, the knowledge of divine and human things, with the lives and actions of illuſtrious men, were written in verſe, and publicly ſung by a chorus to the ſound of inſtruments; which was found the moſt effectual means to impreſs morality on the minds of men, and a right ſenſe of their duty.

Dr. Wallis has endeavoured to account for the ſurpriſing effects attributed to the ancient Muſic; and aſcribes them chiefly to the novelty of the art, and the hyperboles of the ancient writings: nor does he doubt, but the modern Muſic, in like caſes, would produce effects at leaſt as conſiderable as the ancient. The truth is, we can match moſt of the ancient ſtories of this kind in the modern hiſtories. If Timotheus could excite Alexander's fury with the Phrygian mode, and ſooth him into indolence with the Lydian; a more modern muſician has driven Eric, king of Denmark, into ſuch a rage, as to kill his beſt ſervants. Dr. Niewentyt ſpeaks of an Italian who, by varying his Muſic from briſk to ſolemn, and the contrary, could ſo move the ſoul, as to cauſe diſtraction and madneſs; and Dr. South has founded his poem, called Muſica Incantans, on an inſtance he knew of the ſame kind.

Muſic however is found not only to exert its force on the affections, but on the parts of the body alſo: witneſs the Gaſcon knight, mentioned by Mr. Boyle, who could not contain his water at the playing of a bagpipe; and the woman, mentioned by the ſame author, who would burſt into tears at the hearing of a certain tune, with which other people were but a little affected. To ſay nothing of the trite ſtory of the Tarantula, we have an inſtance, in the Hiſtory of the Academy of Sciences, of a muſician being cured of a violent fever,

fever, by a little concert occaſionally played in his room.

Nor are our minds and bodies alone affected with ſounds, but even inanimate bodies are ſo. Kircher ſpeaks of a large ſtone, that would tremble at the ſound of one particular organ pipe; and Morhoff mentions one Petter, a Dutchman, who could break rummer-glaſſes with the tone of his voice. Merſenne alſo mentions a particular part of a pavement, that would ſhake and tremble, as if the earth would open, when the organs played. Mr. Boyle adds, that ſeats will tremble at the ſound of organs; that he has felt his hat do ſo under his hand, at certain notes both of organs and diſcourſe; and that he was well informed every well-built vault would thus anſwer to ſome determinate note.

It has been diſputed among the Learned, whether the Ancients or Moderns beſt underſtood and practiſed Muſic. Some maintain that the ancient art of Muſic, by which ſuch wonderful effects were performed, is quite loſt; and others, that the true ſcience of harmony is now arrived at much greater perfection than was known or practiſed among the Ancients. This point ſeems no other way to be determinable but by comparing the principles and practice of the one with thoſe of the other. As to the theory or principles of harmonics, it is certain we underſtand it better than the Ancients; becauſe we know all that they knew, and have improved conſiderably on their foundations. The great diſpute then lies on the practice; with regard to which it may be obſerved, that among the Ancients, Muſic, in the moſt limited ſenſe of the word, included Harmony, Rythmus, and Verſe; and conſiſted of verſes ſung by one or more voices alternately, or in choirs, ſometimes with the ſound of inſtruments, and ſometimes by voices only. Their muſical faculties, we have juſt obſerved, were Melopœia, Rythmopœia, and Poeſis; the firſt of which may be conſidered under two heads, Melody and Symphony. As to the latter, it ſeems to contain nothing but what relates to the conduct of a ſingle voice, or making what we call Melody. It does not appear that the Ancients ever thought of the concert, or harmony of parts; which is a modern invention, for which we are beholden to Guido Aretine, a Benedictine friar.

Not that the Ancients never joined more voices or inſtruments than one together in the ſame ſymphony; but that they never joined ſeveral voices ſo as that each had a diſtinct and proper melody, which made among them a ſucceſſion of various concords, and were not in every note uniſons, or at the ſame diſtance from each other as octaves. This laſt indeed agrees to the general definition of the word Symphonia; yet it is plain that in ſuch caſes there is but one ſong, and all the voices perform the ſame individual melody. But when the parts differ, not by the tenſion of the whole, but by the different relations of the ſucceſſive notes, this is the modern art, which requires ſo peculiar a genius, and on which account the modern Muſic ſeems to have much the advantage of the ancient. For farther ſatiſfaction on this head, ſee Kircher, Perrault, Wallis, Malcolm, Cerceau, and others; who unanimouſly agree, that after all the pains they have taken to know the true ſtate of the Muſic of the Ancients, they could not find the leaſt reaſon to think there was any ſuch thing in their days as Muſic in parts.

The ancient muſical notes are very myſterious and perplexed: Boethius and Gregory the Great firſt put them into a more eaſy and obvious method. In the year 1204, Guido Aretine, a Benedictine of Arezzo in Tuſcany, firſt introduced the uſe of a ſtaff with five lines, on which, with the ſpaces, he marked his notes by ſetting a point up and down upon them, to denote the riſe and fall of the voice: though Kircher ſays this artifice was in uſe before Guido's time.

Another contrivance of Guido's was to apply the ſix muſical ſyllables, *ut*, *re*, *mi*, *fa*, *ſol*, *la*, which he took out of the Latin hymn,

UT queant laxis RE ſonare fibris
MIra geſtorum FAmuli tuorum,
SOLve polluti LAbii reatum,
O Pater Alme.

We find another application of them in the following lines.

UT RElevit MIſerum FAtum, SOLitoſque LAbores
Aevi, ſit dulcis muſica noſter amor.

Beſides his notes of Muſic, by which, according to Kircher, he diſtinguiſhed the tones, or modes, and the ſeats of the ſemitones, he alſo invented the ſcale, and ſeveral muſical inſtruments, called polyplectra, as ſpinets and harpſichords.

The next conſiderable improvement was in 1330, when Joannes Muria, or de Muris, doctor at Paris (or as Bayle and Geſner make him, an Engliſhman), invented the different figures of notes, which expreſs the times or length of every note, at leaſt their true relative proportions to one another, now called longs, breves, ſemi-breves, crotchets, quavers, &c.

The moſt ancient writer on Muſic was Laſus Hermionenſis; but his works, as well as thoſe of many others, both Greek and Roman, are loſt. Ariſtoxenus, diſciple of Ariſtotle, is the earlieſt author extant on the ſubject: after whom came Euclid, author of the Elements of Geometry; and Ariſtides Quintilianus wrote after Cicero's time. Alypius ſtands next; after him Gaudentius the philoſopher, and Nicomachus the Pythagorean, and Bacchius. Of which ſeven Greek authors we have a fair copy, with a tranſlation and notes, by Meibomius. Ptolomy, the celebrated aſtronomer, wrote in Greek on the principles of harmonics, about the time of the emperor Antoninus Pius. This author keeps a medium between the Pythagoreans and Ariſtoxenians. He was ſucceeded at a conſiderable diſtance by Manuel Bryennius.

Of the Latins, we have Boetius, who wrote in the time of Theodoric the Goth; and one Caſſiodorus, about the ſame time; Martianus, and St. Auguſtine, not far remote.

And of the moderns are Zarlin, Salinas, Vincenzo Galileo, Doni, Kircher, Merſenne, Paran, De Caux, Perrault, Des Cartes, Wallis, Holder, Malcolm, Rouſſeau, &c.

Musical *Numbers*, are the numbers 2, 3, and 5, together with their compoſites. They are ſo called, becauſe all the intervals of muſic may be expreſſed by ſuch numbers. This is now generally admitted by muſical

musical theorists. Mr. Euler seems to suppose, that 7 or other primes might be introduced; but he speaks of this as a doubtful and difficult matter. Here 2 corresponds to the octave, 3 to the fifth, or rather to the 12th, and 5 to the third major, or rather the seventeenth. From these three may all other intervals be found.

Musical *Proportion*, or Harmonical Proportion, is when, of four terms, the first is to the 4th, as the difference of the 1st and 2d is to the difference of the 3d and 4th: as 2, 3, 4, and 8 are in Musical proportion, because 2 : 8 : : 1 : 4. And hence, if there be only three terms, the middle term supplying the place of both the 2d and 3d, the 1st is to the 3d, as the difference of the 1st and 2d, is to the difference of the 2d and 3d: as in these 2, 3, 6; where 2 : 6 : : 1 : 3. See Harmonical *Proportion*.

MUSSCHENBROEK (Peter), a very distinguished natural philosopher and mathematician, was born at Utrecht a little before 1700. He was first professor of these sciences in his own university, and afterwards invited to the chair at Leyden, where he died full of reputation and honours in 1761. He was a member of several academies, particularly the Academy of Sciences at Paris. He published several works in Latin, all of them shewing his great penetration and accuracy. As,

1. His Elements of Physico-Mathematics, in 1726.
2. Elements of Physics, in 1736.
3. Institutions of Physics; containing an abridgment of the new discoveries made by the Moderns; in 1748.
4. Introduction to Natural Philosophy; which he began to print in 1760; and which was completed and published at Leyden, in 1762, by M. Lulofs, after the death of the author. It was translated into French by M. Sigaud de la Fond, and published at Paris in 1769, in 3 vols 4to; under the title of A Course of Experimental and Mathematical Physics.

He had also several papers, chiefly on meteorology, printed in the volumes of Memoirs of the Academy of Sciences, viz, in those of the years 1734, 1735, 1736, 1753, 1756, and 1760.

MUTULE, a kind of square modillion in the Doric frize.

MYRIAD, the number of 10,000, or ten thousand.

N.

NABONASSAR, first king of the Chaldeans; memorable for the Jewish era which bears his name, which began on Wednesday February 26th in the 3967th year of the Julian period, or 747 years before Christ; the years of this epoch being Egyptian ones, of 365 days each. This is a remarkable era in chronology, because Ptolomy assures us there were astronomical observations made by the Chaldeans from Nabonassar to his time; also Ptolomy, and the other astronomers, account their years from that epoch.

Nabonassar was the first king of the Chaldeans or Babylonians. These having revolted from the Medes, who had overthrown the Assyrian monarchy, did, under Nabonassar, found a dominion, which was much increased under Nebuchadnezzar. It is probable this Nabonassar is that Baladan in the 2d Book of Kings, xx, 12, father of Merodach, who sent ambassadors to Hezekiah. See 2 Chron. xxii.

NADIR, that point of the heavens diametrically under our feet, or opposite to the zenith, which is directly over our heads. The zenith and Nadir are the two poles of the horizon, each being 90° distant from it.

The Sun's Nadir, is the axis of the cone projected by the shadow of the earth: so called, because that axis being prolonged, gives a point in the ecliptic diametrically opposite to the sun.

NAKED, in Architecture, as the Naked of a wall, &c, is the surface, or plane, from whence the projectures arise; or which serves as a ground to the projectures.

NAPIER, or Neper (John), baron of Merchiston in Scotland, inventor of the logarithms, was the eldest son of Sir Archibald Napier of Merchiston, and born in the year 1550. Having given early indications of great natural parts, his father was careful to have them cultivated by a liberal education. After going through the ordinary course of education at the university of St. Andrew's, he made the tour of France, Italy, and Germany. On his return to his native country, his literature and other fine accomplishments soon rendered him conspicuous; he however retired from the world to pursue literary researches, in which he made an uncommon progress, as appears by the several useful discoveries with which he afterwards favoured mankind. He chiefly applied himself to the study of mathematics; without however neglecting that of the Scriptures; in both of which he discovered the most extensive knowledge and profound penetration. His Essay upon the book of the Apocalypse indicates the most acute investigation;

investigation; though time hath discovered that his calculations concerning particular events had proceeded upon fallacious data. But what has chiefly rendered his name famous, was his great and fortunate discovery of logarithms in trigonometry, by which the ease and expedition in calculation have so wonderfully assisted the science of astronomy and the arts of practical geometry and navigation. Napier, having a great attachment to astronomy, and spherical trigonometry, had occasion to make many numeral calculations of such triangles, with sines, tangents, &c; and these being expressed in large numbers, they hence occasioned a great deal of labour and trouble: To spare themselves part of this labour, Napier, and other authors about his time, set themselves to find out certain short modes of calculation, as is evident from many of their writings. To this necessity, and these endeavours it is, that we owe several ingenious contrivances; particularly the computation by Napier's Rods, and several other curious and short methods that are given in his *Rabdologia;* and at length, after trials of many other means, the most complete one of logarithms, in the actual construction of a large table of numbers in arithmetical progression, adapted to a set of as many others in geometrical progression. The property of such numbers had been long known, viz, that the addition of the former answered to the multiplication of the latter, &c; but it wanted the necessity of such very troublesome calculations as those above mentioned, joined to an ardent disposition, to make such a use of that property. Perhaps also this disposition was urged into action by certain attempts of this kind which it seems were made elsewhere; such as the following, related by Wood in his Athenæ Oxonienses, under the article Briggs, on the authority of Oughtred and Wingate, viz, "That one Dr. Craig a Scotchman, coming out of Denmark into his own country, called upon John Neper baron of Marcheston near Edinburgh, and told him among other discourses of a new invention in Denmark (by Longomontanus as 'tis said) to save the tedious multiplication and division in astronomical calculations. Neper being solicitous to know farther of him concerning this matter, he could give no other account of it, than that it was by proportionable numbers. Which hint Neper taking, he desired him at his return to call upon him again. Craig, after some weeks had passed, did so, and Neper then shewed him a rude draught of that he called *Canon Mirabilis Logarithmorum.* Which draught, with some alterations, he printing in 1614, it came forthwith into the hands of our author Briggs, and into those of William Oughtred, from whom the relation of this matter came."

Whatever might be the inducement however, Napier published his invention in 1614, under the title of *Logarithmorum Canonis Descriptio, &c,* containing the construction and canon of his logarithms, which are those of the kind that is called hyperbolic. This work coming presently to the hands of Mr. Briggs, then Professor of Geometry at Gresham College in London, he immediately gave it the greatest encouragement, teaching the nature of the logarithms in his public lectures, and at the same time recommending a change in the scale of them, by which they might be advantageously altered to the kind which he afterwards computed himself, which are thence called Briggs's Logarithms, and are those now in common use. Mr. Briggs also presently wrote to lord Napier upon this proposed change, and made journeys to Scotland the two following years, to visit Napier, and consult him about that alteration, before he set about making it. Briggs, in a letter to archbishop Usher, March 10, 1615, writes thus: "Napier lord of Markinston hath set my head and hands at work with his new and admirable logarithms. I hope to see him this summer, if it please God; for I never saw a book which pleased me better, and made me more wonder." Briggs accordingly made him the visit, and staid a month with him.

The following passage, from the life of Lilly the astrologer, contains a curious account of the meeting of those two illustrious men. "I will acquaint you (says Lilly) with one memorable story related unto me by John Marr, an excellent mathematician and geometrician, whom I conceive you remember. He was servant to King James and Charles the First. At first when the lord Napier, or Marchiston, made public his logarithms, Mr. Briggs, then reader of the astronomy lectures at Gresham College in London, was so surprised with admiration of them, that he could have no quietness in himself until he had seen that noble person the lord Marchiston, whose only invention they were: he acquaints John Marr herewith, who went into Scotland before Mr. Briggs, purposely to be there when these two so learned persons should meet. Mr. Briggs appoints a certain day when to meet at Edinburgh; but failing thereof, the lord Napier was doubtful he would not come. It happened one day as John Marr and the lord Napier were speaking of Mr. Briggs; 'Ah, John (said Marchiston), Mr. Briggs will not now come.' At the very instant one knocks at the gate; John Marr hasted down, and it proved Mr. Briggs to his great contentment. He brings Mr. Briggs up into my lord's chamber, where almost one quarter of an hour was spent, each beholding other almost with admiration before one word was spoke. At last Mr. Briggs began: 'My lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help into astronomy, viz, the logarithms; but, my lord, being by you found out, I wonder no body else found it out before, when now known it is so easy.' He was nobly entertained by the lord Napier; and every summer after that, during the lord's being alive, this venerable man Mr. Briggs went purposely into Scotland to visit him."

Napier made also considerable improvements in spherical trigonometry &c, particularly by his Catholic or Universal Rule, being a general theorem by which he resolves all the cases of right-angled spherical triangles in a manner very simple, and easy to be remembered, namely, by what he calls the Five Circular Parts. His Construction of Logarithms too, beside the labour of them, manifests the greatest ingenuity. Kepler dedicated his Ephemerides to Napier, which were published in the year 1617; and it appears from many passages in his letter about this time, that he accounted Napier to be the greatest man of his age in the particular department to which he applied his abilities.

The

The laſt literary exertion of this eminent perſon was the publication of his *Rabdology and Promptuary*, in the year 1617; ſoon after which he died at Marchiſton, the 3d of April in the ſame year, in the 68th year of his age.—The liſt of his works is as follows:

1. A Plain Diſcovery of the Revelation of St. John; 1593.

2. *Logarithmorum Canonis Deſcriptio*; 1614.

3. *Mirifici Logarithmorum Canonis Conſtructio; et eorum ad Naturales ipſorum numeros habitudines; una cum appendice, de alia eaque præſtantiore Logarithmorum ſpecie condenda. Quibus acceſſere propoſitiones ad triangula ſphærica faciliore calculo reſolvenda. Una cum Annotationibus aliquot doctiſſimi D. Henrici Briggii in eas, & memoratam appendicem.* Publiſhed by the author's ſon in 1619.

4. *Rabdologia, ſeu Numerationis per Virgulas, libri duo*; 1617. This contains the deſcription and uſe of the Bones or Rods; with ſeveral other ſhort and ingenious modes of calculation.

5. His Letter to Anthony Bacon (the original of which is in the archbiſhop's library at Lambeth), intitled, Secret Inventions, Profitable and Neceſſary in theſe days for the Defence of this Iſland, and withſtanding Strangers Enemies to God's Truth and Religion; dated June 2, 1596.

Napier's *Bones*, or *Rods*, an inſtrument contrived by lord Napier, for the more eaſy performing of the arithmetical operations of multiplication, diviſion, &c. Theſe rods are five in number, made of Bone, ivory, horn, wood, or paſteboard, &c. Their faces are divided into nine little ſquares (fig. 7, pl. 16); each of which is parted into two triangles by diagonals. In theſe little ſquares are written the numbers of the multiplication-table; in ſuch manner as that the units, or right-hand figures, are found in the right-hand triangle: and the tens, or the left-hand figures, in the left-hand triangle; as in the figure.

To Multiply Numbers by Napier's *Bones*. Diſpoſe the rods in ſuch manner, as that the top figures may exhibit the multiplicand; and to theſe, on the left-hand, join the rod of units: in which ſeek the right-hand figure of the multiplier: and the numbers correſponding to it, in the ſquares of the other rods, write out, by adding the ſeveral numbers occurring in the ſame rhomb together, and their ſums. After the ſame manner write out the numbers correſponding to the other figures of the multiplier; diſpoſing them under one another as in the common multiplication; and laſtly add the ſeveral numbers into one ſum.

```
   5978
    937
 ------
  41846
 17934
53802
-------
5601386
```

For example, ſuppoſe the multiplicand 5978, and the multiplier 937. From the outermoſt triangle on the right-hand (fig. 8, pl. 16) which correſponds to the right-hand figure of the multiplier 7, write out the figure 6, placing it under the line. In the next rhomb towards the left, add 9 and 5; their ſum being 14, write the right-hand figure 4, againſt 6; carrying the left-hand figure 1 to 4 and 3, which are found in the next rhomb: oin the ſum 8 to 46, already ſet down. After the ſame manner, in the laſt rhomb, add 6 and 5, and the latter figure of the ſum 11, ſet down as before, and carry 1 to the 3 found in the left-hand triangle; the ſum 4 join as before on the left-hand of 1846. Thus you will have 41846 for the product of 5978 by 7. And in the ſame manner are to be found the products for the other figures of the multiplier; after which the whole is to be added together as uſual.

To perform Diviſion by Napier's *Bones*. Diſpoſe the rods ſo, as that the uppermoſt figures may exhibit the diviſor; to theſe on the left-hand, join the rod of units. Deſcend under the diviſor, till you meet thoſe figures of the dividend in which it is firſt required how oft the diviſor is found, or at leaſt the next leſs number, which is to be ſubtracted from the dividend; then the number correſponding to this, in the place of units, ſet down for a quotient. And by determining the other parts of the quotient after the ſame manner, the diviſion will be completed.

```
5978)5601386(937
     53802
     -----
      22118
      17934
      -----
       41846
       41846
       -----
```

For example; ſuppoſe the dividend 5601386, and the diviſor 5978; ſince it is firſt enquired how often 5978 is found in 56013, deſcend under the diviſor (fig. 8) till in the loweſt ſeries you find the number 53802, approaching neareſt to 56013; the former of which is to be ſubtracted from the latter, and the figure 9 correſponding to it in the rod of units ſet down for the quotient. To the remainder 2211 join the following figure 8 of the dividend; and the number 17934 being found as before for the next leſs number to it, the correſponding number 3 in the rod of units is to be ſet down for the next figure of the quotient. After the ſame manner the third and laſt figure of the quotient will be found to be 7; and the whole quotient 937.

NATIVITY, in Aſtrology, the ſcheme or figure of the heavens, and particularly of the twelve houſes, at the moment when a perſon was born; called alſo the Horoſcope.

To Caſt the Nativity, is to calculate the poſition of the heavens, and erect the figure of them for the time of birth.

NATURAL *Day, Year, &c.* See Day, Year, &c.

Natural *Horizon*, is the ſenſible or phyſical horizon.

Natural *Magic*, is that which only makes uſe of natural cauſes; ſuch as the Treatiſe of J. Bapt. Porta, Magia Naturalia.

Natural *Philoſophy*, otherwiſe called *Phyſics*, is that ſcience which conſiders the powers of nature, the properties of natural bodies, and their actions upon one another.

Laws of Nature, are certain axioms, or general rules, of motion and reſt, obſerved by natural bodies in their actions upon one another. Of theſe Laws, Sir I. Newton has eſtabliſhed three:

1ſt Law.—That every body perſeveres in the ſame ſtate, either of reſt, or uniform rectilinear motion; unleſs it is compelled to change that ſtate by the action of ſome foreign force or agent. Thus, projectiles perſevere in their motions, except ſo far as they are

retarded by the resistance of the air, and the action of gravity: and thus a top, once set up in motion, only ceases to turn round, because it is resisted by the air, and by the friction of the plane upon which it moves. Thus also the larger bodies of the planets and comets preserve their progressive and circular motions a long time undiminished, in regions void of all sensible resistance.—As body is passive in receiving its motion, and the direction of its motion, so it retains them, or perseveres in them, without any change, till it be acted upon by something external.

2d Law.—The Motion, or Change of Motion, is always proportional to the moving force by which it is produced, and in the direction of the right line in which that force is impressed. If a certain force produce a certain motion, a double force will produce double the motion, a triple force triple the motion, and so on. And this motion, since it is always directed to the same point with the generating force, if the body were in motion before, is either to be added to it, as where the motions conspire; or subtracted from it, as when they are opposite; or combined obliquely, when oblique: being always compounded with it according to the determination of each.

3d Law.—Re-action is always contrary, and equal to action; or the actions of two bodies upon one another, are always mutually equal, and directed contrary ways; and are to be estimated always in the same right line. Thus, whatever body presses or draws another, is equally pressed or drawn by it. So, if I press a stone with my finger, the finger is equally pressed by the stone: if a horse draw a weight forward by a rope, the horse is equally opposed or drawn back towards the weight; the equal tension or stretch of the rope hindering the progress of the one, as it promotes that of the other. Again, if any body, by striking on another, do in any manner change its motion, it will itself, by means of the other, undergo also an equal change in its own motion, by reason of the equality of the pressure. When two bodies meet, each endeavours to persevere in its state, and resists any change: and because the change which is produced in either may be equally measured by the action which it excites upon the other, or by the resistance which it meets with from it, it follows that the changes produced in the motions of each are equal, but are made in contrary directions: the one acquires no new force but what the other loses in the same direction; nor does this last lose any force but what the other acquires; and hence, though by their collisions, motion passes from the one to the other, yet the sum of their motions, estimated in a given direction, is preserved the same, and is unalterable by their mutual actions upon each other. In these actions the changes are equal; not those, we mean, of the velocities, but those of the motions, or momentums; the bodies being supposed free from any other impediments. For the changes of velocities, which are likewise made contrary ways, inasmuch as the motions are equally changed, are reciprocally proportional to the bodies or masses.

This law obtains also in attractions.

NAVIGATION, is the art of conducting a ship at sea from one port or place to another.

This is perhaps the most useful of all arts, and is of the highest antiquity. It may be impossible to say who were the inventors of it; but it is probable that many people cultivated it, independent of each other, who inhabited the coasts of the sea, and had occasion, or found it convenient, to convey themselves upon the water from place to place; beginning from rafts and logs of wood, and gradually improving in the structure and management of their vessels, according to the length of time, and extent of their voyages. Writers however ascribe the invention of this art to different persons, or nations, according to their different sources of information. Thus,

The poets refer the invention of Navigation to Neptune, some to Bacchus, others to Hercules, to Jason, or to Janus, who it is said made the first ship. Historians ascribe it to the Æginetes, the Phœnicians, Tyrians, and the ancient inhabitants of Britain. Some are of opinion that the first hint was taken from the flight of the kite; and some, as Oppian (De Piscibus, lib. 1) from the fish called Nautilus; while others ascribe it to accident; and others again deriving the hint and invention from Noah's ark.

However, history represents the Phœnicians, especially those of the capital Tyre, as the first navigators that made any extensive progress in the art, so far as has come to our knowledge; and indeed it must have been this very art that made their city what it was. For this purpose, Lebanon, and the other neighbouring mountains, furnishing them with excellent wood for ship-building, they were speedily masters of a numerous fleet, with which constantly hazarding new navigations, and settling new trades, they soon arrived at an incredible pitch of opulence and populousness; so as to be in a condition to send out colonies, the principal of which was that of Carthage; which, keeping up their Phœnician spirit of commerce, in time far surpassed Tyre itself; sending their merchant ships through Hercules's pillars, now the straits of Gibraltar, and thence along the western coasts of Africa and Europe; and even, according to some authors, to America itself. The city of Tyre being destroyed by Alexander the Great, its Navigation and commerce were transferred by the conqueror to Alexandria, a new city, well situated for these purposes, and proposed for the capital of the empire of Asia, the conquest of which Alexander then meditated. And thus arose the Navigation of the Egyptians; which was afterwards so cultivated by the Ptolomies, that Tyre and Carthage were quite forgotten.

Egypt being reduced to a Roman province after the battle of Actium, its trade and Navigation fell into the hands of Augustus; in whose time Alexandria was only inferior to Rome; and the magazines of the capital of the world were wholly supplied with merchandizes from the capital of Egypt.

At length, Alexandria itself underwent the fate of Tyre and Carthage; being surprised by the Saracens, who, in spite of the emperor Heraclius, overspread the northern coasts of Africa, &c; whence the merchants being driven, Alexandria has ever since been in a languishing state, though still it has a considerable part of the commerce of the christian merchants trading to the Levant.

The fall of Rome and its empire drew along with it not only that of learning and the polite arts, but that of Naviga-

Navigation alſo; the barbarians, into whoſe hands it fell, contenting themſelves with the ſpoils of the induſtry of their predeceſſors.

But no ſooner were the brave among thoſe nations well ſettled in their new provinces; ſome in Gaul, as the Franks; others in Spain, as the Goths; and others in Italy, as the Lombards; but they began to learn the advantages of Navigation and commerce, with the methods of managing them, from the people they ſubdued; and this with ſo much ſucceſs, that in a little time ſome of them became able to give new leſſons, and ſet on foot new inſtitutions for its advantage. Thus it is to the Lombards we uſually aſcribe the invention and uſe of banks, book-keeping, exchanges, rechanges, &c.

It does not appear which of the European people, after the ſettlement of their new maſters, firſt betook themſelves to Navigation and commerce.—Some think it began with the French; though the Italians ſeem to have the juſter title to it, and are uſually conſidered as the reſtorers of them, as well as of the polite arts, which had been baniſhed together from the time the empire was torn aſunder. It is the people of Italy then, and particularly thoſe of Venice and Genoa, who have the glory of this reſtoration; and it is to their advantageous ſituation for Navigation that they in a great meaſure owe their glory. From about the time of the 6th century, when the inhabitants of the iſlands in the bottom of the Adriatic began to unite together, and by their union to form the Venetian ſtate, their fleets of merchantmen were ſent to all the parts of the Mediterranean; and at laſt to thoſe of Egypt, particularly Cairo, a new city, built by the Saracen princes on the eaſtern banks of the Nile, where they traded for their ſpices and other products of the Indies. Thus they flouriſhed, increaſed their commerce, their Navigation, and their conqueſts on the terra firma, till the league of Cambray in 1508, when a number of jealous princes conſpired to their ruin; which was the more eaſily effected by the diminution of their Eaſt-India commerce, of which the Portugueſe had got one part, and the French another. Genoa too, which had cultivated Navigation at the ſame time with Venice, and that with equal ſucceſs, was a long time its dangerous rival, diſputed with it the empire of the ſea, and ſhared with it the trade of Egypt; and other parts both of the eaſt and weſt.

Jealouſy ſoon began to break out; and the two republics coming to blows, there was almoſt continual war for three centuries, before the ſuperiority was aſcertained; when, towards the end of the 14th century, the battle of Chioza ended the ſtrife: the Genoeſe, who till then had uſually the advantage, having now loſt all; and the Venetians almoſt become deſperate, at one happy blow, beyond all expectation, ſecured to themſelves the empire of the ſea, and the ſuperiority in commerce.

About the ſame time that Navigation was retrieved in the ſouthern parts of Europe, a new ſociety of merchants was formed in the north, which not only carried commerce to the greateſt perfection it was capable of, till the diſcovery of the Eaſt and Weſt Indies, but alſo formed a new ſcheme of laws for the regulation of it, which ſtill obtain under the name of, *Uſes and Cuſtoms of the Sea.* This ſociety is that celebrated league of the Hanſe-towns, begun about the year 1164.

The art of Navigation has been greatly improved in modern times, both in reſpect of the form of the veſſels themſelves, and the methods of working or conducting them. The uſe of rowers is now entirely ſuperceded by the improvements made in the ſails, rigging, &c. It is alſo very probable, that the Ancients were neither ſo well ſkilled as the Moderns, in finding the latitudes, nor in ſteering their veſſels in places of difficult Navigation, as the Moderns. But the greateſt advantage which theſe have over the Ancients, is from the mariner's compaſs, by which they are enabled to find their way with as much facility in the midſt of an immeaſurable ocean, as the Ancients could have done by creeping along the coaſt, and never going out of ſight of land. Some people indeed contend, that this is no new invention, but that the Ancients were acquainted with it. They ſay, it was impoſſible for Solomon's ſhips to go to Ophir, Tarſhiſh, and Parvaim, which laſt they will have to be Peru, without this uſeful inſtrument. They inſiſt, that it was impoſſible for the Ancients to be acquainted with the attractive virtue of the magnet, without knowing its polarity. They even affirm, that this property of the magnet is plainly mentioned in the book of Job, where the loadſtone is called topaz, or the ſtone that turns itſelf. But, not to mention that Mr. Bruce has lately made it appear highly probable that Solomon's ſhips made no more than coaſting voyages, it is certain that the Romans, who conquered Judea, were ignorant of this inſtrument; and it is very probable, that ſo uſeful an invention, if once it had been commonly known to a nation, would never have been forgotten, or perfectly concealed from ſo prudent a people as the Romans, who were ſo much intereſted in the diſcovery of it.

Among thoſe who do agree that the mariner's compaſs is a modern invention, it has been much diſputed who was the inventor. Some give the honour of it to Flavio Gioia of Amalfi in Campania, about the beginning of the 14th century; while others ſay that it came from the eaſt, and was earlier known in Europe. But, at whatever time it was invented, it is certain, that the mariner's compaſs was not commonly uſed in Navigation before the year 1420. In that year the ſcience was conſiderably improved under the auſpices of Henry duke of Viſeo, brother to the king of Portugal. In the year 1485, Roderic and Joſeph, phyſicians to king John the 2d of Portugal, together with one Martin de Bohemia, a Portugueſe native of the iſland of Fayal, and pupil to Regiomontanus, calculated tables of the ſun's declination for the uſe of ſailors, and recommended the aſtrolabe for taking obſervations at ſea. The celebrated Columbus, it is ſaid, availed himſelf of Martin's inſtructions, and improved the Spaniards in the knowledge of this art; for the farther progreſs of which, a lecture was afterwards founded at Seville by the emperor Charles the 5th.

The diſcovery of the variation of the compaſs, is claimed by Columbus, and by Sebaſtian Cabot. The former certainly did obſerve this variation without having heard of it from any other perſon, on the 14th of September 1492, and it is very probable that Cabot might do the ſame. At that time it was found that there was no variation at the Azores, for which rea-

ſon

fon fome geographers made that the firft meridian, though it has fince been difcovered that the variation alters in time. The ufe of the crofs-ftaff now began to be introduced among failors. This ancient inftrument is defcribed by John Werner of Nuremberg, in his annotations on the firft book of Ptolomy's Geography, printed in 1514: he recommends it for obferving the diftance between the moon and fome ftar, from which to determine the longitude.

At this time the art of Navigation was very imperfect, from the ufe of the plane chart, which was the only one then known, and which, by its grofs errors, muft have greatly mifled the mariner, efpecially in places far diftant from the equator; and alfo from the want of books of inftruction for feamen.

At length two Spanifh treatifes came out, the one by Pedro de Medina, in 1545; and the other by Martin Cortes, or Curtis as it is printed in Englifh, in 1556, though the author fays he compofed it at Cadiz in 1545, containing a complete fyftem of the art as far as it was then known. Medina, in his dedication to Philip prince of Spain, laments that multitudes of fhips daily perifhed at fea, becaufe there were neither teachers of the art, nor books by which it might be learned; and Cortes, in his dedication, boafts to the emperor, that he was the firft who had reduced Navigation into a compendium, valuing himfelf much on what he had performed. Medina defended the plane chart; but he was oppofed by Cortes, who fhewed its errors, and endeavoured to account for the variation of the compafs, by fuppofing the needle was influenced by a magnetic pole, different from that of the world, and which he called the *point attractive*: which notion has been farther profecuted by others. Medina's book was foon tranflated into Italian, French, and Flemifh, and ferved for a long time as a guide to foreign navigators. However, Cortes was the favourite author of the Englifh nation, and was tranflated in 1561, by Richard Eden, while Medina's work was much neglected, though tranflated alfo within a fhort time of the other. At that time a fyftem of Navigation confifted of materials fuch as the following: An account of the Ptolomaic hypothefis, and the circles of the fphere; of the roundnefs of the earth, the longitudes, latitudes, climates, &c, and eclipfes of the luminaries; a calendar; the method of finding the prime, epact, moon's age, and tides; a defcription of the compafs, an account of its variation, for the difcovering of which Cortes faid an inftrument might eafily be contrived; tables of the fun's declination for 4 years, in order to find the latitude from his meridian altitude; directions to find the fame by certain ftars: of the courfe of the fun and moon; the length of the days; of time and its divifions; the method of finding the hour of the day and night; and laftly, a defcription of the fea-chart, on which to difcover where the fhip is; they made ufe alfo of a fmall table, that fhewed, upon an alteration of one degree of the latitude, how many leagues were run on each rhumb, together with the departure from the meridian; which might be called a table of diftance and departure, as we have now a table of difference of latitude and departure. Befides, fome inftruments were defcribed, efpecially by Cortes; fuch as, one to find the place and declination of the fun, with the age and place of the moon; certain dials, the aftrolabe, and crofs-ftaff; with a complex machine to difcover the hour and latitude at once.

About the fame time propofals were made for finding the longitude by obfervations of the moon. In 1530, Gemma Frifius advifed the keeping of the time by means of fmall clocks or watches, then newly invented, as he fays. He alfo contrived a new fort of crofs-ftaff, and an inftrument called the Nautical Quadrant; which laft was much praifed by William Cuningham, in his Cofmographical Glafs, printed in the year 1559.

In the year 1537 Pedro Nunez, or Nonius, publifhed a book in the Portuguefe language, to explain a difficulty in Navigation, propofed to him by the commander Don Martin Alphonfo de Sufa. In this work he expofes the errors of the plane chart, and gives the folution of feveral curious aftronomical problems; among which is that of determining the latitude from two obfervations of the fun's altitude and the intermediate azimuth being given. He obferved, that though the rhumbs are fpiral lines, yet the direct courfe of a fhip will always be in the arch of a great circle, by which the angle with the meridians will continually change: all that the fteerfman can here do for preferving the original rhumb, is to correct thefe deviations as foon as they appear fenfible. But thus the fhip will in reality defcribe a courfe without the rhumb-line intended; and therefore his calculations for affigning the latitude, where any rhumb-line croffes the feveral meridians, will be in fome meafure erroneous. He invented a method of dividing a quadrant by means of concentric circles, which, after being much improved by Dr. Halley, is ufed at prefent, and is called a Nonius.

In 1577, Mr William Bourne publifhed a treatife, in which, by confidering the irregularities in the moon's motion, he fhews the errors of the failors in finding her age by the epact, and alfo in determining the hour from obferving on what point of the compafs the fun and moon appeared. In failing towards high latitudes, he advifes to keep the reckoning by the globe, as the plane chart is moft erroneous in fuch fituations. He defpairs of our ever being able to find the longitude, unlefs the variation of the compafs fhould be occafioned by fome fuch attractive point as Cortes had imagined; of which however he doubts: but as he had fhewn how to find the variation at all times, he advifes to keep an account of the obfervations, as ufeful for finding the place of the fhip; which advice was profecuted at large by Simon Stevin in a treatife publifhed at Leyden in 1599; the fubftance of which was the fame year printed at London in Englifh by Mr. Edward Wright, intitled the *Haven-finding Art*. In the fame old tract alfo is defcribed the way by which our failors eftimate the rate of a fhip in her courfe, by the inftrument called the Log. The author of this contrivance is not known; neither was it farther noticed till 1607, when it is mentioned in an Eaft-India voyage publifhed by Purchas: but from this time it became common, and mentioned by all authors on Navigation; and it ftill continues to be ufed as at firft, though many attempts have been made to improve it, and contrivances propofed to fupply its place; fome of which have fucceeded in ftill water, but proved ufelefs in a ftormy fea.

In

In 1581 Michael Coignet, a native of Antwerp, published a Treatife, in which he animadverted on Medina. In this he fhewed, that as the rhumbs are fpirals, making endlefs revolutions about the poles, numerous errors muft arife from their being reprefented by ftraight lines on the fea-charts; but though he hoped to find a remedy for thefe errors, he was of opinion that the propofals of Nonius were fcarcely practicable, and therefore in a great meafure ufelefs. In treating of the fun's declination, he took notice of the gradual decreafe in the obliquity of the ecliptic; he alfo defcribed the Crofs-Staff with three tranfverfe pieces, as it was then in common ufe among the failors. He likewife gave fome inftruments of his own invention; but all of them are now laid afide, excepting perhaps his Nocturnal. He conftructed a fea-table, to be ufed by fuch as failed beyond the 60th degree of latitude; and at the end of the book is delivered a Method of Sailing on a Parallel of Latitude, by means of a ring dial and a 24 hour glafs.

In the fame year Mr. Robert Norman publifhed his Difcovery of the Dipping-needle, in a pamphlet called the New Attractive; to which is always fubjoined Mr. William Burroughs's Difcourfe of the Variation of the Compafs.—In 1594, Capt. John Davis publifhed a fmall treatife, entitled the Seaman's Secrets, which was much efteemed in its time.

The writers of this period complained much of the errors of the plane chart, which continued ftill in ufe, though they were unable to difcover a proper remedy: till Gerrard Mercator contrived his Univerfal Map, which he publifhed in 1569, without clearly underftanding the principles of its conftruction: thefe were firft difcovered by Mr. Edward Wright, who fent an account of the true method of dividing the meridian from Cambridge, where he was a Fellow, to Mr. Blundeville, with a fhort table for that purpofe, and a fpecimen of a chart fo divided. Thefe were publifhed by Blundeville in 1594, among his Exercifes; to the later editions of which was added his Difcourfe of Univerfal Maps, firft printed in 1589. However, in 1599 Mr. Wright printed his Correction of certain Errors in Navigation, in which work he fhews the reafon of this divifion, the manner of conftructing his table, and its ufes in Navigation. A fecond edition of this treatife, with farther improvements, was printed in 1610, and a third edition by Mr. Moxon, in 1657.—The Method of Approximation, by what is called the middle latitude, now ufed by our failors, occurs in Gunter's works, firft printed in 1623.—About this time Logarithms began to be introduced, which were applied to Navigation in a variety of ways by Mr. Edmund Gunter; though the firft application of the Logarithmic Tables to the Cafes of Sailing, was by Mr. Thomas Addifon, in his Arithmetical Navigation, printed in 1625.—In 1635 Mr. Henry Gellibrand printed a Difcourfe Mathematical on the Variation of the Magnetical Needle, containing his difcovery of the changes to which the variation is fubject.—In 1631, Mr. Richard Norwood publifhed an excellent Treatife of Trigonometry, adapted to the invention of logarithms, particularly in applying Napier's general canons; and for the farther improvement of Navigation, he undertook the laborious work of meafuring a degree of the meridian, for examining the divifions of the log-line. He has given a full and clear account of this operation in his Seaman's Practice, firft publifhed in 1637; where he alfo defcribes his own excellent method of fetting down and perfecting a fea-reckoning, &c. This treatife, and that of Trigonometry, were often reprinted, as the principal books for learning fcientifically the art of Navigation. What he had delivered, efpecially in the latter of them, concerning this fubject, was contracted as a manual for failors in a very fmall piece, called his Epitome, which has gone through a great number of editions.—About the year 1645, Mr. Bond publifhed, in Norwood's Epitome, a very great improvement in Wright's method, by a property in his meridian line, by which its divifions are more fcientifically affigned than the author was able to effect; which he deduced from this theorem, that thefe divifions are analogous to the exceffes of the logarithmic tangents of half the refpective latitudes increafed by 45 degrees, above the logarithm of the radius: this he afterwards explained more fully in the 3d edition of Gunter's works, printed in 1653; and the demonftration of the general theorem was fupplied by Mr. James Gregory of Aberdeen, in his Exercitationes Geometricæ, printed at London in 1668, and afterwards by Dr. Halley, in the Philof. Tranf. numb. 219, as alfo by Mr. Cotes, numb. 388.—In 1700, Mr. Bond, who imagined that he had difcovered the longitude, by having difcovered the true theory of the magnetic variation, publifhed a general map, on which curve lines were drawn, expreffing the paths or places where the magnetic needle had the fame variation. The pofitions of thefe curves will indeed continually fuffer alterations; and therefore they fhould be corrected from time to time, as they have already been for the years 1744, and 1756, by Mr. William Mountaine, and Mr. James Dodfon.—The allowances proper to be made for lee-way, are very particularly fet down by Mr. John Buckler, and publifhed in a fmall tract firft printed in 1702, intitled a New Compendium of the whole Art of Navigation, written by Mr. William Jones.

As it is now generally agreed that the earth is a fpheroid, whofe axis or polar diameter is fhorter than the equatorial diameter, Dr. Murdoch publifhed a tract in 1741, in which he adapted Wright's, or Mercator's failing to fuch a figure; and in the fame year Mr. Maclaurin alfo, in the Philof. Tranf. numb. 461, for determining the meridional parts of a fpheroid; and he has farther profecuted the fame fpeculation in his Fluxions, printed in 1742.

The method of finding the longitude at fea, by the obferved diftances of the moon from the fun and ftars, commonly called the Lunar method, was propofed at an early ftage in the Art of Navigation, and has now been happily carried into effectual execution by the encouragement of the Board of Longitude, which was eftablifhed in England in the year 1714, for rewarding any fuccefsful endeavours to keep the longitude at fea. In the year 1767, this Board publifhed a Nautical Almanac, which has been continued annually ever fince, by the advice, and under the direction of the aftronomer royal at Greenwich: this work is purpofely adapted to the ufe of navigators in long voyages, and, among a great many ufeful articles, contains tables of the

lunar diſtances accurately computed for every 3 hours in the year, for the purpoſe of comparing the diſtance thus known for any time, with the diſtance obſerved in an unknown place, from whence to compute the longitude of that place. Under the auſpices of this Board too, beſides giving encouragement to the authors of many uſeful tables and other works, which would otherwiſe have been loſt, time-keepers have been brought to a wonderful degree of perfection, by Mr. Harriſon, Mr. Arnold, and many other perſons, which have proved highly advantageous in keeping the time during long voyages at ſea, and thence giving the longitude.

Some of the other principal writers on Navigation are Bartholomew Creſcenti, of Rome, in 1607; Willebrord Snell, at Leyden, in 1624, his Typhis Batavus; Geo. Fournier, at Paris, 1633; John Baptiſt Riccioli, at Bologna, in 1661; Dechales, in 1674 and 1677; the Sieur Blondel St. Aubin, in 1671 and 1673; M. Daſſier, in 1683; M. Sauveur, in 1692; M. John Bouguer, in 1698; F. Pezenas, in 1733 and 1741; and M. Peter Bouguer, who, in 1753, publiſhed a very elaborate treatiſe on this ſubject, intitled, Nouveau Traité de Navigation; in which he gives a variation compaſs of his own invention, and attempts to reform the Log, as he had before done in the Memoirs of the Academy of Sciences for 1747. He is alſo very particular in determining the lunations more accurately than by the common methods, and in deſcribing the corrections of the dead reckoning. This book was abridged and improved by M. de la Caille, in 1760. To theſe may be added the Navigation of Don George Juan of Spain, in 1757. And, in our own nation, the ſeveral treatiſes of Meſſieurs Newhouſe, Seller, Hodgſon, Atkinſon, Harris, Patoun, Hauxley, Wilſon, Moore, Nicholſon, &c; but, over all, The Elements of Navigation, in 2 vols, by Mr. John Robertſon, firſt printed about the year 1750, and ſince often re-printed; which is the moſt complete work of the kind extant; and to which work is prefixed a Diſſertation on the Riſe and Progreſs of the modern Art of Navigation, by Dr. James Wilſon, containing a very learned and elaborate hiſtory of the writings and improvements in this art.

For an account of the ſeveral inſtruments uſed in this art, with the methods for the longitude, and the various kinds and methods of Navigation, &c, ſee the reſpective articles themſelves.

NAVIGATION is either Proper or Common.

NAVIGATION, *Common*, uſually called Coaſting, in which the places are at no great diſtance from one another, and the ſhip ſails uſually in ſight of land, and moſtly within ſoundings. In this, little elſe is required beſides an acquaintance with the lands, the compaſs, and ſounding-line; each of which, ſee in its place.

NAVIGATION, *Proper*, is where the voyage is long, and purſued through the main ocean. And here, beſides the requiſites in the former caſe, are likewiſe required the uſe of Mercator's Chart, the azimuth and amplitude compaſſes, the log-line, and other inſtruments for celeſtial obſervations; as foreſtaffs, quadrants, and other ſectors, &c.

Navigation turns chiefly upon four things; two of which being given or known, the reſt are thence eaſily found out. Theſe four things are, the difference of latitude, difference of longitude, the reckoning or diſtance run, and the courſe or rhumb ſailed on. The latitudes are eaſily found, and that with ſufficient accuracy: the courſe and diſtance are had by the log-line, or dead reckoning, together with the compaſs. Nor is there any thing wanting to the perfection of Navigation, but to determine the longitude. The mathematicians and aſtronomers of many ages have applied themſelves, with great aſſiduity, to ſupply this grand deſideratum, but not altogether with the ſucceſs that was deſired, conſidering the importance of the object, and the magnificent rewards offered by ſeveral ſtates to the diſcoverer. See LONGITUDE.

Sub-Marine NAVIGATION, or the art of ſailing under water, is mentioned by Mr. Boyle, as the deſideratum of the art of Navigation. This, he ſays, was ſucceſsfully attempted, by Cornelius Drebbel; ſeveral perſons who were in the boat breathing freely all the time. See DIVING-*bell*.

Inland NAVIGATION, is that performed by ſmall craft, upon canals &c, cut through a country.

NAVIGATOR, a perſon capable of conducting a ſhip at ſea to any place propoſed.

NAUTICAL *Chart*, the ſame as Sea-Chart.

NAUTICAL *Compaſs*, the ſame as Sea-Compaſs.

NAUTICAL *Planiſphere*, a projection or conſtruction of the terreſtrial globe upon a plane, for the uſe of mariners; ſuch as the Plane Chart, and Mercator's Chart.

NEAP, or NEEP-*Tides*, are thoſe that happen at equal diſtances between the ſpring tides. The Neap tides are the loweſt, as the ſpring tides are the higheſt ones, being the oppoſites to them. And as the higheſt of the ſpring tides happens about three days after the full or change of the moon, ſo the loweſt of the Neap tides fall about three days after the quarters, or four days before the full and change; when the ſeamen ſay it is Deep Neap.

NEAPED. When a ſhip wants water, ſo that ſhe cannot get out of the harbour, out of the dock, or off the ground, the ſeamen ſay, ſhe is Neaped, or Beneaped.

NEBULOUS, or Cloudy, a term applied to certain fixed ſtars, which ſhew a dim, hazy light; being leſs than thoſe of the 6th magnitude, and therefore ſcarcely viſible to the naked eye, to which at beſt they only appear like little duſky ſpecks or clouds.

Through a moderate teleſcope, theſe Nebulous ſtars plainly appear to be congeries or cluſters of ſeveral little ſtars. In the Nebulous ſtar called Præſepe, in the breaſt of Cancer, there are reckoned 36 little ſtars, 3 of which Mr. Flamſteed ſets down in his catalogue. In the Nebulous ſtar of Orion, are reckoned 21. F. le Compte adds, that there are 40 in the Pleiades; 12 in the ſtar in the middle of Orion's ſword; 500 in the extent of two degrees of the ſame conſtellation; and 2500 in the whole conſtellation. It may farther be obſerved, that the galaxy, or milky-way, is a continued aſſemblage of Nebulæ, or vaſt cluſters of ſmall ſtars.

NEEDHAM (JOHN TUBERVILLE), a reſpectable philoſopher and catholic divine, was born at London December 10, 1713. His father poſſeſſed a conſiderable

able patrimony at Hilſton, in the county of Monmouth, being of the younger or catholic branch of the Needham family, and who died young, leaving but a ſmall fortune to his four children. Our author, who was the eldeſt ſon, ſtudied in the Engliſh college of Douai, where he took orders, taught rhetoric for ſeveral years, and ſurpaſſed all the other profeſſors of that ſeminary in the knowledge of experimental philoſophy.

In 1740, he was engaged by his ſuperiors in the ſervice of the Engliſh miſſion, and was entruſted with the direction of the ſchool erected at Twyford, near Wincheſter, for the education of the Roman Catholic youth.—In 1744 he was appointed profeſſor of philoſophy in the Engliſh college at Liſbon, where, on account of his bad health, he remained only 15 months. After his return, he paſſed ſeveral years at London and Paris, which were chiefly employed in microſcopical obſervations, and in other branches of experimental philoſophy. The reſults of theſe obſervations and experiments were publiſhed in the Philoſophical Tranſactions of the Royal Society of London in the year 1749, and in a volume in 12mo at Paris in 1750; and an account of them was alſo given by M. Buffon, in the firſt volumes of his natural hiſtory. There was an intimate connection ſubſiſted between Mr. Needham and this illuſtrious French naturaliſt: they made their experiments and obſervations together; though the reſults and ſyſtems which they deduced from the ſame objects and operations were totally different.

Mr. Needham was elected a member of the Royal Society of London in the year 1747, and of the Antiquarian Society ſome time after.—From the year 1751 to 1767 he was chiefly employed in finiſhing the education of ſeveral Engliſh and Iriſh noblemen, by attending them as tutor in their travels through France, Italy, and other countries. He then retired from this wandering life to the Engliſh ſeminary at Paris, and in 1768 was choſen by the Royal Academy of Sciences in that city a correſponding member.

When the regency of the Auſtrian Netherlands, for the revival of philoſophy and literature in that country, formed the project of an Imperial Academy, which was preceded by the erection of a ſmall literary ſociety to prepare the way for its execution, Mr. Needham was invited to Bruſſels, and was appointed ſucceſſively chief director of both theſe foundations; an appointment which he held, together with ſome eccleſiaſtical preferments in the Low Countries, till his death, which happened December the 30th 1781.

Mr. Needham's papers inſerted in the Philoſophical Tranſactions, were the following, viz:

1. Account of Chalky Tubulous Concretions, called Malm: vol. 42.
2. Microſcopical Obſervations on Worms in Smutty Corn: vol. 42.
3. Electrical Experiments lately made at Paris: vol. 44.
4. Account of M. Buffon's Mirror, which burns at 66 feet: ib.
5. Obſervations upon the Generation, Compoſition, and Decompoſition of Animal and Vegetable Subſtances: vol. 45.
6. On the Diſcovery of Aſbeſtos in France: vol. 51.

Other works printed at Paris, in French, are,

1. New Microſcopical Diſcoveries: 1745.
2. The ſame enlarged: 1750.
3. On Microſcopical, and the Generation of Organized Bodies: 2 vols, 1769.

NEEDLE, *Magnetical*, denotes a Needle, or a ſlender piece of iron or ſteel, touched with a loadſtone; which, when ſuſtained on a pivot or centre, upon which it plays round at liberty, it ſettles at length in a certain direction, either duly, or nearly north-and-ſouth, and called the magnetic meridian.

Magnetical Needles are of two kinds; Horizontal and Inclinatory.

Horizontal NEEDLES, are thoſe equally balanced on each ſide of the pivot which ſuſtains them; and which, playing horizontally, with their two extremes point out the north and ſouth parts of the horizon.

Conſtruction of a Horizontal NEEDLE. Having procured a thin light piece of pure ſteel, about 6 inches long, a perforation is made in the middle, over which a braſs cap is ſoldered on, having its inner cavity conical, ſo as to play freely on the ſtyle or pivot, which has a fine ſteel point. To give the Needle its verticity, or directive faculty, it is rubbed or ſtroked leiſurely on each pole of a magnet, from the ſouth pole towards the north; firſt beginning with the northern end, and going back at each repeated ſtroke towards the ſouth; being careful not to give a ſtroke in a contrary direction, which would take away the power again. Alſo the hand ſhould not return directly back again the ſame way it came, but ſhould return in a kind of oval figure, carrying the hand about 6 or 8 inches beyond the point where the touch ended, but not beyond on the ſide where the touch begins.

Before touching, the north end of the Needle, in our hemiſphere, is made a little lighter than the other end; becauſe the touch always deſtroys an exact balance, rendering the north end heavier than the ſouth, and thus cauſing the Needle to dip. And if, after touching, the Needle be out of its equilibrium, ſomething muſt be filed off from the heavier ſide, till it be found to balance evenly.

Needles may alſo acquire the magnetic virtue by means of artificial magnetic bars in the following manner: Lay two equal Needles parallel and about an inch aſunder, with the north end of one and the ſouth end of the other pointing the ſame way, and apply two conductors in contact with their ends: then, with two magnetic hard bars, one in each hand, and held as nearly horizontal as can be, with the upper ends, of contrary names, turned outwards to the right and left, let a Needle be ſtroked or rubbed from the middle to both ends at the ſame time, for ten or twelve times, the north end of a bar going over the ſouth end of a Needle, and the ſouth end of a bar going over the north end of a Needle: then, without moving from the place, change hands with the bars, or in the ſame hands turn the other ends downwards, and ſtroke the other Needle in like manner; ſo will they both be magnetical. But to make them ſtill ſtronger, repeat the operation three or four times from Needle to Needle, and at

at laſt turn the lower ſide of each Needle upwards, and repeat the operations of ſtroking them, as on the former ſides.

The Needles that were formerly applied to the compaſs, on board merchant ſhips, were formed of two pieces of ſteel wire, each being bent in the middle, ſo as to form an obtuſe angle, while their ends, being applied together, made an acute one, ſo that the whole repreſented the form of a lozenge. Dr. Knight, who has ſo much improved the compaſs, found, by repeated experiments, that partly from the foregoing ſtructure, and partly from the unequal hardening of the ends, theſe Needles not only varied from the true direction, but from one another, and from themſelves.

Alſo the Needles formerly uſed on board the men of war, and ſome of the larger trading ſhips, were made of one piece of ſteel, of a ſpring temper, and broad towards the ends, but tapering towards the middle. Every Needle of this form is found to have ſix poles inſtead of two, one at each end, two where it becomes tapering, and two at the hole in the middle.

To remedy theſe errors and inconveniences, the Needle which Dr. Knight contrived for his compaſs, is a ſlender parallelopipedon, being quite ſtraight and ſquare at the ends, and ſo has only two poles, although the curves are a little confuſed about the hole in the middle; though it is, upon the whole, the ſimpleſt and beſt.

Mr. Michell ſuggeſts, that it would be uſeful to increaſe the weight and length of magnetic Needles, which would render them both more accurate and permanent; alſo to cover them with a coat of linſeed oil, or varniſh, to preſerve them from any ruſt.

A Needle on occaſion may be prepared without touching it on a loadſtone: for a fine ſteel ſewing Needle, gently laid on the water, or delicately ſuſpended in the air, will take the north-and-ſouth direction.— Thus alſo a Needle heated in the fire, and cooled again in the direction of the meridian, or only in an erect poſition, acquires the ſame faculty.

Declination or Variation of the NEEDLE, is the deviation of the horizontal Needle from the meridian; or the angle it makes with the meridian, when freely ſuſpended in an horizontal plane.

A Needle is always changing the line of its direction, traverſing ſlowly to certain limits towards the eaſt and weſt ſides of the meridian. It was at firſt thought that the magnetic Needle pointed due north; but it was obſerved by Cabot and Columbus that it had a deviation from the north, though they did not ſuſpect that this deviation had itſelf a variation, and was continually changing. This change in the Variation was firſt found out, according to Bond, by Mr. John Mair, ſecondly by Mr. Gunter, and thirdly by Mr. Gellibrand, by comparing together the obſervations made at different times near the ſame place by Mr. Burrowes, Mr. Gunter, and himſelf, and he publiſhed a Diſcourſe upon it in 1635. Soon after this, Mr. Bond ventured to deliver the rate at which the Variation changes for ſeveral years; by which he foretold that at London in 1657 there would be no Variation of the compaſs, and from that time it would gradually increaſe the other way, or towards the weſt, making certain revolutions; which happened accordingly: and upon this Variation he propoſed a method of finding the longitude, which has been farther improved by many others ſince his time, though with very little ſucceſs. See VARIATION.

The period or revolution of the Variation, Henry Philips made only 370 years, but according to Henry Bond it is 600 years, and their yearly motion 36 minutes. The firſt good obſervations of the Variation were by Burrowes, about the year 1580, when the Variation at London was 11° 15′ eaſt; and ſince that time the Needle has been moving to the weſtward at that place; alſo by the obſervations of different perſons, it has been found to point, at different times, as below:

Years.	*Obſervers.*	*Variat. E. or W.*
1580	Burrowes -	11° 15′ Eaſt.
1622	Gunter - -	5 56
1634	Gellibrand -	4 3
1640	Bond - -	3 7
1657	Bond - -	0 0
1665	Bond - -	1 23 Weſt.
1666	Bond - -	1 36
1672	- - -	2 30
1683	- - -	4 30
1692	- - -	6 00
1723	Graham - -	14 17
1747	- - -	17 40
1774	Royal Society -	21 16
1775	Royal Society -	21 43
1776	Royal Society -	21 47
1777	Royal Society -	22 12
1778	Royal Society -	22 20
1779	Royal Society -	22 28
1780	Royal Society -	22 41

By this Table it appears that, from the firſt obſervations in 1580 till 1657, the change in the Variation was 11° 15′ in 77 years, which is at the rate nearly of 9′ a year; and from 1657 till 1780, or the ſpace of 123 years, it changed 22° 41′, which is at the rate of 11′ a year nearly; which it may be preſumed is very near the truth.

The Variation and Dip of the Needle was for many years carefully obſerved by the Royal Society while they met at Crane Court; and it is a pity that ſuch obſervations have not been continued ſince that time.

Dipping, or Inclinatory NEEDLE, is a Needle to ſhew the Dip of the Magnetic Needle, or how far it points below the horizon.

The Inclination or Dip of the Needle was firſt obſerved by Robert Norman, a compaſs-maker at Ratcliffe; and according to him, the dip at that place, in the year 1576, was 71° 50′; and at the Royal Society it was obſerved for ſome years lately as follows:

viz in	1776	-	72° 30′
	1778	-	72 25
	1780	-	72 17.

Mr. Henry Bond makes the Variation and Dip of the Needle depend on the ſame motion of the magnetic poles in their revolution, and upon it he founded a method of diſcovering the longitude at ſea.

NEEP

NEEP *Tides*. See NEAP *Tides*.

NEGATIVE, in Algebra, ſomething marked with the ſign −, or minus, as being contrary to ſuch as are poſitive, or marked with the ſign plus +. As Negative powers and roots, Negative quantities, &c. See POWER, ROOT, QUANTITY, &c.

NEGATIVE *Sign*, the ſign of ſubtraction −, or that which denotes ſomething in defect. Stifel is the firſt author I find who uſed this mark − for ſubtraction, or negation; before his time, the word minus itſelf was uſed, or elſe its initial *m*.

The uſe of the Negative ſign in algebra, is attended with ſeveral conſequences that at firſt ſight are admitted with ſome difficulty, and has ſometimes given occaſion to notions that ſeem to have no real foundation. This ſign implies, that the real value of the quantity repreſented by the letter to which it is prefixed, is to be ſubtracted; and it ſerves, with the poſitive ſign, to keep in view what elements or parts enter into the compoſition of quantities, and in what manner, whether as increments or decrements, that is whether by addition or ſubtraction, which is of the greateſt uſe in this art.

Hence it ſerves to expreſs a quantity of an oppoſite quality to a poſitive; ſuch as a line in a contrary poſition, a motion with oppoſite direction, or a centrifugal force in oppoſition to gravity; and thus it often ſaves the trouble of diſtinguiſhing, and demonſtrating ſeparately, the various caſes of proportions, and preſerves their analogy in view. But as the proportions of lines depend on their magnitude only, without regard to their poſition; and motions and forces are ſaid to be equal or unequal, in any given ratio, without regard to their directions; and in general the proportion of quantities relates to their magnitude only, without determining whether they are to be conſidered as increments or decrements; ſo there is no ground to imagine any other proportion of $+a$ and $-b$, than that of the real magnitudes of the quantities repreſented by a and b, whether theſe quantities are, in any particular caſe, to be added or ſubtracted.

As to the uſual arithmetical operations of addition, ſubtraction, &c, the caſe is different, as the effect of the Negative ſign is here to be carefully attended to, and is to be conſidered always as producing, in thoſe operations, an effect juſt oppoſite to the poſitive ſign. Thus, it is the ſame thing to ſubtract a decrement as to add an equal increment, or to ſubtract $-b$ from $a-b$, is to add $+b$ to it: and becauſe multiplying a quantity by a Negative number, implies only a repeated ſubtraction of it, the multiplying $-b$ by $-n$, is ſubtracting $-b$ as often as there are units in n, and is therefore equivalent to adding $+b$ ſo many times, or the ſame as adding $+nb$. But if we infer from this, that 1 is to $-n$ as $-b$ to nb, according to the rule, that unit is to one of the factors as the other factor is to the product, there is not ground to imagine that there is any myſtery in this, or any other meaning than that the real quantities repreſented by 1, n, b, and nb are proportional. For that rule relates only to the magnitude of the factors and product, without determining whether any factor, or the product, is additive or ſubtractive. But this likewiſe muſt be determined in algebraic computations; and this is the proper uſe concerning the ſigns, without which the operation could not proceed. Becauſe a quantity to be ſubtracted is never produced, in compoſition, by any repeated addition of a poſitive, or repeated ſubtraction of a Negative, a Negative ſquare number is never produced by compoſition from a root. Hence the $\surd-1$, or the ſquare root of a Negative, implies an imaginary quantity, and in reſolution is a mark or character of the impoſſible caſes of a problem, unleſs it is compenſated by another imaginary ſymbol or ſuppoſition, for then the whole expreſſion may have a real ſignification. Thus $1+\surd-1$, and $1-\surd-1$, taken ſeparately, are both imaginary, but yet their ſum is the number 2: as the conditions that ſeparately would render the ſolution of a problem impoſſible, in ſome caſes deſtroy each others effect when conjoined. In the purſuit of general concluſions, and of ſimple forms for repreſenting them, expreſſions of this kind muſt ſometimes ariſe, where the imaginary ſymbol is compenſated in a manner that is not always ſo obvious.

By proper ſubſtitutions, however, the expreſſion may be transformed into another, wherein each particular term may have a real ſignification, as well as the whole expreſſion.

The theorems that are ſometimes briefly diſcovered by the uſe of this ſymbol, may be demonſtrated without it by the inverſe operation, or ſome other way; and though ſuch ſymbols are of ſome uſe in the computations in the method of fluxions, &c, its evidence cannot be ſaid to depend upon any arts of this kind. See Maclaurin's Fluxions, book 2, chap. 1.

Mr. Baron Maſeres publiſhed a pretty large book in quarto, on the uſe of the Negative Sign in algebra.

For the rules or ways of uſing the Negative ſign in the ſeveral rules of Algebra, ſee thoſe rules ſeverally, viz, ADDITION, SUBTRACTION, MULTIPLICATION, &c. And for the method of managing the roots of Negative quantities, ſee IMPOSSIBLES.

NEPER. See NAPIER.

NEWEL, the upright poſt that ſtairs turn about; being that part of the ſtaircaſe which ſuſtains the ſteps.

NEWTON (DR. JOHN), an eminent Engliſh mathematician and divine, was the grandſon of John Newton of Axmouth in Devonſhire, and ſon of Humphrey Newton of Oundle in Northamptonſhire, where he was born in 1622. After receiving the proper foundation of a grammar education, he was ſent to Oxford, where he was entered a commoner of St. Edmund's Hall in 1637. He took the degree of bachelor of arts in 1641; and the year following he was created maſter, in precedence to many ſtudents of quality, on account of his diſtinguiſhed talents in the great branches of literature. His genius leading him ſtrongly to aſtronomy and mathematics, he applied himſelf diligently to thoſe ſciences, as well as to divinity, and made a great proficiency in them, which he found of ſome ſervice to him during Cromwell's government.

After the reſtoration of Charles the 2d, he reaped the fruits of his loyalty: being created doctor of divinity at Oxford, Sept. 1661, he was made one of the king's chaplains, and rector of Roſs in Herefordſhire, inſtead of Mr. John Toombes, ejected for nonconformity. He held this living till his death, which happened at Roſs on Chriſtmas day 1678, at 56 years of age.

Mr. Wood gave him the character of a capricious and humoursome person. However that be, his writings are a proof of his great application to study, and a sufficient monument of his genius and skill in the mathematical sciences. These are,

1. Astronomia Britannica, &c: in 4to, 1656.
2. Help to Calculation; with Tables of Declination, &c: 4to, 1657.
3. Trigonometria Britannica, in two books; the one composed by our author, and the other translated from the Latin of Henry Gellibrand: folio, 1658.
4. Chiliades Centum Logarithmorum, printed with,
5. Geometrical Trigonometry: 1659.
6. Mathematical Elements, three parts: 4to, 1660.
7. A Perpetual Diary, or Almanac: 1662.
8. Description of the Use of the Carpenter's Rule: 1667.
9. Ephemerides, shewing the interest and rate of money at 6 per cent. &c: 1667.
10. Chiliades Centum Logarithmorum et Tabula Partium Proportionalium: 1667.
11. The Rule of Interest, or the Case of Decimal Fractions, &c, part 2: 8vo, 1668.
12. School-pastimes for young children, &c: 8vo, 1669.
13. Art of Practical Gauging, &c: 1669.
14. Introduction to the art of Rhetoric: 1671.
15. The Art of Natural Arithmetic in Whole Numbers, and Fractions Vulgar and Decimal: 8vo, 1671.
16. The English Academy: 8vo, 1677.
17. Cosmography.
18. Introduction to Astronomy.
19. Introduction to Geography: 8vo, 1678.

NEWTON (Sir Isaac), one of the greatest philosophers and mathematicians the world has produced, was born at Woolstrop in Lincolnshire on Christmas day 1642. He was descended from the eldest branch of the family of Sir John Newton, bart. who were lords of the manor of Woolstrop, and had been possessed of the estate for about two centuries before; to which they had removed from Westley in the same county, but originally they came from the town of Newton in Lancashire. Other accounts say, I think more truly, that he was the only child of Mr. John Newton of Colesworth, near Grantham in Lincolnshire, who had there an estate of about 120l. a year, which he kept in his own hands. His mother was of the ancient and opulent family of the Ayscoughs, or Askews, of the same county. Our author losing his father while he was very young, the care of his education devolved on his mother, who, though she married again after his father's death, did not neglect to improve by a liberal education the promising genius that was observed in her son. At 12 years of age, by the advice of his maternal uncle, he was sent to the grammar school at Grantham, where he made a good proficiency in the languages, and laid the foundation of his future studies. Even here was observed in him a strong inclination to figures and philosophical subjects. One trait of this early disposition is told of him: he had then a rude method of measuring the force of the wind blowing against him, by observing how much farther he could leap in the direction of the wind, or blowing on his back, than he could leap the contrary way, or opposed to the wind: an early mark of his original infantine genius.

After a few years spent here, his mother took him home; intending, as she had no other child, to have the pleasure of his company; and that, after the manner of his father before him, he should occupy his own estate.

But instead of minding the markets, or the business of the farm, he was always studying and poring over his books, even by stealth, from his mother's knowledge. On one of these occasions his uncle discovered him one day in a hay-loft at Grantham, whither he had been sent to the market, working a mathematical problem; and having otherwise observed the boy's mind to be uncommonly bent upon learning, he prevailed upon his sister to part with him; and he was accordingly sent, in 1660, to Trinity College in Cambridge, where his uncle, having himself been a member of it, had still many friends. Isaac was soon taken notice of by Dr. Barrow, who was soon after appointed the first Lucasian professor of mathematics; and observing his bright genius, contracted a great friendship for him. At his outsetting here, Euclid was first put into his hands, as usual, but that author was soon dismissed; seeming to him too plain and easy, and unworthy of taking up his time. He understood him almost before he read him; and a cast of his eye upon the contents of his theorems, was sufficient to make him master of them: and as the analytical method of Des Cartes was then much in vogue, he particularly applied to it, and Kepler's Optics, &c, making several improvements on them, which he entered upon the margins of the books as he went on, as his custom was in studying any author.

Thus he was employed till the year 1664, when he opened a way into his new method of Fluxions and Infinite Series; and the same year took the degree of bachelor of arts. In the mean time, observing that the mathematicians were much engaged in the business of improving telescopes, by grinding glasses into one of the figures made by the three sections of a cone, upon the principle then generally entertained, that light was homogeneous, he set himself to grinding of optic glasses, of other figures than spherical, having as yet no distrust of the homogeneous nature of light: but not hitting presently upon any thing in this attempt to satisfy his mind, he procured a glass prism, that he might try the celebrated phenomena of colours, discovered by Grimaldi not long before. He was much pleased at first with the vivid brightness of the colours produced by this experiment; but after a while, considering them in a philosophical way, with that circumspection which was natural to him, he was surprised to see them in an oblong form, which, according to the received rule of refractions, ought to be circular. At first he thought the irregularity might possibly be no more than accidental; but this was what he could not leave without further enquiry: accordingly, he soon invented an infallible method of deciding the question, and the result was, his *New Theory of Light and Colours*.

However, the theory alone, unexpected and surprising as it was, did not satisfy him; he rather considered the

the proper ufe that might be made of it for improving telefcopes, which was his firft defign. To this end, having now difcovered that light was not homogeneous, but an heterogeneous mixture of differently refrangible rays, he computed the errors arifing from this different refrangibility; and, finding them to exceed fome hundreds of times thofe occafioned by the circular figure of the glaffes, he threw afide his glafs works, and took reflections into confideration. He was now fenfible that optical inftruments might be brought to any degree of perfection defired, in cafe there could be found a reflecting fubftance which would polifh as finely as glafs, and reflect as much light as glafs tranfmits, and the art of giving it a parabolical figure he alfo attained: but thefe feemed to him very great difficulties; nay, he almoft thought them infuperable, when he further confidered, that every irregularity in a reflecting fuperficies makes the rays ftray five or fix times more from their due courfe, than the like irregularities in a refracting one.

Amidft thefe fpeculations, he was forced from Cambridge, in 1665, by the plague; and it was more than two years before he made any further progrefs in the fubject. However, he was far from paffing his time idly in the country; on the contrary, it was here, at this time, that he firft ftarted the hint that gave rife to the fyftem of the world, which is the main fubject of the *Principia*. In his retirement, he was fitting alone in a garden, when fome apples falling from a tree, led his thoughts upon the fubject of gravity; and, reflecting on the power of that principle, he began to confider, that, as this power is not found to be fenfibly diminifhed at the remoteft diftance from the centre of the earth to which we can rife, neither at the tops of the loftieft buildings, nor on the fummits of the higheft mountains, it appeared to him reafonable to conclude, that this power muft extend much farther than is ufually thought. "Why not as high as the moon? faid he to himfelf; and if fo, her motion muft be influenced by it; perhaps fhe is retained in her orbit by it: however, though the power of gravity is not fenfibly weakened in the little change of diftance at which we can place ourfelves from the centre of the earth, yet it is very poffible that, at the height of the moon, this power may differ in ftrength much from what it is here." To make an eftimate what might be the degree of this diminution, he confidered with himfelf, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried about the fun by the like power; and, by comparing the periods of the feveral planets with their diftances from the fun, he found, that if any power like gravity held them in their courfes, its ftrength muft decreafe in the duplicate proportion of the increafe of diftance. This he concluded, by fuppofing them to move in perfect circles, concentric to the fun, from which the orbits of the greateft part of them do not much differ. Suppofing therefore the force of gravity, when extended to the moon, to decreafe in the fame manner, he computed whether that force would be fufficient to keep the moon in her orbit.

In this computation, being abfent from books, he took the common eftimate in ufe among the geographers and our feamen, before Norwood had meafured the earth, namely that 60 miles make one degree of latitude; but as that is a very erroneous fuppofition, each degree containing about 69½ of our Englifh miles, his computation upon it did not make the power of gravity, decreafing in a duplicate proportion to the diftance, anfwerable to the power which retained the moon in her orbit: whence he concluded, that fome other caufe muft at leaft join with the action of the power of gravity on the moon. For this reafon he laid afide, for that time, any further thoughts upon the matter. Mr. Whifton (in his Memoirs, pa. 33) fays, he told him that he thought Des Cartes's vortices might concur with the action of gravity.

Nor did he refume this enquiry on his return to Cambridge, which was fhortly after. The truth is, his thoughts were now engaged upon his newly projected reflecting telefcope, of which he made a fmall fpecimen, with a metallic reflector fpherically concave. It was but a rude effay, chiefly defective by the want of a good polifh for the metal. This inftrument is now in the poffeffion of the Royal Society. In 1667 he was chofen Fellow of his college, and took the degree of mafter of arts. And in 1669 Dr. Barrow refigned to him the mathematical chair at Cambridge, the bufinefs of which appointment interrupted for a while his attention to the telefcope: however, as his thoughts had been for fome time chiefly employed upon optics, he made his difcoveries in that fcience the fubject of his lectures, for the firft three years after he was appointed Mathematical Profeffor: and having now brought his *Theory of Light and Colours* to a confiderable degree of perfection, and having been elected a Fellow of the Royal Society in Jan. 1672, he communicated it to that body, to have their judgment upon it; and it was afterwards publifhed in their Tranfactions, viz, of Feb. 19, 1672. This publication occafioned a difpute upon the truth of it, which gave him fo much uneafinefs, that he refolved not to publifh any thing further for a while upon the fubject; and in that refolution he laid up his *Optical Lectures*, although he had prepared them for the prefs. And the *Analyfis by Infinite Series*, which he had intended to fubjoin to them, unhappily for the world, underwent the fame fate, and for the fame reafon.

In this temper he refumed his telefcope; and obferving that there was no abfolute neceffity for the parabolic figure of the glaffes, fince, if metals could be ground truly fpherical, they would be able to bear as great apertures as men could give a polifh to, he completed another inftrument of the fame kind. This anfwering the purpofe fo well, as, though only half a foot in length, to fhew the planet Jupiter diftinctly round, with his four fatellites, and alfo Venus horned, he fent it to the Royal Society, at their requeft, together with a defcription of it, with further particulars; which were publifhed in the Philofophical Tranfactions for March 1672. Several attempts were alfo made by that fociety to bring it to perfection; but, for want of a proper compofition of metal, and a good polifh, nothing fucceeded, and the invention lay dormant, till Hadley made his Newtonian telefcope in 1723. At the requeft of Leibnitz, in 1676, he explained his invention of Infinite Series, and took notice how far he had improved it by his Method of Fluxions, which however

he

he ſtill concealed, and particularly on this occaſion, by a tranſpoſition of the letters that make up the two fundamental propoſitions of it, into an alphabetical order; the letters concerning which are inſerted in Collins's Commercium Epiſtolicum, printed 1712. In the winter between the years 1676 and 1677, he found out the grand propoſition, that, by a centripetal force acting reciprocally as the ſquare of the diſtance, a planet muſt revolve in an ellipſis, about the centre of force placed in its lower focus, and, by a radius drawn to that centre, deſcribe areas proportional to the times. In 1680 he made ſeveral aſtronomical obſervations upon the comet that then appeared; which, for ſome conſiderable time, he took not to be one and the ſame, but two different comets; and upon this occaſion ſeveral letters paſſed between him and Mr. Flamſteed.

He was ſtill under this miſtake, when he received a letter from Dr. Hook, explaining the nature of the line deſcribed by a falling body, ſuppoſed to be moved circularly by the diurnal motion of the earth, and perpendicularly by the power of gravity. This letter put him upon enquiring anew what was the real figure in which ſuch a body moved; and that enquiry, convincing him of another miſtake which he had before fallen into concerning that figure, put him upon reſuming his former thoughts with regard to the moon; and Picart having not long before, viz, in 1679, meaſured a degree of the earth with ſufficient accuracy, by uſing his meaſures, that planet appeared to be retained in her orbit by the ſole power of gravity; and conſequently that this power decreaſes in the duplicate ratio of the diſtance; as he had formerly conjectured. Upon this principle, he found the line deſcribed by a falling body to be an ellipſis, having one focus in the centre of the earth. And finding by this means, that the primary planets really moved in ſuch orbits as Kepler had ſuppoſed, he had the ſatisfaction to ſee that this enquiry, which he had undertaken at firſt out of mere curioſity, could be applied to the greateſt purpoſes. Hereupon he drew up about a dozen propoſitions, relating to the motion of the primary planets round the ſun, which were communicated to the Royal Society in the latter end of 1683. This coming to be known to Dr. Halley, that gentleman, who had attempted the demonſtration in vain, applied, in Auguſt 1684, to Newton, who aſſured him that he had abſolutely completed the proof. This was alſo regiſtered in the books of the Royal Society; at whoſe earneſt ſolicitation Newton finiſhed the work, which was printed under the care of Dr. Halley, and came out about midſummer 1687, under the title of, *Philoſophiæ naturalis Principia mathematica*, containing in the third book, the Cometic Aſtronomy, which had been lately diſcovered by him, and now made its firſt appearance in the world: a work which may be looked upon as the production of a celeſtial intelligence rather than of a man.

This work however, in which the great author has built a new ſyſtem of natural philoſophy upon the moſt ſublime geometry, did not meet at firſt with all the applauſe it deſerved, and was one day to receive. Two reaſons concurred in producing this effect: Des Cartes had then got full poſſeſſion of the world. His philoſophy was indeed the creature of a fine imagination, gaily dreſſed out: he had given her likewiſe ſome of nature's fine features, and painted the reſt to a ſeeming likeneſs of her. On the other hand, Newton had with an unparalleled penetration, and force of genius, purſued nature up to her moſt ſecret abode, and was intent to demonſtrate her reſidence to others, rather than anxious to deſcribe particularly the way by which he arrived at it himſelf: he finiſhed his piece in that elegant conciſeneſs, which had juſtly gained the Ancients an univerſal eſteem. In fact, the conſequences flow with ſuch rapidity from the principles, that the reader is often left to ſupply a long chain of reaſoning to connect them: ſo that it required ſome time before the world could underſtand it. The beſt mathematicians were obliged to ſtudy it with care, before they could make themſelves maſter of it; and thoſe of a lower rank durſt not venture upon it, till encouraged by the teſtimonies of the more learned. But at laſt, when its value came to be ſufficiently known, the approbation which had been ſo ſlowly gained, became univerſal, and nothing was to be heard from all quarters, but one general burſt of admiration. "Does Mr. Newton eat, drink, or ſleep like other men?" ſays the marquis de l'Hoſpital, one of the greateſt mathematicians of the age, to the Engliſh who viſited him. "I repreſent him to myſelf as a celeſtial genius intirely diſengaged from matter."

In the midſt of theſe profound mathematical reſearches, juſt before his Principia went to the preſs in 1686, the privileges of the univerſity being attacked by James the 2d, Newton appeared among its moſt ſtrenuous defenders, and was on that occaſion appointed one of their delegates to the high-commiſſion court; and they made ſuch a defence, that James thought proper to drop the affair. Our author was alſo choſen one of their members for the Convention-Parliament in 1688, in which he ſat till it was diſſolved.

Newton's merit was well known to Mr. Montague, then chancellor of the exchequer, and afterwards earl of Halifax, who had been bred at the ſame college with him; and when he undertook the great work of recoining the money, he fixed his eye upon Newton for an aſſiſtant in it; and accordingly, in 1696, he was appointed warden of the mint, in which employment, he rendered very ſignal ſervice to the nation. And three years after he was promoted to be maſter of the mint, a place worth 12 or 15 hundred pounds per annum, which he held till his death. Upon this promotion, he appointed Mr. Whiſton his deputy in the mathematical profeſſorſhip at Cambridge, giving him the full profits of the place, which appointment itſelf he alſo procured for him in 1703. The ſame year our author was choſen preſident of the Royal Society, in which chair he ſat for 25 years, namely till the time of his death; and he had been choſen a member of the Royal Academy of Sciences at Paris in 1699, as ſoon as the new regulation was made for admitting foreigners into that ſociety.

Ever ſince the firſt diſcovery of the heterogeneous mixture of light, and the production of colours thence ariſing, he had employed a good part of his time in bringing the experiment, upon which the theory is founded, to a degree of exactneſs that might ſatisfy himſelf. The truth is, this ſeems to have been his favourite invention; 30 years he had ſpent in this arduous

ous task, before he published it in 1704. In infinite series and fluxions, and in the power and rule of gravity in preserving the solar system, there had been some, though distant hints, given by others before him: whereas in dissecting a ray of light into its primary constituent particles, which then admitted of no further separation; in the discovery of the different refrangibility of these particles thus separated; and that these constituent rays had each its own peculiar colour inherent in it; that rays falling in the same angle of incidence have alternate fits of reflection and refraction; that bodies are rendered transparent by the minuteness of their pores, and become opaque by having them large; and that the most transparent body, by having a great thinness, will become less pervious to the light: in all these, which make up his new theory of light and colours, he was absolutely and entirely the first starter; and as the subject is of the most subtle and delicate nature, he thought it necessary to be himself the last finisher of it.

In fact, the affair that chiefly employed his researches for so many years, was far from being confined to the subject of light alone. On the contrary, all that we know of natural bodies, seemed to be comprehended in it; he had found out, that there was a natural action at a distance between light and other bodies, by which both the reflections and refractions, as well as inflections, of the former, were constantly produced. To ascertain the force and extent of this principle of action, was what had all along engaged his thoughts, and what after all, by its extreme subtlety, escaped his most penetrating spirit. However, though he has not made so full a discovery of this principle, which directs the course of light, as he has in regard to the power by which the planets are kept in their courses; yet he gave the best directions possible for such as should be disposed to carry on the work, and furnished matter abundantly sufficient to animate them to the pursuit. He has indeed hereby opened a way of passing from optics to an entire system of physics; and, if we look upon his queries as containing the history of a great man's first thoughts, even in that view they must be always at least entertaining and curious.

This same year, and in the same book with his Optics, he published, for the first time, his Method of Fluxions. It has been already observed, that these two inventions were intended for the public so long before as 1672; but were laid by then, in order to prevent his being engaged on that account in a dispute about them. And it is not a little remarkable, that even now this last piece proved the occasion of another dispute, which continued for many years. Ever since 1684, Leibnitz had been artfully working the world into an opinion, that he first invented this method.—Newton saw his design from the beginning, and had sufficiently obviated it in the first edition of the Principia, in 1687 (viz, in the Scholium to the 2d lemma of the 2d book): and with the same view, when he now published that method, he took occasion to acquaint the world, that he invented it in the years 1665 and 1666. In the Acta Eruditorum of Leipsic, where an account is given of this book, the author of that account ascribed the invention to Leibnitz, intimating that Newton borrowed it from him. Dr. Keill, the astronomical professor at Oxford, undertook Newton's defence; and after several answers on both sides, Leibnitz complaining to the Royal Society, this body appointed a committee of their members to examine the merits of the case. These, after considering all the papers and letters relating to the point in controversy, decided in favour of Newton and Keill; as is related at large in the life of this last mentioned gentleman; and these papers themselves were published in 1712, under the title of Commercium Epistolicum Johannis Collins, 8vo.

In 1705, the honour of knighthood was conferred upon our author by queen Anne, in consideration of his great merit. And in 1714 he was applied to by the House of Commons, for his opinion upon a new method of discovering the longitude at sea by signals, which had been laid before them by Ditton and Whiston, in order to procure their encouragement; but the petition was thrown aside upon reading Newton's paper delivered to the committee.

The following year, 1715, Leibnitz, with the view of bringing the world more easily into the belief that Newton had taken the method of fluxions from his Differential method, attempted to foil his mathematical skill by the famous problem of the trajectories, which he therefore proposed to the English by way of challenge; but the solution of this, though the most difficult proposition he was able to devise, and what might pass for an arduous affair to any other, yet was hardly any more than an amusement to Newton's penetrating genius: he received the problem at 4 o'clock in the afternoon, as he was returning from the Mint; and, though extremely fatigued with business, yet he finished the solution before he went to bed.

As Leibnitz was privy-counsellor of justice to the elector of Hanover, so when that prince was raised to the British throne, Newton came more under the notice of the court; and it was for the immediate satisfaction of George the First, that he was prevailed on to put the last hand to the dispute about the invention of Fluxions. In this court, Caroline princess of Wales, afterwards queen consort to George the Second, happened to have a curiosity for philosophical enquiries; no sooner therefore was she informed of our author's attachment to the house of Hanover, than she engaged his conversation, which soon endeared him to her. Here she found in every difficulty that full satisfaction, which she had in vain sought for elsewhere; and she was often heard to declare publicly, that she thought herself happy in coming into the world at a juncture of time, which put it in her power to converse with him. It was at this princess's solicitation, that he drew up an abstract of his Chronology; a copy of which was at her request communicated, about 1718, to signior Conti, a Venetian nobleman, then in England, upon a promise to keep it secret. But notwithstanding this promise, the abbé, who while here had also affected to shew a particular friendship for Newton, though privately betraying him as much as lay in his power to Leibnitz, was no sooner got across the water into France, than he dispersed copies of it, and procured an antiquary to translate it into French, as well as to write a confutation of it. This, being printed at Paris in 1725, was delivered as a present from the bookseller that printed it to our author, that he might obtain, as was said, his consent

to

to the publication; but though he exprefsly refufed fuch confent, yet the whole was publifhed the fame year. Hereupon Newton found it neceffary to publifh a Defence of himfelf, which was inferted in the Philofophical Tranfactions. Thus he, who had fo much all his life long been ftudious to avoid difputes, was unavoidably all his life time, in a manner, involved in them; nor did this laft difpute even finifh at his death, which happened the year following. Newton's paper was republifhed in 1726 at Paris, in French, with a letter of the abbé Conti in anfwer to it; and the fame year fome differtations were printed there by father Souciet againft Newton's Chronological Index, an anfwer to which was inferted by Halley in the Philofophical Tranfactions, numb. 397.

Some time before this bufinefs, in his 80th year, our author was feized with an incontinence of urine, thought to proceed from the ftone in the bladder, and deemed to be incurable. However, by the help of a ftrict regimen and other precautions, which till then he never had occafion for, he procured confiderable intervals of eafe during the five remaining years of his life. Yet he was not free from fome fevere paroxyfms, which even forced out large drops of fweat that ran down his face. In thefe circumftances he was never obferved to utter the leaft complaint, nor exprefs the leaft impatience; and as foon as he had a moment's eafe, he would fmile and talk with his ufual chearfulnefs. He was now obliged to rely upon Mr. Conduit, who had married his niece, for the difcharge of his office in the Mint. Saturday morning March 18, 1727, he read the newfpapers, and difcourfed a long time with Dr. Mead his phyfician, having then the perfect ufe of all his fenfes and his underftanding; but that night he entirely loft them all, and, not recovering them afterwards, died the Monday following, March 20, in the 85th year of his age. His corpfe lay in ftate in the Jerufalem-chamber, and on the 28th was conveyed into Weftminfter-abbey, the pall being fupported by the lord chancellor, the dukes of Montrofe and Roxburgh, and the earls of Pembroke, Suffex, and Macclesfield. He was interred near the entrance into the choir on the left hand, where a ftately monument is erected to his memory, with a moft elegant infcription upon it.

Newton's character has been attempted by M. Fontenelle and Dr. Pemberton, the fubftance of which is as follows. He was of a middle ftature, and fomewhat inclined to be fat in the latter part of his life. His countenance was pleafing and venerable at the fame time; efpecially when he took off his peruke, and fhewed his white hair, which was pretty thick. He never made ufe of fpectacles, and loft but one tooth during his whole life. Bifhop Atterbury fays, that, in the whole air of Sir Ifaac's face and make, there was nothing of that penetrating fagacity which appears in his compofitions; that he had fomething rather languid in his look and manner, which did not raife any great expectation in thofe who did not know him.

His temper it is faid was fo equal and mild, that no accident could difturb it. A remarkable inftance of which is related as follows. Sir Ifaac had a favourite little dog, which he called Diamond. Being one day called out of his ftudy into the next room, Diamond was left behind. When Sir Ifaac returned, having been abfent but a few minutes, he had the mortification to find, that Diamond having overfet a lighted candle among fome papers, the nearly finifhed labour of many years was in flames, and almoft confumed to afhes. This lofs, as Sir Ifaac was then very far advanced in years, was irretrievable; yet, without once ftriking the dog, he only rebuked him with this exclamation, "Oh Diamond! Diamond! thou little knoweft the mifchief thou haft done!"

He was indeed of fo meek and gentle a difpofition, and fo great a lover of peace, that he would rather have chofen to remain in obfcurity, than to have the calm of life ruffled by thofe ftorms and difputes, which genius and learning always draw upon thofe that are the moft eminent for them.

From his love of peace, no doubt, arofe that unufual kind of horror which he felt for all difputes: a fteady unbroken attention, free from thofe frequent recoilings infeparably incident to others, was his peculiar felicity; he knew it, and he knew the value of it. No wonder then that controverfy was looked on as his bane. When fome objections, haftily made to his difcoveries concerning light and colours, induced him to lay afide the defign he had taken of publifhing his Optical Lectures, we find him reflecting on that difpute, into which he had been unavoidably drawn, in thefe terms: "I blamed my own imprudence for parting with fo real a bleffing as my quiet, to run after a fhadow." It is true this fhadow, as Fontenelle obferves, did not efcape him afterwards, nor did it coft him that quiet which he fo much valued, but proved as much a real happinefs to him as his quiet itfelf; yet this was a happinefs of his own making: he took a refolution from thefe difputes, not to publifh any more concerning that theory, till he had put it above the reach of controverfy, by the exacteft experiments, and the ftricteft demonftrations; and accordingly it has never been called in queftion fince. In the fame temper, after he had fent the manufcript to the Royal Society, with his confent to the printing of it by them; yet upon Hook's injurioufly infifting that he himfelf had demonftrated Kepler's problem before our author, he determined, rather than be involved again in a controverfy, to fuppress the third book; and he was very hardly prevailed upon to alter that refolution. It is true, the public was thereby a gainer; that book, which is indeed no more than a corollary of fome propofitions in the firft, being originally drawn up in the popular way, with a defign to publifh it in that form; whereas he was now convinced that it would be beft not to let it go abroad without a ftrict demonftration.

In contemplating his genius, it prefently becomes a doubt, which of thefe endowments had the greateft fhare, fagacity, penetration, ftrength, or diligence; and, after all, the mark that feems moft to diftinguifh it is, that he himfelf made the jufteft eftimation of it, declaring, that if he had done the world any fervice, it was due to nothing but induftry and patient thought; that he kept the fubject of confideration conftantly before him, and waited till the firft dawning opened gradually, by little and little, into a full and clear light. It is faid, that when he had any mathematical problems or folutions in his mind, he would never quit the fubject on any account. And his fervant has faid,

said, when he has been getting up in a morning, he has sometimes begun to dress, and with one leg in his breeches, sat down again on the bed, where he has remained for hours before he has got his clothes on: and that dinner has been often three hours ready for him before he could be brought to table. Upon this head several little anecdotes are related; among which is the following: Doctor Stukely coming in accidentally one day, when Newton's dinner was left for him upon the table, covered up, as usual, to keep it warm till he could find it convenient to come to table; the doctor lifting the cover, found under it a chicken, which he presently ate, putting the bones in the dish, and replacing the cover. Some time after Newton came into the room, and after the usual compliments sat down to his dinner; but on taking up the cover, and seeing only the bones of the fowl left, he observed with some little surprise, "I thought I had not dined, but I now find that I have."

After all, notwithstanding his anxious care to avoid every occasion of breaking his intense application to study, he was at a great distance from being steeped in philosophy. On the contrary, he could lay aside his thoughts, though engaged in the most intricate researches, when his other affairs required his attention; and, as soon as he had leisure, resume the subject at the point where he had left off. This he seems to have done not so much by any extraordinary strength of memory, as by the force of his inventive faculty, to which every thing opened itself again with ease, if nothing intervened to ruffle him. The readiness of his invention made him not think of putting his memory much to the trial; but this was the offspring of a vigorous intenseness of thought, out of which he was but a common man. He spent therefore the prime of his age in those abstruse researches, when his situation in a college gave him leisure, and while study was his proper business. But as soon as he was removed to the mint, he applied himself chiefly to the duties of that office; and so far quitted mathematics and philosophy, as not to engage in any pursuits of either kind afterwards.

Dr. Pemberton observes, that though his memory was much decayed in the last years of his life, yet he perfectly understood his own writings, contrary to what I had formerly heard, says the doctor, in discourse from many persons. This opinion of theirs might arise perhaps from his not being always ready at speaking on these subjects, when it might be expected he should. But on this head it may be observed, that great geniuses are often liable to be absent, not only in relation to common life, but with regard to some of the parts of science that they are best informed of: inventors seem to treasure up in their minds what they have found out, after another manner, than those do the same things, who have not this inventive faculty. The former, when they have occasion to produce their knowledge, are in some measure obliged immediately to investigate part of what they want; and for this they are not equally fit at all times: from whence it has often happened, that such as retain things chiefly by means of a very strong memory, have appeared off-hand more expert than the discoverers themselves.

It was evidently owing to the same inventive faculty that Newton, as this writer found, had read fewer of the modern mathematicians than one could have expected; his own prodigious invention readily supplying him with what he might have occasion for in the pursuit of any subject he undertook. However, he often censured the handling of geometrical subjects by algebraic calculations; and his book of algebra he called by the name of *Universal Arithmetic*, in opposition to the injudicious title of *Geometry* which Des Cartes had given to the treatise in which he shews how the geometrician may assist his invention by such kind of computations. He frequently praised Slusius, Barrow, and Huygens, for not being influenced by the false taste which then began to prevail. He used to commend the laudable attempt of Hugo d'Omerique to restore the ancient analysis; and very much esteemed Apollonius's book *De Sectione Rationis*, for giving us a clearer notion of that analysis than we had before. Dr. Barrow may be esteemed as having shewn a compass of invention equal, if not superior, to any of the Moderns, our author only excepted; but Newton particularly recommended Huygens's style and manner: he thought him the most elegant of any mathematical writer of modern times, and the truest imitator of the Ancients. Of their taste and mode of demonstration our author always professed himself a great admirer; and even censured himself for not following them yet more closely than he did; and spoke with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of Des Cartes, and other algebraic writers, before he had considered the Elements of Euclid with that attention which so excellent a writer deserves.

But if this was a fault, it is certain it was a fault to which we owe both his great inventions in speculative mathematics, and the doctrine of Fluxions and Infinite Series. And perhaps this might be one reason why his particular reverence for the Ancients is omitted by Fontenelle, who however certainly makes some amends by that just elogium which he makes of our author's modesty, which amiable quality he represents as standing foremost in the character of this great man's mind and manners. It was in reality greater than can be easily imagined, or will be readily believed: yet it always continued so without any alteration; though the whole world, says Fontenelle, conspired against it; let us add, though he was thereby robbed of his invention of Fluxions. Nicholas Mercator publishing his *Logarithmotechnia* in 1668, where he gave the quadrature of the hyperbola by an infinite series, which was the first appearance in the learned world of a series of this sort drawn from the particular nature of the curve, and that in a manner very new and abstracted; Dr. Barrow, then at Cambridge, where Mr. Newton, then about 26 years of age, resided, recollected, that he had met with the same thing in the writings of that young gentleman; and there not confined to the hyperbola only, but extended, by general forms, to all sorts of curves, even such as are mechanical; to their quadratures, their rectifications, and their centres of gravity; to the solids formed by their rotations, and to the superficies of those solids; so that, when their determinations were possible, the series stopped at a certain point, or at least their sums were given by stated rules: and if the absolute determinations were impossible, they could yet be infinitely approximated; which is the happiest and most refined

refined method, says Fontenelle, of supplying the defects of human knowledge that man's imagination could possibly invent. To be master of so fruitful and general a theory was a mine of gold to a geometrician; but it was a greater glory to have been the discoverer of so surprising and ingenious a system. So that Newton, finding by Mercator's book, that he was in the way to it, and that others might follow in his track, should naturally have been forward to open his treasures, and secure the property, which consisted in making the discovery; but he contented himself with his treasure which he had found, without regarding the glory. What an idea does it give us of his unparalleled modesty, when we find him declaring, that he thought Mercator had entirely discovered his secret, or that others would, before he should become of a proper age for writing! His manuscript upon Infinite Series was communicated to none but Mr. John Collins and the lord Brounker, then President of the Royal Society, who had also done something in this way himself; and even that had not been complied with, but for Dr. Barrow, who would not suffer him to indulge his modesty so much as he desired.

It is further observed, concerning this part of his character, that he never talked either of himself or others, nor ever behaved in such a manner, as to give the most malicious censurers the least occasion even to suspect him of vanity. He was candid and affable, and always put himself upon a level with his company. He never thought either his merit or his reputation sufficient to excuse him from any of the common offices of social life. No singularities, either natural or affected, distinguished him from other men. Though he was firmly attached to the church of England, he was averse to the persecution of the non-conformists. He judged of men by their manners; and the true schismatics, in his opinion, were the vicious and the wicked. Not that he confined his principles to natural religion, for it is said he was thoroughly persuaded of the truth of Revelation; and amidst the great variety of books which he had constantly before him, that which he studied with the greatest application was the Bible, at least in the latter years of his life: and he understood the nature and force of moral certainty as well as he did that of a strict demonstration.

Sir Isaac did not neglect the opportunities of doing good, when the revenues of his patrimony and a profitable employment, improved by a prudent œconomy, put it in his power. We have two remarkable instances of his bounty and generosity; one to Mr. Maclaurin, extra professor of mathematics at Edinburgh, to encourage whose appointment he offered 20 pounds a year to that office; and the other to his niece Barton, upon whom he had settled an annuity of 100 pounds per annum. When decency upon any occasion required expence and shew, he was magnificent without grudging it, and with a very good grace: at all other times, that pomp which seems great to low minds only, was utterly retrenched, and the expence reserved for better uses.

Newton never married; and it has been said, that "perhaps he never had leisure to think of it; that, being immersed in profound studies during the prime of his age, and afterwards engaged in an employment of great importance, and even quite taken up with the company which his merit drew to him, he was not sensible of any vacancy in life, nor of the want of a companion at home." These however do not appear to be any sufficient reasons for his never marrying, if he had had an inclination so to do. It is much more likely that he had a constitutional indifference to the state, and even to the sex in general; and it has even been said of him, that he never once knew woman.—He left at his death, it seems, 32 thousand pounds; but he made no will; which, Fontenelle tells us, was because he thought a legacy was no gift.—As to his works, besides what were published in his life-time, there were found after his death, among his papers, several discourses upon the subjects of Antiquity, History, Divinity, Chemistry, and Mathematics; several of which were published at different times, as appears from the following catalogue of all his works; where they are ranked in the order of time in which those upon the same subject were published.

1. Several Papers relating to his *Telescope*, and his *Theory of Light and Colours*, printed in the Philosophical Transactions, numbs. 80, 81, 82, 83, 84, 85, 88, 96, 97, 110, 121, 123, 128; or vols 6, 7, 8, 9, 10, 11.

2. *Optics*, or a *Treatise of the Reflections, Refractions, and Inflections, and the Colours of Light*; 1704, 4to.—A Latin translation by Dr. Clarke; 1706, 4to.—And a French translation by Pet. Coste, Amst. 1729, 2 vols 12mo.—Beside several English editions in 8vo.

3. *Optical Lectures*; 1728, 8vo. Also in several Letters to Mr. Oldenburg, secretary of the Royal Society, inserted in the General Dictionary, under our author's article.

4. *Lectiones Opticæ*; 1729, 4to.

5. *Naturalis Philosophiæ Principia Mathematica*; 1687, 4to.—A second edition in 1713, with a Preface, by Roger Cotes.—The 3d edition in 1726, under the direction of Dr. Pemberton.—An English translation, by Motte, 1729, 2 volumes 8vo, printed in several editions of his works, in different nations, particularly an edition, with a large Commentary, by the two learned Jesuits, Le Seur and Jacquier, in 4 volumes 4to, in 1739, 1740, and 1742.

6. *A System of the World*, translated from the Latin original; 1727, 8vo.—This, as has been already observed, was at first intended to make the third book of his Principia.—An English translation by Motte, 1729, 8vo.

7. *Several Letters* to Mr. Flamsteed, Dr. Halley, and Mr. Oldenburg.—See our author's article in the General Dictionary.

8. *A Paper concerning the Longitude*; drawn up by order of the House of Commons; ibid.

9. *Abregé de Chronologie*, &c; 1726, under the direction of the abbé Conti, together with some Observations upon it.

10. *Remarks upon the Observations made upon a Chronological Index of Sir I. Newton, &c.* Philos. Trans. vol. 33. See also the same, vol. 34 and 35, by Dr. Halley.

11. The Chronology of Ancient Kingdoms amended, &c; 1728, 4to.

12. *Arithmetica Universalis, &c*; under the inspection

tion of Mr. Whiſton, Cantab. 1707, 8vo. Printed I think without the author's conſent, and even againſt his will: an offence which it ſeems was never forgiven. There are alſo Engliſh editions of the ſame, particularly one by Wilder, with a Commentary, in 1769, 2 vols 8vo. And a Latin edition, with a Commentary, by Caſtilion, 2 vols 4to, Amſt. &c.

13. *Analyſis per Quantitatum Series, Fluxiones, et Differentias, cum Enumeratione Linearum Tertii Ordinis;* 1711, 4to; under the inſpection of W. Jones, Eſq. F. R. S.—The laſt tract had been publiſhed before, together with another on the *Quadrature of Curves*, by the Method of Fluxions, under the title of *Tractatus duo de Speciebus & Magnitudine Figurarum Curvilinearum;* ſubjoined to the firſt edition of his Optics in 1704; and other letters in the Appendix to Dr. Gregory's Catoptrics, &c, 1735, 8vo.—Under this head may be ranked *Newtoni Geneſis Curvarum per Umbras;* Leyden, 1740.

14. *Several Letters relating to his Diſpute with Leibnitz,* upon his Right to the Invention of Fluxions; printed in the *Commercium Epiſtolicum D. Johannis Collins & aliorum de Analyſi Promota, juſſu Societatis Regiæ editum;* 1712, 8vo.

15. Poſtſcript and Letter of M. Leibnitz to the Abbé Conti, with Remarks, and a Letter of his own to that Abbé; 1717, 8vo. To which was added, Raphſon's Hiſtory of Fluxions, as a Supplement.

16. *The Method of Fluxions, and Analyſis by Infinite Series,* tranſlated into Engliſh from the original Latin; to which is added, a Perpetual Commentary, by the tranſlator Mr. John Colſon; 1736, 4to.

17. *Several Miſcellaneous Pieces, and Letters,* as follow:—(1). A Letter to Mr. Boyle upon the ſubject of the Philoſopher's Stone. Inſerted in the General Dictionary, under the article BOYLE.—(2). A Letter to Mr. Aſton, containing directions for his travels; ibid. under our author's article.—(3). An Engliſh Tranſlation of a Latin Diſſertation upon the Sacred Cubit of the Jews. Inſerted among the miſcellaneous works of Mr. John Greaves, vol. 2, publiſhed by Dr. Thomas Birch, in 1737, 2 vols 8vo. This Diſſertation was found ſubjoined to a work of Sir Iſaac's, not finiſhed, intitled *Lexicon Propheticum.*—(4). Four Letters from Sir Iſaac Newton to Dr. Bentley, containing ſome arguments in proof of a Deity; 1756, 8vo.—(5). Two Letters to Mr. Clarke, &c.

18. *Obſervations on the Prophecies of Daniel and the Apocalypſe of St. John;* 1733, 4to.

19. *Iſ. Newtoni Elementa Perſpectivæ Univerſalis;* 1746, 8vo.

20. *Tables for purchaſing College Leaſes;* 1742, 12mo.

21. Corollaries, by Whiſton.

22. A Collection of ſeveral pieces of our author's, under the following title, *Newtoni Iſ. Opuſcula Mathematica Philoſ. & Philol.* collegit J. Caſtilioneus; Lauſ. 1744, 4to, 8 tomes.

23. *Two Treatiſes* of the Quadrature of Curves, and Analyſis by Equations of an Infinite Number of Terms, explained: tranſlated by John Stewart, with a large Commentary; 1745, 4to.

24. *Deſcription of an Inſtrument* for obſerving the Moon's Diſtance from the Fixed Stars at Sea. Philoſ. Tranſ. vol. 42.

25. Newton alſo publiſhed *Barrow's Optical Lectures,* in 1699, 4to: and *Bern. Varenii Geographia, &c;* 1681, 8vo.

26. The whole works of Newton, publiſhed by Dr. Horſley; 1779, 4to, in 5 volumes.

The following is a liſt of the papers left by Newton at his death, as mentioned above.

A Catalogue of Sir Iſaac Newton's Manuſcripts and Papers, as annexed to a Bond, given by Mr. Conduit, to the Adminiſtrators of Sir Iſaac; by which he obliges himſelf to account for any profit he ſhall make by publiſhing any of the papers.

Dr. Pellet, by agreement of the executors, entered into Acts of the Prerogative Court, being appointed to peruſe all the papers, and judge which were proper for the preſs.

No.

1. Viaticum Nautarum; by Robert Wright.
2. Miſcellanea; not in Sir Iſaac's hand writing.
3. Miſcellanea; part in Sir Iſaac's hand.
4. Trigonometria; about 5 ſheets.
5. Definitions.
6. Miſcellanea; part in Sir Iſaac's hand.
7. 40 ſheets in 4to, relating to Church Hiſtory.
8. 126 ſheets written on one ſide, being foul draughts of the Prophetic Stile.
9. 88 ſheets relating to Church Hiſtory.
10. About 70 looſe ſheets in ſmall 4to, of Chemical papers; ſome of which are not in Sir Iſaac's hand.
11. About 62 ditto, in folio.
12. About 15 large ſheets, doubled into 4to; Chemical.
13. About 8 ſheets ditto, written on one ſide.
14. About 5 ſheets of foul papers, relating to Chemiſtry.
15. 12 half-ſheets of ditto.
16. 104 half-ſheets, in 4to, ditto.
17. About 22 ſheets in 4to, ditto.
18. 24 ſheets, in 4to, upon the Prophecies.
19. 29 half-ſheets; being an anſwer to Mr. Hook, on Sir Iſaac's Theory of Colours.
20. 87 half-ſheets relating to the Optics, ſome of which are not in Sir Iſaac's hand.

From No. 1 to No. 20 examined on the 20th of May 1727, and judged not fit to be printed.

T. Pellet.

Witneſs, *Tho. Pilkington.*

21. 328 half-ſheets in folio, and 63 in ſmall 4to; being looſe and foul papers relating to the Revelations and Prophecies.
22. 8 half-ſheets in ſmall 4to, relating to Church Matters.
23. 24 half-ſheets in ſmall 4to; being a diſcourſe relating to the 2d of Kings.
24. 353 half-ſheets in folio, and 57 in ſmall 4to; being foul and looſe papers relating to Figures and Mathematics.
25. 201 half-ſheets in folio, and 21 in ſmall 4to; looſe and foul papers relating to the Commercium Epiſtolicum.

26. 91 half-ſheets in ſmall 4to, in Latin, upon the Temple of Solomon.
27. 37 half-ſheets in folio, upon the Hoſt of Heaven, the Sanctuary, and other Church Matters.
28. 44 half-ſheets in folio, upon Ditto.
29. 25 half-ſheets in folio; being a farther account of the Hoſt of Heaven.
30. 51 half-ſheets in folio; being an Hiſtorical Account of two notable Corruptions of Scripture.
31. 88 half-ſheets in ſmall 4to; being Extracts of Church Hiſtory.
32. 116 half-ſheets in folio; being Paradoxical Queſtions concerning Athanaſius, of which ſeveral leaves in the beginning are very much damaged.
33. 56 half-ſheets in folio, De Motu Corporum; the greateſt part not in Sir Iſaac's hand.
34. 61 half-ſheets in ſmall 4to; being various ſections on the Apocalypſe.
35. 25 half-ſheets in folio, of the Working of the Myſtery of Iniquity.
36. 20 half-ſheets in folio, of the Theology of the Heathens.
37. 24 half-ſheets in folio; being an Account of the Conteſt between the Hoſt of Heaven, and the Tranſgreſſors of the Covenant.
38. 31 half-ſheets in folio; being Paradoxical Queſtions concerning Athanaſius.
39. 107 quarter-ſheets in ſmall 4to, upon the Revelations.
40. 174 half-ſheets in folio; being looſe papers relating to Church Hiſtory.

May 22, 1727, examined from No. 21 to No. 40 incluſive, and judged them not fit to be printed; only No. 33 and No. 38 ſhould be reconſidered.

T. Pellet.

Witneſs, *Tho. Pilkington.*

41. 167 half-ſheets in folio; being looſe and foul papers relating to the Commercium Epiſtolicum.
42. 21 half-ſheets in folio; being the 3d letter upon Texts of Scripture, very much damaged.
43. 31 half-ſheets in folio; being foul papers relating to Church Matters.
44. 495 half-ſheets in folio; being looſe and foul papers relating to Calculations and Mathematics.
45. 335 half-ſheets in folio; being looſe and foul papers relating to the Chronology.
46. 112 ſheets in ſmall 4to, relating to the Revelations and other Church Matters.
47. 126 half-ſheets in folio; being looſe papers relating to the Chronology, part in Engliſh and part in Latin.
48. 400 half-ſheets in folio; being looſe Mathematical papers.
49. 109 ſheets in 4to, relating to the Prophecies, and Church Matters.
50. 127 half-ſheets in folio, relating to the Univerſity; great part not in Sir Iſaac's hand.
51. 18 ſheets in 4to; being Chemical papers.
52. 255 quarter-ſheets; being Chemical papers.
53. An Account of Corruptions of Scripture; not in Sir Iſaac's hand.
54. 31 quarter-ſheets; being Flammell's Explication of Hieroglyphical Figures.
55. About 350 half-ſheets; being Miſcellaneous papers.
56. 6 half-ſheets; being An Account of the Empires &c repreſented by St. John.
57. 9 half-ſheets folio, and 71 quarter-ſheets 4to; being Mathematical papers.
58. 140 half-ſheets, in 9 chapters, and 2 pieces in folio, titled, Concerning the Language of the Prophets.
59. 606 half-ſheets folio, relating to the Chronology; 9 more in Latin.
60. 182 half-ſheets folio; being looſe papers relating to the Chronology and Prophecies.
61. 144 quarter-ſheets, and 95 half-ſheets folio; being looſe Mathematical papers.
62. 137 half-ſheets folio; being looſe papers relating to the Diſpute with Leibnitz.
63. A folio Common-place book; part in Sir Iſaac's hand.
64. A bundle of Engliſh Letters to Sir Iſaac, relating to Mathematics.
65. 54 half-ſheets; being looſe papers found in the Principia.
66. A bundle of looſe Mathematical Papers; not Sir Iſaac's.
67. A bundle of French and Latin Letters to Sir Iſaac.
68. 136 ſheets folio, relating to Optics.
69. 22 half-ſheets folio, De Rationibus Motuum &c; not in Sir Iſaac's hand.
70. 70 half-ſheets folio; being looſe Mathematical Papers.
71. 38 half-ſheets folio; being looſe papers relating to Optics.
72. 47 half-ſheets folio; being looſe papers relating to Chronology and Prophecies.
73. 40 half-ſheets folio; Proceſtus Myſterii Magni Philoſophicus, by Wm. Yworth; not in Sir Iſaac's hand.
74. 5 half-ſheets; being a letter from Rizzetto to Martine, in Sir Iſaac's hand.
75. 41 half-ſheets; being looſe papers of ſeveral kinds, part in Sir Iſaac's hand.
76. 40 half-ſheets; being looſe papers, foul and dirty, relating to Calculations.
77. 90 half-ſheets folio; being looſe Mathematical papers.
78. 176 half-ſheets folio; being looſe papers relating to Chronology.
79. 176 half-ſheets folio; being looſe papers relating to the Prophecies.
80. { 12 half-ſheets folio; An Abſtract of the Chronology.
92 half-ſheets, folio; The Chronology. }
81. 40 half-ſheets folio; The Hiſtory of the Prophecies, in 10 chapters, and part of the 11th unfiniſhed.
82. 5 ſmall bound books in 12mo, the greateſt part not in Sir Iſaac's hand, being rough Calculations.

May

May 26th 1727, Examined from No. 41 to No. 82 inclusive, and judged not fit to be printed, except No. 80, which is agreed to be printed, and part of No. 61 and 81, which are to be reconsidered.

Th. Pellet.
Witness, *Tho. Pilkington.*

It is astonishing what care and industry Sir Isaac had employed about the papers relating to Chronology, Church History, &c; as, on examining the papers themselves, which are in the possession of the family of the earl of Portsmouth, it appears that many of them are copies over and over again, often with little or no variation; the whole number being upwards of 4000 sheets in folio, or 8 reams of folio paper; beside the bound books &c in this catalogue, of which the number of sheets is not mentioned. Of these there have been published only the Chronology, and Observations on the Prophecies of Daniel and the Apocalypse of St. John.

NEWTONIAN *Philosophy*, the doctrine of the universe, or the properties, laws, affections, actions, forces, motions, &c of bodies, both celestial and terrestrial, as delivered by Newton.

This term however is differently applied; which has given occasion to some confused notions relating to it. For, some authors, under this term, include all the corpuscular philosophy, considered as it now stands reformed and corrected by the discoveries and improvements made in several parts of it by Newton. In which sense it is, that Gravesande calls his Elements of Physics, Introductio ad Philosophiam Newtonianam. And in this sense the Newtonian is the same as the new philosophy; and stands contradistinguished from the Cartesian, the Peripatetic, and the ancient Corpuscular.

Others, by Newtonian Philosophy, mean the method or order used by Newton in philosophising; viz, the reasoning and inferences drawn directly from phenomena, exclusive of all previous hypotheses; the beginning from simple principles, and deducing the first powers and laws of nature from a few select phenomena, and then applying those laws &c to account for other things. In this sense, the Newtonian Philosophy is the same with the Experimental Philosophy, or stands opposed to the ancient Corpuscular, and to all hypothetical and fanciful systems of Philosophy.

Others again, by this term, mean that Philosophy in which physical bodies are considered mathematically, and where geometry and mechanics are applied to the solution of phenomena. In which sense, the Newtonian is the same with the Mechanical and Mathematical Philosophy.

Others, by Newtonian Philosophy, understand that part of physical knowledge which Newton has handled, improved, and demonstrated.

And lastly, others, by this Philosophy, mean the new principles which Newton has brought into Philosophy; with the new system founded upon them, and the new solutions of phenomena thence deduced; or that which characterizes and distinguishes his Philosophy from all others. And this is the sense in which we shall here chiefly consider it.

As to the history of this Philosophy, consult the foregoing article. It was first published in the year 1687, the author being then professor of mathematics in the university of Cambridge; a 2d edition, with considerable additions and improvements, came out in 1713; and a 3d in 1726. An edition, with a very large Commentary, came out in 1739, by Le Seur and Jacquier; besides the complete edition of all Newton's works, with notes, by Dr. Horsley, in 1779 &c. Several authors have endeavoured to make it plainer; by setting aside many of the more sublime mathematical researches, and substituting either more obvious reasonings or experiments instead of them; particularly Whiston, in his Prælect. Phys. Mathem.; Gravesande, in Elem. & Inst.; Pemberton, in his View &c; and Maclaurin, in his Account of Newton's Philosophy.

The chief parts of the Newtonian Philosophy, as delivered by the author, except his Optical Discoveries &c, are contained in his Principia, or Mathematical Principles of Natural Philosophy. He founds his system on the following definitions.

1. Quantity of Matter, is the measure of the same, arising from its density and bulk conjointly.—Thus, air of a double density, in the same space, is double in quantity; in a double space, is quadruple in quantity; in a triple space, is sextuple in quantity, &c.

2. Quantity of Motion, is the measure of the same, arising from the velocity and quantity of matter conjunctly.—This is evident, because the motion of the whole is the motion of all its parts; and therefore in a body double in quantity, with equal velocity, the Motion is double, &c.

3. The Vis Insita, Vis Inertiæ, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or moving uniformly forward in a right line.—This definition is proved to be just by experience, from observing the difficulty with which any body is moved out of its place, upwards, or obliquely, or even downwards when acted on by a body endeavouring to urge it quicker than the velocity given it by gravity; and any how to change its state of motion or rest. And therefore this force is the same, whether the body have gravity or not; and a cannon ball, void of gravity, if it could be, being discharged horizontally, will go the same distance in that direction, in the same time, as if it were endued with gravity.

4. An Impressed Force, is an action exerted upon a body, in order to change its state, whether of rest or motion.—This force consists in the action only; and remains no longer in the body when the action is over. For a body maintains every new state it acquires, by its vis inertiæ only.

5. A Centripetal Force, is that by which bodies are drawn, impelled, or any way tend towards a point, as to a centre.—This may be considered of three kinds, absolute, accelerative, and motive.

6. The Absolute quantity of a centripetal force, is a measure of the same, proportional to the efficacy of the cause that urges it to the centre.

7. The Accelerative quantity of a centripetal force, is the measure of the same, proportional to the velocity which it generates in a given time.

8. The

8. The Motive quantity of a centripetal force, is a meafure of the fame, proportional to the motion which it generates in a given time.—This is always known by the quantity of a force equal and contrary to it, that is juft fufficient to hinder the defcent of the body.

After thefe definitions, follow certain Scholia, treating of the nature and diftinctions of Time, Space, Place, Motion, Abfolute, Relative, Apparent, True, Real, &c. After which, the author propofes to fhew how we are to collect the true motions from their caufes, effects, and apparent differences; and vice verfa, how, from the motions, either true or apparent, we may come to the knowledge of their caufes and effects. In order to this, he lays down the following axioms or laws of motion.

1ft Law. Every body perfeveres in its ftate of reft, or of uniform motion in a right line, unlefs it be compelled to change that ftate by forces impreffed upon it.—Thus, "Projectiles perfevere in their motions, fo far as they are not retarded by the refiftance of the air, or impelled downwards by the force of gravity. A top, whofe parts, by their cohefion, are perpetually drawn afide from rectilinear motions, does not ceafe its rotation otherwife than as it is retarded by the air. The greater bodies of the planets and comets, meeting with lefs refiftance in more free fpaces, preferve their motions, both progreffive and circular, for a much longer time."

2d Law. The Alteration of motion is always proportional to the motive force impreffed; and is made in the direction of the right line in which that force is impreffed. Thus, if any force generate a certain quantity of motion, a double force will generate a double quantity, whether that force be impreffed all at once, or in fucceffive moments.

3d Law. To every action there is always oppofed an equal re-action: or the mutual actions of two bodies upon each other, are always equal, and directed to contrary parts. Thus, whatever draws or preffes another, is as much drawn or preffed by that other. If you prefs a ftone with your finger, the finger is alfo preffed by the ftone: &c.

From this axiom, or law, Newton deduces the following corollaries.

1. A body by two forces conjoined will defcribe the diagonal of a parallelogram, in the fame time that it would defcribe the fides by thofe forces apart.

2. Hence is explained the compofition of any one direct force out of any two oblique ones, viz, by making the two oblique forces the fides of a parallelogram, and the diagonal the direct one.

3. The quantity of motion, which is collected by taking the fum of the motions directed towards the fame parts, and the difference of thofe that are directed to contrary parts, fuffers no change from the action of bodies among themfelves; becaufe the motion which one body lofes, is communicated to another.

4. The common centre of gravity of two or more bodies does not alter its ftate of motion or reft by the actions of the bodies among themfelves; and therefore the common centre of gravity of all bodies, acting upon each other, (excluding external actions and impediments) is either at reft, or moves uniformly in a right line.

5. The motions of bodies included in a given fpace are the fame among themfelves, whether that fpace be at reft, or move uniformly forward in a right line without any circular motion. The truth of this is evident from the experiment of a fhip; where all motions are juft the fame, whether the fhip be at reft, or proceed uniformly forward in a ftraight line.

6. If bodies, any how moved among themfelves, be urged in the direction of parallel lines by equal accelerative forces, they will all continue to move among themfelves, after the fame manner as if they had not been urged by fuch forces.

The mathematical part of the Newtonian Philofophy depends chiefly on the following lemmas; efpecially the firft; containing the doctrine of prime and ultimate ratios.

Lem. 1. Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer the one to the other than by any given difference, become ultimately equal.

Lem. 2 fhews, that in a fpace bounded by two right lines and a curve, if an infinite number of parallelograms be infcribed, all of equal breadth; then the ultimate ratio of the curve fpace and the fum of the parallelograms, will be a ratio of equality.

Lem. 3 fhews, that the fame thing is true when the breadths of the parallelograms are unequal.

In the fucceeding lemmas it is fhewn, in like manner, that the ultimate ratios of the fine, chord, and tangent of arcs infinitely diminifhed, are ratios of equality, and therefore that in all our reafonings about thefe, we may fafely ufe the one for the other:—that the ultimate form of evanefcent triangles, made by the arc, chord, or tangent, is that of fimilitude, and their ultimate ratio is that of equality; and hence, in reafonings about ultimate ratios, thefe triangles may fafely be ufed one for another, whether they are made with the fine, the arc, or the tangent.—He then demonftrates fome properties of the ordinates of curvilinear figures; and fhews that the fpaces which a body defcribes by any finite force urging it, whether that force is determined and immutable, or continually varied, are to each other, in the very beginning of the motion, in the duplicate ratio of the forces:—and laftly, having added fome demonftrations concerning the evanefcence of angles of contact, he proceeds to lay down the mathematical part of his fyftem, which depends on the following theorems.

Theor. 1. The areas which revolving bodies defcribe by radii drawn to an immoveable centre of force, lie in the fame immoveable planes, and are proportional to the times in which they are defcribed.—To this prop. are annexed feveral corollaries, refpecting the velocities of bodies revolving by centripetal forces, the directions and proportions of thofe forces, &c; fuch as, that the velocity of fuch a revolving body, is reciprocally as the perpendicular let fall from the centre of force upon the line touching the orbit in the place of the body, &c.

Theor. 2. Every body that moves in any curve line

line described in a plane, and, by a radius drawn to a point either immoveable or moving forward with an uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force directed to that point.—With corollaries relatinig to such motions in resisting mediums, and to the direction of the forces when the areas are not proportional to the times.

Theor. 3. Every body that, by a radius drawn to the centre of another body, any how moved, describes areas about that centre proportional to the times, is urged by a force compounded of the centripetal forces tending to that other body, and of the whole accelerative force by which that other body is impelled.—With several corollaries.

Theor. 4. The centripetal forces of bodies, which by equal motions describe different circles, tend to the centres of the same circles; and are one to the other as the squares of the arcs described in equal times, applied to the radii of the circles.—With many corollaries, relating to the velocities, times, periodic forces, &c. And, in scholium, the author farther adds, Moreover, by means of the foregoing proposition and its corollaries, we may discover the proportion of a centripetal force to any other known force, such as that of gravity. For if a body by means of its gravity revolve in a circle, concentric to the earth, this gravity is the centripetal force of that body. But from the descent of heavy bodies, the time of one entire revolution, as well as the arc described in any given time, is given by a corol. to this prop. And by such propositions, Mr. Huygens, in his excellent book De Horologio Oscillatorio, has compared the force of gravity with the centrifugal forces of revolving bodies.

On these, and such-like principles, depends the Newtonian Mathematical Philosophy. The author farther shews how to find the centre to which the forces impelling any body are directed, having the velocity of the body given: and finds that the centrifugal force is always as the versed sine of the nascent arc directly, and as the square of the time inversely; or directly as the square of the velocity, and inversely as the chord of the nascent arc. From these premises, he deduces the method of finding the centripetal force directed to any given point when the body revolves in a circle; and this whether the central point be near hand, or at immense distance; so that all the lines drawn from it may be taken for parallels. And he shews the same thing with regard to bodies revolving in spirals, ellipses, hyperbolas, or parabolas. He shews also, having the figures of the orbits given, how to find the velocities and moving powers; and indeed resolves all the most difficult problems relating to the celestial bodies with a surprising degree of mathematical skill. These problems and demonstrations are all contained in the first book of the Principia: but an account of them here would neither be generally understood, nor easily comprized in the limits of this work.

In the second book, Newton treats of the properties and motion of fluids, and their powers of resistance, with the motion of bodies through such resisting mediums, those resistances being in the ratio of any powers of the velocities; and the motions being either made in right lines or curves, or vibrating like pendulums. And here he demonstrates such principles as entirely overthrow the doctrine of Des Cartes's vortices, which was the fashionable system in his time; concluding the book with these words: "So that the hypothesis of vortices is utterly irreconcileable with astronomical phenomena, and rather serves to perplex than explain the heavenly motions. How these motions are performed in free spaces without vortices, may be understood by the first book; and I shall now more fully treat of it in the following book Of the System of the World."—In this second book he makes great use of the doctrine of Fluxions, then lately invented; for which purpose he lays down the principles of that doctrine in the 2d Lemma, in these words: "The moment of any Genitum is equal to the moments of each of the generating sides drawn into the indices of the powers of those sides, and into their coefficients continually:" which rule he demonstrates, and then adds the following scholium concerning the invention of that doctrine: "In a letter of mine, says he, to Mr. J. Collins, dated December 10, 1672, having described a method of tangents, which I suspected to be the same with Slusius's method, which at that time was not made public; I subjoined these words: 'This is one particular, or rather a corollary, of a general method which extends itself, without any troublesome calculation, not only to the drawing of tangents to any curve lines, whether geometrical or mechanical, or any how respecting right lines or other curves, but also to the resolving other abstruser kinds of problems about the curvature, areas, lengths, centres of gravity of curves, &c; nor is it (as Hudden's method de Maximis & Minimis) limited to equations which are free from surd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite series.' So far that letter. And these last words relate to a Treatise I composed on that subject in the year 1671." Which, at least, is therefore the date of the invention of the doctrine of Fluxions.

On entering upon the 3d book of the Principia, Newton briefly recapitulates the contents of the two former books in these words: "In the preceding books I have laid down the principles of philosophy; principles not philosophical, but mathematical; such, to wit, as we may build our reasonings upon in philosophical enquiries. These principles are, the laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy. But lest they should have appeared of themselves dry and barren, I have illustrated them here and there with some philosophical scholiums, giving an account of such things, as are of a more general nature, and which philosophy seems chiefly to be founded on; such as the density and the resistance of bodies, spaces void of all matter, and the motion of light and sounds. It remains, he adds, that from the same principles I now demonstrate the frame of the system of the world. Upon this subject, I had indeed composed the 3d book in a popular method, that it might be read by many. But afterwards considering that such as had not sufficiently entered into the principles could not easily discern the strength of the consequences, nor lay aside the prejudices to which they had been many years accustomed; therefore to prevent the disputes which

might

might be raiſed upon ſuch accounts, I choſe to reduce the ſubſtance of that book into the form of propoſitions, in the mathematical way, which ſhould be read by thoſe only, who had firſt made themſelves maſters of the principles eſtabliſhed in the preceding books."

As a neceſſary preliminary to this 3d part, Newton lays down the following rules for reaſoning in natural philoſophy:

1. We are to admit no more cauſes of natural things, than ſuch as are both true and ſufficient to explain their natural appearances.

2. Therefore to the ſame natural effects we muſt always aſſign, as far as poſſible, the ſame cauſes.

3. The qualities of bodies which admit neither intenſion nor remiſſion of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be eſteemed the univerſal qualities of all bodies whatſoever.

4. In experimental philoſophy, we are to look upon propoſitions collected by general induction from phenomena, as accurately or very nearly true, notwithſtanding any contrary hypotheſes that may be imagined, till ſuch time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.

The phenomena firſt conſidered are, 1. That the ſatellites of Jupiter, by radii drawn to his centre, deſcribe areas proportional to the times of deſcription; and that their periodic times, the fixed ſtars being at reſt, are in the ſeſquiplicate ratio of their diſtances from that centre. 2. The ſame thing is likewiſe obſerved of the phenomena of Saturn. 3. The five primary planets, Mercury, Venus, Mars, Jupiter, and Saturn, with their ſeveral orbits, encompaſs the ſun. 4. The fixed ſtars being ſuppoſed at reſt, the periodic times of the ſaid five primary planets, and of the earth, about the ſun, are in the ſeſquiplicate proportion of their mean diſtances from the ſun. 5. The primary planets, by radii drawn to the earth, deſcribe areas no ways proportional to the times: but the areas which they deſcribe by radii drawn to the ſun are proportional to the times of deſcription. 6. The moon, by a radius drawn to the centre of the earth, deſcribes an area proportional to the time of deſcription. All which phenomena are clearly evinced by aſtronomical obſervations. The mathematical demonſtrations are next applied by Newton in the following propoſitions.

Prop. 1. The forces by which the ſatellites of Jupiter are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the centre of that planet; and are reciprocally as the ſquares of the diſtances of thoſe ſatellites from that centre.

Prop. 2. The ſame thing is true of the primary planets, with reſpect to the ſun's centre.

Prop. 3. The ſame thing is alſo true of the moon, in reſpect of the earth's centre.

Prop. 4. The moon gravitates towards the earth; and by the force of gravity is continually drawn off from a rectilinear motion, and retained in her orbit.

Prop. 5. The ſame thing is true of all the other planets, both primary and ſecondary, each with reſpect to the centre of its motion.

Prop. 6. All bodies gravitate towards every planet; and the weights of bodies towards any one and the ſame planet, at equal diſtances from its centre, are proportional to the quantities of matter they contain.

Prop. 7. There is a power of gravity tending to all bodies, proportional to the ſeveral quantities of matter which they contain.

Prop. 8. In two ſpheres mutually gravitating each towards the other, if the matter in places on all ſides, round about and equidiſtant from the centres, be ſimilar; the weight of either ſphere towards the other, will be reciprocally as the ſquare of the diſtance between their centres.—Hence are compared together the weights of bodies towards different planets: hence alſo are diſcovered the quantities of matter in the ſeveral planets: and hence likewiſe are found the denſities of the planets.

Prop. 9. The force of gravity, in parts downwards from the ſurface of the planets towards their centres, decreaſes nearly in the proportion of the diſtances from thoſe centres.

Theſe, and many other propoſitions and corollaries, are proved or illuſtrated by a great variety of experiments, in all the great points of phyſical aſtronomy; ſuch as, That the motions of the planets in the heavens may ſubſiſt an exceeding long time:—That the centre of the ſyſtem of the world is immoveable:—That the common centre of gravity of the earth, the ſun, and all the planets, is immoveable:—That the ſun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the planets:—That the planets move in ellipſes which have their common focus in the centre of the ſun; and, by radii drawn to that centre, they deſcribe areas proportional to the times of deſcription:—The aphelions and nodes of the orbits of the planets are fixt:—To find the aphelions, eccentricities, and principal diameters of the orbits of the planets:—That the diurnal motions of the planets are uniform, and that the libration of the moon ariſes from her diurnal motion:—Of the proportion between the axes of the planets and the diameters perpendicular to thoſe axes:—Of the weights of bodies in the different regions of our earth:—That the equinoctial points go backwards, and that the earth's axis, by a nutation in every annual revolution, twice vibrates towards the ecliptic, and as often returns to its former poſition:—That all the motions of the moon, and all the inequalities of thoſe motions, follow from the principles above laid down:—Of the unequal motions of the ſatellites of Jupiter and Saturn:—Of the flux and reflux of the ſea, as ariſing from the actions of the ſun and moon:—Of the forces with which the ſun diſturbs the motions of the moon; of the various motions of the moon, of her orbit, variation, inclinations of her orbit, and the ſeveral motions of her nodes:—Of the tides, with the forces of the ſun and moon to produce them:—Of the figure of the moon's body:—Of the preceſſion of the equinoxes:—And of the motions and trajectory of comets. The great author then concludes with a General Scholium, containing reflections on the principal parts of the great and beautiful ſyſtem of the univerſe, and of the infinite, eternal Creator and Governor of it.

"The hypotheſis of vortices, ſays he, is preſſed with

with many difficulties. That every planet by a radius drawn to the ſun may deſcribe areas proportional to the times of deſcription, the periodic times of the ſeveral parts of the vortices ſhould obſerve the duplicate proportion of their diſtances from the ſun. But that the periodic times of the planets may obtain the ſeſquiplicate proportion of their diſtances from the ſun, the periodic times of the parts of the vortex ought to be in the ſeſquiplicate proportion of their diſtances. That the ſmaller vortices may maintain their leſſer revolutions about Saturn, Jupiter, and other planets, and ſwim quietly and undiſturbed in the greater vortex of the ſun, the periodic times of the parts of the ſun's vortex ſhould be equal. But the rotation of the ſun and planets about their axes, which ought to correſpond with the motions of their vortices, recede far from all theſe proportions. The motions of the comets are exceeding regular, are governed by the ſame laws with the motions of the planets, and can by no means be accounted for by the hypotheſis of vortices. For comets are carried with very eccentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a vortex.

" Bodies, projected in our air, ſuffer no reſiſtance but from the air. Withdraw the air, as is done in Mr. Boyle's vacuum, and the reſiſtance ceaſes. For in this void a bit of fine down and a piece of ſolid gold deſcend with equal velocity. And the parity of reaſon muſt take place in the celeſtial ſpaces above the earth's atmoſphere ; in which ſpaces, where there is no air to reſiſt their motions, all bodies will move with the greateſt freedom ; and the planets and comets will conſtantly purſue their revolutions in orbits given in kind and poſition, according to the laws above explained. But though theſe bodies may indeed perſevere in their orbits by the mere laws of gravity, yet they could by no means have at firſt derived the regular poſition of the orbits themſelves from thoſe laws.

" The ſix primary planets are revolved about the ſun, in circles concentric with the ſun, and with motions directed towards the ſame parts, and almoſt in the ſame plane. Ten moons are revolved about the earth, Jupiter and Saturn, in circles concentric with them, with the ſame direction of motion, and nearly in the planes of the orbits of thoſe planets. But it is not to be conceived that mere mechanical cauſes could give birth to ſo many regular motions: ſince the comets range over all parts of the heavens, in very eccentric orbits. For by that kind of motion they paſs eaſily through the orbs of the planets, and with great rapidity ; and in their aphelions, where they move the ſloweſt, and are detained the longeſt, they recede to the greateſt diſtances from each other, and thence ſuffer the leaſt diſturbance from their mutual attractions. This moſt beautiful ſyſtem of the ſun, planets, and comets, could only proceed from the counſel and dominion of an intelligent and powerful Being. And if the fixed ſtars are the centres of other like ſyſtems, theſe being formed by the like wiſe counſel, muſt be all ſubject to the dominion of one ; eſpecially, ſince the light of the fixed ſtars is of the ſame nature with the light of the ſun, and from every ſyſtem light paſſes into all the other ſyſtems. And leſt the ſyſtem of the fixed ſtars ſhould, by their gravity, fall on each other mutually, he hath placed thoſe ſyſtems at immenſe diſtances one from another."

Then, after a truly pious and philoſophical deſcant on the attributes of the Being who could give exiſtence and continuance to ſuch prodigious mechaniſm, and with ſo much beautiful order and regularity, the great author proceeds,

" Hitherto we have explained the phenomena of the heavens and of our ſea, by the power of gravity, but have not yet aſſigned the cauſe of this power. This is certain, that it muſt proceed from a cauſe that penetrates to the very centres of the ſun and planets, without ſuffering the leaſt diminution of its force ; that operates, not according to the quantity of the ſurfaces of the particles upon which it acts, (as mechanical cauſes uſe to do,) but according to the quantity of the ſolid matter which they contain, and propagates its virtue on all ſides, to immenſe diſtances, decreaſing always in the duplicate proportion of the diſtances. Gravitation towards the ſun, is made up out of the gravitations towards the ſeveral particles of which the body of the ſun is compoſed ; and in receding from the ſun, decreaſes accurately in the duplicate proportion of the diſtances, as far as the orb of Saturn, as evidently appears from the quieſcence of the aphelions of the planets ; nay, and even to the remoteſt aphelions of the comets, if thoſe aphelions are alſo quieſcent. But hitherto I have not been able to diſcover the cauſe of thoſe properties of gravity from phenomena, and I frame no hypotheſes. For whatever is not deduced from the phenomena, is to be called an hypotheſis ; and hypotheſes, whether metaphyſical or phyſical, whether of occult qualities or mechanical, have no place in experimental philoſophy. In this philoſophy particular propoſitions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulſive force of bodies, and the laws of motion and of gravitation, were diſcovered. And to us it is enough, that gravity does really exiſt, and act according to the laws which we have explained, and abundantly ſerves to account for all the motions of the celeſtial bodies, and of our ſea.

" And now we might add ſomething concerning a certain moſt ſubtle ſpirit, which pervades and lies hid in all groſs bodies, by the force and action of which ſpirit, the particles of bodies mutually attract one another at near diſtances, and cohere, if contiguous, and electric bodies operate to greater diſtances, as well repelling as attracting the neighbouring corpuſcles ; and light is emitted, reflected, refracted, inflected, and heats bodies ; and all ſenſation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this ſpirit, mutually propagated along the ſolid filaments of the nerves, from the outward organs of ſenſe to the brain, and from the brain into the muſcles. But theſe are things that cannot be explained in few words, nor are we furniſhed with that ſufficiency of experiments which is required to an accurate determination and demonſtration of the laws by which this electric and elaſtic ſpirit operates."

NICHE, a cavity, or hollow part, in the thickness of a wall, to place a figure or statue in.

NICOLE (FRANCIS), a very celebrated French mathematician, was born at Paris December the 23d, 1683. His early attachment to the mathematics induced M. Montmort to take the charge of his education: and he opened out to him the way to the higher geometry. He first became publicly remarkable by detecting the fallacy of a pretended quadrature of the circle. This quadrature a M. Mathulon so assuredly thought he had discovered, that he deposited, in the hands of a public notary at Lyons, the sum of 3000 livres, to be paid to any person who, in the judgment of the Academy of Sciences, should demonstrate the falsity of his solution. M. Nicole, piqued at this challenge, undertook the task, and exposing the paralogism, the Academy's judgment was, that Nicole had plainly proved that the rectilineal figure which Mathulon had given as equal to the circle, was not only unequal to it, but that it was even greater than the polygon of 32 sides circumscribed about the circle.—The prize of 3000 livres, Nicole presented to the public hospital of Lyons.

The Academy named Nicole, Eleve-Mechanician, March 12, 1707; Adjunct in 1716, Associate in 1718, and Pensioner in 1724; which he continued till his death, which happened the 18th of January 1758, at 75 years of age.

His works were all inserted in the different volumes of the Memoirs of the Academy of Sciences; and are as follow:

1. A General Method for determining the Nature of Curves formed by the Rolling of other Curves upon any Given Curve; in the volume for the year 1707.

2. A General Method for Rectifying all Roulets upon Right and Circular Bases; 1708.

3. General Method of determining the Nature of those Curves which cut an Infinity of other Curves given in Position, cutting them always in a Constant Angle; 1715.

4. Solution of a Problem proposed by M. de Lagny; 1716.

5. Treatise of the Calculus of Finite Differences; 1717.

6. Second Part of the Calculus of Finite Differences; 1723.

7. Second Section of ditto; 1723.

8. Addition to the two foregoing papers; 1724.

9. New Proposition in Elementary Geometry; 1725.

10. New Solution of a Problem proposed to the English Mathematicians, by the late M. Leibnitz; 1725.

11. Method of Summing an Infinity of New Series, which are not summable by any other known method; 1727.

12. Treatise of the Lines of the Third Order, or the Curves of the Second Kind; 1729.

13. Examination and Resolution of some Questions relating to Play; 1730.

14. Method of determining the Chances at Play.

15. Observations upon the Conic Sections; 1731.

16. Manner of generating in a Solid Body, all the Lines of the Third Order; 1731.

17. Manner of determining the Nature of Roulets formed upon the Convex Surface of a Sphere; and of determining which are Geometric, and which are Rectifiable; 1732.

18. Solution of a Problem in Geometry; 1732.

19. The Use of Series in resolving many Problems in the Inverse Method of Tangents; 1737.

20. Observations on the Irreducible Case in Cubic Equations; 1738.

21. Observations upon Cubic Equations; 1738.

22. On the Trisection of an Angle; 1740.

23. On the Irreducible Case in Cubic Equations; 1741.

24. Addition to ditto; 1743.

25. His Last Paper upon the same; 1744.

26. Determination, by Incommensurables and Decimals, the Values of the Sides and Areas of the Series in a Double Progression of Regular Polygons, inscribed in and circumscribed about a Circle; 1747.

NIEUWENTYT (BERNARD), an eminent Dutch philosopher and mathematician, was born on the 10th of August 1654, at Westgraafdyk in North Holland, where his father was minister. He discovered very early a good genius and a strong inclination for learning; which was carefully improved by a suitable education. He had also that prudence and sagacity, which led him to pursue literature by sure and proper steps, acquiring a kind of mastery in one science before he proceeded to another. His father had designed him for the ministry; but seeing his inclination did not lie that way, he prudently left him to pursue the bent of his genius. Accordingly young Nieuwentyt apprehending that nothing was more useful than fixing his imagination and forming his judgment well, applied himself early to logic, and the art of reasoning justly, in which he grounded himself upon the principles of Des Cartes, with whose philosophy he was greatly delighted. From thence he proceeded to the mathematics, in which he made a considerable proficiency; though the application he gave to that branch of learning did not hinder him from studying both law and physic. In fact he succeeded in all these sciences so well, as deservedly to acquire the character of a good philosopher, a great mathematician, an expert physician, and an able and just magistrate.

Although he was naturally of a grave and serious disposition, yet he was very affable and agreeable in conversation. His engaging manner procured the affection of every one; and by this means he often drew over to his opinion those who before differed very widely from him. Thus accomplished, he acquired a great esteem and credit in the council of the town of Puremerende, where he resided; as he did also in the states of that province, who respected him the more, inasmuch as he never engaged in any cabals or factions, in order to secure it; regarding in his conduct, an open, honest, upright behaviour, as the best source of satisfaction, and relying solely on his merit. In fact, he was more attentive to cultivate the sciences, than eager to obtain the honours of the government; contenting himself with being counsellor and burgomaster, without courting or accepting any other posts, which might interfere with his studies, and draw him too much out of his library.—Nieuwentyt died the 7th of May 1730, at 76 years

years of age—having been twice married.—He was author of ſeveral works, in the Latin, French, and Dutch languages, the principal of which are the following:

1. A Treatiſe in Dutch, *proving the Exiſtence of God by the Wonders of Nature;* a much eſteemed work, and went through many editions. It was tranſlated alſo into ſeveral languages, as the French, and the Engliſh, under the title of, *The Religious Philoſopher, &c.*

2. A Refutation of Spinoza, in the Dutch language.

3. *Analyſis Infinitorum*; 1695, 4to.

4. *Conſiderationes ſecundæ circa Calculi Differentialis Principia*; 1696, 8vo.—In this work he attacked Leibnitz, and was anſwered by John Bernoulli and James Herman.

5. A Treatiſe on the New Uſe of the Tables of Sines and Tangents.

6. A Letter to Bothnia or Burmania, upon the Subject of Meteors.

NIGHT, that part of the natural day, during which the ſun is below the horizon: though ſometimes it is underſtood that the twilight is referred to the day, or time the ſun is above the horizon; the remainder only being the Night.

Under the equator, the Nights, in the former ſenſe, are always equal to the days; each being 12 hours long. But under the poles, the Night continues half a year. —The ancient Gauls and Germans divided their time not by days, but Nights; as appears from Cæſar and Tacitus; alſo the Arabs and the Icelanders do the ſame. The ſame may alſo be obſerved of our Saxon anceſtors: whence our cuſtom of ſaying, Sevennight, Fortnight, &c.

NOCTILUCA, a ſpecies of phoſphorus, ſo called becauſe it ſhines in the night, without any light being thrown on it: ſuch is the phoſphorus made of urine. By which it ſtands diſtinguiſhed from ſome other ſpecies of phoſphorus, which require to be expoſed to the ſunbeams before they will ſhine; as the Bononian-ſtone, &c. —Mr. Boyle has a particular Treatiſe on this ſubject.

NOCTURNAL *Arch*, is the arch of a circle deſcribed by the ſun, or a ſtar, in the night.

Nocturnal, or Nocturlabium, denotes an inſtrument, chiefly uſed at ſea, to take the altitude or depreſſion of the pole ſtar, and ſome other ſtars about the pole, for finding the latitude, and the hour of the night.

There are ſeveral kinds of this inſtrument; ſome of which are projections of the ſphere; ſuch as the hemiſpheres, or planiſpheres, on the plane of the equinoctial. The ſeamen commonly uſe two kinds; the one adapted to the pole ſtar and the firſt of the guards of the Little Bear; the other to the pole ſtar and the pointers of the Great Bear.

The Nocturnal conſiſts of two circular plates (fig. 15, pl. xiii) applied over each other. The greater, which has a handle to hold the inſtrument, is about $2\frac{1}{2}$ inches diameter, and is divided into 12 parts, anſwering to the 12 months; alſo each month ſubdivided into every 5th day; and in ſuch manner, that the middle of the handle correſponds to that day of the year in which the ſtar here reſpected has the ſame right aſcenſion with the ſun.

When the inſtrument is fitted for two ſtars, the handle is made moveable. The upper circle is divided into 24 equal parts, for the 24 hours of the day, and each hour ſubdivided into quarters, as in the figure. Theſe 24 hours are noted by 24 teeth; to be told in the night. In the centre of the two circular plates is adjuſted a long index A, moveable upon the upper plate. And the three pieces, viz, the two circles and index, are joined by a rivet which is pierced through the centre, with a hole 2 inches in diameter, for the ſtar to be obſerved through.

To Uſe the Nocturnal. Turn the upper plate till the longeſt tooth, marked 12, be againſt the day of the month on the under plate; and bringing the inſtrument near the eye, ſuſpend it by the handle, with the plane nearly parallel to the equinoctial; then viewing the pole-ſtar through the hole in the centre, turn the index about till, by the edge coming from the centre, you ſee the bright ſtar or guard of the Little Bear, if the inſtrument be fitted to that ſtar: then that tooth of the upper circle, under the edge of the index, is at the hour of the night on the edge of the hour-circle: which may be known without a light, by counting the teeth from the longeſt, which is for the hour of 12.

NODATED *Hyperbola*, one, ſo called by Newton, which by turning round decuſſates or croſſes itſelf: as in the 2d, and ſeveral other ſpecies, of his Enumeratio Linearum Tertii Ordinis.

NODES, the two oppoſite points where the orbit of a planet interſects the ecliptic. That, where the planet aſcends from the ſouth to the north ſide of the ecliptic, is called the Aſcending Node, or the Dragon's Head, and marked thus ☊: and the oppoſite point, where the planet deſcends from the north to the ſouth ſide of the ecliptic, is called the Deſcending Node, or Dragon's Tail, and is thus marked ☋. Alſo the right line drawn from the one Node to the other, is called the Line of the Nodes.

By obſervation it appears that, in all the planets, the Line of the Nodes continually changes its place, its motion being *in antecedentia*; i. e. contrary to the order of the ſigns, or from eaſt to weſt; with a peculiar degree of motion for each planet. Thus, by a retrograde motion, the line of the moon's nodes completes its circuit in 18 years and 225 days, in which time the Node returns again to the ſame point of the ecliptic. Newton has not only ſhewn, that this motion ariſes from the action of the ſun, but, from its cauſe, he has with great ſkill calculated all the elements and varieties in this motion. See his Princip. lib. 3, prop. 30, 31, &c.

The moon muſt be in or near one of the Nodes to make an eclipſe either of the ſun or moon.

NODUS, or *Node*, in Dialling, denotes a point or hole in the gnomon of a dial, by the ſhadow or light of which is ſhewn, either the hour of the day in dials without furniture, or the parallels of the ſun's declination, and his place in the ecliptic, &c. in dials with furniture.

NOLLET (the Abbé John Anthony), a conſiderable French philoſopher, and a member of moſt of the philoſophical ſocieties and academies of Europe; was born at Pimpré, in the diſtrict of Noyon, the 19th of November 1700. From the profound retreat, in which the mediocrity of his fortune obliged him to live, his reputation continually increaſed from day to day.

 M. Dufay

M. Dufay aſſociated him in his Electrical Reſearches; and M. de Reaumur reſigned to him his laboratory. It was under theſe maſters that he developed his talents. M. Dufay took him along with him in a journey he made into England; and Nollet profited ſo well of this opportunity, as to inſtitute a friendly and literary correſpondence with ſome of the moſt celebrated men in this country.

The king of Sardinia gave him an invitation to Turin, to perform a courſe of experimental philoſophy to the duke of Savoy. From thence he travelled into Italy, where he collected ſome good obſervations concerning the natural hiſtory of the country.

In France he was maſter of philoſophy and natural hiſtory to the royal family; and profeſſor royal of experimental philoſophy to the college of Navarre, and to the ſchools of artillery and engineers. The Academy of Sciences appointed him adjunct-mechanician in 1739, aſſociate in 1742, and penſioner in 1757. Nollet died the 24th of April 1770, regretted by all his friends, but eſpecially by his relations, whom he always ſuccoured with an affectionate attention. The works publiſhed by Nollet, are the following:

1. Recueils de Lettres ſur l'Electricité; 1753, 3 vols in 12mo.

2. Eſſai ſur l'Electricité des Corps; 1 vol. in 12mo.

3. Recherches ſur les Cauſes particulieres des Phenomenes Electriques; 1 vol. in 12mo.

4. L'Art des Experiences; 1770, 3 vols in 12mo.

His papers printed in the different volumes of the Memoirs of the Academy of Sciences, are much too numerous to be particularized here; they are inſerted in all or moſt of the volumes from the year 1740 to the year 1767 incluſive, moſtly ſeveral papers in each volume.

NONAGESIMAL, or Nonagesimal *Degree*, called alſo the Mid-heaven, is the higheſt point, or 90th degree of the ecliptic, reckoned from its interſection with the horizon at any time; and its altitude is equal to the angle that the ecliptic makes with the horizon at their interſection, or equal to the diſtance of the zenith from the pole of the ecliptic. It is much uſed in the calculation of ſolar eclipſes.

NONAGON, a figure having nine ſides and angles.—In a regular Nonagon, or that whoſe angles, and ſides, are all equal, if each ſide be 1, its area will be $6{\cdot}1818242 = \frac{9}{4}$ of the tangent of 70°, to the radius 1. See my Menſuration, p. 114, 2d edit.

NONES, in the Roman Calendar, the 5th day of the months January, February, April, June, Auguſt, September, November, and December; and the 7th of the other months March, May, July, and October: theſe laſt four months having 6 days before the Nones, and the others only four.—They had this name probably, becauſe they were always 9 days incluſively, from the firſt of the Nones to the Ides, i. e. reckoning incluſively both thoſe days.

NONIUS, or Nunez (Peter), a very eminent Portugueſe mathematician and phyſician, was born in 1497, at Alcazar in Portugal, anciently a remarkable city, known by the name of Salacia, from whence he was ſurnamed Salacienſis. He was profeſſor of mathematics in the univerſity of Coimbra, where he publiſhed ſome pieces which procured him great reputation. He was mathematical preceptor to Don Henry, ſon to king Emanuel of Portugal, and principal coſmographer to the king. Nonius was very ſerviceable to the deſigns, which this court entertained of carrying on their maritime expeditions into the Eaſt, by the publication of his book *Of the Art of Navigation*, and various other works. He died in 1577, at 80 years of age.

Nonius was the author of ſeveral ingenious works and inventions, and juſtly eſteemed one of the moſt eminent mathematicians of his age. Concerning his *Art of Navigation*, father Dechales ſays, "In the year 1530, Peter Nonius, a celebrated Portugueſe mathematician, upon occaſion of ſome doubts propoſed to him by Martinus Alphonſus Soſa, wrote a Treatiſe on Navigation, divided into two books; in the firſt, he anſwers ſome of thoſe doubts, and explains the nature of Loxodromic lines. In the ſecond book, he treats of rules and inſtruments proper for navigation, particularly ſea-charts, and inſtruments ſerving to find the elevation of the pole; but ſays he is rather obſcure in his manner of writing."—Furetiere, in his Dictionary, takes notice that Peter Nonius was the firſt who, in 1530, invented the angles which the Loxodromic curves make with each meridian, calling them in his language Rhumbs, and which he calculated by ſpherical triangles.—Stevinus acknowledges, that Peter Nonius was ſcarce inferior to the very beſt mathematicians of the age. And Schottus ſays, he explained a great many problems, and particularly the mechanical problem of Ariſtotle on the motion of veſſels by oars. His Notes upon Purbach's Theory of the Planets, are very much to be eſteemed: he there explains ſeveral things, which had either not been noticed before, or not rightly underſtood.

In 1542 he publiſhed a Treatiſe on the Twilight, which he dedicated to John the 3d, king of Portugal; to which he added what Alhazen, an Arabian author, has compoſed on the ſame ſubject. In this work he deſcribes the method or inſtrument called, from him, a Nonius, a particular account of which ſee in the following article.—He corrected ſeveral mathematical miſtakes of Orontius Finæus.—But the moſt celebrated of all his works, or that at leaſt he appeared moſt to value, was his *Treatiſe of Algebra*, which he had compoſed in Portugueſe, but tranſlated it into the Caſtilian tongue, when he reſolved upon making it public, which he thought would render his book more uſeful, as this language was more generally known than the Portugueſe. The dedication, to his former pupil, prince Henry, was dated from Liſbon, Dec. 1, 1564. This work contains 341 pages in the Antwerp edition of 1567, in 8vo.

The catalogue of his works, chiefly in Latin, is as follows:

1. *De Arte Navigandi*, libri duo; 1530.
2. *De Crepuſculis*; 1542.
3. *Annotationes in Ariſtotelem.*
4. Problema Mechanicum de Motu Navigii ex Remis.
5. Annotationes in Planetarum Theorias Georgii Purbachii, &c.
6. Libro de Algebra en Arithmetica y Geometra; 1564.

Nonius,

Nonius, is a name alſo erroneouſly given to the method of graduation now generally uſed in the diviſion of the ſcales of various inſtruments, and which ſhould be called Vernier, from its real inventor. The method of Nonius, ſo called from its inventor Pedro Nunez, or Nonius, and deſcribed in his treatiſe De Crepuſculis, printed at Liſbon in 1542, conſiſts in deſcribing within the ſame quadrant, 45 concentric circles, dividing the outermoſt into 90 equal parts, the next within into 89, the next into 88, and ſo on, till the innermoſt was divided into 46 only. By this means, in moſt obſervations, the plumb-line or index muſt croſs one or other of thoſe circles in or very near a point of diviſion: whence by calculation the degrees and minutes of the arch might eaſily be obtained. This method is alſo deſcribed by Nunez, in his treatiſe De Arte et Ratione Navigandi, lib. 2, cap. 6, where he imagines it was not unknown to Ptolomy. But as the degrees are thus divided unequally, and it is very difficult to attain exactneſs in the diviſion, eſpecially when the numbers, into which the arches are to be divided, are incompoſite, of which there are no leſs than nine, the method of diagonals, firſt publiſhed by Thomas Digges, Eſq. in his treatiſe Alæ ſeu Scalæ Mathematicæ, printed at Lond. in 1573, and ſaid to be invented by one Richard Chanſeler, a very ſkilful artiſt, was ſubſtituted in its ſtead. However, Nonius's method was improved at different times; but the admirable diviſion now ſo much in uſe, is the moſt conſiderable improvement of it. See Vernier.

NORMAL, is uſed ſometimes for a perpendicular.

NORTH *Star*, called alſo the Pole-ſtar, is the laſt in the tail of the Little Bear.

Northern *Signs*, are thoſe ſix that are in the north ſide of the equator; viz, Aries, Taurus, Gemini, Cancer, Leo, Virgo.

NORTHING, in Navigation, is the difference of latitude, which a ſhip makes in ſailing northwards.

NOSTRADAMUS (Michel), an able phyſician and celebrated aſtrologer, was born at St. Remy in Provence in the dioceſe of Avignon, December 14, 1503. His father was a notary public, and his grandfather a phyſician, from whom he received ſome tincture of the mathematics. He afterwards completed his courſes of languages and philoſophy at Avignon. From hence, going to Montpelier, he there applied himſelf to phyſic; but being forced away by the plague, he travelled through different places till he came to Bourdeaux, undertaking all ſuch patients as were willing to put themſelves under his care. This courſe occupied him five years; after which he returned to Montpelier, and was created doctor of his faculty in 1529; after which he reviſited the ſame places where he had practiſed phyſic before. At Agen he formed an acquaintance with Julius Cæſar Scaliger, and married his firſt wife; but having buried her, and two children which ſhe brought him, he quitted Agen after a reſidence of about four years. He fixed next at Marſeilles; but, his friends having provided an advantageous match for him at Salon, he repaired thither about the year 1544, and married accordingly his ſecond wife, by whom he had ſeveral children.

In 1546, Aix being afflicted with the plague, he went thither at the ſolicitation of the inhabitants, to whom he rendered great ſervice, particularly by a powder of his own invention: ſo that the town, in gratitude, gave him a conſiderable penſion for ſeveral years after the contagion ceaſed. In 1547 the city of Lyons, being viſited with the ſame diſtemper, had recourſe to our phyſician, who attended them alſo. Afterwards returning to Salon, he began a more retired courſe of life, and in this time of leiſure applied himſelf cloſely to his ſtudies. He had for a long time followed the trade of a conjurer occaſionally; and now he began to fancy himſelf inſpired, and miraculouſly illuminated with a proſpect into futurity. As faſt as theſe illuminations had diſcovered to him any future event, he entered it in writing, in ſimple proſe, though in enigmatical ſentences; but reviſing them afterwards, he thought the ſentences would appear more reſpectable, and ſavour more of a prophetic ſpirit, if they were expreſſed in verſe. This opinion determined him to throw them all into quatrains, and he afterward ranged them into centuries. For ſome time he could not venture to publiſh a work of this nature; but afterwards perceiving that the time of many events foretold in his quatrains was very near at hand, he reſolved to print them, as he did, with a dedication addreſſed to his ſon Cæſar, an infant only ſome months old, and dated March 1, 1555. To this firſt edition, which compriſes but ſeven centuries, he prefixed his name in Latin, but gave to his ſon Cæſar the name as it is pronounced in French, Notradame.

The public were divided in their ſentiments of this work: many looked upon the author as a ſimple viſionary; by others he was accuſed of magic or the black art, and treated as an impious perſon who held a commerce with the devil; while great numbers believed him to be really endued with the ſupernatural gift of prophecy. However, Henry the 2d, and queen Catharine of Medicis, his mother, were reſolved to ſee our prophet, who receiving orders to that effect, he preſently repaired to Paris. He was very graciouſly received at court, and received a preſent of 200 crowns. He was ſent afterwards to Blois, to viſit the king's children there, and report what he ſhould be able to diſcover concerning their deſtinies. It is not known what his ſentence was; however he returned to Salon loaded with honour, and good preſents.

Animated with this ſucceſs, he augmented his work to the number of 1000 quatrains, and publiſhed it with a dedication to the king in 1558. That prince dying the next year of a wound which he received at a tournament, our prophet's book was immediately conſulted; and this unfortunate event was found in the 35th quatrain of the firſt century, which runs thus in the London edition of 1672:

> Le Lion jeune le vieux ſurmontera,
> En champ bellique, par ſingulier duelle,
> Dans cage d'or l'œil il lui crevera,
> Deux playes une, puis mourir mort cruelle.

In English thus, from the same edition:

The young Lion shall overcome the old one,
In martial field by a single duel,
In a golden cage he shall put out his eye,
Two wounds from one, then he shall die a cruel death.

So remarkable a prediction added new wings to his fame; and he was honoured soon after with a visit from Emanuel duke of Savoy, and the princess Margaret of France, his consort. From this time Nostradamus found himself even overburdened with visitors, and his fame made every day new acquisitions. Charles the 9th, coming to Salon, was eager above all things to have a sight of him: Nostradamus, who then was in waiting as one of the retinue of the magistrates, being instantly presented to the king, complained of the little esteem his countrymen had for him; upon which the monarch publicly declared that he should hold the enemies of Nostradamus to be his enemies, and desired to see his children. Nor did that prince's favour stop here; in passing, not long after, through the city of Arles, he sent for Nostradamus, and presented him with a purse of 200 crowns, together with a brevet, constituting him his physician in ordinary, with the same appointment as the rest. But our prophet enjoyed these honours only a short time, as he died 16 months after, viz, July 2, 1566, at Salon, being then in his grand climacteric, or 63d year.—He had published several other pieces, chiefly relating to medicine.

He left three sons and three daughters. Cæsar the eldest son was born at Salon in 1555, and died in 1629: he left a manuscript, giving an account of the most remarkable events in the history of Provence, from 1080 to 1494, in which he inserted the lives of the poets of that country. These memoirs falling into the hands of his nephew Cæsar Nostradamus, gentleman to the duke of Guise, he undertook to complete the work; and being encouraged by the estates of the country, he carried the account up to the Celtic Gauls: the impression was finished at Lyons in 1614, and published under the title of Chronique de l'Histoire de Provence. —The second son, John, exercised with reputation the business of a proctor in the parliament of Provence.—He wrote the Lives of the Ancient Provençal Poets, called Troubadours, and the work was printed at Lyons in 1575, 8vo.—The youngest son it is said undertook the trade of peeping into futurity after his father.

NOTATION, is the representing of numbers, or any other quantities, by Notes, characters, or marks.

The choice of arithmetical, and other, characters, is arbitrary; and hence they are various in various nations: the figures 0, 1, 2, 3, &c, in common use, are derived from the Arabs and Indians, from whom they have their name, and the Notation by them, which forms the decimal or decuple scale, is perhaps the most convenient of any for arithmetical computations.

The Greeks, Hebrews, and other eastern nations, as also the Romans, expressed numbers by the letters of their common alphabet. See CHARACTER.

In Algebra, the quantities are represented mostly by the letters of the alphabet, &c; and that as early as the time of Diophantus. See ALGEBRA.

NOTES, in Music, are characters which mark the tones, i. e. the elevations and fallings of the voice, or sound, and the swiftness or slowness of its motions, &c; and these have undergone various alterations and improvements, before they arrived at their present state of perfection.

NOVEMBER, the eleventh month in the Julian year, but the ninth in the year of Romulus, beginning with March; whence its name. In this month, which contains 30 days, the sun enters the sign ♐, viz, usually about the 21st day of the month.

NUCLEUS, the kernel, is used by Hevelius, and some other astronomers, for the body of a comet, which others call its head, as distinguished from its tail, or beard.

NUCLEUS is also used by some writers for the central parts of the earth, and other planets, which they suppose firmer, and as it were separated from them, considered as a cortex or shell.

NUEL, the same as NEWEL of a Staircase.

NUMBER, a collection or assemblage of several units, or several things of the same kind; as 2, 3, 4, &c, exclusive of the number 1: which is Euclid's definition of Number.—Stevinus defines Number as that by which the quantity of anything is expressed: agreeably to which Newton conceives a Number to consist, not in a multitude of units, as Euclid defines it, but in the abstract ratio of a quantity of any kind to another quantity of the same kind, which is accounted as unity: and in this sense, including all these three species of Number, viz, Integers, Fractions, and Surds.

Wolfius defines Number to be something which refers to unity, as one right line refers to another. Thus, assuming a right line for unity, a Number may likewise be expressed by a right line. And in this way also Des Cartes considers numbers as expressed by lines, where he treats of the arithmetical operations as performed by lines, in the beginning of his Geometry.

For the manner of characterizing NUMBERS, see NOTATION. And

For reading and expressing NUMBERS in combination, see NUMERATION.

Mathematicians consider Number under a great many circumstances, and different relations, accidents, &c.

NUMBERS, *Absolute*, *Abstract*, *Abundant*, *Amicable*, *Applicate*, *Binary*, *Cardinal*, *Circular*, *Composite*, *Concrete*, *Defective*, *Fractional*, *Homogeneal*, *Irrational* or *Surd*, *Linear* or *Mixt*, *Ordinal*, *Polygonal*, *Prime*, *Pyramidal*, *Rational*, *Similar*, *&c*, see the respective adjectives.

Broken NUMBERS, or Fractions, are certain parts of unity, or of some other Number.

Cubic NUMBER, is the product of a square Number multiplied by its root, or the continual product of a Number twice multiplied by itself;

as the Numbers	- -	1,	8,	27,	64,	125, &c,
which are the cubes of	-	1,	2,	3,	4,	5, &c.

This series of the cubes of the ordinal Numbers, may be raised by addition only, viz, adding always the differences; as was first shewn by Peletarius, at the end of his Algebra, first printed in 1558, where he gives a table of the squares and Cubes of the first 140 numbers. See CUBE.

Every Cubic Number whose root is less than 6, viz, the Cubic Numbers 1, 8, 27, 64, 125, being divided by 6, the remainder is the root itself: Thus,

Thus,

$\frac{1}{6} = 0\frac{1}{6}$; $\frac{8}{6} = 1\frac{2}{6}$; $\frac{27}{6} = 4\frac{3}{6}$; $\frac{64}{6} = 10\frac{4}{6}$; $\frac{125}{6} = 20\frac{5}{6}$;
where the remainders, or the numerators of the ſmall fractions, are 0, 1, 2, 3, 4, 5, the ſame as the roots of the Cubes 0, 1, 8, 27, 64, 125. After theſe, the next ſix Cubic Numbers being divided by 6, the remainders will be reſpectively the ſame arithmetical ſeries, viz - - - 0, 1, 2, 3, 4, 5;
to each of which adding 6, gives 6, 7, 8, 9, 10, 11,
for the roots of the next ſix cubes 216, 343, &c.

Then, again dividing the next ſet of ſix Cubic Numbers, viz, - - 1728, 2197, &c,
by 6, the remainders are again the ſame ſeries, viz, 0, 1, 2, 3, 4, 5,
to each of which adding 12, gives 12, 13, 14. 15, 16, 17, for the roots of the ſaid next ſix cubes. And ſo on in infinitum, the ſeries of remainders 0, 1, 2, 3, 4, 5, continually recurring, and to each ſet of theſe remainders the reſpective Numbers 0, 6, 12, 18, 24, &c, being added, the ſums will be the whole ſeries of roots, 0, 1, 2, 3, 4, 5, 6, &c.

M. de la Hire, from conſidering this property of the Number 6, with regard to Cubic Numbers, found that all other Numbers, raiſed to any power whatever, had each their diviſor, which had the ſame effect with regard to them, that 6 has with regard to Cubes. And the general rule he has diſcovered is this: if the exponent of the power of a number be even, i. e. if that number be raiſed to the 2d, 4th, 6th, &c power, it muſt be divided by 2, then the remainder added to 2, or to a multiple of 2, gives the root of the Number correſponding to its power, i. e. the 2d, or 4th, &c, root. But if the exponent of the power of the Number be uneven, viz the 3d, 5th, 7th, &c power, the double of that exponent ſhall be the diviſor, which ſhall have the property here required.

A Determinate NUMBER, is that which is referred to ſome given unit; as a ternary or three.

An *Even* NUMBER, is that which may be divided into two equal parts, without remainder or fraction, as the Numbers 2, 4, 6, 8, 10, &c.—The ſums, differences, products, and powers of Even Numbers, are alſo Even Numbers.

An *Evenly-Even* NUMBER, is ſuch as being divided by an even Number, the quotient is alſo an Even Number without a remainder: as 16, which divided by 8 gives 2 for the quotient.

An *Unevenly-Even* NUMBER, is ſuch as being divided by an Even Number, the quotient is an Uneven one: as 20, which divided by 4, gives 5 for the quotient.

Figurate or *Figural* NUMBERS, are certain ranks of Numbers found by adding together firſt a rank of units, which is the firſt order, which gives the 2d order; then theſe added give the 3d order; and ſo on. Hence, the ſeveral orders of Figurate Numbers, are as follow:

Firſt order	-	1 . 1 . 1 . 1 . 1 . &c.
2d order	-	1 . 2 . 3 . 4 . 5 . &c.
3d order	-	1 . 3 . 6 . 10. 15. &c.
4th order	-	1 . 4 . 10. 20. 35. &c.
5th order	-	1 . 5 . 15. 35. 70. &c.

The firſt order conſiſts all of equals, and the 2d order of the natural arithmetical progreſſion; the 3d order is alſo called triangular Numbers, the 4th order pyramidals, &c.

See FIGURATE *Numbers*.

Heterogeneal NUMBERS, are ſuch as are referred to different units. As three men and 4 trees.

Homogeneal NUMBERS, are ſuch as are referred to the ſame unit. As 3 men and 4 men.

Imperfect NUMBERS, are thoſe whoſe aliquot parts added together, make either more or leſs than the whole of the number itſelf; and are diſtinguiſhed into Abundant and Defective.

Indeterminate NUMBER, is that which is referred to unity in the general; which is what we call Quantity.

Irrational or *Surd* NUMBER, is one that is not commenſurable with unity; as $\sqrt{2}$, or $\sqrt[3]{4}$, &c

Perfect NUMBER, that which is juſt equal to the ſum of its aliquot parts, added together. As, 6, 28, &c: for the aliquot parts of 6 are 1, 2, 3, whoſe ſum is the ſame 6; and the aliquot parts of 28, are
1, 2, 4, 7, 14, whoſe ſum is 28. See PERFECT *Number*.

Plane NUMBER, that which ariſes from the multiplication of two other Numbers: ſo 6 is a plane or rectangle, whoſe two ſides are 2 and 3, for $2 \times 3 = 6$.

Square NUMBER, is a Number produced by multiplying any given Number by itſelf; as the
Square Numbers - - 1, 4, 9, 16, 25, &c,
produced from the roots - 1, 2, 3, 4, 5, &c.

Every Square Number added to its root makes an even Number. See SQUARE.

Uneven NUMBER, or *Odd* NUMBER, that which differs from an even Number by one, or which cannot be divided into two equal integer parts; ſuch as 1, 3, 5, 7, &c. The ſums and differences of Uneven Numbers are even; but all the products and powers of them are Uneven Numbers. On the other hand, the ſum or difference of an even and Uneven Number are both Uneven, but their product is even.

Whole NUMBER, or *Integer*, is unit, or a collection of units.

Golden NUMBER. See GOLDEN *Number* and CYCLE.

NUMBER *of Direction*, in Chronology, ſome one of the 35 Numbers between the Eaſter limits, or between the earlieſt and lateſt day on which it can fall, i. e. between March 22 and April 25, which are 35 days; being ſo called, becauſe it ſerves as a Direction for finding Eaſter for any year; being indeed the Number that expreſſes how many days after March 21, Eaſter-day falls. Thus, Eaſter-day falling as in the firſt line below, the Number of Direction will be as on the lower line:

	March									April		
Eaſter-day,	22,	23,	24,	25,	26,	27,	28,	29,	30,	31,	1,	2, &c.
Nº of Dir.	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12, &c

and ſo on, till the Number of Direction on the lower line be 35, which will anſwer to April 25, being the lateſt that Eaſter can happen. Therefore add 21 to the Number of Direction, and the ſum will be ſo many days in March for the Eaſter-day: if the ſum exceed 31, the exceſs will be the day of April.

To find the NUMBER *of Direction.* Enter the following table (which is adapted to the New Style), with the Dominical Letter on the left hand, and the Golden Number at the top, then where the columns meet is the

the Number of Direction for that year. See Ferguson's Astron. pa. 381, ed. 8vo.

G. N.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Dom. Let.																			
A	29	19	5	26	12	33	19	12	26	19	5	26	12	5	26	12	33	19	12
B	27	13	6	27	13	34	20	13	27	20	6	27	13	6	20	13	34	20	6
C	8	14	7	21	14	35	21	7	28	21	7	28	14	7	21	14	28	21	7
D	19	15	8	22	15	29	22	8	29	15	8	29	15	1	22	15	29	22	8
E	30	16	2	23	16	30	23	9	30	16	9	23	16	2	23	9	30	23	9
F	24	17	3	24	10	31	24	10	31	17	10	24	17	3	24	10	31	17	10
G	25	18	4	25	11	32	18	11	32	18	4	25	18	4	25	11	32	18	11

Thus, for the year 1790, the Dominical Letter being C, and the Golden Number 5; on the line of C, and below 5, is 14 for the Number of Direction. To this add 21, the sum is 35 days from the 1st of March, which, deducting the 31 days of March, leaves 4 for the day of April, for Easter-day that year.

NUMERAL *Characters*. See CHARACTERS.

NUMERAL *Figures*. The antiquity of these in England has, for several reasons, been supposed as high as the eleventh century; in France about the middle of the tenth century; having been introduced into both countries from Spain, where they had been brought by the Moors or Saracens. See Wallis's Algebra, pa. 9 &c, and pa. 153 of additions at the end of the same. See also Philos. Trans. numb. 439 and 475.

NUMERAL *Letters*, those letters of the alphabet that are commonly used for figures or numbers, as I, V, X, L, C, D, M.

NUMERATION, in Arithmetic, the art of estimating or pronouncing any number, or series of numbers.

Numbers are usually expressed by the ten following characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0; the first nine denoting respectively the first nine ordinal numbers; and the last, or cipher 0, joined to any of the others, denotes so many tens. In like manner, two ciphers joined to any one of the first nine significant figures, make it become so many hundreds, three ciphers make it thousands, and so on.

Weigelius indeed shews how to number, without going beyond a quaternary; i. e. by beginning to repeat at each fourth. And Leibnitz and De Lagny, in what they call their binary arithmetic, begin to repeat at every 2d place; using only the two figures 1 and 0. But these are rather matters of curiosity than any real use.

That the nine significant figures may express not only units, but also tens, hundreds, thousands, &c, they have a local value given them, as hinted above; so that, though when alone, or in the right-hand place, they denote only units or ones, yet in the 2d place they denote tens, in the 3d place hundreds, in the 4th place thousands, &c; as the number 5555 is five thousand five hundred fifty and five.

Hence then, to express any written number, or assign the proper value to each character; beginning at the right hand, divide the proposed number into classes, of three characters to each class; and consider two classes as making up a period of six figures or places. Then every period, of six figures, has a name common to all the figures in it; the first being primes or units; the 2d is millions; the 3d is millions of-millions, or billions; the 4th is millions-of-millions-of-millions, or trillions; and so on; also every class, or half-period, of three figures, is read separately by itself, so many hundreds, tens, and units; only, after the left-hand half of each period, the word thousands is added; and at the end of the 2d, 3d, 4th &c period, its common name millions, billions, &c, is expressed.

Thus the number 4,591, is 4 thousand 5 hundred and 91.

The number 210,463, is 2 hundred and 10 thousands, and 463.

The number 281,427,307, is 281 millions, 427 thousands, and 307.

NUMERATOR, of a Fraction, is the number which shews how many of those parts, which the integer is supposed to be divided into, are denoted by the fraction. And, in the notation the Numerator is set over the denominator, or number that shews into how many parts the integer is divided, in the fraction. So, ex. gr. $\frac{3}{4}$ denotes three-fourths, or 3 parts out of 4; where 3 is the numerator, and 4 the denominator.

NUMERICAL, NUMEROUS, or *Numeral*, something that relates to number.

NUMERAL *Algebra*, is that which makes use of numbers, in contradistinction from literal algebra, or that in which the letters of the alphabet are used.

O.

OBELISK, a kind of quadrangular pyramid, very tall and slender, raised as an ornament in some public place, or to serve as a memorial of some remarkable transaction.

OBJECT, something presented to the mind, by sensation, or by imagination. Or something that affects us by its presence, that affects the eye, ear, or some other of the organs of sense.

The

The objects of the eye, or vision, are painted on the retina; though not there erect, but inverted, according to the laws of optics. This is easily shewn from Des Cartes's experiment, of laying bare the vitreous humour on the back part of the eye, and putting over it a bit of white paper, or the skin of an egg, and then placing the fore part of the eye to the hole of a darkened room. By this means there is obtained a pretty landscape of the external objects, painted invertedly on the back of the eye. In this case, how the Objects thus painted invertedly should be seen erect, is matter of controversy.

OBJECT is also used for the subject, or matter of an art or science; being that about which it is employed or concerned.

OBJECT-*Glass*, of a telescope or microscope, is the glass placed at the end of the tube which is next or towards the Object to be viewed.

To prove the goodness and regularity of an Object-glass; on a paper describe two concentric circles, the one having its diameter the same with the breadth of the Object-glass, and the other half that diameter; divide the smaller circumference into 6 equal parts, pricking the points of division through with a fine needle; cover one side of the glass with this paper, and, exposing it to the sun, receive the rays through these 6 holes upon a plane; then by moving the plane nearer to or farther from the glass, it will be found whether the six rays unite exactly together at any distance from the glass; if they do, it is a proof of the regularity and just form of the glass; and the said distance is also the focal distance of the glass.

A good way of proving the excellency of an Object-glass, is by placing it in a tube, and trying it with small eye-glasses, at several distant objects; for that Object-glass is always the best, which represents objects the brightest and most distinct, and which bears the greatest aperture, and the most convex and concave eye-glasses, without colouring or haziness.

A circular Object-glass is said to be truly centred, when the centre of its circumference falls exactly in the axis of the glass; and to be ill centred, when it falls out of the axis.

To prove whether Object-glasses be well centred, hold the glass at a due distance from the eye, and observe the two reflected images of a candle, varying the distance till the two images unite, which is the true centre point: then if this fall in the middle, or central point of the glass, it is known to be truly centred.

As Object-glasses are commonly included in cells that screw upon the end of the tube of a telescope, it may be proved whether they be well centred, by fixing the tube, and observing while the cell is unscrewed, whether the cross-hairs keep fixed upon the same lines of an object seen through the telescope.

For various methods of finding the true centre of an Object-glass, see Smith's Optics, book 3, chap. 3; also the Philos. Trans. vol. 48, pa. 177.

OBJECTIVE *Line*, in Perspective, is any line drawn on the geometrical plane, whose representation is sought for in a draught or picture.

OBJECTIVE *Plane*, in Perspective, is any plane situated in the horizontal plane, whose perspective representation is required.

OBLATE, flatted, or shortened; as an Oblate spheroid, having its axis shorter than its middle diameter; being formed by the rotation of an ellipse about the shorter axis.

OBLATENESS, of the earth, the flatness about the poles, or the diminution of the polar axis in respect of the equatorial. The ratio of these two axes has been determined in various ways; sometimes by the measures of different degrees of latitude, and sometimes by the length of pendulums vibrating seconds in different latitudes, &c; the results of all which, as well as accounts of the means of determining them, see under the articles EARTH and DEGREE. To what is there said, may be added the following, from An Account of the Experiments made in Russia concerning the Length of a Pendulum which swings Seconds, by Mr. Krafft, contained in the 6th and 7th volumes of the New Petersburgh Transactions, for the years 1790 and 1793. These experiments were made at different times, and in various parts of the Russian empire: Mr. Krafft has collected and compared them, with a view to investigate the consequences that may be deduced from them. From the whole he concludes, that the length p of a pendulum, which swings seconds in any given latitude l, and in a temperature of 10 degrees of Reaumur's thermometer, may be determined by the following equation, in lines of a French foot: viz,

$$p = 439{\cdot}178 + 2{\cdot}321 \text{ sine}^2 l.$$

This expression agrees, very nearly, not only with all the experiments made on the pendulum in Russia, but also with those of Mr. Graham, and those of Mr. Lyons in 79° 50′ north latitude, where he found its length to be 441·38 lines.

It also shews the augmentation of gravity from the equator to the parallel of a given latitude l: for, putting g for the gravity under the equator, G for that under the pole, and z for that under the latitude l; Mr. Krafft finds $z = (1 + 0{\cdot}0052848 \text{ sine}^2 l) \times g$; and consequently $G = 1{\cdot}0052848 g$:

From this proportion of Gravity under different latitudes, Mr. Krafft deduces, that on the hypothesis of the earth's being a homogeneous ellipsoid, its oblateness must be $\frac{1}{190}$; instead of $\frac{1}{230}$, which ought to be the result of this hypothesis: but on adopting the supposition that the earth is a heterogeneous ellipsoid, he finds its Oblateness, as deduced from these experiments, to be $\frac{1}{297}$; which agrees with that resulting from the measurement of degrees of the meridian.

This confirms an observation of M. De la Place, that, if the hypothesis of the earth's homogeneity be given up, then do theory, the measurement of degrees of latitude, and experiments with the pendulum, all agree in their result with respect to the Oblateness of the earth.

OBLIQUE, aslant, indirect, or deviating from the perpendicular. As,

OBLIQUE *Angle*, one that is not a right angle, but is either greater or less than this, being either obtuse or acute.

OBLIQUE-*angled Triangle*, that whose angles are all oblique.

OBLIQUE *Ascension*, is that point of the equinoctial which rises with the centre of the sun, or star, or any other point of the heavens, in an Oblique sphere.

OBLIQUE *Circle*, in the stereographic projection, is

is any circle that is Oblique to the plane of projection.

Oblique *Descension*, that point of the equinoctial which sets with the centre of the sun, or star, or other point of the heavens in an Oblique sphere.

Oblique *Direction*, that which is not perpendicular to a line or plane.

Oblique *Force*, or Percussion, or *Power*, or Stroke, is that made in a direction Oblique to a body or plane. It is demonstrated that the effect of such Oblique force &c, upon the body, is to an equal perpendicular one, as the sine of the angle of incidence is to radius.

Oblique *Line*, that which makes an Oblique angle with some other line.

Oblique *Planes*, in Dialling, are such as recline from the zenith, or incline towards the horizon.

Oblique *Projection*, is that where a body is projected or impelled in a line of direction that makes an oblique angle with the horizontal line.

Oblique *Sailing*, in Navigation, is that part which includes the application and calculation of Oblique-angled triangles.

Oblique *Sphere*, in Geography, is that in which the axis is Oblique to the horizon of a place.—In this sphere, the equator and parallels of declination cut the horizon obliquely. And it is this obliquity that occasions the inequality of days and nights, and the variation of the seasons. See Sphere.

OBLIQUITY, that which denotes a thing Oblique.

Obliquity *of the Ecliptic*, is the angle which the ecliptic makes with the equator. See Ecliptic.

OBLONG, sometimes means any figure that is longer than it is broad; but more properly it denotes a rectangle, or a right-angled parallelogram, whose length exceeds its breadth.

Oblong, is also used for the quality or species of a figure that is longer than it is broad: as an Oblong spheroid; formed by an ellipse revolved about its longer or transverse axis; in contradistinction from the oblate spheroid, or that which is flatted at its poles, being generated by the revolution of the ellipse about its conjugate or shorter axis.

OBSCURA *Camera*. See Camera *Obscura*.

Obscura *Clara*. See Clara *Obscure*.

OBSERVATION, in Astronomy and Navigation, is the observing with an instrument some celestial phenomenon; as, the altitude of the sun, moon, or stars, or their distances asunder, &c. But by this term the seamen commonly mean only the taking the meridian altitudes, in order to find the latitude. And the finding the latitude from such observed altitude, they call *working an observation*.

OBSERVATORY, a place destined for observing the heavenly bodies; or a building, usually in form of a tower, erected on some eminence, and covered with a terrace, for making astronomical observations.

Most nations, at almost all times, have had their observatories, either public or private ones, and in various degrees of perfection. A description of a great many of them may be seen in a dissertation of Weidler's, De præsenti Specularum Astronomicarum Statu, printed in 1727, and in different articles of his History of Astronomy, printed in 1741, viz, pa. 86 &c; as also in La Lande's Astronomy, the preface pa. 34. The chief among these are the following:

I. The Greenwich Observatory, or Royal Observatory of England. This was built and endowed in the year 1676, by order of King Charles the 2d, at the instance of Sir Jonas Moore, and Sir Christopher Wren: the former of these gentlemen being Surveyor General of the Ordnance, the office of Astronomer Royal was placed under that department, in which it has continued ever since.

This observatory was at first furnished with several very accurate instruments; particularly a noble sextant of 7 feet radius, with telescopic sights. And the first Astronomer Royal, or the person to whom the province of observing was first committed, was Mr. John Flamsteed; a man who, as Dr. Halley expresses it, seemed born for the employment. During 14 years he watched the motions of the planets with unwearied diligence, especially those of the moon, as was given him in charge; that a new theory of that planet being found, shewing all her irregularities, the longitude might thence be determined.

In the year 1690, having provided himself with a mural arch of near 7 feet radius, made by his Assistant Mr. Abraham Sharp, and fixed in the plane of the meridian, he began to verify his catalogue of the fixed stars, which had hitherto depended altogether on the distances measured with the sextant, after a new and very different manner, viz, by taking the meridian altitudes, and the moments of culmination, or in other words the right ascension and declination. And he was so well pleased with this instrument, that he discontinued almost entirely the use of the sextant.

Thus, in the space of upwards of 40 years, the Astronomer Royal collected an immense number of good observations; which may be found in his Historia Cœlestis Britannica, published in 1725; the principal part of which is the Britannic catalogue of the fixed stars.

Mr. Flamsteed, on his death in 1719, was succeeded by Dr. Halley, and he by Dr. Bradley in 1742, and this last by Mr. Bliss in 1762; but none of the observations of these gentlemen have yet been given to the public.

On the demise of Mr. Bliss, in 1765, he was succeeded by Dr. Nevil Maskelyne, the present worthy astronomer royal, whose valuable observations have been published, from time to time, under the direction of the Royal Society, in several folio volumes.

The Greenwich Observatory is found, by very accurate observations, to lie in 51° 28′ 40″ north latitude, as settled by Dr. Maskelyne, from many of his own observations, as well as those of Dr. Bradley.

II. The Paris Observatory was built by Louis the 14th, in the fauxbourg St. Jaques, being begun in 1664, and finished in 1672. It is a singular but magnificent building, of 80 feet in height, with a terrace at top; and here M. De la Hire, M. Cassini, &c, the king's astronomers, have made their observations. Its latitude is 48° 50′ 14″ north, and its longitude 9′ 20″ east of Greenwich Observatory.

In the Observatory of Paris is a cave, or pit, 170 feet deep, with subterraneous passages, for experiments that are to be made out of the reach of the sun, especially such as relate to congelations, refrigerations, &c. In this cave there is an old thermometer of M. De la Hire, which stands always at the same height; thereby shewing

ſhewing that the temperature of the place remains always the ſame. From the top of the platform to the bottom of the cave is a perpendicular well or pit, uſed formerly for experiments on the fall of bodies; being alſo a kind of long teleſcopical tube, through which the ſtars are ſeen at mid-day.

III. Tycho Brahe's Obſervatory was in the little iſland Ween, or the Scarlet Iſland, between the coaſts of Schonen and Zealand, in the Baltic ſea. This Obſervatory was not well ſituated for ſome kinds of obſervations, particularly the riſings and ſettings; as it lay too low, and was landlocked on all the points of the compaſs except three; and the land horizon being very rugged and uneven.

IV. Pekin Obſervatory. Father Le Compte deſcribes a very magnificent Obſervatory, erected and furniſhed by the late emperor of China, in his capital, at the interceſſion of ſome Jeſuit miſſionaries, chiefly father Verbieſt, whom he appointed his chief obſerver. The inſtruments here are exceeding large; but the diviſions are leſs accurate, and in ſome reſpects the contrivance is leſs commodious than in thoſe of the Europeans. The chief are, an armillary zodiacal ſphere, of 6 Paris feet diameter, an azimuthal horizon 6 feet diameter, a large quadrant 6 feet radius, a ſextant 8 feet radius, and a celeſtial globe 6 feet diameter.

V. Bramins' Obſervatory at Benares, in the Eaſt Indies, which is ſtill one of the principal ſeminaries of the Bramins or prieſts of the original Gentoos of Hindoſtan. This Obſervatory at Benares it is ſaid was built about 200 years ſince, by order of the emperor Ackbar: for as this wiſe prince endeavoured to improve the arts, ſo he wiſhed alſo to recover the ſciences of Hindoſtan, and therefore ordered that three ſuch places ſhould be erected; one at Delhi, another at Agra, and the third at Benares.

Wanting the uſe of optical glaſſes, to magnify very diſtant or very ſmall objects, theſe people directed their attention to the increaſing the ſize of their inſtruments, for obtaining the greater accuracy and number of the diviſions and ſubdiviſions in their inſtruments. Accordingly, the Obſervatory contains ſeveral huge inſtruments, of ſtone, very nicely erected and divided, conſiſting of circles, columns, gnomons, dials, quadrants, &c, ſome of them of 20 feet radius, the circle divided firſt into 360 equal parts, and ſometimes each of theſe into 20 other equal parts, each anſwering to 3′, and of about two-tenths of an inch in extent. And although theſe wonderful inſtruments have been built upwards of 200 years, the graduations and diviſions on the ſeveral arcs appear as well cut, and as accurately divided, as if they had been the performance of a modern artiſt. The execution, in the conſtruction of theſe inſtruments, exhibits an extraordinary mathematical exactneſs in the fixing, bearing, fitting of the ſeveral parts, in the neceſſary and ſufficient ſupports to the very large ſtones that compoſe them, and in the joining and faſtening them into each other by means of lead and iron.

See a farther deſcription, and drawing, of this Obſervatory, by Sir Robert Barker, in the Philoſ. Tranſ. vol. 67, pa. 598.

OBSERVATORY *Portable.* See EQUATORIAL.

OBTUSE *Angle*, one that is greater than a right angle.

OBTUSE-*angled Triangle*, is a triangle that has one of its angles Obtuſe: and it can have only one ſuch.

OBTUSE *Cone*, or OBTUSE-*Angled Cone*, one whoſe angle at the vertex, by a ſection through the axis, is Obtuſe.

OBTUSE *Hyperbola*, one whoſe aſymptotes form an Obtuſe angle.

OBTUSE-*angular Section of a Cone*, a name given to the hyperbola by the ancient geometricians, becauſe they conſidered this ſection only in the Obtuſe cone.

OCCIDENT, or OCCIDENTAL, weſt, or weſtward, in Aſtronomy; a planet is ſaid to be Occident, when it ſets after the ſun.

OCCIDENT, in Geography, the weſtward quarter of the horizon, or that part of the horizon where the ecliptic, or the ſun's place in it, deſcends into the lower hemiſphere.

OCCIDENT *Equinoctial*, that point of the horizon where the ſun ſets, when he croſſes the equinoctial, or enters the ſign Aries or Libra.

OCCIDENT *Eſtival*, that point of the horizon where the ſun ſets at his entrance into the ſign Cancer, or in our ſummer when the days are longeſt.

OCCIDENT *Hybernal*, that point of the horizon where the ſun ſets at midwinter, when entering the ſign Capricorn.

OCCIDENTAL *Horizon*. See HORIZON.

OCCULT, in Geometry, is uſed for a line that is ſcarce perceivable, drawn with the point of the compaſſes, or a black-lead pencil. Occult or dry lines, are uſed in ſeveral operations; as the raiſing of plans, deſigns of building, pieces of perſpective, &c. They are to be effaced or rubbed out when the work is finiſhed.

OCCULTATION, the obſcuration, or hiding from our ſight, any ſtar or planet, by the interpoſition of the body of the moon, or of ſome other planet.—The Occultation of a ſtar by the moon, if obſerved in a place whoſe latitude and longitude are well determined, may be applied to the correction of the lunar tables; but if obſerved in a place whoſe latitude only is well known, may be applied to the determining the longitude of the place.

Circle of Perpetual OCCULTATION. See CIRCLE.

OCEAN, the vaſt collection of ſalt and navigable water, which encompaſſes moſt parts of the earth.

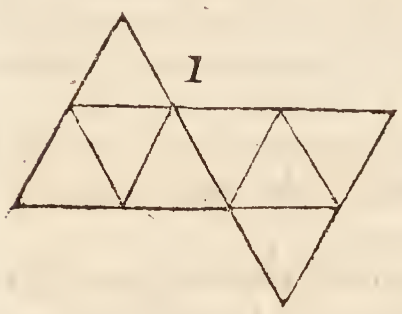
By computation it appears that the Ocean takes up conſiderably more of what we know of the terreſtrial globe, than the dry land does. This is perhaps eaſieſt known, by taking a good map of the world, and with a pair of ſciſſars clipping out all the water from the land, and weighing the two parts ſeparately: by which means it has been found, that the water occupies about two-thirds of the whole ſurface of the globe.

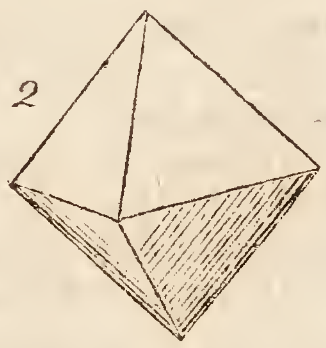
The great and univerſal Ocean is ſometimes, by geographers, divided into three parts. As, 1ſt, the Atlantic and European Ocean, lying between part of Europe, Africa, and America; 2d, the Indian Ocean, lying between Africa, the Eaſt-Indian iſlands, and New Holland; 3d, the Pacific Ocean, or great ſouth ſea, which lies between the Philippine iſlands, China, Japan, and New Holland on the weſt, and the coaſt of America on the eaſt. The Ocean alſo takes divers other names, according

cording to the different countries it borders upon: as the Britiſh Ocean, German Ocean, &c. Alſo according to the poſition on the globe; as the northern, ſouthern, eaſtern, and weſtern Oceans.

The Ocean, penetrating the land at ſeveral ſtreights, quits its name of Ocean, and aſſumes that of ſea or gulph; as the Mediterranean ſea, the Perſian gulph, &c. In very narrow places, it is called a ſtreight, &c.

OCTAEDRON, or Octahedron, one of the five regular bodies; contained under 8 equal and equilateral triangles.—It may be conceived as conſiſting of two quadrilateral pyramids joined together at their baſes.





To form an Octaedron. Join together 8 equal and equilateral triangles, as in fig. 1; then cut the lines half through, and fold the figure up by theſe cut lines, till the extreme edges meet, and form the Octaedron, as in figure 2.

In an Octaedron, if

A be the linear edge or ſide,
B its whole ſurface,
C its ſolidity, or ſolid content,
R the radius of the circumſcribed ſphere, and
r the radius of the inſcribed ſphere: Then

$$A = r\sqrt{6} = R\sqrt{2} = \frac{B\sqrt{3}}{6} = \sqrt[3]{\frac{3C\sqrt{2}}{2}}.$$

$$B = 12r^2\sqrt{3} = 4R^2\sqrt{3} = 2A^2\sqrt{3} = 6\sqrt[3]{\frac{C^2\sqrt{3}}{2}}.$$

$$C = 4r^3\sqrt{3} = \tfrac{4}{3}R^3 = \tfrac{1}{3}A^3\sqrt{2} = \frac{B\sqrt{B\sqrt{3}}}{18}.$$

$$R = r\sqrt{3} = \tfrac{1}{2}A\sqrt{2} = \sqrt{\frac{B\sqrt{3}}{12}} = \sqrt[3]{\tfrac{3}{4}C}.$$

$$r = \tfrac{1}{3}R\sqrt{3} = \tfrac{1}{6}A\sqrt{6} = \tfrac{1}{6}\sqrt{B\sqrt{3}} = \sqrt[3]{\frac{C\sqrt{3}}{12}}.$$

See my Menſuration, pa. 251 &c, 2d edition.

OCTAGON, is a figure of 8 ſides and angles; which, when theſe are all equal, is alſo called a regular one, or may be inſcribed in a circle.

If the ſide of a regular Octagon be s; then

Its area $= 2s^2 \times \overline{1 + \sqrt{2}} = 4{\cdot}8284271s^2$; and the Radius of its circumſc. circle $= \dfrac{s}{\sqrt{2 - \sqrt{2}}}$.

Octagon, in Fortification, denotes a place that has 8 ſides, or 8 baſtions.

OCTANT, the 8th part of a circle.

Octant, or Octile, means alſo an aſpect, or poſition of two planets, when their places are diſtant by the 8th part of a circle, or 45 degrees.

OCTAVE, or 8th, in Muſic, is an interval of 8 ſounds; every 8th note in the ſcale of the gamut being the ſame, as far as the compaſs of muſic requires.

Tones, or ſounds, that are Octaves to each other, or at an Octave's diſtance, are alike, or the ſame nearly as the uniſon. In this caſe, the more acute of the two makes exactly two vibrations while the deeper or graver makes but one; whence, they coincide at every two vibrations of the acuter, which, being more frequent, makes this concord more perfect than any other, and as it were an uniſon. Hence alſo, it happens, that two chords or ſtrings, of the ſame matter, thickneſs, and tenſion, but the one double the length of the other, produce the Octave.

The Octave containing in it all the other ſimple concords, and the degrees being the differences of theſe concords; it is evident, that the diviſion of the Octave comprehends the diviſion of all the reſt.

By joining therefore all the ſimple concords to a common fundamental, we have the following ſeries:

$$1 : \tfrac{5}{6} : \tfrac{4}{5} : \tfrac{3}{4} : \tfrac{2}{3} : \tfrac{5}{8} : \tfrac{3}{5} : \tfrac{1}{2}$$

Fund. 3d *l*, 3d *g*, 4th, 5th, 6th *l*, 6th *g*, 8ve.

Mr. Malcolm obſerves, that any wind inſtrument being over-blown, the ſound will riſe to an Octave, and no other concord; which he aſcribes to the perfection of the Octave, and its being next to uniſon.

Des Cartes, from an obſervation of the like kind, viz, that the ſound of a whiſtle, or organ pipe, will riſe to an Octave, if forcibly blown, concludes, that no ſound is heard, but its acute Octave ſeems ſome way to echo or reſound in the ear.

OCTILE. See Octant.

OCTOBER, the 8th month of the year, in Romulus's calendar; but the tenth in that of Numa, Julius Cæſar, &c, after the addition of January and February. This month contains 31 days; about the 22d of which, the ſun enters the ſign Scorpio ♏.

OCTOGON. See Octagon.

OCTOSTYLE, in Architecture, the face of a building adorned with 8 columns.

ODD, in Arithmetic, is ſaid of a number that is not even. The ſeries of Odd numbers is 1, 3, 5, 7, &c.

ODDLY-Odd. A number is ſaid to be Oddly-Odd, when an Odd number meaſures it by an Odd number. So 15 is a number Oddly-odd, becauſe the Odd number 3 meaſures it by the Odd number 5.

OFFING, or Offin, in Navigation, that part of the ſea which is at a good diſtance from ſhore; where there is deep water, and no need of a pilot to conduct the ſhip into port.

OFFSETS, in Surveying are the perpendiculars let fall, and meaſured from the ſtation lines, to the corners or bends in the hedge, fence, or boundary of any ground.

Offset-*Staff*, a ſlender rod or ſtaff, of 10 links, or other convenient length. Its uſe is for meaſuring the Offſets, and other ſhort lines and diſtances.

OFFWARD, in Navigation, the ſame with from the ſhore, &c.

OGEE, or OG, an ornamental moulding in the ſhape of an S; conſiſting of two members, the one concave and the other convex.

OLDENBURG (Henry), who wrote his name ſometimes Grubendol, reverſing the letters, was a learned German gentleman, and born in the Duchy of Bremen in the Lower Saxony, about the year 1626, being deſcended from the counts of Aldenburg in Weſtphalia; whence his name. During the long Engliſh parliament in the time of Charles the 1ſt, he came to England as conſul for his countrymen; in which capacity he remained at London in Cromwell's adminiſtration. But being diſcharged of that employment, he was engaged as tutor to the lord Henry Obryan, an Iriſh nobleman, whom he attended to the univerſity of Oxford; and in 1656 he entered himſelf a ſtudent in that univerſity, chiefly to have the benefit of conſulting the Bodleian library. He was afterwards appointed tutor to lord William Cavendiſh, and became intimately acquainted with Milton the poet. During his reſidence at Oxford, he became alſo acquainted with the members of that ſociety there, which gave birth to the Royal Society; and upon the foundation of this latter, he was elected a member of it: and when the Society found it neceſſary to have two ſecretaries, he was choſen aſſiſtant to Dr. Wilkins. He applied himſelf with extraordinary diligence to the duties of this office, and began the publication of the Philoſophical Tranſactions with No. 1, in 1664. In order to diſcharge this taſk with more credit to himſelf and the Society, he held a correſpondence with more than ſeventy learned perſons, and others, upon a great variety of ſubjects, in different parts of the world. This fatigue would have been inſupportable, had he not, as he told Dr. Liſter, managed it ſo as to make one letter anſwer another; and that, to be always freſh, he never read a letter before he was ready immediately to anſwer it: ſo that the multitude of his letters did not clog him, nor ever lie upon his hands. Among others, he was a conſtant correſpondent of Mr. Robert Boyle, and he tranſlated many of that ingenious gentleman's works into Latin.

About the year 1674 he was drawn into a diſpute with Mr. Hook, who complained, that the ſecretary had not done him juſtice, in the Hiſtory of the Tranſactions, with reſpect to the invention of the ſpiral ſpring for pocket watches; the conteſt was carried on with ſome warmth on both ſides, but was at length terminated to the honour of Mr. Oldenburg; for, purſuant to an open repreſentation of the affair to the Royal Society, the council thought fit to declare, in behalf of their ſecretary, that they knew nothing of Mr. Hook having printed a book intitled *Lampas*, *&c*; but that the publiſher of the Tranſactions had conducted himſelf faithfully and honeſtly in managing the intelligence of the Royal Society, and given no juſt cauſe for ſuch reflections.

Mr. Oldenburg continued to publiſh the Tranſactions as before, to No. 136, June 25, 1677; after which the publication was diſcontinued till the January following; when they were again reſumed by his ſucceſſor in the ſecretary's office, Mr. Nehemiah Grew, who carried them on till the end of February 1678. Mr. Oldenburg died at his houſe at Charlton, between Greenwich and Woolwich, in Kent, Auguſt 1678, and was interred there, being 52 years of age.

He publiſhed, beſides what has been already mentioned, 20 tracts, chiefly on theological and political ſubjects; in which he principally aimed at reconciling differences, and promoting peace.

OLYMPIAD, in Chronology, a revolution or period of four years, by which the Greeks reckoned their time: ſo called from the Olympic games, which were celebrated every fourth year, during 5 days, near the ſummer ſolſtice, upon the banks of the river Alpheus, near Olympia, a town of Elis. As each Olympiad conſiſted of 4 years, theſe were called the 1ſt, 2d, 3d, and 4th year of each Olympiad; the firſt year commencing with the neareſt new moon to the ſummer ſolſtice.

The firſt Olympiad began the 3938 year of the Julian period, the 3208 of the creation, 776 years before the birth of Chriſt, and 24 years before the foundation of Rome. And the computation by theſe, ended with the 404th Olympiad, being the 440th year of the preſent vulgar Chriſtian era.

OMBROMETER, a name given by Mr. Roger Pickering (Philoſ. Tranſ. No. 473, or Abridg. V, 456) to what is more commonly, though leſs properly, called a Pluviameter or Rain-gage. See Pluviameter.

OMPHALOPTER, or Omphaloptic, in Optics, a glaſs that is convex on both ſides, popularly called a Convex Lens.

OPACITY, a quality of bodies which renders them opake, or the contrary of tranſparency.

The Carteſians make opacity to conſiſt in this; that the pores of the body are not all ſtraight, or directly before each other; or rather not pervious every way.

This doctrine however is deficient: for though, to have a body tranſparent, its pores muſt be ſtraight, or rather open every way; yet it is inconceivable how it ſhould happen, that not only glaſs and diamonds, but even water, whoſe parts are ſo very moveable, ſhould have all their pores open and pervious every way; while the fineſt paper, or the thinneſt gold leaf, ſhould exclude the light, for want of ſuch pores. So that another cauſe of Opacity muſt be ſought for.

Now all bodies have vaſtly more pores or vacuities than are neceſſary for an infinite number of rays to paſs freely through them in right lines, without ſtriking on any of the parts themſelves. For ſince water is 19 times lighter or rarer than gold; and yet gold itſelf is ſo very rare, that magnetic effluvia paſs freely through it, without any oppoſition; and quickſilver is readily received within its pores, and even water itſelf by compreſſion; it muſt have much more pores than ſolid parts: conſequently water muſt have at leaſt 40 times as much vacuity as ſolidity.

The cauſe therefore, why ſome bodies are opake, does not conſiſt in the want of rectilinear pores, pervious every way; but either in the unequal denſity of the parts, or in the magnitude of the pores; and to their being either empty, or filled with a different matter; by means of which, the rays of light, in their paſ-
ſage,

ſage, are arreſted by innumerable refractions and reflections, till at length falling on ſome ſolid part, they become quite extinct, and are utterly abſorbed.

Hence cork, paper, wood, &c, are opake; while glaſs, diamonds, &c, are pellucid. For in the confines or joining of parts alike in denſity, ſuch as thoſe of glaſs, water, diamonds, &c, among themſelves, no refraction or reflection takes place, becauſe of the equal attraction every way; ſo that ſuch of the rays of light as enter the firſt ſurface, paſs ſtraight through the body, excepting ſuch as are loſt and abſorbed, by ſtriking on ſolid parts: but in the bordering of parts of unequal denſity, ſuch as thoſe of wood and paper, both with regard to themſelves, and with regard to the air or empty ſpace in their larger pores, the attraction being unequal, the reflections and refractions will be very great; and thus the rays will not be able to paſs through ſuch bodies, being continually driven about, till they become extinct.

That this interruption or diſcontinuity of parts is the chief cauſe of Opacity, Sir Iſaac Newton argues, appears from hence; that all opake bodies immediately begin to be tranſparent, when their pores become filled with a ſubſtance of nearly equal denſity with their parts. Thus, paper dipped in water or oil, ſome ſtones ſteeped in water, linen cloth dipped in oil or vinegar, &c, become more tranſparent than before.

OPAKE, not tranſlucent, nor tranſparent, or not admitting a free paſſage to the rays of light.

OPEN *Flank*, in Fortification, is that part of the flank which is covered by the orillon or ſhoulder.

OPENING *of the Trenches*, is the firſt breaking of ground by the beſiegers, in order to carry on their approaches towards a place.

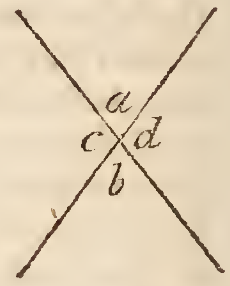
Opening *of Gates*, in Aſtrology, is when one planet ſeparates from another, and preſently applies to a third, bearing rule in a ſign oppoſite to that ruled by the planet with which it was before joined.

OPERA-*Glaſs*, in Optics, is ſo called from its uſe in play-houſes, and ſometimes a *Diagonal Perſpective*, from its conſtruction, which is as follows. ABCD (fig. 5, pl. xvii) repreſents a tube about 4 inches long; in each ſide of which there is a hole EF and GH, exactly againſt the middle of a plane mirror IK, which reflects the rays falling upon it to the convex glaſs LM; through which they are refracted to the concave eye-glaſs NO, whence they emerge parallel to the eye at the hole *rs*, in the end of the tube. Let P*a*Q be an object to be viewed, from which proceed the rays P*c*, *ab*, and Q*d*: theſe rays, being reflected by the plane mirror IK, will ſhew the object in the direction *cp*, *ba*, *dq*, in the image *pq*, equal to the object PQ, and as far behind the mirror as the object is before it: the mirror being placed ſo as to make an angle of 45 degrees with the ſides of the tube. And as, in viewing near objects, it is not neceſſary to magnify them, the focal diſtances of both the glaſſes may be nearly equal; or if that of LM be 3 inches, and that of NO one inch, the diſtance between them will be but 2 inches, and the object will be magnified 3 times, being ſufficient for the purpoſes to which this glaſs is applied.

When the object is very near, as XY, it is viewed through a hole *xy*, at the other end of the tube AB, without an eye glaſs; the upper part of the mirror being poliſhed for that purpoſe, as well as the under. The tube unſcrews near the object-glaſs LM, for taking out and cleanſing the glaſſes and mirror. The poſition of the object will be erect through the concave eye-glaſs.

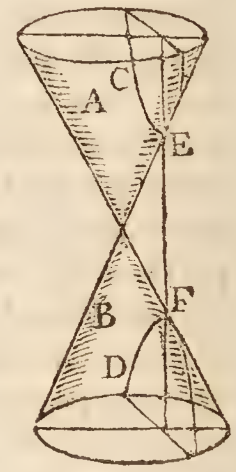
The peculiar artifice of this glaſs is to view a perſon at a ſmall diſtance, ſo that no one ſhall know who is obſerved; for the inſtrument points to a different object from that which is viewed; and as there is a hole on each ſide, it is impoſſible to know on which hand the object is ſituated, which you are viewing.

OPHIUCUS, a conſtellation of the northern hemiſphere; called alſo Serpentarius.



OPPOSITE *Angles*, or Vertical Angles, are thoſe oppoſite to each other, made by two interſecting lines; as *a* and *b*, or *c* and *d*.—The oppoſite angles are equal to each other.

Opposite *Cones*, denote two ſimilar cones vertically oppoſite, having the ſame common vertex and axis, and the ſame ſides produced; as the cones A and B.



Opposite *Sections*, or *Hyperbolas*, are thoſe made by cutting the Oppoſite cones by the ſame plane; as the hyperbolas C and D.—Theſe are always equal and ſimilar, and have the ſame tranſverſe axis EF, as alſo the ſame conjugate axis.

OPPOSITION, is that aſpect or ſituation of two planets or ſtars, when they are diametrically oppoſite to each other; being 180°, or a ſemi-circle apart; and marked thus ☍.

The moon is in Oppoſition to the ſun when ſhe is at the full.

OPTIC, or Optical, ſomething that relates to viſion, or the ſenſe of ſeeing, or the ſcience of optics.

Optic *Angle*. See Angle.

Optic *Axis*. See Axis.

Optic *Chamber*. See Camera *Obſcura*.

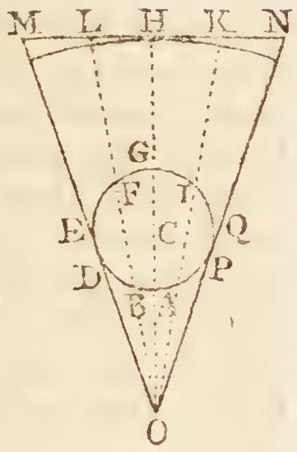
Optic *Glaſſes*, are glaſſes ground either concave or convex; ſo as either to collect or diſperſe the rays of light; by which means viſion is improved, and the eye ſtrengthened, preſerved, &c.

Among theſe, the principal are ſpectacles, reading glaſſes, teleſcopes, microſcopes, magic lanterns, &c.

Optic *Inequality*, in Aſtronomy, is an apparent irregularity in the motions of far diſtant bodies; ſo called, becauſe it is not really in the moving bodies, but ariſing from the ſituation of the obſerver's eye. For if the eye were in the centre, it would always ſee the motions as they really are.

The

The Optic Inequality may be thus illustrated. Suppose a body revolving with a real uniform motion, in the periphery of a circle ABD &c; and suppose the eye in the plane of the same circle, but at a distance from it, viewing the motion of the body from O. Now when the body goes from A to B; its apparent motion is measured by the angle AOB or the arch or line HL, which it will seem to describe. But while it moves through the arch BD in an equal time, its apparent motion will be determined by the angle BOD, or the arch or line LM, which is less than the former LH. But it spends the same time in describing DE, as it does in AB or BD; during all which time of describing DE it appears stationary in the point M. When it really describes EFGIQ, it will appear to pass over MLHKN; so that it will seem to have gone retrograde. And lastly, from Q to P it will again appear stationary in the point N.



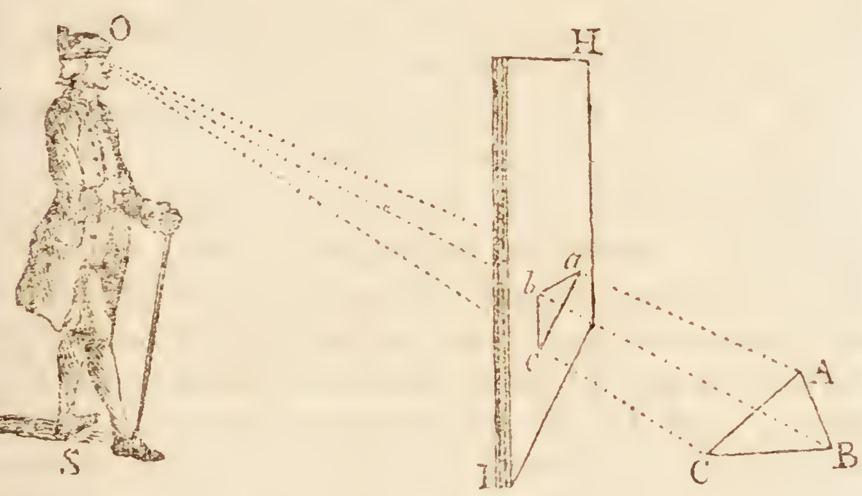
Optic *Nerves*, the second pair of nerves, springing from the crura of the medulla oblongata, and passing thence to the eye.

These are covered with two coats, which they take from the dura and pia mater; and which, by their expansions, form the two membranes of the eye, called the uvea and cornea. And the retina, which is a third membrane, and the immediate organ of sight, is only an expansion of the fibrous, or inner, and medullary part of these nerves.

Optic *Pencil*. See Pencil *of Rays*.

Optic *Place*, of a star &c, is that point or part of its orbit, which is determined by our sight, when the star is seen there. This is either true or apparent; true, when the observer's eye is supposed to be at the centre of the motion; or apparent, when his eye is at the circumference of the earth. See also Place.

Optic *Pyramid*, in Perspective, is the pyramid ABCO, whose base is the visible object ABC, and the vertex is in the eye at O; being formed by rays



drawn from the several points of the perimeter to the eye.

Hence also may appear what is meant by Optic triangle.

Optic *Rays*, particularly means those by which an Optic pyramid, or Optic triangle, is terminated. As OA, OB, OC, &c.

OPTICS, the science of vision; including Catoptrics, and Dioptrics; and even Perspective; as also the whole doctrine of light and colours, and all the phenomena of visible objects.

Optics, in its more extensive acceptation, is a mixed mathematical science; which explains the manner in which vision is performed in the eye; treats of sight in general; gives the reasons of the several modifications or alterations, which the rays of light undergo in the eye; and shews why objects appear sometimes greater, sometimes smaller, sometimes more distinct, sometimes more confused, sometimes nearer and sometimes more remote. In this extensive signification it is considered by Newton, in his excellent work called Optics.

Indeed Optics makes a considerable branch of natural philosophy; both as it explains the laws of nature, according to which vision is performed; and as it accounts for abundance of physical phenomena, otherwise inexplicable.

The Principal Authors and Discoveries in Optics, are the following:

Euclid seems to be the earliest author on Optics that we have. He composed a treatise on the ancient Optics and catoptrics; dioptrics being less known to the Ancients; though it was not quite unnoticed by them, for among the phenomena, at the beginning of that work, Euclid remarks the effect of bringing an object into view, by refraction, in the bottom of a vessel, by pouring water into it, which could not be seen over the edge of the vessel, before the water was poured in; and other authors speak of the then known effects of glass globes &c, both as burning glasses, and as to bodies seen through them. Euclid's work however is chiefly on catoptrics, or reflected rays; in which he shews, in 31 propositions, the chief properties of them, both in plane, convex, and concave surfaces, in his usual geometrical manner; beginning with that concerning the equality of the angles of incidence and reflection, which he demonstrates; and in the last proposition, shewing the effect of a concave speculum, as a burning glass, when exposed to the rays of the sun.

The effects of burning glasses, both by refraction and reflection, are noticed by several others of the Ancients, and it is probable that the Romans had a method of lighting their sacred fire by some such means. Aristophanes, in one of his comedies, introduces a person as making use of a globe filled with water to cancel a bond that was against him, by thus melting the wax of the seal. And if we give but a small degree of credit to what some ancient historians are said to have written concerning the exploits of Archimedes, we shall be induced to think that he constructed some very powerful burning mirrors. It is even allowed that this eminent geometrician wrote a treatise on the subject of them, though it be not now extant; as also concerning the appearance of a ring or circle under water, and therefore could not have been ignorant of the common phenomena of refraction. We find many questions concerning such optical appearances in Aristotle. This author was also sensible that it is the reflection of light from the atmosphere which prevents total darkness after the sun sets, and in places where he does not shine in the day time. He was also of opinion, that rainbows, halos,

halos, and mock ſuns, were all occaſioned by the reflection of the ſunbeams in different circumſtances, by which an imperfect image of his body was produced, the colour only being exhibited, and not his proper figure.

The Ancients were not only acquainted with the more ordinary appearances of refraction, but knew alſo the production of colours by refracted light. Seneca ſays, that when the light of the ſun ſhines through an angular piece of glaſs, it ſhews all the colours of the rainbow. Theſe colours however, he ſays, are falſe, ſuch as are ſeen in a pigeon's neck when it changes its poſition; and of the ſame nature he ſays is a ſpeculum, which, without having any colour of its own, aſſumes that of any other body.

It appears alſo, that the Ancients were not unacquainted with the magnifying power of glaſs globes filled with water, though it does not appear that they knew any thing of the reaſon of this power: and it is ſuppoſed that the ancient engravers made uſe of a glaſs globe filled with water to magnify their figures, that they might work to more advantage.

Ptolomy, about the middle of the ſecond century, wrote a conſiderable treatiſe on Optics. The work is loſt; but from the accounts of others, it appears that he there treated of aſtronomical refractions. The firſt aſtronomers were not aware that the intervals between ſtars appear leſs when near the horizon than in the meridian; and on this account they muſt have been much embarraſſed in their obſervations: but it is evident that Ptolomy was aware of this circumſtance by the caution which he gives to allow ſomething for it, whenever recourſe is had to ancient obſervations. This philoſopher alſo advances a very ſenſible hypotheſis to account for the remarkably great apparent ſize of the ſun and moon when ſeen near the horizon. The mind, he ſays, judges of the ſize of objects by means of a preconceived idea of their diſtance from us: and this diſtance is fancied to be greater when a number of objects are interpoſed between the eye and the body we are viewing; which is the caſe when we ſee the heavenly bodies near the horizon. In his Almageſt, however, he aſcribes this appearance to a refraction of the rays by vapours, which actually enlarge the angle under which the luminaries appear; juſt as the angle is enlarged by which an object is ſeen from under water.

Alhazen, an Arabian writer, was the next author of conſequence, who wrote about the year 1100. Alhazen made many experiments on refraction, at the ſurface between air and water, air and glaſs, and water and glaſs; and hence he deduced ſeveral properties of atmoſpherical refraction; ſuch as, that it increaſes the altitudes of all objects in the heavens; and he firſt advanced that the ſtars are ſometimes ſeen above the horizon by means of refraction, when they are really below it: which obſervation was confirmed by Vitello, Walther, and eſpecially by the obſervations of Tycho Brahe. Alhazen obſerved, that refraction contracts the diameters and diſtances of the heavenly bodies, and that it is the cauſe of the twinkling of the ſtars. This refractive power he aſcribed, not to the vapours contained in the air, but to its different degrees of tranſparency. And it was his opinion, that ſo far from being the cauſe of the heavenly bodies appearing larger near the horizon, that it would make them appear leſs; obſerving that two ſtars appear nearer together in the horizon, than near the meridian. This phenomenon he ranks among optical deceptions. We judge of diſtance, he ſays, by comparing the angle under which objects appear, with their ſuppoſed diſtance; ſo that if theſe angles be nearly equal, and the diſtance of one object be conceived greater than that of the other, this will be imagined to be the larger. And he farther obſerves, that the ſky near the horizon is always imagined to be farther from us than any other part of the concave ſurface.

In the writings of Alhazen too, we find the firſt diſtinct account of the magnifying power of glaſſes; and it is not improbable that his writings on this head gave riſe to the uſeful invention of ſpectacles: for he ſays, that if an object be applied cloſe to the baſe of the larger ſegment of a ſphere of glaſs, it will appear magnified. He alſo treats of the appearance of an object through a globe, and ſays that he was the firſt who obſerved the refraction of rays into it.

In 1270, Vitello, a native of Poland, publiſhed a treatiſe on Optics, containing all that was valuable in Alhazen, and digeſted in a better manner. He obſerves, that light is always loſt by refraction, which makes objects appear leſs luminous. He gave a table of the reſults of his experiments on the refractive powers of air, water, and glaſs, correſponding to different angles of incidence. He aſcribes the twinkling of the ſtars to the motion of the air in which the light is refracted; and he illuſtrates this hypotheſis, by obſerving that they twinkle ſtill more when viewed in water put in motion. He alſo ſhews, that refraction is neceſſary as well as reflection, to form the rainbow; becauſe the body which the rays fall upon is a tranſparent ſubſtance, at the ſurface of which one part of the light is always reflected, and another refracted. And he makes ſome ingenious attempts to explain refraction, or to aſcertain the law of it. He alſo conſiders the foci of glaſs ſpheres, and the apparent ſize of objects ſeen through them; though with but little accuracy.

To Vitello may be traced the idea of ſeeing images in the air. He endeavours to ſhew, that it is poſſible, by means of a cylindrical convex ſpeculum, to ſee the images of objects in the air, out of the ſpeculum, when the objects themſelves cannot be ſeen.

The Optics of Alhazen and Vitello were publiſhed at Baſil in 1572, by Fred. Riſner.

Contemporary with Vitello, was Roger Bacon, a man of very extenſive genius, who wrote upon almoſt every branch of ſcience; though it is thought his improvements in Optics were not carried far beyond thoſe of Alhazen and Vitello. He even aſſents to the abſurd notion, held by all philoſophers down to his time, that viſible rays proceed *from* the eye, inſtead of *towards* it. From many ſtories related of him however, it would ſeem, that he made greater improvements than appear in his writings. It is ſaid he had the uſe of ſpectacles: that he had contrivances, by reflection from glaſſes, to ſee what was doing at a great diſtance, as in an enemy's camp. And lord chancellor Bacon relates a ſtory, of his having apparently walked in the air between two ſteeples, and which he ſuppoſed was effected

by

by reflection from glasses while he walked upon the ground.

About 1279 was written a treatise on Optics by Peccam, archbishop of Canterbury.

One of the next who distinguished himself in this way, was Maurolycus, teacher of mathematics at Messina. In a treatise, De Lumine et Umbra, published in 1575, he demonstrates, that the crystalline humour of the eye is a lens that collects the rays of light issuing from the objects, and throws them upon the retina, where the focus of each pencil is. From this principle he discovered the reason why some people are short-sighted, and others long-sighted; also why the former are relieved by concave glasses, and the others by convex ones.

Contemporary with Maurolycus, was John Baptista Porta, of Naples. He discovered the Camera Obscura, which throws considerable light on the nature of vision. His house was the constant resort of all the ingenious persons at Naples, whom he formed into what he called An Academy of Secrets; each member being obliged to contribute something that was not generally known, and might be useful. By this means he was furnished with materials for his Magia Naturalis, which contains his account of the Camera Obscura, and the first edition of which was published, as he informs us, when he was not quite 15 years old. He also gave the first hint of the Magic Lantern; which Kircher afterwards followed and improved. His experiments with the camera obscura convinced him, that vision is performed by the intromission of something into the eye, and not by visual rays proceeding from it, as had been formerly imagined; and he was the first who fully satisfied himself and others upon this subject. He justly considered the eye as a camera obscura, and the pupil the hole in the window-shutter; but he was mistaken in supposing that the crystalline humour corresponds to the wall which receives the images; nor was it discovered till the year 1604, that this office is performed by the retina. He made a variety of just remarks concerning vision; and particularly explained several cases in which we imagine things to be without the eye, when the appearances are occasioned by some affection of the eye itself, or by some motion within the eye. —He remarked also that, in certain circumstances, vision will be assisted by convex or concave glasses; and he seems even to have made some small advances towards the discovery of telescopes.

Other treatises on Optics, with various and gradual improvements, were afterwards successively published by several authors: as Aguilon, Opticorum libr. 6, Antv. 1613; L'Optique, Catoptrique, & Dioptrique of Herigone, in his Cursus Math. Paris 1637; the Dioptrics of Des Cartes, 1637; L'Optique & Catoptrique of Mersenne, Paris 1651: Scheiner, Optica, Lond. 1652: Manchini, Dioptrica Practica, Bologna, 1660: Barrow, Lectiones Opticæ, London 1663: James Gregory, Optica Promota, Lond. 1663: Grimaldi, Physico-mathesis de Lumine, Coloribus, & Iride, Bononia, 1665: Scaphusa, Cogitationes Physico-mechanicæ de Natura Visionis, Heidel. 1670: Kircher, Ars Magna Lucis & Umbræ, Rome 1671: Cherubin, Dioptrique Oculaire, Paris 1671: Leibnitz, Principe Generale de l'Optique, Leipsic Acts 1682: Newton's Optics and Lectiones Opticæ, 4to and 8vo, 1704 &c: Molyneux, Dioptrics, Lond. 1692: Dr. Jurin's Theory of Distinct and Indistinct Vision.—There is also a large and excellent work on Optics, by Dr. Smith, 2 vols 4to; and an elaborate History of the Present State of Discoveries relating to Vision, Light, and Colours, by Dr. Priestley, 4to, 1772; with a multitude of other authors of inferior note; besides lesser and occasional tracts and papers in the Memoirs of the several learned Academies and Societies of Europe; with improvements by many other persons, among whom are the respectable names of Snell, Fermat, Kepler, Huygens, Hortensius, Boyle, Hook, De la Hire, Lowthorp, Cassini, Halley, Delisle, Euler, Dollond, Clairaut, D'Alembert, Zeiher, Bouguer, Buffon, Nollet, Baume; but the particular improvements by each author must be referred to the history of his life, under the article of their names; while the history and improvements of the several branches are to be found under the various particular articles, as, Light, Colours, Reflection, Refraction, Inflection, Transmission, &c, Spectacles, Telescope, Microscope, &c, &c.

ORB, a spherical shell, hollow sphere, or space contained between two concentric spherical surfaces.—The ancient astronomers conceived the heavens as consisting of several vast azure transparent Orbs or spheres, inclosing one another, and including the bodies of the planets.

The Orbis *Magnus*, or *Great* Orb, is that in which the sun is supposed to revolve; or rather it is that in which the earth makes its annual circuit.

Orb, in Astrology, or Orb *of Light*, is a certain sphere or extent of light, which the astrologers allow a planet beyond its centre. They pretend that, provided the aspects do but fall within this Orb, they have almost the same effect as if they pointed directly against the centre of the planet.—The Orb of Saturn's light they make to be 10 degrees; that of Jupiter 12 degrees; that of Mars $7\frac{1}{2}$; that of the Sun 17 degrees; that of Venus 8 degrees; that of Mercury 7 degrees; and that of the Moon $12\frac{1}{2}$ degrees.

ORBIT, is the path of a planet or comet; being the curve line described by its centre, in its proper motion in the heavens. So the earth's Orbit, is the ecliptic, or the curve it describes in its annual revolution about the sun.

The ancient astronomers made the planets describe circular Orbits, with an uniform velocity. Copernicus himself could not believe they should do otherwise; being unable to disentangle himself entirely from the excentrics and epicycles to which they had recourse, to account for the inequalities in their motions.

But Kepler found, from observations, that the Orbit of the earth, and that of every primary planet, is an ellipsis, having the sun in one of its foci; and that they all move in these ellipses by this law, that a radius drawn from the centre of the sun to the centre of the planet, always describes equal areas in equal times; or, which is the same thing, in unequal times, it describes areas that are proportional to those times. And Newton has since demonstrated, from the nature of universal gravitation, and projectile motion, that the Orbits must of necessity be ellipses, and the motions observe that law,

law, both of the primary and fecondary planets; excepting in fo far as their motions and paths are difturbed by their mutual actions upon one another; as the Orbit of the earth by that of the moon; or that of Saturn by the action of Jupiter; &c.

Of thefe elliptic Orbits, there have been two kinds affigned: the firft that of Kepler and Newton, which is the common or conical ellipfe; for which Seth Ward, though he himfelf keeps to it, thinks we might venture to fubftitute circular Orbits, by ufing two points, taken at equal diftances from the centre, on one of the diameters, as is done in the foci of the ellipfis, and which is called his Circular Hypothefis. The fecond is that of Caffini, of this nature, viz, that the products of the two lines drawn from the two foci, to any point in the circumference, are everywhere equal to the fame conftant quantity; whereas, in the common ellipfe, it is the fum of thofe two lines that is always a conftant quantity.

The Orbits of the planets are not all in the fame plane with the ecliptic, which is the earth's Orbit round the fun, but are varioufly inclined to it, and to each other: but ftill the plane of the ecliptic, or earth's Orbit, interfects the plane of the Orbit of every other planet, in a right line which paffes through the fun, called the line of the nodes, and the points of interfection of the Orbits themfelves are called the nodes.

The mean femidiameters of the feveral Orbits, or the mean diftances of the planets from the fun, with the excentricities of the Orbits, their inclination to the ecliptic, and the places of their nodes, are as in the following table; where the 2d column contains the proportions of femidiameters of the Orbits, the true femidiameter of that of the earth being 95 millions of miles; and the 3d column fhews what part of the femidiameters the excentricities are equal to.

	Propor. femid.	Excentr. pts. of femidiam.	Inclina. of Orbit.	Afcending Node, 1790.
Mercury	387	$\frac{4}{19}$	6° 54′	♉ 14° 43
Venus	723	$\frac{1}{138}$	3 20	♊ 13 59
Earth	1000	$\frac{1}{59}$	0 0	- -
Mars	1524	$\frac{1}{11}$	1 52	♉ 17 17
Jupiter	5201	$\frac{1}{21}$	1 20	♋ 7 29
Saturn	9539	$\frac{1}{18}$	2 30	♋ 21 13
Georgian	19034	$\frac{1}{21}$	0 48	♊ 12 54

The Orbits of the comets are alfo very excentric ellipfes.

ORDER, in Architecture, a fyftem of the feveral members, ornaments, and proportions of a column and pilafter.

There are five Orders of columns, of which three are Greek, viz, the Doric, Ionic, and Corinthian; and two Italic, viz, the Tufcan and Compofite. The three Greek Orders reprefent the three different manners of building, viz, the folid, the delicate, and the middling: the two Italic ones are imperfect productions of thefe.

ORDER, in Aftronomy. A planet is faid to go according to the order of the figns, when it is direct; proceeding from Aries to Taurus, thence to Gemini, &c. As, on the contrary, it goes contrary to the Order of the figns, when it is retrograde, or goes backward, from Pifces to Aquarius, &c.

ORDER, in the Geometry of Curve Lines, is denominated from the rank or Order of the equation by which the geometrical line is expreffed; fo the fimple equation, or 1ft power, denotes the 1ft Order of lines, which is the right line; the quadratic equation, or 2d power, defines the 2d Order of lines, which are the conic fections and circle; the cubic equation, or 3d power, defines the 3d Order of lines; and fo on.

Or, the Orders of lines are denominated from the number of points in which they may be cut by a right line. Thus, the right line is of the 1ft Order, becaufe it can be cut only in one point by a right line; the circle and conic fections are of the 2d Order, becaufe they can be cut in two points by a right line; while thofe of the 3d Order, are fuch as can be cut in 3 points by a right line; and fo on.

It is to be obferved, that the Order of curves is always one degree lower than the correfponding line; becaufe the 1ft Order, or right line, is no curve; and the circle and conic fections, which are the 2d Order of lines, are only the 1ft Order of curves; &c.

See Newton's Enumeratio Linearum Tertii Ordinis.

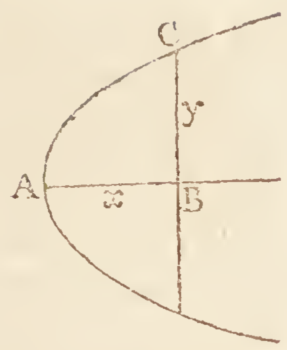
ORDINATES, in the Geometry of Curve Lines, are right lines drawn parallel to each other, and cutting the curve in a certain number of points.

The parallel Ordinates are ufually all cut by fome other line, which is called the abfcifs, and commonly the Ordinates are perpendicular to the abfciffal line. When this line is a diameter of the curve, the property of the Ordinates is then the moft remarkable; for, in the curves of the firft kind, or the conic fections and circle, the Ordinates are all bifected by the diameter, making the part on one fide of it equal to the part on the other fide of it; and in the curves of the 2d order, which may be cut in three points by an Ordinate, then of the three parts of the Ordinate, lying between thefe three interfections of the curve and the interfection with the diameter, the part on one fide the diameter is equal to both the two parts on the other fide of it. And fo for curves of any order, whatever the number of interfections may be, the fum of the parts of any Ordinate, on one fide of the diameter, is equal to the fum of the parts on the other fide of it.

The ufe of Ordinates in a curve, and their abfciffes, is to define or exprefs the nature of a curve, by means of the general relation or equation between them; and the greateft number of factors, or the dimenfions of the higheft term, in fuch equation, is always the fame as the order of the line; that equation being a quadratic, or its higheft term of two dimenfions, in the lines of the 2d order, being the circle and conic fections; and a cubic equation, or its higheft term containing 3 dimenfions, in the lines of the 3d order; and fo on.

Thus,

Thus, y denoting an Ordinate BC, and x its abscifs AB; alfo a, b, c, &c, given quantities: then $y^2 = ax^2 + bx + c$ is the general equation for the lines of the 2d order; and $xy^2 - ey = ax^3 + bx^2 + cx + d$ is the equation for the lines of the 3d order; and fo on.



ORDNANCE, are all forts of great guns, ufed in war; fuch as cannon, mortars, howitzers, &c.

ORFFYREUS's *Wheel*, in Mechanics, is a machine fo called from its inventor, which he afferted to be a perpetual motion. This machine, according to the account given of it by Gravefande, in his Oeuvres Philofophiques, publifhed by Allemand, Amft. 1774, confifted externally of a large circular wheel, or rather drum, 12 feet in diameter, and 14 inches deep; being very light, as it was formed of an affemblage of deals, having the intervals between them covered with waxed cloth, to conceal the interior parts of it. The two extremities of an iron axis, on which it turned, refted on two fupports. On giving a flight impulfe to the wheel, in either direction, its motion was gradually accelerated; fo that after two or three revolutions it acquired fo great a velocity as to make 25 or 26 turns in a minute. This rapid motion it actually preferved during the fpace of 2 months, in a chamber of the landgrave of Heffe, the door of which was kept locked, and fealed with the landgrave's own feal. At the end of that time it was ftopped, to prevent the wear of the materials. The profeffor, who had been an eye-witnefs to thefe circumftances, examined all the external parts of it, and was convinced that there could not be any communication between it and any neighbouring room. Orffyreus however was fo incenfed, or pretended to be fo, that he broke the machine in pieces, and wrote on the wall, that it was the impertinent curiofity of profeffor Gravefande which made him take this ftep. The prince of Heffe, who had feen the interior parts of this wheel, but fworn to fecrefy, being afked by Gravefande, whether, after it had been in motion for fome time, there was any change obfervable in it, and whether it contained any pieces that indicated fraud or deception, anfwered both queftions in the negative, and declared that the machine was of a very fimple conftruction.

ORGANICAL *Defcription of Curves*, is the defcription of them upon a plane, by means of inftruments, and commonly by a continued motion. The moft fimple conftruction of this kind, is that of a circle by means of a pair of compaffes. The next is that of an ellipfe by means of a thread and two pins in the foci, or the ellipfe and hyperbola, by means of the elliptical and hyperbolic compaffes.

A great variety of defcriptions of this fort are to be found in Schooten De Organica Conic. Sect. in Plano Defcriptione; in Newton's Arithmetica Univerfalis, De Curvarum Defcriptione Organica; Maclaurin's Geometria Organica; Brackenridge's Defcriptio Linearum Curvarum; &c.

ORGUES, or ORGANS, in Fortification, long and thick pieces of wood, fhod with pointed iron, and hung each by a feparate rope over the gate-way of a town, ready on any furprife or attempt of the enemy to be let down to ftop up the gate. The ends of the feveral ropes are wound about a windlafs, fo as to be let down all together.

ORGUES is alfo ufed for a machine compofed of feveral harquebuffes or mufket-barrels, bound together; fo as to make feveral explofions at the fame time. They are ufed to defend breaches and other places attacked.

ORIENT, the eaft, or the eaftern point of the horizon.

ORIENT *Equinoctial*, is ufed for that point of the horizon where the fun rifes when he is in the equinoctial, or when he enters the figns Aries and Libra

ORIENT *Aeftival*, is the point where the fun rifes in the middle of fummer, when the days are longeft.

ORIENT *Hybernal*, is the point where the fun rifes in the middle of winter, when the days are fhorteft.

ORIENTAL, fituated towards the eaft with regard to us: in oppofition to occidental or the weft.

ORIENTAL *Aftronomy*, *Philofophy*, &c. ufed for thofe of the eaft, or of the Arabians, Chaldeans, Perfians, Indians. &c.

ORILLON, in Fortification, a fmall rounding of earth, lined with a wall, raifed on the fhoulder of thofe baftions that have cafemates, to cover the cannon in the retired flank, and prevent their being difmounted by the enemy.

There are other forts of Orillons, properly called Epaulements, or Shoulderings, which are almoft of a fquare figure.

ORION, a conftellation of the fouthern hemifphere, with refpect to the ecliptic, but half in the northern, and half on the fouthern fide of the equinoctial, which runs acrofs the middle of his body.

The ftars in this conftellation are, 38 in Ptolomy's catalogue, 42 in Tycho's, 62 in Hevelius's, and 78 in Flamfteed's. But fome telefcopes have difcovered feveral thoufands of ftars in this conftellation.

Of thefe ftars, there are no lefs than two of the firft magnitude, and four of the fecond, befide a great many of the third and fourth. One of thofe two ftars of the firft magnitude is upon the middle of the left foot, and is called *Regel;* the other is on the right fhoulder, and called *Betelgueufe;* of the four of the fecond magnitude, one is on the left fhoulder, and called *Bellatrix*, and the other three are in the belt, lying nearly in a right line and at equal diftances from each other, forming what is popularly called the *Yardwand*.

This conftellation is one of the 48 old afterifms, and one of the moft remarkable in the heavens. It is in the figure of a man, having a fword by his fide, and feems attacking the bull with a club in his right hand, his left bearing a fhield.

This conftellation is particularly mentioned by many of the ancient authors, and even in the Scriptures themfelves. The Greeks, according to their cuftom, give feveral fabulous accounts of him. One is, that this Orion was a fon of their fea-god Neptune by Euryale, the famous huntrefs. The fon poffeffed the difpofition of his mother, and became the greateft hunter in the world; and Neptune gave him the fingular privilege, that he fhould walk upon the furface of the fea as well

as if it were on dry land. Another account of his origin is, that one Hyreius in Thebes, having entertained Jupiter and Mercury with great hoſpitality, requeſted of them the favour that he might have a ſon. The ſkin of the ox which he had ſacrificed to them, was buried in the ground, with certain ceremonies, and the ſon ſo much deſired was produced from it, a youth of promiſing ſpirit, and named Orion.

They farther tell us, that he viſited Chios when grown up, and raviſhed Penelope the daughter of Œnopion, for which the father put out his eyes, and baniſhed him the iſland: he thence went to Lemnos, where Vulcan received him, and gave him Cedalion for a companion. Afterwards, being reſtored to ſight by the ſun, he returned to Chios, and would have revenged himſelf on the king, but the people hid him. After this it ſeems he hunted with Diana, and was ſo exalted with his ſucceſs, that he uſed to ſay he would deſtroy every creature on the earth: the Earth, irritated at this, produced a Scorpion, which ſtung him to death, and both he and the reptile were taken up to the ſkies, the Scorpion making one of the twelve ſigns of the zodiac.

Others give a different account of his deſtruction: they tell us that he would have raviſhed the goddeſs of chaſtity Diana herſelf, and that ſhe killed him with her arrow. All the writers, however, are not agreed about this: they who make him the ſacrifice to the vengeance of the offended goddeſs, ſay, that herſelf afterwards placed his figure in the ſkies as a memorial of the attempt, and a terror to all ages. But there are ſome who ſay ſhe loved him ſo well that ſhe had thoughts of marrying him: theſe add, that Apollo could not bear ſo diſhonourable an alliance for his ſiſter, for which reaſon he killed him; and that Diana, after ſhedding ſhowers of tears over his corps, obtained of Jupiter a place for him in the heavens.

No conſtellation was ſo terrible to the mariners of the early periods, as this of Orion. He is mentioned in this way by all the Greek and Latin poets, and even by their hiſtorians; his riſing and ſetting being attended by ſtorms and tempeſts: and as the northern conſtellations are made the followers of the Pleiades; ſo are the ſouthern ones made the attendants of Orion.

The name of this conſtellation is alſo met with in Scripture ſeveral times, viz, in the books of Job, Amos, and Iſaiah. In Job it is aſked, "Canſt thou bind the ſweet influence of the Pleiades, or looſe the bands of Orion?" And Amos ſays, "Seek him that maketh the Seven Stars and Orion, and turneth the ſhadow of death into morning."

Orion's *River*, the ſame as the conſtellation Eridanus.

ORLE, Orlet, or Orlo, in Architecture, a fillet under the ovolo, or quarter-round of a capital.—When it is at the top or bottom of the ſhaft, it is called the cincture.—Palladio alſo uſes Orlo for the plinth of the baſes of columns and pedeſtals.

ORRERY, an aſtronomical machine, for exhibiting the various motions and appearances of the ſun and planets; and hence often called a Planetarium.

The reaſon of the name Orrery was this: Mr. Rowley, a mathematical inſtrument-maker, having got one from Mr. George Graham, the original inventor, to be ſent abroad with ſome of his own inſtruments, he copied it, and made the firſt for the earl of Orrery, Sir Richard Steel, who knew nothing of Mr. Graham's machine, thinking to do juſtice to the firſt encourager, as well as to the inventor of ſuch a curious inſtrument, called it an Orrery, and gave Rowley the praiſe due to Mr. Graham. Deſaguliers' Experim. Philoſ. vol. 1, pa. 430. The figure of this grand Orrery is exhibited at fig. 1, pl. 19. It is ſince made in various other figures.

ORTEIL, in Fortification. See Berme.

ORTELIUS (Abraham), a celebrated geographer, was born at Antwerp, in 1527. He was well ſkilled in the languages and mathematics, and acquired ſuch reputation by his ſkill in geography, that he was ſurnamed the *Ptolomy of his time*. Juſtus Lipſius, and moſt of the great men of the 16th century, were our author's intimate friends. He paſſed ſome time at Oxford in the reign of Edward the 6th; and he viſited England a ſecond time in 1577.

His *Theatrum Orbis Terræ* was the completeſt work of the kind that had ever been publiſhed, and gained our author a reputation adequate to his immenſe labour in compiling it. He wrote alſo ſeveral other excellent geographical works; the principal of which are, his *Theſaurus*, and his *Synonyma Geographica*.—The world is alſo obliged to him for the *Britannia*, which was undertaken by Cambden at his requeſt.—He died at Antwerp, 1598, at 71 years of age.

ORTHODROMICS, in Navigation, is Great-circle ſailing, or the art of ſailing in the arch of a great circle, which is the ſhorteſt courſe: For the arch of a great circle is Orthodromia, or the ſhorteſt diſtance between two points or places.

ORTHOGONIAL, in Geometry, is the ſame as rectangular, or right-angled.—When the term refers to a plane figure, it ſuppoſes one leg or ſide to ſtand perpendicular to the other: when ſpoken of ſolids, it ſuppoſes their axis to be perpendicular to the plane of the horizon.

ORTHOGRAPHIC or Orthographical *Projection of the Sphere*, is the projection of its ſurface or of the ſphere on a plane, paſſing through the middle of it, by an eye vertically at an infinite diſtance. See Projection.

ORTHOGRAPHY, in Geometry, is the drawing or delineating the fore-right plan or ſide of any object, and of expreſſing the heights or elevations of every part. Being ſo called from its determining things by perpendicular right lines falling on the geometrical plan; or rather, becauſe all the horizontal lines are here ſtraight and parallel, and not oblique as in repreſentations of perſpective.

Orthography, in Architecture, is the profile or elevation of a building, ſhewing all the parts in their true proportion. This is either external or internal.

External Orthography, is a delineation of the outer face or front of a building; ſhewing the principal wall with its apertures, roof, ornaments, and every thing viſible to an eye placed before the building. And

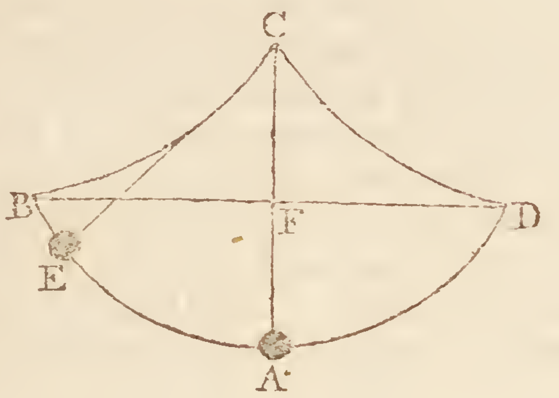
Internal Orthography, called alſo a Section, is a delineation or draught of a building, ſuch as it would appear if the external wall were removed.

Orthography, in Fortification, is the profile, or repreſentation

representation of a work; or a draught so conducted, as that the length, breadth, height, and thickness of the several parts are expressed, such as they would appear, if it were perpendicularly cut from top to bottom.

ORTIVE, or *Eastern Amplitude*, in Astronomy, is an arch of the horizon intercepted between the point where a star rises, and the east point of the horizon.

OSCILLATION, in Mechanics, vibration, or the reciprocal ascent and descent of a pendulum.



If a simple pendulum be suspended between two semicycloids BC, CD, that have the diameter CF of the generating circle equal to half the length of the string, so that the string, as the body E Oscillates, folds about them, then will the body Oscillate in another cycloid BEAD, similar and equal to the former. And the time of the Oscillation in any arc AE, measured from the lowest point A, is always the same constant quantity, whether that arc be larger or smaller. But the Oscillations in a circle are unequal, those in the smaller arcs being less than those in the larger; and so always less and less as the arcs are smaller, but still greater than the time of Oscillation in a cycloidal arc; till the circular arc becomes very small, and then the time of Oscillation in it is very nearly equal to the time in the cycloid, because the circle and cycloid have the same curvature at the vertex, the length of the string being the common radius of curvature to them there.

The time of one whole Oscillation in the cycloid, or of an ascent and descent in any arch of it, is to the time in which a heavy body would fall freely through CF or FA, the diameter of the generating circle, or through half the length of the pendulum string, as the circumference of a circle is to its diameter, that is as 3·1416 to 1. So that if l denote the length of the pendulum CA, and $g = 16\frac{1}{12}$ feet $= 193$ inches, the space a heavy body falls in the 1st second of time, and $p = 3{\cdot}1416$ the circumference of a circle whose diameter is 1: then by the laws of falling bodies,

it is $\sqrt{g} : \sqrt{\frac{1}{2}l} :: 1'' : \sqrt{\frac{l}{2g}}$, the time of falling through CF or $\frac{1}{2}l$; therefore $1 : p :: \sqrt{\frac{l}{2g}} : p\sqrt{\frac{l}{2g}}$, which is the time of one vibration in any arch of the cycloid which has the diameter of its generating circle equal to $\frac{1}{2}l$. Or, by extracting the known numbers, the same time of an Oscillation becomes barely $\frac{4}{25}\sqrt{l}$ or $\frac{16}{100}\sqrt{l}$ very nearly, l being the length of the pendulum in inches. And therefore this is also very nearly the time of an Oscillation in a small circular arc, whose radius is l inches.

Hence the times of the Oscillation of pendulums of different lengths, are directly in the subduplicate ratio of their lengths, or as the square roots of their lengths.

The more exact time of Oscillating in a circular arc, when this is of some finite small length, is

$\frac{4}{25}\sqrt{l} \times (1 + \frac{h}{8l})$; where h is the height of the vibration, or the versed sine of the single arc of ascent, or descent, to the radius l.

The celebrated Huygens first resolved the problem concerning the Oscillations of pendulums, in his book De Horologio Oscillatorio, reducing compound pendulums to simple ones. And his doctrine is founded on this hypothesis, that the common centre of gravity of several bodies, connected together, must ascend exactly to the same height from which it fell, whether those bodies be united, or separated from one another in ascending again, provided that each begin to ascend with the velocity acquired by its descent.

This supposition was opposed by several, and very much suspected by others. And those even who believed the truth of it, yet thought it too daring to be admitted without proof into a science which demonstrates every thing.

At length Mr. James Bernoulli demonstrated it, from the nature of the lever; and published his solution in the Mem. Acad. of Scienc. of Paris, for the year 1703. After his death, which happened in 1705, his brother John Bernoulli gave a more easy and simple solution of the same problem, in the same Memoirs for 1714; and about the same time, Dr. Brook Taylor published a similar solution in his Methodus Incrementorum: which gave occasion to a dispute between these two mathematicians, who accused each other of having stolen their solutions. The particulars of which dispute may be seen in the Leipsic Acts for 1716, and in Bernoulli's works, printed in 1743.

Axis of OSCILLATION, is a line parallel to the horizon, supposed to pass through the centre or fixed point about which the pendulum oscillates, and perpendicular to the plane in which the Oscillation is made.

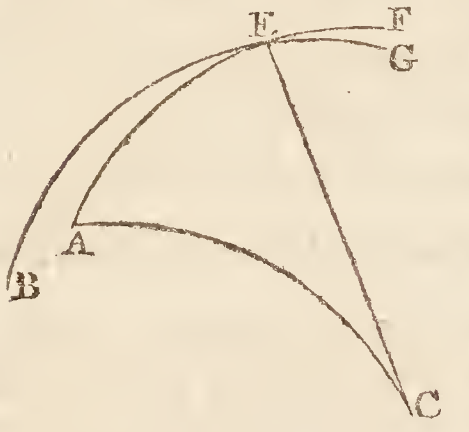
Centre of OSCILLATION, in a suspended body, is a certain point in it, such that the Oscillations of the body will be made in the same time as if that point alone were suspended at that distance from the point of suspension. Or it is the point into which if the whole weight of the body be collected, the several Oscillations will be performed in the same time as before: the Oscillations being made only by the force of gravity of the oscillating body. See CENTRE *of Oscillation*.

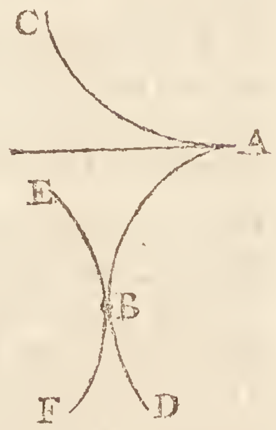
OSCULATION, in Geometry, denotes the contact between any curve and its osculatory circle, that is, the circle of the same curvature with the given curve, at the point of contact or of Osculation. If AC be the evolute of the involute curve AEF, and the tangent CE the radius of curvature at the point E, with which, and the centre C, if the circle BEG be described; this circle is said to osculate or kiss the curve AEF in the point E, which point E Mr. Huygens calls the point of Osculation, or kissing point.

The line CE is called the osculatory radius, or the radius of curvature; and the circle BEG the osculatory or kissing circle.

The

The evolute AC is the locus of the centres of all the circles that oſculate the involute curve AEF.





Osculation alſo means the point of concourſe of two branches of a curve which touch each other. For example, if the equation of a curve be $y = \sqrt{x} + \sqrt[4]{x^3}$, it is eaſy to ſee that the curve has two branches touching one another at the point where $x = 0$, becauſe the roots have each the ſigns + and −.

The point of Oſculation differs from the cuſp or point of retroceſſion (which is alſo a kind of point of contact of two branches) in this, that in this latter caſe the two branches terminate, and paſs no farther, but in the former the two branches exiſt on both ſides of the point of Oſculation. Thus, in the ſecond figure above, the point B is the Oſculation of the two branches ABD, EBF; but C, though it is alſo a tangent point, is a cuſp or point of retroceſſion, of AC and AB, the branches not paſſing beyond the point A.

OSCULATORY *Circle*, or *Kiſſing Circle*, is the ſame as the circle of curvature; that is, the circle having the ſame curvature with any curve at a given point. See the foregoing article, Oſculation, where BEG, in the laſt figure but one, is the Oſculatory circle of the curve AEF at the point E; and CE the Oſculatory radius, or the radius of curvature.

This circle is called Oſculatory, or kiſſing, becauſe that, of all the circles that can touch the curve in the ſame point, that one touches it the cloſeſt, in ſuch manner that no other ſuch tangent circle can be drawn between it and the curve; ſo that, in touching the curve, it embraces it as it were, both touching and cutting it at the ſame time, being on one ſide at the convex part of the curve, and on the other at the concave part of it.

In a circle, all the Oſculatory radii are equal, being the common radius of the circle; the evolute of a circle being only a point, which is its centre. See ſome properties of the Oſculatory circle in Maclaurin's Algebra, Appendix De Linearum Geometricarum Proprietatibus generalibus Tractatus, Theor. 2, § 15 &c, treated in a pure geometrical manner.

Osculatory *Parabola*. See Parabola.

Osculatory *Point*, the Oſculation, or point of contact between a curve and its Oſculatory circle.

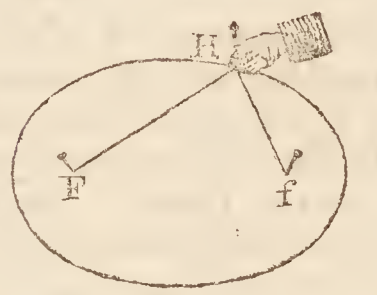
OSTENSIVE *Demonſtrations*, ſuch as plainly and directly demonſtrate the truth of any propoſition. In which they ſtand diſtinguiſhed from Apagogical ones, or reductions ad abſurdum, or ad impoſſibile, which prove the truth propoſed by demonſtrating the abſurdity or impoſſibility of the contrary.

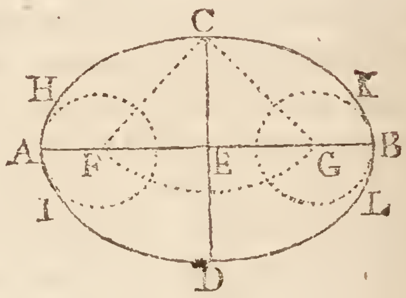
OTACOUSTIC, an inſtrument that aids or improves the ſenſe of hearing. See Acoustics.

OVAL, an oblong curvilinear figure, having two unequal diameters, and bounded by a curve line returning into itſelf. Or a figure contained by a ſingle curve line, imperfectly round, its length being greater than its breadth, like an egg: whence its name.

The proper Oval, or egg-ſhape, is an irregular figure, being narrower at one end than the other; in which it differs from the ellipſe, which is the mathematical Oval, and is equally broad at both ends.—The common people confound the two together: but geometricians call the Oval a Falſe Ellipſe.

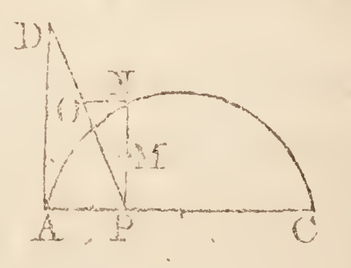
The method of deſcribing an Oval chiefly uſed among artificers, is by a cord or ſtring, as FHf, whoſe length is equal to the greater diameter of the intended Oval, and which is faſtened by its extremes to two points or pins, F and f, planted in its longer diameter; then, holding it always ſtretched out as at H, with a pin or pencil carried round the inſide, the Oval is deſcribed: which will be ſo much the longer and narrower as the two fixed points are farther apart. This Oval ſo deſcribed is the true mathematical ellipſe, the points F and f being the two foci.





Another popular way to deſcribe an Oval of a given length and breadth, is thus: Set the given length and breadth, AB and CD, to biſect each other perpendicularly at E; with the centre C, and radius AE, deſcribe an arc to croſs AB in F and G; then with theſe centres, F and G, and radii AF and BG, deſcribe two little arcs HI and KL for the ſmaller ends of the Oval; and laſtly, with the centres C and D, and radius CD, deſcribe the arcs HK and IL, for the flatter or longer ſides of the Oval.— Sometimes other points, inſtead of C and D, are to be taken by trial, as centres in the line CD, produced if neceſſary, ſo as to make the two laſt arcs join beſt with the two former ones.

Oval denotes alſo certain roundiſh figures, of various and pleaſant ſhapes, among curve lines of the higher kinds. Theſe figures are expreſſed by equations of all dimenſions above the 2d, and more eſpecially the even dimenſions, as the 4th, 6th, &c. Of this kind is the equation $a^2y^2 = -x^4 + ax^3$, which denotes the



Oval B, in ſhape of the ſection of a pear through the middle, and is eaſily deſcribed by means of points. For, if

a circle

a circle be defcribed whofe diameter AC is $= a$, and AD be perpendicular and equal to AC; then taking any point P in AC, joining DP, and drawing PN parallel to AD, and NO parallel to AC; and laftly taking PM $=$ NO, the point M will be one point of the Oval fought.

In like manner the equation

$$y^4 - 4y^2 = - ax^4 + bx^3 + cx^2 + dx + e$$

expreffes feveral very pretty Ovals, among which the following 12 are fome of the moft remarkable. For when the equation

$$ax^4 = bx^3 + cx^2 + dx + e$$

has four real unequal roots, the given equation will denote the three following fpecies, in fig. 1, 2, 3:

Fig. 1.

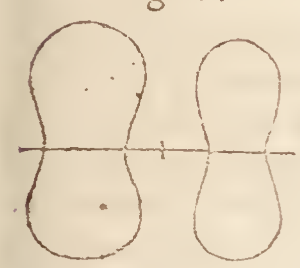


Fig. 2.

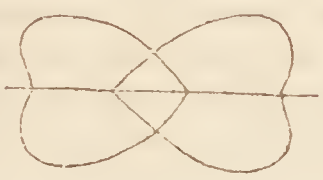
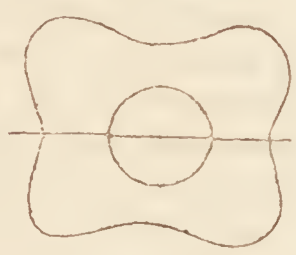


Fig. 3.



When the two lefs roots are equal, the three fpecies will be expreffed as in fig. 4, 5, 6, thus:

Fig. 4.

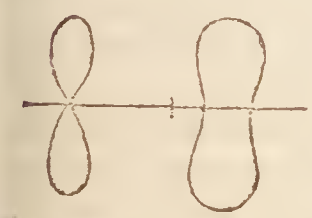


Fig. 5.

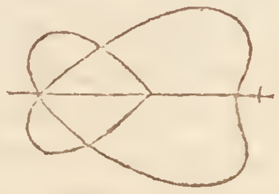
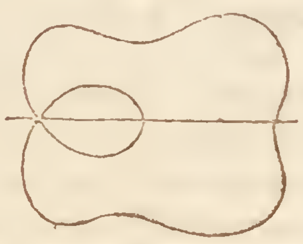


Fig. 6.



When the two lefs roots become imaginary, it will denote the three fpecies as exhibited in fig. 7, 8, 9:

Fig. 7.

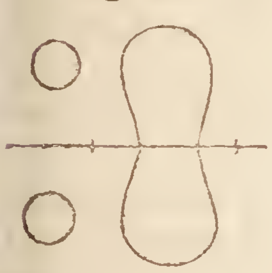


Fig. 8.

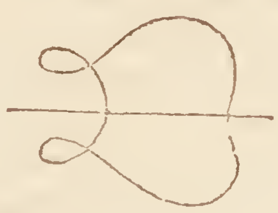


Fig. 9.

When the two middle roots are equal, the fpecies will be as appears in fig. 10: when two roots are equal, and two more fo, the fpecies will be as in fig. 11: and when the two middle roots become imaginary, the fpecies will be as appears in fig. 12:

Fig. 10.

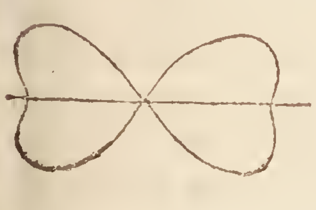


Fig. 11.

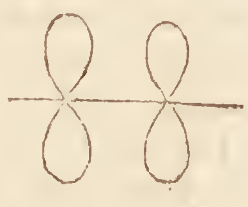
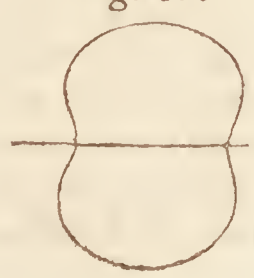


Fig. 12.



OUGHTRED (WILLIAM), an eminent Englifh mathematician and divine, was born at Eton in Buckinghamfhire, 1573, and educated in the fchool there; whence he was elected to King's-college in Cambridge in 1592, where he continued about 12 years, and became a fellow; employing his time in clofe application to ufeful ftudies, particularly the mathematical fciences, which he contributed greatly, by his example and exhortation, to bring into vogue among his acquaintances there.

About 1603 he quitted the univerfity, and was prefented to the rectory of Aldbury, near Guildford in Surry, where he lived a long retired and ftudious life, feldom travelling fo far as London once a year; his recreation being a diverfity of ftudies: "as often, fays he, as I was tired with the labours of my own profeffion, I have allayed that tedioufnefs by walking in the pleafant, and more than Elyfian Fields of the diverfe and various parts of human learning, and not of the mathematics only." About the year 1628 he was appointed by the earl of Arundel tutor to his fon lord William Howard, in the mathematics, and his Clavis was drawn up for the ufe of that young nobleman. He always kept up a correfpondence by letters with fome of the moft eminent fcholars of his time, upon mathematical fubjects: the originals of which were preferved, and communicated to the Royal Society, by William Jones, Efq. The chief mathematicians of that age owed much of their fkill to him; and his houfe was always full of young gentlemen who came from all parts to receive his inftruction: nor was he without invitations to fettle in France, Italy, and Holland. "He was as facetious, fays Mr. David Lloyd, in Greek and Latin, as folid in arithmetic, geometry, and the fphere, of all meafures, mufic, &c; exact in his ftyle as in his judgment; handling his tube and other inftruments at 80 as fteadily as others did at 30; owing this, as he faid, to temperance and exercife; principling his people with plain and folid truths, as he did the world with great and ufeful arts; advancing new inventions in all things but religion, which he endeavoured to promote in its primitive purity, maintaining that prudence, meeknefs, and fimplicity were the great ornaments of his life.

Notwithftanding Oughtred's great merit, being a ftrong royalift, he was in danger, in 1646, of a fequeftration by the committee for plundering minifters; feveral articles being depofed and fworn againft him: but upon his day of hearing, William Lilly, the famous aftrologer, applied to Sir Bulftrode Whitlocke and all his old friends; who appeared fo numerous in his behalf, that though the chairman and many other Prefbyterian members were active againft him, yet he was cleared by the majority. This is told us by Lilly himfelf, in the Hiftory of his own Life, where he ftyles Oughtred the moft famous mathematician then of Europe.—He died in 1660, at 86 years of age, and was buried at Aldbury. It is faid he died of a fudden ecftafy of joy, about the beginning of May, on hearing the news of the vote at Weftminfter, which paffed for the reftoration of Charles the 2d.—He left one fon, whom he put apprentice to a watch-maker, and wrote a book of inftructions in that art for his ufe.

He publifhed feveral works in his life time; the principal of which are the following:

1. *Arithmetica*

1. *Arithmeticæ in Numero & Speciebus Inſtitutio*, in 8vo, 1631. This treatiſe he intended ſhould ſerve as a general Key to the Mathematics. It was afterwards reprinted, with conſiderable alterations and additions, in 1648, under the title of *A Key to the Mathematics*. It was alſo publiſhed in Engliſh, with ſeveral additional tracts; viz, one on the Reſolution of all ſorts of Affected Equations in Numbers; a ſecond on Compound Intereſt; a third on the eaſy Art of Delineating all manner of Plain Sun-dials; alſo a Demonſtration of the Rule of Falſe-Poſition. A 3d edition of the ſame work was printed in 1652, in Latin, with the ſame additional tracts, together with ſome others, viz, On the Uſe of Logarithms; A Declaration of the 10th book of Euclid's Elements; a treatiſe of Regular Solids; and the Theorems contained in the books of Archimedes.

2. *The Circles of Proportion*, and a *Horizontal Inſtrument*; in 1633, 4to; publiſhed by his ſcholar Mr. William Foſter.

3. *Deſcription and Uſe of the Double Horizontal Dial*; 1636, 8vo.

4. *Trigonometria*: his treatiſe on Trigonometry, in Latin, in 4to, 1657: And another edition in Engliſh, together with Tables of Sines, Tangents, and Secants.

He left behind him a great number of papers upon mathematical ſubjects; and in moſt of his Greek and Latin mathematical books, there were found notes in his own hand writing, with an abridgment of almoſt every propoſition and demonſtration in the margin, which came into the muſeum of the late William Jones Eſq. F. R. S. Theſe books and manuſcripts then paſſed into the hands of his friend Sir Charles Scarborough the phyſician; the latter of which were carefully looked over, and all that were found fit for the preſs, printed at Oxford in 1676, in 8vo, under the title of

5. *Opuſcula Mathematica hactenus inedita*. This collection contains the following pieces: (1), Inſtitutiones Mechanicæ: (2), De Variis Corporum Generibus Gravitate & Magnitudine comparatis: (3), Automata: (4), Quæſtiones Diophanti Alexandrini, libri tres: (5), De Triangulis Planis Rectangulis: (6), De Diviſione Superficierum: (7), Muſicæ Elementa: (8), De Propugnaculorum Munitionibus: (9), Sectiones Angulares.

6. In 1660, Sir Jonas Moore annexed to his Arithmetic a treatiſe entitled, "*Conical Sections*; or, The ſeveral Sections of a Cone; being an Analyſis or Methodical Contraction of the two firſt books of Mydorgius, and whereby the nature of the Parabola, Hyperbola, and Ellipſis, is very clearly laid down. Tranſlated from the papers of the learned William Oughtred."

Oughtred, though undoubtedly a very great mathematician, was yet far from having the happieſt method of treating the ſubjects he wrote upon. His ſtyle and manner were very conciſe, obſcure, and dry; and his rules and precepts ſo involved in ſymbols and abbreviations, as rendered his mathematical writings very troubleſome to read, and difficult to be underſtood. Beſide the characters and abbreviations before made uſe of in Algebra, he introduced ſeveral others; as

× to denote multiplication;

:: for proportion or ſimilitude of ratios;

÷ for continued proportion;

⊐ ⊏ for greater and leſs; &c.

OUNCE, a ſmall weight, being the 16th part of a pound avoirdupois; and the 12th part of a pound troy.—The avoirdupois Ounce is divided into 16 drachms or drams; alſo the Ounce troy into 24 pennyweights, and the pennyweight into 24 grains.

OVOLO, in Architecture, a round moulding, whoſe profile or ſweep, in the Ionic and Compoſite capital, is uſually a quadrant of a circle; whence it is alſo popularly called the Quarter round.

OUTWARD *Flanking Angle*, or the *Angle of the Tenaille*, is that comprehended by the two flanking lines of defence.

OUTWORKS, in Fortification, all thoſe works made on the outſide of the ditch of a fortified place, to cover and defend it.

Outworks, called alſo Advanced and Detached Works, are thoſe which not only ſerve to cover the body of the place, but alſo to keep the enemy at a diſtance, and prevent them from taking advantage of the cavities and elevations uſually found in the places about the counterſcarp; which might ſerve them either as lodgments, or as rideaux, to facilitate the carrying on their trenches, and planting their batteries againſt the place. Such are ravelins, tenailles, hornworks, queue d'arondes, envelopes, and crownworks. Of theſe, the moſt uſual are ravelins, or halfmoons, formed between the two baſtions, on the flanking angle of the counterſcarp, and before the curtain, to cover the gates and bridges.

It is a general rule in all Outworks, that if there be ſeveral of them, one before another, to cover one and the ſame tenaille of a place, the nearer ones muſt gradually, and one after another, command thoſe which are fartheſt advanced out into the campagne; that is, muſt have higher ramparts, that ſo they may overlook and fire upon the beſiegers, when they are maſters of the more outward works.

The gorges alſo of all Outworks ſhould be plain, and without parapets; leſt, when taken, they ſhould ſerve to ſecure the beſiegers againſt the fire of the retiring beſieged; whence the gorges of Outworks are only paliſadoed, to prevent a ſurprize.

OX-EYE, in Optics. See Scioptic, and Camera *Obſcura*.

OXGANG, or Oxgate, of land, is uſually taken for 15 acres; being as much land as it is ſuppoſed one ox can plow in a year. In Lincolnſhire they ſtill corruptly call it Oſkin of land.—In Scotland, the term is uſed for a portion of arable land, containing 13 acres.

OXYGONE, in Geometry, is acute-angled, meaning a figure conſiſting wholly of acute angles, or ſuch as are leſs than 90 degrees each.—The term is chiefly applied to triangles, where the three angles are all acute.

OXYGONIAL, is acute-angular.

OZANAM (James), an eminent French mathematician, was deſcended from a family of Jewiſh extraction, but which had long been converts to the Romiſh faith; and ſome of whom had held conſiderable places in the parliaments of Provence. He was born at Boligneux in Breſſia, in the year 1640; and being a younger ſon,

ſon, though his father had a good eſtate, it was thought proper to breed him to the church, that he might enjoy ſome ſmall benefices which belonged to the family, to ſerve as a proviſion for him. Accordingly he ſtudied divinity four years; but then, on the death of his father, he devoted himſelf entirely to the mathematics, to which he had always been ſtrongly attached. Some mathematical books, which fell into his hands, firſt excited his curioſity; and by his extraordinary genius, without the aid of a maſter, he made ſo great a progreſs, that at the age of 15 he wrote a treatiſe of that kind.

For a maintenance, he firſt went to Lyons to teach the mathematics; which anſwered very well there; and after ſome time his generous diſpoſition procured him ſtill better ſucceſs elſewhere. Among his ſcholars were two foreigners, who expreſſing their uneaſineſs to him, at being diſappointed of ſome bills of exchange for a journey to Paris; he aſked them how much would do, and being told 50 piſtoles, he lent them the money immediately, even without their note for it. Upon their arrival at Paris, mentioning this generous action to M. Dagueſſeau, father of the chancellor, this magiſtrate was touched with it; and engaged them to invite Ozanam to Paris, with a promiſe of his favour. The opportunity was eagerly embraced; and the buſineſs of teaching the mathematics here ſoon brought him in a conſiderable income: but he wanted prudence for ſome time to make the beſt uſe of it. He was young, handſome, and ſprightly; and much addicted both to gaming and gallantry, which continually drained his purſe. Among others, he had a love intrigue with a woman, who lodged in the ſame houſe with him, and gave herſelf out for a perſon of condition. However, this expence in time led him to think of matrimony, and he ſoon after married a young woman without a fortune. She made amends for this defect however by her modeſty, virtue, and ſweet temper; ſo that though the ſtate of his purſe was not amended, yet he had more home-felt enjoyment than before, being indeed completely happy in her, as long as ſhe lived. He had twelve children by her, who moſtly all died young; and he was laſtly rendered quite unhappy by the death of his wife alſo, which happened in 1701. Neither did this misfortune come ſingle: for the war breaking out about the ſame time, on account of the Spaniſh ſucceſſion, it ſwept away all his ſcholars, who, being foreigners, were obliged to leave Paris. Thus he ſunk into a very melancholy ſtate; under which however he received ſome relief, and amuſement, from the honour of being admitted this ſame year an eleve of the Royal Academy of Sciences.

He ſeems to have had a pre-ſentiment of his death, from ſome lurking diſorder within, of which no outward ſymptoms appeared. In that perſuaſion he refuſed to engage with ſome foreign noblemen, who offered to become his ſcholars; alleging that he ſhould not live long enough to carry them through their intended courſe. Accordingly he was ſeized ſoon after with an apoplexy, which terminated his exiſtence in leſs than two hours, on the 3d of April 1717, at 77 years of age.

Ozanam was of a mild and calm diſpoſition, a chearful and pleaſant temper, endeared by a generoſity almoſt unparalleled. His manners were irreproachable after marriage; and he was ſincerely pious, and zealouſly devout, though ſtudiouſly avoiding to meddle in theological queſtions. He uſed to ſay, that it was the buſineſs of the Sorbonne to diſcuſs, of the pope to decide, and of a mathematician to go ſtraight to heaven in a perpendicular line. He wrote a great number of uſeful books; a liſt of which is as follows:

1. A treatiſe of Practical Geometry; 12mo, 1684.
2. Tables of Sines, Tangents and Secants; with a treatiſe of Trigonometry; 8vo, 1685.
3. A treatiſe of Lines of the Firſt Order; of the Conſtruction of Equations; and of Geometric Lines, &c; 4to, 1687.
4. The Uſe of the Compaſſes of Proportion, &c; with a treatiſe on the Diviſion of Lands; 8vo, 1688.
5. An Univerſal Inſtrument for readily reſolving Geometrical Problems without calculation; 12mo, 1688.
6. A Mathematical Dictionary; 4to, 1690.
7. A General Method for drawing Dials, &c; 12mo, 1693.
8. A Courſe of Mathematics, in 5 volumes, 8vo, 1693.
9. A treatiſe on Fortification, Ancient and Modern; 4to, 1693.
10. Mathematical and Philoſophical Recreations; 2 vols 8vo, 1694; and again with additions in 4 vols, 1724.
11. New Treatiſe on Trigonometry; 12mo, 1699.
12. Surveying, and meaſuring all ſorts of Artificers Works; 12mo, 1699.
13. New Elements of Algebra; 2 vols 8vo, 1702.
14. Theory and Practice of Perſpective; 8vo, 1711.
15. Treatiſe of Coſmography and Geography; 8vo, 1711.
16. Euclid's Elements, by De Chales, corrected and enlarged; 12mo, 1709.
17. Boulanger's Practical Geometry enlarged, &c; 12mo, 1691.
18. Boulanger's treatiſe on the Sphere corrected and enlarged; 12mo.

Ozanam has alſo the following pieces in the *Journal des Sçavans*: viz, (1), Demonſtration of this theorem, that neither the Sum nor the Difference of two Fourth Powers, can be a Fourth Power; Journal of May 1680.—(2), Anſwer to a Problem propoſed by M. Comiers; Journal of Nov. 17, 1681.—(3), Demonſtration of a Problem concerning Falſe and Imaginary Roots; Journal of April 2 and 9, 1685.—(4), Method of finding in Numbers the Cubic and Surſolid Roots of a Binomial, when it has one; Journal of April 9, 1691.

Alſo in the *Memoires de Trevoux*, of December 1703, he has this piece, viz, Anſwer to certain articles of Objection to the firſt part of his Algebra.

And laſtly, in the Memoirs of the Academy of Sciences, of 1707, he has Obſervations on a Problem of Spherical Trigonometry.

P.

PAGAN (Blaise François Comte de), an eminent French mathematician and engineer, was born at Avignon in Provence, 1604; and took to the profession of a soldier at 14 years of age. In 1620 he was employed at the siege of Caen, in the battle of Pont de Cé, and the reduction of the Navareins, and the rest of Béarn; where he signalized himself, and acquired a reputation far above his years. He was present, in 1621, at the siege of St. John d'Angeli, as also that of Clarac and Montauban, where he lost an eye by a musket-shot. After this time, there happened neither siege, battle, nor any other occasion, in which he did not signalize himself by some effort of courage and conduct. At the passage of the Alps, and the barricade of Suza, he put himself at the head of the Forlorn Hope, composed of the bravest youths among the guards; and undertook to arrive the first at the attack, by a private way which was extremely dangerous; when, having gained the top of a very steep mountain, he cried out to his followers, "There lies the way to glory!" Upon which, sliding along this mountain, they came first to the attack; when immediately commencing a furious onset, and the army coming to their assistance, they forced the barricades. When the king laid siege to Nancy in 1633, Pagan attended him, in drawing the lines and forts of circumvallation.—In 1642 he was sent to the service in Portugal, as field-marshal; and the same year he unfortunately lost the sight of his other eye by a distemper, and thus became totally blind.

But though he was thus prevented from serving his country with his conduct and courage in the field, he resumed the vigorous study of fortification and the mathematics; and in 1645 he gave the public a treatise on the former subject, which was esteemed the best extant.—In 1651 he published his *Geometrical Theorems*, which shewed an extensive and critical knowledge of his subject.—In 1655 he printed a *Paraphrase of the Account of the River of Amazons*, by father de Rennes; and, though blind, it is said he drew the chart of the river and the adjacent parts of the country, as in that work.—In 1657 he published *The Theory of the Planets*, cleared from that multiplicity of eccentric cycles and epicycles, which the astronomers had invented to explain their motions. This work distinguished him among astronomers as much as that of Fortification had among engineers. And in 1658 he printed his *Astronomical Tables*, which are plain and succinct.

Few great men are without some foible: Pagan's was that of a prejudice in favour of judicial astrology; and though he is more reserved than most others on that head, yet we cannot place what he did on that subject among those productions which do honour to his understanding. He was beloved and respected by all persons illustrious for rank as well as science; and his house was the rendezvous of all the polite and learned both in city and court.—He died at Paris, universally regretted, Nov. 18, 1665.

Pagan had an universal genius; and, having turned his attention chiefly to the art of war, and particularly to the branch of Fortification, he made extraordinary progress and improvements in it. He understood mathematics not only better than is usual for a gentleman whose view is to push his fortune in the army, but even to a degree of perfection superior to that of the ordinary masters who teach that science. He had so particular a genius for this kind of learning, that he acquired it more readily by meditation than by reading authors upon it; and accordingly he spent less time in such books than he did in those of history and geography. He had also made morality and politics his particular study; so that he may be said to have drawn his own character in his *Homme Heroïque*, and to have been one of the completest gentlemen of his time.—Having never married, that branch of his family, which removed from Naples to France in 1552, became extinct in his person.

PALILICUM, the same as Aldebaran, a fixed star of the first magnitude, in the eye of the Bull, or sign Taurus.

PALISADES, or Palisadoes, in Fortification, stakes or small piles driven into the ground, in various situations, as some defence against the surprize of an enemy. They are usually about 6 or 7 inches square, and 9 or 10 feet long, driven about 3 feet into the ground, and 6 inches apart from each other, being braced together by pieces nailed across them near the tops; and secured by thick posts at the distance of every 4 or 5 yards.

Palisades are placed in the covert-way, parallel to and at 3 feet distance from the parapet or ridge of the glacis, to secure it against a surprize. They are also used to fortify the avenues of open forts, gorges, half-moons, the bottoms of ditches, the parapets of covert-ways; and in general all places liable to surprize, and easy of access.

Palisadoes are usually planted perpendicularly; though some make an angle inclining out towards the enemy, that the ropes cast over them, to tear them up, may slip.

PALLADIO (Andrew), a celebrated Italian architect in the 16th century, was a native of Vicenza in Lombardy, and the disciple of Triffin, a learned man, who was a Patrician, or Roman nobleman, of the same town

town of Vicenza. Palladio was one of thofe, who laboured particularly to reftore the ancient beauties of architecture, and contributed greatly to revive a true tafte in that art. Having learned the principles of it, he went to Rome; where, applying himfelf with great diligence to ftudy the ancient monuments, he entered into the fpirit of their architects, and poffeffed himfelf with all their beautiful ideas. This enabled him to reftore their rules, which had been corrupted by the barbarous Goths. He made exact drawings of the principal works of antiquity which were to be met with at Rome; to which he added *Commentaries*, which went through feveral impreffions, with the figures. This, though a very ufeful work, yet is greatly exceeded by the four books of architecture, which he publifhed in 1570. The laft book treats of the Roman temples, and is executed in fuch a manner, as gives him the preference to all his predeceffors upon that fubject. It was tranflated into French by Roland Friatt, and into Englifh by feveral authors. Inigo Jones wrote fome excellent remarks upon it, which were publifhed in an edition of Palladio by Leoni, 1742, in 2 volumes folio.

PALLETS, in Clock and Watch Work, are thofe pieces or levers which are connected with the pendulum or balance, and receive the immediate impulfe of the fwing-wheel, or balance-wheel, fo as to maintain the vibrations of the pendulum in clocks, and of the balance in watches.—The Pallets in all the ordinary conftructions of clocks and watches, are formed on the verge or axis of the pendulum or balance, and are of various lengths and fhapes, according to the conftruction of the piece, or the fancy of the artift.

PALLIFICATION, or Piling, in Architecture, denotes the piling of the ground-work, or the ftrengthening it with piles, or timber driven into the ground; which is practifed when buildings are erected upon a moift or marfhy foil.

PALLISADES. See Palisades.

PALM, an ancient long meafure, taken from the extent of the hand.

The Roman Palm was of two kinds: the great Palm, taken from the length of the hand, anfwered to our fpan, and contained 12 fingers, digits, or fingers breadths, or 9 Roman inches, equal to about $8\frac{1}{2}$ Englifh inches. The fmall Palm, taken from the breadth of the hand, contained 4 digits or fingers, equal to about 3 Englifh inches.

The Greek Palm, or Doron, was alfo of two kinds. The fmall contained 4 fingers, equal to little more than 3 inches. The great Palm contained 5 fingers. The Greek double Palm, called Dichas, contained alfo in proportion.

The Modern Palm is different in different places where it is ufed. It contains,

	Inc.	Lines
At Rome - - -	8	$3\frac{1}{2}$
At Naples, according to Riccioli,	8	0
Ditto, according to others, -	8	7
At Genoa - - -	9	9
At Morocco and Fez - -	7	2
Languedoc, and fome other parts of France,	9	9
The Englifh Palm is - -	3	0

PALM-SUNDAY, the laft Sunday in Lent, or the Sunday next before Eafter-Day. So called, from the primitive days, on account of a pious ceremony then in ufe, of bearing Palms, in memory of the triumphant entry of Jefus Chrift into Jerufalem, eight days before the feaft of the paffover.

PAPPUS, a very eminent Greek mathematician of Alexandria towards the latter part of the 4th century, particularly mentioned by Suidas, who fays he flourifhed under the emperor Theodofius the Great, who reigned from the year 379 to 395 of Chrift. His writings fhew him to have been a confummate mathematician. Many of his works are loft, or at leaft have not yet been difcovered. Suidas mentions feveral of his works, as alfo Voffius *de Scientiis Mathematicis*. The principal of thefe are, his *Mathematical Collections*, in 8 books, the firft and part of the fecond being loft. He wrote alfo a *Commentary upon Ptolomy's Almageft*; an *Univerfal Chorography*; *A Defcription of the Rivers of Libya*; A Treatife of *Military Engines*; *Commentaries upon Ariftarchus of Samos, concerning the Magnitude and Diftance of the Sun and Moon*; &c. Of thefe, there have been publifhed, The Mathematical Collections, in a Latin tranflation, with a large Commentary, by Commandine, in folio, 1588; and a fecond edition of the fame in 1660. In 1644, Merfenne exhibited a kind of abridgment of them in his Synopfis Mathematica, in 4to: but this contains only fuch propofitions as could be underftood without figures. In 1655, Meibomius gave fome of the Lemmata of the 7th book, in his Dialogue upon Proportions. In 1688, Dr. Wallis printed the laft 12 propofitions of the 2d book, at the end of his Ariftarchus Samius. In 1703, Dr. David Gregory gave part of the preface of the 7th book, in the Prolegomena to his Euclid. And in 1706, Dr. Halley gave that Preface entire, in the beginning of his Apollonius.

As the contents of the principal work, the Mathematical Collections, are exceedingly curious, and no account of them having ever appeared in Englifh, I fhall here give a very brief analyfis of thofe books, extracted from my notes upon this author.

Of the Third Book—The fubjects of the third book confift chiefly of three principal problems; for the folution of which, a great many other problems are refolved, and theorems demonftrated. The firft of thefe three problems is, To find Two Mean Proportionals between two given lines—The 2d problem is, To find, what are called, three Medietates in a femicircle; where, by a Medietas is meant a fet of three lines in continued proportion, whether arithmetical, or geometrical, or harmonical; fo that to find three medietates, is to find an arithmetical, a geometrical, and an harmonical fet of three terms each. And the third problem is, From fome points in the bafe of a triangle, to draw two lines to meet in a point within the triangle, fo that their fum fhall be greater than the fum of the other two fides which are without them. A great many curious properties are premifed to each of thefe problems; then their folutions are given according to the methods of feveral ancient mathematicians, with an hiftorical account of them, and his own demonftrations; and laftly, their applications to various matters of great importance. In his hiftorical anecdotes, many curious things are preferved concerning mathematicians that were ancient

 even

even in his time, which we ſhould otherwiſe have known nothing at all about.

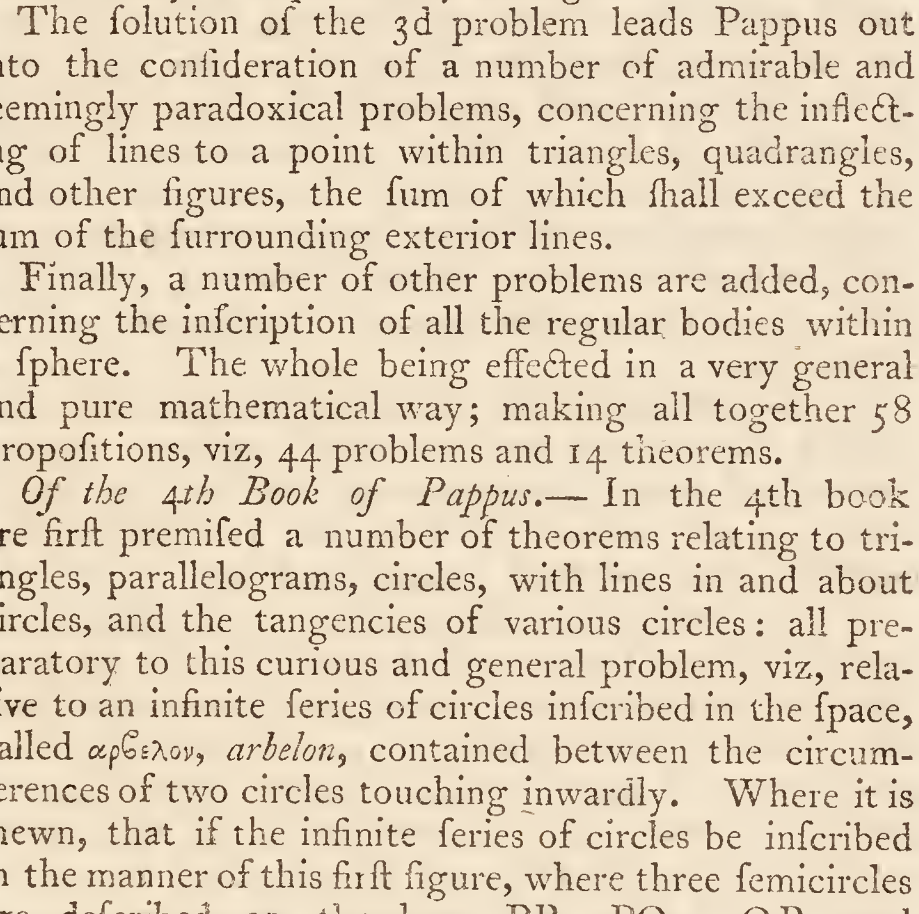
In order to the ſolution of the firſt of the three problems above mentioned, he begins by premiſing four general theorems concerning proportions. Then follows a diſſertation on the nature and diviſion of problems by the Ancients, into Plane, Solid, and Linear, with examples of them, taken out of the writings of Eratoſthenes, Philo, and Hero. A ſolution is then given to the problem concerning two mean proportionals, by four different ways, namely according to Eratoſthenes, Nicomedes, Hero, and after a way of his own, in which he not only doubles the cube, but alſo finds another cube in any proportion whatever to a given cube.

For the ſolution of the ſecond problem, he lays down very curious definitions and properties of *medietates* of all ſorts, and ſhews how to find them all in a great variety of caſes, both as to what the Ancients had done in them, and what was done by others whom he calls the Moderns. *Medietas* ſeems to have been a general term invented to expreſs three lines, having either an arithmetical, or a geometrical, or an harmonical relation; for the words proportion (or ratio), and analogy (or ſimilar proportions), are reſtricted to a geometrical relation only. But he ſhews how all the medietates may be expreſſed by analogies.

The ſolution of the 3d problem leads Pappus out into the conſideration of a number of admirable and ſeemingly paradoxical problems, concerning the inflecting of lines to a point within triangles, quadrangles, and other figures, the ſum of which ſhall exceed the ſum of the ſurrounding exterior lines.

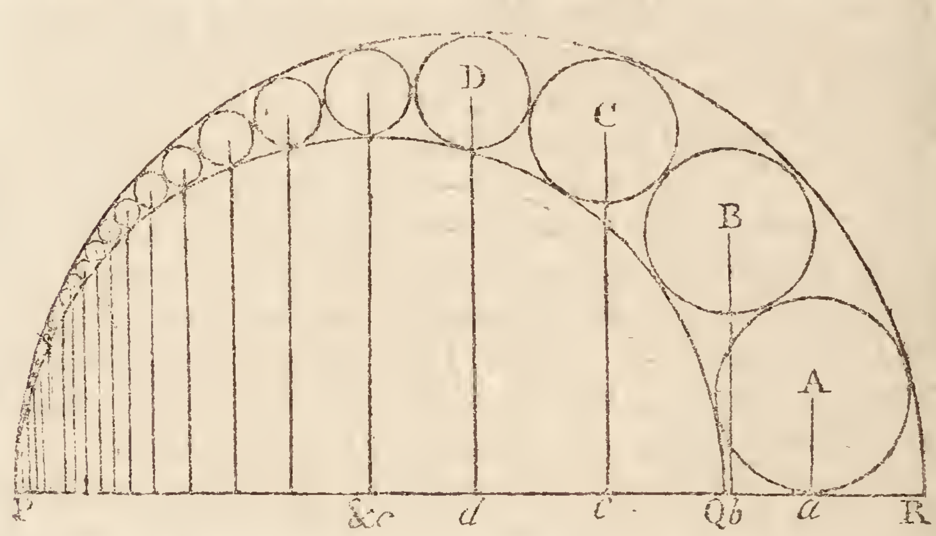
Finally, a number of other problems are added, concerning the inſcription of all the regular bodies within a ſphere. The whole being effected in a very general and pure mathematical way; making all together 58 propoſitions, viz, 44 problems and 14 theorems.

Of the 4th Book of Pappus.—In the 4th book are firſt premiſed a number of theorems relating to triangles, parallelograms, circles, with lines in and about circles, and the tangencies of various circles: all preparatory to this curious and general problem, viz, relative to an infinite ſeries of circles inſcribed in the ſpace, called αρϐελον, *arbelon*, contained between the circumferences of two circles touching inwardly. Where it is ſhewn, that if the infinite ſeries of circles be inſcribed in the manner of this firſt figure, where three ſemicircles are deſcribed on the lines PR, PQ, QR, and



the perpendiculars A*a* B*b*, C*c*, &c, let fall from the centres of the ſeries of inſcribed circles; then the property of theſe perpendiculars is this, viz, that the firſt perpendicular A*a* is equal to the diameter or double the radius of the circle A; the ſecond perpendicular B*b* equal to double the diameter or 4 times the radius of the ſecond circle B; the third perpendicular C*c* equal to 3 times the diameter or 6 times the radius of the third circle C; and ſo on, the ſeries of perpendiculars being to the ſeries of the diameters, as 1, 2, 3, 4, &c, to 1,
or to the ſeries of radii, as 2, 4, 6, 8, &c, to 1.

But if the ſeveral ſmall circles be inſcribed in the



manner of this ſecond circle, the firſt circle of the ſeries touching the part of the line QR; then the ſeries of perpendiculars A*a*, B*b*, C*c*, &c, will be 1, 3, 5, 7, &c, times the radii of the circles A, B, C, D, &c; viz, according to the ſeries of odd numbers; the former proceeding by the ſeries of even numbers.

He next treats of the Helix, or Spiral, propoſed by Conon, and reſolved by Archimedes, demonſtrating its principal properties: in the demonſtration of ſome of which, he makes uſe of the ſame principles as Cavallerius did lately, adding together an infinite number of infinitely ſhort parallelograms and cylinders, which he imagines a triangle and cone to be compoſed of.—He next treats of the properties of the Conchoid which Nicomedes invented for doubling the cube: applying it to the ſolution of certain problems concerning Inclinations, with the finding of two mean proportionals, and cubes in any proportion whatever.—Then of the τετραγωνιζουσα, or Quadratrix, ſo called from its uſe in ſquaring the circle, for which purpoſe it was invented and employed by Dinoſtratus, Nicomedes, and others: the uſe of which however he blames, as it requires poſtulates equally hard to be granted, as the problem itſelf to be demonſtrated by it.—Next he treats of Spirals, deſcribed on planes, and on the convex ſurfaces of various bodies.—From another problem, concerning Inclinations, he there ſhews, how to triſect a given angle; to deſcribe an hyperbola, to two given aſymptotes, and paſſing through a given point; to divide a given arc or angle in any given ratio; to cut off arcs of equal lengths from unequal circles; to take arcs and angles in any proportion, and arcs equal to right lines; with parabolic and hyperbolic loci, which laſt is one of the inclinations of Archimedes.

Of the 5th Book of Pappus.—This book opens with reflections on the different natures of men and brutes, the former acting by reaſon and demonſtration, the latter by inſtinct, yet ſome of them with a certain portion of reaſon or foreſight, as bees, in the curious ſtructure of their cells, which he obſerves are of ſuch a form

a form as to complete the fpace quite around a point, and yet require the leaft materials to build them, to contain the fame quantity of honey. He fhews that the triangle, fquare, and hexagon, are the only regular polygons capable of filling the whole fpace round a point; and remarks that the bees have chofen the fitteft of thefe; proving afterwards, in the propofitions, that of all regular figures of the fame perimeter, that is of the largeft capacity which has the greateft number of fides or angles, and confequently that the circle is the moft capacious of all figures whatever.

And thus he finifhes this curious book on Ifoperimetrical figures, both plane and folid; in which many curious and important properties are ftrictly demonftrated, both of planes and folids, fome of them being old in his time, and many new ones of his own. In fact, it feems he has here brought together into this book, all the properties relating to ifoperimetrical figures then known, and their different degrees of capacity. In the laft theorem of the book, he has a differtation to fhew, that there can be no more regular bodies befide the five Platonic ones, or, that only the regular triangles, fquares, and pentagons, will form regular folid angles.

Of the 6th Book of Pappus.—In this book he treats of certain fpherical properties, which had been either neglected, or improperly and imperfectly treated by fome celebrated authors before his time.—— Such are fome things in the 3d book of Theodofius's Spherics, and in his book on Days and Nights, as alfo fome in Euclid's Phenomena. For the fake of thefe, he premifes and intermixes many curious geometrical properties, efpecially of circles of the fphere, and fpherical triangles. He adverts to fome curious cafes of variable quantities; fhewing how fome increafe and decreafe both ways to infinity; while others proceed only one way by increafe or decreafe, to infinity, and the other way to a certain magnitude; and others again both ways to a certain magnitude, giving a maximum and minimum.—Here are alfo fome curious properties concerning the perfpective of the circles of the fphere, and of other lines. Alfo the locus is determined of all the points from whence a circle may be viewed, fo as to appear an ellipfe, whofe centre is a given point within the circle; which locus is fhewn to be a femicircle paffing through that point.

Of the 7th Book of Pappus.—In the introduction to this book, he defcribes very particularly the nature of the mathematical compofition and refolution of the Ancients, diftinguifhing the particular procefs and ufes of them, in the demonftration of theorems and folution of problems. He then enumerates all the analytical books of the Ancients, or thofe proceeding by refolution, which he does in the following order, viz, 1ft, Euclid's Data, in one book: 2d, Apollonius's Section of a Ratio, 2 books: 3d, his Section of a Space, 2 books: 4th, his Tangencies, 2 books; 5th, Euclid's Porifms, 3 books: 6th, Apollonius's Inclinations, 2 books: 7th, his Plane Loci, 2 books: 8th, his Conics, 8 books: 9th, Ariftæus's Solid Loci, 5 books: 10th, Euclid's Loci in Superficies, 2 books; and 11th, Eratofthenes's Medietates, 2 books. So that all the books are 31, the arguments or contents of which he exhibits, with the number of the Loci, determinations, and cafes, &c; with a multitude of lemmas and propofitions laid down and demonftrated; the whole making 238 propofitions, of the moft curious geometrical principles and properties, relating to thofe books.

Of the 8th Book of Pappus.—The 8th book is altogether on Mechanics. It opens with a general oration on the fubject of mechanics; defining the fcience, enumerating the different kinds and branches of it, and giving an account of the chief authors and writings on it. After an account of the centre of gravity, upon which the fcience of mechanics fo greatly depends, he fhews in the firft propofition, that fuch a point really exifts in all bodies. Some of the following propofitions are alfo concerning the properties of the centre of gravity. He next comes to the Inclined Plane, and in prop. 9, fhews what power will draw a given weight up a given inclined plane, when the power is given which can draw the weight along a horizontal plane. In the 10th prop. concerning the moving a given weight with a given power, he treats of what the Ancients called a Gloffocomum, which is nothing more than a feries of Wheels-and-axles, in any proportions, turning each other, till we arrive at the given power. In this propofition, as well as in feveral other places, he refers to fome books that are now loft; as Archimedes on the Balance, and the Mechanics of Hero and of Philo. Then, from prop. 11 to prop. 19, treats on various mifcellaneous things, as, the organical conftruction of folid problems; the diminution of an architectural column; to defcribe an ellipfe through five given points; to find the axes of an ellipfe organically; to find alfo organically, the inclination of one plane to another, the neareft point of a fphere to a plane, the points in a fpherical furface cut by lines joining certain points, and to infcribe feven hexagons in a given circle. Prop. 20, 21, 22, 23, teach how to conftruct and adapt the *Tympani*, or wheels of the Gloffocomum to one another, fhewing the proportions of their diameters, the number of their teeth, &c. And prop. 24 fhews how to conftruct the fpiral threads of a fcrew.

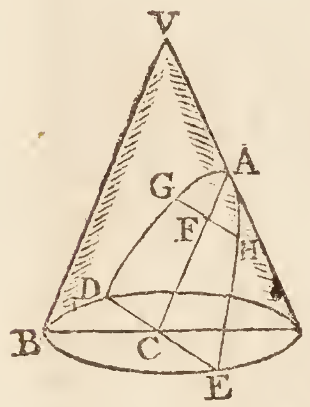
He comes then to the *Five Mechanical Powers, by which a given weight is moved by a given power.* He here propofes briefly to fhew what has been faid of thefe powers by Hero and Philo, adding alfo fome things of his own. Their names are, the Axis-in-peritrochio, the Lever, Pulley, Wedge and Screw; and he obferves, thofe authors fhewed how they are all reduced to one principle, though their figures be very different. He then treats of each of thefe powers feparately, giving their figures and properties, their conftruction and ufes.

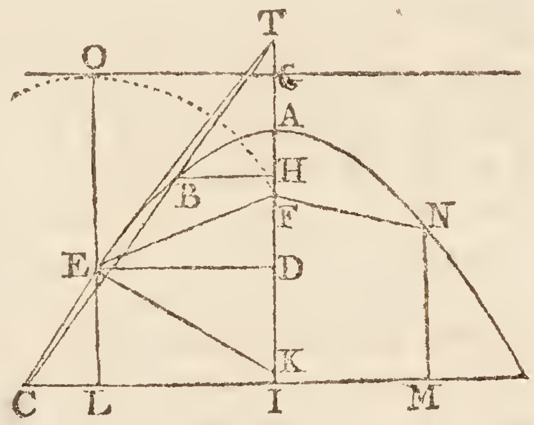
He next defcribes the manner of drawing very heavy weights along the ground, by the machine Chelone, which is a kind of fledge placed upon two loofe rollers, and drawn forward by any power whatever, a third roller being always laid under the fore part of the Chelone, as one of the other two is quitted and left behind by the motion of the Chelone. In fact this is the fame machine as has always been employed upon many occafions in moving very great weights to moderate diftances.

Finally, Pappus defcribes the manner of raifing great weights to a height by the combination of mechanic powers,

powers, as by cranes, and other machines; illustrating this, and the former parts, by drawings of the machines that are described.

PARABOLA, in Geometry, a figure arising from the section of a cone, when cut by a plane parallel to one of its sides, as the section ADE parallel to the side VB of the cone. See CONIC *Sections*, where some general properties are given.





Some other Properties of the Parabola.

1. From the same point of a cone only one Parabola can be drawn; all the other sections between the Parabola and the parallel side of the cone being ellipses, and all without them hyperbolas. Also the Parabola has but one focus, through which the axis AC passes; all the other diameters being parallel to this, and infinite in length also.

2. The parameter of the axis is a third proportional to any abscis and its ordinate; viz, AC : CD : : CD : p the parameter. And therefore if x denote any abscis AC, and y the ordinate CD, it will be $x : y : : y : p = \frac{y^2}{x}$ the parameter; or, by multiplying extremes and means, $px = y^2$, which is the equation of the Parabola.

3. The focus F is the point in the axis where the double ordinate GH is equal to the parameter. Therefore, in the equation of the curve $px = y^2$, taking $p = 2y$, it becomes $2yx = y^2$, or $2x = y$, that is $2AF = FH$, or $AF = \frac{1}{2}FH$, or the focal distance from a vertex AF is equal to half the ordinate there, or $= \frac{1}{4}p$, one-fourth of the parameter.

4. The abscisses of a Parabola are to one another, as the squares of their corresponding ordinates. This is evident from the general equation of the curve $px = y^2$, where, p being constant, x is as y^2.

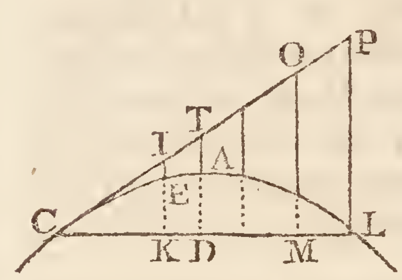
5. The line FE (*fig.* 2 *above*) drawn from the focus to any point of the curve, is equal to the sum of the focal distance and the abscis of the ordinate to that point; that is $FE = FA + AD = GD$, taking $AG = AF = \frac{1}{4}p$. Or EF is always = EO, drawn parallel to DG, to meet the perpendicular GO, called the Directrix.

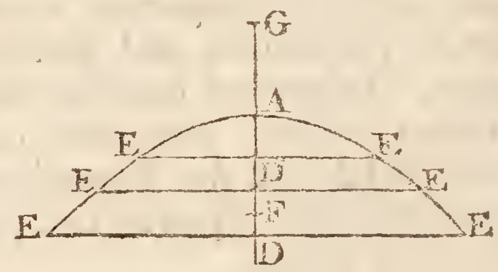
6. If a line TBC cut the curve of a Parabola in two points, and the axis produced in T, and BH and CI be ordinates at those two points; then is AT a mean proportional between the abscisses AH and AI, or $AT^2 = AH . AI$.—And if TE touch the curve, then is AT = AD = the mean between AH and AI.

7. If FE be drawn from the focus to the point of contact of the tangent TE, and EK perpendicular to the same tangent; then is $FT = FE = FK$; and the subnormal DK equal to the constant quantity 2AF or $\frac{1}{2}p$.

8. The diameter EL being parallel to the axis AK, the perpendicular EK, to the curve or tangent at E, bisects the angle LEF. And therefore all rays of light LE, MN, &c, coming parallel to the axis, will be reflected into the point F, which is therefore called the focus, or burning point; for the angle of incidence LEK is = the angle of reflection KEF.

9. If IEK (*next fig. below*) be any line parallel to the axis, limited by the tangent TC and ordinate CKL to the point of contact; then shall IE : EK : : CK : KL. And the same thing holds true when CL is also in any oblique position.





10. The external parts of the parallels IE, TA, ON, PL, &c, are always proportional to the squares of their intercepted parts of the tangent; that is, the external parts IE, TA, ON, PL, are proportional to CI^2, CT^2, CO^2, CP^2, or to the squares CK^2, CD^2, CM^2, CL^2.

And as this property is common to every position of the tangent, if the lines IE, TA, ON, &c, be appended to the points I, T, O, &c, of the tangent, and moveable about them, and of such lengths as that their extremities E, A, N, &c, be in the curve of a Parabola in any one position of the tangent; then making the tangent revolve about the point C, the extremities E, A, N, &c, will always form the curve of some Parabola, in every position of the tangent.

The same properties too that have been shewn of the axis, and its abscisses and ordinates, &c, are true of those of any other diameter. All which, besides many other curious properties of the Parabola, may be seen demonstrated in my Treatise on Conic Sections.

11. *To Construct a Parabola by Points.*

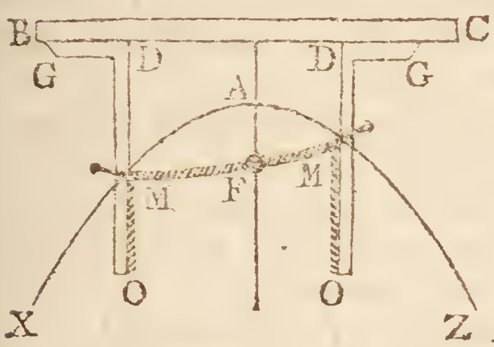
In the axis produced take AG = AF (*last fig. above*) the focal distance, and draw a number of lines EE, EE, &c, perpendicular to the axis AD; then with the distances GD, GD, &c, as radii, and the centre F, describe arcs crossing the parallel ordinates in E, E, &c. Then with a steady hand, or by the side of a slip of bent whale-bone, draw the curve through all the points E, E, E, &c.

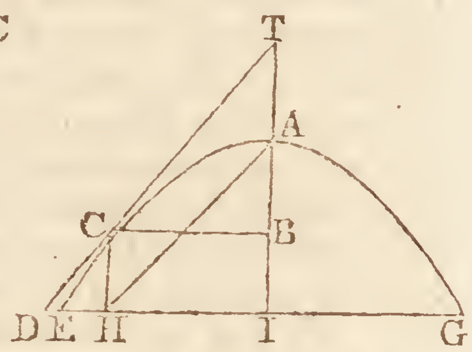
12. *To describe a Parabola by a continued Motion.*

If the rule or the directrix BC be laid upon a plane, (*first fig. below*) with the square GDO, in such manner that one of its sides DG lies along the edge of that rule; and if the thread FMO equal in length to DO, the other side of the square, have one end fixed in the extremity of the rule at O, and the other end in some point

point F: Then slide the side of the square DG along the rule BC, and at the same time keep the thread continually tight by means of the pin M, with its part MO close to the side of the square DO; so shall the curve AMX, which the pin describes by this motion, be one part of a Parabola.

And if the square be turned over, and moved on the other side of the fixed point F, the other part of the same Parabola AMZ will be described.





To draw Tangents to the Parabola.

13. If the point of contact C be given: *(last fig. above)* draw the ordinate CB, and produce the axis till AT be = AB; then join TC, which will be the tangent.

14. Or if the point be given in the axis produced: Take AB = AT, and draw the ordinate BC, which will give C the point of contact; to which draw the line TC as before.

15. If D be any other point, neither in the curve nor in the axis produced, through which the tangent is to pass: Draw DEG perpendicular to the axis, and take DH a mean proportional between DE and DG, and draw HC parallel to the axis, so shall C be the point of contact, through which and the given point D the tangent DCT is to be drawn.

16. When the tangent is to make a given angle with the ordinate at the point of contact: Take the abscis AI equal to half the parameter, or to double the focal distance, and draw the ordinate IE: also draw AH to make with AI the angle HAI equal to the given angle; then draw HC parallel to the axis, and it will cut the curve in C the point of contact, where a line drawn to make the given angle with CB will be the tangent required.

17. *To find the Area of a Parabola.* Multiply the base EG by the perpendicular height AI, and $\frac{2}{3}$ of the product will be the area of the space AEGA; because the Parabolic space is $\frac{2}{3}$ of its circumscribing parallelogram.

18. *To find the Length of the Curve* AC, commencing at the vertex.—Let y = the ordinate BC, p = the parameter, $q = \frac{2y}{p}$, and $s = \sqrt{1 + q^2}$; then shall $\frac{1}{2}p \times (qs + \text{hyp. log. of } q + s)$ be the length of the curve AC.

See various other rules for the areas, and lengths of the curve, &c, in my Treatise on Mensuration, sec. 6, pa. 355, &c, 2d edition.

PARABOLAS *of the Higher Kinds*, are algebraic curves, defined by the general equation $a^{n-1}x = y^n$; that is, either $a^2x = y^3$, or $a^3x = y^4$, or $a^4x = y^5$, &c.

Some call these by the name of Paraboloids: and in particular, if $a^2x = y^3$, they call it a Cubical Paraboloid; if $a^3x = y^4$, they call it a Biquadratical Paraboloid, or a Sursolid Paraboloid. In respect of these, the Parabola of the First Kind, above explained, they call the Apollonian, or Quadratic Parabola.

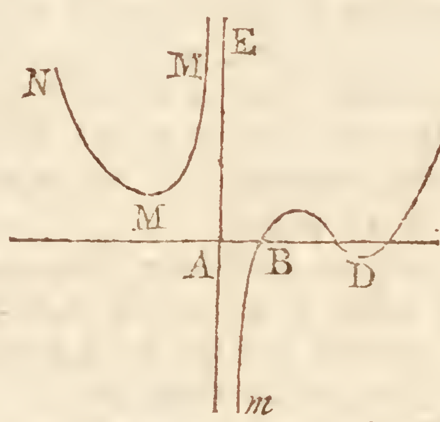
Those curves are also to be referred to Parabolas, that are expressed by the general equation $ax^{n-1} = y^n$, where the indices of the quantities on each side are equal, as before; and these are called Semi-Parabolas: as $ax^2 = y^3$ the Semi-Cubical Parabola; or $ax^3 = y^4$ the Semi-Biquadratical Parabola; &c.

They are all comprehended under the more general equation $a^m x^n = y^{m+n}$, where the two indices on one side are still equal to the index on the other side of the equation; which include both the former kinds of equations, as well as such as these following ones, $a^2x^2 = y^4$, or $a^2x^3 = y^5$, or $a^4x^3 = y^7$, &c.

Cartesian PARABOLA, is a curve of the 2d order expressed by the equation

$$xy = ax^3 + bx^2 + cx + d,$$

containing four infinite legs, viz two hyperbolic ones



MM and Bm, to the common asymptote AE, tending contrary ways, and two Parabolic legs MN and DN joining them, being Newton's, 66th species of lines of the 3d order, and called by him a Trident. It is made use of by Des Cartes in the 3d book of his Geometry, for finding the roots of equations of 6 dimensions, by means of its intersections with a circle. Its most simple equation is $xy = x^3 + g^3$. And points through which it is to pass may be easily found by means of a common Parabola whose abscis is $ax^2 + bx + c$, and an hyperbola whose abscis is $\frac{d}{x}$; for y will be equal to the sum or difference of the corresponding ordinates of this Parabola and hyperbola.

Des Cartes, in the place abovementioned, shews how to describe this curve by a continued motion. And Mr. Maclaurin does the same thing in a different way, in his Organica Geometria.

Diverging PARABOLA, is a name given by Newton to a species of five different lines of the 3d order, expressed by the equation

$$y^2 = ax^3 + bx^2 + cx + d,$$

The

The firft is a bell-form Parabola, with an oval at its head (*fig.* 1.); which is the cafe when the equation $o = ax^3 + bx^2 + cx + d$, has three real and unequal roots; fo that one of the moft fimple equations of a curve of this kind is $py^2 = x^3 + ax^2 + a^2x$.

Fig 1.

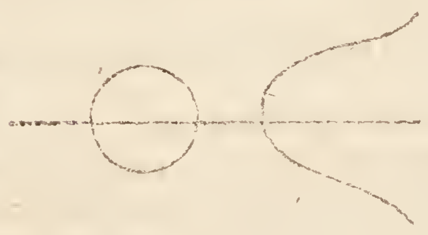


Fig. 2.

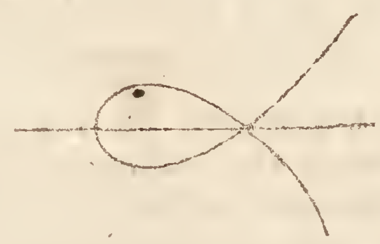


Fig. 3.

Fig. 4.

The 2d is alfo a bell-form Parabola, with a conjugate point, or infinitely fmall oval, at the head (*fig.* 1.); being the cafe when the equation $o = ax^3 + bx^2 + cx + d$ has its two lefs roots equal; the moft fimple equation of which is $py^2 = x^3 + ax^2$.

The third is a Parabola, with two diverging legs, croffing one another like a knot (*fig.* 2.); which happens when the equation $o = ax^3 + bx^2 + cx + d$ has its two greater roots equal; the more fimple equation being $py^2 = x^3 + ax^2$.

The fourth a pure bell-form Parabola (*fig.* 3.); being the cafe when $o = ax^3 + bx^2 + cx + d$ has two imaginary roots; and its moft fimple equation is $py^2 = x^3 + a^3$, or $py^2 = x^3 + a^2x$.

The fifth a Parabola with two diverging legs, forming at their meeting a cufp or double point (*fig.* 4); being the cafe when the equation $o = ax^3 + bx^2 + cx + d$ has three equal roots; fo that $py^2 = x^3$ is the moft fimple equation of this curve, which indeed is the Semicubical, or Neilian Parabola.

If a folid generated by the rotation of a femi-cubical Parabola, about its axis, be cut by a plane, each of thefe five Parabolas will be exhibited by its fections. For, when the cutting plane is oblique to the axis, but falls below it, the fection is a diverging Parabola, with an oval at its head. When oblique to the axis, but paffes through the vertex, the fection is a diverging Parabola, having an infinitely fmall oval at its head. When the cutting is oblique to the axis, falls below it, and at the fame time touches the curve furface of the folid, as well as cuts it, the fection is a diverging Parabola, with a nodus or knot. When the cutting plane falls above the vertex, either parallel or oblique to the axis, the fection is a pure diverging Parabola. And laftly when the cutting plane paffes through the axis, the fection is the femi-cubical Parabola from which the folid was generated.

PARABOLIC *Afymptote*, is ufed for a Parabolic line approaching to a curve, fo that they never meet; yet by producing both indefinitely, their diftance from each other becomes lefs than any given line.

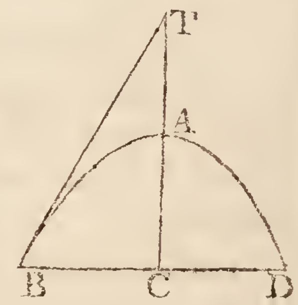
There may be as many different kinds of thefe Afymptotes as there are parabolas of different orders. When a curve has a common parabola for its Afymptote, the ratio of the fubtangent to the abfcifs approaches continually to the ratio of 2 to 1, when the axis of the parabola coincides with the bafe; but this ratio of the fubtangent to the abfcifs approaches to that of 1 to 2, when the axis is perpendicular to the bafe. And by obferving the limit to which the ratio of the fubtangent and abfcifs approaches, Parabolic Afymptotes of various kinds may be difcovered. See Maclaurin's Fluxions, art. 337.

PARABOLIC *Conoid*, is a folid generated by the rotation of a parabola about its axis.

This folid is equal to half its circumfcribed cylinder; and therefore if the bafe be multiplied by the height, half the product will be the folid content.

To find the Curve Surface of a Paraboloid.

Let BAD be the generating parabola, AC = AT, and BT a tangent at B. Put $p = 3 \cdot 1416$, $y = BC$, $x = AC = AT$, and $t = BT = \sqrt{4x^2 + y^2}$; then is the curve furface $= \frac{2}{3}ay \times (y + \frac{tt}{t + y})$.



See various other rules and geometrical conftructions for the furfaces and folidities of Parabolic Conoids, in my Menfuration, part 3, fect. 6, 2d edition.

PARABOLIC *Pyramidoid*, is a folid figure thus named by Dr. Wallis, from its genefis, or formation, which is thus: Let all the fquares of the ordinates of a parabola be conceived to be fo placed, that the axis fhall pafs perpendicularly through all their centres; then the aggregate of all thefe planes will form the Parabolic Pyramidoid.

This figure is equal to half its circumfcribed parallelopipedon. And therefore the folid content is found by multiplying the bafe by the altitude, and taking half the product; or the one of thefe by half the other.

PARABOLIC *Space*, is the fpace or area included by the curve line and bafe or double ordinate of the parabola. The area of this fpace, it has been fhewn under the article Parabola, is $\frac{2}{3}$ of its circumfcribed parallelogram; which is its quadrature, and which was firft found out by Archimedes, though fome fay by Pythagoras.

PARABOLIC *Spindle*, is a folid figure conceived to be formed by the rotation of a parabola about its bafe or double ordinate.

This folid is equal to $\frac{8}{15}$ of its circumfcribed cylinder. See my Menfuration, prob. 15, pa. 390, &c, 2d edition.

PARABOLIC *Spiral*. See HELICOID *Parabola*.

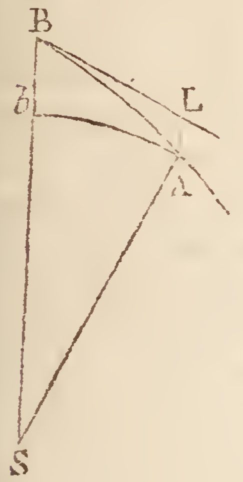
PARABOLIFORM *Curves*, a name fometimes given to the parabolas of the higher orders.

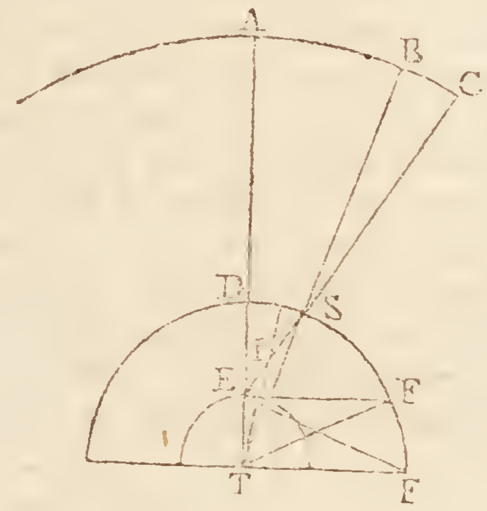
PARABOLOIDES, Parabolas of the higher orders.——The equation for all curves of this kind being $a^{m-n}x^n = y^m$, the proportion of the area of any one to the complement of it to the circumfcribing parallelogram, will be as m to n.

PARA-

PARACENTRIC *Motion*, denotes the space by which a revolving planet approaches nearer to, or recedes farther from, the sun, or centre of attraction.

Thus, if a planet in A move towards B; then is SB − SA = *b*B the Paracentric motion of that planet: where S is the place of the sun.





Paracentric *Solicitation of Gravity*, is the same as the Vis Centripeta; and is expressed by the line AL drawn from the point A, parallel to the ray SB (infinitely near SA), till it intersect the tangent BL.

PARALLACTIC *Angle*, called also simply Parallax, is the angle EST (*last fig. above*) made at the centre of a star, &c, by two lines, drawn, the one from the centre of the earth at T, and the other from its surface at E.—Or, which amounts to the same thing, the Parallactic angle, is the difference of the two angles CEA and BTA, under which the real and apparent distances from the zenith are seen.

The sines of the Parallactic angles ELT, EST, at the same or equal distances DS from the zenith, are in the reciprocal ratio of the distances, TL, and TS, from the centre of the earth.

PARALLAX, is an arch of the heavens intercepted between the true place of a star, and its apparent place.

The true place of a star S, is that point of the heavens B, in which it would be seen by an eye placed in the centre of the earth at T. And the apparent place, is that point of the heavens C, where a star appears to an eye upon the surface of the earth at E.

This difference of places, is what is called absolutely the Parallax, or the Parallax of Altitude; which Copernicus calls the Commutation; and which therefore is an angle formed by two visual rays, drawn, the one from the centre, the other from the circumference of the earth, and traversing the body of the star; being measured by an arch of a great circle intercepted between the two points of true and apparent place, B and C.

The Parallax *of Altitude* CB is properly the difference between the true distance from the zenith AB, and the apparent distance AC. Hence the Parallax diminishes the altitude of a star, or increases its distance from the zenith; and it has therefore a contrary effect to the refraction.

The Parallax is greatest in the horizon, called the Horizontal Parallax EFT. From hence it decreases all the way to the zenith D or A, where it is nothing; the real and apparent places there coinciding.

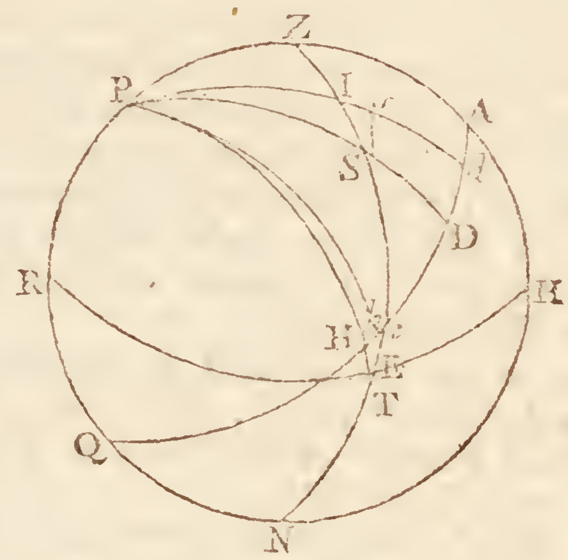
The Horizontal Parallax is the same, whether the star be in the true or apparent horizon.

The fixed stars have no sensible Parallax, by reason of their immense distance, to which the semidiameter of the earth is but a mere point.

Hence also, the nearer a star is to the earth, the greater is its Parallax; and on the contrary, the farther it is off, the less is the Parallax, at an equal elevation above the horizon. So the star at S has a less Parallax than the star at I. Saturn is so high, that it is difficult to observe in him any Parallax at all.

Parallax increases the right and oblique ascension, and diminishes the descension; it diminishes the northern declination and latitude in the eastern part, and increases them in the western; but it increases the southern declination in the eastern and western part; it diminishes the longitude in the western part, and increases it in the eastern. Parallax therefore has just opposite effects to refraction.

The doctrine of Parallaxes is of the greatest importance, in astronomy, for determining the distances of the planets, comets, and other phenomena of the heavens; for the calculation of eclipses, and for finding the longitude.



Parallax *of Right Ascension and Descension*, is an arch of the Equinoctial D*d*, by which the Parallax of altitude increases the ascension, and diminishes the descension.

Parallax *of Declination*, is an arch of a circle of declination *s*I, by which the Parallax of altitude increases or diminishes the declination of a star.

Parallax *of Latitude*, is an arch of a circle of latitude SI, by which the Parallax of altitude increases or diminishes the latitude.

Menstrual Parallax *of the Sun*, is an angle formed by two right lines; one drawn from the earth to the sun, and another from the sun to the moon, at either of their quadratures.

Parallax *of the Annual Orbit of the Earth*, is the difference between the heliocentric and geocentric place of a planet, or the angle at any planet, subtended by the distance between the earth and sun.

There are various methods for finding the Parallaxes of the celestial bodies: some of the principal and easier of which are as follow:

To Observe the Parallax *of a Celestial Body.*—Observe when the body is in the same vertical with a fixed star which is near it, and in that position measure its

apparent

apparent diſtance from the ſtar. Obſerve again when the body and ſtar are at equal altitudes from the horizon; and there meaſure their diſtance again. Then the difference of theſe diſtances will be the Parallax very nearly.

To Obſerve the Moon's PARALLAX.—Obſerve very accurately the moon's meridian altitude, and note the moment of time. To this time, equated, compute her true latitude and longitude, and from theſe find her declination; alſo from her declination, and the elevation of the equator, find her true meridian altitude. Subtract the refraction from the obſerved altitude: then the difference between the remainder and the true altitude, will be the Parallax ſought. If the obſerved altitude be not meridional, reduce it to the true altitude for the time of obſervation.

By this means, in 1583, Oct. 12 day 5 h. 19m. from the moon's meridian altitude obſerved at 13° 38′, Tycho found her Parallax to be 54 minutes.

To Obſerve the Moon's PARALLAX *in an Eclipſe*.—In an eclipſe of the moon obſerve when both horns are in the ſame vertical circle, and at that moment take the altitudes of both horns; then half their ſum will be nearly the apparent altitude of the moon's centre; from which ſubtract the refraction, which gives the apparent altitude freed from refraction. But the true altitude is nearly equal to the altitude of the centre of the ſhadow at that time: now the altitude of the centre of the ſhadow is known, becauſe we know the ſun's place in the ecliptic, and his depreſſion below the horizon, which is equal to the altitude of the oppoſite point of the ecliptic, in which the centre of the ſhadow is. Having thus the true and apparent altitudes, their difference is the Parallax ſought.

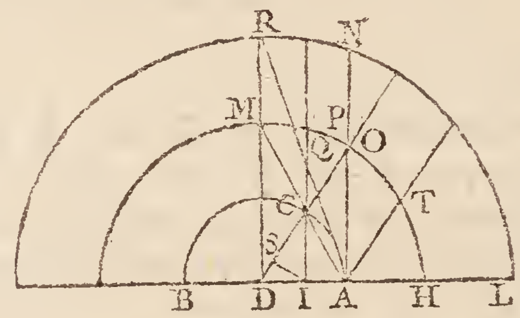
De la Hire makes the greateſt horizontal Parallax 1° 1′ 25″, and the leaſt 54′ 5″. M. le Monnier determined the mean Parallax of the Moon to be 57′ 12″. Others have made it 57′ 18″.

From the Moon's PARALLAX *EST, and altitude SF* (laſt fig. but one); *to find her diſtance from the Earth.*—From her apparent altitude given, there is given her apparent zenith diſtance, i. e. the angle AES; or by her true altitude, the complement angle ATS. Wherefore, ſince at the ſame time, the Parallactic angle S is known, the 3d or ſupplemental angle TES is alſo known. Then, conſidering the earth's ſemidiameter TE as 1, in the triangle TES are given all the angles and the ſide TE, to find ES the moon's diſtance from the ſurface of the earth, or TS her diſtance from the centre.

Thus Tycho, by the obſervation above mentioned, found the moon's diſtance at that time from the earth, was 62 of the earth's ſemidiameters. According to De la Hire's determination, her diſtance when in the perigee is near 56 ſemidiameters, but in her apogee near $63\frac{1}{2}$; and therefore the mean nearly $59\frac{3}{4}$, or in round numbers 60 ſemidiameters.

Hence alſo, ſince, from the moon's theory, there is given the ratio of her diſtances from the earth in the ſeveral degrees of her anomaly; thoſe diſtances being found, by the rule of three, in ſemidiameters of the earth, the Parallax is thence determined to the ſeveral degrees of the true anomaly.

To Obſerve the PARALLAX *of Mars.*—1. Suppoſe Mars in the meridian and equator at H; and that the obſerver, under the equator in A, obſerves him culminating with ſome fixed ſtar. 2. If now the obſerver were in the centre of the earth, he would ſee Mars conſtantly in the ſame point of the heavens with the ſtar; and therefore, together with it, in the plane of the horizon, or of the 6th horary: but ſince Mars here has ſome ſenſible Parallax, and the fixed ſtar has none, Mars will be ſeen in the horizon, when in P, the plane of the ſenſible horizon; and the ſtar, when in R, the plane of the true horizon: therefore obſerve the time between the tranſit of Mars and of the ſtar through the plane of the 6th hour.—3. Convert this time into minutes of the equator, at the rate of 15 degrees to the hour; by which means there will be obtained the arch PM, to which the angle PAM, and conſequently the angle AMD, is nearly equal; which is the horizontal Parallax of Mars.



If the obſerver be not under the equator, but in a parallel IQ, that difference will be a leſs arch QM: wherefore, ſince the ſmall arches QM and PM are nearly as their ſines AD and ID; and ſince ADG is equal to the diſtance of the place from the equator, i. e. to the elevation of the pole, or the latitude; therefore AD to ID, as radius to the coſine of the latitude; ſay, as the coſine of the latitude ID is to radius, ſo is the Parallax obſerved in I, to the Parallax under the equator.

Since Mars and the fixed ſtar cannot be commodiouſly obſerved in the horizon; let them be obſerved in the circle of the 3d hour: and ſince the Parallax obſerved there TO, is to the horizontal one PM, as IS to ID: ſay, as the ſine of the angle IDS, or 45° (ſince the plane DO is in the middle between the meridian DH and the true horizon DM), is to radius, ſo is the Parallax TO to the horizontal Parallax PM.

If Mars be likewiſe out of the plane of the equator, the Parallax found will be an arch of a parallel; which muſt therefore be reduced, as above, to an arch of the equator.

Laſtly, if Mars be not ſtationary, but either direct or retrograde, by obſervations for ſeveral days find out what his motion is every hour, that his true place from the centre may be aſſigned for any given time.

By this method Caſſini, who was the author of it, obſerved the greateſt horizontal Parallax of Mars to be 25″; but Mr. Flamſteed found it near 30″. Caſſini obſerved alſo the Parallax of Venus by the ſame method.

To Find the Sun's PARALLAX.—The great diſtance of the ſun renders his Parallax too ſmall to fail under even the niceſt immediate obſervation. Many attempts have indeed been made, both by the ancients and moderns, and many methods invented for that purpoſe. The firſt was that of Hipparchus, which was followed by Ptolomy, &c, and was founded on the obſervation of lunar eclipſes.

eclipfes. The fecond was that of Ariftarchus, in which the angle fubtended by the femidiameter of the moon's orbit, feen from the fun, was fought from the lunar phafes. But thefe both proving deficient, aftronomers are now forced to have recourfe to the Parallaxes of the nearer planets, Mars and Venus. Now from the theory of the motions of the earth and planets, there is known at any time the proportion of the diftances of the fun and planets from us; and the horizontal Parallaxes being reciprocally proportional to thofe diftances; by knowing the Parallax of a planet, that of the fun may be thence found.

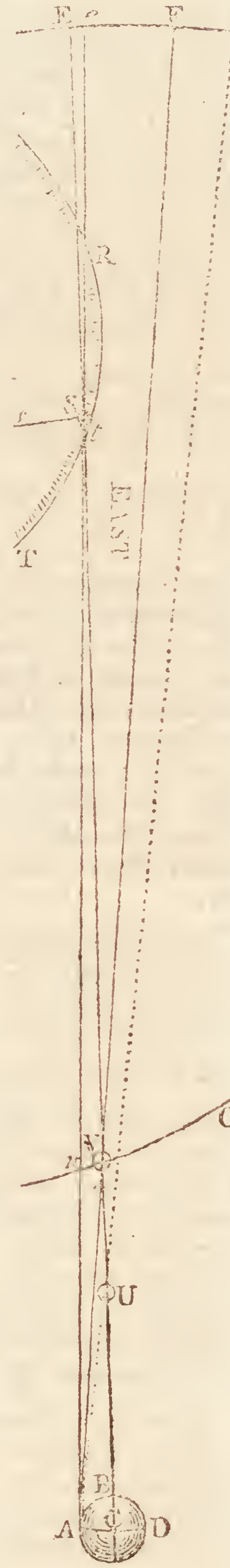
Thus Mars, when oppofite to the fun, is twice as near as the fun is, and therefore his Parallax will be twice as great as that of the fun. And Venus, when in her inferior conjunction with the fun, is fometimes nearer us than he is; and therefore her Parallax is greater in the fame proportion. Thus, from the Parallaxes of Mars and Venus, Caffini found the fun's Parallax to be 10″; from whence his diftance comes out 22000 femidiameters of the earth.

But the moft accurate method of determining the Parallaxes of thefe planets, and thence the Parallax of the fun, is that of obferving their tranfit. However, Mercury, though frequently to be feen on the fun, is not fit for this purpofe; becaufe he is fo near the fun, that the difference of their Parallaxes is always lefs than the folar Parallax required. But the Parallax of Venus, being almoft 4 times as great as the folar Parallax, will caufe very fenfible differences between the times in which fhe will feem to be paffing over the fun at different parts of the earth. With the view of engaging the attention of aftronomers to this method of determining the fun's Parallax, Dr. Halley communicated to the Royal Society, in 1691, a paper, containing an account of the feveral years in which fuch a tranfit may happen, computed from the tables which were then in ufe: thofe at the afcending node occur in the month of November O. S. in the years 918, 1161, 1396, 1631, 1639, 1874, 2109, 2117; and at the defcending node in May O. S. in the years 1048, 1283, 1291, 1518, 1526, 1761, 1769, 1996, 2004. Philof. Tranf. Abr. vol. 1, p. 435 &c.

Dr. Halley even then concluded, that if the interval of time between the two interior contacts of Venus with the fun, could be meafured to the exactnefs of a fecond, in two places properly fituated, the fun's Parallax might be determined within its 500dth part. And this conclufion was more fully explained in a fubfequent paper, concerning the tranfit of Venus in the year 1761, in the Philof. Tranf. numb. 348, or Abr. vol. 4, p. 213.

It does not appear that any of the preceding tranfits had been obferved; except that of 1639, by our ingenious countryman Mr. Horrox, and his friend Mr. Crabtree, of Manchefter. But Mr. Horrox died on the 3d of January, 1641, at the age of 25, juft after he had finifhed his treatife, *Venus in Sole vifa*, in which he difcovers a more accurate knowledge of the dimenfions of the folar fyftem, than his learned commentator Hevelius.

To give a general idea of this method of determining the horizontal Parallax of Venus, and from thence, by analogy, the Parallax and diftance of the fun, and of all the planets from him; let DBA be the earth, V Venus, and TSR the eaftern limb of the fun. To an obferver at B, the point *t* of that limb will be on the meridian, its place referred to the heavens will be at E, and Venus will appear juft within it at S. But to an obferver at A, at the fame inftant, Venus is eaft of the fun, in the right line AVF; the point *t* of the fun's limb appears at *e* in the heavens, and if Venus were then vifible fhe would appear at F. The angle CVA is the horizontal Parallax of Venus; which is equal to the oppofite angle FVE, meafured by the arc FE. ASC is the fun's horizontal Parallax, equal to the oppofite angle *e*SE, meafured by the arc *e*E; and FA*e* or VA*e* is Venus's horizontal Parallax from the fun, which may be found by obferving how much later in abfolute time her total ingrefs on the fun is, as feen from A, than as feen from B, which is the time fhe takes to move from V to *v*, in her orbit OV*v*.



If Venus were nearer the earth, as at U, her horizontal Parallax from the fun would be the arch *fe*, which meafures the angle *f*A*e*; and this angle is greater than the angle FA*e*, by the difference of their meafures F*f*. So that as the diftance of the celeftial object from the earth is lefs, its Parallax is the greater.

Now it has been already obferved, that the horizontal Parallaxes of the planets are inverfely as their diftances from the earth's centre, therefore as the fun's diftance at the time of the tranfit is to Venus's diftance, fo is the Parallax of Venus to that of the fun: and as the fun's mean diftance from the earth's centre, is to his diftance on the day of the tranfit, fo is his horizontal Parallax on that day, to his horizontal Parallax at the time of his mean diftance from the earth's centre. Hence his true diftance in femidiameters of the earth may be obtained by the following analogy, viz, as the fine of the fun's Parallax is to radius, fo is unity or the earth's femidiameter, to the number of femidiameters of the earth in the fun's diftance from the centre; which number multiplied by the number of

 miles

miles in the earth's ſemidiameter, will give the number of miles in the ſun's diſtance. Then from the proportional diſtances of the planets, determined by the theory of gravity, their true diſtances may be found. And from their apparent diameters at theſe known diſtances, their real diameters and bulks may be found.

Mr. Short, with great labour, deduced the quantity of the ſun's Parallax from the beſt obſervations that were made of the tranſit of Venus, on the 6th of June, 1761, (for which ſee Philoſ. Tranſ. vol. 51 and 52) both in Britain and in foreign parts, and found it to have been 8″·52 on the day of the tranſit, when the ſun was very nearly at his greateſt diſtance from the earth; and conſequently 8″·65 when the ſun is at his mean diſtance from the earth. See Philoſ. Tranſ. vol. 52, p. 611 &c. Whence,

As ſin. 8″·65	- - - log.	5·6219140
to radius	- - - -	10·0000000
So is 1 ſemidiameter	- -	0·0000000
to 23882·84 ſemidiameters	-	4·3780860

that is, $23882 \frac{84}{100}$ is the number of the earth's ſemidiameters contained in its diſtance from the ſun; and this number of ſemidiameters being multiplied by 3985, the number of Engliſh miles contained in the earth's ſemidiameter, (though later obſervations make this ſemidiameter only $3956\frac{1}{2}$ miles), there is obtained 95,173,127 miles for the earth's mean diſtance from the ſun. And hence, from the analogies under the article DISTANCE, the mean diſtances of all the reſt of the planets from the ſun, in miles, are found as follow, viz,

Mercury's diſtance	- -	36,841,468
Venus's diſtance	- -	68,891,486
Mars's diſtance	- -	145,014,148
Jupiter's diſtance	- -	494,990,976
Saturn's diſtance	- -	907,956,130.

In another paper (Philoſ. Tranſ. vol. 53, p. 169) Mr. Short ſtates the mean horizontal Parallax of the ſun at 8″·69. And Mr. Hornſby, from ſeveral obſervations of the tranſit of June 3d, 1769 (for which ſee the Philoſ. Tranſ. vol. 59) deduces the ſun's Parallax for that day equal to 8·65, and the mean Parallax 8″·78; whence he makes the mean diſtance of the earth from the ſun to be 93,726,900 Engliſh miles, and the diſtances of the other planets thus:

Mercury's diſtance	- -	36,281,700
Venus's diſtance	- -	67,795,500
Mars's diſtance	- -	142,818,000
Jupiter's diſtance	- -	487,472,000
Saturn's diſtance	- -	894,162,000

See the Philoſ. Tranſ. vol. 61, p. 572.

But others, by taking the reſults of thoſe obſervations that are moſt to be depended on, have made the ſun's Parallax at his mean diſtance from the earth to be 8·6045; and ſome make it only 8·54. According to the former of theſe, the ſun's mean diſtance from the earth is 95,109,736 miles; and according to the latter it is 95,834,742 miles. Upon the whole there ſeems reaſon to conclude that the ſun's horizontal Parallax may be ſtated at 8″·6, and his diſtance near 95 millions of miles. Hence, the following horizontal Parallaxes:

Mean Parallax of the ſun	- -	0′	8″·6
Moon's greateſt	- -	61	32
Moon's leaſt	- - -	54	4
Moon's mean	- - -	57	48
Mars's	- - -	0	25

Of the PARALLAX *of the Fixed Stars.* As to the fixed ſtars, their diſtance is ſo great, that it has never been found that they have any ſenſible Parallax, neither with reſpect to the earth's diameter, nor even with regard to the diameter of the earth's annual orbit round the ſun, although this diameter be about 190 millions of miles. For, any of thoſe ſtars being obſerved from oppoſite ends of this diameter, or at the interval of half a year between the obſervations, when the earth is in oppoſite points of her orbit, yet ſtill the ſtar appears in the ſame place and ſituation in the heavens, without any change that is ſenſible, or meaſurable with the very beſt inſtruments, not amounting to a ſingle ſecond of a degree. That is, the diameter of the earth's annual orbit, at the neareſt of the fixed ſtars, does not ſubtend an angle of a ſingle ſecond; or, in compariſon of the diſtance of the fixed ſtars, the extent of 190 millions of miles is but as a point!

PARALLAX is alſo uſed, in Levelling, for the angle contained between the line of true level, and that of apparent level. And, in other branches of ſcience, for the difference between the true and apparent places.

PARALLEL, in Geometry, is applied to lines, figures, and bodies, which are every where equidiſtant from each other; or which, though infinitely produced, would never either approach nearer, or recede farther from, each other; their diſtance being every where meaſured by a perpendicular line between them. Hence,

PARALLEL *right lines* are thoſe which, though infinitely produced ever ſo far, would never meet: which is Euclid's definition of them.

Newton, in Lemma 22, book 1 of his Principia, defines Parallels to be ſuch lines as tend to a point infinitely diſtant.

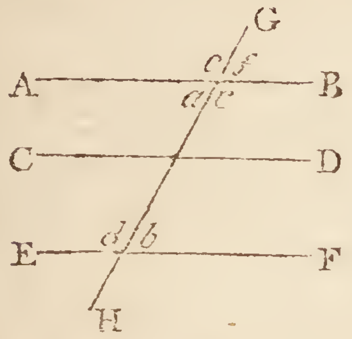
Parallel Lines ſtand oppoſed to lines converging, and diverging.

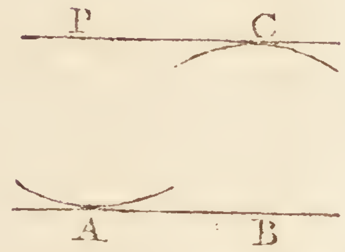
Some define an inclining or converging line, to be that which will meet another at a finite diſtance, and a Parallel line, that which will only meet at an infinite diſtance.

As a perpendicular is by ſome ſaid to be the ſhorteſt of all lines that can be drawn to another; ſo a Parallel is ſaid to be the longeſt.

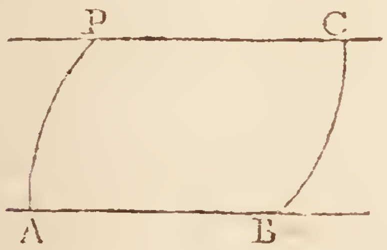
It is demonſtrated by geometricians, that two lines, AB and CD, that are both Parallel to one and the ſame right line EF, are alſo Parallel to each other. And that if two Parallel lines AB and EF be cut by any other line GH; then 1ſt, the alternate angles are equal; viz the angle $a = \angle b$, and $\angle c = \angle d$. 2d, The external angle is equal to the internal one on the ſame ſide of the cutting line; viz the $\angle e = \angle d$, and the $\angle f = \angle b$. 3d, That the two internal ones on the ſame ſide are, taken together, equal to two right

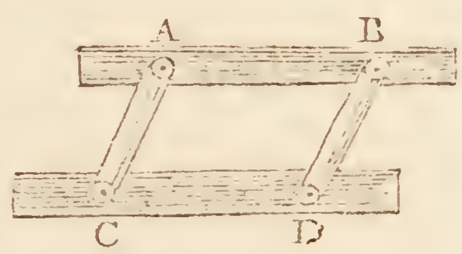
right angles; viz, $\angle a + \angle d = 180^\circ$, or $\angle c + \angle b = 180^\circ$.





To draw a Parallel *Line.*—If the line to be Parallel to AB muſt paſs through a given point P: Take the neareſt diſtance between the point P and the given line AB, by ſetting one foot of the compaſſes in P, and with the other deſcribe an arc juſt to touch the line in A; then with that diſtance as a radius, and a centre B taken any where in the line, deſcribe another arc C; laſtly, through P draw a line PC juſt to touch the arc C, and that will be the Parallel ſought.





Otherwiſe.—With the centre P, and any radius, deſcribe an arc BC, cutting the given line in B. Next, with the ſame radius, and centre B, deſcribe another arc PA, cutting alſo the given line in A. Laſtly, take AP between the compaſſes, and apply it from B to C; and through P and C draw the Parallel PC required.

Or, draw the line with the Parallel Ruler, deſcribed below, by laying one edge of the ruler along AB, and extending the other to the given point or diſtance.

When the one line is to be at a given diſtance from the other; take that diſtance between the compaſſes as a radius, and with two centres taken any where in the given line, deſcribe two arcs; then lay a ruler juſt to touch the arcs, and by it draw the Parallel.

Parallel *Planes*, are every where equidiſtant, or have all the perpendiculars that are drawn between them, everywhere equal.

Parallel *Rays*, in Optics, are thoſe which keep always at an equal diſtance in reſpect to each other, from the viſual object to the eye, from which the object is ſuppoſed to be infinitely diſtant.

Parallel *Ruler*, is a mathematical inſtrument, conſiſting of two equal rulers, AB and CD, either of wood or metal, connected together by two ſlender croſs bars or blades AC and BD, moveable about the points or joints A, B, C, D.

There are other forms of this inſtrument, a little varied from the above; ſome having the two blades croſſing in the middle, and fixed only at one end of them, the other two ends ſliding in grooves along the two rulers; &c.

The uſe of this inſtrument is obvious. For the edge of one of the rulers being applied to any line, the other opened to any extent will be always parallel to the former; and conſequently any Parallels to this may be drawn by the edge of the ruler, opened to any extent.

Parallel *Sailing*, in Navigation, is the ſailing on or under a Parallel of latitude, or Parallel to the equator. —Of this there are three caſes.

1. Given the Diſtance and Difference of Longitude; to find the Latitude.—Rule. As the difference of longitude is to the diſtance, ſo is radius to the coſine of the latitude.

2. Given the Latitude and Difference of Longitude; to find the Diſtance.—Rule. As radius is to the coſine of the latitude, ſo is the difference of longitude to the diſtance.

3. Given the Latitude and Diſtance; to find the difference of longitude.—Rule. As the coſine of latitude is to radius, ſo is the diſtance to the difference of longitude.

Parallel *Sphere*, is that ſituation of the ſphere where the equator coincides with the horizon, and the poles with the zenith and nadir.

In this ſphere all the Parallels of the equator become Parallels of the horizon; conſequently no ſtars ever riſe or ſet, but all turn round in circles Parallel to the horizon, as well as the ſun himſelf, which when in the equinoctial wheels round the horizon the whole day. Alſo, After the ſun riſes to the elevated pole, he never ſets for ſix months; and after his entering again on the other ſide of the line, he never riſes for ſix months longer.

This poſition of the ſphere is theirs only who live at the poles of the earth, if any ſuch there be. The greateſt height the ſun can riſe to them, is $23\frac{1}{2}$ degrees. They have but one day and one night, each being half a year long. See Sphere.

Parallels, or *Places of Arms*, in a Siege, are deep trenches, 15 or 18 feet wide, joining the ſeveral attacks together; and ſerving to place the guard of the trenches in, to be at hand to ſupport the workmen when attacked.

There are uſually three in an attack: the firſt is about 600 yards from the covert-way, the ſecond between 3 and 400, and the third near or on the glacis. —It is ſaid they were firſt invented or uſed by Vauban.

Parallels *of Altitude*, or Almacantars, are circles Parallel to the horizon, conceived to paſs through every degree and minute of the meridian between the horizon and zenith; having their poles in the zenith.

Parallels, or Parallel *Circles*, called alſo Parallels of Latitude, and Circles of Latitude, are leſſer circles of the ſphere, Parallel to the equinoctial or equator.

Parallels *of Declination*, are leſſer circles Parallel to the equinoctial.

Parallels *of Latitude*, in Geography, are leſſer circles Parallel to the equator. But in Aſtronomy they are Parallel to the ecliptic.

PARALLELISM, the quality of a parallel, or that which denominates it ſuch. Or it is that by which two things, as lines, rays, or the like, become equidiſtant from one another.

Parallelism *of the Earth's Axis*, is that invariable ſituation of the axis, in the progreſs of the earth through the annual orbit, by which it always keeps parallel to itſelf; ſo that if a line be drawn parallel to its axis, while

while in any one poſition; the axis, in all other poſitions or parts of the orbit, will always be parallel to the ſame line.

In conſequence of this Parallelîſm, the axis of the earth points always, as to ſenſe, to the ſame place or point in the heavens, viz to the poles. Becauſe, though really the axis, in the annual motion, deſcribes the ſurface of a cylinder, whoſe baſe is the circle of the earth's annual orbit, yet this whole circle is but as a point in compariſon with the diſtance of the fixed ſtars; and therefore all the ſides of the cylinder ſeem to tend to the ſame point, which is the celeſtial pole.—To this Parallelîſm is owing the change and variety of ſeaſons, with the inequality of days and nights.

This Parallelîſm is the neceſſary conſequence of the earth's double motion; the one round the ſun, the other round its own axis. Nor is there any neceſſity to imagine a third motion, as ſome have done, to account for this Parallelîſm.

Parallelism *of Rows of Trees.* The eye placed at the end of an alley bounded by two rows of trees, planted in parallel lines, never ſees them parallel, but always inclining to each other, towards the farther end.

Hence mathematicians have taken occaſion to enquire, in what lines the trees muſt be diſpoſed, to correct this effect of the perſpective, and make the rows ſtill appear parallel. And, to produce this effect, it is evident that the unequal intervals of any two oppoſite or correſponding trees may be ſeen under equal viſual angles.

For this purpoſe, M. Fabry, Tacquet, and Varignon obſerve, that the rows muſt be oppoſite ſemi-hyperbolas. See the Mem. Acad. Sciences, an. 1717.

But notwithſtanding the ingenuity of their ſpeculations, it has been proved by D'Alembert, and Bouguer, that to produce the effect propoſed, the trees are to be ranged merely in two diverging right lines.

PARALLELOGRAM, in Geometry, is a quadrilateral right-lined figure, whoſe oppoſite ſides are parallel to each other.

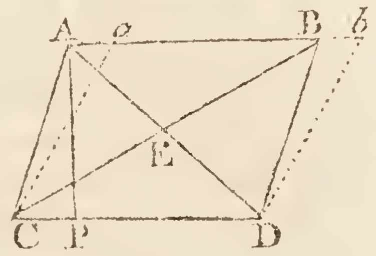
A Parallelogram may be conceived as generated by the motion of a right line, along a plane, always parallel to itſelf.

Parallelograms have ſeveral particular denominations, and are of ſeveral ſpecies, according to certain particular circumſtances, as follow:

When the angles of the Parallelogram are right ones, it is called a Rectangle.—When the angles are right, and all its ſides equal, it is a ſquare.—When the ſides are equal, but the angles oblique ones, the figure is a Rhombus or Lozenge. And when both the ſides and angles are unequal, it is a Rhomboides.

Every other quadrilateral whoſe oppoſite ſides are neither parallel nor equal, is called a Trapezium.

Properties, of the Parallelogram.—1. In every Parallelogram ABDC, the diagonal divides the figure into two equal triangles, ABD, ACD. Alſo the oppoſite angles and ſides are equal, viz, the ſide AB = CD, and AC = BD, alſo the angle A = ∠ D, and the ∠ B = ∠ C. And the ſum of any two ſucceeding angles, or next the ſame ſide, is equal to two right angles, or 180 degrees, as ∠ A + ∠ C = ∠ C + ∠ D = ∠ D + ∠ B = ∠ B + ∠ A = two right-angles.



2. All Parallelograms, as ABDC and *ab*DC, are equal, that are on the ſame baſe CD, and between the ſame parallels A*b*, CD; or that have either the ſame or equal baſes and altitudes; and each is double a triangle of the ſame or equal baſe and altitude.

3. The areas of Parallelograms are to one another in the compound ratio of their baſes and altitudes. If their baſes be equal, the areas are as their altitudes; and if the altitudes be equal, the areas are as the baſes. And when the angles of the one Parallelogram are equal to thoſe of another, the areas are as the rectangles of the ſides about the equal angles.

4. In every Parallelogram, the ſum of the ſquares of the two diagonals, is equal to the ſum of the ſquares of all the four ſides of the figure, viz,

$$AD^2 + BC^2 = AB^2 + BD^2 + DC^2 + CA^2.$$

Alſo the two diagonals biſect each other; ſo that AE = ED, and BE = EC.

5. *To find the Area of a* Parallelogram.—Multiply any one ſide, as a baſe, by the height, or perpendicular let fall upon it from the oppoſite ſide. Or, multiply any two adjacent ſides together, and the product by the ſine of their contained angle, the radius being 1: viz,

The area is = CD × AP = AC × CD × ſin. ∠ C.

Complement of a Parallelogram. See Complement.

Centre of Gravity of a Parallelogram. See Centre *of Gravity*, and Centrobaric *Method*.

Parallelogram, or Parallelism, or Pentagraph, alſo denotes a machine uſed for the ready and exact reduction or copying of deſigns, ſchemes, plans, prints, &c, in any proportion. See Pentagraph.

Parallelogram *of the Hyperbola*, is the Parallelogram formed by the two aſymptotes of an hyperbola, and the parallels to them, drawn from any point of the curve. This term was firſt uſed by Huygens, at the end of his Diſſertatio de Cauſa Gravitatis. This Parallelogram, ſo formed, is of an invariable magnitude in the ſame hyperbola; and the rectangle of its ſides is equal to the power of the hyperbola.

This Parallelogram is alſo the modulus of the logarithmic ſyſtem; and if it be taken as unity or 1, the hyperbolic ſectors and ſegments will correſpond to Napier's or the natural logarithms; for which reaſon theſe have been called the hyperbolic logarithms. If the Parallelogram be taken = ·43429448190 &c, theſe ſectors and ſegments will repreſent Briggs's logarithms; in which caſe the two aſymptotes of the hyperbola make between them an angle of 25° 44′ 25″½.

Newtonian or Analytic Parallelogram, a term uſed for an invention of Sir Iſaac Newton, to find the firſt term of an infinite converging ſeries. It is ſometimes called the Method of the Parallelogram and Ruler; becauſe a ruler or right line is alſo uſed in it.

This Analytical Parallelogram is formed by dividing any geometrical Parallelogram into equal ſmall ſquares or Parallelograms, by lines drawn horizontally and perpendicularly

pendicularly through the equal divisions of the sides of the Parallelogram. The small cells, thus formed, are filled with the dimensions or powers of the species x and y, and their products.

For instance, the powers of y, as y^0 or 1, y, y^2, y^3, y^4, &c, being placed in the lowest horizontal range of cells; and the powers of x, as $x^0 = 1$, x, x^2, x^3, &c, in the vertical column to the left; or vice versa; these powers and their products will stand as in this figure:

A D

x^4	x^4y	x^4y^2	x^4y^3	x^4y^4
x^3	x^3y	x^3y^2	x^3y^3	x^3y^4
x^2	x^2y	x^2y^2	x^2y^3	x^2y^4
x	xy	xy^2	xy^3	xy^4
1	y	y^2	y^3	y^4

B C

Now when any literal equation is proposed, involving various powers of the two unknown quantities x and y, to find the value of one of these in an infinite series of the powers of the other; mark such of the cells as correspond to all its terms, or that contain the same powers and products of x and y; then let a ruler be applied to two, or perhaps more, of the Parallelograms so marked, of which let one be the lowest in the left hand column at AB, the other touching the ruler towards the right hand; and let all the rest, not touching the ruler, lie above it. Then select those terms of the equation which are represented by the cells that touch the ruler, and from them find the first term or quantity to be put in the quotient.

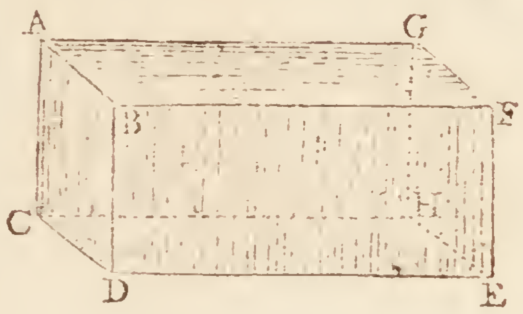
Of the application of this rule, Newton has given several examples in his Method of Fluxions and Infinite Series, p. 9 and 10, but without demonstration; which has been supplied by others. See Colson's Comment on that treatise, p. 192 & seq. Also Newton's Letter to Oldenburg, Oct. 24, 1676. Maclaurin's Algebra, p. 251. And especially Cramer's Analyses des Lignes Courbes, p. 148.—This author observes, that this invention, which is the true foundation of the method of series, was but imperfectly understood, and not valued as it deserved, for a long time. He thinks it however more convenient in practice to use the Analytical Triangle of the abbé de Gua, which takes in no more than the diagonal cells lying between A and C, and those which lie between them and B.

PARALLELOGRAM *Protractor*, a mathematical instrument, consisting of a semicircle of brass, with four rulers in form of a Parallelogram, made to move to any angle. One of these rulers is an index, which shews on the semicircle the quantity of any inward and outward angle.

PARALLELOPIPED, or PARALLELOPIPEDON, is a solid figure contained under six parallelograms, the opposites of which are equal and parallel. Or, it is a prism whose base is a parallelogram.

Properties of the PARALLELOPIPEDON.—All Parallelopipedons, whether right or oblique, that have their bases and altitudes equal, are equal; and each equal to triple a pyramid of an equal base and altitude.—A diagonal plane divides the Parallelopipedon into two equal triangular prisms.—See other properties under the general term PRISM, of which this is only a particular species.

To Measure the Surface and Solidity of a PARALLELOPIPEDON.—Find the areas of the three parallelograms AD, BE, and BG, which add into one sum; and double that sum will be the whole surface of the Parallelopipedon.



For the Solidity; multiply the base by the altitude; that is, any one face or side by its distance from the opposite side; as AD × DE, or AB × BE, or BG × BD.

PARAMETER, a certain constant right line in each of the three Conic Sections; otherwise called also Latus Rectum.

This line is called Parameter, or equal measurer, because it measures the conjugate axis by the same ratio which is between the two axes themselves; being indeed a third proportional to them; viz, a third proportional to the transverse and conjugate axes, in the ellipse and hyperbola; and, which is the same thing, a third proportional to any abscis and its ordinate in the parabola. So if t and c be the two axes in the ellipse and hyperbola, and x and y an abscis and its ordinate in the parabola;

then $t : c :: c : p = \frac{c^2}{t}$ the Param. in the former,

and $x : y :: y : p = \frac{y^2}{x}$ the Param. in the last.

The Parameter is equal to the double ordinate drawn through the focus of any of the three conic sections.

PARAPET, or *Breastwork*, in Fortification, is a defence or screen, on the extreme edge of a rampart, or other work, serving to cover the soldiers and the cannon from the enemy's fire.

The thickness of the Parapet is 18 or 20 feet, commonly lined with masonry; and 7 or 8 feet high, when the enemy has no command above the battery; otherwise, it should be raised higher, to cover the men while they

they load the guns. There are certain openings, called Embrasures, cut in the Parapet, from the top downwards, to within about 2½ or 3 feet of the bottom of it, for the cannon to fire through; the solid pieces of it between one embrasure and another, being called Merlons.

Parapet is also a little breast-wall, raised on the brinks of bridges, quays, or high buildings; to serve as a stay, and prevent people from falling over.

PARDIES (Ignatius Gaston), an ingenious French mathematician and philosopher, was born at Pau, in the province of Gascony, in 1636; his father being a counsellor of the parliament of that city.—At the age of 16 he entered into the order of Jesuits, and made so great a proficiency in his studies, that he taught polite literature, and composed many pieces in prose and verse with a distinguished delicacy of thought and style, before he was well arrived at the age of manhood. Propriety and elegance of language appear to have been his first pursuits; for which purpose he studied the Belles Lettres, and other learned productions. But afterwards he devoted himself to mathematical and philosophical studies, and read, with due attention, the most valuable authors, ancient and modern, in those sciences: so that, in a short time he made himself master of the Peripatetic and Cartesian philosophy, and taught them both with great reputation. Notwithstanding he embraced Cartesianism, yet he affected to be rather an inventor in philosophy himself. In this spirit he sometimes advanced very bold opinions in natural philosophy, which met with opposers, who charged him with starting absurdities: but he was ingenious enough to give his notions a plausible turn, so as to clear them seemingly from contradictions. His reputation procured him a call to Paris, as Professor of Rhetoric in the College of Lewis the Great. He also taught the mathematics in that city, as he had before done in other places. He had from his youth a happy genius for that science, and made a great progress in it; and the glory which his writings acquired him, raised the highest expectations from his future labours; but these were all blasted by his early death, in 1673, at 37 years of age; falling a victim to his zeal, he having caught a contagious disorder by preaching to the prisoners in the Bicetre.

Pardies wrote with great neatness and elegance. His principal works are as follow:

1. Horologium Thaumaticum duplex; 1662, in 4to.

2. Dissertatio de Motu et Natura Cometarum; 1665, 8vo.

3. Discours du Mouvement Local; 1670, 12mo.

4. Elemens de Geometrie; 1670, 12mo.—This has been translated into several languages; in English by Dr. Harris, in 1711.

5. Discours de la Connoissance des Betes; 1672, 12mo.

6. Lettre d'un Philosophe à un Cartesien de ses amis; 1672, 12mo.

7. La Statique ou la Science des Forces Mouvantes; 1673, 12mo.

8. Description et Explication de deux Machines propres à faire des Cadrans avec une grande facilité; 1673, 12mo.

9. Remarques du Mouvement de la Lumiere.

10. Globi Cœlestis in tabula plana redacti Descriptio; 1675, folio.

Part of his works were printed together, at the Hague, 1691, in 12mo; and again at Lyons, 1725.—Pardies had a dispute also with Sir Isaac Newton, about his New Theory of Light and Colours, in 1672. His letters are inserted in the Philosophical Transactions for that year.

PARENT (Anthony), a respectable French mathematician, was born at Paris in 1666. He shewed an early propensity to the mathematics, eagerly perusing such books in that science as fell in his way. His custom was to write remarks in the margins of the books he read; and in this way he had filled a number of books with a kind of commentary by the time he was 13 years of age.

Soon after this he was put under a master, who taught rhetoric at Chartres. Here he happened to see a dodecaedron, upon every face of which was delineated a sun-dial, except the lowest on which it stood. Struck as it were instantaneously with the curiosity of these dials, he attempted drawing one himself: but having only a book which taught the practical part, without the theory, it was not till after his master came to explain the doctrine of the sphere to him, that he began to understand how the projection of the circles of the sphere formed sun-dials. He then undertook to write a treatise upon gnomonics. To be sure the piece was rude and unpolished enough; however, it was entirely his own, and not borrowed. About the same time he wrote a book of geometry, in the same taste, at Beauvais.

His friends then sent for him to Paris to study the law; and in obedience to them he went through a course in that faculty: which was no sooner finished than, urged by his passion for mathematics, he shut himself up in the college of Dormans, that no avocation might take him from his beloved study: and, with an allowance of less than 200 livres a-year, he lived content in this retreat, from which he never stirred but to the Royal College, to hear the lectures of M. de la Hire or M. de Sauveur. When he thought himself capable of teaching others, he took pupils: and fortification being a branch of study which the war had brought into particular notice, he had often occasion to teach it: but after some time he began to entertain scruples about teaching a subject he had never seen, knowing it only by imagination. He imparted this scruple to M. Sauveur, who recommended him to the Marquis d'Aligre, who luckily at that time wanted to have a mathematician with him. M. Parent made two campaigns with the marquis, by which he instructed himself sufficiently in viewing fortified places; of which he drew a number of plans, though he had never learned the art of drawing.

From this period he spent his time in a continual application to the study of natural philosophy, and mathematics in all its branches, both speculative and practical; to which he joined anatomy, botany, and chemistry:—his genius joined with his indefatigable application overcoming every thing.

M. de Billettes being admitted into the Academy of Sciences at Paris in 1699, with the title of their mechanician, he named M. Parent for his eleve or disciple,

ciple, a branch of mathematics in which he chiefly excelled. It was ſoon diſcovered in this ſociety, that he engaged in all the different ſubjects which were brought before them; and indeed that he had a hand in every thing. But this extent of knowledge, joined to a natural warmth and impetuoſity of temper, raiſed a ſpirit of contradiction in him, which he indulged on all occaſions; ſometimes to a degree of precipitancy that was highly culpable, and often with but little regard to decency. Indeed the ſame behaviour was returned to him, and the papers which he brought to the academy were often treated with much ſeverity. In his productions, he was charged with obſcurity; a fault for which he was indeed ſo notorious, that he perceived it himſelf, and could not avoid correcting it.

By a regulation of the academy in 1716, the claſs of eleves was ſuppreſſed, as that diſtinction ſeemed to put too great an inequality between the members. M. Parent was made an adjunct or aſſiſtant member for the claſs of geometry: though he enjoyed this promotion but a very ſhort time; being cut off by the ſmallpox the ſame year, at 50 years of age.

M. Parent, beſides leaving many pieces in manuſcript, publiſhed the following works:

1. Elemens de Mecanique & de Phyſique; in 12mo, 1700.
2. Recherches de Mathematiques & de Phyſique; 3 vols. 4to, 1714.
3. Arithmetique theorico-pratique; in 8vo, 1714.
4. A great multitude of papers in the volumes of the Memoirs of the Academy of Sciences, from the year 1700 to 1714, ſeveral papers in almoſt every volume, upon a variety of branches in the mathematics.

PARGETING, in Building, is uſed for the plaiſtering of walls; ſometimes for plaiſter itſelf.

PARHELION, or PARHELIUM, denotes a mock ſun, or meteor, appearing as a very bright light by the ſide of the ſun; being formed by the reflection of his beams in a cloud properly ſituated.

Parhelia uſually accompany the coronæ, or luminous circles, and are placed in the ſame circumference, and at the ſame height. Their colours reſemble thoſe of the rainbow; the red and yellow are on that ſide towards the ſun, and the blue and violet on the other. Though coronæ are ſometimes ſeen entire, without any Parhelia; and ſometimes Parhelia without coronæ.

The apparent ſize of Parhelia is the ſame as that of the true ſun; but they are not always round, nor always ſo bright as the ſun; and when ſeveral appear, ſome are brighter than others. They are tinged externally with colours like the rainbow, and many of them have a long fiery tail oppoſite to the ſun, but paler towards the extremity. Some Parhelia have been obſerved with two tails and others with three. Theſe tails moſtly appear in a white horizontal circle, commonly paſſing through all the Parhelia, and would go through the centre of the ſun if it were entire. Sometimes there are arcs of leſſer circles, concentric to this, touching thoſe coloured circles which ſurround the ſun: theſe are alſo tinged with colours, and contain other Parhelia.

Parhelia are generally ſituated in the interſections of circles; but Caſſini ſays, thoſe which he ſaw in 1683, were on the outſide of the coloured circle, though the tails were in the circle that was parallel to the horizon. M. Aepinus apprehends, that Parhelia with elliptical coronæ are more frequent in the northern regions, and thoſe with circular ones in the ſouthern. They have been viſible for one, two, three, or four hours together; and it is ſaid that in North America they continue ſeveral days, and are viſible from ſun-riſe to ſun-ſet. When the Parhelia diſappear, it ſometimes rains, or there falls ſnow in the form of oblong ſpiculæ. And Mariotte accounts for the appearance of Parhelia from an infinity of ſmall particles of ice floating in the air, which multiply the image of the ſun, either by refracting or breaking his rays, and thus making him appear where he is not; or by reflecting them, and ſerving as mirrors.

Moſt philoſophers have written upon Parhelia; as Ariſtotle, Pliny, Scheiner, Gaſſendi, Des Cartes, Huygens, Hevelius, De la Hire, Caſſini, Grey, Halley, Maraldi, Muſſchenbroek, &c. See Smith's Optics, book 1, chap. 11. Alſo Prieſtley's Hiſt. of Light &c, p. 613. And Muſſchenbroek's Introduction &c, vol. 2, p. 1038 quarto.

PARODICAL *Degrees*, in an equation, a term that has been ſometimes uſed to denote the ſeveral regular terms in a quadratic, cubic, biquadratic, &c, equation, when the indices of the powers aſcend or deſcend orderly in an arithmetical progreſſion. Thus, $x^3 + mx^2 + nx = p$ is a cubic equation where no term is wanting, but having all its Parodic Degrees; the indices of the terms regularly deſcending thus, 3, 2, 1, 0.

PART, *Aliquant*, *Aliquot*, *Circular Proportional*, *Similar*, &c. See the reſpective adjectives.

PART *of Fortune*, in Judicial Aſtrology, is the lunar horoſcope; or the point in which the moon is, at the time when the ſun is in the aſcending point of the eaſt.

The ſun in the aſcendant is ſuppoſed, according to this ſcience, to give life; and the moon diſpenſes the radical moiſture, and is one of the cauſes of fortune. In horoſcopes the Part of Fortune is repreſented by a circle divided by a croſs.

PARTICLE, the minute part of a body, or an aſſemblage of ſeveral of the atoms of which natural bodies are compoſed. Particle is ſometimes conſidered as ſynonymous with atom, and corpuſcle; and ſometimes they are diſtinguiſhed.

Particles are, as it were, the elements of bodies; by the various arrangement and texture of which, with the difference of the coheſion, &c, are conſtituted the ſeveral kinds of bodies, hard, ſoft, liquid, dry, heavy, light, &c. The ſmalleſt Particles or corpuſcles cohere with the ſtrongeſt attractions, and always compoſe larger Particles of weaker coheſion: and many of theſe, cohering, compoſe ſtill larger Particles, whoſe vigour is ſtill weaker; and ſo on for divers ſucceſſions, till the progreſſion end in the largeſt Particles, upon which the operations in chemiſtry, and the colours of natural bodies, depend; and which, by cohering, compoſe bodies of ſenſible magnitude.

PARTILE *Aſpect*, in Aſtrology, is when the planets are in the exact degree of any particular aſpect. In contradiſtinction to Platic Aſpect, or when they do not regard each other with thoſe very degrees. See ASPECT.

PARTY *Arches*, in Architecture, are arches built between ſeparate tenures, where the property is intermixed, and apartments over each other do not belong to the ſame eſtate.

PARTY *Walls*, are partitions of brick made between buildings in ſeparate occupations, for preventing the ſpread of fire. Theſe are made thicker than the external walls; and their thickneſs in London is regulated by act of parliament of the 14th of George the Third.

PASCAL (BLAISE), a reſpectable French mathematician and philoſopher, and one of the greateſt geniuſes and beſt writers that country has produced. He was born at Clermont in Auvergne, in the year 1623. His father, Stephen Paſcal, was preſident of the Court of Aids in his province: he was alſo a very learned man, an able mathematician, and a friend of Des Cartes. Having an extraordinary tenderneſs for this child, his only ſon, he quitted his office in his province, and ſettled at Paris in 1631, that he might be quite at leiſure to attend to his ſon's education, which he conducted himſelf, and young Paſcal never had any other maſter.

From his infancy Blaiſe gave proofs of a very extraordinary capacity. He was extremely inquiſitive; deſiring to know the reaſon of every thing; and when good reaſons were not given him, he would ſeek for better; nor would he ever yield his aſſent but upon ſuch as appeared to him well grounded. What is told of his manner of learning the mathematics, as well as the progreſs he quickly made in that ſcience, ſeems almoſt miraculous. His father, perceiving in him an extraordinary inclination to reaſoning, was afraid leſt the knowledge of the mathematics might hinder his learning the languages, ſo neceſſary as a foundation to all ſound learning. He therefore kept him as much as he could from all notions of geometry, locked up all his books of that kind, and refrained even from ſpeaking of it in his preſence. He could not however prevent his ſon from muſing on that ſcience; and one day in particular he ſurpriſed him at work with charcoal upon his chamber floor, and in the midſt of figures. The father aſked him what he was doing: I am ſearching, ſays Paſcal, for ſuch a thing; which was juſt the ſame as the 32d propoſition of the 1ſt book of Euclid. He aſked him then how he came to think of this: It was, ſays Blaiſe, becauſe I found out ſuch another thing; and ſo, going backward, and uſing the names of *bar* and *round*, he came at length to the definitions and axioms he had formed to himſelf. Does it not ſeem miraculous, that a boy ſhould work his way into the heart of a mathematical book, without ever having ſeen that or any other book upon the ſubject, or knowing any thing of the terms? Yet we are aſſured of the truth of this by his ſiſter, Madam Perier, and ſeveral other perſons, the credit of whoſe teſtimony cannot reaſonably be queſtioned.

From this time he had full liberty to indulge his genius in mathematical purſuits. He underſtood Euclid's Elements as ſoon as he caſt his eyes upon them. At 16 years of age he wrote a treatiſe on Conic Sections, which was accounted a great effort of genius; and therefore it is no wonder that Des Cartes, who had been in Holland a long time, upon reading it, ſhould chooſe to believe that M. Paſcal the father was the real author of it. At 19 he contrived an admirable arithmetical machine, which was eſteemed a very wonderful thing, and would have done credit as an invention to any man verſed in ſcience, and much more to ſuch a youth.

About this time his health became impaired, ſo that he was obliged to ſuſpend his labours for the ſpace of four years. After this, having ſeen Torricelli's experiment reſpecting a vacuum and the weight of the air, he turned his thoughts towards theſe objects, and undertook ſeveral new experiments, one of which was as follows: Having provided a glaſs tube, 46 feet in length, open at one end, and hermetically ſealed at the other, he filled it with red wine, that he might diſtinguiſh the liquor from the tube, and ſtopped up the orifice; then having inverted it, and placed it in a vertical poſition, with the lower end immerſed into a veſſel of water one foot deep, he opened the lower end, and the wine deſcended to the diſtance of about 32 feet from the ſurface of the veſſel, leaving a conſiderable vacuum at the upper part of the tube. He next inclined the tube gradually, till the upper end became only of 32 feet perpendicular height above the bottom, and he obſerved the liquor proportionally aſcend up to the top of the tube. He made alſo a great many experiments with ſiphons, ſyringes, bellows, and all kinds of tubes, making uſe of different liquors, ſuch as quickſilver, water, wine, oil, &c; and having publiſhed them in 1647, he diſperſed his work through all countries.

All theſe experiments however only aſcertained effects, without demonſtrating the cauſes. Paſcal knew that Torricelli conjectured that thoſe phenomena which he had obſerved were occaſioned by the weight of the air, though they had formerly been attributed to Nature's abhorrence of a vacuum; but if Torricelli's theory were true, he reaſoned that the liquor in the barometer tube ought to ſtand higher at the bottom of a hill, than at the top of it. In order therefore to diſcover the truth of this theory, he made an experiment at the top and bottom of a mountain in Auvergne, called *le Puy de Dome*, the reſult of which gave him reaſon to conclude that the air was indeed heavy. Of this experiment he publiſhed an account, and ſent copies of it to moſt of the learned men in Europe. He alſo renewed it at the top and bottom of ſeveral high towers, as thoſe of Notre Dame at Paris, St. Jaques de la Boucherie, &c; and always remarked the ſame difference in the weight of the air, at different elevations. This fully convinced him of the general preſſure of the atmoſphere; and from this diſcovery he drew many uſeful and important inferences. He compoſed alſo a large treatiſe, in which he fully explained this ſubject, and replied to all the objections that had been ſtarted againſt it. As he afterwards thought this work rather too prolix, and being fond of brevity and preciſion, he divided it into two ſmall treatiſes, one of which he intitled, A Diſſertation on the Equilibrium of Fluids; and the other, An Eſſay on the Weight of the Atmoſphere. Theſe labours procured Paſcal ſo much reputation, that the greateſt mathematicians and philoſophers of the age propoſed various queſtions to him, and conſulted him reſpecting ſuch difficulties as they could not reſolve. Upon one of

of these occasions he discovered the solution of a problem proposed by Mersenne, which had baffled the penetration of all that had attempted it. This problem was to determine the curve described in the air by the nail of a coach-wheel, while the machine is in motion; which curve was thence called a roullette, but now commonly known by the name of cycloid. Pascal offered a reward of 40 pistoles to any one who should give a satisfactory answer to it. No person having succeeded, he published his own at Paris; but as he began now to be disgusted with the sciences, he would not set his real name to it, but sent it abroad under that of A. d'Ettonville.—This was the last work which he published in the mathematics; his infirmities, from a delicate constitution, though still young, now increasing so much, that he was under the necessity of renouncing severe study, and of living so recluse, that he scarcely admitted any person to see him.—Another subject on which Pascal wrote very ingeniously, and in which he has been spoken of as an inventor, was what has been called his Arithmetical Triangle, being a set of figurate numbers disposed in that form. But such a table of numbers, and many properties of them, had been treated of more than a century before, by Cardan, Stifelius, and other arithmetical writers.

After having thus laboured abundantly in mathematical and philosophical disquisitions, he forsook those studies and all human learning at once, to devote himself to acts of devotion and penance. He was not 24 years of age, when the reading some pious books had put him upon taking this resolution; and he became as great a devotee as any age has produced. He now gave himself up entirely to a state of prayer and mortification; and he had always in his thoughts these great maxims of renouncing all pleasure and all superfluity; and this he practised with rigour even in his illnesses, to which he was frequently subject, being of a very invalid habit of body.

Though Pascal had thus abstracted himself from the world, yet he could not forbear paying some attention to what was doing in it; and he even interested himself in the contest between the Jesuits and the Jansenists. Taking the side of the latter, he wrote his *Lettres Provinciales*, published in 1656, under the name of *Louis de Montalte*, making the former the subject of ridicule. "These letters, says Voltaire, may be considered as a model of eloquence and humour. The best comedies of Moliere have not more wit than the first part of these letters; and the sublimity of the latter part of them, is equal to any thing in Bossuet. It is true indeed that the whole book was built upon a false foundation; for the extravagant notions of a few Spanish and Flemish Jesuits were artfully ascribed to the whole society. Many absurdities might likewise have been discovered among the Dominican and Franciscan casuists; but this would not have answered the purpose; for the whole raillery was to be levelled only at the Jesuits. These letters were intended to prove, that the Jesuits had formed a design to corrupt mankind; a design which no sect or society ever had, or can have." Voltaire calls Pascal the first of their satirists; for Despreaux, says he, must be considered as only the second. In another place, speaking of this work of Pascal, he says, that "Examples of all the various species of eloquence are to be found in it. Though it has now been written almost 100 years, yet not a single word occurs in it, favouring of that vicissitude to which living languages are so subject. Here then we are to fix the epoch when our language may be said to have assumed a settled form. The bishop of Lucon, son of the celebrated Bussy, told me, that asking one day the bishop of Meaux what work he would covet most to be the author of, supposing his own performances set aside, Bossu replied, The Provincial Letters." These letters have been translated into all languages, and printed over and over again. Some have said that there were decrees of formal condemnation against them; and also that Pascal himself, in his last illness, detested them, and repented of having been a Jansenist: but both these particulars are false and without foundation. It was supposed that Father Daniel was the anonymous author of a piece against them, intitled *The Dialogues of Cleander and Eudoxus.*

Pascal was but about 30 years of age when these letters were published; yet he was extremely infirm, and his disorders increasing soon after so much, that he conceived his end fast approaching, he gave up all farther thoughts of literary composition. He resolved to spend the remainder of his days in retirement and pious meditation; and with this view he broke off all his former connections, changed his habitation, and spoke to no one, not even to his own servants, and hardly ever even admitted them into his room. He made his own bed, fetched his dinner from the kitchen, and carried back the plates and dishes in the evening; so that he employed his servants only to cook for him, to go to town, and to do such other things as he could not absolutely do himself. In his chamber nothing was to be seen but two or three chairs, a table, a bed, and a few books. It had no kind of ornament whatever; he had neither a carpet on the floor, nor curtains to his bed. But this did not prevent him from sometimes receiving visits; and when his friends appeared surprised to see him thus without furniture, he replied, that he had what was necessary, and that any thing else would be a superfluity, unworthy of a wise man. He employed his time in prayer, and in reading the Scriptures; writing down such thoughts as this exercise inspired. Though his continual infirmities obliged him to use very delicate food, and though his servants employed the utmost care to provide only what was excellent, he never relished what he ate, and seemed quite indifferent whether they brought him good or bad. His indifference in this respect was so great, that though his taste was not vitiated, he forbad any sauce or ragout to be made for him which might excite his appetite.

Though Pascal had now given up intense study, and though he lived in the most temperate manner, his health continued to decline rapidly; and his disorders had so enfeebled his organs, that his reason became in some measure affected. He always imagined that he saw a deep abyss on one side of him, and he never would sit down till a chair was placed there, to secure him from the danger which he apprehended. At another time he pretended that he had a kind of vision or ecstasy; a memorandum of which he preserved

 during

during the remainder of his life on a bit of paper, put between the cloth and the lining of his coat, and which he always carried about him. After languishing for several years in this imbecile state of body and mind, M. Pascal died at Paris the 19th of August 1662, at 39 years of age.

In company, Pascal was distinguished by the amiableness of his behaviour; by great modesty; and by his easy, agreeable, and instructive conversation. He possessed a natural kind of eloquence, which was in a manner irresistible. The arguments he employed for the most part produced the effect which he proposed; and though his abilities intitled him to assume an air of superiority, he never displayed that haughty and imperious tone which may often be observed in men of shining talents. The philosophy of this extraordinary man consisted in renouncing all pleasure, and every superfluity. He not only denied himself the most common gratifications; but he took also without reluctance, and even with pleasure, either as nourishment or as medicine, whatever was disagreeable to the senses; and he every day retrenched some part of his dress, food, or other things, which he considered as not absolutely necessary. Towards the close of his life, he employed himself wholly in devout and moral reflections, writing down those which he deemed worthy of being preserved. The first bit of paper he could find was employed for this purpose; and he commonly set down only a few words of each sentence, as he wrote them merely for his own use. The scraps of paper upon which he had written these thoughts, were found after his death filed upon different pieces of string, without any order or connection; and being copied exactly as they were written, they were afterward arranged and published, under the title of *Pensées, &c.* or *Thoughts upon Religion and other Subjects;* being parts of a work he had intended against atheists and infidels, which has been much admired. After his death appeared also two other little tracts; the one intitled, *The Equilibrium of Fluids;* and the other, *The Weight of the Mass of Air.*

The works of Pascal were collected in 5 volumes 8vo, and published at the Hague, and at Paris, in 1779. This edition of Pascal's works may be considered as the first published; at least the greater part of them were not before collected into one body, and some of them had remained only in manuscript. For this collection, the public were indebted to the Abbé Bossu, and Pascal was deserving of such an editor. "This extraordinary man, says he, inherited from nature all the powers of genius. He was a mathematician of the first rank, a profound reasoner, and a sublime and elegant writer. If we reflect, that in a very short life, oppressed by continual infirmities, he invented a curious arithmetical machine, the elements of the calculation of chances, and a method of resolving various problems, respecting the cycloid; that he fixed in an irrevocable manner the wavering opinions of the learned concerning the weight of the air; that he wrote one of the completest works existing in the French language; and that in his *Thoughts* there are passages the depth and beauty of which are incomparable—we can hardly believe that a greater genius ever existed in any age or nation. All those who had occasion to frequent his company in the ordinary commerce of the world, acknowledged his superiority; but it excited no envy against him, as he was never fond of shewing it. His conversation instructed, without making those who heard him sensible of their own inferiority; and he was remarkably indulgent towards the faults of others. It may be easily seen by his Provincial Letters, and by some of his other works, that he was born with a great fund of humour, which his infirmities could never entirely destroy. In company, he readily indulged in that harmless and delicate raillery which never gives offence, and which greatly tends to enliven conversation; but its principal object was generally of a moral nature. For example, ridiculing those authors who say, *My Book, my Commentary, my History,* they would do better (added he) to say, *Our book, our Commentary, our History;* since there is in them much more of other people's than their own."

The celebrated Baley too, speaking of this great man, says, a hundred volumes of sermons are not of so much avail as a simple account of the life of Pascal. His humanity and his devotion mortified the libertines more than if they had been attacked by a dozen of missionaries. In short, Bayle had so high an idea of this philosopher, that he calls him *a paradox in the human species.* "When we consider his character, says he, we are almost inclined to doubt whether he was born of a woman, like the man mentioned by Lucretius;

"*Ut vix humana videatur stirpe creatus.*"

PATE, in Fortification, a kind of platform, like what is called a Horse-shoe; not always regular, but commonly oval, encompassed only with a parapet, and having nothing to flank it. It is usually erected in marshy grounds, to cover a gate of a town, or the like.

PATH *of the Vertex*, a term frequently used by Mr. Flamsteed, in his Doctrine of the Sphere, denoting a circle, described by any point of the earth's surface, as the earth turns round its axis.

This point is considered as vertical to the earth's centre; and is the same with what is called the vertex or zenith in the Ptolomaic projection.

The semidiameter of this Path of the vertex, is always equal to the complement of the latitude of the point or place that describes it; that is, to the place's distance from the pole of the world.

PAVILION, in Architecture, is a kind of turret, or building usually insulated, and contained under a single roof; sometimes square and sometimes in form of a dome: thus called from the resemblance of its roof to a tent.

PAVO, *Peacock*, a new constellation, in the southern hemisphere, added by the modern astronomers. It contains 14 stars.

PAUSE, or REST, in Music, a character of silence and rest; called also by some a Mute Figure; because it shews that some part or person is to be silent, while the others continue the song.

PECK, a measure or vessel used in measuring grain, pulse, and the like dry substances.

The standard, or Winchester Peck, contains two gallons, or the 4th part of a bushel.

PEDESTAL,

PEDESTAL, in Architecture, the loweſt part of an order of columns; being that which ſuſtains the column, and ſerves it as a foot to ſtand upon. It is a ſquare body or dye, with a cornice and baſe.

The proportions and ornaments of the Pedeſtal are different in the different orders. Vignola indeed, and moſt of the moderns, make the Pedeſtal, and its ornaments, in all the orders, one third of the height of the column, including the baſe and capital. But ſome deviate from this rule.

Perrault makes the proportions of the three conſtituent parts of Pedeſtals, the ſame in all the orders; viz, the baſe one fourth of the Pedeſtal; the cornice an eighth part; and the ſocle or plinth of the baſe, two thirds of the baſe itſelf. The height of the dye is what remains of the whole height of the Pedeſtal.

The *Tuſcan* Pedestal is the ſimpleſt and loweſt of all; from 3 to 5 modules high. It has only a plinth for its baſe, and an aſtragal crowned for its cornice.

The *Doric* Pedestal is made 4 or 5 modules in height, by the moderns; for no ancient columns, of this order, are found with any Pedeſtal, or even with any baſe.

The *Ionic* Pedestal is from 5 to 7 modules high.

The *Corinthian* Pedestal is the richeſt and moſt delicate of all, and is from 4 to 7 modules high.

The *Compoſite* Pedestal is of 6 or 7 modules in height.

Square Pedestal, is one whoſe breadth and height are equal.

Double Pedestal, is that which ſupports two columns, being broader than it is high.

Continued Pedestal, is that which ſupports a row of columns without any break or interruption.

Pedestals *of Statues*, are thoſe ſerving to ſupport figures or ſtatues.

PEDIMENT, in Architecture, a kind of low pinnacle; ſerving to crown porticos, or finiſh a frontiſpiece; and placed as an ornament over gates, doors, windows, niches, altars, &c; being uſually of a triangular form, but ſometimes an arch of a circle. Its height is various, but it is thought moſt beautiful when the height is one fifth of the length of its baſe.

PEDOMETER, or Podometer, foot-meaſurer, or way-wiſer; a mechanical inſtrument, in form of a watch, and conſiſting of various wheels and teeth; which, by means of a chain, or ſtring, faſtened to a man's foot, or to the wheel of a chariot, advance a notch each ſtep, or each revolution of the wheel: by which it numbers the paces or revolutions, and ſo the diſtance from one place to another.

Pedometer is alſo ſometimes uſed for the common ſurveying wheel, an inſtrument chiefly uſed in meaſuring roads; popularly called the way-wiſer. See Perambulator.

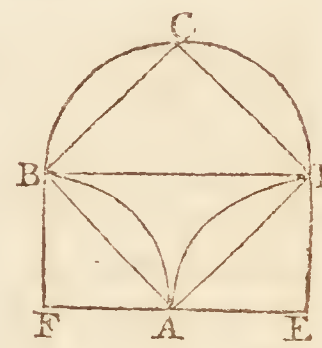
PEER, in Building. See Pier.

PEGASUS, the Horſe, a conſtellation of the northern hemiſphere, figured in the form of a flying horſe; being one of the 48 ancient conſtellations.

It is fabled, by the Greeks, to have been the offſpring of an amour between Neptune and the Gorgon Meduſa; and to have been that on which Bellerophon rode when he overcame the Chimera; and that flying from mount Helicon to heaven, he there became a conſtellation; having thrown his rider in the flight; and that the ſtroke of his hoof on the mount opened the ſacred fountain Hippocrene.

The ſtars in this conſtellation, in Ptolomy's catalogue, are 20, in Tycho's 19, in Hevelius's 38, and in the Britannic catalogue 89.

PELECOIDES, or *Hatchet-form*, in Geometry, a figure in form of a hatchet.



As the figure ABCDA, contained under the ſemicircle BCD and the two quadrantal arcs AB and AD.

The area of the Pelecoides is equal to the ſquare AC, and this again is equal to the rectangle BE. It is equal to the ſquare, becauſe the two ſegments AB and AD, which it wants of the ſquare on the lower part, are compenſated by the two equal ſegments BC and CD, by which it exceeds on the upper part. And the ſquare is equal to the rectangle BE, becauſe the triangle ABD, which is half the ſquare, is alſo half the rectangle BE of the ſame baſe and height with it.

PELL (Dr. John), an eminent Engliſh mathematician, deſcended from an ancient family in Lincolnſhire, was born at Southwick in Suſſex, March 1, 1610, where his father was miniſter. He received his grammar education at the free-ſchool at Stenning in that county. At the age of 13 he was ſent to Trinity College in Cambridge, being then as good a ſcholar as moſt maſters of arts in that univerſity; but though he was eminently ſkilled in the Greek and Hebrew languages, he never offered himſelf a candidate at the election of ſcholars or fellows of his college. His perſon was handſome; and being of a ſtrong conſtitution, uſing little or no recreations, he proſecuted his ſtudies with the more application and intenſeneſs.

In 1629 he drew up the "*Deſcription and Uſe of the Quadrant, written for the Uſe of a Friend,*" *in two books;* the original manuſcript of which is ſtill extant among his papers in the Royal Society. And the ſame year he held a correſpondence with Mr. Briggs on the ſubject of logarithms.

In 1630 he wrote, *Modus ſupputandi Ephemerides Aſtronomicas, &c, ad an.* 1630 *accommodatus;* and, *A Key to unlock the meaning of Johannes Trithemius, in his Diſcourſe of Steganography:* which Key he imparted to Mr. Samuel Hartlib and Mr. Jacob Homedæ. The ſame year he took the degree of Maſter of Arts at Cambridge. And the year following he was incorporated in the Univerſity of Oxford. June the 7th, he wrote *A Letter to Mr. Edmund Wingate on Logarithms;* and Oct. 5, 1631, *Commentationes in Coſmographiam Alſtedii.*

In 1632 he married Ithamaria, ſecond daughter of Mr. Henry Reginolles of London, by whom he had four ſons and four daughters.—March 6, 1634, he finiſhed his *Aſtronomical Hiſtory of Obſervations of Heavenly Motions and Appearances;* and April the 10th, his *Ecliptica Prognoſtica, or Foreknower of the Eclipſes, &c.*—In 1634 he tranſlated *The Everlaſting Tables of*

Heavenly Motions, grounded upon the Obſervations of all Times, and agreeing with them all, by Philip Lanſberg, of Ghent in Flanders. And June the 12th, the ſame year, he committed to writing, *The Manner of Deducing his Aſtronomical Tables out of the Tables and Axioms of Philip Lanſberg.*—March the 9th, 1635, he wrote *A Letter of Remarks on Gellibrand's Mathematical Diſcourſe on the Variation of the Magnetic Needle.* And the 3d of June following, another on the ſame ſubject.

His eminence in mathematical knowledge was now ſo great, that he was thought worthy of a profeſſor's chair in that ſcience; and, upon the vacancy of one at Amſterdam in 1639, Sir William Boſwell, the Engliſh Reſident with the States General, uſed his intereſt, that he might ſucceed in that profeſſorſhip: it was not filled up however till 1643, when Pell was choſen to it; and he read with great applauſe public lectures upon Diophantus.—In 1644 he printed at Amſterdam, in two pages 4to, *A Refutation of Longomontanus's Diſcourſe, De Vera Circuli Menſura.*

In 1646, on the invitation of the Prince of Orange, he removed to the new college at Breda, as Profeſſor of Mathematics, with a ſalary of 1000 guilders a year.—His *Idea Matheſeos*, which he had addreſſed to Mr. Hartlib, who in 1639 had ſent it to Des Cartes and Merſenne, was printed 1650 at London, in 12mo, in Engliſh, with the title of *An Idea of Mathematics*, at the end of Mr. John Durie's Reformed Library-keeper. It is alſo printed by Mr. Hook, in his Philoſophical Collections, No. 5, p. 127; and is eſteemed our author's principal work.

In 1652 Pell returned to England; and in 1654 he was ſent by the protector Cromwell agent to the Proteſtant Cantons in Switzerland; where he continued till June 23, 1658, when he ſet out for England, where he arrived about the time of Cromwell's death. His negociations abroad gave afterwards a general ſatisfaction, as it appeared he had done no ſmall ſervice to the intereſt of king Charles the Second, and of the church of England; ſo that he was encouraged to enter into holy orders; and in the year 1661 he was inſtituted to the rectory of Fobbing in Eſſex, given him by the king. In December that year, he brought into the upper houſe of convocation the calendar reformed by him, aſſiſted by Sancroft, afterwards archbiſhop of Canterbury.—In 1673 he was preſented by Sheldon, biſhop of London, to the rectory of Laingdon in Eſſex; and, upon the promotion of that biſhop to the ſee of Canterbury ſoon after, became one of his domeſtic chaplains. He was then doctor of divinity, and expected to be made a dean; but his improvement in the philoſophical and mathematical ſciences was ſo much the bent of his genius, that he did not much purſue his private advantage. The truth is, he was a helpleſs man, as to worldly affairs, and his tenants and relations impoſed upon him, cozened him of the profits of his parſonage, and kept him ſo indigent, that he wanted neceſſaries, even ink and paper, to his dying day. He was for ſome time confined to the King's-bench priſon for debt; but, in March 1682, was invited by Dr. Whitler to live in the college of phyſicians. Here he continued till June following; when he was obliged, by his ill ſtate of health, to remove to the houſe of a grandchild of his in St. Margaret's Church-yard, Weſtminſter. But he died at the houſe of Mr. Cothorne, reader of the church of St. Giles's in the Fields, December the 12th, 1685, in the 74th year of his age, and was interred at the expence of Dr. Buſby, maſter of Weſtminſter ſchool, and Mr. Sharp, rector of St. Giles's, in the rector's vault under that church.——Dr. Pell publiſhed ſome other things not yet mentioned, a liſt of which is as follows: viz,

1. An Exercitation concerning Eaſter; 1644, in 4to.
2. A Table of 10,000 ſquare numbers, &c; 1672, folio.
3. An Inaugural Oration at his entering upon the Profeſſorſhip at Breda.
4. He made great alterations and additions to Rhonius's Algebra, printed at London 1668, 4to, under the title of, An Introduction to Algebra; tranſlated out of the High-Dutch into Engliſh by Thomas Branker, much altered and augmented by D. P. (Dr. Pell). Alſo a Table of Odd Numbers, leſs than 100,000, ſhewing thoſe that are incompoſite, &c, ſupputated by the ſame Thomas Branker.
5. His Controverſy with Longomontanus concerning the Quadrature of the Circle; Amſterdam, 1646, 4to.

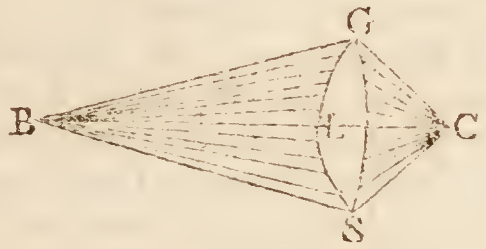
He likewiſe wrote a Demonſtration of the 2d and 10th books of Euclid; which piece was in MS. in the library of lord Brereton in Cheſhire: as alſo Archimedes's Arenarius, and the greateſt part of Diophantus's 6 books of Arithmetic; of which author he was preparing, Aug. 1644, a new edition, in which he intended to have corrected the tranſlation, and made new illuſtrations. He deſigned likewiſe to publiſh an edition of Apollonius, but laid it aſide, in May, 1645, at the deſire of Golius, who was engaged in an edition of that author from an Arabic manuſcript given him at Aleppo 18 years before. Letters of Dr. Pell to Sir Charles Cavendiſh, in the Royal Society.

Some of his manuſcripts he left at Brereton in Cheſhire, where he reſided ſome years, being the ſeat of William lord Brereton, who had been his pupil at Breda. A great many others came into the hands of Dr. Buſby; which Mr. Hook was deſired to uſe his endeavours to obtain for the Society. But they continued buried under duſt, and mixed with the papers and pamphlets of Dr. Buſby, in four large boxes, till 1755; when Dr. Birch, ſecretary to the Royal Society, procured them for that body, from the truſtees of Dr. Buſby. The collection contains not only Pell's mathematical Papers, letters to him, and copies of thoſe from him, &c, but alſo ſeveral manuſcripts of Walter Warner, the mathematician and philoſopher, who lived in the reigns of James the Firſt and Charles the Firſt.

Dr. Pell invented the method of ranging the ſeveral ſteps of an algebraical calculus, in a proper order, in ſo many diſtinct lines, with the number affixed to each ſtep, and a ſhort deſcription of the operation or proceſs in the line. He alſo invented the character ÷ for diviſion, ⊛ for involution, and ⧢ for evolution.

PENCIL *of Rays*, in Optics, is a double cone, or pyramid, of rays, joined together at the baſe; as BGSC;

BGSC: the one cone having its vertex in ſome point of the object at B, and the cryſtalline humour, or the glaſs GLS for its baſe; and the other having its baſe on the ſame glaſs, or cryſtalline, but its vertex in the point of convergence, as at C.



PENDULUM, in Mechanics, any heavy body, ſo ſuſpended as that it may ſwing backwards and forwards, about ſome fixed point, by the force of gravity.

Theſe alternate aſcents and deſcents of the Pendulum, are called its Oſcillations, or Vibrations; each complete oſcillation being the deſcent from the higheſt point on one ſide, down to the loweſt point of the arch, and ſo on up to the higheſt point on the other ſide. The point round which the Pendulum moves, or vibrates, is called its Centre of Motion, or Point of Suſpenſion; and a right line drawn through the centre of motion, parallel to the horizon, and perpendicular to the plane in which the Pendulum moves, is called the Axis of Oſcillation. There is alſo a certain point within every Pendulum, into which, if all the matter that compoſes the Pendulum were collected, or condenſed as into a point, the times in which the vibrations would be performed, would not be altered by ſuch condenſation; and this point is called Centre of Oſcillation. The length of the Pendulum is always eſtimated by the diſtance of this point below the centre of motion; being uſually near the bottom of the Pendulum; but in a cylinder, or any other uniform priſm or rod, it is at the diſtance of one third from the bottom, or two-thirds from and below the centre of motion.

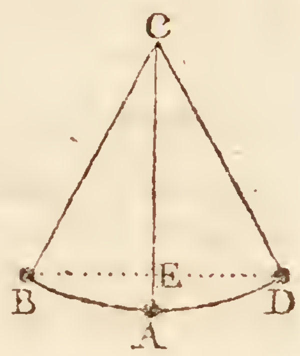
The length of a Pendulum, ſo meaſured to its centre of oſcillation, that it will perform each vibration in a ſecond of time, thence called the ſecond's Pendulum, has, in the latitude of London, been generally taken at $39\frac{2}{10}$ or $39\frac{1}{5}$ inches; but by ſome very ingenious and accurate experiments, the late celebrated Mr. George Graham found the true length to be $39\frac{126}{1000}$, inches, or $39\frac{1}{8}$ inches very nearly.

The length of the Pendulum vibrating ſeconds at Paris, was found by Varin, Des Hays, De Glos, and Godin, to be $440\frac{5}{9}$ lines; by Picard $440\frac{1}{2}$ lines; and by Mairan $440\frac{17}{30}$ lines.

Galileo was the firſt who made uſe of a heavy body annexed to a thread, and ſuſpended by it, for meaſuring time, in his experiments and obſervations. But according to Sturmius, it was Riccioli who firſt obſerved the iſochroniſm of Pendulums, and made uſe of them in meaſuring time. After him, Tycho, Langrene, Wendeline, Merſenne, Kircher, and others, obſerved the ſame thing; though, it is ſaid, without any intimation of what had been done by Riccioli. But it was the celebrated Huygens who firſt demonſtrated the principles and properties of Pendulums, and probably the firſt who applied them to clocks. He demonſtrated, that if the centre of motion were perfectly fixed and immoveable, and all manner of friction, and reſiſtance of the air, &c, removed, then a Pendulum, once ſet in motion, would for ever continue to vibrate without any decreaſe of motion, and that all its vibrations would be perfectly iſochronal, or performed in the ſame time. Hence the Pendulum has univerſally been conſidered as the beſt chronometer or meaſurer of time. And as all Pendulums of the ſame length perform their vibrations in the ſame time, without regard to their different weights, it has been ſuggeſted, by means of them, to eſtabliſh an univerſal ſtandard for all countries. On this principle Mouton, canon of Lyons, has a treatiſe, De Menſura poſteris tranſmittenda; and ſeveral others ſince, as Whitehurſt, &c. See *Univerſal* MEASURE.

Pendulums are either ſimple or compound, and each of theſe may be conſidered either in theory, or as in practical mechanics among artiſans.

A Simple PENDULUM, in Theory, conſiſts of a ſingle weight, as A, conſidered as a point, and an inflexible right line AC, ſuppoſed void of gravity or weight, and ſuſpended from a fixed point or centre C, about which it moves.



A Compound PENDULUM, in Theory, is a Pendulum conſiſting of ſeveral weights moveable about one common centre of motion, but connected together ſo as to retain the ſame diſtance both from one another, and from the centre about which they vibrate.

The Doctrine and Laws of PENDULUMS.—1. A Pendulum raiſed to B, through the arc of the circle AB, will fall, and riſe again, through an equal arc, to a point equally high, as D; and thence will fall to A, and again riſe to B; and thus continue riſing and falling perpetually. For it is the ſame thing, whether the body fall down the inſide of the curve BAD, by the force of gravity, or be retained in it by the action of the ſtring; for they will both have the ſame effect; and it is otherwiſe known, from the oblique deſcents of bodies, that the body will deſcend and aſcend along the curve in the manner above deſcribed.

Experience alſo confirms this theory, in any finite number of oſcillations. But if they be ſuppoſed infinitely continued, a difference will ariſe. For the reſiſtance of the air, and the friction and rigidity of the ſtring about the centre C, will take off part of the force acquired in falling; whence it happens that it will not riſe preciſely to the ſame point from whence it fell.

Thus, the aſcent continually diminiſhing the oſcillation, this will be at laſt ſtopped, and the Pendulum will hang at reſt in its natural direction, which is perpendicular to the horizon.

Now as to the real time of oſcillation in a circular arc BAD: it is demonſtrated by mathematicians, that if $p = 3\cdot1416$, denote the circumference of a circle whoſe diameter is 1; $g = 16\frac{1}{12}$ feet or 193 inches, the ſpace a heavy body falls in the firſt ſecond of time; and $r =$ CA the length of the Pendulum; alſo $a =$ AE the height of the arch of vibration; then the time

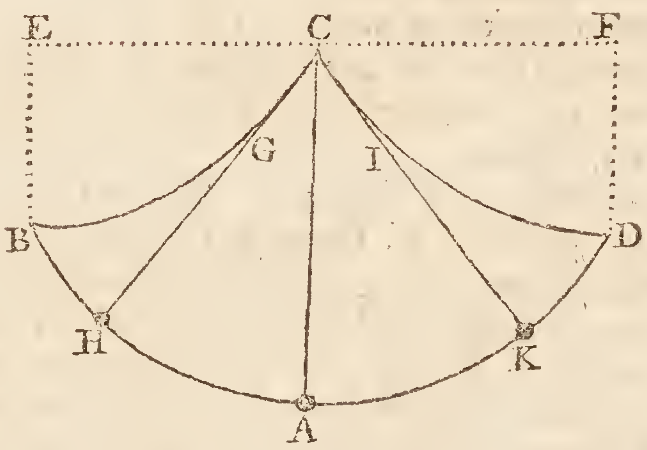
time of each oscillation in the arc BAD will be equal to $p\sqrt{\frac{r}{2g}}$ × into the infinite series

$$1 + \frac{1^2 a}{2^2 d} + \frac{1^2.3^2 a^2}{2^2.4^2 d^2} + \frac{1^2.3^2.5^2 a^3}{2^2.4^2.6^2 d^3} \ \&c,$$

where $d = 2r$ is the diameter of the arc described, or twice the length of the Pendulum.

And here, when the arc is a small one, as in the case of the vibrating Pendulum of a clock, all the terms of this series after the 2d may be omitted, on account of their smallness; and then the time of a whole vibration will be nearly equal to $p\sqrt{\frac{r}{2g}} \times (1 + \frac{a}{8r})$. So that the times of vibration of a Pendulum in different small arcs of the same circle, are as $8r + a$, or 8 times the radius, added to the versed sine of the semiarc.

And farther, if D denote the number of degrees in the semiarc AB, whose versed sine is a, then the quantity last mentioned, for the time of a whole vibration, is changed to $p\sqrt{\frac{r}{2g}} \times (1 + \frac{D^2}{52524})$. And therefore the times of vibration in different small arcs, are as $52524 + D^2$, or as the number 52524 added to the square of the number of degrees in the semiarc AB. See my Conic Sections and Select Exercises, p. 190.



2. Let CB be a semicycloid, having its base EC parallel to the horizon, and its vertex B downwards; and let CD be the other half of the cycloid, in a similar position to the former. Suppose a Pendulum string, of the same length with the curve of each semicycloid BC, or CD, having its end fixed in C, and the thread applied all the way close to the cycloidal curve BC, and consequently the body or Pendulum weight coinciding with the point B. If now the body be let go from B, it will descend by its own gravity, and in descending it will unwind the string from off the arch BC, as at the position CGH; and the ball G will describe a semicycloid BHA, equal and similar to BGC, when it has arrived at the lowest point A; after which, it will continue its motion, and ascend, by another equal and similar semicycloid AKD, to the same height D, as it fell from at B, the string now wrapping itself upon the other arch CID. From D it will descend again, and pass along the whole cycloid DAB, to the point B; and thus perform continual successive oscillations between B and D, in the curve of a cycloid; as it before oscillated in the curve of a circle, in the former case.

This contrivance to make the Pendulum oscillate in the curve of a cycloid, is the invention of the celebrated Huygens, to make the Pendulum perform all its vibrations in equal times, whether the arch, or extent of the vibration be great or small; which is not the case in a circle, where the larger arcs take a longer time to run through them, than the smaller ones do, as is well known both from theory and practice.

The chief properties of the cycloidal Pendulum then, as demonstrated by Huygens, are the following. 1st, That the time of an oscillation in all arcs, whether larger or smaller, is always the same quantity, viz, whether the body begin to descend from the point B, and describe the semiarch BA; or that it begins at H, and describes the arch HA; or that it sets out from any other point; as it will still descend to the lowest point A in exactly the same time. And it is farther proved, that the time of a whole vibration through any double arc BAD, or HAK, &c, is in proportion to the time in which a heavy body will freely fall, by the force of gravity, through a space equal to $\frac{1}{2}$AC, half the length of the Pendulum, as the circumference of a circle is to its diameter. So that, if $g = 16\frac{1}{12}$ feet denote the space a heavy body falls in the first second of time, $p = 3{\cdot}1416$ the circumference of a circle whose diameter is 1, and $r = AC$ the length of the Pendulum; then, because, by the nature of descents by gravity, $\sqrt{g} : \sqrt{\frac{1}{2}r} :: 1'' : \sqrt{\frac{r}{2g}}$ that is the time in which a body will fall through $\frac{1}{2}r$, or half the length of the Pendulum; therefore, by the above proportion, as $1 : p :: \sqrt{\frac{r}{2g}} : p\sqrt{\frac{r}{2g}}$, which is the time of an entire oscillation in the cycloid.

And this conclusion is abundantly confirmed by experience. For example, if we consider the time of a vibration as 1 second, to find the length of the Pendulum that will so oscillate in 1 second; this will give the equation $p\sqrt{\frac{r}{2g}} = 1$; which reduced, gives $r = \frac{2g}{p^2} = \frac{386}{3{\cdot}1416^2}$ inches $= 39{\cdot}11$ or $39\frac{1}{9}$ inches, for the length of the second's Pendulum; which the best experiments shew to be about $39\frac{1}{8}$ inches.

3. Hence also, we have a method of determining, from the experimented length of a Pendulum, the space a heavy body will fall perpendicularly through in a given time: for, since $p\sqrt{\frac{r}{2g}} = 1$, therefore, by reduction, $g = \frac{1}{2}p^2 r$ is the space a body will fall through in the first second of time, when r denotes the length of the second's Pendulum; and as constant experience shews that this length is nearly $39\frac{1}{8}$ inches, in the latitude of London, in this case g or $\frac{1}{2}p^2 r$ becomes $\frac{1}{2} \times 3{\cdot}1416^2 \times 39\frac{1}{8} = 193{\cdot}07$ inches $= 16\frac{1}{12}$ feet, very nearly, for the space a body will fall in the first second of time, in the latitude of London: a fact which has been abundantly confirmed by experiments made there. And in the same manner, Mr. Huygens found the same space fallen through at Paris, to be 15 French feet.

The whole doctrine of Pendulums, oscillating between two semicycloids, both in theory and practice, was

was delivered by that author, in his Horologium Oscillatorium, five Demonstrationes de Motu Pendulorum. And every thing that regards the motion of Pendulums has since been demonstrated in different ways, and particularly by Newton, who has given an admirable theory on the subject, in his Principia, where he has extended to epicycloids the properties demonstrated by Huygens of the cycloids.

4. As the cycloid may be considered as coinciding, in A, with any small arc of a circle described from the centre C, passing through A, where it is known the two curves have the same radius and curvature; therefore the time in the small arc of such a circle, will be nearly equal to the time in the cycloid; so that the times in very small circular arcs are equal, because these small arcs may be considered as portions of the cycloid, as well as of the circle. And this is one great reason why the Pendulums of clocks are made to oscillate in as small arcs as possible, viz, that their oscillations may be the nearer to a constant equality.

This may also be deduced from a comparison of the times of vibration in the circle, and in the cycloid, as laid down in the foregoing articles. It has there been shewn, that the times of vibration in the circle and cycloid are thus, viz,

time in the circle nearly $p\sqrt{\frac{r}{2g}} \times (1 + \frac{a}{8r})$,

time in the cycloidal arc $p\sqrt{\frac{r}{2g}}$;

where it is evident, that the former always exceeds the latter in the ratio of $1 + \frac{a}{8r}$ to 1; but this ratio always approaches nearer to an equality, as the arc, or as its versed sine a, is smaller; till at length, when it is very small, the term $\frac{a}{8r}$ may be omitted, and then the times of vibration become both the same quantity, viz $p\sqrt{\frac{r}{2g}}$.

Farther, by the same comparison, it appears, that the time lost in each second, or in each vibration of the second's Pendulum, by vibrating in a circle, instead of a cycloid, is $\frac{a}{8r}$, or $\frac{D^2}{52524}$; and consequently the time lost in a whole day of 24 hours, is $\frac{5}{3}D^2$ nearly. In like manner, the seconds lost per day by vibrating in the arc of Δ degrees, is $\frac{5}{3}\Delta^2$. Therefore if the Pendulum keep true time in one of these arcs, the seconds lost or gained per day, by vibrating in the other, will be $\frac{5}{3}(D^2 - \Delta^2)$. So, for example, if a Pendulum measure true time in an arc of 3 degrees, on each side of the lowest point, it will lose $11\frac{2}{3}$ seconds a day by vibrating 4 degrees; and $26\frac{2}{3}$ seconds a day by vibrating 5 degrees; and so on.

5. The action of gravity is less in those parts of the earth where the oscillations of the same Pendulum are slower, and greater where these are swifter; for the time of oscillation is reciprocally proportional to $\sqrt{g}$. And it being found by experiment, that the oscillations of the same Pendulum are slower near the equator, than in places farther from it; it follows that the force of gravity is less there; and consequently the parts about the equator are higher or farther from the centre, than the other parts; and the shape of the earth is not a true sphere, but somewhat like an oblate spheroid, flatted at the poles, and raised gradually towards the equator. And hence also the times of the vibration of the same Pendulum, in different latitudes, afford a method of determining the true figure of the earth, and the proportion between its axis and the equatorial diameter.

Thus, M. Richer found by an experiment made in the island Cayenna, about 4 degrees from the equator, where a Pendulum 3 feet $8\frac{1}{2}$ lines long, which at Paris vibrated seconds, required to be shortened a line and a quarter to make it vibrate seconds. And many other observations have confirmed the same principle. See Newton's Principia, lib. 3, prop. 20. By comparing the different observations of the French astronomers, Newton apprehends that 2 lines may be considered as the length a seconds Pendulum ought to be decreased at the equator.

From some observations made by Mr. Campbell, in 1731, in Black-river, in Jamaica, 18° north latitude, it is collected, that if the length of a simple Pendulum that swings seconds in London, be 39·126 English inches, the length of one at the equator would be 39·00, and at the poles 39·206. Philos. Trans. numb. 432; or Abr. vol. 8, part 1, pa. 238.

And hence Mr. Emerson has computed the following Table, shewing the length of a Pendulum that swings seconds at every 5th degree of latitude, as also the length of the degree of latitude there, in English miles.

Degrees of Lat.	Length of Pendulum.	Length of the Degree.
	inches.	miles.
0	39·027	68·723
5	39·029	68·730
10	39·032	68·750
15	39·036	68·783
20	39·044	68·830
25	39·057	68·882
30	39·070	68·950
35	39·084	69·020
40	39·097	69·097
45	39·111	69·176
50	39·126	69·256
55	39·142	69·330
60	39·158	69·401
65	39·168	69·467
70	39·177	69·522
75	39·185	69·568
80	39·191	69·601
85	39·195	69·620
90	39·197	69·628

6. If two Pendulums vibrate in similar arcs, the times of vibration are in the sub-duplicate ratio of their lengths. And the lengths of Pendulums vibrating in similar arcs, are in the duplicate ratio of the times of

of a vibration directly; or in the reciprocal duplicate ratio of the number of oscillations made in any one and the same time. For, the time of vibration t being as $p\sqrt{\frac{r}{2g}}$, where p and g are constant or given, therefore t is as $\sqrt{r}$, and r as t^2. Hence therefore the length of a half-second Pendulum will be $\frac{1}{4}r$ or $\frac{39\frac{1}{8}}{4} = 9{\cdot}781$ inches; and the length of the quarter-second Pendulum will be $\frac{1}{16}r = \frac{39\frac{1}{8}}{16} = 2{\cdot}445$ inches; and so of others.

7. The foregoing laws, &c, of the motion of Pendulums, cannot strictly hold good, unless the thread that sustains the ball be void of weight, and the gravity of the whole ball be collected into a point. In practice therefore, a very fine thread, and a small ball, but of a very heavy matter, are to be used. But a thick thread, and a bulky ball, disturb the motion very much; for in that case, the simple Pendulum becomes a compound one; it being much the same thing, as if several weights were applied to the same inflexible rod in several places.

8. M. Krafft in the new Petersburgh Memoirs, vols 6 and 7, has given the result of many experiments upon Pendulums, made in different parts of Russia, with deductions from them, from whence he derives this theorem: If x be the length of a Pendulum that swings seconds in any given latitude l, and in a temperature of 10 degrees of Reaumur's thermometer, then will the length of that Pendulum, for that latitude, be thus expressed, viz,

$$x = (439{\cdot}178 + 2{\cdot}321 \times \text{sin.}^2 l) \text{ lines of a French foot.}$$

And this expression agrees very nearly, not only with all the experiments made on the Pendulum in Russia, but also, with those of Mr. Graham, and those of Mr. Lyons in 79° 50′ north latitude, where he found its length to be 441·38 lines. See OBLATENESS.

Simple PENDULUM, in Mechanics, an expression commonly used among artists, to distinguish such Pendulums as have no provision for correcting the effects of heat and cold, from those that have such provision. Also Simple Pendulum, and Detached Pendulum, are terms sometimes used to denote such Pendulums as are not connected with any clock, or clock-work.

Compound PENDULUM, in Mechanics, is a Pendulum whose rod is composed of two or more wires or bars of metal. These, by undergoing different degrees of expansion and contraction, when exposed to the same heat or cold, have the difference of expansion or contraction made to act in such manner as to preserve constantly the same distance between the point of suspension, and centre of oscillation, although exposed to very different and various degrees of heat or cold. There are a great variety of constructions for this purpose; but they may be all reduced to the Gridiron, the Mercurial, and the Lever Pendulum.

It may be just observed by the way, that the vulgar method of remedying the inconvenience arising from the extension and contraction of the rods of common Pendulums, is by applying the bob, or small ball, with a screw, at the lower end; by which means the Pendulum is at any time made longer or shorter, as the ball is screwed downwards or upwards, and thus the time of its vibration is kept continually the same.

The *Gridiron* PENDULUM was the invention of Mr. John Harrison, a very ingenious artist, and celebrated for his invention of the watch for finding the difference of longitude at sea, about the year 1725; and of several other time-keepers and watches since that time; for all which he received the parliamentary reward of between 20 and 30 thousand pounds. It consists of 5 rods of steel, and 4 of brass, placed in an alternate order, the middle rod being of steel, by which the Pendulum ball is suspended; these rods of brass and steel, thus placed in an alternate order, and so connected with each other at their ends, that while the expansion of the steel rods has a tendency to lengthen the Pendulum, the expansion of the brass rods, acting upwards, tends to shorten it. And thus, when the lengths of the brass and steel rods are duly proportioned, their expansions and contractions will exactly balance and correct each other, and so preserve the Pendulum invariably of the same length. The simplicity of this ingenious contrivance is much in its favour; and the difficulty of adjustment seems the only objection to it.

Mr. Harrison in his first machine for measuring time at sea, applied this combination of wires of brass and steel, to prevent any alterations by heat or cold: and in the machines or clocks he has made for this purpose, a like method of guarding against the irregularities arising from this cause is used.

The *Mercurial* PENDULUM was the invention of the ingenious Mr. Graham, in consequence of several experiments relating to the materials of which Pendulums might be formed, in 1715. Its rod is made of brass, and branched towards its lower end, so as as to embrace a cylindric glass vessel 13 or 14 inches long, and about 2 inches diameter; which being filled about 12 inches deep with mercury, forms the weight or ball of the Pendulum. If upon trial the expansion of the rod be found too great for that of the mercury, more mercury must be poured into the vessel: if the expansion of the mercury exceeds that of the rod, so as to occasion the clock to go fast with heat, some mercury must be taken out of the vessel, so as to shorten the column. And thus may the expansion and contraction of the quicksilver in the glass be made exactly to balance the expansion and contraction of the Pendulum rod, so as to preserve the distance of the centre of oscillation from the point of suspension invariably the same.

Mr. Graham made a clock of this sort, and compared it with one of the best of the common sort, for 3 years together; when he found the errors of his but about one-eighth part of those of the latter. Philos. Trans. numb. 392.

The *Lever* PENDULUM. From all that appears concerning this construction of a Pendulum, we are inclined to believe that the idea of making the difference of the expansion of different metals operate by means of a lever, originated with Mr. Graham, who in the year 1737 constructed a Pendulum, having its rod composed of one bar of steel between two of brass, which acted upon the short end of a lever, to the other end of which, the ball or weight of the Pendulum was suspended.

This

This Pendulum however was, upon trial, found to move by jerks; and therefore laid aside by the inventor, to make way for the mercurial Pendulum, juſt mentioned.

Mr. Short informs us in the Philoſ. Tranſ. vol. 47, art. 88, that a Mr. Frotheringham, a quaker in Lincolnſhire, cauſed a Pendulum of this kind to be made: it conſiſted of two bars, one of braſs, and the other of ſteel, faſtened together by ſcrews, with levers to raiſe or let down the bulb; above which theſe levers were placed. M. Caſſini too, in the Hiſtory of the Royal Academy of Sciences at Paris, for 1741, deſcribes two ſorts of Pendulums for clocks, compounded of bars of braſs and ſteel, and in which he applies a lever to raiſe or let down the bulb of the Pendulum, by the expanſion or contraction of the bar of braſs.

Mr. John Ellicott alſo, in the year 1738, conſtructed a Pendulum on the ſame principle, but differing from Mr. Graham's in many particulars. The rod of Mr. Ellicott's Pendulum was compoſed of two bars only; the one of braſs, and the other of ſteel. It had two levers, each ſuſtaining its half of the ball or weight; with a ſpring under the lower part of the ball to relieve the levers from a conſiderable part of its weight, and ſo to render their motion more ſmooth and eaſy. The one lever in Mr. Graham's conſtruction was above the ball: whereas both the levers in Mr. Ellicott's were within the ball; and each lever had an adjuſting ſcrew, to lengthen or ſhorten the lever, ſo as to render the adjuſtment the more perfect. See the Philoſ. Tranſ. vol. 47, p. 479; where Mr. Ellicott's methods of conſtruction are deſcribed, and illuſtrated by figures.

Notwithſtanding the great ingenuity diſplayed by theſe very eminent artiſts on this conſtruction, it muſt farther be obſerved, in the hiſtory of improvements of this nature, that Mr. Cumming, another eminent artiſt, has given, in his Eſſays on the Principles of Clock and Watch-work, Lond. 1766, an ample deſcription, with plates, of a conſtruction of a Pendulum with levers, in which it ſeems he has united the properties of Mr. Graham's and Mr. Ellicott's, without being liable to any of the defects of either. The rod of this Pendulum is compoſed of one flat bar of braſs, and two of ſteel; he uſes three levers within the ball of the Pendulum; and, among many other ingenious contrivances, for the more accurate adjuſting of this Pendulum to mean time, it is provided with a ſmall ball and ſcrew below the principal ball or weight, one entire revolution of which on its ſcrew will only alter the rate of the clock's going one ſecond per day; and its circumference is divided into 30, one of which diviſions will therefore alter its rate of going one ſecond in a month.

Pendulum *Clock*, is a clock having its motion regulated by the vibration of a Pendulum.

It is controverted between Galileo and Huygens, which of the two firſt applied the Pendulum to a clock. For the pretenſions of each, ſee Clock.

After Huygens had diſcovered, that the vibration made in arcs of a cycloid, however unequal they might be in extent, were all equal in time; he ſoon perceived, that a Pendulum applied to a clock, ſo as to make it deſcribe arcs of a cycloid, would rectify the otherwiſe unavoidable irregularities of the motion of the clock; ſince, though the ſeveral cauſes of thoſe irregularities ſhould occaſion the Pendulum to make greater or ſmaller vibrations, yet, by virtue of the cycloid, it would ſtill make them perfectly equal in point of time; and the motion of the clock governed by it, would therefore be preſerved perfectly equable. But the difficulty was, how to make the Pendulum deſcribe arcs of a cycloid; for naturally the Pendulum, being tied to a fixed point, can only deſcribe circular arcs about it.

Here M. Huygens contrived to fix the iron rod or wire, which bears the ball or weight, at the top to a ſilken thread, placed between two cycloidal cheeks, or two little arcs of a cycloid, made of metal. Hence the motion of vibration, applying ſucceſſively from one of thoſe arcs to the other, the thread, which is extremely flexible, eaſily aſſumes the figure of them, and by that means cauſes the ball or weight at the bottom to deſcribe a juſt cycloidal arc.

This is doubtleſs one of the moſt ingenious and uſeful inventions many ages have produced: by means of which it has been aſſerted there have been clocks that would not vary a ſingle ſecond in ſeveral days: and the ſame invention alſo gave riſe to the whole doctrine of involute and evolute curves, with the radius and degree of curvature, &c.

It is true, the Pendulum is ſtill liable to its irregularities, how minute ſoever they may be. The ſilken thread by which it was ſuſpended, ſhortens in moiſt weather, and lengthens in dry; by which means the length of the whole Pendulum, and conſequently the times of the vibrations, are ſomewhat varied.

To obviate this inconvenience, M. De la Hire, inſtead of a ſilken thread, uſed a little fine ſpring; which was not indeed ſubject to ſhorten and lengthen, from thoſe cauſes; yet he found it grew ſtiffer in cold weather, and then made its vibrations faſter than in warm; to which alſo we may add its expanſion and contraction by heat and cold. He therefore had recourſe to a ſtiff wire or rod, firm from one end to the other. Indeed by this means he renounced the advantages of the cycloid; but he found, as he ſays, by experience, that the vibrations in circular arcs are performed in times as equal, provided they be not of too great extent, as thoſe in cycloids. But the experiments of Sir Jonas Moore, and others, have demonſtrated the contrary.

The ordinary cauſes of the irregularities of Pendulums Dr. Derham aſcribes to the alterations in the gravity and temperature of the air, which increaſe and diminiſh the weight of the ball, and by that means make the vibrations greater and leſs; an acceſſion of weight in the ball being found by experiment to accelerate the motion of the Pendulum; for a weight of 6 pounds added to the ball, Dr. Derham found made his clock gain 13 ſeconds every day.

A general remedy againſt the inconveniences of Pendulums, is to make them long, the ball heavy, and to vibrate but in ſmall arcs. Theſe are the uſual means employed in England; the cycloidal cheeks being generally neglected. See the foregoing article.

Pendulum clocks reſting againſt the ſame rail have been found to influence each other's motion. See the Philoſ. Tranſ. numb. 453, ſect. 5 and 6, where Mr. Ellicott has given a curious and exact account of this phenomenon.

PENDULUM *Royal*, a name ufed among us for a clock, whofe Pendulum fwings feconds, and goes 8 days without winding up; fhewing the hour, minute, and fecond. The numbers in fuch a piece are thus calculated. Firft caft up the feconds in 12 hours, which are the beats in one turn of the great wheel; and they will be found to be 43200 = 12 × 60 × 60. The fwing wheel muft be 30, to fwing 60 feconds in one of its revolutions; now let the half of 43200, viz 21600, be divided by 30, and the quotient will be 720, which muft be feparated into quotients. The firft of thefe muft be 12, for the great wheel, which moves round once in 12 hours. Now 720 divided by 12, gives 60, which may alfo be conveniently broken into two quotients, as 10 and 6, or 12 and 5, or 8 and $7\frac{1}{2}$; which laft is moft convenient: and if the pinions be all taken 8, the work will ftand thus:

$$\begin{array}{r}
8\ )\ 96\ (\ 12 \\
8\ )\ 64\ (\ \ 8 \\
8\ )\ 60\ (\ 7\frac{1}{2} \\
\hline
30
\end{array}$$

According to this computation, the great wheel will go round once in 12 hours, to fhew the hour; the next wheel once in an hour, to fhew the minutes; and the fwing-wheel once in a minute, to fhew the feconds. See CLOCK-WORK.

Balliftic PENDULUM. See BALLISTIC *Pendulum*.

Level PENDULUM. See LEVEL.

PENDULUM *Watch*. See WATCH.

PENETRABILITY, capability of being penetrated. See IMPENETRABILITY.

PENETRATION, the act by which one thing enters another, or takes up the place already poffeffed by another.

The fchoolmen define Penetration the co-exiftence of two or more bodies, fo that one is prefent, or has its extenfion in the fame place as the other.

Moft philofophers hold the penetration of bodies abfurd, i. e. that two bodies fhould be at the fame time in the fame place; and accordingly impenetrability is laid down as one of the effential properties of matter.

What is popularly called Penetration, only amounts to the matter of one body's being admitted into the vacuity of another. Such is the Penetration of water through the fubftance of gold.

PENINSULA, is a portion or extent of land which is almoft furrounded with water, being joined to the continent only by an ifthmus, or narrow neck. Such is Africa, the greateft Peninfula in the world, which is joined to Afia, by the neck at the end of the Red Sea; fuch alfo is Peloponnefus, or the Morea, joined to Greece: and Jutland, &c. Peninfula is the fame with what is otherwife called Cherfonefus.

PENNY, formerly a piece of filver coin, but now an imaginary fum, equal to two copper coins called a halfpenny.

The Penny was the firft filver coin ftruck in England by our Saxon anceftors, being the 240th part of their pound, and its true weight was about $22\frac{1}{2}$ grains Troy.

In Etheldred's time, the Penny was the 20th part of the Troy ounce, and equal in weight to our three pence; which value it retained till the time of Edward the Third.

Till the time of King Edward the Firft, the Penny was ftruck with a crofs fo deeply funk in it, that it might, on occafion, be eafily broken, and parted into two halves, thence called Halfpennies; or into four, thence called Fourthings, or Farthings. But that Prince coined it without the crofs; inftead of which he ftruck round Halfpence and Farthings. Though there are faid to be inftances of fuch round Halfpence having been made in the reign of Henry the Firft, if not alfo in that of the two Williams.

Edward the Firft alfo reduced the weight of the Penny to a ftandard; ordering that it fhould weigh 32 grains of wheat, taken out of the middle of the ear. This Penny was called the Penny Sterling; and 20 of them were to weigh an ounce; whence the Penny became a weight as well as a coin.

By the 9th of Edward the Third, it was diminifhed to the 26th part of the Troy ounce; by the 2d of Henry the Sixth it was the 32d part; by the 5th of Edward the Fourth, it became the 40th, and alfo by the 36th of Henry the Eighth, and afterwards, the 45th; but by the 2d of Elizabeth, 60 Pence were coined out of the ounce, and during her reign 62, which laft proportion is ftill obferved in our times.

The Penny Sterling is now difufed as a coin; and fcarce fubfifts, but as a money of account, containing two copper Halfpence, or the 12th part of a fhilling, or the 240th part of a pound.

The French Penny, or Denier, is of two kinds; the Paris Penny, called Denier Parifis; and the Penny of Tours, called Denier Tournois.

The Dutch Penny, called Pennink, or Pening, is a real money, worth about one-fifth more than the French Penny Tournois. The Pennink is alfo ufed as a money of account, in keeping books by pounds, florins, and patards; 12 Penninks make the patard, and 20 patards the florin.

At Hamburg, Nuremberg, &c, the Penny or Pfennig of account is equal to the French Penny Tournois. Of thefe, 8 make the krieuk; and 60 the florin of thofe cities; alfo 90 the French crown, or 4s 6d fterling.

PENNY-*Weight*, a Troy weight, being the 20th part of an ounce, containing 24 grains; each grain weighing a grain of wheat gathered out of the middle of the ear, well dried. The name took its rife from its being actually the weight of one of our ancient filver Pennies. See the foregoing article.

PENTAGON, in Geometry, a plane figure confifting of five angles, and confequently five fides alfo. If the angles be all equal, it is a regular Pentagon.

It is a remarkable property of the Pentagon, that its fide is equal in power to the fides of a hexagon and a decagon infcribed in the fame circle; that is, the fquare of the fide of the Pentagon, is equal to both the fquares taken together of the fides of the other two figures; and confequently thofe three fides will confti-

tute a right-angled triangle. Euclid, book 13, prop. 10.

Pappus has alfo demonftrated, that 12 regular Pentagons contain more than 20 triangles infcribed in the fame circle; lib. 5, prop. 45.

The dodecahedron, which is the fourth regular body or folid, is contained under 12 equal and regular Pentagons.

To find the Area of a Regular PENTAGON. Multiply the fquare of its fide by 1·7204774, or by $\frac{5}{4}$ of the tangent of 54°, or by $\frac{5}{4}\sqrt{1 + \frac{2}{5}\sqrt{5}}$. Hence if s denote the fide of the Pentagon, its area will be $1{\cdot}7204774s^2 = \frac{5}{4}s^2\sqrt{1 + \frac{2}{5}\sqrt{5}} = \frac{5}{4}s^2 \times \text{tang. } 54°$.

PENTAGRAPH, otherwife called a Parallelogram, a mathematical inftrument for copying defigns, prints, plans, &c, in any proportion.

The common Pentagraph (Plate xix, fig. 2) confifts of four rulers or bars, of metal or wood, two of them from 15 to 18 inches long, the other two half that length. At the ends, and in the middle, of the long rulers, as alfo at the ends of the fhorter ones, are holes upon the exact fixing of which the perfection of the inftrument chiefly depends. Thofe in the middle of the long rulers are to be at the fame diftance from thofe at the end of the long ones, and thofe of the fhort ones; fo that, when put together, they may always make a parallelogram.

The inftrument is fitted together for ufe, by feveral little pieces, particularly a little pillar, number 1, having at one end a nut and fcrew, joining the two long rulers together; and at the other end a fmall knot for the inftrument to flide on. The piece numb. 2 is a rivet with a fcrew and nut by which each fhort ruler is faftened to the middle of each long one. The piece numb. 3 is a pillar, one end of which, being hollowed into a fcrew, has a nut fitted to it; and at the other end is a worm to fcrew into the table; when the inftrument is to be ufed, it joins the ends of the two fhort rulers. The piece numb. 4 is a pen, or pencil, or portcrayon, fcrewed into a little pillar. Laftly, the piece numb. 5 is a brafs point, moderately blunt, fcrewed likewife into a little pillar.

Ufe of the PENTAGRAPH.—1. To copy a defign in the fame fize or fcale as the original. Screw the worm numb. 3 into the table; lay a paper under the pencil numb. 4, and the defign under the point numb. 5. This done, conducting the point over the feveral lines and parts of the defign, the pencil will draw or repeat the fame on the paper.

2. When the defign is to be reduced — ex. gr. to half the fcale; the worm muft be placed at the end of the long ruler numb. 4, and the paper and pencil in the middle. In this fituation conduct the brafs point over the feveral lines of the defign, as before; and the pencil at the fame time will draw its copy in the proportion required; the pencil here only moving half the lengths that the point moves.

3. On the contrary, when the defign is to be enlarged to a double fize; the brafs point, with the defign, muft be placed in the middle at numb. 3, the pencil and paper at the end of the long ruler, and the worm at the other end.

4. To reduce or enlarge in other proportions, there are holes drilled at equal diftances on each ruler; viz, all along the fhort ones, and half way of the long ones, for placing the brafs point, pencil, and worm, in a right line in them; i. e. if the piece carrying the point be put in the third hole, the other two pieces muft be put each in its third hole; &c.

PENTANGLE, a plane figure of five angles, or the fame as the PENTAGON.

PENUMBRA, in Aftronomy, a faint or partial fhade, in an eclipfe, obferved between the perfect fhadow, and the full light.

The Penumbra arifes from the magnitude of the fun's body: were he only a luminous point, the fhadow would be all perfect; but by reafon of the diameter of the fun it happens, that a place which is not illuminated by the whole body of the fun, does yet receive rays from fome part of it.

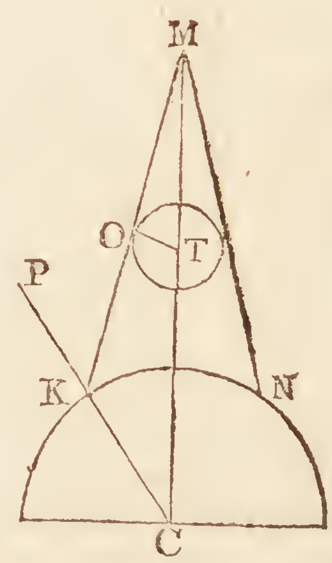
Thus, fuppofe S the fun, and T the moon, and the fhadow of the latter projected on a plane, as GH (Plate xix, fig. 3). The true proper fhadow of T, viz GH, will be encompaffed with an imperfect fhadow, or Penumbra, HL and GE, each portion of which is illuminated by an entire hemifphere of the fun.

The degree of light or fhade of the Penumbra, will be more or lefs in different parts, as thofe parts lie open to the rays of a greater or lefs part of the fun's body; thus from L to H, and from E to G, the light continually diminifhes; and in the confines of G and H, the Penumbra is darkeft, and becomes loft and confounded with the total fhade: as near E and L it is thin and confounded with the total light.

A Penumbra muft be found in all eclipfes, whether of the fun, the moon, or the other planets, primary or fecondary; but it is moft confiderable with us in eclipfes of the fun; which is the cafe here referred to.

The Penumbra extends infinitely in length, and grows ftill wider and wider; two rays drawn from the two extremities of the earth's diameter, and which proceed always diverging, form its two edges; all that infinite diverging fpace, included between lines paffing through E and L, is the Penumbra, except the cone of the fhadow in the middle of it.

To determine how much of the furface of the earth can be involved in the Penumbra, let the apparent femidiameter of the fun be fuppofed the greateft, or about 16′ 20″, which is when the earth is in her perihelion; alfo let the moon be in her apogee, and therefore at her greateft diftance from the earth, or about 64 of the earth's femidiameters. Let KNC be the earth, T the moon, and MKN the Penumbra, involving the part of the earth from K to N, which it is required to find.



Here then are given the angle KMC = 16′ 20″, TC = 64, KC = 1, and OT = $\frac{11}{40}$ of KC. Hence, in the right-angled triangle OTM, as fin. OMT : radius :: OT : TM = $210\frac{1}{2}$OT = 58KC nearly. Therefore MC = MT + TC = 58 + 64 = 122 femidiameters of the earth. Then, in the triangle KMC, there are given KC =

KC = 1, and MC = 122, also the angle KMC = 16′ 20″, to find the angle C; thus, as

KC : MC :: sin. ∠ KMC : sin. ∠ MKP = 35° 25′ 35″;
from this take the ∠ KMC - - 0 16 20,
leaves the ∠ C - - - 35 9 11,
the double of which is the arc KN 70 18 22,
or nearly a space of 4866 miles in diameter.

PERAMBULATOR, an instrument for measuring distances; called also Pedometer, Waywiser, and Surveying Wheel.

This wheel is contrived to measure out a pole, or $16\frac{1}{2}$ feet, in making two revolutions; consequently its circumference is $8\frac{1}{4}$ feet, and its diameter 2·626 feet, or 2 feet $5\frac{1}{2}$ inches and $\frac{11}{1000}$ parts, very nearly. It is either driven forward by two handles, by a person walking; or is drawn by a coach wheel, &c, to which it is attached by a pole. It contains various movements, by wheels, or clock-work, with indices on its face, which is like that of a clock, to point out the distance passed over, in miles, furlongs, poles, yards, &c.

Its advantages are its readiness and expedition; being very useful for measuring roads, and great distances on level ground. See the fig. Plate xvii, fig. 6.

PERCH, in Surveying, a square measure, being the 40th part of a rood, or the 160th part of an acre; that is, the square of a pole or rod, of the length of $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet.

Perch is by some also made to mean a measure of length; being the same as the rod or pole of $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet long. But it is better, for preventing confusion, to distinguish them.

PERCUSSION, in Physics, the impression a body makes in falling or striking upon another; or the shock or collision of two bodies, which meeting alter each other's motion.

Percussion is either Direct or Oblique. It is also either of Elastic or Nonelastic bodies, which have each their different laws. It is true, we know of no bodies in nature that are either perfectly elastic or the contrary; but all partaking that property in different degrees; even the hardest and the softest being not entirely divested of it. But, for the sake of perspicuity, it is usual, and proper, to treat of these two separately and apart.

Direct Percussion is that in which the impulse is made in the direction of a line perpendicular at the place of impact, and which also passes through the common centre of gravity of the two striking bodies. As is the case in two spheres, when the line of the direction of the stroke passes through the centres of both spheres; for then the same line, joining their centres, passes perpendicularly through the point of impact. And

Oblique Percussion, is that in which the impulse is made in the direction of a line that does not pass through the common centre of gravity of the striking bodies; whether that line of direction is perpendicular to the place of impact, or not.

The force of Percussion is the same as the momentum, or quantity of motion, and is represented by the product arising from the mass or quantity of matter moved, multiplied by the velocity of its motion; and that without any regard to the time or duration of action; for its action is considered totally independent of time, or but as for an instant, or an infinitely small time.

This consideration will enable us to resolve a question that has been greatly canvassed among philosophers and mathematicians, viz, what is the relation between the force of Percussion and mere pressure or weight? For we hence infer, that the former force is infinitely, or incomparably, greater than the latter. For, let M denote any mass, body, or weight, having no motion or velocity, but simply its pressure; then will that pressure or force be denoted by M itself, if it be considered as acting for some certain finite assignable time; but, considered as a force of Percussion, that is, as acting but for an infinitely small time, its velocity being 0, or nothing, its percussive force will be 0 × M, that is 0, or nothing; and is therefore less than any the smallest percussive force whatever. Again, let us consider the two forces, viz, of Percussion and pressure, with respect to the effects they produce: Now the intensity of any force is very well measured and estimated by the effect it produces in a given time: But the effect of the pressure M, in 0 time, or an infinitely small time, is nothing at all; that is, it will not, in an infinitely small time, produce, for example, any motion, either in itself, or in any other body: its intensity therefore, as its effect, is infinitely less than any the smallest force of Percussion. It is true, indeed, that we see motion and other considerable effects produced by mere pressure, and to counteract which it will require the opposition of some considerable percussive force: but then it must be observed, that the former has been an infinitely longer time than the latter in producing its effect; and it is no wonder in mathematics that an infinite number of infinitely small quantities makes up a finite one. It has therefore only been for want of considering the circumstance of *time*, that any question could have arisen on this head. Hence the two forces are related to each other, only as a surface is to a solid or body: by the motion of the surface through an infinite number of points, or through a finite right line, a solid or body is generated: and by the action of the pressure for an infinite number of moments, or for some finite time, a quantity equal to a given percussive force is generated: but the surface itself is infinitely less than any solid, and the pressure infinitely less than any percussive force. This point may be easily illustrated by some familiar instances, which prove at least the enormous disproportion between the two forces, if not also their absolute incomparability. And first, the blow of a small hammer, upon the head of a nail, will drive the nail into a board; when it is hard to conceive any weight so great as will produce a like effect, i. e. that will sink the nail as far into the board, at least unless it is left to act for a very considerable time: and even after the greatest weight has been laid as a pressure on the head of the nail, and has sunk it as far as it can as to sense, by remaining for a long time there without producing any farther sensible effect; let the weight be removed from the head of the nail, and instead of it, let it be struck a small blow with a hammer, and the nail will immediately sink farther into the wood. Again, it is also well known, that a ship-carpenter, with a blow of his mallet, will drive a wedge in below the greatest ship whatever, lying aground, and so overcome her weight, and lift her up. Lastly, let us consider a man with a club to strike a small ball, upwards or in any

any other direction; it is evident that the ball will acquire a certain determinate velocity by the blow, suppose that of 10 feet per second, or minute, or any other time whatever: now it is a law, universally allowed in the communication of motion, that when different bodies are struck with equal forces, the velocities communicated are reciprocally as the weights of the bodies that are struck; that is, that a double body, or weight, will acquire half the velocity from an equal blow; a body 10 times as great, one 10th of the velocity; a body 100 times as great, the 100th part of the velocity; a body a million times as great, the millionth part of the velocity; and so on without end: from whence it follows, that there is no body or weight, how great soever, but will acquire some finite degree of velocity, and be overcome, by any given small finite blow, or Percussion.

It appears that Des Cartes, first of any, had some ideas of the laws of Percussion; though it must be acknowledged, in some cases perhaps wide of the truth. The first who gave the true laws of motion in nonelastic bodies, was Doctor Wallis, in the Philos. Trans. numb. 43, where he also shews the true cause of reflections in other bodies, and proves that they proceed from their elasticity. Not long after, the celebrated Sir Christopher Wren and Mr. Huygens imparted to the Royal Society the laws that are observed by perfectly elastic bodies, and gave exactly the same construction, though each was ignorant of what the other had done. And all those laws, thus published in the Philos. Trans. without demonstration, were afterwards demonstrated by Dr. Keill, in his Philos. Lect. in 1700; and they have since been followed by a multitude of other authors.

In Percussion, we distinguish at least three several sorts of bodies; the perfectly hard, the perfectly soft, and the perfectly elastic. The two former are considered as utterly void of elasticity; having no force to separate them, or throw them off from each other again, after collision; and therefore either remaining at rest, or else proceeding uniformly forward together as one body or mass of matter.

The laws of Percussion therefore to be considered, are of two kinds: those for elastic, and those for nonelastic bodies.

The one only general principle, for determining the motions of bodies from Percussion, and which belongs equally to both the sorts of bodies, i. e. both the elastic and nonelastic, is this: viz, that there exists in the bodies the same momentum, or quantity of motion, estimated in any one and the same direction, both before the stroke and after it. And this principle is the immediate result of the third law of nature or motion, that reaction is equal to action, and in a contrary direction; from whence it happens, that whatever motion is communicated to one body by the action of another, exactly the same motion doth this latter lose in the same direction, or exactly the same does the former communicate to the latter in the contrary direction.

From this general principle too it results, that no alteration takes place in the common centre of gravity of bodies by their actions upon one another; but that the said common centre of gravity perseveres in the same state, whether of rest or of uniform motion, both before and after the shock of the bodies.

Now, from either of these two laws, viz, that of the preservation of the same quantity of motion, in one and the same direction, and that of the preservation of the same state of the centre of gravity, both before and after the shock, all the circumstances of the motions of both the kinds of bodies after collision may be made out; in conjunction with their own peculiar and separate constitutions, namely, that of the one sort being elastic, and the other nonelastic.

The effects of these different constitutions, here alluded to, are these; that nonelastic bodies, on their shock, will adhere together, and either remain at rest, or else move together as one mass with a common velocity; or if elastic, they will separate after the shock with the very same relative velocity with which they met and shocked. The former of these consequences is evident, viz, that nonelastic bodies keep together as one mass after they meet; because there exists no power to separate them; and without a cause there can be no effect. And the latter consequence results immediately from the very definition and essence of elasticity itself, being a power always equal to the force of compression, or shock; and which restoring force therefore, acting the contrary way, will generate the same relative velocity between the bodies, or the same quantity of matter, as before the shock, and the same motion also of their common centre of gravity.

To apply now the general principle to the determination of the motions of bodies after their shock; let B and b be any two bodies, and V and v their respective velocities, estimated in the direction AD; which quantities V and v will be both positive if the bodies both move towards D, but one of them as v will be negative if the body b move towards A, and v will be $= 0$ if the body b be at rest. Hence then BV is the momentum of B towards D, and bv is the momentum of b towards D, whose sum is $BV + bv$, which is the whole quantity of motion in the direction AD, and which momentum must also be preserved after the shock.

Now if the bodies have no elasticity, they will move together as one mass $B + b$ after they meet, with some common velocity, which call y, in the direction AD; therefore the momentum in that direction after the shock, being the product of the mass and velocity, will be $(B + b) \times y$. But the momenta, in the same direction, before and after the impact, are equal, that is $BV + bv = (B + b)y$; from which equation any one of the quantities may be determined when the rest are given. So, if we would find the common velocity after the stroke, it will be $y = \frac{BV + bv}{B + b}$, equal to the sum of the momenta divided by the sum of the bodies; which is also equal to the velocity of the common centre of gravity of the two bodies, both before and after the collision. The signs of the terms, in this value of y, will be all positive, as above,

above, when the bodies move both the fame way AD; but one term bv muft be made negative when the motion of b is the contrary way; and that term will be abfent or nothing, when b is at reft, before the fhock.

Again, for the cafe of elaftic bodies, which will feparate after the ftroke, with certain velocities, x and z, viz, x the velocity of B, and z the velocity of b after the collifion, both eftimated in the direction AD, which quantities will be either pofitive, or negative, or nothing, according to the circumftances of the maffes B and b, with thofe of their celerities before the ftroke. Hence then Bx and bz are the feparate momenta after the fhock, and $Bx + bz$ their fum, which muft be equal to the fum $BV + bv$ in the fame direction before the ftroke: alfo $z - x$ is the relative velocity with which the bodies feparate after the blow, and which muft be equal to $V - v$ the fame with which they meet; or, which is the fame thing, that $V + x = v + z$; that is, the fum of the two velocities of the one body, is equal to the fum of the velocities of the other, taken before and after the ftroke; which is another notable theorem. Hence then, for determining the two unknown quantities x and z, there are thefe two equations,

$$\text{viz, } BV + bv = Bx + bz,$$
$$\text{and } V - v = z - x;$$
$$\text{or } V + x = v + z;$$

the refolution of which equations gives thofe two velocities as below,

$$\text{viz, } x = \frac{2bv + (B - b)V}{B + b},$$
$$\text{and } z = \frac{2BV - (B - b)v}{B + b}.$$

From thefe general values of the velocities, which are to be underftood in the direction AD, any particular cafes may eafily be drawn. As, if the two bodies B and b be equal, then $B - b = 0$, and $B + b = 2B$, and the two velocities in that cafe become, after impulfe, $x = v$, and $z = V$, the very fame as they were before, but changed to the contrary bodies, i. e. the bodies have taken each other's velocity that it had before, and with the fame fign alfo. So that, if the equal bodies were before both moving the fame way, or towards D, they will do the fame after, but with interchanged velocities. But if they before moved contrary ways, B towards D, and b towards A, they will rebound contrary ways, B back towards A, and b towards D, each with the other's velocity. And, laftly, if one body, as b, were at reft before the ftroke, then the other B will be at reft after it, and b will go on with the motion that B had before. And thus may any other particular cafes be deduced from the firft general values of x and z.

We may now conclude this article with fome remarks on thefe motions, and the miftakes of fome authors concerning them. And firft, we obferve this ftriking difference between the motions that are communicated by elaftic and by nonelaftic bodies, viz, that a nonelaftic body, by ftriking, communicates to the body it ftrikes, exactly its whole momentum; as is evident. But the ftroke of an elaftic body may either communicate its whole motion to the body it ftrikes, or it may communicate only a part of it, or it may even communicate more than it had. For, if the ftriking body remain at reft after the ftroke, it has juft loft all its motion, and therefore has communicated all it had; but if it ftill move forward in the fame direction, it has ftill fome motion left in that direction, and therefore has only communicated a part of what motion it had; and if the ftriking body rebound back, and move in the contrary direction, the other body has received not only the whole of the motion that the firft had, but alfo as much more as the firft has acquired in the contrary direction.

It has been denied by fome authors, and in the Encyclopédie, that the fame quantity of motion remains after the fhock, as before it; and hence they feize an opportunity to reprehend the Cartefians for making that affertion, which they do, not only with refpect to the cafe of two bodies, but alfo of all the bodies in the whole univerfe. And yet nothing is more true, if the motion be confidered as eftimated always in one and the fame direction, efteeming that as negative, which is in the contrary or oppofite direction. For it is a general law of nature, that no motion, nor force, can be generated, nor deftroyed, nor changed, but by fome caufe which muft produce an equal quantity in the oppofite direction. And this being the cafe in one body, or two bodies, it muft neceffarily be the cafe in all bodies, and in the whole folar fyftem, fince all bodies act upon one another. And hence alfo it is manifeft, that the common centre of gravity of the whole folar fyftem muft always preferve its original condition, whether it be of reft or of uniform motion; fince the ftate of that centre is not changed by the mutual actions of bodies upon one another, any more than their quantity of motion, in one and the fame direction.

What may have led authors into the miftake above alluded to, which they bring no proof of, feems to be the difcovery of M. Huygens, that the fums of the two products are equal, both before and after the fhock, that are made by multiplying each body by the fquare of its velocity, viz, that $BV^2 + bv^2 = Bx^2 + bz^2$, where V and v are the velocities before the fhock, and x and z the velocities after it. Such an expreffion, namely the product of the mafs by the fquare of the velocity, is called the vis viva, or living force; and hence it has been inferred that the whole vis viva before the fhock, or $BV^2 + bv^2$, is equal to that after the ftroke, or $Bx^2 + bz^2$; which is indeed very true, as will be fhewn prefently. But when they hence infer, both that therefore the forces of bodies in motion are as the fquares of the velocities, and that there is not the fame quantity of motion between the two ftriking bodies, both before and after the fhock, they are grofsly miftaken, and thereby fhew that they are ignorant of the true derivation of the equation $BV^2 + bv^2 = Bx^2 + bz^2$. For this equation is only a confequence of the very principle above laid down, and which is not acceded to by thofe authors, viz, that the quantity of motion is the fame before and after the fhock, or that $BV + bv = Bx + bz$, the truth of which laft equation they deny, becaufe they think the former one is true, never dreaming that they may be both true, and much lefs that the one is a confequence of the other, and derived from it; which however is now found to be the cafe, as is proved in this manner:

It has been fhewn that the fum of the two momenta, in

in the ſame direction, before and after the ſtroke, are equal, or that $BV + bv = Bx + bz$; and alſo that the ſum of the two velocities of the one body, is equal to the ſum of thoſe of the other, or that $V + x = v + z$; and it is now propoſed to ſhew that from theſe two equations there reſults the third equation $BV^2 + bv^2 = Bx^2 + bz^2$, or the equation of the living forces.

Now becauſe $BV + bx = Bx + bz$, by tranſpoſition it is - $BV - Bz = bz - bv$; which ſhews that the difference between the two momenta of the one body, before and after the ſtroke, is equal to the difference between thoſe of the other body; which is another notable theorem. But now, to derive the equation of the vis viva, ſet down the two foregoing equations, and multiply them together, ſo ſhall the products give the ſaid equation required; thus

Mult. $BV - Bx = bz - bv$, the equat. of the momenta, by $\underline{V + x = z + v}$, the equat. of the velocities,

produc. $BV^2 - Bx^2 = bz^2 - bv^2$,

or $BV^2 + bv^2 = Bx^2 + bz^2$,

the very equation of the vis viva required. Which was to be proved.

When the elaſticity of the bodies is not perfect, but only partially ſo, as is the caſe with all the bodies we know of, the determination of the motions after colliſion may be determined in a ſimilar manner. See Keill's Lect. Philoſ. lect. 14, theor. 29, at the end. And for the geometrical determinations after impact, ſee the article Collision.

Centre of Percussion, is the point in which the ſhock or impulſe of a body which ſtrikes another is the greateſt that it can be. See Centre.

The Centre of Percuſſion is the ſame as the centre of oſcillation, when the ſtriking body moves round a fixed axis. See Oscillation.

But if all the parts of the ſtriking body move with a parallel motion, and with the ſame velocity, then the Centre of Percuſſion is the ſame as the centre of gravity.

PERFECT Number, is one that is equal to the ſum of all its aliquot parts, when added together. Eucl. lib. 7, def. 22. As the number 6, which is $= 1 + 2 + 3$, the ſum of all its aliquot parts; alſo 28, for $28 = 1 + 2 + 4 + 7 + 14$, the ſum of all its aliquot parts.

It is proved by Euclid, in the laſt prop. of book the 9th, that if the common geometrical ſeries of numbers 1, 2, 4, 8, 16, 32, &c, be continued to ſuch a number of terms, as that the ſum of the ſaid ſeries of terms ſhall be a prime number, then the product of this ſum by the laſt term of the ſeries will be a perfect number.

This ſame rule may be otherwiſe expreſſed thus: If n denote the number of terms in the given ſeries 1, 2, 4, 8, &c; then it is well known that the ſum of all the terms of the ſeries is $2^n - 1$, and it is evident that the laſt term is 2^{n-1}: conſequently the rule becomes thus, viz, $2^{n-1} \times \overline{2^n - 1}$ = a perfect number, whenever $2^n - 1$ is a prime number.

Now the ſums of one, two, three, four, &c, terms of the ſeries 1, 2, 4, 8, &c, form the ſeries 1, 3, 7, 15, 31, &c; ſo that the number will be found perfect whenever the correſponding term of this ſeries is a prime, as 1, 3, 7, 31, &c. Whence the table of perfect numbers may be found and exhibited as follows; where the 1ſt column ſhews the number of terms, or the value of n; the 2d column is the laſt term of the ſeries 1, 2, 4, 8, &c, and is expreſſed by 2^{n-1}; the 3d column contains the correſponding ſums of the ſaid ſeries, or the values of the quantity $2^n - 1$; which numbers in this 3d column are eaſily conſtructed by adding always the laſt number in this column to the next following number in the 2d column: and laſtly, the 4th column ſhews the correſpondent Perfect Numbers, or the values of $2^{n-1} \times \overline{2^n - 1}$, the product of the numbers in the 2d and 3d columns, when $2^n - 1$, or the number in the 3d column, is a prime number; the products in the other caſes being omitted, as not Perfect Numbers.

Values of n	Values of 2^{n-1}	Values of $2^n - 1$	Perf. Numbers, or $2^{n-1} \times (2^n - 1)$
1	1	1	1
2	2	3	6
3	4	7	28
4	8	15	·
5	16	31	496
6	32	63	·
7	64	127	8128

Hence the firſt four Perfect Numbers are found to be 6, 28, 496, 8128; and thus the table might be continued to find others, but the trouble would be very great, for want of a general method to diſtinguiſh which numbers are primes, as the caſe requires. Several learned mathematicians have endeavoured to facilitate this buſineſs, but hitherto with only a ſmall degree of perfection. After the foregoing four Perfect Numbers, there is a long interval before any more occur. The firſt eight are as follow, with the factors and products which produce them:

The firſt Perfect Numbers.	Their values.
6 - -	$= (2^2 - 1)\,2$
28 - -	$= (2^3 - 1)\,2^2$
496 - -	$= (2^5 - 1)\,2^4$
8128 - -	$= (2^7 - 1)\,2^6$
33550336 - -	$= (2^{13} - 1)\,2^{12}$
8589869056 - -	$= (2^{17} - 1)\,2^{16}$
137438691328 -	$= (2^{19} - 1)\,2^{18}$
2305843008139952128	$= (2^{31} - 1)\,2^{30}$

See ſeveral conſiderable tracts on the ſubject of Perfect Numbers in the Memoirs of the Peterſburgh Academy, vol. 2 of the new vols, and in ſeveral other volumes.

PERIŒCI. See Perioeci.

PERIGÆUM, or Perigee, is that point of the orbit of the ſun or moon, which is the neareſt to the earth. In which ſenſe it ſtands oppoſed to Apogee, which is the moſt diſtant point from the earth.

PERIGEE, in the Ancient Aftronomy, denotes a point in a planet's orbit, where the centre of its epicycle is at the leaft diftance from the earth.

PERIHELION, PERIHELIUM, that point in the orbit of a planet or comet which is neareft to the fun. In which fenfe it ftands oppofed to Aphelion, or Aphelium, which is the higheft or moft diftant point from the fun.

Inftead of this term, the Ancients ufed Perigeum; becaufe they placed the earth in the centre.

PERIMETER, in Geometry, the ambit, limit, or outer bounds of a figure; being the fum of all the lines by which it is inclofed or formed.

In circular figures, &c, inftead of this term, the word circumference or periphery is ufed.

PERIOD, in Aftronomy, the time in which a ftar or planet makes one revolution, or returns again to the fame point in the heavens.

The fun's, or properly the earth's tropical period, is 365 days 5 hours 48 minutes 45 feconds 30 thirds. That of the moon is 27 days 7 hours 43 minutes. That of the other planets as below.

There is a wonderful harmony between the diftances of the planets from the fun, and their Periods round him; the great law of which is, that the fquares of the Periodic times are always proportional to the cubes of their mean diftances from the fun.

The Periods, both tropical and fydereal, with the proportions of the mean diftances of the feveral planets are as follow:

Planets	Tropical Periods	Sydereal Periods	Proport. Difts.
Mercury	$87^d\ 23^h\ 14'$	$87^d\ 23^h\ 16'$	36710
Venus	224 16 42	224 16 49	72333
Earth	365 5 49	365 6 9	100000
Mars	686 22 18	686 23 31	152369
Jupiter	4330 8 58	4332 8 51	520110
Saturn	10749 7 22	10761 14 37	953800
Georgian or Herfchel	30456 1 41		1908180

As to the comets, the Periods of very few of them are known. There is one however of between 75 and 76 years, which appeared for the laft time in 1759; another was fuppofed to have its Period of 129 years, which was expected to appear in 1789 or 1790, but it did not; and the comet which appeared in 1680 it is thought has its Period of 575 years.

PERIOD, in Chronology, denotes an epoch, or interval of time, by which the years are reckoned; or a feries of years by which time is meafured, in different nations. Such are the Calippic and Metonic Periods, two different corrections of the Greek calendar, the Julian Period, invented by Jofeph Scaliger; the Victorian Period, &c.

Calippic PERIOD. See CALIPPIC *Period.*

Conftantinopolitan PERIOD, is that ufed by the Greeks, and is the fame as the *Julian* PERIOD, which fee.

Chaldaic PERIOD. See SAROS.

Dionyfian PERIOD. See *Victorian* PERIOD.

Hipparchus's PERIOD, is a feries or cycle of 304 folar years, returning in a conftant round, and reftoring the new and full moons to the fame day of the folar year; as Hipparchus thought.

This Period arifes by multiplying the Calippic Period by 4. Hipparchus affumed the quantity of the folar year to be 365d. 5h. 55m. 12 fec. and hence he concluded, that in 304 years Calippus's Period would err a whole day. He therefore multiplied the Period by 4, and from the product caft away an entire day. But even this does not reftore the new and full moons to the fame day throughout the whole Period: but they are fometimes anticipated 1d. 8h. 23 m. 29 fec. 20 thirds.

Julian PERIOD, fo called as being adapted to the Julian year, is a feries of 7980 Julian years; arifing from the multiplications of the cycles of the fun, moon, and indiction together, or the numbers 28, 19, 15; commencing on the 1ft day of January in the 764th Julian year before the creation, and therefore is not yet completed. This comprehends all other cycles, Periods and epochs, with the times of all memorable actions and hiftories; and therefore it is not only the moft general, but the moft ufeful of all Periods in Chronology.

As every year of the Julian Period has its particular folar, lunar, and indiction cycles, and no two years in it can have all thefe three cycles the fame, every year of this Period becomes accurately diftinguifhed from another.

This Period was invented by Jofeph Scaliger, as containing all the other epochs, to facilitate the reduction of the years of one given epoch to thofe of another. It agrees with the Conftantinopolitan Period, ufed by the Greeks, except in this, that the cycles of the fun, moon, and indiction, are reckoned differently; and alfo in that the firft year of the Conftantinopolitan Period differs from that of the Julian Period.

To find the year anfwering to any given year of the Julian Period, and vice verfa; fee EPOCH.

Metonic PERIOD. See CYCLE *of the Moon.*

Victorian PERIOD, an interval of 532 Julian years; at the end of which, the new and full moons return again on the fame day of the Julian year, according to the opinion of the inventor Victorinus, or Victorius, who lived in the time of pope Hilary.

Some afcribe this Period to Dionyfius Exiguus, and hence they call it the Dionyfian Period: others again call it the Great Pafchal Cycle, becaufe it was invented for computing the time of Eafter.

The Victorian Period is produced by multiplying the folar cycle 28 by the lunar cycle 19, the product being 532. But neither does this reftore the new and full moons to the fame day throughout its whole duration, by 1d. 16h. 58m. 59s. 40 thirds.

PERIOD, in Arithmetic, is a diftinction made by a point, or a comma, after every 6th place, or figure; and is ufed in numeration, for the readier diftinguifhing and naming the feveral figures or places, which are thus diftinguifhed into Periods of fix figures each. See NUMERATION.

PERIOD is alfo ufed in Arithmetic, in the extraction of

of roots, to point off, or separate the figures of the given number into Periods, or parcels, of as many figures each as are expressed by the degree of the root to be extracted, viz, of two places each for the square root, three places for the cube root, and so on.

PERIODIC, or PERIODICAL, appertaining to Period, or going by periods. Thus, the Periodical motion of the moon, is that of her monthly period or course about the earth, called her Periodical month, containing 27 days 7 hours 45 minutes.

PERIODICAL *Month*. See MONTH.

PERIŒCI, or PERIOECIANS, in Geography, are such as live in opposite points of the same parallel of latitude. Hence they have the same seasons at the same time, with the same phenomena of the heavenly bodies; but their times of the day are opposite, or differ by 12 hours, being noon with the one when it is midnight with the other.

PERIPATETIC *Philosophy*, the system of philosophy taught and established by Aristotle, and maintained by his followers, the Peripatetics. See ARISTOTLE.

PERIPATETICS, the followers of Aristotle. Though some derive their establishment from Plato himself, the master of both Xenocrates and Aristotle.

PERIPHERY, in Geometry, is the circumference, or bounding line, of a circle, ellipse, or other regular curvilineal figure. See CIRCUMFERENCE, and CIRCLE.

PERISCII, or PERISCIANS, those inhabitants of the earth, whose shadows do, in one and the same day, turn quite round to all the points of the compass, without disappearing.

Such are the inhabitants of the two frozen zones, or who live within the compass of the arctic and antarctic circles; for, as the sun never sets to them, after he is once up, but moves quite round about, so do their shadows also.

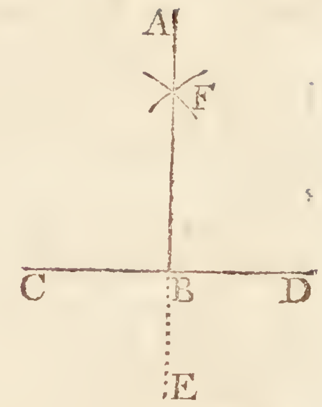
PERISTYLE, in the ancient Architecture, a place or building encompassed with a row of columns on the inside; by which it is distinguished from the periptere, where the columns are disposed on the outside.

PERISTYLE is also used, by modern writers, for a range of columns, either within or without a building.

PERITROCHIUM, in Mechanics, is a wheel or circle, concentric with the base of a cylinder, and moveable together with it, about an axis. The axis, with the wheel, and levers fixed in it to move it, make that mechanical power, called AXIS *in Peritrochio*, which see.

PERMUTATIONS *of Quantities*, in Algebra, the ALTERNATIONS, CHANGES, or different COMBINATIONS of any number of things. See those terms.

PERPENDICULAR, in Geometry, or NORMAL. One line is Perpendicular to another, when the former meets the latter so as to make the angles on both sides of it equal to each other. And those angles are called right angles. And hence, to be Perpendicular to, or to make right-angles with, means one and the same thing. So, when the angle ABC is equal to the angle ABD, the line AB is said to be Perpendicular, or normal, or at right angles to the line CD.



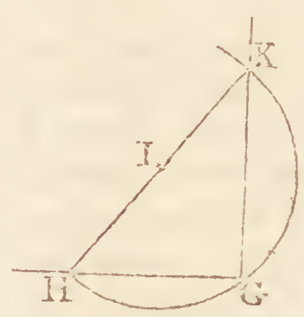
A line is Perpendicular to a curve, when it is perpendicular to the tangent of the curve at the point of contact.

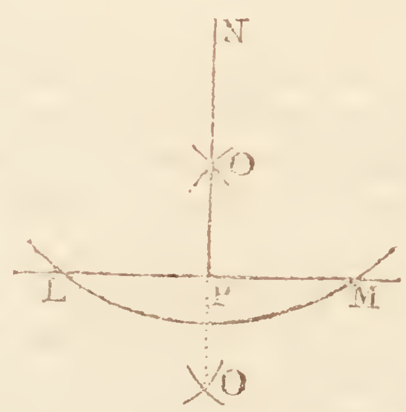
A line is Perpendicular to a plane, when it is Perpendicular to every line drawn in the plane through the bottom of the Perpendicular. And one plane is Perpendicular to another, when a line in the one plane is Perpendicular to the other plane.

From the very principle and motion of a Perpendicular, it follows, 1. That the Perpendicularity is mutual; if the first AB is perpendicular to the second CD, then is the second Perpendicular to the first.—2. That only one Perpendicular can be drawn from one point in the same place.—3. That if a Perpendicular be continued through the line it was drawn Perpendicular to; the continuation BE will also be Perpendicular to the same.—4. That if there be two points, A and E, of a right line, each of which is at an equal distance from two points, C and D, of another right line; those lines are Perpendiculars.—5. That a line which is Perpendicular to another line, is also Perpendicular to all the parallels of the other.—6. That a Perpendicular is the shortest of all those lines which can be drawn from the same point to the same right line. Hence the distance of a point from a line or plane, is a line drawn from the point Perpendicular to the line or plane: and hence also the altitude of a figure is a Perpendicular let fall from the vertex to the base.

To Erect a Perpendicular from a given point in a line.—1. When the given point B is near the middle of the line; with any interval in the compasses take the two equal parts BC, BD: and from the two centres C and D, with any radius greater than BC or BD, strike two arcs intersecting in F; then draw BFA the Perpendicular required.

2. When the given point G is at or near the end of the line; with any centre I and radius IG describe an arc HGK through G; then a ruler laid by H and I will cut the arc in the point K, through which the Perpendicular GK must be drawn.





To let fall a Perpendicular upon a given line LM from a given point N. With the centre N, and a convenient radius, describe an arc cutting the given line in L and M; with these two centres, and any other convenient radius, strike

two

two other arcs interfecting in O, the point through which the Perpendicular NOP muft be drawn.

Note, that Perpendiculars are beft drawn, in practice, by means of a fquare, laying one fide of it along the given line, and the other to pafs through the given point.

PERPENDICULAR, in Gunnery, is a fmall inftrument ufed for finding the centre line of a piece, in the operation of pointing it to a given object. See *Pointing of a* GUN.

PERPETUAL *Motion*. See MOTION.

Circle of PERPETUAL *Occultation and Apparition*. See CIRCLE.

PERPETUAL, or *Endlefs Screw*. See SCREW.

PERPETUITY, in the Doctrine of Annuities, is the number of years in which the fimple intereft of any principal fum will amount to the fame as the principal itfelf. Or it is the quotient arifing by dividing 100, or any other principal, by its intereft for one year. Thus, the Perpetuity, at the rate of 5 per cent. intereft, is $\frac{100}{5} = 20$; at 4 per cent. $\frac{100}{4} = 25$; &c.

PERRY (Captain JOHN), was a celebrated Englifh engineer. After acquiring great reputation for his fkill in this country, he refided many years in Ruffia, having been recommended to the czar Peter while in England, as a perfon capable of ferving him on a variety of occafions relating to his new defign of eftablifhing a fleet, making his rivers navigable, &c. His falary in this fervice was to be 300l. per annum, befides travelling expences and fubfiftence money on whatever fervice he fhould be employed, with a farther reward to his fatisfaction at the conclufion of any work he fhould finifh.

After fome converfation with the czar himfelf, particularly refpecting a communication between the rivers Volga and Don, he was employed on that work for three fummers fucceffively; but not being well fupplied with men, partly on account of the ill fuccefs of Peter's arms againft the Swedes at the battle of Narva, and partly by the difcouragement of the governor of Aftracan, he was ordered at the end of 1707 to ftop, and next year was employed in refitting the fhips at Veronife, and 1709 in making the river of that name navigable. But after repeated difappointments, and a variety of fruitlefs applications for his falary, he at length quitted the kingdom, under the protection of Mr. Whitworth, the Englifh ambaffador, in 1712. (See his Narrative in the Preface to *The State of Ruffia.*)

In 1721 he was employed in ftopping the breach at Dagenham, made in the bank of the river Thames, near the village of that name in Effex, and about 3 miles below Woolwich, in which he happily fucceeded, after feveral other perfons had failed in that undertaking. He was alfo employed, the fame year, about the harbour at Dublin, and publifhed at that time an Anfwer to the objections made againft it.—Befide this piece, Captain Perry was author of, *The State of Ruffia*, 1716, 8vo; and *An Account of the Stopping of Dagenham Breach*, 1721, 8vo.—He died February the 11th 1733.

PERSEUS, a conftellation of the northern hemifphere, being one of the 48 ancient afterifms.

The Greeks fabled that this is Perfeus, whom they make the fon of Jupiter by Danae. The father of that lady had been told, that he fhould be killed by his grandchild, and having only Danae to take care of, he locked her up; but Jupiter found his way to her in a fhower of gold, and Perfeus verified the oracle. He cut off alfo the head of the gorgon, and affixed it to his fhield; and after many other great exploits he refcued Andromeda, the daughter of Caffiopeia, whom the fea nymphs, in revenge for that lady's boafting of fuperior beauty, had faftened to a rock to be devoured by a monfter. Jupiter his father in honour of the exploit, they fay, afterwards took up the hero, and the whole family with him, into the fkies.

The number of ftars in this conftellation, in Ptolomy's catalogue, are 29; in Tycho's 29, in Hevelius's 46, and in the Britannic catalogue 59.

PERSIAN *Wheel*, in Mechanics, a machine for raifing a quantity of water, to ferve for various purpofes. Such a wheel is reprefented in plate xx, fig. 1; with which water may be raifed by means of a ftream AB turning a wheel CDE, according to the order of the letters, with buckets *a*, *a*, *a*, *a*, &c, hung upon the wheel by ftrong pins *b*, *b*, *b*, *b*, &c, fixed in the fide of the rim; which muft be made as high as the water is intended to be raifed above the level of that part of the ftream in which the wheel is placed. As the wheel turns, the buckets on the right hand go down into the water, where they are filled, and return up full on the left hand, till they come to the top at K; where they ftrike againft the end *n* of the fixed trough M, by which they are overfet, and fo empty the water into the trough; from whence it is to be conveyed in pipes to any place it is intended for: and as each bucket gets over the trough, it falls into a perpendicular pofition again, and fo goes down empty till it comes to the water at A, where it is filled as before. On each bucket is a fpring *r*, which going over the top or crown of the bar *m* (fixed to the trough M) raifes the bottom of the bucket above the level of its mouth, and fo caufes it to empty all its water into the trough.

Sometimes this wheel is made to raife water no higher than its axis; and then inftead of buckets hung upon it, its fpokes C, *d*, *e*, *f*, *g*, *h*, are made of a bent form, and hollow within; thefe hollows opening into the holes C, D, E, F, in the outfide of the wheel, and alfo into thofe at O in the box N upon the axis. So that, as the holes C, D, &c, dip into the water, it runs into them; and as the wheel turns, the water rifes in the hollow fpokes, *c*, *d*, &c, and runs out in a ftream P from the holes at O, and falls into the trough Q, from whence it is conveyed by pipes.

PERSIAN, or PERSIC, in Architecture, a name common to all ftatues of men; ferving inftead of columns to fupport entablatures.

PERSIAN *Era* and *Year*. See EPOCH and YEAR.

PERSPECTIVE, the art of delineating vifible objects on a plane furface, fuch as they appear at a given diftance, or height, upon a tranfparent plane, placed commonly perpendicular to the horizon, between the eye and the object. This is particularly called

Linear PERSPECTIVE, as regarding the pofition, magnitude, form, &c, of the feveral lines, or contours of objects, and expreffing their diminution.

Some make this a branch of Optics; others an art and

and ſcience derived from it: its operations however are all geometrical.

Hiſtory of PERSPECTIVE. This art derives its origin from painting, and particularly from that branch of it which was employed in the decorations of the theatre, where landſcapes were chiefly introduced. Vitruvius, in the proem to his 7th book, ſays that Agatharchus, at Athens, was the firſt author who wrote upon this ſubject, on occaſion of a play exhibited by Æſchylus, for which he prepared a tragic ſcene; and that afterwards the principles of the art were more diſtinctly taught in the writings of Democritus and Anaxagoras, the diſciples of Agatharchus, which are not now extant.

The Perſpective of Euclid and of Heliodorus Lariſſeus contains only ſome general elements of optics, that are by no means adapted to any particular practice; though they furniſh ſome materials that might be of ſervice even in the linear Perſpective of painters.

Geminus, of Rhodes, a celebrated mathematician, in Cicero's time, alſo wrote upon this ſcience.

It is alſo evident that the Roman artiſts were acquainted with the rules of Perſpective, from the account which Pliny (Nat. Hiſt. lib. 35, cap. 4) gives of the repreſentation on the ſcene of thoſe plays given by Claudius Pulcher; by the appearance of which the crows were ſo deceived, that they endeavoured to ſettle on the fictitious roofs. However, of the theory of this Art among the Ancients we know nothing; as none of their writings have eſcaped the general wreck of ancient literature in the dark ages of Europe. Doubtleſs this art muſt have been loſt, when painting and ſculpture no longer exiſted. However, there is reaſon to believe that it was practiſed much later in the Eaſtern empire.

John Tzetzes, in the 12th century, ſpeaks of it as well acquainted with its importance in painting and ſtatuary. And the Greek painters, who were employed by the Venetians and Florentines, in the 13th century, it ſeems brought ſome optical knowledge along with them into Italy: for the diſciples of Giotto are commended for obſerving Perſpective more regularly than any of their predeceſſors in the art had done; and he lived in the beginning of the 14th century.

The Arabians were not ignorant of this art; as may be preſumed from the optical writings of Alhazen, about the year 1100. And Vitellus, a Pole, about the year 1270, wrote largely and learnedly on optics. And, of our own nation, friar Bacon, as well as John Peckham, archbiſhop of Canterbury, treated this ſubject with ſurpriſing accuracy, conſidering the times in which they lived.

The firſt authors who profeſſedly laid down rules of Perſpective, were Bartolomeo Bramantino, of Milan, whoſe book, Regole di Perſpectiva, e Miſure delle Antichita di Lombardia, is dated 1440; and Pietro del Borgo, likewiſe an Italian, who was the moſt ancient author met with by Ignatius Danti, and who it is ſuppoſed died in 1443. This laſt writer ſuppoſed objects placed beyond a tranſparent tablet, and ſo to trace the images, which rays of light, emitted from them, would make upon it. And Albert Durer conſtructed a machine upon the principles of Borgo, by which he could trace the Perſpective appearance of objects.

Leon Battiſta Alberti, in 1450, wrote his treatiſe De Pictura, in which he treats chiefly of Perſpective.

Balthazar Perazzi, of Siena, who died in 1536, had diligently ſtudied the writings of Borgo; and his method of Perſpective was publiſhed by Serlio in 1540. To him it is ſaid we owe the diſcovery of points of diſtance, to which are drawn all lines that make an angle of 45° with the ground line.

Guido Ubaldi, another Italian, ſoon after diſcovered, that all lines that are parallel to one another, if they be inclined to the ground line, converge to ſome point in the horizontal line; and that through this point alſo will paſs a line drawn from the eye parallel to them. His Perſpective was printed at Piſaro in 1600, and contained the firſt principles of the method afterwards diſcovered by Dr. Brook Taylor.

In 1583 was publiſhed the work of Giacomo Barozzi, of Vignola, commonly called Vignola, intitled The two Rules of Perſpective, with a learned commentary by Ignatius Danti. In 1615 Marolois' work was printed at the Hague, and engraved and publiſhed by Hondius. And in 1625, Sirigatti publiſhed his treatiſe of Perſpective, which is little more than an abſtract of Vignola's.

Since that time the art of Perſpective has been gradually improved by ſubſequent geometricians, particularly by profeſſor Graveſande, and ſtill more by Dr. Brook Taylor, whoſe principles are in a great meaſure new, and far more general than thoſe of any of his predeceſſors. He did not confine his rules, as they had done, to the horizontal plane only, but made them general, ſo as to affect every ſpecies of lines and planes, whether they were parallel to the horizon or not; and thus his principles were made univerſal. Beſides, from the ſimplicity of his rules, the tedious progreſs of drawing out plans and elevations for any object, is rendered uſeleſs, and therefore avoided; for by this method, not only the feweſt lines imaginable are required to produce any Perſpective repreſentation, but every figure thus drawn will bear the niceſt mathematical examination. Farther, his ſyſtem is the only one calculated for anſwering every purpoſe of thoſe who are practitioners in the art of deſign; for by it they may produce either the whole, or only ſo much of an object as is wanted; and by fixing it in its proper place, its apparent magnitude may be determined in an inſtant. It explains alſo the Perſpective of ſhadows, the reflection of objects from poliſhed planes, and the inverſe practice of Perſpective.

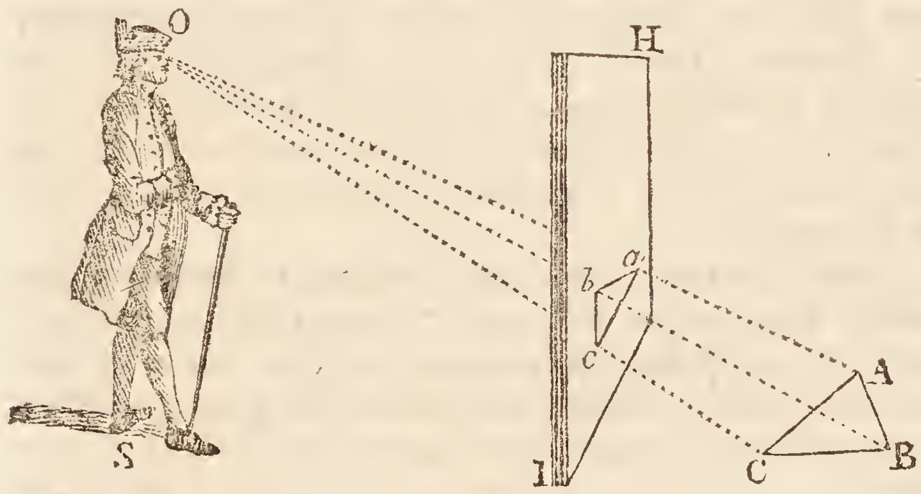
His Linear Perſpective was firſt publiſhed in 1715; and his New Principles of Linear Perſpective in 1719, which he intended as an explanation of his firſt treatiſe. And his method has been chiefly followed by all others ſince.

In 1738 Mr. Hamilton publiſhed his Stereography, in 2 vols folio, after the manner of Dr. Taylor. But the neateſt ſyſtem of Perſpective, both as to theory and practice, on the ſame principles, is that of Mr. Kirby. There are alſo good treatiſes on the ſubject, by Deſargues, de Boſſe, Albertus, Lamy, Niceron, Pozzo the Jeſuit, Ware, Cowley, Prieſtley, Ferguſon, Emerſon, Malton, Henry Clarke, &c, &c.

Of the Principles of PERSPECTIVE. To give an idea of

of the firſt principles and nature of this art; ſuppoſe a tranſparent plane, as of glaſs &c, HI raiſed perpendicularly on a horizontal plane; and the ſpectator S directing his eye O to the triangle ABC: if now we conceive the rays AO, BO, CO, &c, in their paſſage through the plane, to leave their traces or veſtiges in *a*, *b*, *c*, &c, on the plane; there will appear the triangle *abc*; which, as it ſtrikes the eye by the ſame rays *a*O, *b*O, *c*O, by which the reflected particles of light from the triangle are tranſmitted to the ſame, it will exhibit the true appearance of the triangle ABC, though the object ſhould be removed, the ſame diſtance and height of the eye being preſerved.

The buſineſs of Perſpective then, is to ſhew by what certain rules the points *a*, *b*, *c*, &c, may be found geometrically: and hence alſo we have a mechanical method of delineating any object very accurately.



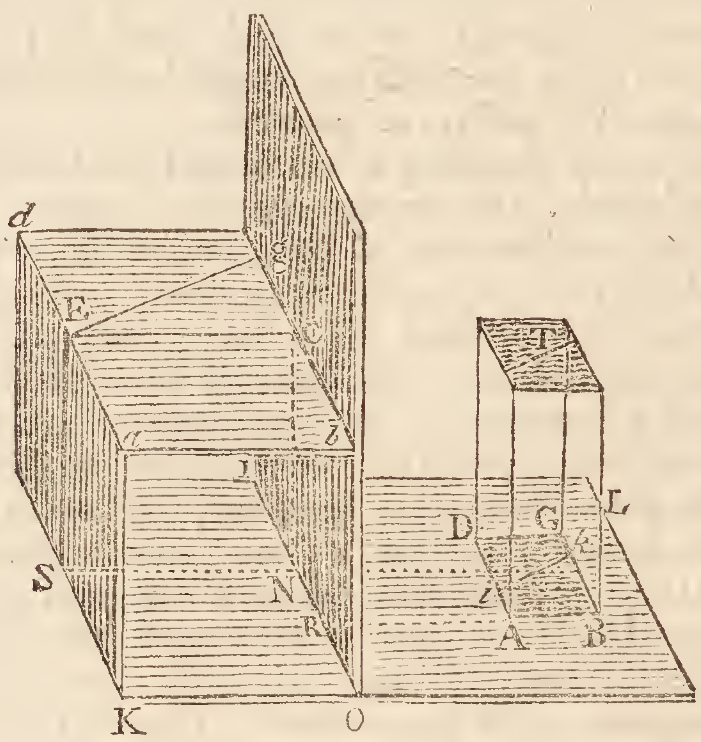
Hence it appears that *abc* is the ſection of the plane of the picture with the rays, which proceed from the original object to the eye: and therefore, when this is parallel to the picture, its repreſentation will be both parallel to the original, and ſimilar to it, though ſmaller in proportion as the original object is farther from the picture. When the original object is brought to coincide with the picture, the repreſentation is equal to the original; but as the object is removed farther and farther from the picture, its image will become ſmaller and ſmaller, and alſo riſe higher and higher in the picture, till at laſt, when the object is ſuppoſed to be at an infinite diſtance, its image will vaniſh in an imaginary point, exactly as high above the bottom of the picture as the eye is above the ground plane, upon which the ſpectator, the picture, and the original object are ſuppoſed to ſtand.

This may be familiarly illuſtrated in the following manner: Suppoſe a perſon at a window looks through an upright pane of glaſs at any object beyond; and, keeping his head ſteady, draws the figure of the object upon the glaſs, with a black-lead pencil, as if the point of the pencil touched the object itſelf; he would then have a true repreſentation of the object in Perſpective, as it appears to his eye. For properly drawing upon the glaſs, it is neceſſary to lay it over with ſtrong gum water, which will be fit for drawing upon when dry, and will then retain the traces of the pencil. The perſon ſhould alſo look through a ſmall hole in a thin plate of metal, fixed about a foot from the glaſs, between it and his eye; keeping his eye cloſe to the hole, otherwiſe he might ſhift the poſition of his head, and ſo make a falſe delineation of the object.

Having traced out the figure of the object, he may go over it again, with pen and ink; and when that is dry, cover it with a ſheet of paper, tracing the image upon this with a pencil; then taking away the paper, and laying it upon a table, he may finiſh the picture, by giving it the colours, lights, and ſhades, as he ſees them in the object itſelf; and thus he will have a true reſemblance of the object on the paper.

Of certain Definitions in PERSPECTIVE.

The point of ſight, in Perſpective, is the point E, where the ſpectator's eye ſhould be placed to view the



picture. And the *point of ſight*, in the picture, called alſo the *centre of the picture*, is the point C directly oppoſite to the eye, where a perpendicular from the eye at E meets the picture. Alſo this perpendicular EC is the *diſtance of the picture*: and if this diſtance be transferred to the horizontal line on each ſide of the point C, as is ſometimes done, the extremes are called the points of diſtance.

The *original plane*, or *geometrical plane*, is the plane KL upon which the real or original object ABGD is ſituated. The line OI, where the ground plane cuts the bottom of the picture, is called the *ſection* of the original plane, the *ground-line*, the *line of the baſe*, or the *fundamental line*.

If an original line AB be continued, ſo as to interſect the picture, the point of interſection R is called the interſection of that original line, or its *interſecting point*. The *horizontal plane* is the plane *abgd*, which paſſes through the eye, parallel to the horizon, and cuts the Perſpective plane or picture at right angles; and the *horizontal line bg* is the common interſection of the horizontal plane with the picture.

The *vertical plane* is that which paſſes through the eye at right angles both to the ground plane and to the picture, as ECSN. And the *vertical line* is the common ſection of the vertical plane and the picture, as CN.

The *line of ſtation* SN is the common ſection of the vertical plane with the ground plane, and perpendicular to the ground line OI.

The

The *line of the height of the eye* is a perpendicular, as ES, let fall from the eye upon the ground plane.

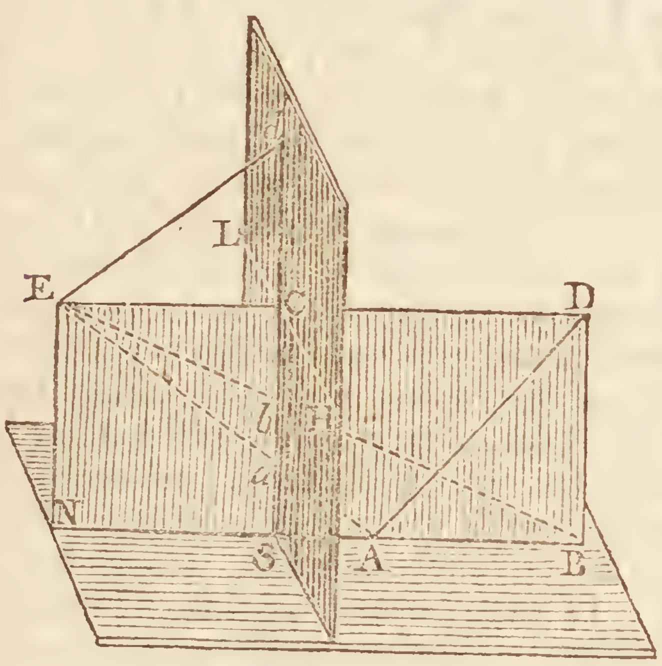
The *vanishing line* of the original plane, is that line where a plane passing through the eye, parallel to the original plane, cuts the picture: thus *bg* is the vanishing line of ABGD, being the greatest height to which the image can rise, when the original object is infinitely distant.

The *vanishing point* of the original line, is that point where a line drawn from the eye, parallel to that original line, intersects the picture: thus C and *g* are the vanishing points of the lines AB and *ki*. All lines parallel to each other have the same vanishing point.

If from the point of sight a line be drawn perpendicular to any vanishing line, the point where that line intersects the vanishing line, is called the centre of that vanishing line: and the *distance of a vanishing line* is the length of the line which is drawn from the eye, perpendicular to the said line.

Measuring points are points from which any lines in the Perspective plane are measured, by laying a ruler from them to the divisions laid down upon the ground line. The measuring point of all lines parallel to the ground line, is either of the points of distance on the horizontal line, or point of sight. The measuring point of any line perpendicular to the ground line, is in the point of distance on the horizontal line; and the measuring point of a line oblique to the ground line is found by extending the compasses from the vanishing point of that line to the point of distance on the perpendicular, and setting off on the horizontal line.

Some general Maxims or Theorems in PERSPECTIVE.

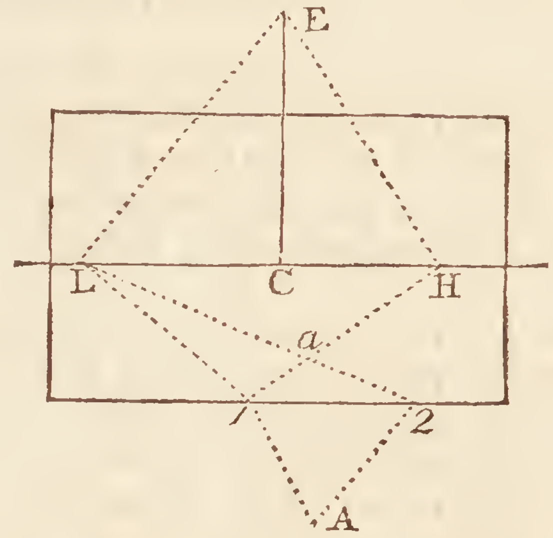


1. The representation *ab*, of a line AB, is part of a line SC, which passes through the intersecting point S, and the vanishing point C, of the original line AB.

2. If the original plane be parallel to the picture, it can have no vanishing line upon it; consequently the representation will be parallel. When the original is perpendicular to the ground line, as AB, then its vanishing point is in C, the centre of the picture, or point of sight; because EC is perpendicular to the picture, and therefore parallel to AB.

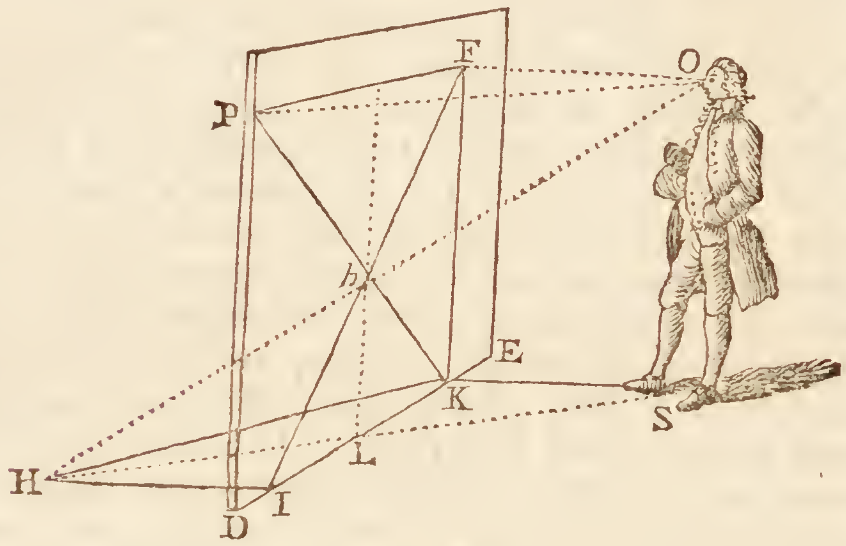
3. The image of a line bears a certain proportion to its original. And the image may be determined by transferring the length or distance of the given line to the intersecting line; and the distance of the vanishing point to the horizontal line; i. e. by bringing both into the plane of the picture.

PROB. *To find the representation of an Objective point* A. —Draw A1 and A2 at pleasure, intersecting the bot-



tom of the picture in 1 and 2; and from the eye E draw EH parallel to A1, and EL parallel to A2; then draw H1 and L2, which will intersect each other in *a*, the representation of the point A.

OTHERWISE. Let H be the given objective point.



From which draw HI perpendicular to the fundamental line DE. From the fundamental line DE cut off IK = IH: through the point of sight F draw a horizontal line FP, and make FP equal to the distance of the eye SK: lastly, join FI and PK, and their intersection *h* will be the appearance of the given objective point H, as required.

And thus, by finding the representations of the two points, which are the extremes of a line, and connecting them together, there will be formed the representation of the line itself. In like manner, the representations of all the lines or sides of any figure or solid, determine those of the solid itself; which therefore are thus put into Perspective.

Aerial PERSPECTIVE, is the art of giving a due diminution or gradation to the strength of light, shade, and colours of objects, according to their different distances, the quantity of light which falls upon them, and the medium through which they are seen.

PERSPECTIVE *Machine*, is a machine for readily and easily making the Perspective drawing and appearance of any object, with little or no skill in the art. There have been invented various machines of this kind. One of which may even be seen in the works of Albert Durer.

Durer. A very convenient one was invented by Dr. Bevis, and is defcribed by Mr. Ferguſon, in his Perſpective, pa. 113. And another is deſcribed in Kirby's Perſpective, pa. 65.

Perspective *Plan*, or *Plane*, is a glaſs or other tranſparent ſurface ſuppoſed to be placed between the eye and the object, and uſually perpendicular to the horizon.

Scenographic Perspective. See Scenography.

Perspective *of Shadows*. See Shadow.

Specular Perspective, is that which repreſents the objects in cylindrical, conical, ſpherical, or other mirrors.

PERTICA, a ſort of comet, being the ſame with Veru.

PETARD, a military engine, ſomewhat reſembling in ſhape a high-crowned hat; ſerving formerly to break down gates, barricades, draw-bridges, or the like works intended to be ſurpriſed. It is about 8 or 9 inches wide, and weighs from 55 to 70 pounds. Its uſe was chiefly in a clandeſtine or private attack, to break down the gates &c. It has alſo been uſed in countermines, to break through the enemies galleries, and give vent to their mines: but the uſe of Petards is now diſcontinued.——Their invention is aſcribed to the French Hugonots in the year 1579. Their moſt ſignal exploit was the taking the city Cahors by means of them, as we are told by d'Aubigné.

PETIT (Peter), a conſiderable mathematician and philoſopher of France, was born at Montluçon in the dioceſe of Bourges, in the year 1589 according to ſome, but in 1600 according to others. — He firſt cultivated the mathematics and philoſophy in the place of his nativity; but in 1633 he repaired to Paris, to which place his reputation had procured him an invitation. Here he became highly celebrated for his ingenious writings, and for his connections with Paſcal, Des Cartes, Merſenne, and the other great men of that time. He was employed on ſeveral occaſions by cardinal Richelieu; he was commiſſioned by this miniſter to viſit the ſea-ports, with the title of the king's engineer; and was alſo ſent into Italy upon the king's buſineſs. He was at Tours in 1640, where he married; and was afterwards made intendant of the fortifications. Baillet, in his Life of Des Cartes, ſays, that Petit had a great genius for mathematics; that he excelled particularly in aſtronomy; and had a ſingular paſſion for experimental philoſophy. About 1637 he returned to Paris from Italy, when the Dioptrics of Des Cartes were much ſpoken of. He read them, and communicated his objections to Merſenne, with whom he was intimately acquainted. And yet he ſoon after embraced the principles of Des Cartes, becoming not only his friend, but his partiſan and defender alſo. He was intimately connected with Paſcal, with whom he made at Rouen the ſame experiments concerning the vacuum, which Torricelli had before made in Italy; and was aſſured of their truth by frequent repetitions. This was in 1646 and 1647; and though there appears to be a long interval from this date to the time of his death, we meet with no other memoirs of his life. He died Auguſt the 20th 1667 at Lagny, near Paris, whither he had retired for ſome time before his deceaſe.

Petit was the author of ſeveral works upon phyſical and aſtronomical ſubjects; the principal of which are,

1. Chronological Diſcourſe, &c, 1636, 4to. In defence of Scaliger.
2. Treatiſe on the Proportional Compaſſes.
3. On the Weight and Magnitude of Metals.
4. Conſtruction and Uſe of the Artillery Calipers.
5. On a Vacuum.
6. On Eclipſes.
7. On Remedies againſt the Inundations of the Seine at Paris.
8. On the Junction of the Ocean with the Mediterranean ſea, by means of the rivers Aude and Garonne.
9. On Comets.
10. On the proper Day for celebrating Eaſter.
11. On the Nature of Heat and Cold, &c.

PETTY (Sir William), a ſingular inſtance of a univerſal genius, was the elder ſon of Anthony Petty, a clothier at Rumſey in Hampſhire, where he was born May the 16th, 1623. While a boy he took great delight in ſpending his time among the artificers there, whoſe trades he could work at when but 12 years of age. He then went to the grammar-ſchool in that place, where at 15 he became maſter of the Latin, Greek, and French languages, with arithmetic and thoſe parts of practical geometry and aſtronomy uſeful in navigation. Soon after, he went to the univerſity of Caen in Normandy; and after ſome ſtay there he returned to England, where he was preferred in the king's navy. In 1643, when the civil war grew hot, and the times troubleſome, he went into the Netherlands and France for three years; and having vigorouſly proſecuted his ſtudies, eſpecially in phyſic, at Utrecht, Leyden, Amſterdam, and Paris, he returned home to Rumſey. In 1647 he obtained a patent to teach the art of double writing for 17 years. In 1648 he publiſhed at London, "Advice to Mr. Samuel Hartlib, for the advancement of ſome particular parts of learning." At this time he adhered to the prevailing party of the nation; and went to Oxford, where he taught anatomy and chemiſtry, and was created a doctor of phyſic, and grew into ſuch repute that the philoſophical meetings, which preceded and laid the foundation of the Royal Society, were firſt held at his houſe. In 1650 he was made profeſſor of anatomy there; and ſoon after a member of the college of phyſicians in London, as alſo profeſſor of muſic at Greſham college London. In 1652 he was appointed phyſician to the army in Ireland; as alſo to three lord lieutenants ſucceſſively, Lambert, Fleetwood, and Henry Cromwell. In Ireland he acquired a great fortune, but not without ſuſpicions and charges of unfair practices in his offices. After the rebellion was over in Ireland, he was appointed one of the commiſſioners for dividing the forfeited lands to the army who ſuppreſſed it. When Henry Cromwell became lieutenant of that kingdom, in 1655, he appointed Dr. Petty his ſecretary, and clerk of the council: he likewiſe procured him to be elected a burgeſs for Weſtloo in Cornwall, in Richard Cromwell's parliament, which met in January 1658. But, in March following, Sir Hierom Sankey, member for Woodſtock in Oxfordſhire, impeached him of high crimes and miſdemeanors in the execution of his office.

This gave the doctor a great deal of trouble, as he was ſummoned before the Houſe of Commons; and notwithſtanding the ſtrenuous endeavours of his friends, in their recommendations of him to ſecretary Thurloe, and the defence he made before the houſe, his enemies procured his diſmiſſion from his public employments, in 1659. He then retired to Ireland, till the reſtoration of king Charles the Second; ſoon after which he came into England, where he was very graciouſly received by the king, reſigned his profeſſorſhip at Greſham college, and was appointed one of the commiſſioners of the Court of Claims. Likewiſe, April the 11th, 1661, he received the honour of knighthood, and the grant of a new patent, conſtituting him ſurveyor-general of Ireland, and was choſen a member of parliament there.

Upon the incorporating of the Royal Society, he was one of the firſt members, and of its firſt council. And though he had left off the practice of phyſic, his name was continued as an honorary member of the college of phyſicians in 1663.

About this time he invented his double bottomed ſhip, to ſail againſt wind and tide, and afterwards preſented a model of this ſhip to the Royal Society; to whom alſo, in 1665, he communicated "A Diſcourſe about the Building of Ships," containing ſome curious ſecrets in that art. But, upon trial, finding his ſhip failed in ſome reſpects, he at length gave up that project.

In 1666 Sir William drew up a treatiſe, called *Verbum Sapienti*, containing an account of the wealth and expences of England, and the method of raiſing taxes in the moſt equal manner.—The ſame year, 1666, he ſuffered a conſiderable loſs by the fire of London.—The year following he married Elizabeth, daughter of Sir Hardreſſe Waller; and afterwards ſet up iron works and pilchard fiſhing, opened lead mines and a timber trade in Kerry, which turned to very good account. But all theſe concerns did not hinder him from the purſuit of both political and philoſophical ſpeculations, which he thought of public utility, publiſhing them either ſeparately or by communication to the Royal Society, particularly on finances, taxes, political arithmetic, land carriage, guns, pumps, &c.

Upon the firſt meeting of the Philoſophical Society at Dublin, upon the plan of that at London, every thing was ſubmitted to his direction; and when it was formed into a regular ſociety, he was choſen preſident in Nov. 1684. Upon this occaſion he drew up a "Catalogue of mean, vulgar, cheap, and ſimple Experiments," proper for the infant ſtate of the ſociety, and preſented it to them; as he did alſo his *Supellex Philoſophica*, conſiſting of 45 inſtruments requiſite to carry on the deſign of their inſtitution. In 1685 he made his will; in which he declares, that being then about 60, his views were fixed upon improving his lands in Ireland, and to promote the trade of iron, lead, marble, fiſh, and timber, which his eſtate was capable of. And as for ſtudies and experiments, "I think now, ſays he, to confine the ſame to the anatomy of the people, and political arithmetic; as alſo the improvement of ſhips, land-carriages, guns, and pumps, as of moſt uſe to mankind, not blaming the ſtudy of other men." But a few years after, all his purſuits were determined by the effects of a gangrene in his foot, occaſioned by the ſwelling of the gout, which put a period to his life, at his houſe in Piccadilly, Weſtminſter, Dec. 16, 1687, in the 65th year of his age. His corpſe was carried to Rumſey, and there interred, near thoſe of his parents.

Sir William Petty died poſſeſſed of a very large fortune, as appears by his will; where he makes his real eſtate about 6,500l. per annum, his perſonal eſtate about 45,000l. his bad and deſperate debts 30,000l. and the demonſtrable improvements of his Iriſh eſtate, 4000l. per annum; in all, at 6 per cent. intereſt, 15,000l. per annum. This eſtate came to his family, which conſiſted of his widow and three children, Charles, Henry, and Anne: of whom Charles was created baron of Shelbourne, in the county of Waterford in Ireland, by king William the Third; but dying without iſſue, was ſucceeded by his younger brother Henry, who was created viſcount Dunkeron, in the county of Kerry, and earl of Shelbourne Feb. 11, 1718. He married the lady Arabella Boyle, ſiſter of Charles earl of Cork, who brought him ſeveral children. He was member of parliament for Great Marlow in Buckinghamſhire, and a fellow of the Royal Society: he died April 17, 1751. Anne was married to Thomas Fitzmorris, baron of Kerry and Lixnaw, and died in Ireland in the year 1737.

The variety of purſuits, in which Sir William Petty was engaged, ſhews him to have had a genius capable of any thing to which he choſe to apply it: and it is very extraordinary, that a man of ſo active and buſy a ſpirit could find time to write ſo many things, as it appears he did, by the following catalogue.

1. Advice to Mr. S. Hartlib &c; 1648, 4to.—2. A Brief of Proceedings between Sir Hierom Sankey and the author &c; 1659, folio.—3. Reflections upon ſome perſons and things in Ireland, &c; 1660, 8vo.—4. A Treatiſe of Taxes and Contribution, &c; 1662, 1667, 1685, 4to, all without the author's name. This laſt was re-publiſhed in 1690, with two other anonymous pieces, "The Privileges and Practice of Parliaments," and "The Politician Diſcovered;" with a new title-page, where it is ſaid they were all written by Sir William, which, as to the firſt, is a miſtake.—5. Apparatus to the Hiſtory of the Common Practice of Dyeing;" printed in Sprat's Hiſtory of the Royal Society, 1667, 4to.—6. A Diſcourſe concerning the Uſe of Duplicate Proportion, together with a New Hypotheſis of Springing or Elaſtic Motions; 1674, 12mo.—7. Colloquium Davidis cum Anima ſua, &c; 1679, folio.—8. The Politician Diſcovered, &c; 1681, 4to.—9. An Eſſay in Political Arithmetic; 1682, 8vo.—10. Obſervations upon the Dublin Bills of Mortality in 1681, &c; 1683, 8vo.—11. An Account of ſome Experiments relating to Land-carriage, Philoſ. Tranſ. numb. 161.—12. Some Queries for examining Mineral Waters, ibid. numb. 166.—13. A Catalogue of Mean, Vulgar, Cheap, and Simple Experiments, &c; ibid. numb. 167.—14. Maps of Ireland, being an Actual Survey of the whole Kingdom, &c; 1685, folio.—15. An Eſſay concerning the Multiplication of Mankind; 1686, 8vo.—16. A further Aſſertion concerning the magnitude of London, vindicating it, &c; Philoſ. Tranſ. numb. 185.—17. Two Eſſays in Politi-

cal

cal Arithmetic; 1687, 8vo.—18. Five Eſſays in Political Arithmetic; 1687, 8vo.—19. Obſervations upon London and Rome; 1687, 8vo.

His poſthumous pieces are, (1), Political Arithmetic; 1690, 8vo, and 1755, with his life prefixed.—(2), The Political Anatomy of Ireland, with Verbum Sapienti, 1691, 1719.—(3), A Treatiſe of Naval Philoſophy; 1691, 12mo.—(4), What a complete Treatiſe of Navigation ſhould contain; Philoſ. Tranſ. numb. 198.—(5), A Diſcourſe of making Cloth with Sheep's Wool; in Birch's Hiſt. of the Roy. Soc.—(6), Supellex Philoſophica; ibid.

PHÆNOMENON. See PHENOMENON.

PHARON, the name of a game of chance. See De Moivre's Doctrine of Chances, pa. 77 and 105.

PHASES, in Aſtronomy, the various appearances, or quantities of illumination of the moon, Venus, Mercury, and the other planets, by the ſun. Theſe Phaſes are very obſervable in the moon with the naked eye; by which ſhe ſometimes increaſes, ſometimes wanes, is now bent into horns, and again appears a half circle; at other times ſhe is gibbous, and again a full circular face. And by help of the teleſcope, the like variety of Phaſes is obſerved in Venus, Mars, &c.

Copernicus, a little before the uſe of teleſcopes, foretold, that after ages would find that Venus underwent all the changes of the moon; which prophecy was firſt fulfilled by Galileo, who, directing his teleſcope to Venus, obſerved her Phaſes to emulate thoſe of the moon; being ſometimes full, ſometimes horned, and ſometimes gibbous.

PHASES *of an Eclipſe*. To determine theſe for any time: Find the moon's place in her viſible way for that moment; and from that point as a centre, with the interval of the moon's ſemidiameter, deſcribe a circle: In like manner find the ſun's place in the ecliptic, from which, with the ſemidiameter of the ſun, deſcribe another circle: The interſection of the two circles ſhews the Phaſes of the eclipſe, the quantity of obſcuration, and the poſition of the cuſps or horns.

PHENOMENON, or PHÆNOMENON, an appearance in phyſics, an extraordinary appearance in the heavens, or on earth; either diſcovered by obſervation of the celeſtial bodies, or by phyſical experiments, the cauſe of which is not obvious. Such are meteors, comets, uncommon appearance of ſtars and planets, earthquakes, &c. Such alſo are the effects of the magnet, phoſphorus, &c.

PHILOLAUS, of Crotona, was a celebrated philoſopher of the Ancients. He was of the ſchool of Pythagoras, to whom that philoſopher's Golden Verſes have been aſcribed. He made the heavens his chief object of contemplation; and has been ſaid to be the author of that true ſyſtem of the world which Copernicus afterwards revived; but erroneouſly, becauſe there is undoubted evidence that Pythagoras learned that ſyſtem in Egypt. On that erroneous ſuppoſition however it was, that Bulliald placed the name of Philolaus at the head of two works, written to illuſtrate and confirm that ſyſtem.

"He was (ſays Dr. Enfield, in his Hiſtory of Philoſophy) a diſciple of Archytas, and flouriſhed in the time of Plato. It was from him that Plato purchaſed the written records of the Pythagorean ſyſtem, contrary to an expreſs oath taken by the ſociety of Pythagoreans, pledging themſelves to keep ſecret the myſteries of their ſect. It is probable that among theſe books were the writings of Timæus, upon which Plato formed the dialogue which bore his name. Plutarch relates, that Philolaus was one of the perſons who eſcaped from the houſe which was burned by Cylon, during the life of Pythagoras; but this account cannot be correct. Philolaus was contemporary with Plato, and therefore certainly not with Pythagoras. Interfering in affairs of ſtate, he fell a ſacrifice to political jealouſy.

"Philolaus treated the doctrine of nature with great ſubtlety, but at the ſame time with great obſcurity; referring every thing that exiſts to mathematical principles. He taught, that reaſon, improved by mathematical learning, is alone capable of judging concerning the nature of things: that the whole world conſiſts of infinite and finite; that number ſubſiſts by itſelf, and is the chain by which its power ſuſtains the eternal frame of things; that the Monad is not the ſole principle of things, but that the Binary is neceſſary to furniſh materials from which all ſubſequent numbers may be produced; that the world is one whole, which has a fiery centre, about which the ten celeſtial ſpheres revolve, heaven, the ſun, the planets, the earth, and the moon; that the ſun has a vitreous ſurface, whence the fire diffuſed through the world is reflected, rendering the mirror from which it is reflected viſible; that all things are preſerved in harmony by the law of neceſſity; and that the world is liable to deſtruction both by fire and by water. From this ſummary of the doctrine of Philolaus it appears probable that, following Timæus, whoſe writings he poſſeſſed, he ſo far departed from the Pythagorean ſyſtem as to conceive two independent principles in nature, God and matter, and that it was from the ſame ſource that Plato derived his doctrine upon this ſubject."

PHILOSOPHER, a perſon well verſed in philoſophy; or who makes a profeſſion of, or applies himſelf to, the ſtudy of nature or of morality.

PHILOSOPHICAL TRANSACTIONS, thoſe of the Royal Society. See TRANSACTIONS.

PHILOSOPHIZING, the act of conſidering ſome object of our knowledge, examining its properties, and the phenomena it exhibits, and enquiring into their cauſes or effects, and the laws of them; the whole conducted according to the nature and reaſon of things, and directed to the improvement of knowledge.

The Rules of PHILOSOPHIZING, as eſtabliſhed by Sir Iſaac Newton, are, 1. That no more cauſes of a natural effect be admitted than are true, and ſuffice to account for its phenomena. This agrees with the ſentiments of moſt philoſophers, who hold that nature does nothing in vain; and that it were vain to do that by many things, which might be done by fewer.

2. That natural effects of the ſame kind, proceed from the ſame cauſes. Thus, for inſtance, the cauſe of reſpiration is one and the ſame in man and brute; the cauſe of the deſcent of a ſtone, the ſame in Europe as in America; the cauſe of light, the ſame in the ſun and in culinary fire; and the cauſe of reflection, the ſame in the planets as the earth.

3. Thoſe qualities of bodies which are not capable of being heightened, and remitted, and which are found

in all bodies on which experiments can be made, muſt be conſidered as univerſal qualities of all bodies. Thus, the extenſion of body is only perceived by our ſenſes, nor is it perceivable in all bodies: but ſince it is found in all that we have perception of, it may be affirmed of all. So we find that ſeveral bodies are hard; and argue that the hardneſs of the whole only ariſes from the hardneſs of the parts: whence we infer that the particles, not only of thoſe bodies which are ſenſible, but of all others, are likewiſe hard. Laſtly, if all the bodies about the earth gravitate towards the earth, and this according to the quantity of matter in each; and if the moon gravitate towards the earth alſo, according to its quantity of matter; and the ſea again gravitate towards the moon; and all the planets and comets gravitate towards each other: it may be affirmed univerſally, that all bodies in the creation gravitate towards each other. This rule is the foundation of all natural philoſophy.

PHILOSOPHY, the knowledge or ſtudy of nature or morality, founded on reaſon and experience. Literally and originally, the word ſignified a love of wiſdom. But by Philoſophy is now meant the knowledge of the nature and reaſons of things; as diſtinguiſhed from hiſtory, which is the bare knowledge of facts; and from mathematics, which is the knowledge of the quantity and meaſures of things.

Theſe three kinds of knowledge ought to be joined as much as poſſible. Hiſtory furniſhes matter, principles, and practical examinations; and mathematics completes the evidence.

Philoſophy being the knowledge of the reaſons of things, all arts muſt have their peculiar Philoſophy which conſtitutes their theory: not only law and phyſic, but the loweſt and moſt abject arts are not without their reaſons. It is to be obſerved that the bare intelligence and memory of philoſophical propoſitions, without any ability to demonſtrate them, is not Philoſophy, but hiſtory only. However, where ſuch propoſitions are determinate and true, they may be uſefully applied in practice, even by thoſe who are ignorant of their demonſtrations. Of this we ſee daily inſtances in the rules of arithmetic, practical geometry, and navigation; the reaſons of which are often not underſtood by thoſe who practiſe them with ſucceſs. And this ſucceſs in the application produces a conviction of mind, which is a kind of medium between Philoſophical or ſcientific knowledge, and that which is hiſtorical only.

If we conſider the difference there is between natural philoſophers, and other men, with regard to their knowledge of phenomena, we ſhall find it conſiſts not in an exacter knowledge of the efficient cauſe that produces them, for that can be no other than the will of the Deity; but only in a greater and more enlarged comprehenſion, by which analogies, harmonies, and agreements are deſcribed in the works of nature, and the particular effects explained; that is, reduced to general rules, which rules grounded on the analogy and uniformneſs obſerved in the production of natural effects, are more agreeable, and ſought after by the mind; for that they extend our proſpect beyond what is preſent, and near to us, and enable us to make very probable conjectures, touching things that may have happened at very great diſtances of time and place, as well as to predict things to come; which ſort of endeavour towards omniſcience is much affected by the mind. Berkley, Princip. of Hum. Knowledge, ſect. 104, 105.

From the firſt broachers of new opinions, and the firſt founders of ſchools, Philoſophy is become divided into ſeveral ſects, ſome ancient, others modern; ſuch are the Platoniſts, Peripatetics, Epicureans, Stoics, Pyrrhonians, and Academics; alſo the Carteſians, Newtonians, &c. See the particular articles for each.

Philoſophy may be divided into two branches, or it may be conſidered under two circumſtances, theoretical and practical.

Theoretical or *Speculative* PHILOSOPHY, is employed in mere contemplation. Such is phyſics, which is a bare contemplation of nature, and natural things.

Theoretical Philoſophy again is uſually ſubdivided into three kinds, viz, pneumatics, phyſics or ſomatics, and metaphyſics or ontology.

The firſt conſiders being, abſtractedly from all matter: its objects are ſpirits, their natures, properties, and effects. The ſecond conſiders matter, and material things: its objects are bodies, their properties, laws, &c.

The third extends to each indifferently: its objects are body or ſpirit.

In the order of our diſcovery, or arrival at the knowledge of them, phyſics is firſt, then metaphyſics; the laſt ariſes from the two firſt conſidered together.

But in teaching, or laying down theſe ſeveral branches to others, we obſerve a contrary order; beginning with the moſt univerſal, and deſcending to the more particular. And hence we ſee why the Peripatetics call metaphyſics, and the Carteſians pneumatics, the *prima philoſophia.*

Others prefer the diſtribution of Philoſophy into four parts, viz, 1. Pneumatics, which conſiders and treats of ſpirits. 2. Somatics, of bodies. 3. The third compounded of both, anthropology, which conſiders man, in whom both body and ſpirit are found. 4. Ontoſophy, which treats of what is common to all the other three.

Again, Philoſophy may be divided into three parts; intellectual, moral, and phyſical: the intellectual part compriſes logic and metaphyſics; the moral part contains the laws of nature and nations, ethics and politics; and laſtly the phyſical part comprehends the doctrine of bodies, animate or inanimate: theſe, with their various ſubdiviſions, will comprize the whole of Philoſophy.

Practical PHILOSOPHY, is that which lays down the rules of a virtuous and happy life; and excites us to the practice of them. Moſt authors divide it into two kinds, anſwerable to the two ſorts of human actions to be directed by it; viz, Logic, which governs the operations of the underſtanding; and Ethics, properly ſo called, which direct thoſe of the will.

For the ſeveral particular ſorts of Philoſophy, ſee the articles, Arabian, Ariſtotelian, Atomical, Carteſian, Corpuſcular, Epicurean, Experimental, Hermetical, Leibnitzian, Mechanical, Moral, Natural, Newtonian, Oriental, Platonic, Scholaſtic, Socratic, &c. &c.

 PHOENIX,

PHOENIX, a conſtellation of the ſouthern hemiſphere. This is one of the new-added aſteriſms, unknown to the Ancients, and is not viſible in our northern parts of the globe. There are 13 ſtars in this conſtellation.

PHONICS, otherwiſe called Acoustics, is the doctrine or ſcience of ſounds.

Phonics may be conſidered as an art analogous to Optics; and may be divided, like that, into Direct, Refracted, and Reflected. Theſe branches, the biſhop of Ferns, in alluſion to the parts of Optics, denominates Phonics, Diaphonics, and Cataphonics. See Acoustics.

PHOSPHORUS, a matter which ſhines, or even burns ſpontaneouſly, and without the application of any ſenſible fire.

Phoſphori are either natural or artificial.

Natural Phosphori, are matters which become luminous at certain times, without the aſſiſtance of any art or preparation. Such are the glow-worms, frequent in our colder countries; lantern-flies, and other ſhining inſects, in hot countries; rotten-wood; the eyes, blood, ſcales, fleſh, ſweat, feathers, &c, of ſeveral animals; diamonds, when rubbed after a certain manner, or after having been expoſed to the ſun or light; ſugar and ſulphur, when pounded in a dark place; ſea water, and ſome mineral waters, when briſkly agitated; a cat's or horſe's back, duly rubbed with the hand, &c, in the dark; nay Dr. Croon tells us, that upon rubbing his own body briſkly with a well-warmed ſhirt, he has frequently made both to ſhine; and Dr. Sloane adds, that he knew a gentleman of Briſtol, and his ſon, both whoſe ſtockings would ſhine much after walking.

All natural Phoſphori have this in common, that they do not ſhine always, and that they never give any heat.

Of all the natural Phoſphori, that which has occaſioned the greateſt ſpeculation, is the

Barometrical or *Mercurial* Phosphorus. M. Picard firſt obſerved, that the mercury of his barometer, when ſhaken in a dark place, emitted light. And many fanciful explanations have been given of this phenomenon, which however is now found to be a mere electrical effect.

Mr. Hawkſbee has ſeveral experiments on this appearance. Paſſing air forcibly through the body of quickſilver, placed in an exhauſted receiver, the parts were violently driven againſt the ſide of the receiver, and gave all around the appearance of fire; continuing thus till the receiver was half full again of air.

From other experiments he found, that though the appearance of light was not producible by agitating the mercury in the ſame manner in the common air, yet that a very fine medium, nearly approaching to a vacuum, was not at all neceſſary. And laſtly, from other experiments he found that mercury incloſed in water, which communicated with the open air, by a violent ſhaking of the veſſel in which it was incloſed, emitted particles of light in great plenty, like little ſtars.

By including the veſſel of mercury, &c, in a receiver, and exhauſting the air, the phenomenon was changed; and upon ſhaking the veſſel, inſtead of ſparks of light, the whole maſs appeared one continued circle of light.

Farther, if mercury be incloſed in a glaſs tube, cloſe ſtopped, that tube is found, on being rubbed, to give much more light, than when it had no mercury in it. When this tube has been rubbed, after raiſing ſucceſſively its extremities, that the mercury might flow from one end to the other, a light is ſeen creeping in a ſerpentine manner all along the tube, the mercury being all luminous. By making the mercury run along the tube afterwards without rubbing it, it emitted ſome light, though much leſs than before; this proves that the friction of the mercury againſt the glaſs, in running along, does in ſome meaſure electrify the glaſs, as the rubbing it with the hand does, only in a much leſs degree. This is more plainly proved by laying ſome very light down near the tube, for this will be attracted by the electricity raiſed by the running of the mercury, and will riſe to that part of the glaſs along which the mercury runs; from which it is plain, that what has been long known in the world under the name of the Phoſphorus of the barometer, is not a Phoſphorus, but merely a light raiſed by electricity, the mercury electrifying the tube. Philoſ. Tranſ. numb. 484.

Artificial Phosphori, are ſuch as owe their luminous quality to ſome art or preparation. Some of theſe are made by the maceration of plants alone, and without any fire; ſuch as thread, linen cloth, but above all paper: the luminous appearance of this laſt, which it is now known is an electrical phenomenon, is greatly increaſed by heat. Almoſt all bodies, by a proper treatment, have that power of ſhining in the dark, which at firſt was ſuppoſed to be the property of one, and afterwards only of a few. See Philoſ. Tranſ. numb. 478, in vol. 44, pa. 83.

Of Artificial Phoſphori there are three principal kinds: the firſt *burning*, which conſumes every combuſtible it touches; the other two have no ſenſible heat, and are called the *Bononian* and *Hermetic* Phoſphorus; to which claſs others of a ſimilar kind may be referred.

The Burning Phosphorus, is a combination of phlogiſton with a peculiar acid, and conſequently a ſpecies of ſulphur, tending to decompoſe itſelf, and ſo as to take fire on the acceſs of air only. This may be made of urine, blood, hairs, and generally of any part of an animal that yields an oil by diſtillation, and moſt eaſily of urine. It is of a yellowiſh colour, and of the conſiſtence of hard wax, in the condition it is left by the diſtillation; in which ſtate it is called *phoſphorus fulgurans*, from its corruſcations; and *phoſphorus ſmaragdinus*, becauſe its light is often green or blue, eſpecially in places that are not very dark; and from its conſiſtence it is called ſolid Phoſphorus. It diſſolves in all kinds of diſtilled oils, in which ſtate it is called liquid Phoſphorus. And it may be ground in all kinds of fat pomatums, in which way it makes a luminous unguent.—So that theſe ſorts are all the ſame preparation, under different circumſtances.

The diſcovery of this Phoſphorus was made in 1677, by one Brandt, a citizen of Hamburgh, in his reſearches for the philoſopher's ſtone. And the method was afterwards found out both by Kunckel, and Mr. Boyle, from only learning that urine was the chief ſubſtance

ſtance of it; ſince then it has been called Kunckell's Phoſphorus. It is prepared by firſt evaporating the urine to a rob, or the conſiſtence of honey, and afterwards diſtilling it in a very ſtrong heat, &c. See Mem. Acad. Paris 1737; Philoſ. Tranſ. numb. 196, or Abr. vol. 3, pa. 346; Mem. Acad. Berlin 1743.

Many curious and amuſing experiments are made with Phoſphorus; as by writing with it, when the letters will appear like flame in the dark, though in the light nothing appears but a dim ſmoke; alſo a little bit of it rubbed between two papers, preſently takes fire, and burns vehemently; &c. By waſhing the face, or hands, &c, with liquid Phoſphorus, they will ſhine very conſiderably in the dark, and the luſtre will be communicated to adjacent objects, yet, without hurting the ſkin: on bringing in the candle, the ſhining diſappears, and no change is perceivable.

Bolognian or *Bononian* Phosphorus, is a preparation of a ſtone called the Bononian ſtone, from Bologna, a city in Italy, near which it is found. This Phoſphorus has no ſenſible heat, and only becomes luminous after being expoſed to the ſun or day light. For the method of preparing it, ſee the Mem. Acad. Berlin 1749 and 1750.

The *Hermetic* Phosphorus, or third kind, is a preparation of Engliſh chalk, with aqua fortis, or ſpirit of nitre, by the fire. It makes a body conſiderably ſofter than the Bolognian ſtone, but having otherwiſe all the ſame qualities. It is alſo called Baldwin's Phoſphorus, from its inventor, a German chemiſt, called alſo Hermes in the ſociety of the Naturæ Curioſorum, whence its other name Hermetic: it was diſcovered a little before the year 1677. See Acad. Par. 1693, pa. 271; and Grew's Muſ. Reg. Soc. p. 353.

Ammoniacal Phosphorus, firſt diſcovered by Homberg, is a combination of quick-lime with the acid of ſal ammoniac, from which it receives its phlogiſton. Mem. Acad. Par. 1693.

Antimonial Phosphorus, is a kind diſcovered by Mr. Geoffroy in his experiments on antimony. Mem. Acad. Par. 1736.

Phosphorus *of the Berne-ſtone*, a name given to a ſtone from Berne, in Switzerland, where it is found, and which becomes a kind of Phoſphorus when heated. Mem. Acad. Paris 1724.

Canton's Phosphorus, a very good kind, prepared by Mr. Canton, an ingenious philoſopher, from calcined oyſter ſhells. Philoſ. Tranſ. vol. 58, pa. 337.

Phopshorus *Fæcalis*, a very good kind, exhibiting many wonderful phenomena, and prepared, by Mr. Homberg, from human dung mixed with alum. Mem. Acad. Par. 1711.

Phosphorus *Metallorum*, a name given by ſome chemiſts to a preparation of a certain mineral ſpar, found in the mines of Saxony, and other places where there is copper. Philoſ. Tranſ. numb. 244, p. 365.

Phosphorus *of Sulphur*, a new-diſcovered ſpecies, which readily takes fire on being expoſed to the open air, and invented by M. Le Fevre. Mem. Acad. Par. 1728.

Phosphorus, in Aſtronomy, is the morning ſtar, or the planet Venus, when ſhe riſes before the ſun. The Latins call it Lucifer, the French Etoile de berger, and the Greeks Phoſphorus.

PHYSICAL, ſomething belonging to nature, or exiſting in it. Thus, we ſay a Phyſical point, in oppoſition to a mathematical one, which laſt only exiſts in the imagination. Or a Phyſical ſubſtance or body, in oppoſition to ſpirit, or metaphyſical ſubſtance, &c.

Physical, or *Senſible Horizon.* See Horizon.

Physico-*Mathematics*, or *Mixed Mathematics*, includes thoſe branches of Phyſics which, uniting obſervation and experiment to mathematical calculation, explain mathematically the phenomena of nature.

PHYSICS, called alſo *Phyſiology*, and *Natural Phileſophy*, is the doctrine of natural bodies, their phenomena, cauſes, and effects, with their various affections, motions, operations, &c. So that the immediate and proper objects of Phyſics, are body, ſpace, and motion.

The origin of Phyſics is referred, by the Greeks, to the Barbarians, viz, the brachmans, the magi, and the Hebrew and Egyptian prieſts. From theſe it paſſed to the Greek ſages or ſophi, particularly to Thales, who it is ſaid firſt profeſſed the ſtudy of nature in Greece. Hence it deſcended into the ſchools of the Pythagoreans, the Platoniſts, and the Peripatetics; from whence it paſſed into Italy, and thence through the reſt of Europe. Though the druids, bards, &c, had a kind of ſyſtem of Phyſics of their own.

Phyſics may be divided, with regard to the manner in which it has been handled, and the perſons by whom, into

Symbolical Physics, or ſuch as was couched under ſymbols: ſuch was that of the old Egyptians, Pythagoreans, and Platoniſts; who delivered the properties of natural bodies under arithmetical and geometrical characters, and hieroglyphics.

Peripatetical Physics, or that of the Ariſtotelians, who explained the nature of things by matter, form, and privation, elementary and occult qualities, ſympathies, antipathies, attractions, &c.

Experimental Physics, which enquires into the reaſons and natures of things from experiments: ſuch as thoſe in chemiſtry, hydroſtatics, pneumatics, optics, &c. And

Mechanical or *Corpuſcular* Physics, which explains the appearances of nature from the matter, motion, ſtructure, and figure of bodies and their parts: all according to the ſettled laws of nature and mechanics. See each of theſe articles under its own head.

PIASTER, a Spaniſh money, more uſually called Piece of Eight, about the value of 4s. 6d.

PIAZZA, popularly called Piache, an Italian name for a portico, or covered walk, ſupported by arches.

PICARD (John), an able mathematician of France, and one of the moſt learned aſtronomers of the 17th century, was born at Fleche, and became prieſt and prior of Rillie in Anjou. Coming afterwards to Paris, his ſuperior talents for mathematics and aſtronomy ſoon made him known and reſpected. In 1666 he was appointed aſtronomer in the Academy of Sciences. And five years after, he was ſent, by order of the king, to the caſtle of Uraniburgh, built by Tycho Brahe in Denmark, to make aſtronomical obſervations there; and from thence he brought the original manuſcripts, written by Tycho Brahe; which are the more valuable, as they differ in many places from the printed copies, and

contain a book more than has yet appeared. Thefe difcoveries were followed by many others, particularly in aftronomy: He was one of the firft who applied the telefcope to aftronomical quadrants: he firft executed the work called, *La Connoiffance des Temps*, which he calculated from 1679 to 1683 inclufively: he firft obferved the light in the vacuum of the barometer, or the mercurial phofphorus: he alfo firft of any went through feveral parts of France, to meafure the degrees of the French meridian, and firft gave a chart of the country, which the Caffinis afterwards carried to a great degree of perfection. He died in 1682 or 1683, leaving a name dear to his friends, and refpectable to his contemporaries and to pofterity. His works are,

1. A treatife on Levelling.
2. Practical Dialling by calculation.
3. Fragments of Dioptrics.
4. Experiments on Running Water.
5. Of Meafurements.
6. Menfuration of Fluids and Solids.
7. Abridgment of the Meafure of the Earth.
8. Journey to Uraniburgh, or Aftronomical Obfervations made in Denmark.
9. Aftronomical Obfervations made in divers parts of France.
10. La Connoiffance des Temps, from 1679 to 1683.

All thefe, and fome other of his works, which are much efteemed, are given in the 6th and 7th volumes of the Memoirs of the Academy of Sciences.

PICCOLOMINI (Alexander), archbifhop of Patras, and a native of Sienna, where he was born about the year 1508. He was of an illuftrious and ancient family, which came originally from Rome, but afterwards fettled at Sienna. He compofed with fuccefs for the theatre; but he was not more diftinguifhed by his genius, than by the purity of his manners, and his regard to virtue. His charity was great; and was chiefly exerted in relieving the neceffities of men of letters. He was the firft who made ufe of the Italian language in writing upon philofophical fubjects. He died at Sienna the 12th of March 1578, at 70 years of age, leaving behind him a number of works in Italian, on a variety of fubjects. A particular catalogue of them may be feen in the Typographical Dictionary; the principal of which are the following:

1. Various Dramatical pieces.
2. A treatife on the Sphere.
3. A Theory of the Planets.
4. Tranflation of Ariftotle's Art of Rhetoric and Poetry.
5. A Syftem of Morality, publifhed at Venice, 1575, in 4to; tranflated into French by Peter de Larivey, and printed at Paris, 1581, in 4to.

Thefe, with a variety of other works, prove his extenfive knowledge in natural philofophy, mathematics, and theology.

Piccolomini (Francis), of the fame family with the foregoing, was born in 1520, and taught philofophy with fuccefs, for the fpace of 22 years, in the moft celebrated univerfities of Italy, and afterwards retired to Sienna, where he died, in 1604, at 84 years of age. He was fo much and fo generally refpected, that the city went into mourning on his death.

Piccolomini laboured to revive the doctrine of Plato, and endeavoured alfo to imitate the manners of that philofopher. He had for his rival the famous James Zabar Alla, whom he excelled in facility of expreffion and neatnefs of diction; but to whom he was much inferior in point of argument, becaufe he did not examine matters to the bottom as the other did; but paffed too rapidly from one propofition to another.

PICKET, *Picquet*, or *Piquet*, in Fortification &c, a ftake fharp at one end, and ufually fhod with iron; ufed in laying out ground, to mark the feveral bounds and angles of it. There are alfo larger Pickets, driven into the earth, to hold together fafcines or faggots, in works that are thrown up in hafte. As alfo various forts of fmaller Pickets for divers other ufes.

PIECES, in Artillery, include all forts of great guns and mortars; meaning Pieces of ordnance, or of artillery.

PIEDOUCHE, in Architecture, a little ftand, or pedeftal, either oblong or fquare, enriched with mouldings; ferving to fupport a buft, or other little figure; and is more ufually called a bracket pedeftal.

PIEDROIT, in Architecture, a kind of fquare pillar, or pier, partly hid within a wall. Differing from the Pilafter by having no regular bafe nor capital.

Piedroit is alfo ufed for a part of the folid wall annexed to a door or window; comprehending the doorpoft, chambranle, tableau, leaf, &c.

PIER, in Building, denotes a mafs of ftone, &c, oppofed by way of fortrefs, againft the force of the fea, or a great river, for the fecurity of fhips lying in any harbour or haven. Such are the Piers at Dover, or Ramfgate, or Yarmouth, &c.

Piers are alfo ufed in Architecture for a kind of pilafters, or buttreffes, raifed for fupport, ftrength, and fometimes for ornament.

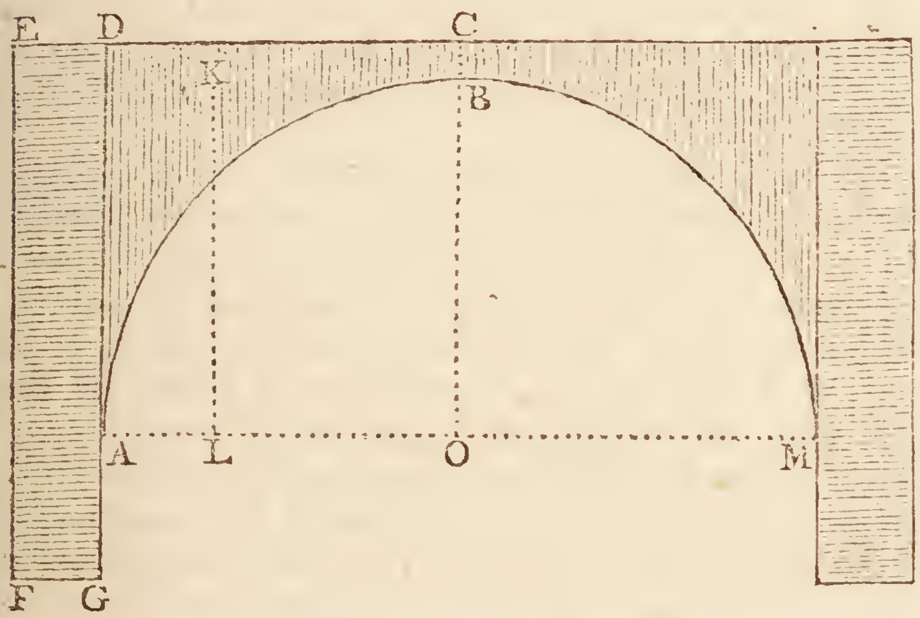
Circular Piers, are called Maffive Columns, and are either with or without caps. Thefe are often feen in Saracenic architecture.

Piers, of a Bridge, are the walls built to fupport the arches, and from which they fpring as bafes, to ftand upon.

Piers fhould be built of large blocks of ftone, folid throughout, and cramped together with iron, which will make the whole as one folid ftone. Their extremities, or ends, from the bottom, or bafe, up to high-water mark, ought to project fharp out with a faliant angle, to divide the ftream. Or perhaps the bottom part of the Pier fhould be built flat or fquare up to about half the height of low-water mark, to encourage a lodgment againft it for the fand and mud, to cover the foundation; left, being left bare, the water fhould in time undermine and ruin it. The beft form of the projection for dividing the ftream, is the triangle; and the longer it is, or the more acute the faliant angle, the better it will divide it, and the lefs will the force of the water be againft the Pier; but it may be fufficient to make that angle a right one, as it will render the mafonry ftronger, and in that cafe the perpendicular projection will be equal to half the breadth or thicknefs of the Pier. In rivers where large heavy craft navigate, and pafs the arches, it may perhaps be better to make the ends femicircular; for though this figure does not divide the water fo well as the triangle, it will better turn off, and bear the fhock of the craft.

 The

The thicknefs of the Piers ought to be fuch as will make them of weight, or ftrength, fufficient to fupport their interjacent arch, independent of the affiftance of any other arches. And then, if the middle of the Pier be run up to its full height, the centring may be ftruck, to be ufed in another arch, before the hanches or fpandrels are filled up. They ought alfo to be made with a broad bottom on the foundation, and gradually diminifhed in thicknefs by offsets up to low-water mark.



To find the thicknefs FG *of the Piers*, neceffary to fupport an arch ABM, this is a general rule. Let K be the centre of gravity of the half arch ADCB, A = its area; KL perpendicular to AM the fpan of the arch, OB its height, and BC its thicknefs at the crown: then is the thicknefs of the pier

$$FG = \sqrt{\frac{2GA \times AL}{EF \times KL} \times A}.$$

Some authors pretend to give numbers, in tables, for this purpofe; but they are very erroneous. See my treatife on the Principles of Bridges, fect. 3.

PIKE, an offenfive weapon, confifting of a fhaft of wood, 12 or 14 feet long, headed with a flat-pointed fteel, called the fpear.

Pliny fays the Lacedemonians were the inventors of the Pike. The Macedonian phalanx was evidently a battalion of Pikemen.

The Pike was long ufed by the infantry, to enable them to fuftain the attack of the cavalry; but it is now taken from them, and the bayonet, fixed to the muzzle of the firelock, is given inftead of it.—It is ftill ufed by fome officers of infantry, under the name of fpontoon.

Half PIKE is the weapon carried by an officer of foot; being only 8 or 9 feet long.

PILASTER, in Architecture, a fquare column, fometimes infulated, but more frequently let within a wall, and only projecting by a 4th or 5th part of its thicknefs.

The Pilafter is different in the different orders; borrowing the name of each order, and having the fame proportions, and the fame capitals, members, and ornaments, with the columns themfelves.

Demi-PILASTER, called alfo *Membretto*, is a Pilafter that fupports an arch; and it generally ftands againft a pier or column.

PILES, in Building, are large ftakes, or beams, fharpened at the end, and fhod with iron, to be driven into the ground, for a foundation to build upon in marfhy places.

Amfterdam, and fome other cities, are wholly built upon Piles. The ftoppage of Dagenham-breach was effected by dove-tail Piles, that is by Piles mortifed into one another by a dovetail joint.

Piles are driven down by blows of a large iron weight, ram, or hammer, dropped continually upon them from a height, till the Pile is funk deep enough into the ground.

Notwithftanding the momentum, or force of a body in motion, is as the weight multiplied by the velocity, or fimply as its velocity, the weight being given, or conftant; yet the effect of the blow will be nearly as the fquare of that velocity, the effect being the quantity the Pile finks in the ground by the ftroke. For the force of the blow, which is transferred to the Pile, being deftroyed, in fome certain definite time, by the friction of the part which is within the earth, which is nearly a conftant quantity; and the fpaces, in conftant forces, being as the fquares of the velocities; therefore the effects, which are thofe fpaces funk, are nearly as the fquare of the velocities; or, which is the fame thing, nearly as the heights fallen by the ram or hammer, to the head of the Pile. See, upon this fubject, Leopold Belidor, alfo Defaguliers's Exper. Philof. vol. 1, pa. 336, and vol. 2, pa. 417: and Philof. Tranf. 1779, pa. 120.

There have been various contrivances for raifing and dropping the hammer, for driving down the Piles; fome fimple and moved by ftrength of men, and fome complex and by machinery; but the completeft Pile-Driver is efteemed that which was employed in driving the Piles in the foundation of Weftminfter bridge. This machine was the invention of a Mr. Vauloue, and the defcription of it is as follows.

Defcription of Vauloue's PILE-*Driver*. See fig. 2, pl. xx. A is the great upright fhaft or axle, carrying the great wheel B and drum C, and turned by horfes attached to the bars S, S. The wheel B turns the trundle X, having a fly O at the top, to regulate the motion, and to act againft the horfes, and keep them from falling when the heavy ram Q is difengaged to drive the Pile P down into the mud &c. in the bottom of the river. The drum C is loofe upon the fhaft A, but is locked to the wheel B by the bolt Y. On this drum the great rope HH is wound; one end of it being fixed to the drum, and the other to the follower G, paffing over the pulleys I and K. In the follower G are contained the tongs F, which take hold of the ram Q, by the ftaple R for drawing it up. D is a fpiral or fufee fixed to the drum, on which winds the fmall rope T which goes over the pulley U, under the pulley V, and is faftened to the top of the frame at 7. To the pulley-block V is hung the counterpoife W, which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its defcent, the line T winds downwards upon the fufee, on a larger and larger radius, by which means the counterpoife W acts ftronger and ftronger againft it; and fo allows it to come down with only a moderate and uniform velocity. The bolt Y locks the drum

drum to the great wheel, being pushed upward by the small lever 2, which goes through a mortise in the shaft A, turns upon a pin in the bar 3 fixed into the great wheel B, and has a weight 4, which always tends to push up the bolt Y through the wheel into the drum. L is the great lever turning on the axis *m*, and resting upon the forcing bar 5, 5, which goes down through a hollow in the shaft A, and bears upon the little lever 2.

By the horses going round, the great rope H is wound about the drum C, and the ram Q is drawn up by the tongs F in the follower G, till they come between the inclined planes E; which, by shutting the tongs at the top, open them below, and so discharge the ram, which falls down between the guide posts *bb* upon the Pile P, and drives it by a few strokes as far into the ground as it can go, or as is desired; after which, the top part is sawed off close to the mud, by an engine for that purpose. Immediately after the ram is discharged, the piece 6 upon the follower G takes hold of the ropes *aa*, which raise the end of the lever L, and cause its end N to descend and press down the forcing bar 5 upon the little lever 2, which, by drawing down the bolt Y, unlocks the drum C from the great wheel B; and then the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs slip over the staple R, and the weight of their heads causes them to fall outward, and shuts upon it. Then the weight 4 pushes up the bolt Y into the drum, which locks it to the great wheel, and so the ram is drawn up as before.

As the follower comes down, it causes the drum to turn backward, and unwinds the rope from it, while the horses, the great wheel, trundle, and fly, go on with an uninterrupted motion: and as the drum is turning backward, the counterpoise W is drawn up, and its rope T wound upon the spiral fusee D.

There are several holes in the under side of the drum, and the bolt Y always takes the first one that it finds when the drum stops by the falling of the follower upon the ram; till which stoppage, the bolt has not time to slip into any of the holes.

The peculiar advantages of this engine are, that the weight, called the ram, or hammer, may be raised with the least force; that, when it is raised to a proper height, it readily disengages itself and falls with the utmost freedom; that the forceps or tongs are lowered down speedily, and instantly of themselves again lay hold of the ram, and lift it up; on which account this machine will drive the greatest number of piles in the least time, and with the fewest labourers.

This engine was placed upon a barge on the water, and so was easily conveyed to any place desired. The ram was a ton weight; and the guides *b*, *b*, by which it was let fall, were 30 feet high.

A new machine for driving piles has been invented lately by Mr. S. Bunce of Kirby-street, Hatton-street, London. This, it is said, will drive a greater number of Piles in a given time than any other; and that it can be constructed more simply to work by horses than Vauloue's engine above described.

Fig. 3 and 4, plate xx, represent a side and front section of the machine. The chief parts are, A, fig. 3, which are two endless ropes or chains, connected by cross pieces of iron B (fig. 4) corresponding with two cross grooves cut diametrically opposite in the wheel C (fig. 3) into which they are received; and by which means the rope or chain A is carried round. FHK is a side-view of a strong wooden frame moveable on the axis H. D is a wheel, over which the chain passes and turns within at the top of the frame. It moves occasionally from F to G upon the centre H, and is kept in the position F by the weight I fixed to the end K. In fig. 5, L is the iron ram, which is connected with the cross pieces by the hook M. N is a cylindrical piece of wood suspended at the hook at O, which by sliding freely upon the bar that connects the hook to the ram, always brings the hook upright upon the chain when at the bottom of the machine, in the position of GP. See fig. 3.

When the man at S turns the usual crane-work, the ram being connected to the chain, and passing between the guides, is drawn up in a perpendicular direction; and when it is near the top of the machine, the projecting bar Q of the hook strikes against a cross piece of wood at R (fig. 3); and consequently discharges the ram, while the weight I of the moveable frame instantly draws the upper wheel into the position shewn at F, and keeps the chain free of the ram in its descent. The hook, while descending, is prevented from catching the chain by the wooden piece N: for that piece being specifically lighter than the iron weight below, and moving with a less degree of velocity, cannot come into contact with the iron, till it is at the bottom, and the ram stops. It then falls, and again connects the hook with the chain, which draws up the ram, as before.

Mr. Bunce has made a model of this machine, which performs perfectly well; and he observes, that, as the motion of the wheel C is uninterrupted, there appears to be the least possible time lost in the operation.

Pile is also used among Architects, for a mass or body of building.

Pile, in Artillery, denotes a collection or heap of shot or shells, piled up by horizontal courses into either a pyramidal or else a wedge-like form; the base being an equilateral triangle, a square, or a rectangle. In the triangle and square, the Pile terminates in a single ball or point, and forms a pyramid, as in plate xix, fig. 4 and 5, but with the rectangular base, it finishes at top in a row of balls, or an edge, forming a wedge, as in fig. 6.

In the triangular and square Piles, the number of horizontal rows, or courses, or the number counted on one of the angles from the bottom to the top, is always equal to the number counted on one side, in the bottom row. And in rectangular Piles, the number of rows, or courses, is equal to the number of balls in the breadth of the bottom row, or shorter side of the base: also, in this case, the number in the top row, or edge, is one more than the difference between the length and breadth of the base. All which is evident from the inspection of the figures, as above.

The courses in these Piles are figurate numbers.

In a triangular Pile, each horizontal course is a triangular number, produced by taking the successive sums of the ordinate numbers, viz,

$$1 = 1$$
$$1 + 2 = 3$$
$$1 + 2 + 3 = 6$$
$$1 + 2 + 3 + 4 = 10, \text{ \&c.}$$

And

And the number of ſhot in the triangular Pile, is the ſum of all theſe triangular numbers, taken as far, or to as many terms, as the number in one ſide of the baſe. And therefore, to find this ſum, or the number of all the ſhot in the Pile, multiply continually together, the number in one ſide of the baſe row, and that number increaſed by 1, and the ſame number increaſed by 2; then $\frac{1}{6}$ of the laſt product will be the anſwer, or number of all the ſhot in the Pile.

That is, $\frac{n \cdot \overline{n+1} \cdot \overline{n+2}}{6}$ is the ſum;

where n is the number in the bottom row.

Again, in Square Piles, each horizontal courſe is a ſquare number, produced by taking the ſquare of the number in its ſide, or the ſucceſſive ſums of the odd numbers, thus,

$$\begin{aligned} 1 &= 1 \\ 1+3 &= 4 \\ 1+3+5 &= 9 \\ 1+3+5+7 &= 16, \text{ \&c.} \end{aligned}$$

And the number of ſhot in the ſquare Pile is the ſum of all theſe ſquare numbers, continued ſo far, or to as many terms, as the number in one ſide of the baſe. And therefore, to find this ſum, multiply continually together, the number in one ſide of the bottom courſe, and that number increaſed by 1, and double the ſame number increaſed by 1; then $\frac{1}{6}$ of the laſt product will be the ſum or anſwer.

That is, $\frac{n \cdot \overline{n+1} \cdot \overline{2n+1}}{6}$ is the ſum.

In a rectangular Pile, each horizontal courſe is a rectangle, whoſe two ſides have always the ſame difference as thoſe of the baſe courſe, and the breadth of the top row, or edge, being only 1: becauſe each courſe in aſcending has its length and breadth always leſs by 1 than the courſe next below it. And theſe rectangular courſes are found by multiplying ſucceſſively the terms or breadths 1, 2, 3, 4, &c, by the ſame terms added to the conſtant difference of the two ſides d; thus,

$$\begin{aligned} 1 \cdot \overline{1+d} &= 1 + d \\ 2 \cdot \overline{2+d} &= 4 + 2d \\ 3 \cdot \overline{3+d} &= 9 + 3d \\ 4 \cdot \overline{4+d} &= 16 + 4d, \text{ \&c.} \end{aligned}$$

And the number of ſhot in the rectangular Pile is the ſum of all theſe rectangles, which, it is evident, conſiſt of the ſum of the ſquares, together with the ſum of an arithmetical progreſſion, continued till the number of terms be the difference between the length and breadth of the baſe, and 1 leſs than the edge or top row. And therefore, to find this ſum, multiply continually together, the number in the breadth of the baſe row, the ſame number increaſed by 1, and double the ſame number increaſed by 1, and alſo increaſed by triple the difference between the length and breadth of the baſe; then $\frac{1}{6}$ of the laſt product will be the anſwer.

That is, $\frac{b \cdot \overline{b+1} \cdot \overline{2b+3d+1}}{6}$ is the ſum.

where b is the breadth of the baſe, and d the difference between the length and breadth of the bottom courſe.

As to incomplete Piles, which are only fruſtums, wanting a ſimilar ſmall Pile at the top; it is evident that the number in them will be found, by firſt computing the number in the whole Pile, as if it were complete, and alſo the number in the ſmall Pile wanting at top, both by their proper rule; and then ſubtracting the one number from the other.

In piling of ſhot, when room is an object, it may be obſerved that the ſquare Pile is the leaſt eligible, of any, as it takes up more room, in proportion to the number of ſhot contained in it, than either of the other two forms; and that the rectangular Pile is the moſt eligible, as taking up the leaſt room in proportion to the number it contains.

PILLAR, a kind of irregular column, round, and inſulated, or detached from the wall. Pillars are not reſtricted to any rules, their parts and proportions being arbitrary; ſuch for example as thoſe that ſupport Saracenic vaults, and other buildings, &c.

PINION, in Mechanics, is an arbor, or ſpindle, in the body of which are ſeveral notches, which are catched by the teeth of a wheel that ſerves to turn it round. Or a Pinion is any leſſer wheel that plays in the teeth of a larger.

In a watch, &c, the notches of a Pinion are called leaves, and not teeth, as in other wheels; and their number is commonly 4, 5, 6, 8, &c.

Pinion *of Report*, is that Pinion, in a watch, commonly fixed on the arbor of a great wheel: and which uſed to have but four leaves in old watches; it drives the dial-wheel, and carries about the hand.

The number of turns to be laid upon the Pinion of report, is found by this proportion: as the beats in one turn of the great wheel, are to the beats in an hour, ſo are the hours on the face of the clock (viz 12 or 24), to the quotient of the hour-wheel or dial-wheel divided by the Pinion of report, that is, by the number of turns which the Pinion of report hath in one turn of the dial-wheel. Which in numbers is 26928 : 20196 :: 12 : 9. —Or thus; as the hours of the watch's going, are to the numbers of the turns of the fuſee, ſo are the hours of the face, to the quotient of the Pinion of report. So, if the hours be 12, then as 16 : 12 :: 12 : 9; but if 24, then as 16 : 12 :: 24 : 18.

This rule may ſerve to lay the Pinion of report on any other wheel, thus: as the beats in one turn of any wheel, are to the beats in an hour, ſo are the hours of the face, or dial-plate, of the watch, to the quotient of the dial-wheel divided by the Pinion of report, fixed on the ſpindle of the aforeſaid wheel.

PINT, a meaſure of capacity, being the 8th part of a gallon, both in ale and wine meaſure, &c. The wine Pint of pure ſpring water, weighs near 17 ounces avoirdupois, and the ale Pint a little above 20 ounces.

The Paris Pint contains about 2 pounds of common water. And the Scotch Pint contains $108\frac{2}{5}$ cubic inches, and therefore contains 3 Engliſh Pints.

PISCES, the 12th ſign or conſtellation in the zodiac; in the form of two fiſhes tied together by the tails.

The Greeks, who have ſome fable to account for the origin of every conſtellation, tell us, that when Venus and Cupid were one time on the banks of the Euphrates, there appeared before them that terrible giant Typhon, who was ſo long a terror to all the Gods. Theſe deities immediately, they ſay, threw themſelves

into the water, and were there changed into thefe two fifhes, the Pifces, by which they efcaped the danger. But the Egyptians ufed the figns of the zodiac as part of their hieroglyphic language, and by the 12 they conveyed an idea of the proper employment during the 12 months of the year. The Ram and the Bull had, at that time, taken to the increafe of their flock, the young of thofe animals being then growing up; the maid Virgo, a reaper in the field, fpoke the approach of harveft; Sagittary declared autumn the time for hunting; and the Pifces, or fifhes tied together, in token of their being taken, reminded men that the approach of fpring was the time for fifhing.

The Ancients, as they gave one of the 12 months of the year to the patronage of each of the 12 fuperior deities, fo they alfo dedicated to, or put under the tutelage of each, one of the 12 figns of the zodiac. In this divifion, the fifhes naturally fell to the fhare of Neptune; and hence arifes that rule of the aftrologers, which throws every thing that regards the fate of fleets and merchandize, under the more immediate patronage and protection of this conftellation.

The ftars in the fign Pifces are, in Ptolomy's catalogue 38, in Tycho's 36, in Hevelius's 39, and in the Britannic catalogue 113.

PISCIS *Auftralis*, the Southern Fifh, is a conftellation of the fouthern hemifphere, being one of the old 48 conftellations mentioned by the Ancients.

The Greeks have here again the fable of Venus and her fon throwing themfelves into the fea, to efcape from the terrible Typhon. This fable is probably borrowed from the hieroglyphics of the Egyptians. With them, a fifh reprefented the fea, its element; and Typhon was probably a land flood, perhaps reprefented by the fign Aquarius, or water pourer, whofe ftream or river is reprefented as fwallowed up by this fifh, as the land floods and rivers are by the fea. And Venus was fome queen, perhaps Semiramis, otherwife called Hamamah, who took to the river or the fea with her fon, in a veffel, to avoid the flood, &c.

The remarkable ftar Fomahaut, of the 1ft magnitude, is juft in the mouth of this fifh. The ftars of this conftellation are, in Ptolomy's catalogue 18, and in Flamfteed's 24.

PISCIS *Volans*, the Flying Fifh, is a fmall conftellation of the fouthern hemifphere, unknown to the Ancients, but added by the Moderns. It is not vifible in our latitude, and contains only 8 ftars.

PISTOLE, a gold coin in Spain, Italy, Switzerland, &c, of the value of about 16s. 6d.

PISTON, a part or member in feveral machines, particularly pumps, air-pumps, fyringes, &c; called alfo the Embolus, and popularly the Sucker.

The Pifton of a pump is a fhort cylinder of wood or metal, fitted exactly to the cavity of the barrel, or body; and which, being worked up and down alternately, raifes the water; and when raifed, preffes it again, fo as to make it force up a valve with which it is furnifhed, and fo efcape through the fpout of the pump.

There are two forts of Piftons ufed in pumps; the one with a valve, called a bucket; and the other without a valve, called a forcer.

PLACE, in Philofophy, that part of infinite fpace which any body poffeffes.

Ariftotle and his followers divide Place into External and Internal.

Internal PLACE, is that fpace or room which the body contains. And

External PLACE, is that which includes or contains the body; and is by Ariftotle called the firft or concave and immoveable furface of the ambient body.

Newton better, and more intelligibly, diftinguifhes Place into Abfolute and Relative.

Abfolute and *Primary* PLACE, is that part of infinite and immoveable fpace which a body poffeffes. And

Relative, or *Secondary* PLACE, is the fpace it poffeffes confidered with regard to other adjacent objects.

Dr. Clark adds another kind of Relative Place, which he calls Relatively Common Place; and defines it, that part of any moveable or meafurable fpace which a body poffeffes; which Place moves together with the body.

PLACE, Mr. Locke obferves, is fometimes likewife taken for that portion of infinite fpace poffeffed by the material world; though this, he adds, were more properly called extenfion. The proper idea of Place, according to him, is the relative pofition of any thing, with regard to its diftance from certain fixed points; whence it is faid a thing has or has not changed Place, when its diftance is or is not altered with refpect to thofe bodies.

PLACE, in Optics, or *Optical* PLACE, is the point to which the eye refers an object.

Optic PLACE of a ftar, is a point in the furface of the mundane fphere in which a fpectator fees the centre of the ftar, &c.—This is divided into True and Apparent.

True, or *Real Optic* PLACE, is that point of the furface of the fphere, where a fpectator at the centre of the earth would fee the ftar, &c.

Apparent, or *Vifible Optic* PLACE, is that point of the furface of the fphere, where a fpectator at the furface of the earth fees the ftar, &c.

The diftance between thefe two optic Places makes what is called the Parallax.

PLACE *of the Sun*, or *Moon*, or *Star*, or *Planet*, in Aftronomy, fimply denotes the fign and degree of the zodiac which the luminary is in; and is ufually expreffed either by its latitude and longitude, or by its right afcenfion and declination.

PLACE *of Radiation*, in Optics, is the interval or fpace in a medium, or tranfparent body, through which any vifible object radiates.

PLACE, in Geometry, ufually called *Locus*, is a line ufed in the folution of problems, being that in which the determination of every cafe of the problem lies. See LOCUS, *Plane*, *Simple*, *Solid*, &c.

PLACE, in War and Fortification, a general name for all kinds of fortreffes, where a party may defend themfelves.

PLACE *of Arms*, a ftrong part where the arms &c are depofited, and where ufually the foldiers affemble and are drawn up.

PLAFOND, or PLATFOND, in Architecture, the cieling of a room.

PLAIN &c. See PLANE.

PLAN, a reprefentation of fomething, drawn on a plane. Such as maps, charts, and ichnogrophies.

PLAN, in Architecture, is particularly ufed for a draught of a building; fuch as it appears, or is intended to appear, on the ground; fhewing the extent, divifion, and diftribution of its area into apartments, rooms, paffages, &c. It is alfo called the Ground Plot, Platform, and Ichnography of the building; and is the firft device or fketch the architect makes.

Geometrical PLAN, is that in which the folid and vacant parts are reprefented in their natural proportion.

Raifed PLAN, is that where the elevation, or upright, is fhewn upon the geometrical Plan, fo as to hide the diftribution.

Perfpective PLAN, is that which is conducted and exhibited by degradations, or diminutions, according to the rules of Perfpective.

PLANE, or FLAIN, in Geometry, denotes a Plane figure, or a furface lying evenly between its bounding lines. Euclid.

Some define a Plane, a furface, from every point of whofe perimeter a right line may be drawn to every other point in the fame, and always coinciding with it.

As the right line is the fhorteft extent from one point to another, fo is a Plane the fhorteft extenfion between one line and another.

PLANES are much ufed in Aftronomy, conic fections, fpherics, &c, for imaginary furfaces, fuppofed to cut and pafs through folid bodies.

When a Plane cuts a cone parallel to one fide, it makes a parabola; when it cuts the cone obliquely, an ellipfe or hyperbola; and when parallel to its bafe, a circle. Every fection of a fphere is a circle.

The fphere is wholly explained by Planes, conceived to cut the celeftial bodies, and to fill the areas or circumferences of the orbits. They are differently inclined to each other; and by us the inhabitants of the earth, the Plane of whofe orbit is the Plane of the ecliptic, their inclination is eftimated with regard to this Plane.

PLANE *of a Dial*, is the furface on which a dial is fuppofed to be defcribed.

PLANE, in Mechanics. A *Horizontal* PLANE, is a Plane that is level, or parallel to the horizon.

Inclined PLANE, is one that makes an oblique angle with a horizontal Plane.

The doctrine of the motion of bodies on Inclined Planes, makes a very confiderable article in mechanics, and has been fully explained under the articles, MECHANICAL *Powers*, and INCLINED *Plane*.

PLANE *of Gravity*, or *Gravitation*, is a Plane fuppofed to pafs through the centre of gravity of the body, and in the direction of its tendency; that is, perpendicular to the horizon.

PLANE *of Reflection*, in Catoptrics, is a Plane which paffes through the point of reflection; and is perpendicular to the Plane of the glafs, or reflecting body.

PLANE *of Refraction*, is a Plane paffing through the incident and refracted ray.

Perfpective PLANE, is a Plane tranfparent furface, ufually perpendicular to the horizon, and placed between the fpectator's eye and the object he views; through which the optic rays, emitted from the feveral points of the object, are fuppofed to pafs to the eye, and in their paffage to leave marks that reprefent them on the faid Plane.—Some call this the Table, or Picture, becaufe the draught or Perfpective of the object is fuppofed to be upon it. Others call it the Section, from its cutting the vifual rays; and others again the Glafs, from its fuppofed tranfparency.

Geometrical PLANE, in Perfpective, is a Plane parallel to the horizon, upon which the object is fuppofed to be placed that is to be drawn.

Horizontal PLANE, in Perfpective, is a Plane paffing through the fpectator's eye, parallel to the horizon.

Vertical PLANE, in Perfpective, is a Plane paffing through the fpectator's eye, perpendicular to the geometrical Plane, and ufually at right angles to the perfpective Plane.

Objective PLANE, in Perfpective, is any Plane fituate in the horizontal Plane, of which the reprefentation in perfpective is required.

PLANE *of the Horopter*, in Optics, is a Plane paffing through the horopter AB, and perpendicular to a Plane paffing through the two optic axes CH and CI. See the fig. to the article HOROPTER.

PLANE *of the Projection*, is the Plane upon which the fphere is projected.

PLANE *Angle*, is an angle contained under two lines or furfaces.—It is fo called in contradiftinction to a folid angle, which is formed by three or more Planes.

PLANE *Triangle*, is a triangle formed by three right lines; in oppofition to a fpherical and a mixt triangle.

PLANE *Trigonometry* is the doctrine of Plane triangles, their meafures, proportions, &c. See TRIGONOMETRY.

PLANE *Glafs*, or *Mirror*, in Optics, is a glafs or mirror having a flat or even furface.

PLANE *Chart*, in Navigation, is a fea-chart, having the meridians and parallels reprefented by parallel ftraight lines; and confequently having the degrees of longitude the fame in every part. See CHART.

PLANE *Number*, is that which may be produced by the multiplication of two numbers the one by the other. Thus, 6 is a plane number, being produced by the multiplication of the two numbers 2 and 3; alfo 15 is a Plane number, being produced by the multiplication of the numbers 3 and 5. See NUMBER.

PLANE *Place*, *Locus Planus*, or *Locus ad Planum*, is a term ufed by the ancient geometricians, for a geometrical locus, when it was a right line or a circle, in oppofition to a folid place, which was one of the conic fections.

Thefe Plane Loci are diftinguifhed by the Moderns into Loci ad Rectum, and Loci ad Circulum. See LOCUS.

PLANE *Problem*, is fuch a one as cannot be refolved geometrically, but by the interfection either of a right line and a circle, or of the circumferences of two circles. Such as this problem following: viz, Given the hypothenufe, and the fum of the other two fides, of a right-angled triangle; to find the triangle. Or this: Of four given lines to form a trapezium of a given area.

PLANE *Sailing*, in Navigation, is the art of working the feveral cafes and varieties in a fhip's motion on a Plane chart; or of navigating a fhip upon principles

 deduced

deduced from the notion of the earth's being an extended Plane.

This principle, though notoriously false, yet places being laid down accordingly, and a long voyage broken into many short ones, the voyage may be performed tolerably well by it, especially near the same meridian.

In Plain Sailing it is supposed that these three, the rhumb line, the meridian, and parallel of latitude, will always form a right-angled triangle; and so posited, as that the perpendicular side will represent part of the meridian, or north and south line, containing the difference of latitude; the base of the triangle, the departure, or east-and-west line; and the hypothenuse the distance sailed. The angle at the vertex is the course; and the angle at the base, the complement of the course; any two of which, besides the right angle, being given, the triangle may be protracted, and the other three parts found.

For the doctrine of Plane Sailing, see SAILING.

PLANE *Scale*, is a thin ruler, upon which are graduated the lines of chords, sines, tangents, secants, leagues, rhumbs, &c; being of great use in most parts of the mathematics, but especially in navigation. See its description and use under SCALE.

PLANE *Table*, an instrument much used in land-surveying; by which the draught, or plan, is taken upon the spot, as the survey or measurement goes on, without any future protraction, or plotting.

This instrument consists of a Plane rectangular board, of any convenient size, the centre of which, when used, is fixed by means of screws to a three-legged stand, having a ball and socket, or universal joint, at the top, by means of which, when the legs are fixed on the ground, the table is inclined in any direction. To the table belongs,

1. A frame of wood, made to fit round its edges, for the purpose of fixing a sheet of paper upon the table. The one side of this frame is usually divided into equal parts, by which to draw lines across the table, parallel or perpendicular to the sides; and the other side of the frame is divided into 360 degrees, from a centre which is in the middle of the table; by means of which the table is to be used as a theodolite, &c.

2. A magnetic needle and compass screwed into the side of the table, to point out directions and be a check upon the sights.

3. An index, which is a brass two-foot scale, either with a small telescope, or open sights erected perpendicularly upon the ends. These sights and the fiducial edge of the index are parallel, or in the same Plane.

General Use of the PLANE *Table.*

To use this instrument properly, take a sheet of writing or drawing paper, and wet it to make it expand; then spread it flat upon the table, pressing down the frame upon the edges, to stretch it, and keep it fixed there; and when the paper is become dry, it will, by shrinking again, stretch itself smooth and flat from any cramps or unevenness. Upon this paper is to be drawn the plan or form of the thing measured.

The general use of this instrument, in land-surveying, is to begin by setting up the table at any part of the ground you think the most proper, and make a point upon a convenient part of the paper or table, to represent that point of the ground; then fix in that point of the paper one leg of the compasses, or a fine steel pin, and apply to it the fiducial edge of the index, moving it round the table, close by the pin, till through the sights you perceive some point desired, or remarkable object, as the corner of a field, or a picket set up, &c; and from the station point draw a dry or obscure line along the fiducial edge of the index. Then turn the index to another object, and draw a line on the paper towards it. Do the same by another; and so on till as many objects are set as may be thought necessary. Then measure from your station towards as many of the objects as may be necessary, and no more, taking the requisite offsets to corners or crooks in the hedges, &c; laying the measured distances, from a proper scale, down upon the respective lines on the paper. Then move the table to any of the proper places measured to, for a second station, fixing it there in the original position, turning it about its centre for that purpose, both till the magnetic needle point to the same degree of the compass as at first, and also by laying the fiducial edge of the index along the line between the two stations, and turning the table till through the index the former station can be seen; and then fix the table there: from this new station repeat the same operations as at the former; setting several objects, that is, drawing lines towards them, on the paper, by the edge of the index, measuring and laying off the distances. And thus proceed from station to station; measuring only such lines as are necessary, and determining as many as you can by intersecting lines of direction drawn from different stations.

Of Shifting the Paper on the PLANE *Table.* When one paper is full of the lines &c measured, and the survey is not yet completed; draw a line in any manner through the farthest point of the last station line to which the work can be conveniently laid down; then take the sheet off the table, and fix another fair sheet in its place, drawing a line upon it, in a part of it the most convenient for the rest of the work, to represent the line drawn at the end of the work on the former paper. Then fold or cut the old sheet by the line drawn upon it; apply it so to the line on the new sheet, and, as they lie together in that position, continue or produce the last station line of the old sheet upon the new one; and place upon it the remainder of the measurement of that line, beginning at where the work left off on the old sheet. And so on, from one sheet to another, till the whole work is completed.

But it is to be noted, that if the said joining lines, upon the old and new sheet, have not the same inclination to the side of the table, the needle will not respect or point to the original degree of the compass, when the table is rectified. But if the needle be required to respect still the same degree of the compass, the easiest way then of drawing the lines in the same position, is to draw them both parallel to the same sides of the table, by means of the equal parallel divisions marked on the other two sides of the frame.

When the work of surveying is done, and you would fasten all the sheets together into one piece, or rough plan, the aforesaid lines are to be accurately joined together, in the same manner as when the lines were transferred from the old sheets to the new ones.

See more full directions for the use of the Plane-Table,

Table, illustrated with various examples, in my Treatise on Mensuration, 2d edit. pa. 509 &c.

PLANET, literally a wanderer, or a wandering ſtar, in oppoſition to a ſtar, properly ſo called, which remains fixed. It is a celeſtial body, revolving around the ſun, or ſome other planet, as a centre, or at leaſt as a focus, and with a moderate degree of excentricity, ſo that it never is ſo much farther from the ſun at one time than at another, but that it can be ſeen as well from one part of its orbit as another; as diſtinguiſhed from the comets, which on the fartheſt part of their trajectory go off to ſuch vaſt diſtances, as to remain a long time inviſible.

The Planets are uſually diſtinguiſhed into Primary and Secondary.

Primary PLANETS, called alſo ſimply Planets, are thoſe which move round the ſun, as their centre, or focus of their orbit. Such as Mercury, Venus, the Earth, Mars, Jupiter, Saturn, the Georgian or Herſchel, and perhaps others. And the

Secondary PLANETS, are ſuch as move round ſome primary one, as their centre, in the ſame manner as the primary ones do about the ſun. Such as the moon, which moves round the earth, as a ſecondary; and the three, Jupiter, Saturn, and Georgian, have each ſeveral ſecondary Planets, or moons, moving round them.

Till very lately the number of the primary Planets was eſteemed only ſix, which it was thought conſtituted the whole number of them in the ſolar ſyſtem; viz, Mercury, Venus, the Earth, Mars, Jupiter, and Saturn; all of which it appears were known to the aſtronomers of all ages, who never dreamt of an increaſe to their number. But a ſeventh has been lately diſcovered, by Dr. Herſchel, viz, on March the 13th, 1781, lying beyond all the reſt, and now called the Georgian, or Herſchel: and poſſibly others may ſtill remain undiſcovered to this day.

The primary Planets are again diſtinguiſhed into Superior and Inferior.

The Superior Planets are thoſe that are above the earth, or farther from the ſun than the earth is; as, Mars, Jupiter, Saturn, and the Georgian or Herſchel. And

The Inferior Planets are thoſe that are below the earth, or that are nearer the ſun than the earth is; which are Venus and Mercury.

The Planets were repreſented by the ſame characters as the chemiſts uſe to repreſent their metals by, on account of ſome ſuppoſed analogy between thoſe celeſtial and the ſubterraneous bodies. Thus,

Mercury, the meſſenger of the Gods, repreſented by ☿, the ſame as that metal, imitating a man with wings on his head and feet, is a ſmall bright planet, with a light tinct of blue, the ſun's conſtant attendant, from whoſe ſide it never departs above 28°, and by that means is uſually hid in his ſplendor. It performs its courſe around him in about 3 months.

Venus, the goddeſs of love, marked ♀, from the figure of a woman, the ſame as denotes copper, from a ſlight tinge of that colour, or verging to a light ſtraw colour. She is a very bright Planet, revolving next above Mercury, and never appears above 48 degrees from the ſun, finiſhing her courſe about him in about ſeven months. When this Planet goes before the ſun, or is a morning ſtar, it has been called Phoſphorus, and alſo Lucifer; and when following him, or when it ſhines in the evening as an evening ſtar, it is called Heſperus.

Tellus, the Earth, next above Venus, is denoted by ⊖, and performs its courſe about the ſun in the ſpace of a year.

Mars, the god of war, characterized ♂, a man holding out a ſpear, the ſame as iron, is a ruddy fiery-coloured Planet, and finiſhes his courſe about the ſun in about 2 years.

Jupiter, the chief god, or thunderer, marked ♃, to repreſent the thunderbolts, denoting the ſame as tin, from his pure white brightneſs. This Planet is next above Mars, and completes its courſe round the ſun in about 12 years.

Saturn, the father of the Gods, is expreſſed by ♄, to imitate an old man ſupporting himſelf with a ſtaff, and is the ſame as denotes lead, from his feeble light and duſky colour. He revolves next above Jupiter, and performs his courſe in about 30 years.

Laſtly, the Georgian, or Herſchel, is denoted by ♅, the initial of his name, with a croſs for the chriſtian Planet, or that diſcovered by the chriſtians. This is the higheſt, or outermoſt, of the known Planets, and revolves around the ſun in the ſpace of about 90 years.

From theſe deſcriptions a perſon may eaſily diſtinguiſh all the Planets, except the laſt, which requires the aid of a teleſcope. For if after ſun-ſet he ſees a Planet nearer the eaſt than the weſt, he may conclude it is neither Venus nor Mercury; and he may determine whether it is Saturn, Jupiter, or Mars, by the colour, light, and magnitude: by which alſo he may diſtinguiſh between Venus and Mercury.

It is probable that all the Planets are dark opake bodies, ſimilar to the earth, and for the following reaſons.

1. Becauſe, in Mercury, Venus, and Mars, only that part of the diſk is found to ſhine which is illuminated by the ſun; and again, Venus and Mercury, when between the ſun and the earth, appear like maculæ or dark ſpots on the ſun's face: from which it is evident, that thoſe three Planets are opake bodies, illuminated by the borrowed light of the ſun. And the ſame appears of Jupiter, from his being void of light in that part to which the ſhadow of his ſatellites reaches as well as in that part turned from the ſun: and that his ſatellites are opake, and reflect the ſun's light, like the moon, is abundantly ſhewn. Moreover, ſince Saturn, with his ring and ſatellites, and alſo Herſchel, with his ſatellites, only yield a faint light, conſiderably fainter than that of the reſt of the Planets, and than that of the fixed ſtars, though theſe be vaſtly more remote; it is paſt a doubt that theſe Planets too, with their attendants, are opake bodies.

2. Since the ſun's light is not tranſmitted through Mercury or Venus, when placed againſt him, it is plain they are denſe opake bodies; which is likewiſe evident of Jupiter, from his hiding the ſatellites in his ſhadow; and therefore, by analogy, the ſame may be concluded of Saturn and Herſchel.

3. From the variable ſpots of Venus, Mars, and Jupiter, it is evident that theſe Planets have a changeable atmoſphere; which ſort of atmoſphere may, by a like argument, be inferred of the ſatellites of Jupiter; and therefore,

therefore, by fimilitude, the fame may be concluded of the other Planets.

4. In like manner, from the mountains obferved in the moon and Venus, the fame may be fuppofed in the other Planets.

5. Laftly, fince all thefe Planets are opake bodies, fhining with the fun's borrowed light, are furnifhed with mountains, and are encompaffed with a changeable atmofphere; they confequently have waters, feas &c, as well as dry land, and are bodies like the moon, and therefore like the earth. And hence, it feems alfo probable, that the other Planets have their animal inhabitants, as well as our earth has.

Of the Orbits of the PLANETS.

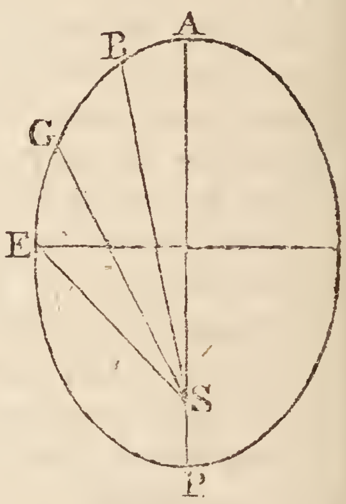
Though all the primary Planets revolve about the fun, their orbits are not circles, but ellipfes, having the fun in one of the foci. This circumftance was firft found out by Kepler, from the obfervations of Tycho Brahe: before that, all aftronomers took the planetary orbits for eccentric circles.

The Planes of thefe orbits do all interfect in the fun; and the line in which the plane of each orbit cuts that of the earth, is called the Line of the nodes; and the two points in which the orbits themfelves touch that plane, are the Nodes; alfo the angle in which each plane cuts that of the ecliptic, is called the Inclination of the plane or orbit.—The diftance between the centre of the fun, and the centre of each orbit, is called the excentricity of the Planet, or of its orbit.

The Motions of the PLANETS.

The motions of the primary Planets are very fimple, and tolerably uniform, as being compounded only of a projectile motion, forward in a right line, which is a tangent to the orbit, and a gravitation towards the fun at the centre. Befides, being at fuch vaft diftances from each other, the effects of their mutual gravitation towards one another are in a confiderable degree, though not altogether, infenfible; for the action of Jupiter upon Saturn, for ex. is found to be $\frac{1}{204}$ of the action of the fun upon Saturn, by comparing the matter of Jupiter with that of the fun, and the fquare of the diftance of each from Saturn. So that the elliptic orbit of Saturn will be found more juft, if its focus be fuppofed not in the centre of the fun, but in the common centre of gravity of the fun and Jupiter, or rather in the common centre of gravity of the fun and all the Planets below Saturn. And in like manner, the elliptic orbit of any other Planet will be found more accurate, by fuppofing its focus to be in the common centre of gravity of the fun and all the Planets that are below it. But the matter is far otherwife, in refpect of the fecondary Planets: for every one of thefe, though it chiefly gravitates towards its refpective primary one, as its centre, yet at equal diftances from the fun, it is alfo attracted towards him with an equally accelerated gravity, as the primary one is towards him; but at a greater diftance with lefs, and at a nearer diftance with greater: from which double tendency towards the fun, and towards their own primary Planets, it happens, that the motion of the fatellites, or fecondary Planets, comes to be very much compounded, and affected with various inequalities.

The motions even of the primary Planets, in their elliptic orbits, are not equable, becaufe the fun is not in their centre, but their focus. Hence they move; fometimes fafter, and fometimes flower, as they are nearer to or farther from the fun; but yet thefe irregularities are all certain, and follow according to an immutable law. Thus, the ellipfis PEA &c reprefenting the orbit of a Planet, and the focus S the fun's place: the axis of the ellipfe AP, is the line of the apfes; the point A, the higher apfis or aphelion; P the lower apfis or perihelion; CS the eccentricity; and ES the Planet's mean diftance from the fun. Now the motion of the Planet in its perihelion P is fwifteft, but in its aphelion A it is floweft; and at E the motion as well as the diftance is a mean, being there fuch as would defcribe the whole orbit in the fame time it is really defcribed in. And the law by which the motion in every point is regulated, is this, that a line or radius drawn from the centre of the fun to the centre of the Planet, and thus carried along with an angular motion, does always defcribe an elliptic area proportional to the time; that is, the trilineal area ASB, is to the area ASG, as the time the Planet is in moving over AB, to the time it is in moving over AG. This law was firft found out by Kepler, from obfervations; and has fince been accounted for and demonftrated by Sir Ifaac Newton, from the general laws of attraction and projectile motion.



As to the periods and velocities of the Planets, or the times in which they perform their courfes, they are found to have a wonderful harmony with their diftances from the fun, and with one another: the nearer each Planet being to the fun, the quicker ftill is its motion, and its period the fhorter, according to this general and regular law; viz, that the fquares of their periodical times are as the cubes of their mean diftances from the fun or focus of their orbits. The knowledge of this law we owe alfo to the fagacity of Kepler, who found that it obtained in all the primary Planets; as aftronomers have fince found it alfo to hold good in the fecondary ones. Kepler indeed deduced this law merely from obfervation, by a comparifon of the feveral diftances of the Planets with their periods or times: the glory of inveftigating it from phyfical principles is due to Sir Ifaac Newton, who has demonftrated that, in the prefent ftate of nature, fuch a law was inevitable.

The phenomena of the Planets are, their Conjunctions, Oppofitions, Elongations, Stations, Retrogradations, Phafes, and Eclipfes; for which fee the refpective articles.

For a view of the comparative magnitudes of the Planets; and for a view of their feveral diftances, &c; fee the articles ORBIT and SOLAR SYSTEM, as alfo Plate xxi, fig. 1.

The following Table contains a fynopfis of the diftances, magnitudes, periods, &c, of the feveral Planets, according to the lateft obfervations and improvements.

TABLE *of the* PLANETARY MOTIONS, DISTANCES, &c.

Anno 1784.	MERCURY.	VENUS.	EARTH.	MARS.	JUPITER.	SATURN.	HERSCHEL, or GEORGIAN, 1782.
Greateſt Elongation of Inferior, and Parallax of Superior Planets.	28° 20′	47° 48′	* *	47° 24′	11° 51′	6° 29′	3° 4¼′
Periodical Revotions round the Sun.	$87^d\ 23^h\ 15\frac{1}{2}^m$	$224^d\ 16^h\ 49\frac{1}{4}^m$	$365^d\ 6^h\ 9\frac{1}{4}^m$	$686^d\ 23^h\ 30\frac{3}{4}^m$	$4332^d\ 8^h\ 51\frac{1}{2}^m$	$10761^d\ 14^h\ 36\frac{3}{4}^m$	$30445^d\ 18^h$
Diurnal Rotations upon their Axes.	* * *	$23^h\ 22^m$	$23^h\ 56^m\ 4^s$	$24^h\ 39^m\ 22^s$	$9^h\ 56^m$	* *	* *
Inclinations of their Orbits to the Ecliptic.	7° 0′	3° 23⅓′	* *	1° 51′	1° 19¼′	2° 30⅓′	48′ 0″
Place of the Aſcending Node.	1^s 15° 46¾′	2^s 14° 44′	* * *	1^s 17° 59′	3^s 8° 50′	3^s 21° 48¼′	3^s 13° 1′
Place of the Aphelion, or point fartheſt from the Sun.	8^s 14° 13′	10^s 9° 38′	9^s 9° 15¼′	5^s 2° 6¼′	6^s 10° 57½′	9^s 0° 45½′	11^s 23° 23′
Greateſt Apparent Diameters, ſeen from the Earth.	11″	58″	*	25″	46″	20″	4″
Diameters in Engliſh Miles; that of the Sun being 883217.	3222	7687	7964	4189	89170	79042	35109
Proportional Mean Diſtances from the Sun.	38710	72333	100000	152369	520098	953937	1903421
Mean Diſtances from the Sun in Semidiameters of the Earth.	9210	17210	23799	36262	123778	227028	453000
Mean Diſtances from the Sun in Engliſh Miles.	37 millions	68 millions	95 millions	144 millions	490 millions	900 millions	1800 millions
Eccentricities or Diſtance of the Focus from the Centre.	7960	510	1680	14218	25277	53163	4759
Proportion of Light and Heat; that of the Earth being 100.	668	191	100	43	3·7	1·1	0·276
Proportion of Bulk; that of the Sun being 1380000.	$\frac{1}{15}$	$\frac{8}{9}$	1	$\frac{7}{25}$	$1\frac{2}{5}$	1000	90
Proportion of Denſity; that of the Sun being ¼.	2	1¼	1	·7	·23	·02	*

A Planet's motion, or diſtance from its apogee, is called the mean anomaly of the Planet, and is meaſured by the area it deſcribes in the given time: when the Planet arrives at the middle of its orbit, or the point E, the area or time is called the true anomaly. When the Planet's motion is reckoned from the firſt point of Aries, it is called its motion in longitude; which is either mean or true; viz, mean, which is ſuch as it would have were it to move uniformly in a circle; and true, which is that with which the Planet actually deſcribes its orbit, and is meaſured by the arc of the ecliptic it deſcribes. And hence may be found the Planet's place in its orbit for any given time after it has left the aphelion: for ſuppoſe the area of the ellipſis be ſo divided by the line SG, that the whole elliptic area may have the ſame proportion to the part ASG, as the whole periodical time in which the Planet deſcribes its whole orbit, has to the given time; then will G be the Planet's place in its orbit ſought.

PLANETARIUM, an aſtronomical machine, contrived to repreſent the motions, orbits, &c, of the planets, as they really are in nature, or according to the Copernican ſyſtem. The larger ſort of them are called Orreries. See ORRERY.

A very remarkable machine of this ſort was invented by Huygens, and deſcribed in his Opuſc. Poſth. tom. 2. p. 157, edit. Amſt. 1728. And it is ſtill preſerved among the curioſities of the univerſity at Leyden.

In this Planetarium, the five primary planets perform their revolutions about the ſun, and the moon performs her revolution about the earth, in the ſame time that they are really performed in the heavens. Alſo the orbits of the moon and planets are repreſented with their true proportions, eccentricity, poſition, and declination from the ecliptic or orbit of the earth. So that by this machine the ſituation of the planets, with the conjunctions, oppoſitions, &c, may be known, not only for the preſent time, but for any other time either paſt or yet to come; as in a perpetual ephemeris.

There was exhibited in London, viz. in the year 1791, a ſtill much more complete Planetarium of this ſort; called "a Planetarium or aſtronomical machine, which exhibits the moſt remarkable phenomena, motions, and revolutions of the univerſe. Invented, and partly executed, by the celebrated M. Phil. Matthew Hahn, member of the academy of ſciences at Erfurt. But finiſhed and completed by Mr. Albert de Mylius." This is a moſt ſtupendous and elaborate machine; conſiſting of the ſolar ſyſtem in general, with all the orbits and planets in their due proportions and poſitions; as alſo the ſeveral particular planetary ſyſtems of ſuch as have ſatellites, as of the earth, Jupiter, &c; the whole kept in continual motion by a chronometer, or grand eight-day clock; by which all theſe ſyſtems are made perpetually to perform all their motions exactly as in nature, exhibiting at all times the true and real motions, poſitions, aſpects, phenomena, &c, of all the celeſtial bodies, even to the very diurnal rotation of the planets, and the unequal motions in their elliptic orbits. A deſcription was publiſhed of this moſt ſuperb machine; and it was purchaſed and ſent as one of the preſents to the emperor of China, in the embaſſy of Lord Macartney, in the year 1793.

But the Planetariums or orreries now moſt commonly uſed, do not repreſent the true times of the celeſtial motions, but only their proportions; and are not kept in continual motion by a clock, but are only turned round occaſionally with the hand, to help to give young beginners an idea only of the planetary ſyſtem; as alſo, if conſtructed with ſufficient accuracy, to reſolve problems, in a coarſe way, relating to the motions of the planets, and of the earth and moon, &c.

Dr. Deſaguliers (Exp. Philoſ. vol. 1, p. 430.) deſcribes a Planetarium of his own contrivance, which is one of the beſt of the common ſort. The machine is contrived to be rectified or ſet to any latitude; and then by turning the handle of the Planetarium, all the planets perform their revolutions round the ſun in proportion to their periodical times, and they carry indices which ſhew the longitudes of the planets, by pointing to the diviſions graduated on circles for that purpoſe.

The Planetarium repreſented in fig. 1, plate xxii. is an inſtrument contrived by Mr. Wm. Jones, of Holborn, London, mathematical inſtrument maker, who has paid conſiderable attention to ſuch machines, to bring them to a great degree of ſimplicity and perfection. It repreſents in a general manner, by various parts of its machinery, all the motions and phenomena of the planetary ſyſtem. This machine conſiſts of, the Sun in the centre, with the Planets in the order of their diſtance from him, viz. Mercury, Venus, the Earth and Moon, Mars, Jupiter with his moons, and Saturn with his ring and moons; and to it is alſo occaſionally applied an extra long arm for the Georgian Planet and his two moons. To the earth and moon is applied a frame CD, containing only four wheels and two pinions, which ſerve to preſerve the earth's axis in its due paralleliſm in its motion round the ſun, and to give the moon at the ſame time her due revolution about the earth. Theſe wheels are connected with the wheelwork in the round box below, and the whole is ſet in motion by the winch H. The arm M that carries round the moon, points out on the plate C her age and phaſes for any ſituation in her orbit, upon which they are engraved. In like manner the arm points out her place in the ecliptic B, in ſigns and degrees, called her geocentric place, that is, as ſeen from the earth. The moon's orbit is repreſented by the flat rim A; the two joints of it, upon which it turns, denoting her nodes; and the orbit being made to incline to any required angle. The terrella, or little earth, of this machine, is uſually made of a three inch globe papered, &c, for the purpoſe; and by means of the terminating wire that goes over it, points out the changes of the ſeaſons, and the different lengths of days and nights more conſpicuouſly. By this machine are ſeen at once all the Planets in motion about the Sun, with the ſame reſpective velocities and periods of revolution which they have in the heavens; the wheelwork being calculated to a minute of time, from the lateſt diſcoveries. See Mr. Jones's Deſcription of his new portable Orrery.

PLANETARY, ſomething that relates to the planets. Thus, we ſay Planetary worlds, Planetary inhabitants, Planetary motions, &c. Huygens and Fontenelle bring ſeveral probable arguments for the reality of Planetary worlds, animals, plants, men, &c.

PLANETARY *Syſtem*, is the ſyſtem or aſſemblage of the Planets, primary and ſecondary, moving in their reſpective

respective orbits, round their common centre the sun. See *Solar* SYSTEM.

PLANETARY *Days*. With the Ancients, the week was shared among the seven planets, each planet having its day. This we learn from Dion Cassius and Plutarch, Sympos. lib. 4. q. 7. Herodotus adds, that it was the Egyptians who first discovered what god, that is what planet, presides over each day; for that among this people the planets were directors. And hence it is, that in most European languages the days of the week are still denominated from the planets; as Sunday, Monday, &c.

PLANETARY *Dials*, are such as have the Planetary hours inscribed on them.

PLANETARY *Hours*, are the 12th parts of the artificial day and night. See *Planetary* HOUR.

PLANETARY *Squares*, are the squares of the seven numbers from 3 to 9, disposed magically. Cornelius Agrippa, in his book of magic, has given the construction of the seven Planetary squares. And M. Poignard, canon of Brussels, in his treatise on sublime squares, gives new, general, and easy methods, for making the seven Planetary squares, and all others to infinity, by numbers in all sorts of progressions. See MAGIC *square*.

PLANETARY *Years*, the periods of time in which the several planets make their revolutions round the sun, or earth.—As from the proper revolution of the earth, or the apparent revolution of the sun, the solar year takes its original; so from the proper revolutions of the rest of the planets about the earth, as many sorts of years do arise; viz, the Saturnian year, which is defined by 29 Egyptian years 174 days 58 minutes, equivalent in a round number to 30 solar years. The Jovial year, containing 11 years 317 days 14 hours 59 minutes. The Martial year, containing 1 year 321 days 23 hours 31 minutes. For Venus and Mercury, as their years, when judged of with regard to the earth, are almost equal to the solar year; they are more usually estimated from the sun, the true centre of their motions: in which case the former is equal to 224 days 16 hours 49 minutes; and the latter to 87 days 23 hours 16 minutes.

PLANIMETRY, that part of geometry which considers lines and plane figures, without any regard to heights or depths.—Planimetry is particularly restricted to the mensuration of planes and other surfaces; as contradistinguished from Stereometry, or the mensuration of solids, or capacities of length, breadth and depth.

Planimetry is performed by means of the squares of long measures, as square inches, square feet, square yards, &c; that is, by squares whose side is an inch, a foot, a yard, &c. So that the area or content of any surface is said to be found, when it is known how many such square inches, feet, yards, &c, it contains. See MENSURATION and SURVEYING.

PLANISPHERE, a projection of the sphere, and its various circles, on a plane; as upon paper or the like. In this sense, maps of the heavens and the earth, exhibiting the meridians and other circles of the sphere, may be called Planispheres.

Planisphere is sometimes also considered as an astronomical instrument, used in observing the motions of the heavenly bodies; being a projection of the celestial sphere upon a plane, representing the stars, constellations, &c, in their proper situations, distances, &c. As the Astrolabe, which is a common name for all such projections.

In all Planispheres, the eye is supposed to be in a point, viewing all the circles of the sphere, and referring them to a plane beyond them, against which the sphere is as it were flattened: and this plane is called the Plane of Projection, which is always some one of the circles of the sphere itself, or parallel to some one.

Among the infinite number of Planispheres which may be furnished by the different planes of projection, and the different positions of the eye, there are two or three that have been preferred to the rest. Such as that of Ptolomy, where the plane of projection is parallel to the equator: that of Gemma Frisius, where the plane of projection is the colure, or solstitial meridian, and the eye the pole of the meridian, being a stereographical projection: or that of John de Royas, a Spaniard, whose plane of projection is a meridian, and the eye placed in the axis of that meridian, at an infinite distance; being an orthographical projection, and called the Analemma.

PLANO-*Concave* glass or lens, is that which is plane on one side, and concave on the other. And

PLANO-*Convex* glass or lens, is that which is plane on one side, and convex on the other. See LENS.

PLAT-BAND, in Architecture, is any flat square moulding, whose height much exceeds its projecture. Such are the faces of an architrave, and the Platbands of the modillions of a cornice.

PLATFORM, in Artillery and Gunnery, a small elevation, or a floor of wood, stone, or the like, on which cannon, &c, are placed, for more conveniently working and firing them.

PLATFORM, in Architecture, a row of beams that support the timber-work of a roof, lying on the top of the walls, where the entablature ought to be raised. Also a kind of flat walk, or plane floor, on the top of a building; from whence a fair view may be taken of the adjacent grounds. So, an edifice is said to be covered with a Platform, when it has no arched roof.

PLATO, one of the most celebrated among the ancient philosophers, being the founder of the sect of the Academics, was the son of Aristo, and born at Athens, about 429 years before Christ. He was of a royal and illustrious family, being descended by his father from Codrus, and by his mother from Solon. The name given him by his parents was *Aristocles*; but being of a robust make, and remarkably broad-shouldered, from this circumstance he was nick-named *Plato* by his wrestling-master, which name he retained ever after.

From his infancy, Plato distinguished himself by his lively and brilliant imagination. He eagerly imbibed the principles of poetry, music, and painting. The charms of philosophy however prevailing, drew him from those of the fine arts; and at the age of twenty he attached himself to Socrates only, who called him the *Swan of the Academy*. The disciple profited so well of his master's lessons, that at twenty-five years of age he had the reputation of a consummate sage. He lived with Socrates for eight years, in which time he committed to writing, according to the custom of the students, the purport of a great number of his master's excellent lectures, which he digested by way of philoso-

phical converfations; but made fo many judicious additions and improvements of his own, that Socrates, hearing him one day recite his Lyfis, cried out, O Hercules! how many fine fentiments does this young man afcribe to me, that I never thought of! And Laertius affures us, that he compofed feveral difcourfes which Socrates had no manner of hand in. At the time when Socrates was firft arraigned, Plato was a junior fenator, and he affumed the orator's chair to plead his mafter's caufe, but was interrupted in that defign, and the judges paffed fentence of condemnation upon Socrates. Upon this occafion Plato begged him to accept from him a fum of money fufficient to purchafe his enlargement, but Socrates peremptorily refufed the generous offer, and fuffered himfelf to be put to death.

The philofophers who were at Athens were fo alarmed at the death of Socrates, that moft of them fled, to avoid the cruelty and injuftice of the government. Plato retired to Megara, where he was kindly entertained by Euclid the philofopher, who had been one of the firft fcholars of Socrates, till the ftorm fhould be over. Afterwards he determined to travel in purfuit of knowledge; and from Megara he went to Italy, where he conferred with Eurytus, Philolaus, and Archytas, the moft celebrated of the Pythagoreans, from whom he learned all his natural philofophy, diving into the moft profound and myfterious fecrets of the Pythagoric doctrines. But perceiving other knowledge to be connected with them, he went to Cyrene, where he ftudied geometry and other branches of mathematics under Theodorus, a celebrated mafter.

Hence he travelled into Egypt, to learn the theology of their priefts, with the fciences of arithmetic, aftronomy, and the nicer parts of geometry. Having taken alfo a furvey of the country, with the courfe of the Nile and the canals, he fettled fome time in the province of Sais, learning of the fages there their opinions concerning the univerfe, whether it had a beginning, whether it moved wholly or in part, &c; alfo concerning the immortality and tranfmigration of fouls: and here it is alfo thought he had fome communication with the books of Mofes.

Plato's curiofity was not yet fatisfied. He travelled into Perfia, to confult the magi as to the religion of that country. He defigned alfo to have penetrated into India, to learn of the Brachmans their manners and cuftoms; but was prevented by the wars in Afia.

Afterwards, returning to Athens, he applied himfelf to the teaching of philofophy, opening his fchool in the Academia, a place of exercife in the fuburbs of the city; from whence it was that his followers took the name of Academics.

Yet, fettled as he was, he made feveral excurfions abroad: one in particular to Sicily, to view the fiery ebullitions of Mount Etna. Dionyfius the tyrant then reigned at Syracufe; a very bad man. Plato however went to vifit him; but, inftead of flattering him like a courtier, reproved him for the diforders of his court, and the injuftice of his government. The tyrant, not ufed to difagreeable truths, grew enraged at Plato, and would have put him to death, if Dion and Ariftomenes, formerly his fcholars, and then favourites of that prince, had not powerfully interceded for him. Dionyfius however delivered him into the hands of an envoy of the Lacedemonians, who were then at war with the Athenians: and this envoy, touching upon the coaft of Ægina, fold him for a flave to a merchant of Cyrene; who, as foon as he had bought him, liberated him, and fent him home to Athens.

Some time after, he made a fecond voyage into Sicily, in the reign of Dionyfius the younger; who fent Dion, his minifter and favourite, to invite him to court, that he might learn from him the art of governing his people well. Plato accepted the invitation, and went; but the intimacy between Dion and Plato raifing jealoufy in the tyrant, the former was difgraced, and the latter fent back to Athens. But Dion, being taken into favour again, perfuaded Dionyfius to recall Plato, who received him with all the marks of goodwill and friendfhip that a great prince could give. He fent out a fine galley to meet him, and went himfelf in a magnificent chariot, attended by all his court, to receive him. But this prince's uneven temper hurried him into new fufpicions. It feems indeed that thefe apprehenfions were not altogether groundlefs: for Ælian fays, and Cicero was of the fame opinion, that Plato taught Dion how to difpatch the tyrant, and to deliver the people from oppreffion. However this may be, Plato was offended and complained; and Dionyfius, incenfed at thefe complaints, refolved to put him to death: but Archytas, who had great intereft with the tyrant, being informed of it by Dion, interceded for the philofopher, and obtained leave for him to retire.

The Athenians received him joyfully at his return, and offered him the adminiftration of the government; but he declined that honour, choofing rather to live quietly in the Academy, in the peaceable contemplation and ftudy of philofophy; being indeed fo defirous of a private retirement that he never married. His fame drew difciples from all parts, when he would admit them, as well as invitations to come to refide in many of the other Grecian ftates; but the three that moft diftinguifhed themfelves, were Speufippus his nephew, who continued the Academy after him, Xenocrates the Caledonian, and the celebrated Ariftotle. It is faid alfo that Theophraftus and Demofthenes were two of his difciples. He had it feems fo great a refpect for the fcience of geometry and the mathematics, that he had the following infcription painted in large letters over the door of his academy; Let no one enter here, unless he has a taste for Geometry and the Mathematics!

But as his great reputation gained him on the one hand many difciples and admirers, fo on the other it raifed him fome emulators, efpecially among his fellow-difciples, the followers of Socrates. Xenophon and he were particularly difaffected to each other. Plato was of fo quiet and even a temper of mind, even in his youth, that he never was known to exprefs a pleafure with any greater emotion than that of a fmile; and he had fuch a perfect command of his paffions, that nothing could provoke his anger or refentment; from hence, and the fubject and ftyle of his writings, he acquired the appellation of the *Divine Plato*. But although he was naturally of a referved and very penfive difpofition; yet, according to Ariftotle, he was affable, courteous, and perfectly good-natured; and fometimes would condefcend

ſcend to crack little innocent jokes on his intimate acquaintances. Of his affability there needs no greater proof than his civil manner of converſing with the philoſophers of his own times, when pride and envy were at their height. His behaviour to Diogenes is always mentioned in his hiſtory. This Cynic was greatly offended, it ſeems, at the politeneſs and fine taſte of Plato, and uſed to catch all opportunities of ſnarling at him. Dining one day at his table with other company, when trampling upon the tapeſtry with his dirty feet, he uttered this brutiſh ſarcaſm, "I trample upon the pride of Plato:" to which the latter wiſely and calmly replied, "with a greater pride."

This extraordinary man, being arrived at 81 years of age, died a very eaſy and peaceable death, in the midſt of an entertainment, according to ſome; but, according to Cicero, as he was writing. Both the life and death of this philoſopher were calm and undiſturbed; and indeed he was finely compoſed for happineſs. Beſide the advantages of a noble birth, he had a large and comprehenſive underſtanding, a vaſt fund of wit and good taſte, great evenneſs and ſweetneſs of temper, all cultivated and refined by education and travel; ſo that it is no wonder he was honoured by his countrymen, eſteemed by ſtrangers, and adored by his ſcholars. Tully perfectly adored him: he tells us that he was juſtly called by Panætius, the divine, the moſt wiſe, the moſt ſacred, the Homer of philoſophers; thinks, that if Jupiter had ſpoken Greek, he would have done it in Plato's ſtyle, &c. But, panegyric aſide, Plato was certainly a very wonderful man, of a large and comprehenſive mind, an imagination infinitely fertile, and of a moſt flowing and copious eloquence. However, the ſtrength and heat of fancy prevailing over judgment in his compoſition, he was too apt to ſoar beyond the limits of earthly things, to range in the imaginary regions of general and abſtracted ideas; on which account, though there is always a greatneſs and ſublimity in his manner, he did not philoſophize ſo much according to truth and nature as Ariſtotle, though Cicero did not ſcruple to give him the preference.

The writings of Plato are all in the way of dialogue, where he ſeems to deliver nothing from himſelf, but every thing as the ſentiments and opinions of others, of Socrates chiefly, of Timæus, &c. His ſtyle, as Ariſtotle obſerved, is between proſe and verſe: on which account ſome have not ſcrupled to rank him among the poets: and indeed, beſide the elevation and grandeur of his ſtyle, his matter is frequently the offspring of imagination, inſtead of doctrines or truths deduced from nature. The firſt edition of Plato's works in Greek, was printed by Aldus at Venice in 1513: but a Latin verſion of them by Marſilius Ficinus had been printed there in 1491. They were reprinted together at Lyons in 1588, and at Francfort in 1602. The famous printer Henry Stephens, in 1578, gave a beautiful and correct edition of Plato's works at Paris, with a new Latin verſion by Serranus, in three volumes folio.

PLATONIC, ſomething that relates to Plato, his ſchool, philoſophy, opinions, or the like.

PLATONIC *Bodies*, ſo called from Plato who treated of them, are what are otherwiſe called the regular bodies. They are five in number; the tetraedron, the hexaedron, the octaedron, the dodecaedron, and the icoſaedron. See each of theſe articles, as alſo REGULAR BODIES.

PLATONIC *Year*, or the *Great Year*, is a period of time determined by the revolution of the equinoxes, or the time in which the ſtars and conſtellations return to their former places, in reſpect of the equinoxes.

The Platonic year, according to Tycho Brahe, is 25816 ſolar years, according to Riccioli 25920, and according to Caſſini 24800 years.

This period being once accompliſhed, it was an opinion among the ancients, that the world was to begin anew, and the ſame ſeries of things to return over again.

PLATONISM, the doctrine and ſentiments of Plato and his followers, with regard to philoſophy, &c. His diſciples were called Academics, from Academia, the name of a villa in the ſuburbs of Athens where he opened his ſchool. Among theſe were Xenocrates, Ariſtotle, Lycurgus, Demoſthenes, and Iſocrates. In phyſics, he chiefly followed Heraclitus; in ethics and politics, Socrates; and in metaphyſics, Pythagoras.

After his death, two of the principal of his diſciples, Xenocrates and Ariſtotle, continuing his office, and teaching, the one in the Academy, the other in the Lycæum, formed two ſects, under different names, though in other reſpects the ſame; the one retaining the denomination of ACADEMICS, the other aſſuming that of PERIPATETICS. See theſe two articles.

Afterwards, about the time of the firſt ages of Chriſtianity, the followers of Plato quitted the title of *Academiſts*, and took that of *Platoniſts*. It is ſuppoſed to have been at Alexandria, in Egypt, that they firſt aſſumed this new title, after having reſtored the ancient academy, and re-eſtabliſhed Plato's ſentiments; which had many of them been gradually dropped and laid aſide. Porphyry, Plotin, Iamblichus, Proclus, and Plutarch, are thoſe who acquired the chief reputation among the Greek Platoniſts; Apuleius and Chalcidius, among the Latins; and Philo Judæus, among the Hebrews. The modern Platoniſts own Plotin the founder, or at leaſt the reformer, of their ſect.

The Platonic philoſophy appears very conſiſtent with the Moſaic; and many of the primitive fathers follow the opinions of that philoſopher, as being favourable to Chriſtianity. Juſtin is of opinion that there are many things in the works of Plato which this philoſopher could not learn from mere natural reaſon; but thinks he muſt have learnt them from the books of Moſes, which he might have read when in Egypt. Hence Numenius the Pythagorean expreſsly calls Plato the *Attic Moſes*, and upbraids him with plagiariſm; becauſe he ſtole his doctrine concerning God and the world from the books of Moſes. Theodoret ſays expreſsly, that he has nothing good and commendable concerning the Deity and his worſhip, but what he took from the Hebrew theology; and Clemens Alexandrinus calls him the *Hebrew Philoſopher*. Gale is very particular in his proof of the point, that Plato borrowed his philoſophy from the Scriptures, either immediately, or by means of tradition; and, beſide the authority of the ancient writers, he brings ſome arguments from the thing itſelf. For example, Plato's confeſſion, that the Greeks borrowed their knowledge of the one infinite God, from an ancient people, better and

 nearer

nearer to God than they; by which people, our author makes no doubt, he meant the Jews, from his account of the ſtate of innocence; as, that man was born of the earth, that he was naked, that he enjoyed a truly happy ſtate, that he converſed with brutes, &c. In fact, from an examination of all the parts of Plato's philoſophy, phyſical, metaphyſical, and ethical, this author finds, in every one, evident marks of its ſacred original.

As to the manner of the creation, Plato teaches, that the world was made according to a certain exemplar, or idea, in the divine architect's mind. And all things in the univerſe, in like manner, he ſhews, do depend on the efficacy of internal ideas. This ideal world is thus explained by Didymus: 'Plato ſuppoſes certain patterns, or exemplars, of all ſenſible things, which he calls ideas; and as there may be various impreſſions taken off from the ſame ſeal, ſo he ſays are there a vaſt number of natures exiſting from each idea.' This idea he ſuppoſes to be an eternal eſſence, and to occaſion the ſeveral things in nature to be ſuch as itſelf is. And that moſt beautiful and perfect idea, which comprehends all the reſt, he maintains to be the world.

Farther, Plato teaches that the univerſe is an intelligent animal, conſiſting of a body and a ſoul, which he calls *the generated God*, by way of diſtinction from what he calls the *immutable eſſence*, who was the cauſe of the generated God, or the univerſe.

According to Plato, there were two ſorts of inferior and derivative gods; the mundane gods, all of which had a temporary generation with the world; and the ſupramundane eternal gods, which were all of them, one excepted, produced from that one, and dependent on it as their cauſe. Dr. Cudworth ſays, that Plato aſſerted a plurality of gods, meaning animated or intellectual beings, or dæmons, ſuperior to men, to whom honour and worſhip are due; and applying the appellation to the ſun, moon, and ſtars, and alſo to the earth. He aſſerts however, at the ſame time, that there was one ſupreme God, the ſelf originated being, the maker of the heaven and earth, and of all thoſe other gods. He alſo maintains, that the Pſyche, or univerſal mundane ſoul, which is a ſelf-moving principle, and the immediate cauſe of all the motion in the world, was neither eternal nor ſelf-exiſtent, but made or produced by God in time; and above this ſelf-moving Pſyche, but ſubordinate to the Supreme Being, and derived by emanation from him, he ſuppoſes an immoveable Nous or intellect, which was properly the Demiurgus, or framer of the world.

The firſt matter of which this body of the univerſe was formed, he obſerves, was a rude indigeſted heap, or chaos: Now, adds he, the creation was a mixed production; and the world is the reſult of a combination of neceſſity and underſtanding, that is, of matter, which he calls neceſſity, and the divine wiſdom: yet ſo that mind rules over neceſſity; and to this neceſſity he aſcribes the introduction and prevalence both of moral and natural evil.

The principles, or elements, which Plato lays down, are fire, air, water, and earth. He ſuppoſes two heavens, the Empyrean, which he takes to be of a fiery nature, and to be inhabited by angels, &c; and the Starry heaven, which he teaches is not adamantine, or ſolid, but liquid and ſpirable.

With regard to the human ſoul, Plato maintained its tranſmigration, and conſequently its future immortality and pre-exiſtence. He aſſerted, that human ſouls are here in a lapſed ſtate, and that ſouls ſinning ſhould fall down into theſe earthly bodies. Euſebius expreſsly ſays, that Plato held the ſoul to be ungenerated, and to be derived by emanation from the firſt cauſe.

His phyſics, or doctrine *de corpore*, is chiefly laid down in his Timæus, where he argues on the properties of body in a geometrical manner; which Ariſtotle takes occaſion to reprehend in him. His doctrine *de mente* is delivered in his 10th Book of Laws, and his Parmenides.

St. Auguſtine commends the Platonic philoſophy; and even ſays, that the Platoniſts were not far from Chriſtianity. It is alſo certain that moſt of the celebrated fathers were Platoniſts, and borrowed many of their explanations of ſcripture from the Platonic ſyſtem. To account for this fact, it may be obſerved, that towards the end of the ſecond century, a new ſect of philoſophers, called the modern, or later, Platonics, aroſe of a ſudden, ſpread with amazing rapidity through the greateſt part of the Roman empire, ſwallowed up almoſt all the other ſects, and proved very detrimental to Chriſtianity.

The ſchool of Alexandria in Egypt, inſtituted by Ptolomy Philadelphus, renewed and reformed the Platonic philoſophy. The votaries of this ſyſtem diſtinguiſhed themſelves by the title of Platonics, becauſe they thought that the ſentiments of Plato concerning the Deity and inviſible things, were much more rational and ſublime than thoſe of the other philoſophers. This new ſpecies of Platoniſm was embraced by ſuch of the Alexandrian Chriſtians as were deſirous to retain, with the profeſſion of the goſpel, the title, the dignity, and the habit of philoſophers. Ammonius Saccas was its principal founder, who was ſucceeded by his diſciple Plotinus, as this latter was by Porphyry, the chief of thoſe formed in his ſchool. From the time of Ammonius until the ſixth century, this was almoſt the only ſyſtem of philoſophy publicly taught at Alexandria. It was brought into Greece by Plutarch, who renewed at Athens the celebrated Academy, from whence iſſued many illuſtrious philoſophers. The general principle on which this ſect was founded, was, that truth was to be purſued with the utmoſt liberty, and to be collected from all the different ſyſtems in which it lay diſperſed. But none that were deſirous of being ranked among theſe new Platoniſts, called in queſtion the main doctrines; thoſe, for example, which regarded the exiſtence of one God, the fountain of all things; the eternity of the world; the dependance of matter upon the Supreme Being; the nature of ſouls; the plurality of gods, &c.

In the fourth century, under the reign of Valentinian, a dreadful ſtorm of perſecution aroſe againſt the Platoniſts; many of whom, being accuſed of magical practices, and other heinous crimes, were capitally convicted.

In the fifth century Proclus gave new life to the doctrine of Plato, and reſtored it to its former credit in Greece; with whom concurred many of the Chriſtian doctors, who adopted the Platonic ſyſtem. The

Platonic

Platonic philofophers were generally oppofers of Chriftianity; but in the fixth century. Chalcidius gave the Pagan fyftem an evangelical afpect; and thofe who, before it became the religion of the ftate, ranged themfelves under the ftandard of Plato, now repaired to that of Chrift, without any great change of their fyftem.

Under the emperor Juftinian, who iffued a particular edict, prohibiting the teaching of philofophy at Athens, which edict feems to have been levelled at modern Platonifm, all the celebrated philofophers of this fect took refuge among the Perfians, who were at that time the enemies of Rome; and though they returned from their voluntary exile, when the peace was concluded between the Perfians and Romans, in 533, they could never recover their former credit, nor obtain the direction of the public fchools.

Platonifm however prevailed among the Greeks, and was by them, and particularly by Gemiftius Pletho, introduced into Italy, and eftablifhed, under the aufpices of Cofmo de Medicis, about the year 1439, who ordered Marfilius Ficinus to tranflate into Latin the works of the moft renowned Platonifts.

PLATONISTS, the followers of Plato; otherwife called Academics, from Academia, the name of the place that philofopher chofe for his refidence at Athens.

PLEIADES, an affemblage of feven ftars in the neck of the conftellation Taurus, the bull; although there are now only fix of them vifible to the naked eye. The largeft of thefe is of the third magnitude, and called Lucido Pleiadum.

The Greeks fabled, that the name Pleiades was given to thefe ftars from feven daughters of Atlas and Pleione one of the daughters of Oceanus, who having been the nurfes of Bacchus, were for their fervices taken up to heaven and placed there as ftars, where they ftill fhine. The meaning of which fable may be, that Atlas firft obferved thefe ftars, and called them by the names of the daughters of his wife Pleione.

PLENILUNIUM, the full-moon.

PLENUM, in Phyfics, fignifies that ftate of things, in which every part of fpace, or extenfion, is fuppofed to be full of matter: in oppofition to a Vacuum, which is a fpace devoid of all matter.

The Cartefians held the doctrine of an abfolute Plenum; namely on this principle, that the effence of matter confifts in extenfion; and confequently, there being every where extenfion or fpace, there is every where matter: which is little better than begging the queftion.

PLINTH, in Architecture, a flat fquare member in form of a brick or tile; ufed as the foot or foundation of columns and pillars, &c.

PLOT, in Surveying, the plan or draught of any parcel of ground; as a field, farm, or manor, &c.

PLOTTING, in Surveying, the defcribing or laying down on paper, the feveral angles and lines, &c, of a tract of land, that has been furveyed and meafured.

Plotting is ufually performed by two inftruments, the protractor and Plotting-fcale; the former ferving to lay off all the angles that have been meafured and fet down, and the latter all the meafured lines. See thefe two inftruments under their refpective names.

Plotting *Scale*, a mathematical inftrument chiefly ufed for the plotting of grounds in furveying, or fetting off the lengths of the lines. It is either 6, 9, or 12 inches in length, and about an inch and half broad; being made either of box-wood, brafs, ivory, or filver; thofe of ivory are the neateft.

This inftrument contains various fcales or divided lines, on both fides of it. On the one fide are a number of plane fcales, or fcales of equal divifions, each of a different number to the inch; as alfo fcales of chords, for laying down angles; and fometimes even the degrees of a circle marked on one edge, anfwering to a centre marked on the oppofite edge, by which means it ferves alfo as a protractor. On the other fide are feveral diagonal fcales, of different fizes, or different divifions to the inch; ferving to take off lines expreffed by numbers to three dimenfions, as units, tens, and hundreds; as alfo a fcale of divifions which are the 100th parts of a foot. But the moft ufeful of all the lines that can be laid upon this inftrument, though not always done, is a line or plane fcale upon the two oppofite edges, made thin for that purpofe. This is a very ufeful line in furveying; for by laying the inftrument down upon the paper, with its divided edge along a line upon which are to be laid off feveral diftances, for the places of off-fets, &c; thefe diftances are all transferred at once from the inftrument to the line on the paper, by making fmall marks or points againft the refpective divifions on the edge of the fcale. See fig. 2 & 3, plates xxi and xxii.

Plotting-*Table*, in Surveying, is ufed for a plane table, as improved by Mr. Beighton, who has obviated a good many inconveniencies attending the ufe of the common plane table. See Philof. Tranf. numb. 461, fect. 1.

PLOUGH, or Plow, in Navigation, an ancient mathematical inftrument, made of box or pear-tree, and ufed to take the height of the fun or ftars, in order to find the latitude. This inftrument admits of the degrees to be very large, and has been much efteemed by many artifts; though now quite out of ufe.

PLUMB-Line, a term among artificers for a line perpendicular to the horizon.

PLUMMET, Plumb-rule, or Plumb-line, an inftrument ufed by mafons, carpenters, &c, to draw perpendiculars; in order to judge whether walls, &c, be upright, or planes horizontal, and the like.

PLUNGER, in Mechanics, a folid brafs cylinder, ufed as a forcer in forcing pumps.

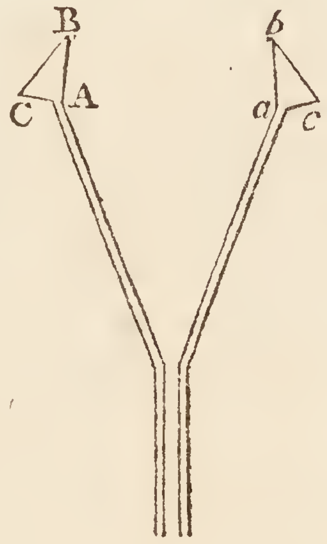
PLUS, in Algebra, the affirmative or pofitive fign, +, fignifying more or addition, or that the quantity following it is either to be confidered as a pofitive or affirmative quantity, or that it is to be added to the other quantities; fo $4 + 6 = 10$, is read thus, 4 plus 6 is equal to 10. See Affirmative *Sign*.

The more early writers of Algebra, as Lucas de Burgo, Cardan, Tartaglia, &c, wrote the word moftly at full length. Afterwards the word was contracted or abbreviated, ufing one or two of its firft letters; which initial was, by the Germans I think, corrupted to the prefent character +; which I find firft ufed by Stifelius, printed in his Arithmetic.

PLUVIAMETER, a machine for meafuring the quantity of rain that falls. There is defcribed in the Philof. Tranf. (numb. 473, or Abridg. x, 456), by Robert Pickering, under the name of an Ombrameter, an

an inſtrument of this kind. It conſiſts of a tin funnel *d*, whoſe ſurface is an inch ſquare (fig. 6, plate xx); a flat board *aa*; and a glaſs tube *bb*, ſet into the middle of it in a groove; and an index with diviſions *cc*; the board and tube being of any length at pleaſure. The bore of the tube is about half an inch, which Mr. Pickering ſays is the beſt ſize. The machine is fixed in ſome free and open place, as the top of the houſe, &c.

The Rain-gage employed at the houſe of the Royal Society is deſcribed by Mr. Cavendiſh, in the Philoſ. Tranſ. for 1776, p. 384. The veſſel which receives the rain is a conical funnel, ſtrengthened at the top by a braſs ring, 12 inches in diameter. The ſides of the funnel and inner lip of the braſs ring are inclined to the horizon, in an angle of above 65°; and the outer lip in an angle of above 50°; which are ſuch degrees of ſteepneſs, that there ſeems no probability either that any rain which falls within the funnel, or on the inner lip of the ring, ſhall daſh out, or that any which falls on the outer lip ſhall daſh into the funnel. The annexed figure is a vertical ſection of the funnel, ABC and *abc* being the braſs ring, BA and *ba* the inner lip, and BC and *bc* the outer.'



Note, that in fixing Pluviameters care ſhould be taken that the rain may have free acceſs to them, without being impeded or overſhaded by buildings, &c; and therefore the tops of houſes are moſtly to be preferred. Alſo when the quantities of rain collected in them, at different places, are compared together, the inſtruments ought to be fixed at the ſame height above the ground at both places; becauſe at different heights the quantities are always different, even in the ſame place. And hence alſo, any regiſter or account of rain in the Pluviameter, ought to be accompanied with a note of the height above the ground the inſtrument is placed at. See *Quantity of* RAIN.

PNEUMATICS, that branch of natural philoſophy which treats of the weight, preſſure, and elaſticity of the air, or elaſtic fluids, with the effects ariſing from them. Wolfius, inſtead of Pneumatics, uſes the term Aerometry.

This is a ſiſter ſcience to Hydroſtatics; the one conſidering the air in the ſame manner as the other does water. And ſome conſider Pneumatics as a branch of mechanics; becauſe it conſiders the air in motion, with the conſequent effects.

For the nature and properties of air, ſee the article AIR, where they are pretty largely treated of. To which may be added the following, which reſpects more particularly the ſcience of Pneumatics, as contained in a few propoſitions, and their corollaries.

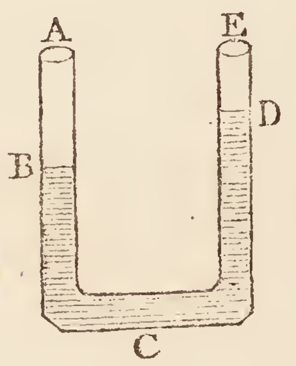
PROP. I. *The Air is a heavy fluid body, which ſurrounds and gravitates upon all parts of the ſurface of the earth.*

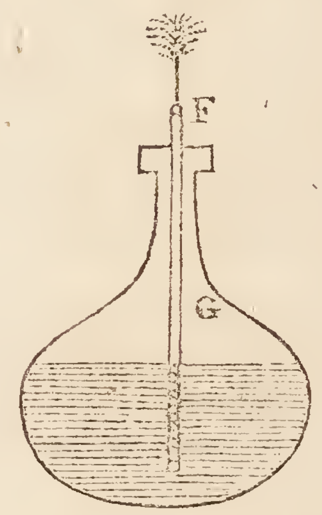
Theſe properties of air are proved by experience. That it is a fluid, is evident from its eaſily yielding to any the leaſt force impreſſed upon it, with little or no ſenſible reſiſtance.

But when it is moved briſkly, by any means, as by a fan, or a pair of bellows; or when any body is moved ſwiftly through it; in theſe caſes we become ſenſible of it as a body, by the reſiſtance it makes in ſuch motions, and alſo by its impelling or blowing away any light ſubſtances. So that, being capable of reſiſting, or moving other bodies by its impulſe, it muſt itſelf be a body, and be heavy, like all other bodies, in proportion to the matter it contains; and therefore it will preſs upon all bodies that are placed under it.

And being a fluid, it will ſpread itſelf all over upon the earth; alſo like other fluids it will gravitate upon, and preſs every where upon the earth's ſurface.

The gravity and preſſure of the air is alſo evident from many experiments. Thus, for inſtance, if water, or quick-ſilver, be poured into the tube ACE, and the





air be ſuffered to preſs upon it, in both ends of the tube; the fluid will reſt at the ſame height in both the legs: but if the air be drawn out of one end as E, by any means; then the air preſſing on the other end A, will preſs down the fluid in this leg at B, and raiſe it up in the other to D, as much higher than at B, as the preſſure of the air is equal to. By which it appears, not only that the air does really preſs, but alſo what the quantity of that preſſure is equal to. And this is the principle of the Barometer.

PROP. II. *The air is alſo an elaſtic fluid, being condenſible and expanſible. And the law it obſerves in this reſpect is this, namely, that its denſity is always proportional to the force by which it is compreſſed.*

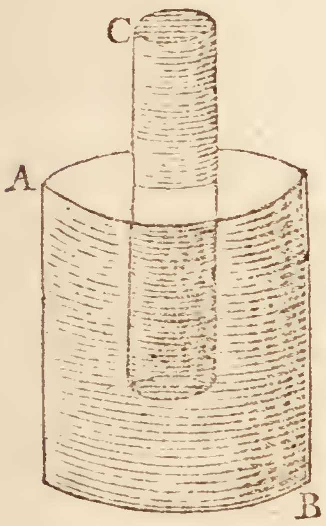
This property of the air is proved by many experiments. Thus, if the handle of a ſyringe be puſhed inwards, it will condenſe the incloſed air into a leſs ſpace; by which it is ſhewn to be condenſible. But the included air, thus condenſed, will be felt to act ſtrongly againſt the hand, and to reſiſt the force compreſſing it more and more; and on withdrawing the hand, the handle is puſhed back again to where it was at firſt. Which ſhews that the air is elaſtic.

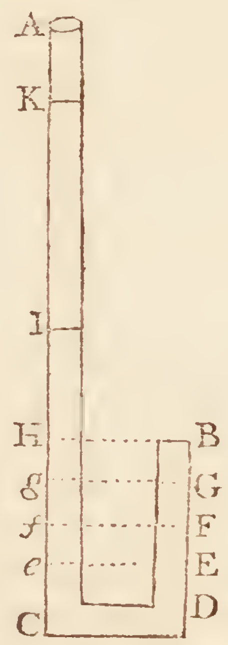
Again, fill a ſtrong bottle half full with water, and then inſert a pipe into it, putting its lower end down near to the bottom, and cementing it very cloſe round the mouth of the bottle. Then if air be ſtrongly injected through the pipe, as by blowing with the mouth or otherwiſe, it will paſs through the water from the lower end, and aſcend up into the part before occupied

by

by the air at G, and the whole maſs of air become there condenſed becauſe the water is not eaſily compreſſed into a leſs ſpace. But on removing the force which injected the air at F, the water will begin to riſe from thence in a jet, being puſhed up the pipe by the increaſed elaſticity of the air G, by which it preſſes on the ſurface of the water, and forces it through the pipe, till as much be expelled as there was air forced in; when the air at G will be reduced to the ſame denſity as at firſt, and, the balance being reſtored, the jet will ceaſe.

Likewiſe, if into a jar of water AB, be inverted an empty glaſs tumbler C, or ſuch like; the water will



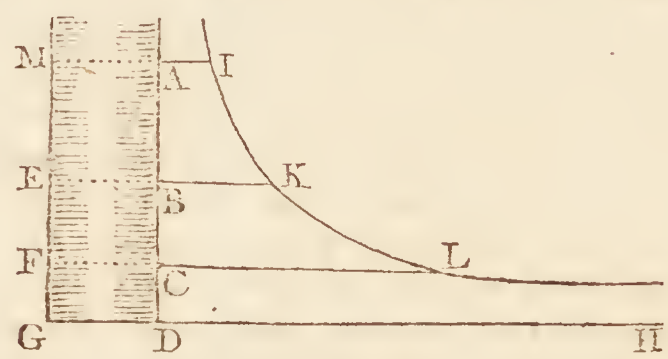


enter it, and partly fill it, but not near ſo high as the water in the jar, compreſſing and condenſing the air into a leſs ſpace in the upper part C, and cauſing the glaſs to make a ſenſible reſiſtance to the hand in puſhing it down. But on removing the hand, the elaſticity of the internal condenſed air throws the glaſs up again.—All theſe ſhewing that the air is condenſible and elaſtic.

Again, to ſhew the rate or proportion of the elaſticity to the condenſation; take a long ſlender glaſs tube, open at the top A, bent near the bottom or cloſe end B, and equally wide throughout, or at leaſt in the part BD (2d fig. above). Pour in a little quickſilver at A, juſt to cover the bottom to the bend at CD, and to ſtop the communication between the external air and the air in BD. Then pour in more quickſilver, and obſerve to mark the correſponding heights at which it ſtands in the two legs: ſo, when it riſes to H in the open leg AC, let it riſe to E in the cloſe one, reducing its included air from the natural bulk BD to the contracted ſpace BE, by the preſſure of the column H*e*; and when the quickſilver ſtands at I and K, in the open leg, let it riſe to F and G in the other, reducing the air to the reſpective ſpaces BF, BG, by the weights of the columns I*f*, K*g*. Then it is always found, that the condenſations and elaſticities are as the compreſſing weights, or columns of the quickſilver and the atmoſphere together. So, if the natural bulk of the air BD be compreſſed into the ſpaces BE, BF, BG, or reduced by the ſpaces DE, DF, DG, which are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of BD, or as the numbers 1, 2, 3; then the atmoſphere, together with the correſponding column H*e*, I*f*, K*g*, will alſo be found to be in the ſame proportion, or as the numbers 1, 2, 3: and then the weights of the quickſilver are thus, viz, $He = \frac{1}{3}A$, $If = A$, and $Kg = 3A$; where A denotes the weight of the atmoſphere. Which ſhews that the condenſations are directly as the compreſſing forces. And the elaſticities are alſo in the ſame proportion, ſince the preſſures in AC are ſuſtained by the elaſticities in BD.

From the foregoing principles may be deduced many uſeful remarks, as in the following corollaries, viz:

Corol. 1. The ſpace that any quantity of air is confined in, is reciprocally as the force that compreſſes it. So, the forces which confine a quantity of air in the



cylindrical ſpaces AG, BG, CG, are reciprocally as the ſame, or reciprocally as the heights AD, BD, CD. And therefore, if to the two perpendicular lines AD, DH, as aſymptotes, the hyperbola IKL be deſcribed, and the ordinates AI, BK, CL be drawn; then the forces which confine the air in the ſpaces AG, BG, CG, will be as the correſponding ordinates AI, BK, CL, ſince theſe are reciprocally as the abſciſſes AD, BD, CD, by the nature of the hyperbola.

Corol. 2. All the air near the earth is in a ſtate of compreſſion, by the weight of the incumbent atmoſphere.

Corol. 3. The air is denſer near the earth, than in high places; or denſer at the foot of a mountain, than at the top of it. And the higher above the earth, the rarer it is.

Corol. 4. The ſpring or elaſticity of the air, is equal to the weight of the atmoſphere above it; and they will produce the ſame effects; ſince they are always ſuſtained and balanced by each other.

Corol. 5. If the denſity of the air be increaſed, preſerving the ſame heat or temperature; its ſpring or elaſticity will likewiſe be increaſed, and in the ſame proportion.

Corol. 6. By the gravity and preſſure of the atmoſphere upon the ſurfaces of fluids, the fluids are made to riſe in any pipes or veſſels, when the ſpring or preſſure within is diminiſhed or taken off.

PROP. III. *Heat increaſes the elaſticity of the air, and cold diminiſhes it. Or heat expands, and cold contracts and condenſes the air.*

This property is alſo proved by experience.

Thus, tie a bladder very cloſe, with ſome air in it; and lay it before the fire; then as it warms, it will more and more diſtend the bladder, and at laſt burſt it, if the heat be continued and increaſed high enough. But if the bladder be removed from the fire; it will contract again to its former ſtate by cooling.——It was upon this principle that the firſt air-balloons were made by Montgolfier: for by heating the air within them, by a fire underneath, the hot air diſtends them to a ſize which occupies a ſpace in the atmoſphere whoſe weight of common air exceeds that of the balloon.

Alſo, if a cup or glaſs, with a little air in it, be inverted into a veſſel of water; and the whole be heated

over

over the fire, or otherwise: the air in the top will expand till it fill the glass, and expel the water out of it; and part of the air itself will follow, by continuing or increasing the heat.

Many other experiments to the same effect might be adduced, all proving the properties mentioned in the proposition.

Schol. Hence, when the force of the elasticity of the air is considered, regard must be had to its heat or temperature; the same quantity of air being more or less elastic, as its heat is more or less. And it has been found by experiment that its elasticity is increased at the following rate, viz, by the 435th part, by each degree of heat expressed by Fahrenheit's thermometer, of which there are 180 between the freezing and boiling heat of water. It has also been found (Philos. Transf. 1777, pa. 560 &c), that water expands the 6666th part, with each degree of heat; and mercury the 9600th part by each degree. Moreover, the relative or specific gravities of these three substances, are as follow: viz,

Air	1·232	when the barom. is at 30,
Water	1000	and the thermom. at 55.
Mercury	13600	

Also these numbers are the weights of a cubic foot of each, in the same circumstances of the barometer and thermometer.

PROP. IV. *The weight or pressure of the atmosphere, upon any base at the surface of the earth, is equal to the weight of a column of quicksilver of the same base, and its height between* 28 *and* 31 *inches.*

This is proved by the barometer, an instrument which measures the pressure of the air; the description of which see under its proper article. For at some seasons, and in some places, the air sustains and balances a column of mercury of about 28 inches; but at others, it balances a column of 29, or 30, or near 31 inches high; seldom in the extremes 28 or 31, but commonly about the means 29 or 30, and indeed mostly near 30. A variation which depends partly on the different degrees of heat in the air near the surface of the earth, and partly on the commotions and changes in the atmosphere, from winds and other causes, by which it is accumulated in some places, and depressed in others, being thereby rendered denser and heavier, or rarer and lighter; which changes in its state are almost continually happening in any one place. But the medium state is from $29\frac{1}{2}$ to 30 inches.

Corol. 1. Hence the pressure of the atmosphere upon every square inch at the earth's surface, at a medium, is very near 15 pounds avoirdupois. For, a cubic foot of mercury weighing nearly 13600 ounces, a cubic inch of it will weigh the 1728th part of it, or almost 8 ounces, or half a pound, which is the weight of the atmosphere for every inch of the barometer upon a base of a square inch; and therefore $29\frac{3}{4}$ inches, the medium height of the barometer, weighs almost 15 pounds, or rather $14\frac{3}{4}$lb very nearly.

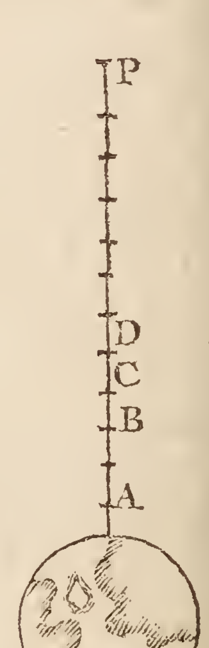
Corol. 2. Hence also the weight or pressure of the atmosphere, is equal to that of a column of water from 32 to 35 feet high, or on a medium 33 or 34 feet high. For water and quicksilver are in weight nearly as 1 to 13·6; so that the atmosphere will balance a column of water 13·6 times higher than one of quicksilver; consequently $13·6 \times 30$ inches $= 408$ inches or 34 feet, is near the medium height of water, or it is more nearly $33\frac{3}{4}$ feet. And hence it appears that a common sucking pump will not raise water higher than about 34 feet. And that a syphon will not run if the perpendicular height of the top of it be more than 33 or 34 feet.

Corol. 3. If the air were of the same uniform density, at every height, up to the top of the atmosphere, as at the surface of the earth; its height would be about $5\frac{1}{4}$ miles at a medium. For the weights of the same volume of air and water, are nearly as 1·232 to 1000; therefore as 1·232 : 1000 : : 34 feet : 27600 feet, or $5\frac{1}{4}$ miles very nearly. And so high the atmosphere would be, if it were all of uniform density, like water. But, instead of that, from its expansive and elastic quality, it becomes continually more and more rare the farther above the earth, in a certain proportion which will be treated of below.

Corol. 4. From this prop. and the last, it follows that the height is always the same, of an uniform atmosphere above any place, which shall be all of the uniform density with the air there, and of equal weight or pressure with the real height of the atmosphere above that place, whether it be at the same place at different times, or at any different places or heights above the earth; and that height is always about 27600 feet, or $5\frac{1}{4}$ miles, as found above in the 3d corollary. For, as the density varies in exact proportion to the weight of the column, it therefore requires a column of the same height in all cases, to make the respective weights or pressures. Thus, if W and w be the weights of atmosphere above any places, D and d their densities, and H and h the heights of the uniform columns, of the same densities and weights: Then $H \times D = W$, and $h \times d = w$; therefore $\frac{W}{D}$ or H is equal to $\frac{w}{d}$ or h; the temperature being the same.

PROP. V. *The density of the atmosphere, at different heights above the earth, decreases in such sort, that when the heights increase in arithmetical progression, the densities decrease in geometrical progression.*

Let the perpendicular line AP, erected on the earth, be conceived to be divided into a great number of very small parts A, B, C, D, &c, forming so many thin strata of air in the atmosphere, all of different density, gradually decreasing from the greatest at A: then the density of the several strata A, B, C, D, &c, will be in geometrical progression decreasing.



For, as the strata A, B, C, &c, are all of equal thickness, the quantity of matter in each of them, is as the density there; but the density in any one, being as the compressing force, is as the weight or quantity of matter from that place upward to the top of the atmosphere; therefore the quantity of matter in each stratum, is also as the whole quantity from that place upwards. Now if from the whole weight at any

place

place as B, the weight or quantity in the ſtratum B be ſubtracted, the remainder will be the weight at the next higher ſtratum C; that is, from each weight ſubtracting a part which is proportional to itſelf, leaves the next weight; or, which is the ſame thing, from each denſity ſubtracting a part which is always proportional to itſelf, leaves the next denſity. But when any quantities are continually diminiſhed by parts which are proportional to themſelves, the remainders then form a ſeries of continued proportionals; and conſequently theſe denſities are in geometrical progreſſion.

Thus, if the firſt denſity be D, and from each there be taken its nth part; then there remains its $\frac{n-1}{n}$ part, or the $\frac{m}{n}$ part, putting m for $n-1$; and therefore the ſeries of denſities will be D, $\frac{m}{n}$D, $\frac{m^2}{n^2}$D, $\frac{m^3}{n^3}$D, &c, $\frac{m}{n}$ being the common ratio of the ſeries.

Schol. Becauſe the terms of an arithmetical ſeries, are proportional to the logarithms of the terms of a geometrical ſeries; therefore different altitudes above the earth's ſurface, are as the logarithms of the denſities, or weights of air, at thoſe altitudes. So that,

if D denote the denſity at the altitude A,
and d the denſity at the altitude a;
then A being as the logarithm of D,
and a as the logarithm of d,
the dif. of altitude $A - a$ will be as

the log. of D — log. of d, or as log. of $\frac{D}{d}$.

And if A = 0, or D the denſity at the ſurface of the earth, then any altitude above the ſurface a, is as the log. of $\frac{D}{d}$. Or, in general, the log. of $\frac{D}{d}$ is as the altitude of the one place above the other, whether the lower place be at the ſurface of the earth, or any where elſe.

And from this property is derived the method of determining the heights of mountains, and other eminences, by the barometer, which is an inſtrument that meaſures the weight or denſity of the air at any place. For by taking with this inſtrument, the preſſure or denſity at the foot of a hill for inſtance, and again at the top of it, the difference of the logarithms of theſe two preſſures, or the logarithms of their quotient, will be as the difference of altitude, or as the height of the hill; ſuppoſing the temperatures of the air to be the ſame at both places, and the gravity of air not altered by the different diſtances from the earth's centre.

But as this formula expreſſes only the relations between different altitudes, with reſpect to their denſities, recourſe muſt be had to ſome experiment, to obtain the real altitude which correſponds to any given denſity, or the denſity which correſponds to a given altitude. Now there are various experiments by which this may be done. The firſt, and moſt natural, is that which reſults from the known ſpecific gravity of air, with reſpect to the whole preſſure of the atmoſphere on the ſurface of the earth. Now, as the altitude a is always as log. $\frac{D}{d}$, aſſume h ſo that a may be $= h \times$ log. $\frac{D}{d}$, where h will be of one conſtant value for all altitudes; and to determine that value, let a caſe be taken in which we know the altitude a correſponding to a known denſity d: as for inſtance take $a = 1$ foot, or 1 inch, or ſome ſuch ſmall altitude; and becauſe the denſity D may be meaſured by the preſſure of the atmoſphere, or the uniform column of 27600 feet, when the temperature is 55°; therefore 27600 feet will denote the denſity D at the lower place, and 27599 the leſs denſity d at one foot above it; conſequently this equation ariſes, viz, $1 = h \times$ log. of $\frac{27600}{27599}$, which, by the nature of logarithms, is nearly

$= h \times \frac{\cdot 43429448}{27600} = \frac{h}{63551}$ nearly; and hence $h =$ 63551 feet; which gives for any altitude whatever, this general theorem, viz, $a =$

$63551 \times$ log. $\frac{D}{d}$, or $= 63551 \times$ log. $\frac{M}{m}$ feet, or

$10592 \times$ log. $\frac{M}{m}$ fathoms; where M is the column of mercury which is equal to the preſſure or weight of the atmoſphere at the bottom, and m that at the top of the altitude a; and where M and m may be taken in any meaſure, either feet, or inches, &c.

Note, that this formula is adapted to the mean temperature of the air 55°. But for every degree of temperature different from this, in the medium between the temperatures at the top and bottom of the altitude a, that altitude will vary by its 435th part; which muſt be added when the medium exceeds 55°, otherwiſe ſubtracted.

Note alſo, that a column of 30 inches of mercury varies its length by about the 320th part of an inch for every degree of heat, or rather the 9600th part of the whole volume.

But the ſame formula may be rendered much more convenient for uſe, by reducing the factor 10592 to 10000, by changing the temperature proportionably from 55°: thus, as the difference 592 is the 18th part of the whole factor 10592; and as 18 is the 24th part of 435; therefore the correſponding change of temperature is 24°, which reduces the 55° to 31°. So that the formula becomes $a = 10000 \times$ log. of $\frac{M}{m}$ fathoms when the temperature is 31 degrees; and for every degree above that, the reſult muſt be increaſed by ſo many times its 435th part.

See more on this head under the article Barometer, at the end.

By the weight and preſſure of the atmoſphere, the effect and operations of Pneumatic engines may be accounted for, and explained; ſuch as ſyphons, pumps, barometers, &c. See each of theſe articles, alſo Air.

Pneumatic *Engine*, the ſame as the Air-Pump.

POCKET *Electrical Apparatus*.—This is a contrivance of Mr. William Jones, in Holborn, the form of which is repreſented in plate xxiii, fig. 4.

This ſmall machine is capable of a tolerably ſtrong charge, or accumulation of electricity, and will give a ſmall ſhock to one, two, three, or a greater number of perſons.

A is the Leyden phial or jar that holds the charge. B is the diſcharger to diſcharge the jar when required without electrifying the perſon that holds it. C is a ribbon prepared in a peculiar manner ſo as to be excited, and communicate its electricity to the jar. D are two hair, &c, ſkin rubbers, which are to be placed on the firſt and middle fingers of the left hand.

To charge the Jar.

Place the two finger-caps D on the firſt and middle finger of the left hand; hold the jar A at the ſame time, at the joining of the red and black on the outſide between the thumb and firſt finger of the ſame hand; then take the ribbon in your right hand, and ſteadily and gently draw it upwards between the two rubbers D, on the two fingers; taking care at the ſame time, the braſs ball of the jar is kept nearly cloſe to the ribbon, while it is paſſing through the fingers. By repeating this operation twelve or fourteen times, the electrical fire will paſs into the jar which will become charged, and by placing the diſcharger C againſt it, as in the plate, you will ſee a ſenſible ſpark paſs from the ball of the jar to that of the diſcharger. If the apparatus is dry and in good order, you will hear the crackling of the fire when the ribbon is paſſing through the fingers, and the jar will diſcharge at the diſtance repreſented in the figure.

To electrify a Perſon.

You muſt deſire him to take the jar in one hand, and with the other touch the nob of it: or, if diverſion is intended, deſire the perſon to ſmell at the nob of it, in expectation of ſmelling the ſcent of a roſe or a pink; this laſt mode has occaſioned it to be ſometimes called the Magic Smelling Bottle.

POETICAL *Numbers*. See NUMBERS.

POETICAL *Riſing* and *Setting*. See RISING and SETTING.

The ancient poets, referring the riſing and ſetting of the ſtars to that of the ſun, make three kinds of riſing and ſetting, viz, Coſmical, Acronical, and Heliacal. See each of theſe words in its place.

POINT, a term uſed in various arts and ſciences.

POINT, in Architecture. Arches of the third Point, and Arches of the fourth Point. See ARCHES.

POINT, in Aſtronomy, is a term applied to certain parts or places marked in the heavens, and diſtinguiſhed by proper epithets.

The four grand points or diviſions of the horizon, viz, the eaſt, weſt, north, and ſouth, are called the Cardinal Points.—The zenith and nadir are the Vertical Points.—The Points where the orbits of the planets cut the plane of the ecliptic, are called the Nodes.—The Points where the ecliptic and equator interſect, are called the Equinoctial Points. In particular, that where the ſun aſcends towards the north pole is called the Vernal Point; and that where he deſcends towards the ſouth, the Autumnal Point.—The higheſt and loweſt Points of the ecliptic are called the Solſtitial Points. Particularly, the former of them the Eſtival or Summer Point; the latter, the Brumal or Winter Point.

POINTS, in Electricity, are thoſe acute terminations of bodies which facilitate the paſſage of the electrical fluid either *from* or *to* ſuch bodies.

Mr. Jallabert was probably the firſt perſon who obſerved that a body pointed at one end, and round at the other, produced different appearances upon the ſame body, according as the pointed or round end was preſented to it. But Dr. Franklin firſt obſerved and evinced the whole effect of pointed bodies, both in drawing and throwing off electricity at greater diſtances than other bodies could do it; though he candidly acknowledges, that the power of Points to throw off the electric fire was communicated to him by his friend Mr. Thomas Hopkinſon.

Dr. Franklin electrified an iron ſhot, 3 or 4 inches in diameter, and obſerved that it would not attract a thread when the Point of a needle, communicating with the earth, was preſented to it; and he found it even impoſſible to electrify an iron ſhot when a ſharp needle lay upon it. This remarkable property, poſſeſſed by pointed bodies, of gradually and ſilently receiving or throwing off the electric fluid, has been evinced by a variety of other familiar experiments.

Thus, if one hand be applied to the outſide coating of a large jar fully charged, and the Point of a needle held in the other, be directed towards the knob of the jar, and moved gradually near it, till the Point of the needle touch the knob or ball, the jar will be entirely diſcharged, ſo as to give no ſhock at all, or one that is hardly ſenſible. In this caſe the Point of the needle has gradually and ſilently drawn away the ſuperabundant electricity from the electrified jar.

Farther, if the knob of a braſs rod be held at ſuch a diſtance from the prime conductor, that ſparks may eaſily eſcape from the latter to the former, whilſt the machine is in motion; then if the Point of a needle be preſented, though at twice the diſtance of the rod from the conductor, no more ſparks will be ſeen paſſing to the rod. When the needle is removed, the ſparks will be ſeen; but upon preſenting it again, they will again diſappear. So that the Point of the needle draws off ſilently almoſt all the fluid, which is thrown by the cylinder or globe of the machine upon the prime conductor. This experiment may be varied, by fixing the needle upon the prime conductor with the point upward; and then, though the knob of a diſcharging rod, or the knuckle of the finger, be brought very near the prime conductor, and the excitation be very ſtrong, little or no ſpark will be perceived.

The influence of points is alſo evinced in the amuſing experiment, commonly called the electrical horſe-race, and many others. See THUNDER-*houſe*.

The late Mr. Henly exhibited the efficacy of pointed bodies, by ſuſpending a large bladder, well blown, and covered with gold, ſilver, or braſs leaf, by means of gum-water, at the end of a ſilken thread 6 or 7 feet long, hanging from the cieling of a room, and electrifying the bladder by giving it a ſtrong ſpark with the knob of a charged bottle: upon preſenting to it the knob of a wire, it cauſed the bladder to move towards the knob, and when nearly in contact gave it a ſpark, thus diſcharging its electricity. By giving the bladder another charge, and preſenting the Point of a needle to it, the bladder was not attracted by the Point, but

rather receded from it, eſpecially when the needle was ſuddenly preſented towards it.

But experiments evincing the efficacy of pointed bodies for ſilently receiving or throwing off the electric fluid, may be infinitely diverſified, according to the fancy or convenience of the electrician.

It may be obſerved, that in the caſe of points throwing off or receiving electricity, a current of air is ſenſible at an electrified Point, which is always in the direction of the Point, whether the electricity be poſitive or negative. A fact which has been well aſcertained by many electricians, and particularly by Dr. Prieſtley and Sig. Beccaria. The former contrived to exhibit the influence of this current on the flame of a candle, preſented to a pointed wire, electrified negatively, as well as poſitively. The blaſt was in both caſes alike, and ſo ſtrong as to lay bare the greateſt part of the wick, the flame being driven from the Point; and the effect was the ſame whether the electric fluid iſſued out of the Point or entered into it. He farther evinced this phenomenon by means of thin light vanes; and he found, as Mr. Wilſon had before obſerved, that the vanes would not turn in vacuo, nor in a cloſe unexhauſted receiver where the air had no free circulation. And in much the ſame manner, Beccaria exhibited to ſenſe the influence of the wind or current of air driven from points.

As to the *Theory* of the phenomena of Points, theſe are accounted for in a variety of ways, by different authors, though perhaps by none with perfect ſatisfaction. See Franklin's writings on Electricity; Lord Mahon's Principles of Electricity, 1779; Beccaria's Artificial Electricity, 1776, pa. 331; and Prieſtley's Hiſtory of Electricity, vol. 2, pa. 191, edit. 1775.

As to the *Application* of the doctrine of Points; it may be obſerved that there is not a more important fact in the hiſtory of electricity, than the uſe to which the diſcovery of the efficacy of pointed bodies has been applied.

Dr. Franklin, having aſcertained the identity of electricity and lightning, was preſently led to propoſe a cheap and eaſy method of ſecuring buildings from the damage of lightning, by fixing a pointed metal rod higher than any part of the building, and communicating with the ground, or with the neareſt water. And this contrivance was actually executed in a variety of caſes; and has uſually been thought an excellent preſervative againſt the terrible effects of lightning.

Some few inſtances however having occurred, in which buildings have been ſtruck and damaged, though provided with theſe conductors; a controverſy aroſe with regard to their expediency and utility. In this controverſy Mr. Benjamin Wilſon took the lead, and Dr. Muſgrave, and ſome few other electricians, the leaſt acquainted with the ſubject, concurred with him in their oppoſition to pointed elevated conductors. Theſe alledge, that every Point, as ſuch, ſolicits the lightning, and thus contributes not only to increaſe the quantity of every actual diſcharge, but alſo frequently to occaſion a diſcharge when it might not otherwiſe have happened: whereas, ſay they, if inſtead of pointed conductors, thoſe with blunted terminations were uſed, they would as effectually anſwer the purpoſe of conveying away the lightning ſafely, without the ſame tendency to increaſe or invite it. Accordingly, Mr. Wilſon, in a letter to the marquis of Rockingham (Philoſ. Tranſ. vol. 54, art. 44), expreſſes his opinion, that, in order to prevent lightning from doing miſchief to high buildings, large magazines, and the like, inſtead of the elevated external conductors, that, on the inſide of the higheſt part of ſuch building, and within a foot or two of the top, it may be proper to fix a rounded bar of metal, and to continue it down along the ſide of the wall to any kind of moiſture in the ground.

On the other hand, it is urged by the advocates for pointed conductors, that Points, inſtead of increaſing an actual diſcharge, really prevent a diſcharge where it would otherwiſe happen, and that blunted conductors tend to invite the clouds charged with lightning. And it ſeems to be a certain fact, that though a ſharp Point will draw off a charge of electricity ſilently at a much greater diſtance than a knob, yet a knob will be ſtruck with a full exploſion or ſhock, the charge being the ſame in both caſes, at a greater diſtance than a ſharp Point.

The efficacy of pointed bodies for preventing a ſtroke of lightning, is ingeniouſly explained by Dr. Franklin in the following manner:—An eye, he ſays, ſo ſituated as to view horizontally the underſide of a thunder-cloud, will ſee it very ragged, with a number of ſeparate fragments or ſmall clouds one under another; the loweſt ſometimes not far from the earth. Theſe, as ſo many ſtepping-ſtones, aſſiſt in conducting a ſtroke between a cloud and a building. To repreſent theſe by an experiment, he directs to take two or three locks of fine looſe cotton, and connect one of them with the prime conductor by a fine thread of 2 inches, another to that, and a third to the ſecond, by like threads, which may be ſpun out of the ſame cotton. He then directs to turn the globe, and ſays we ſhall ſee theſe locks extending themſelves towards the table, as the lower ſmall clouds do towards the earth; but that, on preſenting a ſharp Point, erect under the loweſt, it will ſhrink up to the ſecond, the ſecond up to the firſt, and all together to the prime conductor, where they will continue as long as the Point continues under them. May not, he adds, in like manner, the ſmall electrified clouds, whoſe equilibrium with the earth is ſoon reſtored by the Point, riſe up to the main body, and by that means occaſion ſo large a vacancy, as that the grand cloud cannot ſtrike in that place? Letters, pa. 121.

Mr. Henly too, as well as ſeveral other perſons, with a view of determining the queſtion, whether Points or knobs are to be preferred for the terminations of conductors, made ſeveral experiments, ſhewing in a variety of inſtances, the efficacy of Points in ſilently drawing off the electricity, and preventing ſtrokes which would happen to knobs in the ſame ſituation. Philoſ. Tranſ. vol. 64, part 2, art. 18. See alſo THUNDER-*Houſe*.

Indeed it has been univerſally allowed, that in caſes where the quantity of electricity, with which thunder-clouds are charged, is ſmall, or when they move ſlowly in their paſſage to and over a building, pointed conductors, which draw off the electrical fluid ſilently, within the diſtance at which rounded ends will explode, will gradually exhauſt them, and thus contribute to prevent a ſtroke and preſerve the buildings to which they are annexed.

But it has been said by those who are averse to the use of such conductors, that if clouds, of great extent, and highly electrified, should be driven directly over them with great velocity, or if a cloud hanging directly over buildings to which they are annexed, suddenly receives a charge by explosion from another cloud at a distance, so as to enable it instantly to strike into the earth, these pointed conductors must take the explosion; on account of their greater readiness to admit electricity at a much greater distance than those that are blunted, and in proportion to the difference of that striking distance, do mischief instead of good: and therefore, they add, that such pointed conductors, though they may be sometimes advantageous, are yet at other times prejudicial; and that, as the purpose for which conductors are fixed upon buildings, is not to protect them from one particular sort of clouds only, but if possible from all, it cannot be advisable to use that kind of conductors which, if they diminish danger on the one hand, will increase it on the other. Besides, it is alleged, that if pointed conductors are attended with any the slightest degree of danger, that danger must be considerably augmented by carrying them high up into the air, and by fixing them upon every angle of a building, and by making them project in every direction. Such is the reasoning of Dr. Musgrave: see his paper in the Philos. Trans. vol. 68, part 2, art. 36.

Mr. Wilson too, dissenting from the report of a committee of the Royal Society, appointed to inspect the damage done by lightning to the house of the Board of Ordnance, at Purfleet, in 1777, was led to justify his dissent, and to disparage the use of pointed and elevated conductors, by means of a magnificent apparatus he constructed, with which he might produce effects similar to those that had happened in the case referred to the consideration and decision of the committee. With this view he procured a model of the Board-house at Purfleet, resembling it as nearly as possible in every essential appendage, and furnished with conductors of different lengths and terminations. And to construct a substitute for a cloud, he joined together the broad rims of 120 drums, forming together a cylinder of 155 feet in length, and above 16 inches in diameter; and this immense cylinder, of about 600 square feet of coated surface, was connected occasionally with one end of a wire 4800 feet long. As this bulky apparatus, representing the thunder-cloud, could not conveniently be put in motion, he contrived to accomplish the same end by moving the model of the building, with a velocity answering to that of the cloud, which he states, at a moderate computation, to be about 4 or 5 miles an hour. This apparatus was charged by a machine with one glass cylinder, about 10 or 11 feet from its nearest end; and the whole of the apparatus was disposed in the great room of the Pantheon, and applied to use in a variety of experiments. But it is impossible within the limits of this article to do justice to Mr. Wilson's experiments, or to the inferences which he deduces from them. Suffice it just to observe, that most of his experiments, in which the model of the house, which was passed swiftly under the artificial cloud, and having annexed to it either the pointed or blunt conductors at the same or different heights, were intended to shew, that pointed conductors are struck at a greater distance, and with a higher elevation, than the blunted ones: and from all his experiments made with pointed and rounded conductors, provided the circumstances be the same in both, he infers, that the rounded ones are much the safer of the two; whether the lightning proceeds from one cloud or from several; that those are still safer which rise little or nothing above the highest part of the building; and that this safety arises from the greatest resistance exerted at the larger surface. See Philos. Trans. for 1778, pa. 232.

The committee of the Royal Society however, which was composed of nine of the most distinguished electricians in the kingdom, and to whom was referred the consideration of the most effectual method of securing the powder-magazines at Purfleet against the effects of lightning, express their united opinion, that elevated sharp rods, constructed and disposed in the manner which they direct, are preferable to low conductors terminated in rounded ends, knobs, or balls of metal; and that the experiments and reasonings, made and alleged to the contrary, by Mr. Wilson, are inconclusive.

Mr. Nairne also, in order to obviate the objections of Mr. Wilson and others, and to vindicate the preference generally given to high and pointed conductors, constructed a much more simple apparatus than that of Mr. Wilson, with which he made a number of well-designed and well-conducted experiments, which seem to prove the point as far as it is capable of being proved by an artificial electrical apparatus. From these last experiments it appears, that though the point was struck by means of a swift motion of the artificial cloud, yet a small ball of 3 tenths of an inch diameter was struck farther off than the Point, and a larger ball at a much greater distance than either, even with the swiftest motion. Upon the whole, Mr. Nairne seems to be justified in preferring elevated pointed conductors; next to them, those that are pointed, though they rise but little above the highest part of a building; and after them, those that are terminated in a ball, and placed even with the highest part of the building. See Philos. Trans. 1778, pa. 823.

On the other part, Dr. Musgrave, not yet satisfied, gave in another paper, being "Reasons for dissenting from the Report of the Committee appointed to consider of Mr. Wilson's Experiments; including Remarks on some Experiments exhibited by Mr. Nairne;" which is inserted, by mistake, before Mr. Nairne's paper, being at pa. 801 of the same volume.

And farther, Mr. Wilson has another paper, on the same subject, at pa. 999 of the same vol. of Philos. Trans. for 1778, entitled, "New Experiments upon the Leyden Phial, respecting the termination of conductors;" repeating and asserting his former objections and reasonings.

In the Philos. Trans. too for 1779, pa. 454, Mr. William Swift has a paper, farther prosecuting this subject; making various experiments with simple and ingenious machinery, with models of houses and clouds, and with various sorts of conductors. From the experiments he infers in general, that "the whole current

of these experiments tends to shew the preference of Points to balls, in order to diminish and draw off the electric matter when excited, or to prevent it from accumulating; and consequently the propriety or even necessity of terminating all conductors with Points, to make them useful to prevent damage to buildings from lightning. Nay the very construction of all electrical machines, in which it is necessary to round all the parts, and to avoid making edges and points which would hinder the matter from being excited, will, I imagine, on reflection, be another corroborating proof of the result of the experiments themselves."

There were other communications made to the Royal Society upon the important subject of conductors, some of which were received, and others rejected. Upon the whole, this contest turned out one of the most extraordinary that ever was agitated in the Society; producing the most remarkable disputes, differences, and strange consequences, that ever the Society experienced since it had existence; consequences which manifested themselves in various instances for many years after, and which continue to this very day. All which, with the various secret springs and astonishing intrigues, may probably be given to the public on some other occasion.

Point, in Geometry, according to Euclid, is that which has no parts, or is indivisible; being void of all extension, both as to length, breadth, and depth.

This is what is otherwise called the Mathematical Point, being the intersection of two lines, and is only conceived by the imagination; yet it is in this that all magnitude begins and ends; the extremes of a line being Points; the extremes of a surface, Lines; and the extremes of a solid, Surfaces. And hence some define a Point, the inceptive of magnitude.

Proportion of Mathematical Points. It is a popular maxim, that all infinites are equal; yet is the maxim false, whether of quantities infinitely great, or infinitely little. Dr. Halley instances in several infinite quantities which are in a finite proportion to each other; and some that are infinitely greater than others. See Infinite *Quantity*.

And the same is shewn by Mr. Robarts, of infinitely small quantities, or mathematical Points. He demonstrates, for instance, that the Points of contact between circles and their tangents, are in the subduplicate ratio of the diameters of the circles; that the Point of contact between a sphere and a plane is infinitely greater than between a circle and a line; and that the Points of contact in spheres of different magnitudes, are to each other as the diameters of the spheres. Philos. Trans. vol. 27, pa. 470.

Conjugate Point, is used for that Point into which the conjugate oval, belonging to some kind of curves, vanishes. Maclaurin's Alg. pa. 308.

Point *of Contrary Flexure*, &c. See Inflexion, Retrogradation or Retrogression, &c, of curves.

Points *of the Compass*, or *Horizon*, &c, in Geography and Navigation, are the Points of division when the whole circle, quite around, is divided into 32 equal parts. These Points are therefore at the distance of the 32d part of the circle, or $11° 15'$, from each other; hence $5° 37'\frac{1}{2}$ is the distance of the half points, and $2° 48'\frac{3}{4}$ is the distance of the quarter Points. See Compass. The principal of these are the four cardinal Points, east, west, north and south.

Point is also used for a cape or headland, jutting out into the sea.——The seamen say two Points of land are one in another, when they are in a right line, the one behind the other.

Point, in Optics. As the

Point *of Concourse* or *Concurrence*, is that in which converging rays meet; and is usually called focus.

Point of *Dispersion*, *Incidence*, *Reflection*, *Refraction*, and *Radiant* Point. See these several articles.

Point, in Perspective, is a term used for various parts or places, with regard to the perspective plane. As, the

Point *of Sight*, or *of the eye*, called also the Principal Point, is the Point on a plane where a perpendicular from the eye meets it. See Perspective.

Some authors, however, by the Point of Sight, or Vision, mean the Point where the eye is actually placed, and where all the rays terminate. See Perspective.

Point *of Distance*, is a Point in a horizontal line, at the same distance from the principal Point as the eye is from the same. See Perspective.

Third Point, is a Point taken at discretion in the line of distance, where all the diagonals meet that are drawn from the divisions of the geometrical plane.

Objective Point, is a Point on a geometrical plane, whose representation on the perspective plane is required.

Accidental Point, and *Visual* Point. See Accidental and Visual.

Point *of View*, with regard to Building, Painting, &c, is a Point at a certain distance from a building, or other object, where the eye has the most advantageous view or prospect of the same. And this Point is usually at a distance equal to the height of the building.

Point, in Physics, is the smallest or least sensible object of sight, marked with a pen, or point of a compass, or the like. This is popularly called a Physical Point, and of such does all physical magnitude consist.

Point-Blanc, *Point-Blank*, in Gunnery, denotes the horizontal or level position of a gun, or having its muzzle neither elevated nor depressed. And the Point-blanc range, is the distance the shot goes, before it strikes the level ground, when discharged in the horizontal or Point-blanc direction. Or sometimes this means the distance the ball goes horizontally in a straight-lined direction.

POINTING, in Artillery and Gunnery, is the laying a piece of ordnance in any proposed direction, either horizontal, or elevated, or depressed, to any angle. This is usually effected by means of the gunner's quadrant, which, being applied to, or in, the muzzle of the piece, shews by a plummet the degree of elevation or depression.

Pointing, in Navigation, is the marking on the chart in what Point, or place, the vessel is.——This is done by means of the latitude and longitude, after these are known, or found by observation or computation. Thus, draw a line, with a pencil, across the chart according to the latitude; and another across the other way according to the longitude; then the intersection

fection of thefe two lines, is the Point or place on the chart where the fhip is; which is then marked black with a pen, and the pencil lines rubbed out. From the Point or place, thus found, the chart readily fhews the direct diftance and courfe run, as alfo yet to run to the intended port, &c.

POLAR, fomething that relates to the poles of the world: as polar virtue, polar tendency.

POLAR *Circles*, are two leffer circles of the fphere, or globe, one round each pole, and at the fame diftance from it as is equal to the fun's greateft declination or the obliquity of the ecliptic; that is, at prefent 23° 28'.—The fpace included within each polar circle, is the frigid zone; and to every part of this fpace, the fun never fets at fome time of the year, and never rifes at another time; each of thefe being a longer duration as the place is nearer the pole.

POLAR *Dials*, are fuch as have their planes parallel to fome great circle paffing through the poles, or to fome one of the hour-circles; fo that the pole is neither elevated above the plane, nor depreffed below it.—This dial, therefore, can have no centre; and confequently its ftyle, fubftyle, and hour-lines, are parallel.—This will therefore be an horizontal dial to thofe who live at the equator.

POLAR *Projection*, is a reprefentation of the earth, or heavens, projected on the plane of one of the polar circles.

POLAR *Regions*, are thofe parts of the earth which lie near the north and fouth poles.

POLARITY, the quality of a thing having poles, or pointing to, or refpecting fome pole: as the magnetic needle, &c.

By heating an iron bar, and letting it cool again in a vertical pofition, it acquires a polarity, or magnetic virtue: the lower end becoming the north pole, and the upper end the fouth pole. But iron bars acquire a polarity by barely continuing a long time in an erect pofition, even without heating them. Thus, the upright iron bars of fome windows, &c, are often found to have poles: Nay, an iron rod acquires a polarity, by the mere holding it erect; the lower end, in that cafe, attracting the fouth end of a magnetic needle; and the upper, the north end. But thefe poles are mutable, and fhift with the fituation of the rod.

Some modern writers, particularly Dr. Higgins, in his Philofophical Effay concerning Light, have maintained the polarity of the parts of matter, or that their fimple attractions are more forcible in one direction, or axis of each atom, than in any other.

POLES, in Aftronomy, the extremities of the axis upon which the whole fphere of the world revolves; or the points on the furface of the fphere through which the axis paffes. Thefe are on every fide at the diftance of a quadrant, or 90°, from every point of the equinoctial, and are called, by way of eminence, the poles of the world. That which is vifible to us in Europe, or raifed above our horizon, is called the Arctic or North Pole; and its oppofite one, the Antarctic or South Pole.

POLES, in Geography, are the extremities of the earth's axis; or the points on the furface of the earth through which the axis paffes. Of which, that elevated above our horizon is called the Arctic or North Pole; and the oppofite one, the Antarctic or South Pole.

In confequence of the fituation of the Poles, with the inclination of the earth's axis, and its parallelifm during the annual motion of our globe round the fun, the Poles have only one day and one night throughout the year, each being half a year in length. And becaufe of the obliquity with which the rays of the fun fall upon the polar regions, and the great length of the night in the winter feafon, it is commonly fuppofed the cold is fo intenfe, that thofe parts of the globe which lie near the Poles have never been fully explored, though the attempt has been repeatedly made by the moft celebrated navigators. And yet Dr. Halley was of opinion, that the folftitial day, at the Pole, is as hot as at the equator when the fun is in the zenith; becaufe all the 24 hours of that day under the Pole the fun-beams are inclined to the horizon in an angle of 23° 28'; whereas at the equator, though the fun becomes vertical, yet he fhines no more than 12 hours, being abfent the other 12 hours: and befides, that during 3 hours 8 minutes of the 12 hours which he is above the horizon there, he is not fo much elevated as at the Pole. Experience however feems to fhew that this opinion and reafoning of Dr. Halley are not well founded: for in all the parts of the earth that we know, the middle of fummer is always the lefs hot the farther the place is from the equator, or the nearer it is to the Pole.

The great object for which navigators have ventured themfelves in the frozen feas about the north pole, was to find out a more quick and ready paffage to the Eaft Indies. And this has been attempted three feveral ways: one by coafting along the northern parts of Europe and Afia, called the north-eaft paffage; another, by failing round the northern part of the American continent, called the north-weft paffage; and the third, by failing directly over the pole itfelf.

The poffibility of fucceeding in the north-eaft was for a long time believed; and in the laft century many navigators, particularly the Hollanders, attempted it with great fortitude and perfeverance. But it was always found impoffible to furmount the obftacles which nature had thrown in the way; and fubfequent attempts have in a manner demonftrated the impoffibility of ever failing eaftward along the northern coaft of Afia. The reafon of this impoffibility is, that in proportion to the extent of land, the cold is always greater in winter, and vice verfa. This is the cafe even in temperate climates; but much more fo in thofe frozen regions when the fun's influence, even in fummer, is but fmall. Hence, as the continent of Afia extends a vaft way from weft to eaft, and has befides the continent of Europe joined to it on the weft, it follows, that about the middle part of that tract of land the cold fhould be greater than any where elfe. Experience has determined this to be fact; and it now appears, that about the middle of the northern part of Afia, the ice never thaws; neither have even the hardy Ruffians and Siberians themfelves been able to overcome the difficulties they meet with in that part of their voyages.

With regard to the north-weft paffage, the fame difficulties occur as in the other. According to Captain Cook's voyage, it appears that if there is any ftrait which

which divides the continent of America into two, it muſt lie in a higher latitude than 70°, and conſequently be perpetually frozen up. And therefore if a north-weſt paſſage can be found, it muſt be by ſailing round the whole American continent, inſtead of ſeeking a paſſage through it, which ſome have ſuppoſed to exiſt in the bottom of Baffin's Bay. But the extent of the American continent to the northward is yet unknown; and there is a poſſibility of its being joined to that part of Aſia between the Piaſida and Chatanga, which has never yet been circumnavigated. Indeed a rumour has lately gone abroad of ſome remarkable inlet being obſerved on the weſtern coaſt of North America, which it is gueſſed may poſſibly lead to ſome communication with the eaſtern ſide, by the lakes, or a paſſage into Hudſon's Bay: but there ſeems little or no probability of any ſucceſs this way, in which many fruitleſs attempts have been made at various times. It remains therefore to conſider, whether there is any probability of attaining the wiſhed-for paſſage by ſailing directly north, between the eaſtern and weſtern continents.

The late celebrated mathematician, Mr. Maclaurin, was ſo fully perſuaded of the practicability of paſſing by this way to the South and Indian ſeas, that he uſed to ſay, if his other avocations would permit, he would undertake the voyage of trial, even at his own expence.

The practicability of this method, which would lead directly to the Pole itſelf, has alſo been ingeniouſly ſupported by Mr. Daines Barrington, in ſome tracts publiſhed in the years 1775 and 1776, in conſequence of the unſucceſsful attempt made by captain Phipps in the year 1773, to reach a higher northern latitude than 81°. Mr. Barrington inſtances a great number of navigators who have reached very high northern latitudes; nay, ſome who have been at the Pole itſelf, or gone beyond it. From all which he concludes, that if the voyage be attempted at a proper time of the year, there would not be any great difficulty in reaching the Pole. Thoſe vaſt pieces of ice which commonly obſtruct the navigators, he thinks, proceed from the mouths of the great Aſiatic rivers which run northward into the frozen ocean, and are driven eaſtward and weſtward by the currents. But, though we ſhould ſuppoſe them to come directly from the Pole, ſtill our author thinks that this affords an undeniable proof that the Pole itſelf is free from ice; becauſe, when the pieces leave it, and come to the ſouthward, it is impoſſible that they can at the ſame time accumulate at the Pole.

The Altitude or Elevation of the Pole, is an arch of the meridian intercepted between the Pole and the horizon of any places, and is equal to the latitude of the place.

To obſerve the Altitude of the Pole. With a quadrant, obſerve both the greateſt and leaſt meridian altitude of the Pole ſtar. Then half the ſum of the two altitudes, will be the height of the Pole, or the latitude of the place; and half the difference of the ſame will be the diſtance of the ſtar from the Pole. But, for accuracy, the obſerved altitudes ſhould be corrected on account of refraction, before their ſum or difference is taken: See Refraction.

Pole, in Spherics, or the Pole of a great circle, is a point upon the ſphere equally diſtant from every part of the circumference of the great circle; or a point 90° diſtant from the circumference in any part of it.—— The zenith and nadir are the Poles of the horizon; and the Poles of the equator are the ſame with thoſe of the ſphere or globe.

Poles, in Magnetics, are two points in a loadſtone, correſponding to the Poles of the world; one pointing to the north, and the other to the ſouth.

If the ſtone be broken in ever ſo many pieces, every fragment will ſtill have its two Poles. And if a magnet be biſected by a plane perpendicular to the axis; the two points before joined will become oppoſite Poles, one in each ſegment.

To touch a needle, &c, with a magnet, that part intended for the north end is touched with the ſouth Pole of the magnet; and that intended for the ſouth end, with the north Pole; for the Poles of the needle become contrary to thoſe of the magnet.

A piece of iron acquires a polarity by only holding it upright; though its Poles are not fixed, but ſhift, and are inverted as the iron is. Fire deſtroys all fixed Poles; but it ſtrengthens the mutable ones.

Dr. Gilbert ſays, the end of a rod being heated, and left to cool pointing northward, it becomes a fixed north Pole; if ſouthward, a fixed ſouth Pole. When the end is cooled, held downward, it acquires rather more magnetiſm than if cooled horizontally towards the north. But the beſt way is to cool it a little inclined to the north. Repeating the operations of heating and cooling does not increaſe the effect.

Dr. Power ſays, if a rod be held northwards, and the north end be hammered in that poſition, it will become a fixed north Pole; and contrarily if the ſouth end be hammered. The heavier the blows are, cæteris paribus, the ſtronger will the magnetiſm be; and a few hard blows have as much effect as a great number. And what is ſaid of hammering, is to be likewiſe underſtood of filing, grinding, ſawing, &c; nay, a gentle rubbing, when long continued, will produce Poles.

Old punches and drills have all fixed north poles; becauſe they are almoſt conſtantly uſed downwards. New drills have either mutable Poles, or weak north ones. Drilling with ſuch a one ſouthward horizontally, it is a chance if you produce a fixed ſouth Pole; much leſs if you drill ſouth downwards; but by drilling ſouth upwards, you always make a fixed ſouth Pole.

Mr. Ballard ſays, that in 6 or 7 drills, made in his preſence, the bit of each became a north Pole, merely by hardening.

A weak fixed Pole may degenerate into a mutable one in a day, or even in a few minutes, by holding it in a poſition contrary to its pole. The loadſtone itſelf will not make a fixed Pole in every piece of iron: if the iron be thick, it is neceſſary that it have ſome conſiderable length.

Pole *of a Glaſs*, in Optics, is the thickeſt part of a convex glaſs, or the thinneſt part of a concave one; being the ſame as what is otherwiſe called the vertex of the glaſs; and which, when truly ground, is exactly in the middle of its ſurface.

Pole, or *Rod*, in Surveying, is a lineal meaſure containing $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet.---The ſquare of it is called a ſquare Pole; but more uſually a perch, or a rod.

Pole.

Pole-Star, is a ſtar of the 2d magnitude near the north Pole, in the end of the tail of Urſa Minor, or the Little Bear. Its mean place in the heavens for the beginning of 1790, was as follows: viz,

Right Aſcenſion	12°	31′	47″
Annual variat. in ditto	0	3	4
Declination	88	11	8
Annual variat. in ditto	0	0	$19\frac{6}{10}$

The nearneſs of this ſtar to the Pole, on which account it is always above the horizon in theſe northern latitudes, makes it very uſeful in Navigation, &c, for determining the meridian line, the elevation of Pole, and conſequently the latitude of the place, &c.

POLEMOSCOPE, in Optics, an oblique kind of proſpective glaſs, contrived for the ſeeing of objects that do not lie directly before the eye. It was invented by Hevelius, in 1637, and is the ſame as Opera *Glaſs*; which ſee.

POLITICAL *Arithmetic*, the application of arithmetical calculations to political uſes and ſubjects; ſuch as the public revenues, the number of people, the extent and value of lands, taxes, trade, commerce, or whatever relates to the power, ſtrength, riches, &c, of a nation or commonwealth. Or, as Davenant conciſely defines it, the art of reaſoning by figures, upon things relating to government.

The chief authors who have attempted calculations of this kind, are, Sir William Petty, Major Graunt, Dr. Halley, Dr. Davenant, Mr. King, and Dr. Price.

Sir William Petty, among many other articles, ſtates that, in his time, the people in England were about ſix millions, and their annual expence about 7l. each; that the rent of the lands was about eight millions, and the intereſts and profits of the perſonal eſtates as much; that the rent of the houſes in England was four millions, and the profits of the labour of all the people twenty-ſix millions yearly; that the corn uſed in England, at 5s. the buſhel for wheat, and 2s. 6d. for barley, amounts to ten millions per annum; that the navy of England required 36,000 men to man it, and the trade and other ſhipping about 48,000; that the whole people in England, Scotland, and Ireland, together, were about nine millions and a half; and thoſe in France about thirteen millions and a half; and in the whole world about 350 millions; alſo that the whole caſh of England, in current money, was then about ſix millions ſterling. See his Political Arith., p. 74, &c.

Mr. Davenant gives ſome good reaſons why many of Sir W. Petty's numbers are not to be entirely depended on; and advances others of his own, founded on the obſervations of Mr. Greg. King. Some of the particulars are, that the land of England is thirty-nine millions of acres; that the number of people in London was about 530,000, and in all England five millions and a half, increaſing 9000 annually, or about the 600th part; the yearly rent of the lands ten millions, and that of the houſes two millions; the produce of all kinds of grain 9 millions. Davenant's Eſſay upon the probable methods &c, in his works, vol. 6.

Major Graunt, in his obſervations on the bills of mortality, computes, that there are 39,000 ſquare miles of land in England, or 25 million acres in England and Wales, and 4,600,000 perſons, making about 5 acres and a half to each perſon; that the people of London were 640,000; and ſtates the ſeveral numbers of perſons living at the different ages.

Sir William Petty, in his diſcourſe about duplicate proportion, farther ſtates, that it is found by experience, that there are more perſons living between 16 and 26 than of any other age; and from thence he infers, that the ſquare roots of every number of men's ages under 16, whoſe root is 4, ſhew the proportion of the probability of ſuch perſons reaching the age of 70 years: thus, the probability of reaching that age by perſons of the

ages of	16,	9,	4,	and 1,	
are as	4,	3,	2,	1,	reſpectively.

Alſo that the probabilities of their order of dying, at ages above that, are as the ſquare-roots of the ages: thus, the probabilities of the order of dying firſt,

of the ages	16,	25,	36,	&c,
are as the roots	4,	5,	6,	&c.

that is, the odds are 5 to 4 that a perſon of 25 dies before one of 16, and ſo on, declining up to 70 years of age.

Dr. Halley has made a very exact eſtimation of the degrees of mortality of mankind, from a curious table of the births and burials, at the city of Breſlau, in Sileſia; with an attempt to aſcertain the price of annuities upon lives, and many other curious particulars. See the Philoſ. Tranſ. vol. 17, pa. 596. Another table of this kind is given by Mr. Simpſon, for the city of London; and ſeveral by Dr. Price, for many different places.

Mr. Kerſeboom, of Holland, has many and curious calculations and tables of the ſame kind. From his obſervations on the births of the people in England, it appears, that the number of males born, is in proportion to that of the females, as 18 to 17; and that the inhabitants living in Holland are in the ſame proportion.

Dr. Brackenridge has given an eſtimate of the number of people in England, formed both from the number of houſes, and alſo from the quantity of bread conſumed. Upon the former principle, he finds the number of houſes in England and Wales to be about 900,000; and, allowing 6 perſons to each houſe, the number of people near 5 millions and a half. And upon the latter principle, eſtimating the quantity of corn conſumed at home at 2 millions of quarters, and 3 perſons to every quarter of corn, makes the number of people 6 millions. See Philoſ. Tranſ. vol. 49, art. 45 and 113.

Dr. Derham, from a great number of regiſters of places, finds the proportions of the marriages to the births and burials; and Dr. Price has done the ſame for ſtill more places; the mediums of all which are,

	Marriages to Births, as
Dr. Derham	1 to 4·7
Dr. Price	1 to 3·9

See Philoſ. Tranſ. Abr. vol. 7, part 4, pa. 46; alſo Dr. Price's Obſervations on Reverſionary Payments; and the articles of this Dictionary, Expectation *of Life*,

Life, LIFE-*Annuities*, MORTALITY, POPULATION, &c.

POLLUX, in Aftronomy, the hind twin, or the pofterior part of the conftellation Gemini.

POLLUX is alfo a fixed ftar of the fecond magnitude, in the conftellation Gemini, or the Twins. See CASTOR and *Pollux*, alfo GEMINI.

POLYACOUSTICS, inftruments contrived to multiply founds, as polyfcopes or multiplying glaffes do the images of objects.

POLYEDRON. See POLYHEDRON.

POLYGON, in Geometry, a figure of many angles; and confequently of many fides alfo; for every figure has as many fides as angles. If the angles be all equal among themfelves, the polygon is faid to be a regular one; otherwife, it is irregular. Polygons alfo take particular names according to the number of their fides; thus a Polygon of

3 fides is called a trigon,
4 fides - a tetragon,
5 fides - a pentagon,
6 fides - a hexagon, &c.

and a circle may be confidered as a Polygon of an infinite number of fmall fides, or as the limit of the Polygons.

Polygons have various properties, as below:

1. Every Polygon may be divided into as many triangles as it hath fides.

2. The angles of any Polygon taken together, make twice as many right angles, wanting 4, as the figure hath fides. Thus, if the Polygon has 5 fides; the double of that is 10, from which fubtracting 4, leaves 6 right angles, or 540 degrees, which is the fum of the 5 angles of the pentagon. And this property, as well as the former, belongs to both regular and irregular Polygons.

3. Every regular Polygon may be either infcribed in a circle, or defcribed about it. But not fo of the irregular ones, except the triangle, and another particular cafe as in the following property.

An equilateral figure infcribed in a circle, is always equiangular.— But an equiangular figure infcribed in a circle is not always equilateral, but only when the number of fides is odd. For if the fides be of an even number, then they may either be all equal; or elfe half of them may be equal, and the other half equal to each other, but different from the former half, the equals being placed alternately.

4. Every Polygon, circumfcribed about a circle, is equal to a right-angled triangle, of which one leg is the radius of the circle, and the other the perimeter or fum of all the fides of the Polygon. Or the Polygon is equal to half the rectangle under its perimeter and the radius of its infcribed circle, or the perpendicular from its centre upon one fide of the Polygon.

Hence, the area of a circle being lefs than that of its circumfcribing Polygon, and greater than that of its infcribed Polygon, the circle is the limit of the infcribed and circumfcribed Polygons: in like manner the circumference of the circle is the limit between the perimeters of the faid Polygons: confequently the circle is equal to a right-angled triangle, having one leg equal to the radius, and the other leg equal to the circumference; and therefore its area is found by multiplying half the circumference by half the diameter. In like manner, the area of any Polygon is found by multiplying half its perimeter by the perpendicular demitted from the centre upon one fide.

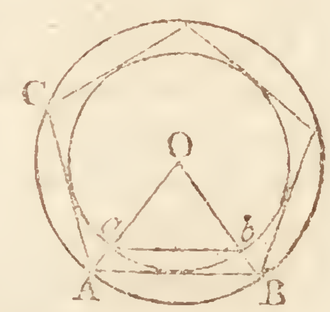
5. The following Table exhibits the moft remarkable particulars in all the Polygons, up to the dodecagon of 12 fides; viz, the angle at the centre AOB, the angle of the Polygon C or CAB or double of OAB, and the area of the Polygon when each fide AB is 1. (See the following figure.)

No. of fides.	Name of Polygon.	Ang. O at cent.	Ang. C. of Polyg.	Area.
3	Trigon	120°	60°	0·4330127
4	Tetragon	90	90	1·0000000
5	Pentagon	72	108	1·7204774
6	Hexagon	60	120	2·5980762
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	3·6339124
8	Octagon	45	135	4·8284271
9	Nonagon	40	140	6·1818242
10	Decagon	36	144	7·6942088
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	9·3656399
12	Dodecagon	30	150	11·1961524

By means of the numbers in this Table, any Polygons may be conftructed, or their areas found: thus, (1ft) *To infcribe a Polygon in a given Circle.* At the centre make the angle O equal to the angle at the centre of the propofed Polygon, found in the 3d column of the Table, the legs cutting the circle in A and B; and join A and B which will be one fide of the Polygon. Then take AB between the compaffes, and apply it continually round the circumference, to complete the Polygon.

(2d) *Upon the given Line* AB *to defcribe a regular Polygon.* From the extremities draw the two lines AO and BO, making the angles A and B each equal to half the angle of the Polygon, found in the 4th column of the Table, and their interfection O will be the centre of the circumfcribed circle: then apply AB continually round the circumference as before.

(3d) *To defcribe a Polygon about a given Circle.*— At the centre O make the angle of the centre as in the 1ft art. its legs cutting the circle in *a* and *b*: join *ab*, and parallel to it draw AB to touch the circle: and meeting O*a* and O*b* produced in A and B: with the radius OA, or OB, defcribe a circle, and around its circumference apply continual AB, which will complete the Polygon as before.



(4th) *To find the Area of any regular Polygon.*— Multiply the fquare of its fide by the tabular area, found on the line of its name in the laft column of the Table, and the product will be the area. Thus, to find

find the area of the trigon, or equilateral triangle, whose side is 20. The square of 20 being 400, multiply the tabular area ·4330127 by 400, as in the margin, and the product 173·20508 will be the area.

$$\begin{array}{r} 0{\cdot}4330127 \\ 400 \\ \hline 173{\cdot}2050800 \end{array}$$

6. There are several curious algebraical theorems for inscribing Polygons in circles, or finding the chord of any proposed part of the circumference, which is the same as angular sections. These kinds of sections, or parts and multiples of arcs, were first treated of by Vieta, as shewn in the Introduction to my Log. pa. 9, and since pursued by several other mathematicians, in whose works they are usually to be found. Many other particulars relating to Polygons may also be seen in my Mensuration, 2d edit. pa. 20, 21, 22, 23, 113, &c.

Polygon, in Fortification, denotes the figure or perimeter of a fortress, or fortified place. This is either Exterior or Interior.

Exterior Polygon is the perimeter or figure formed by lines connecting the points of the bastions to one another, quite round the work. And

Interior Polygon, is the perimeter or figure formed by lines connecting the centres of the bastions, quite around.

Line of Polygons, is a line on some sectors, containing the homologous sides of the first nine regular Polygons inscribed in the same circle; viz, from an equilateral triangle to a dodecagon.

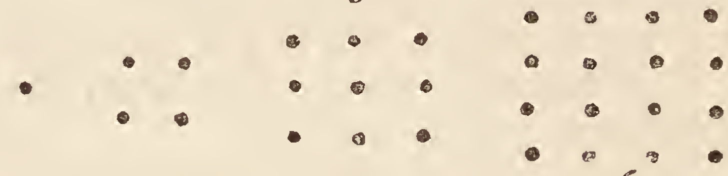
POLYGONAL *Numbers*, are the continual or successive sums of a rank of any arithmeticals beginning at 1, and regularly increasing; and therefore are the first order of figurate numbers; they are called Polygonals, because the number of points in them may be arranged in the form of the several Polygonal figures in geometry, as is illustrated under the article Figurate *Numbers*, which see.

The several sorts of Polygonal numbers, viz, the triangles, squares, pentagons, hexagons, &c, are formed from the addition of the terms of the arithmetical series, having respectively their common difference 1, 2, 3, 4, &c; viz, if the common difference of the arithmeticals be 1, the sums of their terms will form the triangles; if 2, the squares; if 3, the pentagons; if 4, the hexagons, &c. Thus:

Arith. Progref.	1,	2,	3,	4,	5,	6,	7.
Triang. Nos.	1,	3,	6,	10,	15,	21,	28.
Arith. Progref.	1,	3,	5,	7,	9,	11,	13.
Square Numbers	1,	4,	9,	16,	25,	36,	49.
Arith. Progref.	1,	4,	7,	10,	13,	16,	19.
Pentagonal Nos.	1,	5,	12,	22,	35,	51,	70.
Arith. Progref.	1,	5,	9,	13,	17,	21,	25.
Hexagonal Nos.	1,	6,	15,	28,	45,	66,	91.

The *Side* of a Polygonal number is the number of points in each side of the Polygonal figure when the points in the number are ranged in that form. And this is also the same as the number of terms of the arithmeticals that are added together in composing the Polygonal number; or, in short, it is the number of the term from the beginning. So, in the 2d or squares,

1 2 3 4



1 4 9 16

the side of the first (1) is 1, that of the second (4) is 2, that of the third (9) is 3, that of the fourth (16) is 4, and so on. And

The *Angles*, or Numbers of Angles, are the same as those of the figure from which the number takes its name. So the angles of the triangular numbers are 3, of the square ones 4, of the pentagonals 5, of the hexagonals 6, and so on. Hence, the angles are 2 more than the common difference of the arithmetical series from which any rank of Polygonals is formed: so the arithmetical series has for its common difference the number 1 or 2 or 3 &c as follows, viz, 1 in the triangles, 2 in the squares, 3 in the pentagons, &c; and, in general, if a be the number of angles in the Polygon, then $a - 2$ is $= d$ the common difference of the arithmetical series, or $d + 2 = a$ the number of angles.

Prob. 1. *To find any Polygonal Number proposed; having given its side* n *and angles* a. The Polygonal number being evidently the sum of the arithmetical progression whose number of terms is n and common difference $a - 2$, and the sum of an arithmetical progression being equal to half the product of the extremes by the number of terms, the extremes being 1 and $1 + d.\overline{n-1} = 1 + \overline{a-2}.\overline{n-1}$; therefore that number, or this sum, will be

$$\frac{n^2 d - n.\overline{d-2}}{2} \text{ or } \frac{n^2.\overline{a-2} - n.\overline{a-4}}{2}, \text{ where}$$

d is the common difference of the arithmeticals that form the Polygonal number, and is always 2 less than the number of angles a.

Hence, for the several sorts of Polygons, any particular number whose side is n, will be found from either of these two formulæ, by using for d its values 1, 2, 3, 4, &c; which gives these following formulæ for the Polygonal number in each sort, viz, the

Triangular	-	$\frac{n^2 + n}{2}$,
Square	-	$\frac{2n^2 - 0n}{2} = n^2$,
Pentagonal	-	$\frac{3n^2 - n}{2}$,
Hexagonal	-	$\frac{4n^2 - 2n}{2}$,
Heptagonal	-	$\frac{5n^2 - 3n}{2}$,
&c.		

Prob. 2. *To find the Sum of any Number of Polygonal Numbers of any order.*—Let the angles of the Polygon be

be a, or the common difference of the arithmeticals that form the Polygonals, d; and n the number of terms in the Polygonal ſeries, whoſe ſum is ſought: then is

$$\left(\frac{n^2-1}{6}d + \frac{n+1}{2}\right)n \text{ or } \left(\frac{n^2-1}{6}\cdot\overline{a-2} + \frac{n+1}{2}\right)n$$

the ſum of the n terms ſought.

Hence, ſubſtituting ſucceſſively the numbers 1, 2, 3, 4, &c, for d, there is obtained the following particular caſes, or formulæ, for the ſums of n terms of the ſeveral ranks of Polygonal numbers, viz, the ſum of the

Triangulars -	$\frac{n^2+3n+2}{6}n$,
Squares -	$\frac{2n^2+3n+1}{6}n$,
Pentagonals -	$\frac{3n^2+3n+0}{6}n$,
Hexagonals -	$\frac{4n^2+3n-1}{6}n$,
Heptagonals -	$\frac{5n^2+3n-2}{6}n$,

&c

POLYGRAM, in Geometry, a figure conſiſting of many lines.

POLYHEDRON, or Polyedron, a body or ſolid contained by many rectilinear planes or ſides.

When the ſides of the Polyhedron are regular polygons, all ſimilar and equal, then the Polyhedron becomes a regular body, and may be inſcribed in a ſphere; that is, a ſphere may be deſcribed about it, ſo that its ſurface ſhall touch all the angles or corners of the ſolid. There are but five of theſe regular bodies, viz, the tetraedron, the hexaedron or cube, the octaedron, the dodecaedron, and the icoſaedron. See Regular *Body*, and each of thoſe five bodies ſeverally.

Gnomonical Polyhedron, is a ſtone with ſeveral faces, on which are projected various kinds of dials. Of this ſort, that in the Privy-garden, London, now gone to ruin, was eſteemed the fineſt in the world.

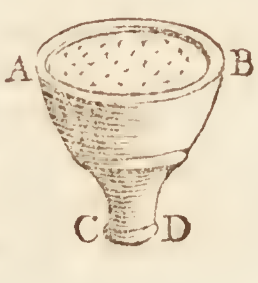
Polyhedron, in Optics. See Polyscope.

POLYHEDROUS *Figure*, in Geometry, a ſolid contained under many ſides or planes. See Polyhedron.

POLYNOMIAL, in Algebra, a quantity of many names or terms, and is otherwiſe called a Multinomial. As $a + 3b - 2c + 4d$, &c. See Multinomial.

POLYOPTRUM, in Optics, a glaſs through which objects appear multiplied, but diminiſhed. The Polyoptrum differs both in ſtructure and phenomena from the common multiplying glaſſes called Polyhedra or Polyſcopes.

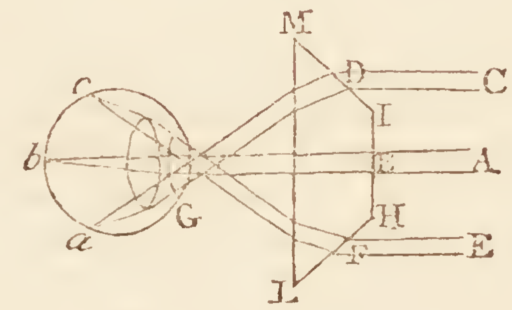
To conſtruct the Polyoptrum.—From a glaſs AB, plane on both ſides, and about 3 fingers thick, cut out ſpherical ſegments, ſcarce a 5th part of a digit in diameter.—If then the glaſs be removed to ſuch a diſtance from the eye, that you can take in all the cavities at one view, you will ſee the ſame object, as if



through ſo many ſeveral concave glaſſes as there are cavities, and all exceeding ſmall.—Fit this, as an object-glaſs, in a tube ABCD, whoſe aperture AB is equal to the diameter of the glaſs, and the other CD is equal to that of an eye-glaſs, as for inſtance about a finger's breadth. The length of the tube AC is to be accommodated to the object-glaſs and eye-glaſs, by trial. In CD fit a convex eye-glaſs, or in its ſtead a meniſcus having the diſtance of its principal focus a little larger than the length of the tube; ſo that the point from which the rays diverge after refraction in the object-glaſs, may be in the focus. If then the eye be applied near the eye-glaſs, a ſingle object will be ſeen repeated as often as there are cavities in the object-glaſs, but ſtill diminiſhed.

POLYSCOPE, or Polyhedron, in Optics, is a multiplying glaſs, being a glaſs or lens which repreſents a ſingle object to the eye as if it were many. It conſiſts of ſeveral plane ſurfaces, diſpoſed into a convex form, through every one of which the object is ſeen.

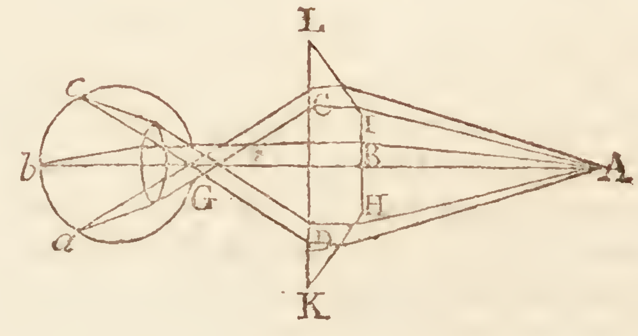
Phenomena of the Polyſcope.—1. If ſeveral rays, as EF, AB, CD, fall parallel on the ſurface of a Poly-



ſcope, they will continue parallel after refraction. If then the Polyſcope be ſuppoſed regular, LH, HI, IM will be as tangents cutting the ſpherical convex lens in F, B, and D; and conſequently, rays falling on the points of contact, interſect the axis. Wherefore, ſince the reſt are parallel to theſe, they will alſo mutually interſect each other in G.

Hence, if the eye be placed where parallel rays decuſſate, rays of the ſame object will be propagated to it ſtill parallel from the ſeveral ſides of the glaſs. Wherefore, ſince the cryſtalline humour, by its convexity, unites parallel rays, the rays will be united in as many different points of the retina, a, b, c, as the glaſs has ſides. Conſequently the eye, through a Polyſcope, ſees the object repeated as many times as there are ſides. And hence, ſince rays coming from very remote objects are parallel, a remote object is ſeen through a Polyſcope as often repeated as that has ſides.

2. If rays AB, AC, AD, coming from a radiant



point A, fall on ſeveral ſides of a regular Polyſcope;

 after

after refraction they will decuſſate in G, and proceed on a little diverging.

Hence, if the eye be placed where the rays decuſſate after coming from the ſeveral planes, the rays will be propagated to it from the ſeveral planes a little diverging, or as if they proceeded from different points. But ſince the cryſtalline humour, by its convexity, collects rays from ſeveral points into the ſame point; the rays will be united in as many different points of the retina, *a*, *b*, *c*, as the glaſs has ſides; and conſequently the eye, being placed in the focus G, will ſee even a near object through the Polyſcope as often repeated as that has ſides.

Thus may the images of objects be multiplied in a camera obſcura, by placing a Polyſcope at its aperture, and adding a convex lens at a due diſtance from it. And it makes a very pleaſant appearance, if a priſm be applied ſo that the coloured rays of the ſun refracted from it be received on the Polyſcope: for by this means they will be thrown on a paper or wall near at hand in little lucid ſpecks, much exceeding the brightneſs of any precious ſtone; and in the focus of the Polyſcope, where the rays decuſſate (for in this experiment they are received on the convex ſide) will be a ſtar of ſurpriſing luſtre.

Farther, if images be painted in water-colours in the areolæ or little ſquares of a Polyſcope, and the glaſs be applied to the aperture of a camera obſcura; the ſun's rays, paſſing through it, will carry with them the images, and project them on the oppoſite wall.—This artifice bears a reſemblance to that other, by which an image on paper is projected on the camera; viz, by wetting the paper with oil, and ſtraining it tight in a frame; then applying it to the aperture of the camera obſcura, ſo that the rays of a candle may paſs through it upon the Polyſcope.

To make an Anamorphoſis, or Deformed Image, which ſhall appear regular and beautiful through a Polyſcope, or Multiplying Glaſs.—At one end of a horizontal table erect another perpendicularly, upon which a figure may be deſigned; and on the other end erect another, to ſerve as a fulcrum or ſupport, moveable on the horizontal one. To the fulcrum apply a plano-convex Polyſcope, conſiſting, for example, of 24 plane triangles; and let the Polyſcope be fitted in a draw-tube, of which that end towards the eye may have only a very ſmall aperture, and a little farther off than the focus. Remove the fulcrum from the other perpendicular table, till it be out of the diſtance of the focus; and the more ſo, as the image is to be greater. Before the little aperture place a lamp; and trace the luminous areolæ projected from the ſides of the Polyſcope, with a black lead pencil, on the vertical plane, or a paper applied upon it.

In the ſeveral areolæ, deſign the different parts of an image, in ſuch a manner as that, when joined together, they may make one whole, looking every now and then through the tube to guide and correct the colours, and to ſee that the ſeveral parts match and fit well together. As to the intermediate ſpace, it may be filled up with any figures or deſigns at pleaſure, contriving it ſo, as that to the naked eye the whole may exhibit ſome appearance very different from that intended to appear through the Polyſcope.

The eye, now looking through the ſmall aperture of the tube, will ſee the ſeveral parts and members diſperſed among the areolæ to exhibit one continued image, all the intermediate parts diſappearing. See ANAMORPHOSIS.

POLYSPASTON, in Mechanics, a machine ſo called by Vitruvius, conſiſting of an aſſemblage of ſeveral pullies, uſed for raiſing heavy weights.

PONTON, or PONTOON, a kind of flat-bottomed boat, whoſe carcaſs of wood is lined within and without with tin. Some nations line them on the outſide only, and that with plates of copper, which is better. Our Pontoons are 21 feet long, nearly 5 feet broad, and 2 feet $1\frac{1}{2}$ inch deep within. They are carried along with an army upon carriages, to make temporary bridges, called Pontoon-bridges. See the next article.

PONTOON-*Bridge*, a bridge made of Pontoons ſlipped into the water, and moored by anchors and otherwiſe faſtened together by ropes, at ſmall diſtances from one another; then covered by beams of timber paſſing over them; upon which is laid a flooring of boards. By this means, whole armies of infantry, cavalry, and artillery are quickly paſſed over rivers.—For want of Pontoons, &c, bridges are ſometimes formed of empty powder caſks, or powder barrels, which ſupport the beams and flooring. Julius Cæſar and Aulus Gellius both mention Pontoons (pontones); but theirs were no more than a kind of ſquare flat veſſels, proper for carrying over horſe &c.

PONT-VOLANT, or *Flying-bridge*, is a kind of bridge uſed in ſieges, for ſurpriſing a poſt or outwork that has but narrow moats. It is made of two ſmall bridges laid over each other, and ſo contrived that, by means of cords and pullies placed along the ſides of the under bridge, the upper may be puſhed forwards, till it join the place where it is deſigned to be fixed. The whole length of both ought not to be above 5 fathoms, leſt it ſhould break with the weight of the men.

PORES, are the ſmall interſtices between the particles of matter which compoſe bodies; and are either empty, or filled with ſome inſenſible medium.

Condenſation and rarefaction are only performed by cloſing and opening the Pores. Alſo the tranſparency of bodies is ſuppoſed to ariſe from their Pores being directly oppoſite to one another. And the matter of inſenſible perſpiration is conveyed through the Pores of the cutis.

Mr. Boyle has a particular eſſay on the poroſity of bodies, in which he proves that the moſt ſolid bodies have ſome kind of Pores: and indeed if they had not, all bodies would be alike ſpecifically heavy.

Sir Iſaac Newton ſhews, that bodies are much more rare and porous than is commonly believed. Water, for example, is 19 times lighter and rarer than gold; and gold itſelf is ſo rare, as very readily, and without the leaſt oppoſition, to tranſmit magnetic effluvia, and eaſily to admit even quickſilver into its pores, and to let water paſs through it: for a concave ſphere of gold hath, when filled with water, and ſoldered up, upon preſſing it with a great force, ſuffered the water to ſqueeze through it, and ſtand all over its outſide, in multitudes of ſmall drops like dew, without burſting or cracking the gold. Whence it may be concluded, that

that gold has more pores than ſolid parts, and conſequently that water has above 40 times more Pores than parts. Hence it is that the magnetic effluvia paſſes freely through all cold bodies that are not magnetic; and that the rays of light paſs, in right lines, to the greateſt diſtances through pellucid bodies.

PORIME, *Porima*, in Geometry, a kind of eaſy lemma, or theorem ſo eaſily demonſtrated, that it is almoſt ſelf-evident: ſuch, for example, as that a chord is wholly within the circle.—Porime ſtands oppoſed to Aporime, which denotes a propoſition ſo difficult, as to be almoſt impoſſible to be demonſtrated, or effected. Such as the quadrature of the circle, &c.

PORISM, *Poriſma*, in Geometry, has by ſome been defined a general theorem, or canon, deduced from a geometrical locus, and ſerving for the ſolution of other general and difficult problems. Proclus derives the word from the Greek πορίζω, *I eſtabliſh*, and conclude from ſomething already done and demonſtrated: and accordingly he defines Poriſma a theorem drawn occaſionally from ſome other theorem already proved: in which ſenſe it agrees with what is otherwiſe called corollary.

Pappus ſays, a Poriſm is that in which ſomething was propoſed to be inveſtigated.

Others derive it from πόρος, *a paſſage*, and make it of the nature of a lemma, or a propoſition neceſſary for paſſing to another more important one.

But Dr. Simſon, rejecting the erroneous accounts that have been given of a Poriſm, defines it a propoſition, either in the form of a problem or a theorem, in which it is propoſed either to inveſtigate, or demonſtrate.

Euclid wrote three books of Poriſms, being a curious collection of various things relating to the analyſis of the more difficult and general problems. Thoſe books however are loſt; and nothing remains in the works of the ancient geometricians concerning this ſubject, beſides what Pappus has preſerved, in a very imperfect and obſcure ſtate, in his Mathematical Collections, viz, in the introduction to the 7th book.

Several attempts have been made to reſtore theſe writings in ſome degree, beſides that which Pappus has left upon the ſubject. Thus, Fermat has given a few propoſitions of this kind; which are to be found in the collection of his works, in folio, 1679, pa. 116. The like was done by Bullialdus, in his Exercitationes Geometricæ, 4to, 1657. Dr. Robert Simſon gave alſo a ſpecimen, in two propoſitions, in the Philoſ. Tranſ. vol. 32, pa. 330; and beſides left behind him a conſiderable treatiſe on the ſubject of Poriſms, which has been printed in an edition of his works, at the expence of the earl of Stanhope, in 4to, 1776.

The whole three books of Euclid were alſo reſtored by that ingenious mathematician Albert Girard, as appears by two notices that he gave, firſt in his Trigonometry, printed in French, at the Hague, in 1629, and alſo in his edition of the works of Stevinus, printed at Leyden in 1634, pa. 459; but whether his intention of publiſhing them was ever carried into execution, I have not been able to learn.

A learned paper on the ſubject of Poriſms, by the very ingenious Profeſſor Playfair, has juſt been inſerted in the 3d volume of the *Tranſactions* of the Royal Society of Edinburgh. As this paper contains a number of curious obſervations on the geometry of the Ancients in general, as well as forms a complete treatiſe as it were on Poriſm in particular, a pretty conſiderable abſtract of it cannot but be deemed in this place very uſeful and important.

"The reſtoration of the ancient books of geometry (ſays the learned profeſſor) would have been impoſſible, without the coincidence of two circumſtances, of which, though the one is purely accidental, the other is eſſentially connected with the nature of the mathematical ſciences. The firſt of theſe circumſtances is the preſervation of a ſhort abſtract of thoſe books, drawn up by Pappus Alexandrinus, together with a ſeries of ſuch lemmata, as he judged uſeful to facilitate the ſtudy of them. The ſecond is, the neceſſary connection that takes place among the objects of every mathematical work, which, by excluding whatever is arbitrary, makes it poſſible to determine the whole courſe of an inveſtigation, when only a few points in it are known. From the union of theſe circumſtances, mathematics has enjoyed an advantage of which no other branch of knowledge can partake; and while the critic or the hiſtorian has only been able to lament the fate of thoſe books of Livy and Tacitus which are loſt, the geometer has had the high ſatisfaction to behold the works of Euclid and Apollonius reviving under his hands.

"The firſt reſtorers of the ancient books were not, however, aware of the full extent of the work which they had undertaken. They thought it ſufficient to demonſtrate the propoſitions, which they knew from Pappus, to have been contained in thoſe books; but they did not follow the antient method of inveſtigation, and few of them appear to have had any idea of the elegant and ſimple analyſis by which theſe propoſitions were originally diſcovered, and by which the Greek Geometry was peculiarly diſtinguiſhed.

"Among theſe few, Fermat and Halley are to be particularly remarked. The former, one of the greateſt mathematicians of the laſt age, and a man in all reſpects of ſuperior abilities, had very juſt notions of the geometrical analyſis, and appears often abundantly ſkilful in the uſe of it; yet in his reſtoration of the Loci Plani, it is remarkable, that in the moſt difficult propoſitions, he lays aſide the analytical method, and contents himſelf with giving the ſynthetical demonſtration. The latter, among the great number and variety of his literary occupations, found time for a moſt attentive ſtudy of the ancient mathematicians, and was an inſtance of, what experience ſhews to be much rarer than might be expected, a man equally well acquainted with the ancient and the modern geometry, and equally diſpoſed to do juſtice to the merit of both. He reſtored the books of Apollonius, on the problem De Sectione Spatii, according to the true principles of the ancient analyſis.

"Theſe books, however, are but ſhort, ſo that the firſt reſtoration of conſiderable extent that can be reckoned complete, is that of the Loci Plani by Dr. Simſon, publiſhed in 1749, which, if it differs at all from the work it is intended to replace, ſeems to do ſo only by its greater excellence. This much at leaſt is certain, that the method of the ancient geometers does not appear to greater advantage in the moſt entire of their writings,

writings, than in the restoration above mentioned; and that Dr. Simson has often sacrificed the elegance to which his own analysis would have led, in order to tread more exactly in what the lemmata of Pappus pointed out to him, as the track which Apollonius had pursued.

" There was another subject, that of Porisms, the most intricate and enigmatical of any thing in the ancient geometry, which was still reserved to exercise the genius of Dr. Simson, and to call forth that enthusiastic admiration of antiquity, and that unwearied perseverance in research, for which he was so peculiarly distinguished. A treatise in three books, which Euclid had composed on Porisms, was lost, and all that remained concerning them was an abstract of that treatise, inserted by Pappus Alexandrinus in his Mathematical Collections, in which, had it been entire, the geometers of later times would doubtless have found wherewithal to console themselves for the loss of the original work. But unfortunately it has suffered so much from the injuries of time, that all which we can immediately learn from it is, that the Ancients put a high value on the propositions which they called Porisms, and regarded them as a very important part of their analysis. The Porisms of Euclid are said to be, " Collectio artificiosissima multarum rerum quæ spectant ad analysin difficiliorum et generalium problematum." The curiosity, however, which is excited by this encomium is quickly disappointed; for when Pappus proceeds to explain what a Porism is, he lays down two definitions of it, one of which is rejected by him as imperfect, while the other, which is stated as correct, is too vague and indefinite to convey any useful information.

" These defects might nevertheless have been supplied, if the enumeration which he next gives of Euclid's Propositions had been entire; but on account of the extreme brevity of his enunciations, and their reference to a diagram which is lost, and for the constructing of which no directions are given, they are all, except one, perfectly unintelligible. For these reasons, the fragment in question is so obscure, that even to the learning and penetration of Dr. Halley, it seemed impossible that it could ever be explained; and he therefore concluded, after giving the Greek text with all possible correctness, and adding the Latin translation, " Hactenus Porismatum descriptio nec mihi intellecta, " nec lectori profutura. Neque aliter fieri potuit, tam " ob defectum schematis cujus fit mentio, quam ob " omissa quædam et transposita, vel aliter vitiata in pro- " positionis generalis expositione, unde quid sibi velit " Pappus haud mihi datum est conjicere. His adde " dictionis modum nimis contractum, ac in re difficili, " qualis hæc est, minime usurpandum."

" It is true, however, that before this time, Fermat had attempted to explain the nature of Porisms, and not altogether without success. Guiding his conjectures by the definition which Pappus censures as imperfect, because it defined Porisms only " ab accidente," viz. " Porisma est quod deficit hypothesi a Theoremate Lo- " cali," he formed to himself a tolerably just notion of these propositions, and illustrated his general description by examples that are in effect Porisms. But he was able to proceed no farther; and he neither proved, that his notion of a Porism was the same with Euclid's, nor attempted to restore, or explain any one of Euclid's propositions; much less did he suppose, that they were to be investigated by an analysis peculiar to themselves. And so imperfect indeed was this attempt, that the complete restoration of the Porisms was necessary to prove, that Fermat had even approximated to the truth.

" All this did not, however, deter Dr. Simson from turning his thoughts to the same subject, which he appears to have done very early, and long before the publication of the Loci Plani in 1749.

" The account he gives of his progress, and of the obstacles he encountered, will be always interesting to mathematicians. " Postquam vero apud Pappum le- " geram, Porismata Euclidis collectionem fuisse artifi- " ciosissimam multarum rerum, quæ spectant ad analysin " difficiliorum et generalium problematum, magno " desiderio tenebar, aliquid de iis cognoscendi; quare " sæpius et multis variisque viis tum Pappi propositio- " nem generalem, mancam et imperfectam, tum pri- " mum lib. i.

" Porisma, quod solum ex omnibus in tribus libris " integrum adhuc manet, intelligere et restituere " conabar; frustra tamen, nihil enim proficiebam. " Cumque cogitationes de hac re multum mihi tempo- " ris consumpserint, atque molestæ admodum evaserint, " firmiter animum induxi hæc nunquam in posterum " investigare; præsertim cum optimus geometra Hal- " leius spem omnem de iis intelligendis abjecisset. Un- " de quoties menti occurrebant, toties eas arcebam. " Postea tamen accidit, ut improvidum et propositi im- " memorem invaserint, meque detinuerint donec tan- " dem lux quædam effulserit, quæ spem mihi faciebat " inveniendi saltem Pappi propositionem generalem, " quam quidem multa investigatione tandem restitui. " Hæc autem paulo post una cum Porismate primo " lib. i. impressa est inter Transactiones Phil. anni 1723, " num. 177."

" The propositions mentioned, as inserted in the Philosophical Transactions for 1723, are all that Dr. Simson published on the subject of Porisms during his life, though he continued his investigations concerning them, and succeeded in restoring a great number of Euclid's propositions, together with their analysis. The propositions thus restored form a part of that valuable edition of the posthumous works of this geometer which the mathematical world owes to the munificence of the late earl Stanhope.

" The subject of Porisms is not, however, exhausted, nor is it yet placed in so clear a light as to need no farther illustration. It yet remains to enquire into the probable origin of these propositions, that is to say, into the steps by which the ancient geometers appear to have been led to the discovery of them.

" It remains also to point out the relations in which they stand to the other classes of geometrical truths; to consider the species of analysis, whether geometrical or algebraical, that belongs to them; and, if possible, to assign the reason why they have so long escaped the notice of modern mathematicians. It is to these points that the following observations are chiefly directed.

" I begin with describing the steps that appear to have led the ancient geometers to the discovery of Porisms; and must here supply the want of express testi-

 mony.

mony by probable reafonings, fuch as are neceffary, whenever we would trace remote difcoveries to their fources, and which have more weight in mathematics than in any other of the fciences.

"It cannot be doubted, that it has been the folution of problems, which, in all ftates of the mathematical fciences, has led to the difcovery of moft geometrical truths. The firft mathematical enquiries, in particular, muft have occurred in the form of queftions, where fomething was given, and fomething required to be done; and by the reafonings neceffary to anfwer thefe queftions, or to difcover the relation between the things that were given, and thofe that were to be found, many truths were fuggefted, which came afterwards to be the fubjects of feparate demonftration. The number of thefe was the greater, that the ancient geometers always undertook the folution of problems with a fcrupulous and minute attention, which would fcarcely fuffer any of the collateral truths to efcape their obfervation. We know from the examples which they have left us, that they never confidered a problem as refolved, till they had diftinguifhed all its varieties, and evolved feparately every different cafe that could occur, carefully remarking whatever change might arife in the conftruction, from any change that was fuppofed to take place among the magnitudes which were given.

"Now as this cautious method of proceeding was not better calculated to avoid error, than to lay hold of every truth that was connected with the main object of enquiry, thefe geometers foon obferved, that there were many problems which, in certain circumftances, would admit of no folution whatever, and that the general conftruction by which they were refolved would fail, in confequence of a particular relation being fuppofed among the quantities which were given.

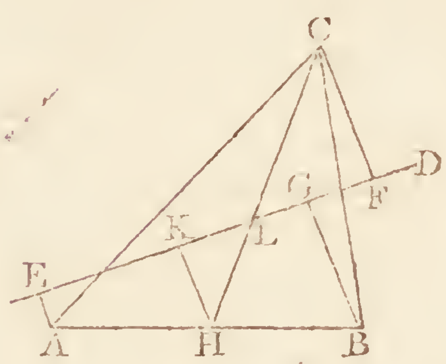
"Such problems were then faid to become impoffible; and it was readily perceived, that this always happened, when one of the conditions prefcribed was inconfiftent with the reft, fo that the fuppofition of their being united in the fame fubject, involved a contradiction. Thus, when it was required to divide a given line, fo that the rectangle under its fegment, fhould be equal to a given fpace, it was evident, that if this fpace was greater than the fquare of half the given line, the thing required could not poffibly be done; the two conditions, the one defining the magnitude of the line, and the other that of the rectangle under its fegments, being then inconfiftent with one another. Hence an infinity of beautiful propofitions concerning the maxima and the minima of quantities, or the limits of the poffible relations which quantities may ftand in to one another.

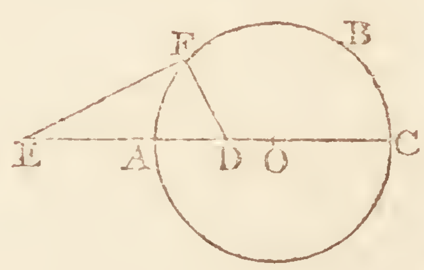
"Such cafes as thefe would occur even in the folution of the fimpleft problems; but when geometers proceeded to the analyfis of fuch as were more complicated, they muft have remarked, that their conftructions would fometimes fail, for a reafon directly contrary to that which has now been affigned. Inftances would be found where the lines that, by their interfection, were to determine the thing fought, inftead of interfecting one another, as they did in general, or of not meeting at all, as in the above-mentioned cafe of impoffibility, would coincide with one another entirely, and leave the queftion of confequence unrefolved. But though this circumftance muft have created confiderable embarraffment to the geometers who firft obferved it, as being perhaps the only inftance in which the language of their own fcience had yet appeared to them ambiguous or obfcure, it would not probably be long till they found out the true interpretation to be put on it. After a little reflexion, they would conclude, that fince, in the general problem, the magnitude required was determined by the interfection of the two lines above mentioned, that is to fay, by the points common to them both; fo, in the cafe of their coincidence, as all their points were in common, every one of thefe points muft afford a folution; which folutions therefore muft be infinite in number; and alfo, though infinite in number, they muft all be related to one another, and to the things given, by certain laws, which the pofition of the two coinciding lines muft neceffarily determine.

"On enquiring farther into the peculiarity in the ftate of the data which had produced this unexpected refult, it might likewife be remarked, that the whole proceeded from one of the conditions of the problem involving another, or neceffarily including it; fo that they both together made in fact but one, and did not leave a fufficient number of independent conditions, to confine the problem to a fingle folution, or to any determinate number of folutions. It was not difficult afterwards to perceive, that thefe cafes of problems formed very curious propofitions, of an intermediate nature between problems and theorems, and that they admitted of being enunciated feparately, in a manner peculiarly elegant and concife. It was to fuch propofitions, fo enunciated, that the ancient geometers gave the name of Porifms.

"This deduction requires to be illuftrated by examples." Mr. Playfair then gives feveral problems by way of illuftration; one of which, which may here fuffice to fhew the method, is as follows:

"A triangle ABC being given, and alfo a point D, to draw through D a ftraight line DG, fuch, that, perpendiculars being drawn to it from the three angles of the triangle, viz, AE, BG, CF, the fum of the two perpendiculars on the fame fide of DG, fhall be equal to the remaining perpendicular: or, that AE and BG together, may be equal to CF.





"Suppofe it done: Bifect AB in H, join CH, and draw HK perpendicular to DG.

"Becaufe AB is bifected in H, the two perpendiculars AE and BG are together double of HK; and as they are alfo equal to CF by hypothefis, CF muft be double of HK; and CL of LH. Now, GH is given in pofition, and magnitude; therefore the point L is given;

given; and the point D being also given, the line DL is given in position, which was to be found.

"The construction was obvious. Bisect AB in H, join CH, and take HL equal to one third of CH; the straight line which joins the points D and L is the line required.

"Now, it is plain, that while the triangle ABC remains the same, the point L also remains the same, wherever the point D may be. The point D may therefore coincide with L; and when this happens, the position of the line to be drawn is left undetermined; that is to say, any line whatever drawn through L will satisfy the conditions of the problem. Here therefore we have another indefinite case of a problem, and of consequence another Porism, which may be thus enunciated: "A triangle being given in position, a point in it may be found, such, that any straight line whatever being drawn through that point, the perpendiculars drawn to this straight line from the two angles of the triangle which are on one side of it, will be together equal to the perpendicular that is drawn to the same line from the angle on the other side of it.

"This Porism may be made much more general; for if, instead of the angles of a triangle, we suppose ever so many points to be given in a plane, a point may be found such, that any straight line being drawn through it, the sum of all the perpendiculars that fall on that line from the given points on one side of it, is equal to the sum of the perpendiculars that fall on it from all the points on the other side of it.

"Or still more generally, any number of points being given not in the same plane, a point may be found, through which if any plane be supposed to pass, the sum of all the perpendiculars which fall on that plane from the points on one side of it, is equal to the sum of all the perpendiculars that fall on the same plane from the points on the other side of it. It is unnecessary to observe, that the point to be found in these propositions, is no other than the centre of gravity of the given points; and that therefore we have here an example of a Porism very well known to the modern geometers, though not distinguished by them from other theorems."

After some examples of other Porisms, and remarks upon them, the author then adds,

"From this account of the origin of Porisms, it follows, that a Porism may be defined, *A proposition affirming the possibility of finding such conditions as will render a certain problem indeterminate, or capable of innumerable solutions.*

"To this definition, the different characters which Pappus has given will apply without difficulty. The propositions described in it like those which he mentions, are, strictly speaking, neither theorems nor problems, but of an intermediate nature between both; for they neither simply enunciate a truth to be demonstrated, nor propose a question to be solved: but are affirmations of a truth, in which the determination of an unknown quantity is involved. In as far therefore as they assert, that a certain problem may become indeterminate, they are of the nature of theorems; and in as far as they seek to discover the conditions by which that is brought about, they are of the nature of problems.

"In the preceding definition also, and the instances from which it is deduced, we may trace that imperfect description of Porisms which Pappus ascribes to the later geometers, viz, "Porisma est quod deficit hypothesi a theoremate locali." Now, to understand this, it must be observed, that if we take the converse of one of the propositions called *Loci*, and make the construction of the figure a part of the hypothesis, we have what was called by the Ancients a Local Theorem. And again, if, in enunciating this theorem, that part of the hypothesis which contains the construction be suppressed, the proposition arising from thence will be a Porism; for it will enunciate a truth, and will also require, to the full understanding and investigation of that truth, that something should be found, viz, the circumstance in the construction, supposed to be omitted.

"Thus when we say; If from two given points E and D (2d fig. above), two lines EF and FD are inflected to a third point F, so as to be to one another in a given ratio, the point F is in the circumference of a circle given in position: we have a *Locus*.

"But when conversely it is said; If a circle ABC, of which the centre is O, be given in position, as also a point E, and if D be taken in the line EO, so that the rectangle EO OD be equal to the square of AO, the semidiameter of the circle; and if from E and D, the lines EF and DF be inflected to any point whatever in the circumference ABC; the ratio of EF to DF will be a given ratio, and the same with that of EA to AD: we have a local theorem.

"And, lastly, when it is said; If a circle ABC be given in position, and also a point E, a point D may be found, such, that if the two lines EF and FD be inflected from E and D to any point whatever F, in the circumference, these lines shall have a given ratio to one another: the proposition becomes a Porism.

"Here it is evident, that the local theorem is changed into a Porism, by leaving out what relates to the determination of the point D, and of the given ratio. But though all propositions formed in this way, from the conversion of *Loci*, be Porisms, yet all Porisms are not formed from the conversion of *Loci*. The first and second of the preceding, for instance, cannot by conversion be changed into *Loci*; and therefore the definition which describes all Porisms as being so convertible, is not sufficiently comprehensive. Fermat's idea of Porisms, as has been already observed, was founded wholly on this definition, and therefore could not fail to be imperfect.

"It appears, therefore, that the definition of Porisms given above agrees with Pappus's idea of these propositions, as far at least as can be collected from the imperfect fragments which contain his general description of them. It agrees also with Dr. Simson's definition, which is this: "Porisma est propositio in qua proponitur demonstrare rem aliquam, vel plures datas esse, cui, vel quibus, ut et cuilibet ex rebus innumeris, non quidem datis, sed quæ ad ea quæ data sunt eandem habent relationem, convenire ostendendum est affectionem quandam communem in propositione descriptam.

"It cannot be denied, that there is a considerable degree of obscurity in this definition; notwithstanding of which it is certain, that every proposition to which it applies

applies muſt contain a *problematical* part, viz "in qua proponitur demonſtrare rem aliquam, vel plures datas eſſe," and alſo a *theoretical* part, which contains the property, or *communis affectio*, affirmed of certain things which have been previouſly deſcribed.

"It is alſo evident, that the ſubject of every ſuch propoſition, is the relation between magnitudes of three different kinds; determinate magnitudes which are given; determinate magnitudes which are to be found; and indeterminate magnitudes which, though unlimited in number, are connected with the others by ſome common property. Now, theſe are exactly the conditions contained in the definitions that have been given here.

"To confirm the truth of this theory of the origin of Poriſms, or at leaſt the juſtneſs of the notions founded on it, I muſt add a quotation from an Eſſay on the ſame ſubject, by a member of this ſociety, the extent and correctneſs of whoſe views make every coincidence with his opinions peculiarly flattering. In a paper read ſeveral years ago before the Philoſophical Society, Profeſſor Dugald Stewart defined a Poriſm to be "A propoſition affirming the poſſibility of finding one or more of the conditions of an indeterminate theorem." Where, by an indeterminate theorem, as he had previouſly explained it, is meant one which expreſſes a relation between certain quantities that are indeterminate, both in magnitude and in number. The near agreement of this with the definition and explanations which have been given above, is too obvious to require to be pointed out; and I have only to obſerve, that it was not long after the publication of Simſon's poſthumous works, when, being both of us occupied in ſpeculations concerning Poriſms, we were led ſeparately to the concluſions which I have now ſtated.

"In an enquiry into the origin of Poriſms, the etymology of the term ought not to be forgotten. The queſtion indeed is not about the derivation of the word Πορισμα, for concerning that there is no doubt; but about the reaſon why this term was applied to the claſs of propoſitions above deſcribed. Two opinions may be formed on this ſubject, and each of them with conſiderable probability: 1*mo*. One of the ſignifications of πορίζω, is *to acquire or obtain*; and hence Πορισμα, *the thing obtained or gained*.

"Accordingly, Scapula ſays, *Eſt vox a geometris deſumpta qui theorema aliquid ex demonſtrativo ſyllogiſmo neceſſario ſequens inferentes, illud quaſi lucrari dicuntur, quod non ex profeſſo quidem theorematis hujus inſtituta ſit demonſtratio, ſed tamen ex demonſtratis recte ſequatur.* In this ſenſe Euclid uſes the word in his Elements of Geometry, where he calls the corollaries of his propoſition, *Poriſmata*. This circumſtance creates a preſumption, that when the word was applied to a particular claſs of propoſitions, it was meant, in both caſes, to convey nearly the ſame idea, as it is not at all probable, that ſo correct a writer as Euclid, and ſo ſcrupulous in his uſe of words, ſhould employ the ſame term to expreſs two ideas which are perfectly different. May we not therefore conjecture, that theſe propoſitions got the name of Poriſms, entirely with a reference to their origin. According to the idea explained above, they would in general occur to mathematicians when engaged in the ſolution of the more difficult problems, and would ariſe from thoſe particular caſes, where one of the conditions of the data involved in it ſome one of the reſt. Thus a particular kind of theorem would be obtained, following as a corollary from the ſolution of the problem: and to this theorem the term Πορισμα might be very properly applied, ſince, in the words of Scapula, already quoted, *Non ex profeſſo theorematis hujus inſtituta ſit demonſtratio, ſed tamen ex demonſtratis recte ſequatur.*

"2do. But though this interpretation agrees ſo well with the ſuppoſed origin of Poriſms, it is not free from difficulty. The verb πορίζω has another ſignification, *to find out, to diſcover, to deviſe*; and is uſed in this ſenſe by Pappus, when he ſays that the propoſitions called Poriſms, afford great delight, τοις δυναμενοις ορᾳν και πορίζμα, *to thoſe who are able to underſtand and inveſtigate.* Hence comes πορισμος, *the act of finding out or diſcovering*, and from πορισμος, in this ſenſe, the ſame author evidently conſiders Πορισμα as being derived. His words are, Εφασαν δε (οι αρχαιοι) Πορισμα ειναι το προτεινομανον εις Πορισμον αυτη προτεινομανη, *the Ancients ſaid, that a Poriſm is ſomething propoſed for the* finding out, *or* diſcovering *of the very thing propoſed.* It ſeems ſingular, however, that Poriſms ſhould have taken their name from a circumſtance common to them with ſo many other geometrical truths; and if this was really the caſe, it muſt have been on account of the enigmatical form of their enunciations, which required, that in the analyſis of theſe propoſitions, a ſort of double diſcovery ſhould be made, not only of the Truth, but alſo of the Meaning *of the very thing which was propoſed.* They may therefore have been called *Poriſmata*, or *inveſtigations*, by way of eminence.

"We might next proceed to conſider the particular Poriſms which Dr. Simſon has reſtored, and to ſhew, that every one of them is the indeterminate caſe of ſome problem. But of this it is ſo eaſy for any one, who has attended to the preceding remarks, to ſatisfy himſelf, by barely examining the enunciations of thoſe propoſitions, that the detail unto which it would lead ſeems to be unneceſſary. I ſhall therefore go on to make ſome obſervations on that kind of analyſis which is particularly adapted to the inveſtigation of Poriſms.

"If the idea which we have given of theſe propoſitions be juſt, it follows, that they are always to be diſcovered by conſidering the caſes in which the conſtruction of a problem fails in conſequence of the lines which, by their interſection, or the points which, by their poſition, were to determine the magnitude required, happening to coincide with one another—a Poriſm may therefore be deduced from the problem it belongs to, in the ſame manner that the propoſitions concerning the *maxima* and *minima* of quantities are deduced from the problems of which they form the limitations; and ſuch no doubt is the moſt natural and moſt obvious analyſis of which this claſs of propoſitions will admit.

"It is not, however, the only one that they will admit of; and there are good reaſons for wiſhing to be provided with another, by means of which, a Poriſm that is any how ſuſpected to exiſt, may be found out, independently of the general ſolution of the problem to which it belongs. Of theſe reaſons, one is, that the Poriſm may perhaps admit of being inveſtigated more eaſily than the general problem admits of being reſolved;

and another is, that the former, in almoſt every caſe, helps to diſcover the ſimpleſt and moſt elegant ſolution that can be given of the latter.

" It is deſirable to have a method of inveſtigating Poriſms, which does not require, that we ſhould have previouſly reſolved the problems they are connected with, and which may always ſerve to determine, whether to any given problem there be attached a Poriſm, or not. Dr. Simſon's Analyſis may be conſidered as anſwering to this deſcription; for as that geometer did not regard theſe propoſitions at all in the light that is done here, nor in relation to their origin, an independent analyſis of this kind, was the only one that could occur to him; and he has accordingly given one which is extremely ingenious, and by no means eaſy to be invented, but which he uſes with great ſkilfulneſs and dexterity throughout the whole of his Reſtoration.

" It is not eaſy to aſcertain whether this be the preciſe method uſed by the Ancients. Dr. Simſon had here nothing to direct him but his genius, and has the full merit of the firſt inventor. It ſeems probable, however, that there is at leaſt a great affinity between the methods, ſince the *lemmata* given by Pappus as neceſſary to Euclid's demonſtrations, are ſubſervient alſo to thoſe of our modern geometer.

" It is, as we have ſeen, a general principle that a problem is converted into a Poriſm, when one, or when two, of the conditions of it, neceſſarily involve in them ſome one of the reſt. Suppoſe then that two of the conditions are exactly in that ſtate which determines the third; then, while they remain fixed or given, ſhould that third one be ſuppoſed to vary, or differ, ever ſo little, from the ſtate required by the other two, a contradiction will enſue. Therefore if, in the hypotheſis of a problem, the conditions be ſo related to one another as to render it indeterminate, a Poriſm is produced; but if, of the conditions thus related to one another, ſome one be ſuppoſed to vary, while the others continue the ſame, an abſurdity follows, and the problem becomes impoſſible. *Wherever therefore any problem admits both of an indeterminate, and an impoſſible caſe, it is certain, that theſe caſes are nearly related to one another, and that ſome of the conditions by which they are produced, are common to both.*

" It is ſuppoſed above, that *two* of the conditions of a problem involve in them a third, and wherever that happens, the concluſion which has been deduced will invariably take place.

" But a Poriſm may ſometimes be ſo ſimple, as to ariſe from the mere coincidence of *one* condition of a problem with another, though in no caſe whatever, any inconſiſtency can take place between them. Thus, in the ſecond of the foregoing propoſitions, the coincidence of the point given in the problem with another point, viz. the centre of gravity of the given triangle, renders the problem indeterminate; but as there is no relation of diſtance, or poſition, between theſe points, that may not exiſt, ſo the problem has no impoſſible caſe belonging to it. There are, however, comparatively but few Poriſms ſo ſimple in their origin as this, or that ariſe from problems in which the conditions are ſo little complicated; for it uſually happens, that a problem which can become indefinite, may alſo become impoſſible; and if ſo, the connection between theſe caſes, which has been already explained, never fails to take place.

" Another ſpecies of impoſſibility may frequently ariſe from the poriſmatic caſe of a problem, which will very much affect the application of geometry to aſtronomy, or any of the ſciences of experiment or obſervation. For when a problem is to be reſolved by help of data furniſhed by experiment or obſervation, the firſt thing to be conſidered is, whether the data ſo obtained, be ſufficient for determining the thing ſought; and in this a very erroneous judgment may be formed, if we reſt ſatisfied with a general view of the ſubject: For though the problem may in general be reſolved from the data that we are provided with, yet theſe data may be ſo related to one another in the caſe before us, that the problem will become indeterminate, and inſtead of one ſolution, will admit of an infinite number.

" Suppoſe, for inſtance, that it were required to determine the poſition of a point F from knowing that it was ſituated in the circumference of a given circle ABC, and alſo from knowing the ratio of its diſtances from two given points E and D; it is certain that in general theſe data would be ſufficient for determining the ſituation of F. But nevertheleſs, if E and D ſhould be ſo ſituated, that they were in the ſame ſtraight line with the centre of the given circle; and if the rectangle under their diſtances from that centre, were alſo equal to the ſquare of the radius of the circle, then, the poſition of F could not be determined.

" This particular inſtance may not indeed occur in any of the practical applications of geometry; but there is one of the ſame kind which has actually occurred in aſtronomy: And as the hiſtory of it is not a little ſingular, affording beſides an excellent illuſtration of the nature of Poriſms, I hope to be excuſed for entering into the following detail concerning it.

" Sir Iſaac Newton having demonſtrated, that the trajectory of a comet is a parabola, reduced the actual determination of the orbit of any particular comet to the ſolution of a geometrical problem, depending on the properties of the parabola, but of ſuch conſiderable difficulty, that it is neceſſary to take the aſſiſtance of a more elementary problem, in order to find, at leaſt nearly, the diſtance of the comet from the earth, at the times when it was obſerved. The expedient for this purpoſe, ſuggeſted by Newton himſelf, was to conſider a ſmall part of the comet's path as rectilineal, and deſcribed with an uniform motion, ſo that four obſervations of the comet being made at moderate intervals of time from one another, four ſtraight lines would be determined, viz. the four lines joining the places of the earth and the comet, at the times of obſervation, acroſs which if a ſtraight line were drawn, ſo as to be cut by them in three parts, in the ſame ratios with the intervals of time abovementioned; the line ſo drawn would nearly repreſent the comet's path, and by its interſection with the given lines, would determine, at leaſt nearly, the diſtances of the comet from the earth at the time of obſervation.

" The geometrical problem here employed, of drawing a line to be divided by four other lines given in poſition, into parts having given ratios to one another, had been already reſolved by Dr. Wallis and Sir Chriſ-

topher

topher Wren, and to their solutions Sir Isaac Newton added three others of his own, in different parts of his works. Yet none of all these geometers observed that peculiarity in the problem which rendered it inapplicable to astronomy. This was first done by M. Boscovich, but not till after many trials, when, on its application to the motion of comets, it had never led to any satisfactory result. The errors it produced in some instances were so considerable, that Zanotti, seeking to determine by it the orbit of the comet of 1739, found, that his construction threw the comet on the side of the sun opposite to that on which he had actually observed it. This gave occasion to Boscovich, some years afterwards, to examine the different cases of the problem, and to remark that, in one of them, it became indeterminate, and that, by a curious coincidence, this happened in the only case which could be supposed applicable to the astronomical problem abovementioned; in other words, he found, that in the state of the data, which must there always take place, innumerable lines might be drawn, that would be all cut in the same ratio, by the four lines given in position. This he demonstrated in a dissertation published at Rome in 1749, and since that time in the third volume of his *Opuscula*. A demonstration of it, by the same author, is also inserted at the end of Castillon's Commentary on the *Arithmetica Universalis*, where it is deduced from a construction of the general problem, given by Mr. Thomas Simpson, at the end of his Elements of Geometry. The proposition, in Boscovich's words, is this: Problema quo quæritur recta linea quæ quatuor rectas positione datas ita secet, ut tria ejus segmenta sint invicem in ratione data, evadit aliquando indeterminatum, ita ut per quodvis punctum cujusvis ex iis quatuor rectis duci possit recta linea, quæ ei conditioni faciat satis.

"It is needless, I believe, to remark, that the proposition thus enunciated is a Porism, and that it was discovered by Boscovich, in the same way, in which I have supposed Porisms to have been first discovered by the geometers of antiquity.

"A question nearly connected with the origin of Porisms still remains to be solved, namely, from what cause has it arisen that propositions which are in themselves so important, and that actually occupied so considerable a place in the ancient geometry, have been so little remarked in the modern? It cannot indeed be said, that propositions of this kind were wholly unknown to the Moderns before the restoration of what Euclid had written concerning them; for besides M. Boscovich's proposition, of which so much has been already said, the theorem which asserts, that in every system of points there is a centre of gravity, has been shewn above to be a Porism; and we shall see hereafter, that many of the theorems in the higher geometry belong to the same class of propositions. We may add, that some of the elementary propositions of geometry want only the proper form of enunciation to be perfect Porisms. It is not therefore strictly true, that none of the propositions called Porisms have been known to the Moderns; but it is certain, that they have not met, from them, with the attention they met with from the Ancients, and that they have not been distinguished as a separate class of propositions. The cause of this difference is undoubtedly to be sought for in a comparison of the methods employed for the solution of geometrical problems in ancient and modern times.

"In the solution of such problems, the geometers of antiquity proceeded with the utmost caution, and were careful to remark every particular case, that is to say, every change in the construction, which any change in the state of the data could produce. The different conditions from which the solutions were derived, were supposed to vary one by one, while the others remained the same; and all their possible combinations being thus enumerated, a separate solution was given, wherever any considerable change was observed to have taken place.

"This was so much the case, that the *Sectio Rationis*, a geometrical problem of no great difficulty, and one of which the solution would be dispatched, according to the methods of the modern geometry, in a single page, was made by Apollonius, the subject of a treatise consisting of two books. The first book has seven general divisions, and twenty-four cases; the second, fourteen general divisions, and seventy-three cases, each of which cases is separately considered. Nothing, it is evident, that was any way connected with the problem, could escape a geometer, who proceeded with such minuteness of investigation.

"The same scrupulous exactness may be remarked in all the other mathematical researches of the Ancients; and the reason doubtless is, that the geometers of those ages, however expert they were in the use of their analysis, had not sufficient experience in its powers, to trust to the more general applications of it. That principle which we call the *law of continuity*, and which connects the whole system of mathematical truths by a chain of insensible gradations, was scarcely known to them, and has been unfolded to us, only by a more extensive knowledge of the mathematical sciences, and by that most perfect mode of expressing the relations of quantity, which forms the language of algebra; and it is this principle alone which has taught us, that though in the solution of a problem, it may be impossible to conduct the investigation without assuming the data in a *particular* state, yet the result may be perfectly *general*, and will accommodate itself to every case with such wonderful versatility, as is scarcely credible to the most experienced mathematician, and such as often forces him to stop, in the midst of his calculus, and look back, with a mixture of diffidence and admiration, on the unforeseen harmony of his conclusions. All this was unknown to the Ancients; and therefore they had no resource, but to apply their analysis separately to each particular case, with that extreme caution which has just been described; and in doing so, they were likely to remark many peculiarities, which more extensive views, and more expeditious methods of investigation, might perhaps have induced them to overlook.

"To rest satisfied, indeed, with too general results, and not to descend sufficiently into particular details, may be considered as a vice that naturally arises out of the excellence of the modern analysis. The effect which this has had, in concealing from us the class of propositions we are now considering, cannot be better illustrated than by the example of the Porism discovered by Boscovich, in the manner related above. Though the problem from which that Porism is derived, was

 resolved

resolved by several mathematicians of the first eminence, among whom also was Sir Isaac Newton, yet the Porism which, as it happens is the most important case of it, was not observed by any of them. This is the more remarkable, that Sir Isaac Newton takes notice of the two most simple cases, in which the problem obviously admits of innumerable solutions, viz, when the lines given in position are either all parallel, or all meeting in a point, and these two hypotheses he therefore expressly excepts. Yet he did not remark, that there are other circumstances which may render the solution of the problem indeterminate as well as these; so that the porismatic case considered above, escaped his observation: and if it escaped the observation of one who was accustomed to penetrate so far into matters infinitely more obscure, it was because he satisfied himself with a general construction, without pursuing it into its particular cases. Had the solution been conducted after the manner of Euclid or Apollonius, the Porism in question must infallibly have been discovered."

PORISTIC *Method*, is that which determines when, by what means, and how many different ways, a problem may be resolved.

PORTA (John Baptista), called also in Italy *Giovan Batista de la Porta*, of Naples, lived about the end of the 16th century, and was famous for his skill in philosophy, mathematics, medicine, natural history, &c, as well as for his indefatigable endeavours to improve and propagate the knowledge of those sciences. With this view, he not only established private schools for particular sciences, but to the utmost of his power promoted public academies. He had no small share in establishing the academy at *Gli Ozioni*, at Naples, and had one in his own house, called *de Secreti*, into which none were admitted members, but such as had made some new discoveries in nature. He died at Pisa, in the kingdom of Naples, in the year 1615.

Porta gave the fullest proof of an extensive genius, and wrote a great many works; the principal of which are as follow:

1. His *Natural Magic*; a book abounding with curious experiments; but containing nothing of magic, the common acceptation of the word, as he pretends to nothing above the power of nature.
2. *Elements of Curve Lines.*
3. *A Treatise of Distillation.*
4. *A Treatise of Arithmetic.*
5. *Concerning Secret Letter-writing.*
6. *Of Optical Refractions.*
7. *A Treatise of Fortification.*
8. *A Treatise of Physiognomy.*

Beside some Plays and other pieces of less note.

PORTAIL, in Architecture, the face or frontispiece of a church, viewed on the side in which the great door is placed. It means also the great door or gate itself of a palace, castle, &c.

PORTAL, in Architecture, a term used for a little square corner of a room, cut off from the rest of the room by the wainscot; frequent in the ancient buildings, but now disused.

Portal is sometimes also used for a little gate, portella; where there are two gates, a large and a small one.

Portal is sometimes also used for a kind of arch of joiner's work before a door.

PORTCULLICE, called also *Herse*, and *Sarrasin*, in Fortification, an assemblage of several large pieces of wood laid or joined across one another, like a harrow, and each pointed at the bottom with iron. These were formerly used to be hung over the gateways of fortified places, to be ready to let down in case of a surprize, when the enemy should come so quick, as not to allow time to shut the gates. But the orgues are now more generally used, being found to answer the purpose better.

PORT-Fire, in Gunnery, a paper tube, about 10 inches long, filled with a composition of meal-powder, sulphur, and nitre, rammed moderately hard; used to fire guns and mortars, instead of match.

PORTICO, in Architecture, is a kind of gallery, raised upon arches, under which people walk for shelter.

POSITION, or *Site*, or *Situation*, in Physics, is an affection of place, expressing the manner of a body's being in it.

Position, in Architecture, denotes the situation of a building, with respect to the points of the horizon. The best it is thought is when the four sides point directly to the four winds.

Position, in Astronomy, relates to the sphere. The position of the sphere is either right, parallel, or oblique; whence arise the inequality of days, the difference of seasons, &c.

Circles of Position, are circles passing through the common intersections of the horizon and meridian, and through any degree of the ecliptic, or the centre of any star, or other point in the heavens; used for finding out the position or situation of any star. These are usually counted six in number, cutting the equator into twelve equal parts, which the astrologers call the celestial houses.

Position, in Arithmetic, called also False Position, or Supposition, or Rule of False, is a rule so called, because it consists in calculating by false numbers supposed or taken at random, according to the process described in any question or problem proposed, as if they were the true numbers, and then from the results, compared with that given in the question, the true numbers are found. It is sometimes also called Trial-and-Error; because it proceeds by trials of false numbers, and thence finds out the true ones by a comparison of the errors.

Position is either Single or Double.

Single Position is when only one supposition is employed in the calculation. And

Double Position is that in which two suppositions are employed.

To the rule of Position properly belong such questions as cannot be resolved from a direct process by any of the other usual rules in arithmetic, and in which the required numbers do not ascend above the first power: such, for example, as most of the questions usually brought to exercise the reduction of simple equations in algebra. But it will not bring out true answers when the numbers sought ascend above the first power; for then the results are not proportional to the Positions,

or

or ſuppoſed numbers, as in the ſingle rule; nor yet the errors to the difference of the true number and each Poſition, as in the double rule. Yet in all ſuch caſes, it is a very good approximation, and in exponential equations, as well as in many other things, it ſucceeds better than perhaps any other method whatever.

Thoſe queſtions, in which the reſults are proportional to their ſuppoſitions, belong to Single Poſition: ſuch are thoſe which require the multiplication or diviſion of the number ſought by any number; or in which it is to be increaſed or diminiſhed by itſelf any number of times, or by any part or parts of it. But thoſe in which the reſults are not proportional to their poſitions, belong to the double rule: ſuch are thoſe, in which the numbers ſought, or their multiples or parts, are increaſed or diminiſhed by ſome given abſolute number, which is no known part of the number ſought.

To work by *the Single Rule of* POSITION. Suppoſe, take, or aſſume any number at pleaſure, for the number ſought, and proceed with it as if it were the true number, that is, perform the ſame operations with it as, in the queſtion, are deſcribed to be performed with the number required: then if the reſult of thoſe operations be the ſame with that mentioned or given in the queſtion, the ſuppoſed number is the ſame as the true one that was required; but if it be not, make this proportion, viz, as your reſult is to that in the queſtion, ſo is your ſuppoſed falſe number, to the true one required.

Example. Suppoſe that a perſon, after ſpending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 60l.; what ſum had he at firſt?

Suppoſe he had at firſt 120l.

Now $\frac{1}{3}$ of 120 is	40
and $\frac{1}{4}$ of it is	30
their ſum is	70
which taken from	120
leaves remaining	50, inſtead of 60.

Therefore as 50 : 60 :: 120 : 144 the ſum at firſt.

Proof.	$\frac{1}{3}$ of 144 is	48
	$\frac{1}{4}$ of it is	36
	their ſum	84
	taken from	144
	leaves juſt	60 as per queſt.

To work by the Double Rule of POSITION.

In this rule, make two different ſuppoſitions, or aſſumptions, and work or perform the operations with each, deſcribed in the queſtion, exactly as in the ſingle rule: and if neither of the ſuppoſed numbers ſolve the queſtion, that is, produce a reſult agreeing with that in the queſtion; then obſerve the errors, or how much each of the falſe reſults differs from the true one, and alſo whether they are too great or too little; marking them with + when too great, and with − when too little. Next multiply, croſſwiſe, each poſition by the error of the other; and if the errors be of the ſame affection, that is both +, or both −, ſubtract the one product from the other, as alſo the one error from the other, and divide the former of theſe two remainders by the latter, for the anſwer, or number ſought. But if the errors be unlike, that is, the one +, and the other −, add the two products together, and alſo the two errors together, and divide the former ſum by the latter, for the anſwer.

And in this rule it is particularly uſeful to remember this part of the rule, viz. to ſubtract when the errors are alike, both + or both −, but to add when unlike, or the one + and the other −.

Example. A ſon aſking his father how old he was, received this anſwer: Your age is now $\frac{1}{4}$ of mine; but 5 years ago your age was only $\frac{1}{5}$ of mine at that time. What then were their ages?

Firſt, ſuppoſe the ſon 15;

then 15 × 4 = 60 the father's;	
alſo, 5 years ago the ſon was	10,
and the father's muſt be	55,
but ought to be 10 × 5 or	50,
therefore the error is	5 −.

Again, ſuppoſe the ſon 22;

then 22 × 4 = 88 is the father's;	
alſo 5 years ago the ſon was	17,
and the father's then	83,
but ought to be 17 × 5, or	85,
therefore the error is	2 +.

And the errors, being unlike, muſt be added, their ſum being 7.

Then 15	22
2	5
30	110
	30

7) 140 (20 the ſon's age, and conſequently 80 the father's.

This rule of Poſition, or trial-and-error, is a good general way of approximating to the roots of the higher equations, to which it may be applied even before the equation is reduced to a final or ſimple ſtate, by which it often ſaves much trouble in ſuch reductions. It is alſo eminently uſeful in reſolving exponential equations, and equations involving arcs, or ſines, &c, or logarithms, and in ſhort in any equations that are very intricate and difficult. And even in the extraction of the higher roots of common numbers, it may be very uſefully applied. As for inſtance, to extract the 3d or cubic root of the number 20.—Here it is evident that the root is greater than 2 and leſs than 3; making theſe two numbers therefore the ſuppoſitions, the proceſs will be thus:

1ſt ſup.	$2^3 = 8$	2d ſup.	$3^3 = 27$
given number	20	given number	20
1ſt error	12 −	2d error	7 +
	3		2
12	36 } add		14
7	14 }		

19) 50 (2·63 the firſt approximation.

Again,

Again, as it thus appears the cube root of 20 is near 2·6 or 2·7, make ſuppoſition of theſe two, and repeat the proceſs with them, thus:

1ſt ſup. $2{\cdot}6^3 =$	17·576	2d ſup. $2{\cdot}7^3 =$	19·683
given number	20·	given number	20·
1ſt error	2·424 −	2d error	0·317 −
	2·7		2·6
	16968		1902
	4848		634
2·424	6·5448 } ſubtr.		·8242
·317	·8242		
2·107)	5·7206 (2·714 root ſought.		

The rule of Poſition paſſed from the Moors into Europe, by Spain and Italy, along with their algebra, or method of equations, which was probably derived from the former.

POSITION, in Geometry, reſpects the ſituation, bearing, or direction of one thing, with regard to another. And Euclid ſays, "Points, lines, and angles, which have and keep always one and the ſame place and ſituation, are ſaid to be given by Poſition or ſituation." Data, def. 4.

POSITIVE *Quantities*, in Algebra, ſuch as are of a real, affirmative, or additive nature; and which either have, or are ſuppoſed to have, the affirmative or poſitive ſign + before them; as a or $+a$, or bc, &c. It is uſed in contradiſtinction from negative quantities, which are defective or ſubductive ones, and marked by the ſign −; as $-a$, or $-ab$.

POSTERN, or *Sally-port*, in Fortification, a ſmall gate, uſually made in the angle of the flank of a baſtion, or in that of the curtain, or near the orillon, deſcending into the ditch; by which the garriſon can march in and out, unperceived by the enemy, either to relieve the works, or to make private ſallies, &c.—It means alſo any private or back door.

POSTICUM, in Architecture, the poſtern gate, or back-door of any fabric.

POSTULATE, a demand, petition, or a problem of ſo obvious a nature, as to need neither demonſtration, nor explication, to render it either more plain or certain. This definition will nearly agree alſo to an axiom, which is a ſelf-evident theorem, as a Poſtulate is a ſelf-evident problem.

Euclid lays down theſe three Poſtulates in his Elements; viz, 1ſt, That from one point to another a line can be drawn. 2d, That a right line can be produced out at pleaſure. 3d, That with any centre and radius a circle may be deſcribed.

As to axioms, he has a great number; as, That two things which are equal to one and the ſame thing, are equal to each other, &c.

POUND, a certain weight; which is of two kinds, viz, the pound troy, and the pound avoirdupois; the former conſiſting of 12 ounces troy, and the latter of 16 ounces avoirdupois.—The pound troy is to the pound avoirdupois as 5760 to $6999\frac{1}{2}$, or nearly 576 to 700.

POUND alſo is an imaginary money uſed in accounting, in ſeveral countries. Thus, in England there is the Pound ſterling, containing 20 ſhillings; in France the Pound or livre Tournois and Pariſis; in Holland and Flanders, a Pound or livre de gros, &c.—The term aroſe from hence, that the ancient pound ſterling, though it only contained 240 pence, as ours does; yet each penny being equal to five of ours, the pound of ſilver weighed a Pound troy.

POUNDER, in Artillery, a term uſed to expreſs a certain weight of ſhot or ball, or how many pounds weight the proper ball is for any cannon: as a 24 pounder, a 12 pounder, &c.

POWDER, *Gun*. See GUNPOWDER.

POWDER-*Triers*. See EPROUVETTE.

POWER, in Mechanics, denotes ſome force which, being applied to a machine, tends to produce motion; whether it does actually produce it or not. In the former caſe, it is called a moving Power; in the latter, a ſuſtaining power.

POWER is alſo uſed in Mechanics, for any of the ſix ſimple machines, viz. the lever, the balance, the ſcrew, the wheel and axle, the wedge, and the pulley.

POWER *of a Glaſs*, in Optics, is by ſome uſed for the diſtance between the convexity and the ſolar focus.

POWER, in Arithmetic, the produce of a number, or other quantity, ariſing by multiplying it by itſelf, any number of times.

Any number is called the firſt power of itſelf. If it be multiplied once by itſelf, the product is the ſecond power, or ſquare; if this be multiplied by the firſt power again, the product is the third power, or cube; if this be multiplied by the firſt power again, the product is the fourth power, or biquadratic; and ſo on; the Power being always denominated from the number which exceeds the multiplications by one or unity, which number is called the index or exponent of the Power, and is now ſet at the upper corner towards the right of the given quantity or root, to denote or expreſs the Power. Thus,

$$\begin{array}{llll} 3 & \text{or } 3^1 = & 3 & \text{is the 1ſt power of } 3, \\ 3 \times 3 & \text{or } 3^2 = & 9 & \text{is the 2d power of } 3, \\ 3^2 \times 3 & \text{or } 3^3 = & 27 & \text{is the 3d power of } 3, \\ 3^3 \times 3 & \text{or } 3^4 = & 81 & \text{is the 4th power of } 3, \\ \text{\&c.} & & & \text{\&c.} \end{array}$$

Hence, to raiſe a quantity to a given Power or dignity, is the ſame as to find the product ariſing from its being multiplied by itſelf a certain number of times; for example, to raiſe 2 to the 3d power, is the ſame thing as to find the factum, or product $8 = 2 \times 2 \times 2$. The operation of raiſing Powers, is called Involution.

Powers, of the ſame degree, are to one another in the ratio of the roots as manifold as their common exponent contains units: thus, ſquares are in a duplicate ratio of the roots; cubes in a triplicate ratio; 4th powers in a quadruplicate ratio.—And the Powers of proportional quantities are alſo proportional to one another: ſo, if $a : b :: c : d$, then, in any Powers alſo, $a^n : b^n :: c^n : d^n$.

The particular names of the ſeveral Powers, as introduced by the Arabians, were, ſquare, cube, quadrato-quadratum or biquadrate, ſurſolid, cube ſquared, ſecond ſurſolid, quadrato-quadrato-quadratum, cube of the cube,

cube, square of the surfolid, third surfolid, and so on, according to the *products* of the indices.

And the names given by Diophantus, who is followed by Vieta and Oughtred, are, the side or root, square, cube, quadrato-quadratum, quadrato-cubus, cubo-cubus, quadrato-quadrato-cubus, quadrato-cubo-cubus, cubo-cubo-cubus, &c, according to the *sums* of the indices.

But the moderns, after Des Cartes, are contented to distinguish most of the Powers by the exponents; as 1st, 2d, 3d, 4th, &c.

The characters by which the several Powers are denoted, both in the Arabic and Cartesian notation are as follow:

Arab.	1	*R*	*q*	*c*	*bq.*	*s*	*qc*	*Bs*	*tq*	*bc*
Cart.	a^0	a^1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9
	1	2	4	8	16	32	64	128	256	512

Hence, 1st. The Powers of any quantity, form a series of geometrical proportionals, and their exponents a series of arithmetical proportionals, in such sort that the addition of the latter answers to the multiplication of the former, and the subtraction of the latter answers to the division of the former, &c; or in short, that the latter, or exponents, are as the logarithms of the former, or Powers.

Thus, $a^2 \times a^3 = a^5$, and $2 + 3 = 5$;
$4 \times 8 = 32$;
also $a^5 \div a^3 = a^2$, and $5 - 3 = 2$;
$32 \div 8 = 4$.

2d. The 0 Power of any quantity, as a^0, is $= 1$.

3d. Powers of the same quantity are multiplied, by adding their exponents: Thus,

Mult.	a^3	x^2	y^m	x^m	a^3
by	a^4	x^4	y^m	x^n	a^n
Prod.	a^7	x^6	y^{2m}	x^{m+n}	a^{3+n}

4th. Powers are divided by subtracting their exponents. Thus,

Div.	a^7	x^6	y^{2m}	x^{m+n}	a^{3+n}
by	a^3	x^2	y^m	x^m	a^3
Quot.	a^4	x^4	y^m	x^n	a^n

5th. Powers are also considered as negative ones, or having negative exponents, when they denote a divisor, or the denominator of a fraction. So $\frac{1}{a^3} = a^{-3}$, and $\frac{2}{a^2} = 2a^{-2}$, and $\frac{a^2}{x^4} = a^2 x^{-4}$, &c. And hence any quantity may be changed from the denominator to the numerator, or from a divisor to a multiplier, or vice versa, by changing the sign of its exponent; and the whole series of Powers proceeds indefinitely both ways from 1 or the 0 Power, positive on the one hand, and negative on the other. Thus,

$$\&c\ a^{-4}\ a^{-3}\ a^{-2}\ a^{-1}\ a^0\ a^1\ a^2\ a^3\ a^4\ \&c,$$

$$\text{or } \&c\ \frac{1}{a^4}\ \frac{1}{a^3}\ \frac{1}{a^2}\ \frac{1}{a}\ 1\ a\ a^2\ a^3\ a^4\ \&c.$$

Powers are also denoted with fractional exponents, or even with surd or irrational ones; and then the numerator denotes the Power raised to, and the denominator the exponent of some root to be extracted: Thus,

$$\sqrt{a} = a^{\frac{1}{2}}, \text{ and } \sqrt{a^3} = a^{\frac{3}{2}}, \text{ and } \sqrt[3]{a^2} = a^{\frac{2}{3}}, \ \&c.$$

And these are sometimes called imperfect powers, or surds.

When the quantity to be raised to any Power is positive, all its Powers must be positive. And when the radical quantity is negative, yet all its even Powers must be positive: because $- \times -$ gives $+$: the odd Powers only being negative, or when their exponents are odd numbers: Thus, the Powers of $-a$,

are $+1,\ -a,\ +a^2,\ -a^3,\ +a^4,\ -a^5,\ +a^6$, &c.

where the even Powers a^2, a^4, a^6 are positive,
and the odd Powers a, a^3, a^5 are negative.

Hence, if a Power have a negative sign, no even root of it can be assigned; since no quantity multiplied by itself an even number of times, can give a negative product. Thus $\sqrt{-a^2}$, or the square or 2d root of $-a^2$, cannot be assigned; and is called an impossible root, or an imaginary quantity.—Every Power has as many roots, real and imaginary, as there are units in the exponent.

M. De la Hire gives a very odd property common to all Powers. M. Carre had observed with regard to the number 6, that all the natural cubic numbers, 8, 27, 64, 125, having their roots less than 6, being divided by 6, the remainder of the division is the root itself; and if we go farther, 216, the cube of 6, being divided by 6, leaves no remainder; but the divisor 6 is itself the root. Again, 343, the cube of 7, being divided by 6, leaves 1; which added to the divisor 6, makes the root 7, &c. M. De la Hire, on considering this, has found that all numbers, raised to any Power whatever, have divisors, which have the same effect with regard to them, that 6 has with regard to cubic numbers. For finding these divisors, he discovered the following general rule, viz, If the exponent of the Power of a number be even, i. e. if the number be raised to the 2d, 4th, 6th, &c Power, it must be divided by 2; the remainder of the division, when there is any, added to 2, or to a multiple of 2, gives the root of this number, corresponding to its Power, i. e. the 2d, 6th, &c root.

But if the exponent of the power be an uneven number, i. e. if the number be raised to the 3d, 5th, 7th, &c Power; the double of this exponent will be the divisor, which has the property abovementioned. Thus is it found in 6, the double of 3, the exponent of the Power of the cubes: so also 10, the double of 5, is the divisor of all 5th Powers; &c.

Any Power of the natural numbers 1, 2, 3, 4, 5, 6, &c, as the *n*th Power, has as many orders of differences as there are units in the common exponent of all the numbers; and the last of those differences is a constant quantity, and equal to the continual product $1 \times 2 \times 3 \times 4 \times \text{-------} \times n$, continued till the last factor, or the number of factors, be n, the exponent of the Powers. Thus,

the 1st Powers 1, 2, 3, 4, 5, &c, have but one order of differences 1 1 1 1 &c, and that difference is 1.

The 2d Pwrs. 1, 4, 9, 16, 25, &c; have two orders of differences

$$\begin{array}{cccc} 3 & 5 & 7 & 9 \\ & 2 & 2 & 2 \end{array}$$

and the last of these is $2 = 1 \times 2$.

The 3d Pwrs. 1, 8, 27, 64, 125, &c, have three orders of differences

$$\begin{array}{cccc} 7 & 19 & 37 & 61 \\ & 12 & 18 & 24 \\ & & 6 & 6 \end{array}$$

and the last of these is $6 = 1 \times 2 \times 3$.

In like manner, the 4th or last differences of the 4th Powers, are each $24 = 1 \times 2 \times 3 \times 4$; and the 5th or last differences of the 5th Powers, are each $120 = 1 \times 2 \times 3 \times 4 \times 5$. And so on. Which property was first noticed by Peletarius.

And the same is true of the Powers of any other arithmetical progression $1, 1 + d, 1 + 2d, 1 + 3d$, &c,

viz, $1, \overline{1 + d}|^n, \overline{1 + 2d}|^n, \overline{1 + 3d}|^n$, &c,

the number of the orders of differences being still the same exponent n, and the last of those orders each equal to $1 \times 2 \times 3 \text{ - - - - - - } \times nd^n$, the same product of factors as before, multiplied by the same Power of the common difference d of the series of roots: as was shewn by Briggs.

And hence arises a very easy and general way of raising all the Powers of all the natural numbers, viz, by common addition only, beginning at the last differences, and adding them all continually, one after another, up to the Powers themselves. Thus, to generate the series of cubes, or 3d Powers, adding always 6, the common 3d difference gives the 2d differences 12, 18, 24, &c; and these added to the 1st of the 1st differences 7, gives the rest of the said 1st differences; and these again added to the 1st cube 1, gives the rest of the series of cubes, 8, 27, 64, &c, as below.

3dD.	2dD.	1stD.	Cubes.
			1
	12	7	
6			8
		19	
6	18		27
		37	
	24		64
6		61	
	30		125
		91	
			216
			&c.

Commensurable in POWER, is said of quantities which, though not commensurable themselves, have their squares, or some other Power of them, commensurable. Euclid confines it to squares. Thus, the diagonal and side of a square are commensurable in Power, their squares being as 2 to 1, or commensurable; though they are not commensurable themselves, being as $\sqrt{2}$ to 1.

POWER *of an Hyperbola*, is the square of the 4th part of the conjugate axis.

PRACTICAL Arithmetic, Geometry, Mathematics, &c, is the part that regards the practice, or application, as contradistinguished from the theoretical part.

PRACTICE, in Arithmetic, is a rule which expeditiously and compendiously answers questions in the golden rule, or rule-of-three, especially when the first term is 1. See rules for this purpose in all the books of practical arithmetic.

PRECESSION *of the Equinoxes*, is a very slow motion of them, by which they change their place, going from east to west, or backward, *in antecedentia*, as astronomers call it, or contrary to the order of the signs.

From the late improvements in astronomy it appears, that the pole, the solstices, the equinoxes, and all the other points of the ecliptic, have a retrograde motion, and are constantly moving from east to west, or from Aries towards Pisces, &c; by means of which, the equinoctial points are carried farther and farther back, among the preceding signs or stars, at the rate of about $50''\frac{1}{4}$ each year; which retrograde motion is called the Precession, Recession, or Retrocession of the Equinoxes.

Hence, as the stars remain immoveable, and the equinoxes go backward, the stars will seem to move more and more eastward with respect to them; for which reason the longitudes of all the stars, being reckoned from the first point of Aries, or the vernal equinox, are continually increasing.

From this cause it is, that the constellations seem all to have changed the places assigned to them by the ancient astronomers. In the time of Hipparchus, and the oldest astronomers, the equinoctial points were fixed to the first stars of Aries and Libra: but the signs do not now answer to the same points; and the stars which were then in conjunction with the sun when he was in the equinox, are now a whole sign, or 30 degrees, to the eastward of it: so, the first star of Aries is now in the portion of the ecliptic, called Taurus; and the stars of Taurus are now in Gemini; and those of Gemini in Cancer; and so on.

This seeming change of place in the stars was first observed by Hipparchus of Rhodes, who, 128 years before Christ, found that the longitudes of the stars in his time were greater than they had been before observed by Tymochares, and than they were in the sphere of Eudoxus, who wrote 380 years before Christ. Ptolomy also perceived the gradual change in the longitudes of the stars; but he stated the quantity at too little, making it but 1° in 100 years, which is at the rate of only $36''$ per year. Y-hang, a Chinese, in the year 721, stated the quantity of this change at 1° in 83 years, which is at the rate of $43''\frac{1}{2}$ per year. Other more modern astronomers have made this precession still more, but with some small differences from each other; and it is now usually taken at $50''\frac{1}{4}$ per year. All these rates are deduced from a comparison of the longitude of certain stars as observed by more ancient astronomers, with the later observations of the same stars; viz, by subtracting the former from the latter, and dividing the remainder by the number of years in the interval between the dates of the observations. Thus, by a medium of a great number of comparisons, the quantity of the annual change has been fixed at $50''\frac{1}{4}$, according to which rate it will require 25791 years for the equinoxes to make their revolution westward quite around the circle, and return to the same point again.

Thus

Thus, by taking the longitudes of the principal ſtars eſtabliſhed by Tycho Brahe, in his book Aſtronomiæ Inſtauratæ Progymnaſmata, pa. 208 and 232, for the beginning of 1586, and comparing them with the ſame as determined for the year 1750, by M. de la Caille, for that interval of 164 years, there will be obtained the following differences of longitude of ſeveral ſtars; viz,

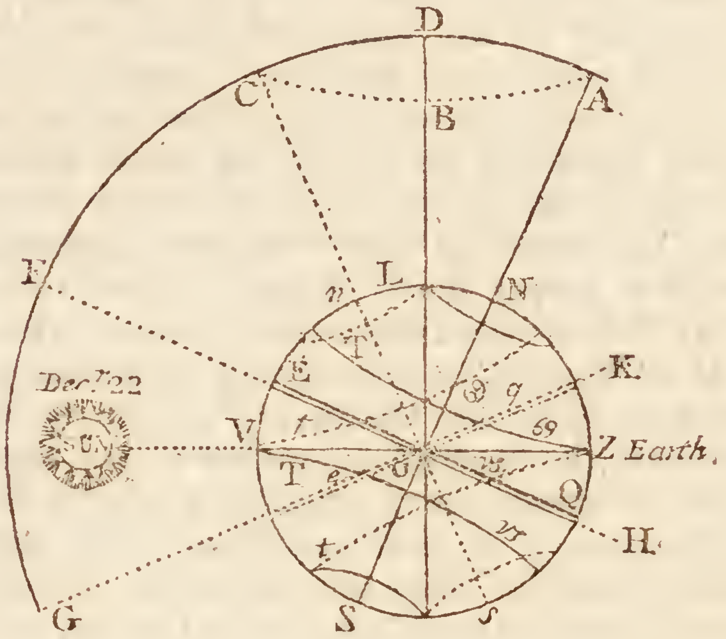
γ Arietis	2°	17′	37″
Aldebaran	2	17	45
μ Geminorum	2	17	1
β Geminorum	2	15	26
Regulus	2	16	32
α Virginis	2	18	18
α Aquilæ	2	19	1
α Pegaſi	2	16	12
β Libræ	2	17	52
Antares	2	16	28
ε Tauri	2	17	58
γ Geminorum	2	18	38
γ Cancri	2	19	12
γ Leonis	2	19	38
γ Capricorni	2	16	10
Medium of theſe 15 ſtars	2	17	35

which divided by 164, the interval of years, gives 50″·336, or nearly $50''\frac{1}{3}$, or after the rate of 1° 23′ $53''\frac{1}{3}$ in 100 years, or 25,748 years for the whole revolution, or circle of 360 degrees. And nearly the ſame concluſion reſults from the longitudes of the ſtars in the Britannic catalogue, compared with thoſe of the ſtill later catalogues. See De la Lande's Aſtronomie, in ſeveral places.

The Ancients, and even ſome of the Moderns, have taken the equinoxes to be immoveable; and aſcribed that change in the diſtance of the ſtars from it, to a real motion of the orb of the fixed ſtars, which they ſuppoſed had a ſlow revolution about the poles of the ecliptic; ſo as that all the ſtars perform their circuits in the ecliptic, or its parallels, in the ſpace of 25,791 years; after which they ſhould all return again to their former places.

This period the Ancients called the Platonic, or great year; and imagined that at its completion every thing would begin as at firſt, and all things come round in the ſame order as they have done before.

The phenomena of this retrograde motion of the equinoxes, or interſections of the equinoctial with the ecliptic, and conſequently of the conical motion of the earth's axis, by which the pole of the equator deſcribes a ſmall circle in the ſame period of time, may be underſtood and illuſtrated by a ſcheme, as follows: Let NZSVL be the earth, SONA its axis produced to the ſtarry heavens, and terminating in A, the preſent north pole of the heavens, which is vertical to N, the north pole of the earth. Let EOQ be the equator, T♋Z the tropic of cancer, and VT♑ the tropic of capricorn; VOZ the ecliptic, and BO its axis, both of which are immoveable among the ſtars. But as the equinoctial points recede in the ecliptic, the earth's axis SON is in motion upon the earth's centre O, in ſuch a manner as to deſcribe the double cone NO*n*



and SO*s*, round the axis of the ecliptic BO, in the time that the equinoctial points move round the ecliptic, which is 25,791 years; and in that length of time, the north pole of the earth's axis, produced, deſcribes the circle ABCDA in the ſtarry heavens, round the pole of the ecliptic, which keeps immoveable in the centre of that circle. The earth's axis being now 23° 28′ inclined to the axis of the ecliptic, the circle ABCDA, deſcribed by the north pole of the earth's axis produced to A, is 46° 56′ in diameter, or double the inclination of the earth's axis. In conſequence of this, the point A, which is at preſent the north pole of the heavens, and near to a ſtar of the 2d magnitude in the end of the Little Bear's tail, muſt be deſerted by the earth's axis; which moving backwards 1 degree every $71\frac{2}{3}$ years nearly, will be directed towards the ſtar or point B in $6447\frac{3}{4}$ years hence; and in double of that time, or $12,895\frac{1}{2}$ years, it will be directed towards the ſtar or point C; which will then be the north pole of the heavens, although it is at preſent $8\frac{1}{2}$ degrees ſouth of the zenith of London L. The preſent poſition of the equator EOQ will then be changed into *eOq*, the tropic of cancer T♋Z into V*t*♋, and the tropic of capricorn VT♑ into *t*V♑Z; as is evident by the figure. And the ſun, in the ſame part of the heavens where he is now over the earthly tropic of capricorn, and makes the ſhorteſt days and longeſt nights in the northern hemiſphere, will then be over the earthly tropic of cancer, and make the days longeſt and nights ſhorteſt. So that it will require $12,895\frac{1}{2}$ years yet more, or from that time, to bring the north pole N quite round, ſo as to be directed toward that point of the heavens which is vertical to it at preſent. And then, and not till then, the ſame ſtars which at preſent deſcribe the equator, tropics, and polar circles, &c, by the earth's diurnal motion, will deſcribe them over again.

From this ſhifting of the equinoctial points, and with them all the ſigns of the ecliptic, it follows, that thoſe ſtars which in the infancy of aſtronomy were in Aries, are now found in Taurus; thoſe of Taurus in Gemini, &c. Hence likewiſe it is, that the ſtars which roſe or ſet at any particular ſeaſon of the year, in the times of Heſiod, Eudoxus, Virgil, Pliny, &c, by

by no means anſwer at this time to their deſcriptions.

As to the phyſical cauſe of the Preceſſion of the equinoxes, Sir Iſaac Newton demonſtrates, that it ariſes from the broad or flat ſpheroidal figure of the earth; which itſelf ariſes from the earth's rotation about its axis: for as more matter has thus been accumulated all round the equatorial parts than any where elſe on the earth, the ſun and moon, when on either ſide of the equator, by attracting this redundant manner, bring the equator ſooner under them, in every return towards it, than if there was no ſuch accumulation.

Sir Iſaac Newton, in determining the quantity of the annual Preceſſion from the theory of gravity, on ſuppoſition that the equatorial diameter of the earth is to the polar diameter, as 230 to 229, finds the ſun's action ſufficient to produce a Preceſſion of $9''\frac{1}{8}$ only; and collecting from the tides the proportion between the ſun's force and the moon's to be as 1 to $4\frac{1}{2}$, he ſettles the mean Preceſſion reſulting from their joint actions, at $50''$; which, it muſt be owned, is nearly the ſame as it has ſince been found by the beſt obſervations; and yet ſeveral other mathematicians have ſince objected to the truth of Sir Iſaac Newton's computation.

Indeed, to determine the quantity of the Preceſſion ariſing from the action of the ſun, is a problem that has been much agitated among modern mathematicians; and although they ſeem to agree as to Newton's miſtake in the ſolution of it, they have yet generally diſagreed from one another. M. D'Alembert, in 1749, printed a treatiſe on this ſubject, and claims the honour of having been the firſt who rightly determined the method of reſolving problems of this kind. The ſubject has been alſo conſidered by Euler, Friſius, Silvabelle, Walmeſley, Simpſon, Emerſon, La Place, La Grange, Landen, Milner, and Vince.

M. Silvabelle, ſtating the ratio of the earth's axis to be that of 178 to 177, makes

the annual Preceſſion cauſed by the ſun	$13''\ 52'''$,
and that of the moon - -	$34\ \ 17$;

making the ratio of the lunar force to the ſolar, to be that of 5 to 2; alſo the nutation of the earth's axis cauſed by the moon, during the time of a ſemirevolution of the pole of the moon's orbit, i. e. in $9\frac{1}{3}$ years, he makes $17''\ 51'''$.—M. Walmeſley, on the ſuppoſition that the ratio of the earth's diameters is that of 230 to 229, and the obliquity of the ecliptic to the equator $23^\circ\ 28'\ 30''$, makes the annual Preceſſion, owing to the ſun's force, equal to $10''\cdot583$; but ſuppoſing the ratio of the diameters to be that of 178 to 177, that Preceſſion will be $13''\cdot675$.—Mr. Simpſon, by a different method of calculation, determines the whole annual preceſſion of the equinoxes cauſed by the ſun, at $21''\ 6'''$; and he has pointed out the errors of the computations propoſed by M. Silvabelle and M. Walmeſley.—Mr. Milner's deduction agrees with that of Mr. Simpſon, as well as Mr. Vince's; and their papers contain beſides ſeveral curious particulars relative to this ſubject. But for the various principles and reaſonings of theſe mathematicians, ſee Philoſ. Tranſ. vol. 48, pa. 385; vol. 49, pa. 704; vol. 69, pa. 505; and vol. 77. pa. 363; as alſo the writings of Simpſon, Emerſon, Landen, &c; alſo De la Lande's Aſtronomie, and the Memoirs of the Acad. Sci. in ſeveral places.

As to the effect of the planets upon the equinoctial points, M. De la Place, in his new reſearches on this article, finds that their action cauſes thoſe points to advance by $0''\cdot2016$ in a year, along the equator, or $0''\cdot1849$ along the ecliptic; from whence it follows that the quantity of the luni-ſolar Preceſſion muſt be $50''\cdot4349$, ſince the total obſerved Preceſſion is $50''\frac{1}{4}$, or $50''\cdot25$.

To find the Preceſſion in right aſcenſion and declination.

Put $d =$ the declination of a ſtar,
and $a =$ its right aſcenſion;

then their annual variations of Preceſſions will be nearly as follow:

viz, $20''\cdot084 \times \text{coſ.}\ a =$ the annual preceſ. in declinat.
and $46''\cdot0619 + 20''\cdot084 \times \text{ſin.}\ a \times \text{tang.}\ d =$ that of right aſcenſion. See the Connoiſſance des Temps for 1792, pa. 206, &c.

PRESS, in Mechanics, is a machine made of iron or wood, ſerving to compreſs or ſqueeze any body very cloſe, by means of ſcrews.

The common Preſſes conſiſt of ſix members, or pieces; viz, two flat and ſmooth planks; between which the things to be preſſed are laid; two ſcrews, or worms, faſtened to the lower plank, and paſſing through two holes in the upper; and two nuts, ſerving to drive the upper plank, which is moveable, againſt the lower, which is ſtable, and without motion.

PRESSION. See PRESSURE.

PRESSURE, is properly the action of a body which makes a continual effort or endeavour to move another; ſuch as the action of a heavy body ſupported by a horizontal table; in contradiſtinction from percuſſion, or a momentary force or action. Preſſure equally reſpects both bodies, that which preſſes, and that which is preſſed; from the mutual equality of action and reaction.

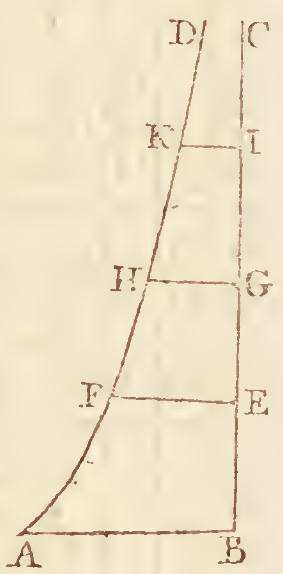
Preſſure, in the Carteſian Philoſophy, is an impulſive kind of motion, or rather an endeavour to move, impreſſed on a fluid medium, and propagated through it. In ſuch a preſſure the Carteſians ſuppoſe the action of light to conſiſt. And in the various modifications of this Preſſure, by the ſurfaces of bodies, on which that medium preſſes, they ſuppoſe the various colours to conſiſt, &c. But Newton ſhews, that if light conſiſted only in a Preſſure, propagated without actual motion, it could not agitate and warm ſuch bodies as reflect and refract it, as we actually find it does; and if it conſiſted in an inſtantaneous motion, or one propagated to all diſtances in an inſtant, as ſuch Preſſure ſuppoſes, there would be required an infinite force to produce that motion every moment, in every lucid particle. Farther, if light conſiſted either in Preſſure, or in motion propagated in a fluid medium, whether inſtantaneouſly, or in time, it muſt follow, that it would inflect itſelf *ad umbram*; for Preſſure, or motion, in a fluid medium, cannot be propagated in right lines, beyond any obſtacle which ſhall hinder any part of the motion; but will inflect and diffuſe itſelf, every way, into thoſe parts of the quieſcent medium which lie beyond the ſaid obſtacle. Thus

Thus the force of gravity tends downward; but the Preſſure which ariſes from that force of gravity, tends every way with an equable force; and, with equal eaſe and force, is propagated in crooked lines, as in ſtraight ones. Waves on the ſurface of water, while they ſlide by the ſides of any large obſtacle, do inflect, dilate, and diffuſe themſelves gradually into the quieſcent water lying beyond the obſtacle. The waves, pulſes, or vibrations of the air, in which ſounds conſiſt, do manifeſtly inflect themſelves, though not ſo much as the waves of water; for the ſound of a bell, or of a cannon, can be heard over a hill, which intercepts the ſonorous object from our ſight; and ſounds are propagated as eaſily through crooked tubes, as through ſtraight ones. But light is never obſerved to go in curved lines, nor to inflect itſelf *ad umbram*; for the fixed ſtars do immediately diſappear on the interpoſition of any of the planets; as well as ſome parts of the ſun's body, by the interpoſition of the Moon, or Venus, or Mercury.

PRESSURE *of Air, Water, &c.* See AIR, WATER, &c.

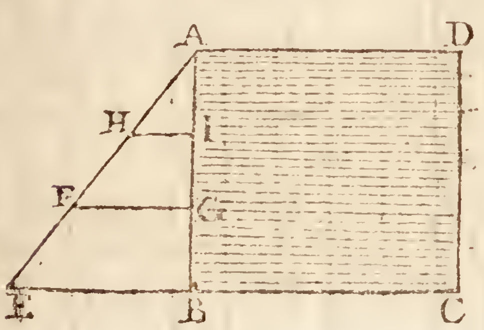
The effects anciently aſcribed to the fuga vacui, are now accounted for from the weight and Preſſure of the air.

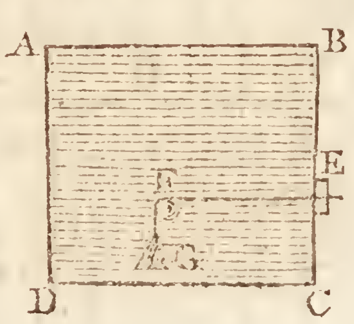
The Preſſure of the air on the ſurface of the earth, is balanced by a column of water of the ſame baſe, and about 34 feet high; or of one of Mercury of near 30 inches high; and upon every ſquare inch at the earth's ſurface, that Preſſure amounts to about $14\frac{3}{4}$ pounds avoirdupois. The elaſticity of the air is equal to that Preſſure, and by means of that Preſſure, or elaſticity, the air would ruſh into a vacuum with a velocity of about 1370 feet per ſecond. At different heights above the earth's ſurface, the Preſſure of the air is as its denſity and elaſticity, and each decreaſes in ſuch ſort, that when the heights above the ſurface increaſe in arithmetical progreſſion, the Preſſure &c decreaſe in geometrical progreſſion: and hence if the axis BC of a logarithmic curve AD be erected perpendicular to the horizon, and if the ordinate AB denote the Preſſure, or elaſticity, or denſity of the air, at the earth's ſurface, then will any other abſciſs



EF, GH, IK } denote the Preſſure &c at the altitude { BE, BG, BI,

The Preſſure of water, as this fluid is every where of the ſame denſity, is as its depth at any place, and in all directions the ſame; and upon a ſquare foot of ſurface, every foot in height preſſes with the force of a weight of 1000 ounces or $62\frac{1}{2}$lbs avoirdupois. And hence, if AB be the depth of water in any veſſel, and BE denote its Preſſure at the depth B; by joining AE and drawing any other ordinates FG, HI; then ſhall theſe ordinates FG, HI, &c, denote the Preſſure at the correſponding depths AG, AI, &c; alſo the area of the triangle ABE will denote the whole Preſſure againſt the whole upright ſide AB, and which therefore is but half the Preſſure on the bottom of the ſame area as the ſide. Moreover, if a hole were opened in the bottom or ſide of the veſſel at B, the water, from the Preſſure of the ſuperincumbent fluid, would iſſue out with the velocity of $8\sqrt{AB}$ feet per ſecond nearly; AB being eſtimated in feet.





Centre of PRESSURE, in Hydroſtatics, is that point of any plane, to which, if the total Preſſure were applied, its effect upon the plane would be the ſame as when it was diſtributed unequally over the whole; or it is that point in which the whole Preſſure may be conceived to be united; or it is that point to which, if a force were applied equal to the total Preſſure, but with an oppoſite direction, it would exactly balance, or reſtrain the effect of the Preſſure, ſo that the body preſſed on will not incline to either ſide. Thus, if ABCD (2d fig. above) be a veſſel of water, and the ſide BC be preſſed upon with a force equivalent to 20 pounds of water, this force is unequally diſtributed over BC, for the parts near B are leſs preſſed than thoſe near C, which are at a greater depth; and therefore the efforts of all the particular Preſſures are united in ſome point E, which is nearer to C than to B; and that point E is called the centre of Preſſure: if to that point a force equivalent to 20 pounds weight be applied, it will affect the plane BC in the ſame manner as by the Preſſure of the water diſtributed unequally over the whole; and if to the ſame point the ſame force be applied in a contrary direction to that of the Preſſure of the water, the force and the Preſſure will balance each other, and by oppoſite endeavours deſtroy each other's effects. Suppoſing a cord EFG fixed at E, and paſſing over the pulley F, has a weight of 20 pounds annexed to it, and that the part of the cord FE is perpendicular to BC; then the effort of the weight G is equal, and its direction contrary, to that of the Preſſure of the water. Now if E be the centre of Preſſure, theſe two powers will be in equilibrio, and mutually defeat each other's endeavours.

This point E, or the centre of Preſſure, is the ſame with the centre of percuſſion of the plane BC, the point of ſuſpenſion being B, the ſurface of the water. And if the plane be oblique, the caſe is ſtill the ſame, taking for the axis of ſuſpenſion, the interſection of that plane and the ſurface of the fluid, both produced if neceſſary. See Cotes's Lectures, pa. 40, &c.

The centre of Preſſure upon a plane parallel to the horizon, or upon any plane where the Preſſure is uniform, is the ſame as the centre of gravity of that plane. For the Preſſure acts upon every part in the ſame manner as gravity does.

PRIMARY *Planets*, are thoſe which revolve round the ſun as a centre. Such are the planets Mercury, Venus, Terra the Earth, Mars, Jupiter, Saturn, and Herſchel, and perhaps others. They are thus called, in contradiſtinction from the ſecondary planets, or ſatellites, which revolve about their reſpective Primaries. See PLANET.

PRIMES, denote the first divisions into which some whole or integer is divided. As, a minute, or Prime minute, the 60th part of a degree; or the first place of decimals, being the 10th parts of units; or the first division of inches in duodecimals, being the 12th parts of inches; &c.

PRIME *Numbers*, are those which can only be measured by unity, or exactly divided without a remainder, 1 being the only aliquot part: as 2, 3, 5, 7, 11, 13, 17, &c. And they are otherwise called Simple, or Incomposite numbers. No even number is a Prime, because every even number is divisible by 2. No number that ends with 0 or 5 is a Prime, the former being divisible by 10, and the latter by 5. The following Table contains all the Prime numbers, and all the odd composite numbers, under 10,000, with the least Prime divisors of these; the description, nature, and use of which, see immediately following the Table.

A Table of Prime and Composite Odd Numbers, under 10,000.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
01			3	7		3			3	17	7	3			3	19	
03			7	3	13		3	19	11	3	17		3		23	3	7
07			3		11	3		7	3		19	3	17		3	11	
09	3		11	3			3			3			3	7		3	
11		3			3	7	13	3			3	11	7	3	17		3
13			3		7	3		23	3	11		3		13	3	17	
17		3	7		3	11		3	19	7	3			3	13	37	3
19		7	3	11		3			3			3	23		3	7	
21	3	11	13	3			3	7		3		19	3		7	3	
23		3		17	3		7	3		13	3			3			3
27	3			3	7	17	3			3	13	7	3			3	
29		3		7	3	23	17	3			3			3		11	3
31			3			3		17	3	7		3		11	3		7
33	3	7		3		13	3		7	3		11	3	31		3	23
37			3		19	3	7	11	3		17	3		7	3	29	
39	3			3		7	3			3		17	3	13		3	11
41		3		11	3			3	29		3	7	17	3	11	23	3
43		11	3	7		3			3	23	7	3	11	17	3		31
47		3	13		3			3	7		3	31	29	3		7	3
49	7		3			3	11	7	3	13		3		19	3		17
51	3			3	11	19	3		23	3			3	7		3	13
53		3	11		3	7		3			3		7	3			3
57	3			3			3			3	7	13	3	23	31	3	
59		3	7		3	13		3		7	3	19		3			3
61		7	3	19		3			3	31		3	13		3	7	11
63	3			3			3	7		3			3	29	7	3	
67			3			3	23	13	3		11	3	7		3		
69	3	13		3	7		3		11	3		7	3	37	13	3	
71		3		7	3		11	3	13		3		31	3			3
73			3		11	3			3	7	29	3	19		3	11	7
77	7	3		13	3			3			3	11		3	7	19	3
79			3			3	7	19	3	11	13	3		7	3		23
81	3			3	13	7	3	11		3	23		3			3	41
83		3			3	11		3			3	7		3			3
87	3	11	7	3			3			3			3	19		3	7
89		3	17		3	19	13	3	7	23	3	29		3		7	3
91	7		3	17		3		7	3			3		13	3	37	19
93	3			3	17		3	13	19	3			3	7		3	
97			3		7	3	17		3			3		11	3		
99	3		13	3			3	17	29	3	7	11	3			3	

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
01	3			3	11	31	3	7	41	3	37		3		7	3	
03	13	3	11		3		7	3		19	3			3	29		3
07	3	13		3	7		3	29	23	3		7	3	31	13	3	
09		3	23	7	3	47		3	13		3	53		3			3
11	29		3			3			3	7		3	41		3	13	7
13	3	7		3			3	19	7	3		29	3	23	11	3	
17	17	23	3		29	3	7		3		11	3		7	3		31
19	3	17	19	3	13	7	3	41	11	3			3			3	
21		3	17	43	3		11	3			3	7	23	3			3
23			3	7	11	3	23		3	43	7	3	37		3	11	
27	11	3	41		3	17	13	3	7	37	3	11		3	53	7	3
29	7	31	3			3	17	7	3	11		3	29	13	3		
31	3			3		23	3	11		3		19	3	7	31	3	
33		3		19	3	7		3	17		3		7	3	13	53	3
37	3	11	13	3			3		43	3	7		3			3	47
39	37	3	7		3			3		7	3	17		3	43	41	3
41		7	3	13		3			3	19		3	17		3	7	13
43	3	19	29	3			3	7		3	13		3	17	7	3	
47			3	23	19	3			3		41	3	7	11	3	17	
49	3	43		3	7	13	3	31		3		7	3		47	3	17
51	17	3		7	3			3		11	3		13	3	23		3
53		17	3			3	13	11	3	7		3		43	3		7
57	7	3	19	11	3	37		3			3			3	7		3
59		11	3	29	17	3	7		3		31	3	11	7	3		
61	3		37	3		7	3	23	13	3	11		3		29	3	
63	41	3	13		3	31	17	3	11		3	7		3		13	3
67	3		7	3	11		3		17	3		47	3			3	7
69	29	3	11		3		23	3	7	17	3	19		3		7	3
71	7		3	19	13	3		7	3		17	3		37	3		
73	3			3	41		3		31	3	47	13		7	19	3	
77			3	31	7	3			3			3	13	17	3	29	11
79	3			3		43	3	37		3	7		3		11	3	31
81	13	3	7		3			3	29	7	3	43	11	3		17	3
83		7	3		37	3		13	3		11	3	19		3	7	17
87		3			3		7	3	13		3		29	3		19	3
89			3		11	3		19	3			3	7		3	11	
91	3	31	11	3	7	29	3	47		3		7	3	11		3	
93	11	3		7	3			3			3	11	41	3	31	37	3
97	3	7		3	13		3	11	7	3			3	19	23	3	43
99	7	3			3	11		3	23		3	13		3	7		3

A Table of Prime and Composite Odd Numbers, under 10,000.

	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
01	19	3	13		3	47		3		11	3	7	43	3		13	3
03	41	31	3	7		3		11	3	13	7	3			3		
07		3		11	3			3	7	59	3		17	3	11	7	3
09	7	11	3		13	3	19	7	3	31		3	11	17	3		
11	3		23	3	37		3			3	11	13	3	7	17	3	
13		3		47	3	7		3	11	19	3		7	3		17	3
17	3			3	11		3	23		3	7		3	53		3	29
19	13	3	7		3			3		7	3		31	3	61		3
21	11	7	3	61		3		13	3	29		3			3	7	
23	3	13		3			3	7	41	3			3		7	3	
27	23		3		43	3			3		19	3	7	29	3	13	11
29	3		19	3	7		3			3	43	7	3		11	3	47
31	47	3		7	3		29	3		61	3	23	11	3			3
33			3			3	37		3	7	11	3	41		3		7
37	7	3		37	3	31	11	3	19		3	13		3	7		3
39	19		3		11	3	7		3		23	3		7	3	11	
41	3		11	3	23	7	3	41		3		19	3	11	47	3	71
43	11	3		19	3		13	3		43	3	7		3	29		3
47	3		7	3			3	11	31	3			3	47	37	3	7
49		3	41	23	3	11		3	7		3			3	13	7	3
51	7	53	3	11		3		7	3	19		3			3		
53	3	11	13	3		59	3			3	61	29	3	7	23	3	31
57			3	13	7	3			3			3		67	3		13
59	3			3	17	37	3			3	7	47	3		43	3	
61		3	7		3	17	31	3		7	3		59	3		11	3
63		7	3	53		3	17	23	3			3		11	3	7	61
67		3	19		3		7	3	17	11	3		13	3	31		3
69		43	3		53	3	13	11	3	17	41	3	7	19	3		37
71	3			3	7	11	3	43		3	17	7	3	13		3	11
73	23	3		7	3	29		3			3	17		3	11		3
77	3	7		3		41	3		7	3	11	23	3	17		3	
79	7	3	13		3	23		3	11	29	3	19		3	7	13	3
81	59		3	19		3	7	37	3	13		3	31	7	3	17	
83	3		29	3	11	7	3	47		3			3		19	3	13
87	11	17	3	7	13	3	61	53	3	41	7	3	43		3		
89	3	37	7	3			3	59		3	67	13	3			3	7
91		3		17	3	13		3	7		3			3	67	7	3
93	7		3		17	3		7	3	23		3	13		3		11
97	13	3			3	7	17	3			3		7	3	59	19	3
99		59	3	29	7	3		13	3	53	11	3	37		3		

	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67
01		7	3	11		3			3	17		3		37	3	7	
03	3	11		3		13	3	7		3	17		3	19	7	3	
07		41	3			3	13		3		31	3	7	43	3		19
09	3			3	7	71	3	37	19	3	41	7	3	13	23	3	
11	19	3	47	7	3	31		3	23		3			3	17	11	3
13		13	3		37	3	29		3	7		3	59	11	3	17	7
17	7	3	13		3	41		3	61	11	3			3	7	13	3
19		17	3			3	7	11	3	13	29	3	71	7	3		
21	3	23	17	3		7	3		31	3			3			3	11
23	47	3		11	3		59	3		19	3	7		3	11	37	3
27	3		7	3		17	3			3	11	13	3		61	3	7
29	23	3	73	61	3	13	17	3	7		3			3		7	3
31	7		3			3	11	7	3	37		3	13	59	3	19	53
33	3			3	11	43	3	19	17	3		23	3	7	47	3	
37	11		3		7	3		13	3		17	3		41	3		
39	3	13	19	3	29		3			3	7	17	3	47	13	3	23
41	53	3	7		3			3	13	7	3	79	17	3	31	29	3
43	37	7	3		23	3			3			3		17	3	7	11
47		3		13	3		7	3	19		3		11	3		17	3
49	19	29	3		31	3			3	23	11	3	7		3	61	17
51	3	59		3	7		3		11	3		7	3			3	43
53		3	53	7	3		11	3			3	13		3			3
57	3	7	11	3			3		7	3	47		3	11	79	3	29
59	7	3	23	53	3		13	3	59	73	3	11		3	7		3
61	13		3	43	67	3	7		3	11	61	3		7	3		
63	3	19	31	3		7	3	11	67	3	7		3	23		3	
67		23	3	7	19	3	73		3			3		29	3	59	67
69	3	11	7	3			3		47	3	31		3			3	7
71		3	41		3	53	29	3	7	13	3		23	3		7	3
73	7		3	13		3	23	7	3			3			3		13
77	31	3	19		3	7	53	3	43	59	3		7	3		11	3
79			3			3			3		37	3		11	3		
81	3			3		13	3			3	7	11	3			3	
83	71	3	7		3			3	31	7	3	61	13	3	29	41	3
87	3	17		3	37	11	3	7		3	23		3	13	7	3	11
89		3	17	11	3		7	3	53		3	19		3	11		3
91	29	11	3	17		3		43	3		41	3	7		3		
93	3	67		3	7		3	71	13	3	11	7	3	43	19	3	
97			3	23	29	3	11		3	7		3		73	3	37	7
99	3	7		3	11	41	3	17		3			3	67		3	13

ment; his text-book was Puffendorff *de Officio Hominis et Civis*; agreeably to the method he purfued through life, of making fact and experiment the bafis of fcience.

Dr. Pringle continued in the practice of Phyfic at Edinburgh, and in duly performing the office of profeffor, till 1742, when he was appointed phyfician to the earl of Stair, who then commanded the Britifh army. By the intereft of this nobleman, Dr. Pringle was conftituted, the fame year, phyfician to the military hofpital in Flanders, with a falary of 20 fhillings a-day, and the right to half-pay for life. On this occafion he was permitted to retain his profefforfhip of moral philofophy; two gentlemen, Meffrs. Muirhead and Cleghorn teaching in his abfence, as long as he requefted it. The great attention which Dr. Pringle paid to his duty as an army phyfician, is evident from every page of his *Treatife on the Difeafes of the Army*, in the execution of which office he was fometimes expofed to very imminent dangers. He foon after alfo met with no fmall affliction in the retirement of his great friend the earl of Stair, from the army. He offered to refign with his noble patron, but was not permitted: he was therefore obliged to content himfelf with teftifying his refpect and gratitude to him, by accompanying the earl 40 miles on his return to England; after which he took leave of him with the utmoft regret.

But though Dr. Pringle was thus deprived of the immediate protection of a nobleman who knew and efteemed his worth, his conduct in the duties of his ftation procured him effectual fupport. He attended the army in Flanders through the campaign of 1744, and fo powerfully recommended himfelf to the duke of Cumberland, that in the fpring following he had a commiffion, appointing him phyfician-general to the king's forces in the Low Countries, and parts beyond the feas; and on the next day he received a fecond commiffion from the duke, conftituting him phyfician to the royal hofpitals in thofe countries. In confequence of thefe promotions, he the fame year refigned his profefforfhip in the univerfity of Edinburgh.

In 1745 he was alfo with the army in Flanders; but was recalled from that country in the latter end of the year, to attend the forces which were to be fent againft the rebels in Scotland. At this time he had the honour of being chofen F. R. S. and the Society had good reafon to be pleafed with the addition of fuch a member. In the beginning of 1746, Dr. Pringle accompanied, in his official capacity, the duke of Cumberland in his expedition againft the rebels; and remained with the forces, after the battle of Culloden, till their return to England the following fummer. In 1747 and 1748, he again attended the army abroad; but in the autumn of 1748, he embarked with the forces for England, on the figning of the treaty of Aix-la-Chapelle.

From that time he moftly refided in London, where, from his known fkill and experience, and the reputation he had acquired, he might reafonably expect to fucceed as a phyfician. In 1749 he was appointed phyfician in ordinary to the duke of Cumberland. And in 1750 he publifhed, in a letter to Dr. Mead, *Obfervations on the Gaol or Hofpital Fever*: this piece, with fome alterations, was afterwards included in his grand work on the *Difeafes of the Army*.

In this, and the two following years Dr. Pringle communicated to the Royal Society his celebrated *Experiments upon Septic and Antifeptic Subftances, with Remarks relating to their Ufe in the Theory of Medicine*; fome of which were printed in the Philofophical Tranfactions, and the whole were fubjoined, as an appendix, to his *Obfervations on the Difeafes of the Army*. Thofe experiments procured for the ingenious author the honour of Sir Godfrey Copley's gold medal; befides gaining him a high and juft reputation as an experimental philofopher. He gave alfo many other curious papers to the Royal Society: thus, in 1753, he prefented, *An Account of feveral Perfons feized with the Gaol Fever by working in Newgate; and of the Manner by which the Infection was communicated to one entire Family*; in the Philof. Tranf. vol. 48. His next communication was, *A remarkable cafe of Fragility, Flexibility, and Diffolution of the Bones*; in the fame vol.—In the 49th volume, are accounts which he gave of an Earthquake felt at Bruffels; of another at Glafgow and Dunbarton; and of the Agitation of the Waters, Nov. 1, 1756, in Scotland and at Hamburgh.—The 50th volume contains his Obfervations on the Cafe of lord Walpole, of Wolleiton; and a Relation of the Virtues of Soap, in Diffolving the Stone.—The next volume is enriched with two of the doctor's articles, of confiderable length, as well as value. In the firft, he hath collected, digefted, and related, the different accounts that had been given of a very extraordinary Fiery Meteor, which appeared the 26th of November 1758; and in the fecond he hath made a variety of remarks upon the whole, difplaying a great degree of philofophical fagacity.—Befides his communications in the Philofophical Tranfactions, he gave, in the 5th volume of the Edinburgh Medical Effays, an Account of the Succefs of the *Vitrum ceratum Antimonii*.

In 1752, Dr. Pringle married Charlotte, the fecond daughter of Dr. Oliver, an eminent phyfician at Bath: a connection which however did not laft long, the lady dying in the fpace of a few years. And nearly about the time of his marriage, he gave to the public the firft edition of his *Obfervations on the Difeafes of the Army*; which afterwards went through many editions with improvements, was tranflated into the French, the German, and the Italian languages, and defervedly gained the author the higheft credit and encomiums. The utility of this work however was of ftill greater importance than its reputation. From the time that the doctor was appointed a phyfician to the army, it feems to have been his grand object to leffen, as far as lay in his power, the calamities of war; nor was he without confiderable fuccefs in his noble and benevolent defign. The benefits which may be derived from our author's great work, are not folely confined to gentlemen of the medical profeffion. General Melville, a gentleman who unites with his military abilities the fpirit of philofophy, and the feelings of humanity, was enabled, when governor of the Neutral Iflands, to be fingularly ufeful, in confequence of the inftructions he had received from Dr. Pringle's book, and from perfonal converfation with him. By taking care to have his men always lodged in large, open, and airy apartments, and by never letting his forces remain long enough in fwampy places to be injured by the noxious air of fuch places, the general was the

happy

happy inftrument of faving the lives of feven hundred foldiers.

Though Dr. Pringle had not for fome years been called abroad, he ftill held his place of phyfician to the army; and in the war that began in 1755, he attended the camps in England during three feafons. In 1758, however, he entirely quitted the fervice of the army; and being now determined to fix wholly in London, he was the fame year admitted a licentiate of the college of phyficians.—After the acceffion of king George the 3d to the throne of Great Britain, Dr. Pringle was appointed, in 1761, phyfician to the queen's houfehold; and this honour was fucceeded, by his being conftituted, in 1763, phyfician extraordinary to the queen. The fame year he was chofen a member of the Academy of Sciences at Haarlem, and elected a fellow of the Royal College of Phyficians in London.—In 1764, on the deceafe of Dr. Wollafton, he was made phyfician in ordinary to the queen. In 1766 he was elected a foreign member, in the phyfical line, of the Royal Society of Sciences at Gottingen, and the fame year he was raifed to the dignity of a baronet of Great Britain. In 1768 he was appointed phyfician in ordinary to the late princefs dowager of Wales.

After having had the honour to be feveral times elected into the council of the Royal Society, Sir John Pringle was at length, viz, Nov. 30, 1772, in confequence of the death of James Weft Efq. elected prefident of that learned body. His election to this high ftation, though he had fo refpectable a character as the late Sir James Porter for his opponent, was carried by a very confiderable majority. Sir John Pringle's conduct in this honourable ftation fully juftified the choice the Society made of him as their prefident. By his equal, impartial, and encouraging behaviour, he fecured the good will and beft exertions of all for the general benefit of fcience, and true interefts of the Society, which in his time was raifed to the pinnacle of honour and credit. Inftead of fplitting the members into oppofite parties, by cruel, unjuft, and tyrannical conduct, as has fometimes been the cafe, to the ruin of the beft interefts of the Society, Sir John Pringle cherifhed and happily united the endeavours of all, collecting and directing the energy of every one to the common good of the whole. He happily alfo ftruck out a new way to diftinction and ufefulnefs, by the difcourfes which were delivered by him, on the annual affignment of Sir Godfrey Copley's medal. This gentleman had originally bequeathed five guineas, to be given at each anniverfary meeting of the Royal Society, by the determination of the prefident and council, to the perfon who fhould be the author of the beft paper of experimental obfervations for the year. In procefs of time, this pecuniary reward, which could never be an important confideration to a man of an enlarged and philofophical mind, however narrow his circumftances might be, was changed into the more liberal form of a gold medal; in which form it is become a truly honourable mark of diftinction, and a juft and laudable object of ambition. No doubt it was always ufual for the prefident, on the delivery of the medal, to pay fome compliment to the gentleman on whom it was beftowed; but the cuftom of making a fet fpeech on the occafion, and of entering into the hiftory of that part of philofophy to which the experiments, or the fubject of the paper related, was firft introduced by Martin Folkes Efq. The difcourfes however which he and his fucceffors delivered, were very fhort, and were only inferted in the minute-books of the Society. None of them had ever been printed before Sir John Pringle was raifed to the chair. The firft fpeech that was made by him being much more elaborate and extended than ufual, the publication of it was defired; and with this requeft, it is faid, he was the more ready to comply, as an abfurd account of what he had delivered had appeared in a newspaper. Sir John was very happy in the fubject of his firft difcourfe. The difcoveries in magnetifm and electricity had been fucceeded by the inquiries into the various fpecies of air. In thefe enquiries, Dr. Prieftley, who had already greatly diftinguifhed himfelf by his electrical experiments, and his other philofophical purfuits and labours, took the principal lead. A paper of his, intitled, *Obfervations on different Kinds of Air*, having been read before the Society in March 1772, was adjudged to be deferving of the gold medal; and Sir John Pringle embraced with pleafure the occafion of celebrating the important communications of his friend, and of relating with accuracy and fidelity what had previoufly been difcovered upon the fubject.

It was not intended, we believe, when Sir John's firft fpeech was printed, that the example fhould be followed; but the fecond difcourfe was fo well received by the Society, that the publication of it was unanimoufly requefted. Both the difcourfe itfelf, and the fubject on which it was delivered, merited fuch a diftinction. The compofition of the fecond fpeech is evidently fuperior to that of the former one; Sir John having probably been animated by the favourable reception of his firft effort. His account of the Torpedo, and of Mr. Walfh's ingenious and admirable experiments relative to the electrical properties of that extraordinary fifh, is fingularly curious. The whole difcourfe abounds with ancient and modern learning, and exhibits the worthy prefident's knowledge in natural hiftory, as well as in medicine, to great advantage.

The third time that he was called upon to difplay his abilities at the delivery of the annual medal, was on a very beautiful and important occafion. This was no lefs than Mr. (now Dr.) Mafkelyne's fuccefsful attempt completely to eftablifh Newton's fyftem of the univerfe, by his *Obfervations made on the Mountain Schehallien, for finding its attraction.* Sir John laid hold of this opportunity to give a perfpicuous and accurate relation of the feveral hypothefes of the Ancients, with regard to the revolutions of the heavenly bodies, and of the noble difcoveries with which Copernicus enriched the aftronomical world. He then traces the progrefs of the grand principle of gravitation down to Sir Ifaac's illuftrious confirmation of it; to which he adds a concife account of Meffrs. Bouguer's and Condamine's experiment at Chimboraço, and of Mr. Mafkelyne's at Schehallien. If any doubts ftill remained with refpect to the truth of the Newtonian fyftem, they were now completely removed.

Sir John Pringle had reafon to be peculiarly fatisfied with the fubject of his fourth difcourfe; that fubject being perfectly congenial to his difpofition and ftudies. His own life had been much employed in pointing out the means which tended not only to cure, but to prevent

vent the diseases of mankind; and it is probable, from his intimate friendship with captain Cook, that he might suggest to that sagacious commander some of the rules which he followed, in order to preserve the health of the crew of his ship, during his voyage round the world. Whether this was the case, or whether the method pursued by the captain to attain so salutary an end, was the result alone of his own reflections, the success of it was astonishing; and this celebrated voyager seemed well entitled to every honour which could be bestowed. To him the Society assigned their gold medal, but he was not present to receive the honour. He was gone out upon the voyage, from which he never returned. In this last voyage he continued equally successful in maintaining the health of his men.

The learned president, in his fifth annual dissertation, had an opportunity of displaying his knowledge in a way in which it had not hitherto appeared. The discourse took its rise from the adjudication of the prize medal to Mr. Mudge, then an eminent surgeon at Plymouth, on account of his valuable paper, containing *Directions for making the best Composition for the Metals of Reflecting Telescopes, together with a Description of the Process for Grinding, Polishing, and giving the Great Speculum the true Parabolic form.* Sir John hath accurately related a variety of particulars, concerning the invention of reflecting telescopes, the subsequent improvements of these instruments, and the state in which Mr. Mudge found them, when he first set about working them to a greater perfection, till he had truly realized the expectation of Newton, who, above an hundred years ago, presaged that the public would one day possess a parabolic speculum, not accomplished by mathematical rules, but by mechanical devices.

Sir John Pringle's sixth and last discourse, to which he was led by the assignment of the gold medal to myself, on account of my paper intitled, *The Force of fired Gunpowder, and the Initial Velocity of Cannon Balls, determined by Experiments,* was on the theory of gunnery. Though Sir John had so long attended the army, this was probably a subject to which he had heretofore paid very little attention. We cannot however help admiring with what perspicuity and judgment he hath stated the progress that was made, from time to time, in the knowledge of projectiles, and the scientific perfection to which it has been said to be carried in my paper. As Sir John Pringle was not one of those who delighted in war, and in the shedding of human blood, he was happy in being able to shew that even the study of artillery might be useful to mankind; and therefore this is a topic which he hath not forgotten to mention. Here ended our author's discourses upon the delivery of Sir Godfrey Copley's medal, and his presidency over the Royal Society at the same time, the delivering that medal into my hand being the last office he ever performed in that capacity; a ceremony which was attended by a greater number of the members, than had ever met together before upon any other occasion. Had he been permitted to preside longer in that chair, he would doubtless have found other occasions of displaying his acquaintance with the history of philosophy. But the opportunities which he had of signalizing himself in this respect were important in themselves, happily varied, and sufficient to gain him a solid and lasting reputation.

Several marks of literary distinction, as we have already seen, had been conferred upon Sir John Pringle, before he was raised to the president's chair. But after that event they were bestowed upon him in great abundance, having been elected a member of almost all the literary societies and institutions in Europe. He was also, in 1774, appointed physician extraordinary to the king.

It was at rather a late period of life when Sir John Pringle was chosen to be president of the Royal Society, being then 65 years of age. Considering therefore the great attention that was paid by him to the various and important duties of his office, and the great pains he took in the preparation of his discourses, it was natural to expect that the burthen of his honourable station should grow heavy upon him in a course of time. This burthen, though not increased by any great addition to his life, for he was only 6 years president, was somewhat augmented by the accident of a fall in the area in the back part of his house, from which he received some hurt. From these circumstances some persons have affected to account for his resigning the chair at the time when he did. But Sir John Pringle was naturally of a strong and robust frame and constitution, and had a fair prospect of being well able to discharge the duties of his situation for many years to come, had his spirits not been broken by the most cruel harassings and baitings in his office. His resolution to quit the chair arose from the disputes introduced into the Society, concerning the question, whether pointed or blunted electrical conductors are the most efficacious in preserving buildings from the pernicious effects of lightning, and from the cruel circumstances attending those disputes. These drove him from the chair. Such of those circumstances as were open and manifest to every one, were even of themselves perhaps quite sufficient to drive him to that resolution. But there were yet others of a more private nature, which operated still more powerfully and directly to produce that event; which may probably hereafter be laid before the public, when I shall give to them the history of the most material transactions of the Royal Society, especially those of the last 22 years, which I have from time to time composed and prepared with that view.

His intention of resigning however, was disagreeable to his friends, and the most distinguished members of the Society, who were many of them perhaps ignorant of the true motive for it. Accordingly, they earnestly solicited him to continue in the chair; but, his resolution being fixed, he resigned it at the anniversary meeting in 1778, immediately on delivering the medal, at the conclusion of his speech, as mentioned above.

Though Sir John Pringle thus quitted his particular relation to the Royal Society, and did not attend its meetings so constantly as he had formerly done, he still retained his literary connections in general. His house continued to be the resort of ingenious and philosophical men, whether of his own country, or from abroad; and he was frequent in his visits to his friends. He was held in particular esteem by eminent and learned foreigners, none of whom came to England without waiting upon him, and paying him the greatest respect. He treated them, in return, with distinguished civility and regard. When a number of gentlemen met at his

his table, foreigners were ufually a part of the company.

In 1780 Sir John fpent the fummer on a vifit to Edinburgh; as he did alfo that of 1781; where he was treated with the greateft refpect. In this laft vifit he prefented to the Royal College of Phyficians in that city, the refult of many years labour, being ten folio volumes of *Medical and Phyfical Obfervations*, in manufcript, on condition that they fhould neither be publifhed, nor lent out of the library of the college on any account whatever. He was at the fame time preparing two other volumes, to be given to the univerfity, containing the formulas referred to in his annotations. He returned again to London, and continued for fome time his ufual courfe of life, receiving and paying vifits to the moft eminent literary men, but languifhing and declining in his health and fpirits, till the 18th of January 1782, when he died, in the 75th year of his age; the account of his death being every where received in a manner which fhewed the high fenfe that was entertained of his merit.

Sir John Pringle's eminent character as a practical phyfician, as well as a medical author, is fo well known, and fo univerfally acknowledged, that an enlargement upon it cannot be neceffary. In the exercife of his profeffion he was not rapacious; being ready, on various occafions, to give his advice without pecuniary views. The turn of his mind led him chiefly to the love of fcience, which he built on the firm bafis of fact. With regard to philofophy in general, he was as averfe to theory, unfupported by experiments, as he was with refpect to medicine in particular. Lord Bacon was his favourite author; and to the method of inveftigation recommended by that great man, he fteadily adhered. Such being his intellectual character, it will not be thought furprifing that he had a diflike to Plato. And to metaphyfical difquifitions he loft all regard in the latter part of his life.

Sir John had no great fondnefs for poetry. He had not even any diftinguifhed relifh for the immortal Shakefpeare: at leaft he feemed too highly fenfible of the defects of that illuftrious bard, to give him the proper degree of eftimation. Sir John had not in his youth been neglectful of philological enquiries, nor did he defert them in the laft ftages of his life, but cultivated even to the laft a knowledge of the Greek language. He paid a great attention to the French language; and it is faid that he was fond of Voltaire's critical writings. Among all his other purfuits, he never forgot the ftudy of the Englifh language. This he regarded as a matter of fo much confequence, that he took uncommon pains with regard to the ftyle of his compofitions; and it cannot be denied, that he excelled in perfpicuity, correctnefs, and propriety of expreffion. His fix difcourfes in particular, delivered at the annual meetings of the Royal Society, on occafion of the prize medals, have been univerfally admired as elegant compofitions, as well as critical and learned differtations. And this characteriftic of them, feemed to increafe and heighten, from year to year: a circumftance which argues rather an improvement of his faculties, than any decline of them, and that even after the accident which it was pretended occafioned his defcent from the prefident's chair. So excellent indeed were thefe compofitions efteemed, that envy ufed to afperfe his character with the imputation of borrowing the hand of another in thofe learned difcourfes. But how falfe fuch afperfion was, I, and I believe moft of the other gentlemen who had the honour of receiving the annual medal from his hands, can fully teftify. For myfelf in particular, I can witnefs for the laft, and perhaps the beft, that on the theory and improvements in gunnery, having been prefent or privy to his compofition of every part of it.—Though our author was not fond of poetry, he had a great affection for the fifter art, mufic. Of this art he was not merely an admirer, but became fo far a practitioner in it, as to be a performer on the violoncello, at a weekly concert given by a fociety of gentlemen at Edinburgh. Befides a clofe application to medical and philofophical fcience, during the latter part of his life, he devoted much time to the ftudy of divinity: this being with him a very favourite and interefting object.

If, from the intellectual, we pafs on to the moral character of Sir John Pringle, we fhall find that the ruling feature of it was integrity. By this principle he was uniformly actuated in the whole of his conduct and behaviour. He was equally diftinguifhed for his fobriety. I and other perfons have heard him declare, that he had never once in his life been intoxicated with liquor. In his friendfhips, he was ardent and fteady. The intimacies which were formed by him, in the early part of his life, continued unbroken to the deceafe of the gentlemen with whom they were made; and were kept up by a regular correfpondence, and by all the good offices that lay in his power.

With regard to Sir John's external manner of deportment, he paid a very refpectful attention to thofe who were honoured with his friendfhip and efteem, and to fuch ftrangers as came to him well recommended. Foreigners in particular had good reafon to be fatisfied with the uncommon pains which he took to fhew them every mark of civility and regard. He had however at times fomewhat of a drynefs and referve in his behaviour, which had the appearance of coldnefs; and this was the cafe when he was not perfectly pleafed with the perfons who were introduced to him, or who happened to be in his company. His fenfe of integrity and dignity would not permit him to adopt that falfe and fuperficial politenefs, which treats all men alike, though ever fo different in point of real eftimation and merit, with the fame fhew of cordiality and kindnefs. He was above affuming the profeffion, without the reality of refpect.

PRISM, in Geometry, is a body, or folid, whofe two ends are any plane figures which are parallel, equal, and fimilar; and its fides, connecting thofe ends, are parallelograms.—Hence, every fection parallel to the ends, is the fame kind of equal and fimilar figure as the ends themfelves are; and the Prifm may be confidered as generated by the parallel motion of this plane figure.

Prifms take their feveral particular names from the figure of their ends. Thus, when the end is a triangle, it is a Triangular Prifm; when a fquare, a Square Prifm; when a pentagon, a Pentagonal Prifm; when a hexagon, a Hexagonal Prifm; and fo on. And hence the denomination Prifm comprifes alfo the cube and parallelopipedon, the former being a fquare Prifm, and

the latter a rectangular one. And even a cylinder may be considered as a round Prism, or one that has an infinite number of sides. Also a Prism is said to be regular or irregular, according as the figure of its end is a regular or an irregular polygon.

The Axis of a Prism, is the line conceived to be drawn lengthways through the middle of it, connecting the centre of one end with that of the other end.

Prisms, again, are either right or oblique.

A *Right* Prism is that whose sides, and its axis, are perpendicular to its ends; like an upright tower. And

An *Oblique* Prism, is when the axis and sides are oblique to the ends; so that, when set upon one end, it inclines on one hand, like an inclined tower.

The principal properties of Prisms, are,

1. That all Prisms are to one another in the ratio compounded of their bases and heights.

2. Similar Prisms are to one another in the triplicate ratio of their like sides.

3. A Prism is triple of a pyramid of equal base and height; and the solid content of a Prism is found by multiplying the base by the perpendicular height.

4. The upright surface of a right Prism, is equal to a rectangle of the same height, and its breadth equal to the perimeter of the base or end. And therefore such upright surface of a right Prism, is found by multiplying the perimeter of the base by the perpendicular height. Also the upright surface of an oblique Prism is found by computing those of all its parallelogram sides separately, and adding them together.

And if to the upright surface be added the areas of the two ends, the sum will be the whole surface of the Prism.

Prism, in Dioptrics, is a piece of glass in form of a triangular Prism: which is much used in experiments concerning the nature of light and colours.

The use and phenomena of the Prism arise from its sides not being parallel to each other; from whence it separates the rays of light in their passage through it, by coming through two sides of one and the same angle.

The more general of these phenomena are enumerated and illustrated under the article Colour; which are sufficient to prove, that colours do not either consist in the contorsion of the globules of light, as Des Cartes imagined; nor in the obliquity of the pulses of the etherial matter, as Hook fancied; nor in the constipation of light, and its greater or less concitation, as Dr. Barrow conjectured; but that they are original and unchangeable properties of light itself.

PRISMOID, is a solid, or body, somewhat resembling a prism, but that its ends are any dissimilar parallel plane figures of the same number of sides; the upright sides being trapezoids.——If the ends of the Prismoid be bounded by dissimilar curves, it is sometimes called a cylindroid.

PROBABILITY *of an Event*, in the Doctrine of Chances, is the ratio of the number of chances by which the event may happen, to the number by which it may both happen and fail. So that, if there be constituted a fraction, of which the numerator is the number of chances for the events happening, and the denominator the number for both happening and failing, that fraction will properly express the value of the Probability of the event's happening. Thus, if an event have 3 chances for happening, and 2 for failing, the sum of which being 5, the fraction $\frac{3}{5}$ will fitly represent the Probability of its happening, and may be taken to be the measure of it. The same thing may be said of the Probability of failing, which will likewise be measured by a fraction, whose numerator is the number of chances by which it may fail, and its denominator the whole number of chances both for its happening and failing: so the Probability of the failing of the above event, which has 2 chances to fail, and 3 to happen, will be expressed or measured by the fraction $\frac{2}{5}$.

Hence, if there be added together the fractions which express the Probability for both happening and failing, their sum will always be equal to unity or 1; since the sum of their numerators will be equal to their common denominator. And since it is a certainty that an event will either happen or fail, it follows that a certainty, which may be considered as an infinitely great degree of Probability, is fitly represented by unity. See Simpson's or Demoivre's Doctrine of Chances; also Bernoulli's Ars Conjectandi; Monmort's Analyse des Jeux de Hasard; or M. De Parcieu's Essais sur les Probabilites de la Vie humaine. See also Expectation, and Gaming.

Probability *of Life*. See Expectation *of Life*, and Life-*Annuities*.

PROBLEM, in Geometry, is a proposition in which some operation or construction is required. As, to bisect a line, to make a triangle, to raise a perpendicular, to draw a circle through three points, &c.

A Problem, according to Wolfius, consists of three parts: The proposition, which expresses what is to be done; the resolution, or solution, in which are orderly rehearsed the several steps of the process or operation; and the demonstration, in which it is shewn, that by doing the several things prescribed in the resolution, the thing required is obtained.

Problem, in Algebra, is a question or proposition which requires some unknown truth to be investigated or discovered; and the truth of the discovery demonstrated.

Problem, *Kepler's*. See Kepler's *Problem*.

Problem, *Determinate*, *Diophantine*, *Indeterminate*, *Limited*, *Linear*, *Local*, *Plane*, *Solid*, *Surfolid*, and *Unlimited*. See the adjectives.

Deliacal Problem, in Geometry, is the doubling of a cube. This amounts to the same thing as the finding of two mean proportionals between two given lines: whence this also is called the Deliacal Problem. See Duplication.

PROCLUS, an eminent philosopher and mathematician among the later Platonists, was born at Constantinople in the year 410, of parents who were both able and willing to provide for his instruction in all the various branches of learning and knowledge. He was first sent to Xanthus, a city of Lycia, to learn grammar: from thence to Alexandria, where he was under the best masters in rhetoric, philosophy, and mathematics: and from Alexandria he removed to Athens, where he attended the younger Plutarch, and Syrian, both of them celebrated philosophers. He succeeded the latter in the

the government of the Platonic ſchool at Athens; where he died in 485, at 75 years of age.

Marinus of Naples, who was his ſucceſſor in the ſchool, wrote his life; the firſt perfect copy of which was publiſhed, with a Latin verſion and notes, by Fabricius at Hamburgh, 1700, in 4to; and afterwards ſubjoined to his *Bibliotheca Latina*, 1703, in 8vo.

Proclus wrote a great number of pieces, and upon many different ſubjects; as, commentaries on philoſophy, mathematics, and grammar; upon the whole works of Homer, Heſiod, and Plato's books of the republic: he wrote alſo on the conſtruction of the Aſtrolabe. Many of his pieces are loſt; ſome have been publiſhed; and a few remain ſtill in manuſcript only. Of the publiſhed, there are four very elegant hymns; one to the Sun, two to Venus, and one to the Muſes. There are commentaries upon ſeveral pieces of Plato; upon the four books of Ptolomy's work *de Judiciis Aſtrorum*; upon the firſt book of Euclid's Elements; and upon Heſiod's *Opera et Dies*. There are alſo works of Proclus upon philoſophical and aſtronomical ſubjects; particularly the piece *De Sphæra*, which was publiſhed, 1620, in 4to, by Bainbridge, the Savilian profeſſor of aſtronomy at Oxford. He wrote alſo 18 arguments againſt the Chriſtians, which are ſtill extant, and in which he attacks them upon the queſtion, whether the world be eternal? the affirmative of which he maintains.

The character of Proclus is the ſame as that of all the later Platoniſts, who it ſeems were not leſs enthuſiaſts and madmen, than the Chriſtians their contemporaries, whom they repreſented in this light. Proclus was not reckoned quite orthodox by his own order: he did not adhere ſo rigorouſly, as Julian and Porphyry, to the doctrines and principles of his maſter: "He had, ſays Cudworth, ſome peculiar fancies and whimſies of his own, and was indeed a confounder of the Platonic theology, and a mingler of much unintelligible ſtuff with it."

PROCYON, in Aſtronomy, a fixed ſtar, of the ſecond magnitude, in Canis Minor, or the Little Dog.

PRODUCING, in Geometry, denotes the continuing a line, or drawing it farther out, till it have an aſſigned length.

PRODUCT, in Arithmetic, or Algebra, is the factum of two numbers, or quantities, or the quantity ariſing from, or produced by, the multiplication of, two or more numbers &c together. Thus, 48 is the product of 6 multiplied by 8.—In multiplication, unity is in proportion to one factor, as the other factor is to the product. So $1 : 6 :: 8 : 48$.

In Algebra, the product of ſimple quantities is expreſſed by joining the letters together like a word, and prefixing the product of the numeral coefficients: So the product of a and b is ab, of $3a$ and $4bc$ is $12abc$. But the product of compound factors or quantities is expreſſed by ſetting the ſign of multiplication between them, and binding each compound factor in a vinculum: ſo the product of $2a + 3b$ and $a - 4c$ is $\overline{2a + 3b} \times \overline{a - 4c}$, or $(2a + 3b) \times (a - 4c)$.

In geometry, a rectangle anſwers to a product, its length and breadth being the two factors; becauſe the numbers expreſſing the length and breadth being multiplied together, produce the content or area of the rectangle.

PROFILE, in Architecture, the figure or draught of a building, fortification, or the like; in which are expreſſed the ſeveral heights, widths, and thickneſſes, ſuch as they would appear, were the building cut down perpendicularly from the roof to the foundation. Whence the Profile is alſo called the Section, and ſometimes the Orthographical Section; and by Vitruvius the Sciography. In this ſenſe, Profile amounts to the ſame thing with Elevation; and ſo ſtands oppoſed to a Plan or Ichnography.

Profile is alſo uſed for the contour, or outline of a figure, building, member of architecture, or the like; as a baſe, a cornice, &c.

PROGRESSION, an orderly advancing or proceeding in the ſame manner, courſe, tenor, proportion, &c.

Progreſſion is either Arithmetical, or Geometrical.

Arithmetical Progression, is a ſeries of quantities proceeding by continued equal differences, either increaſing or decreaſing. Thus,

increaſing	1,	3,	5,	7,	9,	&c, or
decreaſing	21,	18,	15,	12,	9,	&c;

where the former progreſſion increaſes continually by the common difference 2, and the latter ſeries or Progreſſion decreaſes continually by the common difference 3.

1. And hence, to conſtruct an arithmetical Progreſſion, from any given firſt term, and with a given common difference; add the common difference to the firſt term, to give the 2d; to the 2d, to give the 3d; to the 3d, to give the 4th; and ſo on; when the ſeries is aſcending or increaſing: but ſubtract the common-difference continually, when the ſeries is a deſcending one.

2. The chief property of an arithmetical Progreſſion, and which ariſes immediately from the nature of its conſtruction, is this; that the ſum of its extremes, or firſt and laſt terms, is equal to the ſum of every pair of intermediate terms that are equidiſtant from the extremes, or to the double of the middle term when there is an uneven number of the terms.

Thus,	1,	3,	5,	7,	9,	11,	13,
	13,	11,	9,	7,	5,	3,	1,
Sums	14	14	14	14	14	14	14,

where the ſum of every pair of terms is the ſame number 14.

Alſo,	a,	$a + d$,	$a + 2d$,	$a + 3d$,	$a + 4d$,
	$a + 4d$,	$a + 3d$,	$a + 2d$,	$a + d$,	a
ſums	$2a + 4d$	$2a + 4d$	$2a + 4d$	$2a + 4d$	$2a + 4d$

3. And hence it follows, that double the ſum of all the terms in the ſeries, is equal to the ſum of the two extremes multiplied by the number of the terms; and conſequently, that the ſingle ſum of all the terms of the ſeries, is equal to half the ſaid product. So the ſum of the 7 terms

1,

1, 3, 5, 7, 9, 11, 13, is $\overline{1+13}\times\frac{7}{2}=\frac{14}{2}\times 7=49$. And the ſum of the five terms

$a,\ a+d,\ a+2d,\ a+3d,\ a+4d$, is $\overline{a+4d}\times\frac{5}{2}$.

4. Hence alſo, if the firſt term of the Progreſſion be 0, the ſum of the ſeries will be equal to half the product of the laſt term multiplied by the number of terms: i. e. the ſum of

$$0+d+2d+3d+4d\,\text{------}\,\overline{n-1}.d=\tfrac{1}{2}n.\overline{n-1}.d,$$

where n is the number of terms, ſuppoſing 0 to be one of them. That is, in other words, the ſum of an arithmetical Progreſſion, whether finite or infinite, whoſe firſt term is 0, is to the ſum of as many times the greateſt term, in the ratio of 1 to 2.

5. In like manner, the ſum of the ſquares of the terms of ſuch a ſeries, beginning at 0, is to the ſum of as many terms each equal to the greateſt, in the ratio of 1 to 3. And

6. The ſum of the cubes of the terms of ſuch a ſeries, is to the ſum of as many times the greateſt term, in the ratio of 1 to 4.

7. And univerſally, if every term of ſuch a Progreſſion be raiſed to the m power, then the ſum of all thoſe powers will be to the ſum of as many terms equal to the greateſt, in the ratio of $m+1$ to 1. That is,

the ſum $0 + d + 2d + 3d\,\text{-------}\,l$,
is to $l^m + l^m + l^m + l^m\,\text{-------}\,l^m$,

in the ratio of 1 to $m+1$.

8. A ſynopſis of all the theorems, or relations, in an arithmetical Progreſſion, between the extremes or firſt and laſt term, the ſum of the ſeries, the number of terms, and the common difference, is as follows: viz, if

a denote the leaſt term,
z the greateſt term,
d the common difference,
n the number of terms,
s the ſum of the ſeries;

then will each of theſe five quantities be expreſſed in terms of the others, as below:

$$a=z-\overline{n-1}.d=\frac{2s}{n}-z=\frac{s}{n}-\frac{n-1}{2}d=\sqrt{\left(\tfrac{1}{2}d+z\right)^2-2ds}+\tfrac{1}{2}d.$$

$$z=a+\overline{n-1}.d=\frac{2s}{n}-a=\frac{s}{n}+\frac{n-1}{2}d=\sqrt{\left(\tfrac{1}{2}d-a\right)^2+2ds}-\tfrac{1}{2}d.$$

$$d=\frac{z-a}{n-1}=\frac{s-na}{n-1}.\frac{2}{n}=\frac{nz-s}{n-1}.\frac{2}{n}=\frac{z+a.z-a}{2s-a-z}.$$

$$n=\frac{z-a}{d}+1=\frac{2s}{a+z}=\frac{\tfrac{1}{2}d-a+\sqrt{\left(\tfrac{1}{2}d-a\right)^2+2ds}}{d}=\frac{\tfrac{1}{2}d+z-\sqrt{\left(\tfrac{1}{2}d+z\right)^2-2ds}}{d}.$$

$$s=\frac{a+z}{2}n=\frac{a+z}{2}.\frac{z-a+d}{d}=\frac{2a+\overline{n-1}.d}{2}n=\frac{2z-\overline{n-1}.d}{2}n.$$

And moſt of theſe expreſſions will become much ſimpler if the firſt term be 0 inſtead of a.

Geometrical Progression, is a ſeries of quantities proceeding in the ſame continual ratio or proportion, either increaſing or decreaſing; or it is a ſeries of quantities that are continually proportional; or which increaſe by one common multiplier, or decreaſe by one common diviſor; which common multiplier or diviſor is called the common ratio. As,

increaſing, 1, 2, 4, 8, 16, &c,
decreaſing, 81, 27, 9, 3, 1, &c;

where the former progreſſion increaſes continually by the common multiplier 2, and the latter decreaſes by the common diviſor 3.

Or aſcending, $a,\ ra,\ r^2a,\ r^3a$, &c,

or deſcending, $a,\ \frac{a}{r},\ \frac{a}{r^2},\ \frac{a}{r^3}$, &c;

where the firſt term is a, and common ratio r.

1. Hence, the ſame principal properties obtain in a geometrical Progreſſion, as have been remarked of the arithmetical one, uſing only multiplication in the geometricals for addition in the arithmeticals, and diviſion in the former for ſubtraction in the latter. So that, to conſtruct a geometrical Progreſſion, from any given firſt term, and with a given common ratio; multiply the 1ſt term continually by the common ratio, for the reſt of the terms when the ſeries is an aſcending one; or divide continually by the common ratio, when it is a deſcending Progreſſion.

2. In every geometrical Progreſſion, the product of the extreme terms, is equal to the product of every pair of the intermediate terms that are equidiſtant from the extremes, and alſo equal to the ſquare of the middle term when there is a middle one, or an uneven number of the terms.

Thus,	1,	2,	4,	8,	16,
	16	8	4	2	1
prod.	16	16	16	16	16
Alſo	a,	ra,	r^2a,	r^3a,	r^4a,
	r^4a	r^3a	r^2a	ra	a
prod.	r^4a^2	r^4a^2	r^4a^2	r^4a^2	r^4a^2

3. The laſt term of a geometrical Progreſſion, is equal to the firſt term multiplied, or divided, by the ratio raiſed to the power whoſe exponent is leſs by 1 than the number of terms in the ſeries; ſo $z=ar^{n-1}$ when the ſeries is an aſcending one, or $z=\frac{a}{r^{n-1}}$, when it is a deſcending Progreſſion.

4. As the ſum of all the antecedents, or all the terms except the leaſt, is to the ſum of all the conſequents, or all the terms except the greateſt, ſo is 1 to r the ratio. For,

if $a + ra + r^2a + r^3a$ be all except the laſt,
then $ra + r^2a + r^3a + r^4a$ are all except the firſt;

where it is evident that the former is to the latter as 1 to r, or the former multiplied by r gives the latter. So that, z denoting the laſt term, a the firſt term, and r the ratio, alſo s the ſum of all the terms; then $s - z : s - a :: 1 : r$, or $s - a = \overline{s - z} . r$. And from this equation all the relations among the four quantities a, z, r, s, are eaſily derived; ſuch as, $s = \frac{rz - a}{r - 1}$; viz, multiply the greateſt term by the ratio, ſubtract the leaſt term from the product, then the remainder divided by 1 leſs than the ratio, will give the ſum of the ſeries. And if the leaſt term a be 0, which happens when the deſcending Progreſſion is infinitely continued, then the ſum is barely $\frac{rz}{r - 1}$. As in the infinite Progreſſion $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c, where $z = 1$, and $r = 2$, it is s or $\frac{rz}{r - 1} = \frac{2}{2 - 1} = \frac{2}{1} = 2$.

5. The firſt or leaſt term of a geometrical Progreſſion, is to the ſum of all the terms, as the ratio minus 1, to the n power of the ratio minus 1; that is $a : s :: r - 1 : r^n - 1$.

Other relations among the five quantities a, z, r, n, s, where

a denotes the leaſt term,
z the greateſt term,
r the common ratio,
n the number of terms,
s the ſum of the Progreſſion,

are as below; viz,

$$a = \frac{z}{r^{n-1}} = zr - (r - 1)s = \frac{r - 1}{r^n - 1}s.$$

$$z = ar^{n-1} = \frac{a + (r - 1)s}{r} = \frac{r - 1}{r^n - 1}sr^{n-1}.$$

$$r = \frac{s - a}{s - z} = \sqrt[n-1]{\frac{z}{a}}.$$

$$n = \frac{\log. \frac{rz}{a}}{\log. r} = \frac{\log. \frac{a + (r - 1)s}{a}}{\log. r} = \frac{\log. \frac{rz}{rz - (r - 1)s}}{\log. r} = \frac{\log. \frac{s - a}{s - z} \cdot \frac{z}{a}}{\log. \frac{s - a}{s - z}}.$$

$$s = \frac{rz - a}{r - 1} = \frac{r^n - 1}{r - 1}a = \frac{r^n - 1}{r - 1} \cdot \frac{z}{r^{n-1}} = \frac{\sqrt[n-1]{z^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{z} - \sqrt[n-1]{a}}.$$

And the other values of a, z, and r are to be found from theſe equations, viz,

$$(s - z)^{n-1} z = (s - a)^{n-1} a,$$

$$r^n - \frac{s}{a} r = 1 - \frac{s}{a},$$

$$r^n - \frac{s}{s - z} r^{n-1} = \frac{z}{s - z}.$$

For other ſorts of Progreſſions, ſee SERIES.

PROJECTILE, or PROJECT, in Mechanics, is any body which, being put into a violent motion by an external force impreſſed upon it, is diſmiſſed from the agent, and left to purſue its courſe. Such as a ſtone thrown out of the hand or a ſling, an arrow from a bow, a ball from a gun, &c.

PROJECTILES, the ſcience of the motion, velocity, flight, range, &c, of a projectile put into violent motion by ſome external cauſe, as the force of gunpowder, &c. This is the foundation of gunnery, under which article may be found all that relates peculiarly to that branch.

All bodies, being indifferent as to motion or reſt, will neceſſarily continue the ſtate they are put into, except ſo far as they are hindered, and forced to change it by ſome new cauſe. Hence, a Projectile, put in motion, muſt continue eternally to move on in the ſame right line, and with the ſame uniform or conſtant velocity, were it to meet with no reſiſtance from the medium, nor had any force of gravity to encounter.

In the firſt caſe, the theory of Projectiles would be very ſimple indeed; for there would be nothing more to do, than to compute the ſpace paſſed over in a given time by a given conſtant velocity; or either of theſe, from the other two being given.

But by the conſtant action of gravity, the Projectile is continually deflected more and more from its right-lined courſe, and that with an accelerated velocity; which, being combined with its Projectile impulſe, cauſes the body to move in a curvilineal path, with a variable motion, which path is the curve of a parabola, as will be proved below; and the determination of the range, time of flight, angle of projection, and variable velocity, conſtitutes what is uſually meant by the doctrine of Projectiles, in the common acceptation of the word.

What is ſaid above however, is to be underſtood of Projectiles moving in a non-reſiſting medium; for when the reſiſtance of the air is alſo conſidered, which is enormouſly great, and which very much impedes the firſt Projectile velocity, the path deviates greatly from the parabola, and the determination of the circumſtances of its motion becomes one of the moſt complex and difficult problems in nature.

In the firſt place therefore it will be proper to conſider the common doctrine of Projectiles, or that on the parabolic theory, or as depending only on the nature of gravity and the Projectile motion, as abſtracted from the reſiſtance of the medium.

Little more than 200 years ago, philoſophers took the line deſcribed by a body projected horizontally, ſuch as a bullet out of a cannon, while the force of the powder greatly exceeded the weight of the bullet, to be a right line, after which they allowed it became a curve. Nicholas Tartaglia was the firſt who perceived the miſtake, maintaining that the path of the bullet was a curved line through the whole of its extent. But it was Galileo who firſt determined what particular curve it is that a Projectile deſcribes; ſhewing that the path of a bullet projected horizontally from an eminence, was a parabola; the vertex of which is the point where the bullet quits the cannon. And the ſame is proved generally, in the 2d ſection following, when the projection is made in any direction whatever, viz, that the curve

1, 3, 5, 7, 9, 11, 13, is $\overline{1+13} \times \frac{7}{2} = \frac{14}{2} \times 7 = 49$. And the ſum of the five terms

a, $a+d$, $a+2d$, $a+3d$, $a+4d$, is $\overline{a+4d} \times \frac{5}{2}$.

4. Hence alſo, if the firſt term of the Progreſſion be 0, the ſum of the ſeries will be equal to half the product of the laſt term multiplied by the number of terms: i. e. the ſum of

$0+d+2d+3d+4d \text{-------} \overline{n-1}.d = \frac{1}{2}n.\overline{n-1}.d$,

where n is the number of terms, ſuppoſing 0 to be one of them. That is, in other words, the ſum of an arithmetical Progreſſion, whether finite or infinite, whoſe firſt term is 0, is to the ſum of as many times the greateſt term, in the ratio of 1 to 2.

5. In like manner, the ſum of the ſquares of the terms of ſuch a ſeries, beginning at 0, is to the ſum of as many terms each equal to the greateſt, in the ratio of 1 to 3. And

6. The ſum of the cubes of the terms of ſuch a ſeries, is to the ſum of as many times the greateſt term, in the ratio of 1 to 4.

7. And univerſally, if every term of ſuch a Progreſſion be raiſed to the m power, then the ſum of all thoſe powers will be to the ſum of as many terms equal to the greateſt, in the ratio of $m+1$ to 1. That is,

the ſum $0 + d + 2d + 3d \text{-------} l$,
is to $l^m + l^m + l^m + l^m \text{-------} l^m$,

in the ratio of 1 to $m+1$.

8. A ſynopſis of all the theorems, or relations, in an arithmetical Progreſſion, between the extremes or firſt and laſt term, the ſum of the ſeries, the number of terms, and the common difference, is as follows: viz, if

a denote the leaſt term,
z the greateſt term,
d the common difference,
n the number of terms,
s the ſum of the ſeries;

then will each of theſe five quantities be expreſſed in terms of the others, as below:

$$a = z - \overline{n-1}.d = \frac{2s}{n} - z = \frac{s}{n} - \frac{n-1}{2}d = \sqrt{\overline{\tfrac{1}{2}d+z}|^2 - 2ds} + \tfrac{1}{2}d.$$

$$z = a + \overline{n-1}.d = \frac{2s}{n} - a = \frac{s}{n} + \frac{n-1}{2}d = \sqrt{\overline{\tfrac{1}{2}d-a}|^2 + 2ds} - \tfrac{1}{2}d.$$

$$d = \frac{z-a}{n-1} = \frac{s-na}{n-1}.\frac{2}{n} = \frac{nz-s}{n-1}.\frac{2}{n} = \frac{z+a.z-a}{2s-a-z}.$$

$$n = \frac{z-a}{d} + 1 = \frac{2s}{a+z} = \frac{\tfrac{1}{2}d - a + \sqrt{\overline{\tfrac{1}{2}d-a}|^2 + 2ds}}{d} = \frac{\tfrac{1}{2}d + z - \sqrt{\overline{\tfrac{1}{2}d+z}|^2 - 2ds}}{d}.$$

$$s = \frac{a+z}{2}n = \frac{a+z}{2}.\frac{z-a+d}{d} = \frac{2a+\overline{n-1}.d}{2}n = \frac{2z-\overline{n-1}.d}{2}n.$$

And moſt of theſe expreſſions will become much ſimpler if the firſt term be 0 inſtead of a.

Geometrical Progression, is a ſeries of quantities proceeding in the ſame continual ratio or proportion, either increaſing or decreaſing; or it is a ſeries of quantities that are continually proportional; or which increaſe by one common multiplier, or decreaſe by one common diviſor; which common multiplier or diviſor is called the common ratio. As,

increaſing, 1, 2, 4, 8, 16, &c,
decreaſing, 81, 27, 9, 3, 1, &c;

where the former progreſſion increaſes continually by the common multiplier 2, and the latter decreaſes by the common diviſor 3.

Or aſcending, a, ra, r^2a, r^3a, &c,

or deſcending, a, $\frac{a}{r}$, $\frac{a}{r^2}$, $\frac{a}{r^3}$, &c;

where the firſt term is a, and common ratio r.

1. Hence, the ſame principal properties obtain in a geometrical Progreſſion, as have been remarked of the arithmetical one, uſing only multiplication in the geometricals for addition in the arithmeticals, and diviſion in the former for ſubtraction in the latter. So that, to conſtruct a geometrical Progreſſion, from any given firſt term, and with a given common ratio; multiply the 1ſt term continually by the common ratio, for the reſt of the terms when the ſeries is an aſcending one; or divide continually by the common ratio, when it is a deſcending Progreſſion.

2. In every geometrical Progreſſion, the product of the extreme terms, is equal to the product of every pair of the intermediate terms that are equidiſtant from the extremes, and alſo equal to the ſquare of the middle term when there is a middle one, or an uneven number of the terms.

Thus,	1,	2,	4,	8,	16,
	16	8	4	2	1
prod.	16	16	16	16	16
Alſo	a,	ra,	r^2a,	r^3a,	r^4a,
	r^4a	r^3a	r^2a	ra	a
prod.	r^4a^2	r^4a^2	r^4a^2	r^4a^2	r^4a^2

3. The laſt term of a geometrical Progreſſion, is equal to the firſt term multiplied, or divided, by the ratio raiſed to the power whoſe exponent is leſs by 1 than the number of terms in the ſeries; ſo $z = ar^{n-1}$ when the ſeries is an aſcending one, or $z = \frac{a}{r^{n-1}}$, when it is a deſcending Progreſſion.

4. As the ſum of all the antecedents, or all the terms except the leaſt, is to the ſum of all the conſequents, or all the terms except the greateſt, ſo is 1 to r the ratio. For,

if $a + ra + r^2a + r^3a$ be all except the laſt,
then $ra + r^2a + r^3a + r^4a$ are all except the firſt;

where it is evident that the former is to the latter as 1 to r, or the former multiplied by r gives the latter. So that, z denoting the laſt term, a the firſt term, and r the ratio, alſo s the ſum of all the terms; then $s - z : s - a :: 1 : r$, or $s - a = \overline{s - z} . r$. And from this equation all the relations among the four quantities a, z, r, s, are eaſily derived; ſuch as, $s = \frac{rz - a}{r - 1}$; viz, multiply the greateſt term by the ratio, ſubtract the leaſt term from the product, then the remainder divided by 1 leſs than the ratio, will give the ſum of the ſeries. And if the leaſt term a be 0, which happens when the deſcending Progreſſion is infinitely continued, then the ſum is barely $\frac{rz}{r - 1}$. As in the infinite Progreſſion $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c, where $z = 1$, and $r = 2$, it is s or $\frac{rz}{r - 1} = \frac{2}{2 - 1} = \frac{2}{1} = 2$.

5. The firſt or leaſt term of a geometrical Progreſſion, is to the ſum of all the terms, as the ratio minus 1, to the n power of the ratio minus 1; that is $a : s :: r - 1 : r^n - 1$.

Other relations among the five quantities a, z, r, n, s, where

a denotes the leaſt term,
z the greateſt term,
r the common ratio,
n the number of terms,
s the ſum of the Progreſſion,

are as below; viz,

$$a = \frac{z}{r^{n-1}} = zr - (r - 1)s = \frac{r - 1}{r^n - 1}s.$$

$$z = ar^{n-1} = \frac{a + (r - 1)s}{r} = \frac{r - 1}{r^n - 1}sr^{n-1}.$$

$$r = \frac{s - a}{s - z} = \sqrt[n-1]{\frac{z}{a}}.$$

$$n = \frac{\log. \frac{rz}{a}}{\log. r} = \frac{\log. \frac{a + (r - 1)s}{a}}{\log. r} = \frac{\log. \frac{rz}{rz - (r - 1)s}}{\log. r} = \frac{\log. \frac{s - a}{s - z} . \frac{z}{a}}{\log. \frac{s - a}{s - z}}.$$

$$s = \frac{rz - a}{r - 1} = \frac{r^n - 1}{r - 1}a = \frac{r^n - 1}{r - 1} . \frac{z}{r^{n-1}} = \frac{\sqrt[n-1]{z^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{z} - \sqrt[n-1]{a}}.$$

And the other values of a, z, and r are to be found from theſe equations, viz,

$$(s - z)^{n-1} z = (s - a)^{n-1} a,$$

$$r^n - \frac{s}{a} r = 1 - \frac{s}{a},$$

$$r^n - \frac{s}{s - z} r^{n-1} = \frac{z}{s - z}.$$

For other ſorts of Progreſſions, ſee Series.

PROJECTILE, or Project, in Mechanics, is any body which, being put into a violent motion by an external force impreſſed upon it, is diſmiſſed from the agent, and left to purſue its courſe. Such as a ſtone thrown out of the hand or a ſling, an arrow from a bow, a ball from a gun, &c.

PROJECTILES, the ſcience of the motion, velocity, flight, range, &c, of a projectile put into violent motion by ſome external cauſe, as the force of gunpowder, &c. This is the foundation of gunnery, under which article may be found all that relates peculiarly to that branch.

All bodies, being indifferent as to motion or reſt, will neceſſarily continue the ſtate they are put into, except ſo far as they are hindered, and forced to change it by ſome new cauſe. Hence, a Projectile, put in motion, muſt continue eternally to move on in the ſame right line, and with the ſame uniform or conſtant velocity, were it to meet with no reſiſtance from the medium, nor had any force of gravity to encounter.

In the firſt caſe, the theory of Projectiles would be very ſimple indeed; for there would be nothing more to do, than to compute the ſpace paſſed over in a given time by a given conſtant velocity; or either of theſe, from the other two being given.

But by the conſtant action of gravity, the Projectile is continually deflected more and more from its right-lined courſe, and that with an accelerated velocity; which, being combined with its Projectile impulſe, cauſes the body to move in a curvilineal path, with a variable motion, which path is the curve of a parabola, as will be proved below; and the determination of the range, time of flight, angle of projection, and variable velocity, conſtitutes what is uſually meant by the doctrine of Projectiles, in the common acceptation of the word.

What is ſaid above however, is to be underſtood of Projectiles moving in a non-reſiſting medium; for when the reſiſtance of the air is alſo conſidered, which is enormouſly great, and which very much impedes the firſt Projectile velocity, the path deviates greatly from the parabola, and the determination of the circumſtances of its motion becomes one of the moſt complex and difficult problems in nature.

In the firſt place therefore it will be proper to conſider the common doctrine of Projectiles, or that on the parabolic theory, or as depending only on the nature of gravity and the Projectile motion, as abſtracted from the reſiſtance of the medium.

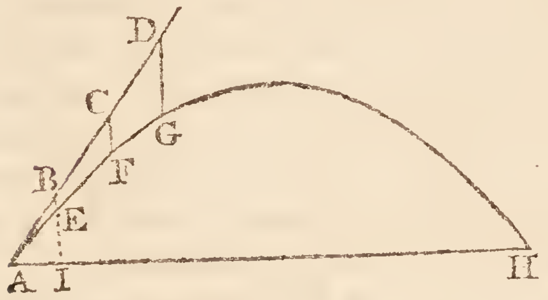
Little more than 200 years ago, philoſophers took the line deſcribed by a body projected horizontally, ſuch as a bullet out of a cannon, while the force of the powder greatly exceeded the weight of the bullet, to be a right line, after which they allowed it became a curve. Nicholas Tartaglia was the firſt who perceived the miſtake, maintaining that the path of the bullet was a curved line through the whole of its extent. But it was Galileo who firſt determined what particular curve it is that a Projectile deſcribes; ſhewing that the path of a bullet projected horizontally from an eminence, was a parabola; the vertex of which is the point where the bullet quits the cannon. And the ſame is proved generally, in the 2d ſection following, when the projection is made in any direction whatever, viz, that the curve

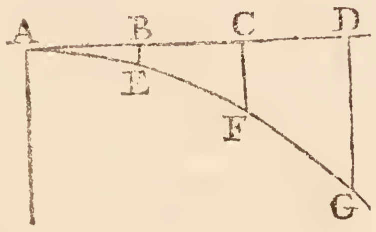
curve is always a parabola, ſuppoſing the body moves in a non-reſiſting medium.

The Laws of the Motion of PROJECTILES.

I. If a heavy body be projected perpendicularly, it will continue to aſcend or deſcend perpendicularly; becauſe both the projecting and the gravitating force are found in the ſame line of direction.

II. If a body be projected in free ſpace, either parallel to the horizon, or in any oblique direction; it will, by this motion, in conjunction with the action of gravity, deſcribe the curve line of a parabola.





For let the body be projected from A, in the direction AD, with any uniform velocity; then in any equal portions of time it would, by that impulſe alone, deſcribe the equal ſpaces AB, BC, CD, &c, in the line AD, if it were not drawn continually down below that line by the action of gravity. Draw BE, CF, DG, &c, in the direction of gravity, or perpendicular to the horizon; and take BE, CF, DG, &c, equal to the ſpaces through which the body would deſcend by its gravity in the ſame times in which it would uniformly paſs over the ſpaces AB, AC, AD, &c, by the Projectile motion. Then, ſince by theſe motions, the body is carried over the ſpace AB in the ſame time as the ſpace BE, and the ſpace AC in the ſame time as the ſpace CF, and the ſpace AD in the ſame time as the ſpace DG, &c; therefore, by the compoſition of motions, at the end of thoſe times the body will be found reſpectively in the points E, F, G, &c, and conſequently the real path of the Projectile will be the curve line AEFG &c. But the ſpaces AB, AC, AD, &c, being deſcribed by uniform motion, are as the times of deſcription; and the ſpaces BE, CF, DG, &c, deſcribed in the ſame times by the accelerating force of gravity, are as the ſquares of the times; conſequently the perpendicular deſcents are as the ſquares of the ſpaces in AD,

that is - - - BE, CF, DG, &c, are reſpectively proportional to AB^2, AC^2, AD^2, &c,

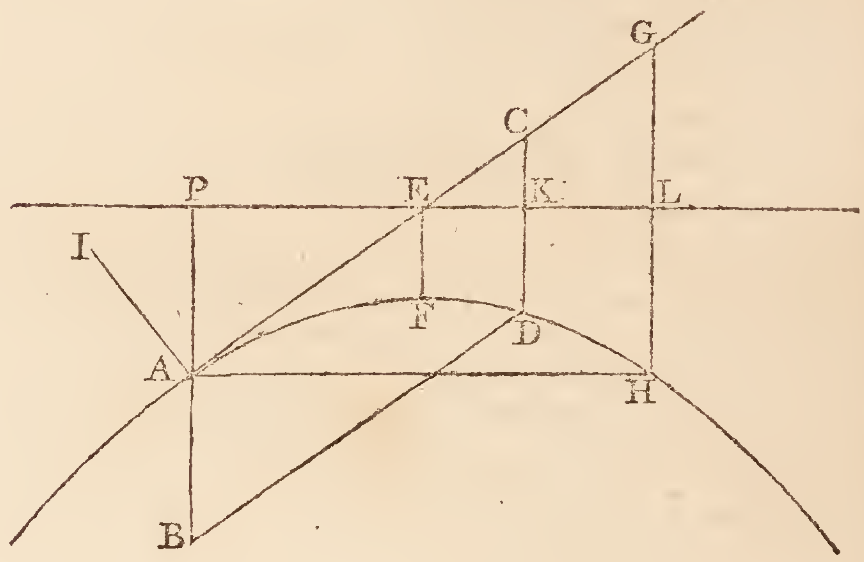
which is the ſame as the property of the parabola. Therefore the path of the Projectile is the parabolic line AEFG &c, to which AD is a tangent at the point A.

Hence, 1. The horizontal velocity of a Projectile is always the ſame conſtant quantity, in every point of the curve; becauſe the horizontal motion is in a conſtant ratio to the motion in AD, which is the uniform Projectile motion; viz, the conſtant horizontal velocity being to the Projectile velocity, as radius to the coſine of the angle DAH, or angle of elevation or depreſſion of the piece above or below the horizontal line AH.

2. The velocity of the Projectile in the direction of the curve, or of its tangent, at any point A, is as the ſecant of its angle BAI of direction above the horizon. For the motion in the horizontal direction AI being conſtant, and AI being to AB as radius to the ſecant of the angle A; therefore the motion at A, in AB, is as the ſecant of the angle A.

3. The velocity in the direction DG of gravity, or perpendicular to the horizon, at any point G of the curve, is to the firſt uniform Projectile velocity at A, as 2GD to AD. For the times of deſcribing AD and DG being equal, and the velocity acquired by freely deſcending through DG being ſuch as would carry the body uniformly over twice DG in an equal time, and the ſpaces deſcribed with uniform motions being as the velocities, it follows that the ſpace AD is to the ſpace 2DG, as the Projectile velocity at A is to the perpendicular velocity at G.

III. The velocity in the direction of the curve, at any point of it, as A, is equal to that which is generated by gravity in freely deſcending through a ſpace which is equal to one-fourth of the parameter of the diameter to the parabola at that point.



Let PA or AB be the height due to the velocity of the Projectile at any point A, in the direction of the curve or tangent AC, or the velocity acquired by falling through that height; and complete the parallelogram ACDB. Then is CD = AB or AP the height due to the velocity in the curve at A; and CD is alſo the height due to the perpendicular velocity at D, which will therefore be equal to the former: but, by the laſt corollary, the velocity at A is to the perpendicular velocity at D, as AC to 2CD; and as theſe velocities are equal, therefore AC or BD is equal to 2CD or 2AB; and hence AB or AP is equal to $\frac{1}{4}$BD or $\frac{1}{4}$ of the parameter of the diameter AB by the nature of the parabola.

Hence, 1. If through the point P, the line PL be drawn perpendicular to AP; then the velocity in the curve at every point, will be equal to the velocity acquired by falling through the perpendicular diſtance of

of the point from the said line PL; that is, a body falling freely through

PA, acquires the velocity in the curve at A,
EF, - - - at F,
KD, - - - at D,
LH, - - - at H.

The reason of which is, that the line PL is what is called the Directrix of the parabola, the property of which is, that the perpendicular to it, from every point of the curve, is equal to one-fourth of the parameter of the diameter at that point, viz,

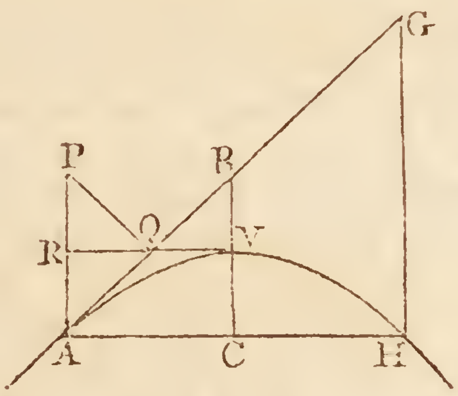
PA $= \frac{1}{4}$ the parameter of the diameter at A,
EF = - - - at F,
KD = - - - at D,
LH = - - - at H.

2. If a body, after falling through the height PA, which is equal to AB, and when it arrives at A if its course be changed, by reflection from a firm plane AI, or otherwise, into any direction AC, without altering the velocity; and if AC be taken equal to 2AP or 2AB, and the parallelogram be completed; the body will describe the parabola passing through the point D.

3. Because AC = 2AB or 2CD or 2AP, therefore $AC^2 = 2AP \cdot 2CD$ or $AP \cdot 4CD$; and because all the perpendiculars EF, CD, GH are as AE^2, AC^2, AG^2; therefore also $AP \cdot 4EF = AE^2$, and $AP \cdot 4GH = AG^2$, &c; and because the rectangle of the extremes is equal to the rectangle of the means, of four proportionals, therefore it is always,

$$AP : AE :: AE : 4EF,$$
and $AP : AC :: AC : 4CD$,
and $AP : AG :: AG : 4GH$,
and so on.

IV. Having given the Direction of a Projectile, and the Impetus or Altitude due to the first velocity; to determine the Greatest Height to which it will rise, and the Random or Horizontal Range.

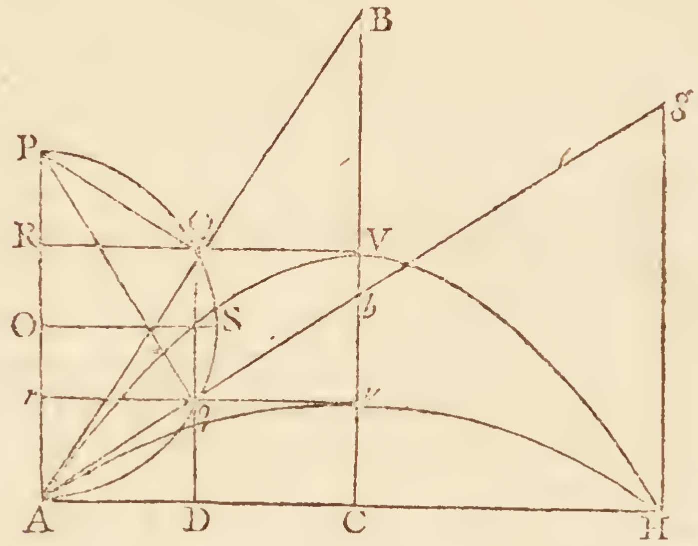


Let AP be the height due to the Projectile velocity at A, or the height which a body must fall to acquire the same velocity as the projectile has in the curve at A; also AG the direction, and AH the horizon. Upon AG let fall the perpendicular PQ, and on AP the perpendicular QR; so shall AR be equal to the greatest altitude CV, and 4RQ equal to the horizontal range AH. Or, having drawn PQ perpendicular to AG, take AG = 4AQ, and draw GH perpendicular to AH; then AH is the range.

For by the last cor. - - - $AP : AG :: AG : 4GH$,
and by sim. triangles, - - - $AP : AG :: AQ : GH$,
or $AP : AG :: 4AQ : 4GH$;
therefore AG = 4AQ; and, by similar triangles, AH = 4RQ.

Also, if V be the vertex of the parabola, then AB or $\frac{1}{2}$AG = 2AQ, or AQ = QB; consequently AR = BV which is = CV by the nature of the parabola.

Hence, 1. Because the angle Q is a right angle, which is the angle in a semicircle, therefore if upon AP as a diameter a semicircle be described, it will pass through the point Q.



2. If the Horizontal Range and the Projectile Velocity be given, the Direction of the piece so as to hit the object H will be thus easily found: Take AD = $\frac{1}{4}$AH, and draw DQ perpendicular to AH, meeting the semicircle described on the diameter AP in Q and q; then either AQ or Aq will be the direction of the piece. And hence it appears, that there are two directions AB and Ab which, with the same Projectile velocity, give the very same horizontal range AH; and these two directions make equal angles qAD and QAP with AH and AP, because the arc PQ is equal to the arc Aq.

3. Or if the Range AH and Direction AB be given; to find the Altitude and Velocity or Impetus: Take AD = $\frac{1}{4}$AH, and erect the perpendicular DQ meeting AB in Q; so shall DQ be equal to the greatest altitude CV. Also erect AP perpendicular to AH, and QP to AQ; so shall AP be the height due to the velocity.

4. When the body is projected with the same velocity, but in different directions; the horizontal ranges AH will be as the sines of double the angles of elevation. Or, which is the same thing, as the rectangle of the sine and cosine of elevation. For AD or RQ, which is $\frac{1}{4}$AH, is the sine of the arc AQ, which measures double the angle QAD of elevation.

And when the direction is the same, but the velocities different, the horizontal ranges are as the square of the velocities, or as the height AP which is as the square of the velocity; for the sine AD or RQ, or $\frac{1}{4}$AH, is as the radius, or as the diameter AP.

Therefore, when both are different, the ranges are in the compound ratio of the squares of the velocities, and the sines of double the angles of elevation.

5. The greatest range is when the angle of elevation is half a right angle, or 45°. For the double of 45 is 90°, which has the greatest sine. Or the radius OS, which is $\frac{1}{4}$ of the range, is the greatest sine.

And hence the greatest range, or that at an elevation of 45°, is just double the altitude AP which is due to the

the velocity. Or equal to 4VC. And consequently, in that case, C is the focus of the parabola, and AH its parameter.

And the ranges are equal at angles equally above and below 45°.

6. When the elevation is 15°, the double of which, or 30°, having its sine equal to half the radius, consequently its range will be equal to AP, or half the greatest range at the elevation of 45°; that is, the range at 15° is equal to the impetus or height due to the projectile velocity.

7. The greatest altitude CV, being equal to AR, is as the versed sine of double the angle of elevation, and also as AP or the square of the velocity. Or as the square of the sine of elevation, and the square of the velocity; for the square of the sine is as the versed sine of the double angle.

8. The time of flight of the projectile, which is equal to the time of a body falling freely through GH or 4CV, 4 times the altitude, is therefore as the square root of the altitude, or as the projectile velocity and sine of the elevation.

9. And hence may be deduced the following set of theorems, for finding all the circumstances relating to projectiles on horizontal planes, having any two of them given. Thus, let

s, c, t = sine, cosine, and tang. of elevation,

S, v = sine and vers. of double the elevation,

R the horizontal range, T the time of flight, V the projectile velocity, H the greatest height of the projectile, $g = 16\frac{1}{12}$ feet, and a = the impetus or the altitude due to the velocity V. Then,

$$R = 2aS = 4asc = \frac{SV^2}{2g} = \frac{scV^2}{g} = \frac{gcT^2}{s} = \frac{gT^2}{t} = \frac{4H}{t}.$$

$$V = \sqrt{4ag} = \sqrt{\frac{2gR}{S}} = \sqrt{\frac{gR}{sc}} = \frac{gT}{s} = \frac{2\sqrt{gH}}{s}.$$

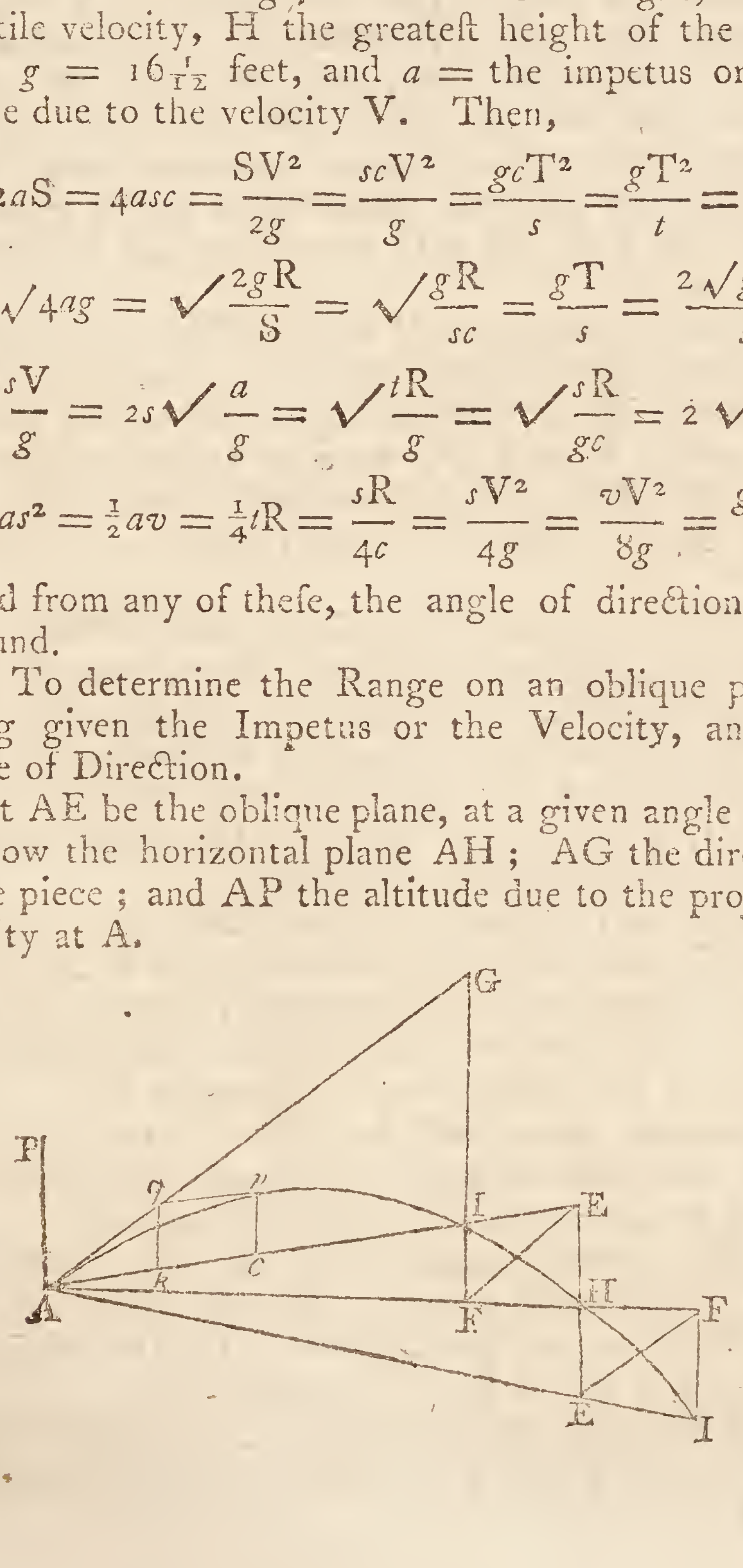
$$T = \frac{sV}{g} = 2s\sqrt{\frac{a}{g}} = \sqrt{\frac{tR}{g}} = \sqrt{\frac{sR}{gc}} = 2\sqrt{\frac{H}{g}}.$$

$$H = as^2 = \tfrac{1}{2}av = \tfrac{1}{4}tR = \frac{sR}{4c} = \frac{sV^2}{4g} = \frac{vV^2}{8g} = \frac{gT^2}{4}.$$

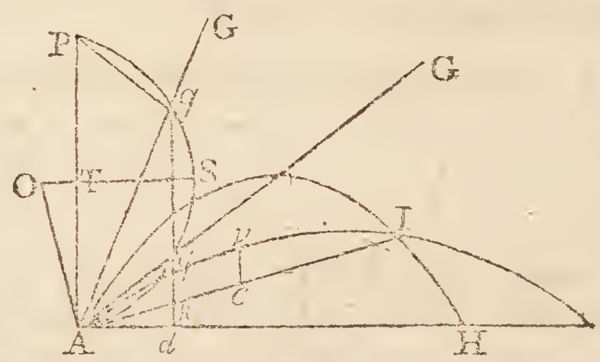
And from any of these, the angle of direction may be found.

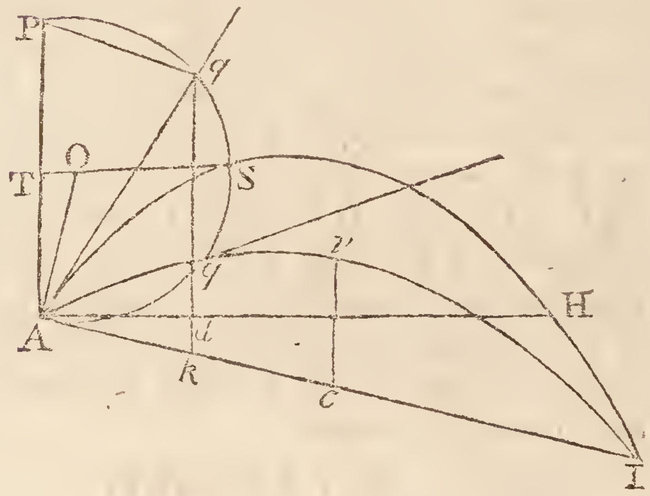
V. To determine the Range on an oblique plane; having given the Impetus or the Velocity, and the Angle of Direction.

Let AE be the oblique plane, at a given angle above or below the horizontal plane AH; AG the direction of the piece; and AP the altitude due to the projectile velocity at A.



By the last prop. find the horizontal range AH to the given velocity and direction; draw HE perpendicular to AH meeting the oblique plane in E; draw EF parallel to the direction AG, and FI parallel to HE; so shall the projectile pass through I, and the range on the oblique plane will be AI. This is evident from prob. 17 of the Parabola in my treatise on Conic Sections, where it is proved, that if AH, AI be any two lines terminated at the curve, and IF, HE be parallel to the axis; then is EF parallel to the tangent AG.





Hence, 1. If AO be drawn perpendicular to the plane AI, and AP be bisected by the perpendicular STO; then with the centre O describing a circle through A and P, the same will also pass through q, because the angle GAI, formed by the tangent AG and AI, is equal to the angle APq, which will therefore stand upon the same arc Aq.

2. If there be given the Range and Velocity, or the Impetus, the Direction will then be easily found thus: Take Ak = $\frac{1}{4}$AI, draw kq perpendicular to AH, meeting the circle described with the radius AO in two points q and q; then Aq or Aq will be the direction of the piece. And hence it appears that there are two directions, which, with the same impetus, give the very same range AI, on the oblique plane. And these two directions make equal angles with AI and AP, the plane and the perpendicular, because the arc Pq = the arc Aq. They also make equal angles with a line drawn from A through S, because the arc Sq = the arc Sq.

3. Or, if there be given the Range AI, and the Direction Aq; to find the Velocity or Impetus. Take Ak = $\frac{1}{4}$AI, and erect kq perpendicular to AH meeting the line of direction in q; then draw qP making the angle AqP = the angle Akq; so shall AP be the impetus, or altitude due to the projectile velocity.

4. The range on an oblique plane, with a given elevation, is directly as the rectangle of the cosine of the direction of the piece above the horizon and the sine of the direction above the oblique plane, and reciprocally as the square of the cosine of the angle of the plane above or below the horizon.

For

For put $s =$ sin. $\angle\, qAI$ or APq,
$c =$ cos. $\angle\, qAH$ or sin. PAq,
$C =$ cos. $\angle$ IAH or sin. Akd or Akq or AqP.
Then, in the tri. APq, - - - $C : s :: AP : Aq$,
and in the tri. Akq, - - - $C : c :: Aq : Ak$,
therefore by compos. - - - $C^2 : cs :: AP : Ak = \frac{1}{4}AI$.

So that the oblique range $AI = \frac{cs}{C^2} \times 4AP$.

Hence the range is the greatest when Ak is the greatest, that is when kq touches the circle in the middle point S, and then the line of direction passes through S, and bisects the angle formed by the oblique plane and the vertex. Also the ranges are equal at equal angles above and below this direction for the maximum.

5. The greatest height cv or kq of the projectile, above the plane, is equal to $\frac{s^2}{c^2} \times$ AP. And therefore it is as the impetus and square of the sine of direction above the plane directly, and square of the cosine of the plane's inclination reciprocally.

For C (sin. AqP) : s (sin. APq) :: $AP : Aq$,
and C (sin. Akq) : s (sin. kAq) :: $Aq : kq$,
therefore by comp. $C^2 : s^2 :: AP : kq$.

6. The time of flight in the curve AvI is $= \frac{2s}{C}\sqrt{\frac{AP}{g}}$, where $g = 16\frac{1}{12}$ feet. And therefore it is as the velocity and sine of direction above the plane directly, and cosine of the plane's inclination reciprocally. For the time of describing the curve, is equal to the time of falling freely through GI or $4kq$ or $\frac{4s^2}{C^2} \times$ AP. Therefore, the time being as the square root of the distance, $\sqrt{g} : \frac{2s}{C}\sqrt{AP} :: 1'' : \frac{2s}{C}\sqrt{\frac{AP}{g}}$ the time of flight.

7. From the foregoing corollaries may be collected the following set of theorems, relating to projects made on any given inclined planes, either above or below the horizontal plane. In which the letters denote as before, namely,

$c =$ cos. of direction above the horizon,
$C =$ cos. of inclination of the plane,
$s =$ sin. of direction above the plane,
R the range on the oblique plane,
T the time of flight,
V the projectile velocity,
H the greatest height above the plane,
a the impetus, or alt. due to the velocity V,
$g = 16\frac{1}{12}$ feet. Then

$$R = \frac{cs}{C^2} \times 4a = \frac{cs}{C^2 g}V^2 = \frac{gc}{s}T^2 = \frac{4c}{s}H.$$

$$H = \frac{s^2}{C^2}a = \frac{s^2V^2}{4gC^2} = \frac{sR}{4c} = \frac{g}{4}T^2.$$

$$V = \sqrt{4ag} = C\sqrt{\frac{gR}{cs}} = \frac{gC}{s}T = \frac{2C}{s}\sqrt{gH}.$$

$$T = \frac{2s}{C}\sqrt{\frac{a}{g}} = \frac{sV}{gC} = \sqrt{\frac{sR}{gc}} = 2\sqrt{\frac{H}{g}}.$$

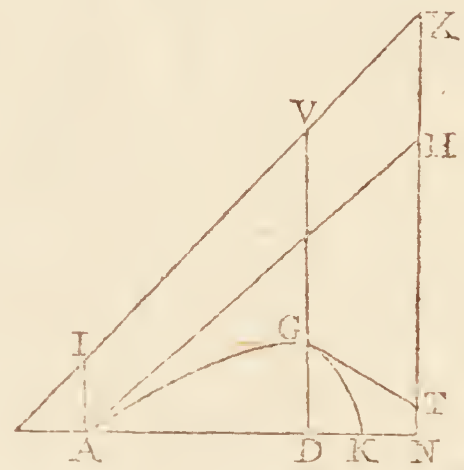
And from any of these, the angle of direction may be found.

Of the Path of PROJECTILES *as depending on the Resistance of the Air.*

For a long time after Galileo, philosophers seemed to be satisfied with the parabolic theory of Projectiles, deeming the effect of the air's resistance on the path as of no consequence. In process of time, however, as the true philosophy began to dawn, they began to suspect that the resistance of the medium might have some effect upon the Projectile curve, and they set themselves to consider this subject with some attention.

Huygens, supposing that the resistance of the air was proportional to the velocity of the moving body, concluded that the line described by it would be a kind of logarithmic curve.

But Newton, having clearly proved, that the resistance to the body is not proportional to the velocity itself, but to the square of it, shews, in his Principia, that the line a Projectile describes, approaches nearer to an hyperbola than a parabola. Schol. prop. 10, lib. 2. Thus, if AGK be a curve of



the hyperbolic kind, one of whose asymptotes is NX, perpendicular to the horizon AK, and the other IX inclined to the same, where VG is reciprocally as DN^n, whose index is n: this curve will nearer represent the path of a Projectile thrown in the direction AH in the air, than a parabola. Newton indeed says, that these hyperbolas are not accurately the curves that a Projectile makes in the air; for the true ones are curves which about the vertex are more distant from the asymptotes, and in the parts remote from the axis approach nearer to the asymptotes than these hyperbolas; but that in practice these hyperbolas may be used instead of those more compounded ones. And if a body be projected from A, in the right line AH, and AI be drawn parallel to the asymptote NX, and GT a tangent to the curve at the vertex: Then the density of the medium in A will be reciprocally as the tangent AH, and the body's velocity will be as $\sqrt{\frac{AH^2}{AI}}$, and the resistance of the medium will be to gravity, as AH to $\frac{2n^2+2n}{n+2} \times$ AI.

M. John Bernoulli constructed this curve by means of the quadrature of some transcendental curves, at the

 request

request of Dr. Keil, who proposed this problem to him in 1718. It was also resolved by Dr. Taylor; and another solution of it may be found in Hermann's Phoronomia.

The commentators Le Sieur and Jacquier say, that the description of the curve in which a Projectile moves, is so very perplexed, that it can scarcely be expected any deduction should be made from it, either to philosophical or mechanical purposes: vol. 2. pa. 118.

Dan. Bernoulli too proved, that the resistance of the air has a very great effect on swift motions, such as those of cannon shot. He concludes from experiment, that a ball which ascended only 7819 feet in the air, would have ascended 58750 feet in vacuo, being near eight times as high. Comment. Acad. Petr. tom. 2.

M. Euler has farther investigated the nature of this curve, and directed the calculation and use of a number of tables for the solution of all cases that occur in gunnery, which may be accomplished with nearly as much expedition as by the common parabolic principles. Memoirs of the Academy of Berlin, for the year 1753.

But how rash and erroneous the old opinion of the inconsiderable resistance of the air is, will easily appear from the experiments of Mr. Robins, who has shewn that, in some cases, this resistance to a cannon ball, amounts to more than 20 times the weight of the ball; and I myself, having prosecuted this subject far beyond any former example, have sometimes found this resistance amount to near 100 times the weight of the ball, viz, when it moved with a velocity of 2000 feet per second, which is a rate of almost 23 miles in a minute. What errors then may not be expected from an hypothesis which neglects this force, as inconsiderable! Indeed it is easy to shew, that the path of such Projectiles is neither a parabola nor nearly a parabola. For, by that theory, if the ball, in the instance last mentioned, flew in the curve of a parabola, its horizontal range, at 45° elevation, will be found to be almost 24 miles; whereas it often happens that the ball, with such a velocity, ranges far short of even one mile.

Indeed the falseness of this hypothesis almost appears at sight, even in Projectiles slow enough to have their motion traced by the eye; for they are seen to descend through a curve manifestly shorter and more inclined to the horizon than that in which they ascended, and the highest point of their flight, or the vertex of the curve, is much nearer to the place where they fall on the ground, than to that from whence they were at first discharged. These things cannot for a moment be doubted of by any one, who in a proper situation views the flight of stones, arrows, or shells, thrown to any considerable distance.

Mr. Robins has not only detected the errors of the parabolic theory of gunnery, which takes no account of the resistance of the air, but shews how to compute the real range of resisted bodies. But for the method which he proposes, and the tables he has computed for this purpose, see his Tracts of Gunnery, pa. 183, &c, vol. 1; and also Euler's Commentary on the same, translated by Mr. Hugh Brown, in 1777.

There is an odd circumstance which often takes place in the motion of bodies projected with considerable force, which shews the great complication and difficulty of this subject; namely, that bullets in their flight are not only depressed beneath their original direction by the action of gravity, but are also frequently driven to the right or left of that direction by the action of some other force.

Now if it were true that bullets varied their direction by the action of gravity only, then it ought to happen that the errors in their flight to the right or left of the mark they were aimed at, should increase in the proportion of the distance of the mark from the piece only. But this is contrary to all experience; the same piece which will carry its bullet within an inch of the intended mark, at 10 yards distance, cannot be relied on to 10 inches in 100 yards, much less to 30 in 300 yards.

And this inequality can only arise from the track of the bullet being incurvated sideways as well as downwards; for by this means the distance between the incurvated line and the line of direction, will increase in a much greater ratio than that of the distance; these lines coinciding at the mouth of the piece, and afterwards separating in the manner of a curve from its tangent, if the mouth of the piece be considered as the point of contact.

This is put beyond a doubt from the experiments made by Mr. Robins; who found also that the direction of the shot in the perpendicular line was not less uncertain, falling sometimes 200 yards short of what it did at other times, although there was no visible cause of difference in making the experiment. And I myself have often experienced a difference of one-fifth or one-sixth of the whole range, both in the deflection to the right or left, and also in the extent of the range, of cannon shot.

If it be asked, what can be the cause of a motion so different from what has been hitherto supposed? It may be answered, that the deflection in question must be owing to some power acting obliquely to the progressive motion of the body, which power can be no other than the resistance of the air. And this resistance may perhaps act obliquely to the progressive motion of the body, from inequalities in the resisted surface; but its general cause is doubtless a whirling motion acquired by the bullet about an axis, by its friction against the sides of the piece; for by this motion of rotation, combined with the progressive motion, each part of the ball's surface will strike the air in a direction very different from what it would do if there was no such whirl; and the obliquity of the action of the air, arising from this cause, will be greater, according as the rotatory motion of the bullet is greater in proportion to its progressive motion. Tracts, vol. 1, p. 149, &c.

M. Euler, on the contrary, attributes this deflection of the ball to its figure, and very little to its rotation: for if the ball was perfectly round, though its centre of gravity did not coincide, the deflection from the axis of the cylinder, or line of direction sideways, would be very inconsiderable. But when it is not round, it will generally

generally go to the right or left of its direction, and so much the more, as its range is greater. From his reasoning on this subject he infers, that cannon shot, which are made of iron, and rounder and less susceptible of a change of figure in passing along the cylinder than those of lead, are more certain than musket shot. True Principles of Gunnery investigated, 1777, p. 304, &c.

PROJECTION, in Mechanics, the act of giving a projectile its motion.

If the direction of the force, by which the projectile is put in motion, be perpendicular to the horizon, the Projection is said to be perpendicular; if parallel to the apparent horizon, it is said to be an horizontal Projection; and if it make an oblique angle with the horizon, the Projection is oblique. In all cases, the angle which the line of direction makes with the horizontal line, is called the angle of Elevation of the projectile, or of Depression when the line of direction points below the horizontal line.

Projection, in Perspective, denotes the appearance or representation of an object on the perspective plane. So, the Projection of a point, is a point, where the optic ray passes from the objective point through the plane to the eye; or it is the point where the plane cuts the optic ray.— And hence it is easy to conceive what is meant by the projection of a line, a plane, or a solid.

Projection *of the Sphere in Plano*, is a representation of the several points or places of the surface of the sphere, and of the circles described upon it, upon a transparent plane placed between the eye and the sphere, or such as they appear to the eye placed at a given distance. For the laws of this Projection, see Perspective; the Projection of the sphere being only a particular case of perspective.

The chief use of the Projection of the sphere, is in the construction of planispheres, maps, and charts; which are said to be of this or that Projection, according to the several situations of the eye, and the perspective plane, with regard to the meridians, parallels, and other points or places to be represented.

The most usual Projection of maps of the world, is that on the plane of the meridian, which exhibits a right sphere; the first meridian being the horizon. The next is that on the plane of the equator, which has the pole in the centre, and the meridians the radii of a circle, &c; and this represents a parallel sphere. See Map.—The primitive circle is that great circle.

The Projection of the sphere is usually divided into Orthographic and Stereographic; to which may be added Gnomonic.

Orthographic Projection, is that in which the surface of the sphere is drawn upon a plane, cutting it in the middle; the eye being placed at an infinite distance vertically to one of the hemispheres. And

Stereographic Projection of the sphere, is that in which the surface and circles of the sphere are drawn upon the plane of a great circle, the eye being in the pole of that circle.

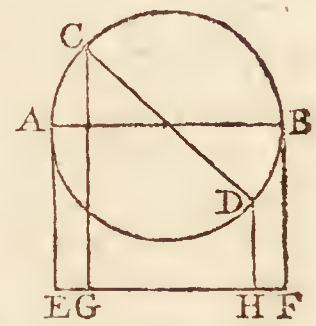
Gnomonical Projection *of the Sphere*, is that in which the surface of the sphere is drawn upon a plane without side of it, commonly touching it, the eye being at the centre of the sphere. See Gnomonical *Projection*.

Laws of the Orthographic Projection.

1. The rays coming from the eye, being at an infinite distance, and making the Projection, are parallel to each other, and perpendicular to the plane of Projection.

2. A right line perpendicular to the plane of Projection, is projected into a point, where that line meets the said plane.

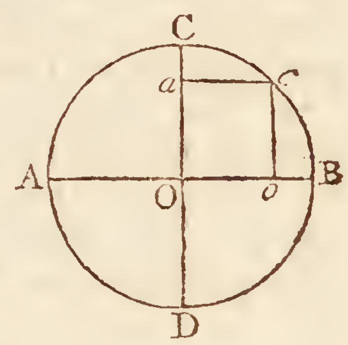
3. A right line, as AB, or CD, not perpendicular, but either parallel or oblique to the plane of the Projection, is projected into a right line, as EF or GH, and is always comprehended between the extreme perpendiculars AE and BF, or CG and DH.



4. The Projection of the right line AB is the greatest, when AB is parallel to the plane of the Projection.

5. Hence it is evident, that a line parallel to the plane of the Projection, is projected into a right line equal to itself; but a line that is oblique to the plane of Projection, is projected into one that is less than itself.

6. A plane surface, as ACBD, perpendicular to the plane of the Projection, is projected into the right line, as AB, in which it cuts that plane. —Hence it is evident, that the circle ACBD perpendicular to the plane of Projection, passing through its centre, is projected into that diameter AB in which it cuts the plane of the Projection. Also any arch as C*c* is projected into O*o*, equal to *ca*, the right sine of that arch; and the complemental arc *c*B is projected into *o*B, the versed sine of the same arc *c*B.



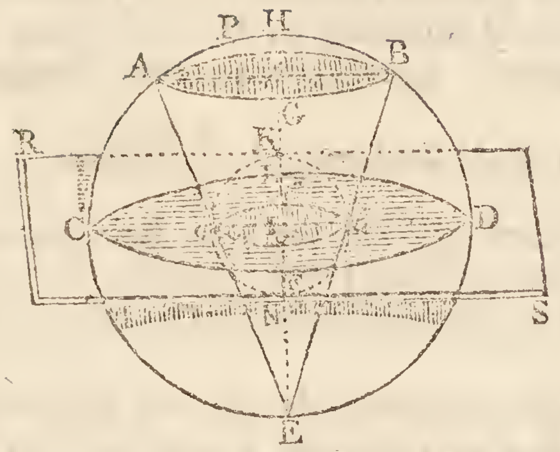
7. A circle parallel to the plane of the Projection, is projected into a circle equal to itself, having its centre the same with the centre of the Projection, and its radius equal to the cosine of its distance from the plane. And a circle oblique to the plane of the Projection, is projected into an ellipsis, whose greater axis is equal to the diameter of the circle, and its less axis equal to double the cosine of the obliquity of the circle, to a radius equal to half the greater axis.

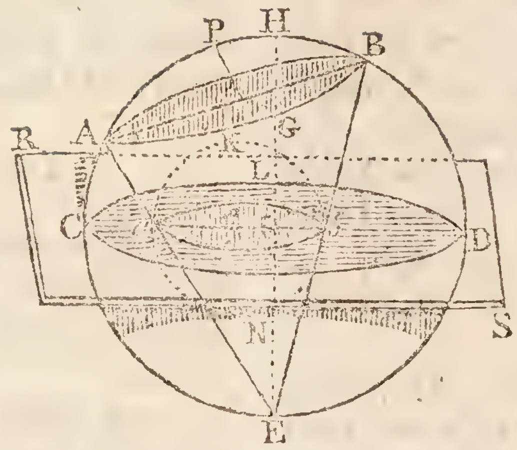
Properties of the Stereographic Projection.

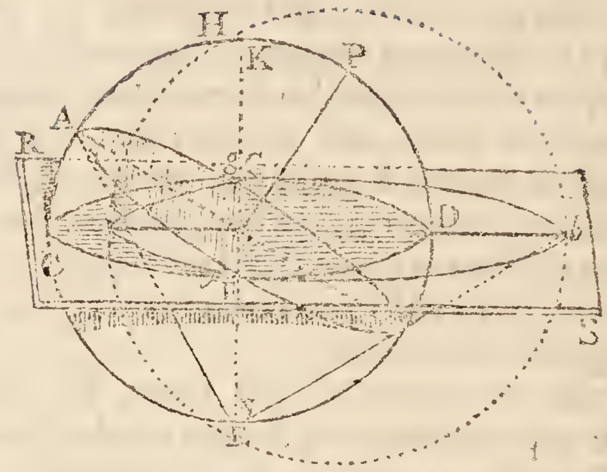
1. In this Projection a right circle, or one perpendicular to the plane of Projection, and passing through the eye, is projected into a line of half tangents.

2. The Projection of all other circles, not passing through the projecting point, whether parallel or oblique, are projected into circles.

Thus,

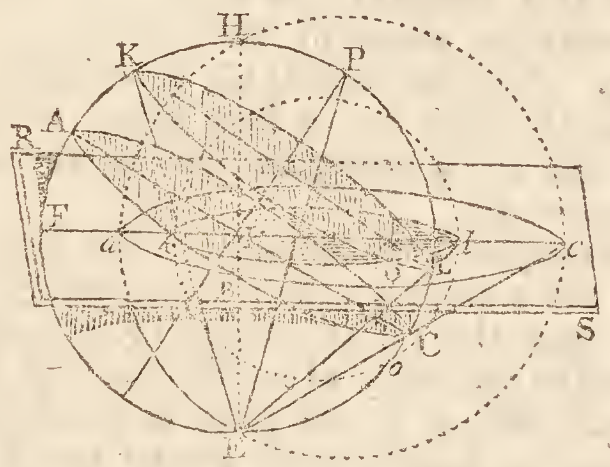






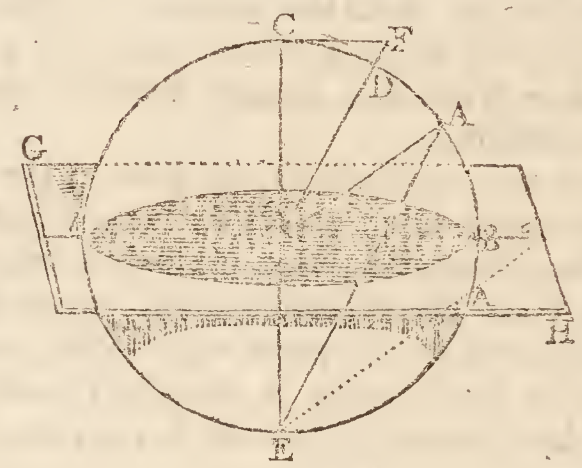
Thus, let ACEDB repreſent a ſphere, cut by a plane RS, paſſing through the centre I, perpendicular to the diameter EH, drawn from E the place of the eye; and let the ſection of the ſphere by the plane RS be the circle CFDL, whoſe poles are H and E. Suppoſe now AGB is a circle on the ſphere to be projected, whoſe pole moſt remote from the eye is P; and the viſual rays from the circle AGB meeting in E, form the cone AGBE, of which the triangle AEB is a ſection through the vertex E, and diameter of the baſe AB: then will the figure *agbf*, which is the Projection of the circle AGB, be itſelf a circle. Hence, the middle of the projected diameter is the centre of the projected circle, whether it be a great circle or a ſmall one: Alſo the poles and centres of all circles, parallel to the plane of Projection, fall in the centre of the Projection: And all oblique great circles cut the primitive circle in two points diametrically oppoſite.

2. The projected diameter of any circle ſubtends an angle at the eye equal to the diſtance of that circle from its neareſt pole, taken on the ſphere; and that angle is biſected by a right line joining the eye and that pole. Thus, let the plane RS cut the ſphere HFEG through



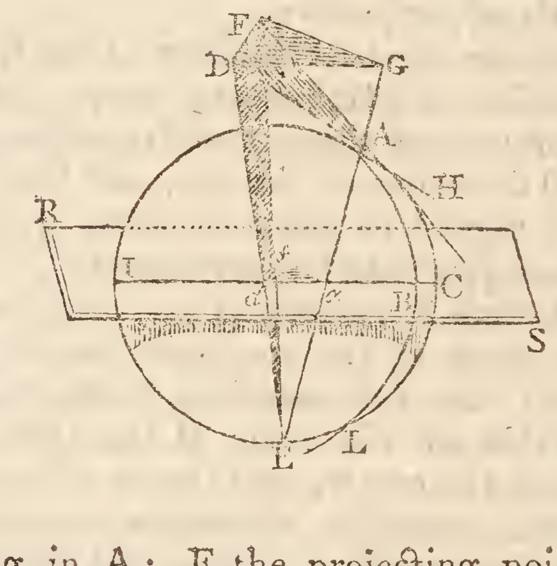
its centre I; and let ABC be any oblique great circle, whoſe diameter AC is projected into *ac*; and KOL any ſmall circle parallel to ABC, whoſe diameter KL is projected in *kl*. The diſtances of thoſe circles from their pole P, being the arcs AHP, KHP; and the angles *a*E*c*, *k*E*l*, are the angles at the eye, ſubtended by their projected diameters, *ac* and *kl*. Then is the angle *a*E*c* meaſured by the arc AHP, and the angle *k*E*l* meaſured by the arc KHP; and thoſe angles are biſected by EP.

3. Any point of a ſphere is projected at ſuch a diſtance from the centre of Projection, as is equal to the tangent of half the arc intercepted between that point and the pole oppoſite to the eye, the ſemidiameter of the ſphere being radius. Thus, let C*b*EB be a great circle of the ſphere, whoſe centre is *c*, GH the plane of Projection cutting the diameter of the ſphere in *b* and B; alſo E and C the poles of the ſection by that plane; and *a* the projection of A. Then *ca* is equal



the tangent of half the arc AC, as is evident by drawing CF = the tangent of half that arc, and joining *c*F.

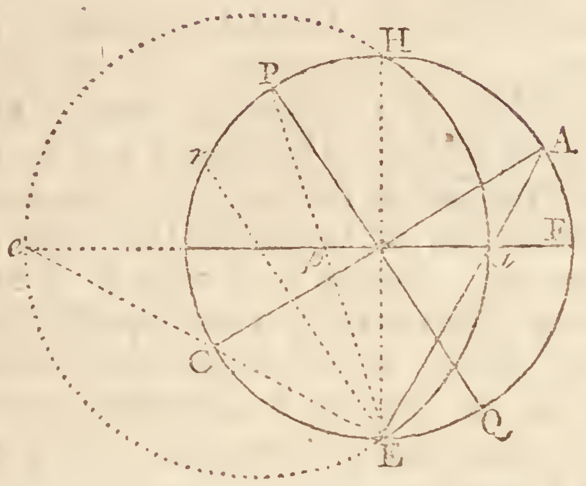
4. The angle made by two projected circles, is equal to the angle which theſe circles make on the ſphere. For let IACE and ABL be two circles on a ſphere



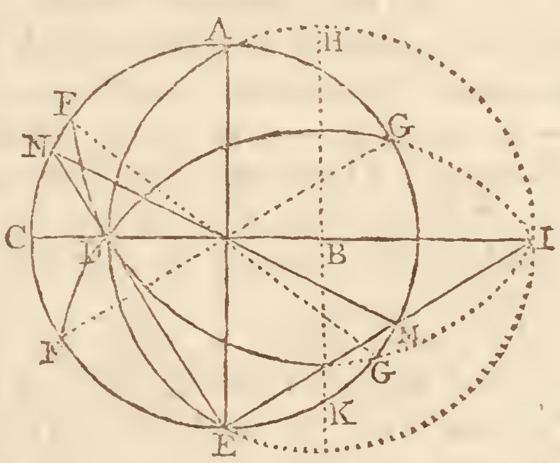
interſecting in A; E the projecting point; and RS the plane of Projection, in which the point A is projected in *a*, in the line IC, the diameter of the circle ACE. Alſo let DH and FA be tangents to the circles ACE and ABL. Then will the projected angle *daf* be equal to the ſpherical angle BAC.

5. The diſtance between the poles of the primitive circle and an oblique circle, is equal to the tangent of half the inclination of thoſe circles; and the diſtance of their centres, is equal to the tangent of their inclination; the ſemidiameter of the primitive being radius. For let AC be the diameter of a circle, whoſe poles are P and Q, and inclined to the plane of Projection in the angle AIF; and let *a*, *c*, *p* be the Projections of the points A, C, P; alſo let H*a*E be the projected oblique circle, whoſe centre is *q*. Now when the plane of Projection becomes the primitive circle, whoſe pole is I; then is I*p* = tangent of half the angle AIF, or of half the

the arch AF; and Iq = tangent of AF, or of the angle FHa = AIF.

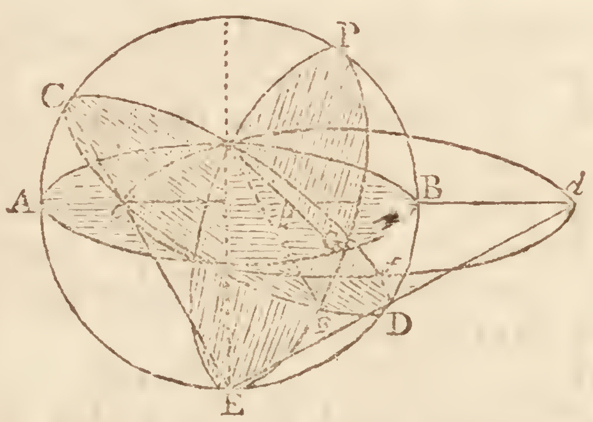


6. If through any given point in the primitive circle, an oblique circle be deſcribed; then the centres of all other oblique circles paſſing through that point, will be in a right line drawn through the centre of the firſt oblique circle, and perpendicular to a line paſſing through that centre, the given point, and the centre of the primitive circle.



Thus, let GACE be the primitive circle, ADEI a great circle deſcribed through D, its centre being B. HK is a right line drawn through B perpendicular to a right line CI paſſing through D and B and the centre of the primitive circle. Then the centres of all other great circles, as FDG, paſſing through D, will fall in the line HK.

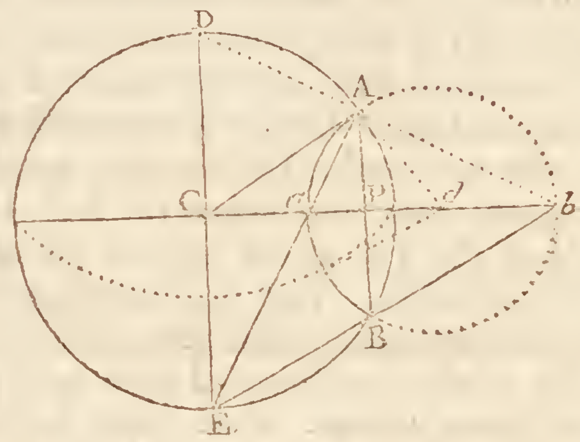
7. Equal arcs of any two great circles of the ſphere will be intercepted between two other circles drawn on the ſphere through the remoteſt poles of thoſe great circles. For let PBEA be a ſphere, on which AGB and



CFD are two great circles, whoſe remoteſt poles are E and P; and through theſe poles let the great circle PBEC and the ſmall circle PGE be drawn, cutting the great circles AGB and CFD in the points B, G, D, F. Then are the intercepted arcs BG and DF equal to one another.

8. If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle and plane of Projection; the intercepted arcs of thoſe peripheries are equal; that is, the arc BG = df.

9. The radius of any leſſer circle, whoſe plane is perpendicular to that of the primitive circle, is equal to the tangent of that leſſer circle's diſtance from its pole; and the ſecant of that diſtance is equal to the diſtance of the centres of the primitive and leſſer circle. For let P be the pole, and AB the diameter of a leſſer circle, its plane being perpendicular to that of the primi-



tive circle, whoſe centre is C: then d being the centre of the projected leſſer circle, da is equal to the tangent of the arc PA, and dC = the ſecant of PA. See STEREOGRAPHIC *Projection*.

Mercator's PROJECTION. See MERCATOR and CHART.

PROJECTION of Globes, &c. See GLOBE, &c.

Polar PROJECTION. See POLAR.

PROJECTION *of Shadows*. See SHADOW.

PROJECTION, or PROJECTURE, in Building, the outjetting or prominency which the mouldings and members have, beyond the plane or naked of the wall, column, &c.

Monſtrous PROJECTION. See ANAMORPHOSIS.

PROJECTIVE *Dialling*, a manner of drawing the hour lines, the furniture &c of dials, by a method of projection on any kind of ſurface whatever, without regard to the ſituation of thoſe ſurfaces, either as to declination, reclination, or inclination. See DIALLING.

PROLATE, or OBLONG *Spheroid*, is a ſpheroid produced by the revolution of a ſemiellipſis about its longer diameter; being longeſt in the direction of that axis, and reſembling an egg, or a lemon.

It is ſo called in oppoſition to the oblate or ſhort ſpheroid, which is formed by the rotation of a ſemiellipſis about its ſhorter axis; being therefore ſhorteſt in the direction of its axis, or flatted at the poles, and ſo reſembling an orange, or perhaps a turnip, according to the degree of flatneſs; and which is alſo the figure of the earth we inhabit, and perhaps of the planets alſo; having their equatorial diameter longer than the polar. See SPHEROID.

PROMONTORY, in Geography, is a rock or high point of land projecting out into the ſea. The extremity of which towards the ſea is uſually called a Cape, or Headland.

PROPORTION, in Arithmetic &c, the equality or

or similitude of ratios. As the four numbers 4, 8, 15, 30 are proportionals, or in proportion, because the ratio of 4 to 8 is equal or similar to the ratio of 15 to 30, both of them being the same as the ratio of 1 to 2.

Euclid, in the 5th definition of the 5th book, gives a general definition of four proportionals, or when, of four terms, the first has the same ratio to the 2d, as the 3d has to the 4th, viz, when any equimultiples whatever of the first and third being taken, and any equimultiples whatever of the 2d and 4th; if the multiple of the first be less than that of the 2d, the multiple of the 3d is also less than that of the 4th; or if the multiple of the first be equal to that of the 2d, the multiple of the 3d is also equal to that of the 4th; or if the multiple of the first be greater than that of the 2d, the multiple of the 3d is also greater than that of the 4th. And this definition is general for all kinds of magnitudes or quantities whatever, though a very obscure one.

Also, in the 7th book, Euclid gives another definition of proportionals, viz, when the first is the same equimultiple of the 2d, as the 3d is of the 4th, or the same part or parts of it. But this definition appertains only to numbers and commensurable quantities.

Proportion is often confounded with ratio; but they are quite different things. For, ratio is properly the relation of two magnitudes or quantities of one and the same kind; as the ratio of 4 to 8, or of 15 to 30, or of 1 to 2; and so implies or respects only two terms or things. But Proportion respects four terms or things, or two ratios which have each two terms. Though the middle term may be common to both ratios, and then the Proportion is expressed by three terms only, as 4, 8, 64, where 4 is to 8 as 8 to 64.

Proportion is also sometimes confounded with progression. In fact, the two often coincide; the difference between them only consisting in this, that progression is a particular species of Proportion, being indeed a continued Proportion, or such as has all the terms in the same ratio, viz, the 1st to the 2d, the 2d to the 3d, the 3d to the 4th, &c; as the terms 2, 4, 8, 16, &c; so that progression is a series or continuation of Proportions.

Proportion is either continual, or discrete or interrupted.

The Proportion is continual when every two adjacent terms have the same ratio, or when the consequent of each ratio is the antecedent of the next following ratio, and so all the terms form a progression; as 2, 4, 8, 16, &c; where 2 is to 4 as 4 to 8, and as 8 to 16, &c.

Discrete or interrupted Proportion, is when the consequent of the first ratio is different from the antecedent of the 2d, &c; as 2, 4, and 3, 6.

Proportion is also either Direct or Inverse.

Direct Proportion is when more requires more, or less requires less. As it will require more men to perform more work, or fewer men for less work, in the same time.

Inverse or *Reciprocal* Proportion, is when more requires less, or less requires more. As it will require more men to perform the same work in less time, or fewer men in more time. Ex. If 6 men can perform a piece of work in 15 days, how many men can do the same in 10 days. Then,

reciprocally - as $\frac{1}{15}$ to $\frac{1}{10}$ so is 6 : 9 } the
or inversely - as 10 to 15 so is 6 : 9 } answer.

Proportion, again, is distinguished into Arithmetical, Geometrical, and Harmonical.

Arithmetical Proportion is the equality of two arithmetical ratios, or differences. As in the numbers 12, 9, 6; where the difference between 12 and 9, is the same as the difference between 9 and 6, viz 3.

And here the sum of the extreme terms is equal to the sum of the means, or to double the single mean when there is but one. As $12 + 6 = 9 + 9 = 18$.

Geometrical Proportion is the equality between two geometrical ratios, or between the quotients of the terms. As in the three 9, 6, 4, where 9 is to 6 as 6 is to 4, thus denoted $9 : 6 :: 6 : 4$; for $\frac{9}{6} = \frac{6}{4}$, being each equal $\frac{3}{2}$ or $1\frac{1}{2}$.

And in this Proportion, the rectangle or product of the extreme terms, is equal to that of the two means, or the square of the single mean when there is but one. For $9 \times 4 = 6 \times 6 = 36$.

Harmonical Proportion, is when the first term is to the third, as the difference between the 1st and 2d is to the difference between the 2d and 3d; or in four terms when the 1st is to the 4th, as the difference between the 1st and 2d is to the difference between the 3d and 4th; or the reciprocals of an arithmetical Proportion are in harmonical Proportion. As 6, 4, 3; because $6 : 3 :: 6 - 4 = 2 : 4 - 3 = 1$; or because $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ are in arithmetical Proportion, making $\frac{1}{6} + \frac{1}{3} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Also the four 24, 16, 12, 9 are in harmonical Proportion, because $24 : 9 :: 8 : 3$.

See Proportionals.

Compass of Proportion, a name by which the French, and some English authors, call the Sector.

Rule of Proportion, in Arithmetic, a rule by which a 4th term is found in Proportion to three given terms. And is popularly called the Golden Rule, or Rule of Three.

PROPORTIONAL, relating to Proportion. As, Proportional Compasses, Parts, Scales, Spirals, &c. See the several terms.

Proportional *Compasses*, are compasses with two pair of opposite legs, like a St. Andrew's cross, by which any space is enlarged or diminished in any proportion.

Proportional *Part*, is a part of some number that is analogous to some other part or number; such as the Proportional parts in the logarithms, and other tables.

Proportional *Scales*, called also Logarithmic Scales, are the logarithms, or artificial numbers, placed on lines, for the ease and advantage of multiplying and dividing &c, by means of compasses, or of sliding rulers. These are in effect so many lines of numbers, as they are called by Gunter, but made single, double, triple, or quadruple; beyond which they seldom go. See Gunter's *Scale*, Scale, &c.

Proportional *Spiral*. See Spiral.

PROPORTIONALITY, the quality of Proportionals. This term is used by Gregory St. Vincent, for the proportion that is between the exponents of four ratios.

PROPORTIONALS, are the terms of a proportion; consisting of two extremes, which are the first

 and

and laſt terms of the ſet, and the means, which are the reſt of the terms. Theſe Proportionals may be either arithmeticals, geometricals, or harmonicals, and in any number above two, and alſo either continued or diſcontinued.

Pappus gives this beautiful and ſimple compariſon of the three kinds of Proportionals, arithmetical, geometrical, and harmonical, viz, a, b, c being the firſt, ſecond and third terms in any ſuch proportion, then

$$\left.\begin{array}{ll}\text{In the arithmeticals,} & a : a \\ \text{in the geometricals,} & a : b \\ \text{in the harmonicals,} & a : c\end{array}\right\} :: a - b : b - c.$$

See MEAN *Proportional.*

Continued Proportionals form what is called a progreſſion; for the properties of which ſee PROGRESSION.

I. *Properties of Arithmetical* PROPORTIONALS.

(For what reſpects Progreſſions and Mean Proportionals of all ſorts, ſee MEAN, and PROGRESSION.)

1. Four Arithmetical Proportionals, as 2, 3, 4, 5,
are ſtill Proportionals when inverſely, 5, 4, 3, 2;
or alternately, thus, - - 2, 4, 3, 5;
or inverſely and alternately, thus - 5, 3, 4, 2.

2. If two Arithmeticals be added to the like terms of other two Arithmeticals, of the ſame difference or arithmetical ratio, the ſums will have double the ſame difference or arithmetical ratio.

So, to	3 and 5,	whoſe difference is	2,
add	7 and 9,	whoſe difference is alſo	2,
the ſums	10 and 14	have a double diff. viz	4.

And if to theſe ſums be added two other numbers alſo in the ſame difference, the next ſums will have a triple ratio or difference; and ſo on. Alſo, whatever be the ratios of the terms that are added, whether the ſame or different, the ſums of the terms will have ſuch arithmetical ratio as is compoſed of the ſums of the others that are added.

So	3,	5,	whoſe dif. is 2
and	7,	10,	whoſe dif. is 3
and	12	16,	whoſe dif. is 4
make	22,	31,	whoſe dif. is 9.

On the contrary, if from two Arithmeticals be ſubtracted others, the difference will have ſuch arithmetical ratio as is equal to the differences of thoſe.

So from	12 and	16,	whoſe dif. is	4
take	7 and	10,	whoſe dif. is	3
leaves	5 and	6,	whoſe dif. is	1
Alſo from	7 and	9,	whoſe dif. is	2
take	3 and	5,	whoſe dif. is	2
leaves	4 and	4,	whoſe dif. is	0

3. Hence, if Arithmetical Proportionals be multiplied or divided by the ſame number, their difference, or arithmetical ratio, is alſo multiplied or divided by the ſame number.

II. *Properties of Geometrical Proportionals.*

The properties relating to mean Proportionals are given under the term MEAN *Proportional;* ſome are alſo given under the article Proportion; and ſome additional ones are as below:

1. To find a 3d Proportional to two given numbers, or a 4th Proportional to three: In the former caſe, multiply the 2d term by itſelf, and divide the product by the 1ſt: and in the latter caſe, multiply the 2d term by the 3d, and divide the product by the 1ſt.

So $2 : 6 :: 6 : 18$, the 3d prop. to 2 and 6:
and $2 : 6 :: 5 : 15$, the 4th prop. to 2, 6, and 5.

2. If the terms of any geometrical ratio be augmented or diminiſhed by any others in the ſame ratio, or proportion, the ſums or differences will ſtill be in the ſame ratio or proportion.

So if $a : b :: c : d$,
then is $a : b :: a \pm c : b \pm d :: c : d$.

And if the terms of a ratio, or proportion, be multiplied or divided by any one and the ſame number, the products and quotients will ſtill be in the ſame ratio, or proportion.

Thus, $a : b :: na : nb :: \frac{a}{n} : \frac{b}{n}$.

3. If a ſet of continued Proportionals be either augmented or diminiſhed by the ſame part or parts of themſelves, the ſums or differences will alſo be Proportionals.

Thus if a, b, c, d, &c be Propors.
then are $a \pm \frac{a}{n}$, $b \pm \frac{b}{n}$, $c \pm \frac{c}{n}$, &c alſo Propors.

where the common ratio is $1 \pm \frac{1}{n}$.

And if any ſingle quantity be either augmented or diminiſhed by ſome part of itſelf, and the reſult be alſo increaſed or diminiſhed by the ſame part of itſelf, and this third quantity treated in the ſame manner, and ſo on; then ſhall all theſe quantities be continued Proportionals. So, beginning with the quantity a, and taking always the nth part, then ſhall

a, $a \pm \frac{a}{n}$, $a \pm \frac{2a}{n} + \frac{a^2}{n^2}$, &c be Proportionals,

or a, $a \pm \frac{a}{n}$, $(a \pm \frac{a}{n})^2$, $(a \pm \frac{a}{n})^3$, &c Propors.

the common ratio being $1 \pm \frac{a}{n}$.

4. If one ſet of Proportionals be multiplied or divided by any other ſet of Proportionals, each term by each, the products or quotients will alſo be Proportionals.

Thus, if $a : na :: b : nb$,
and $c : mc :: d : md$;
then is $ac : mnac :: bd : mnbd$,
and $\frac{a}{c} : \frac{na}{mc} :: \frac{b}{d} : \frac{nb}{mb}$.

5. If there be ſeveral continued Proportionals, then whatever ratio the 1ſt has to the 2d, the 1ſt to the 3d

ſhall have the duplicate of the ratio, the 1ſt to the 4th the triplicate of it, and ſo on.

So in $a,\ na,\ n^2a,\ n^3a$, &c, the ratio being n;
then $a : n^2a$, or 1 to n^2, the duplicate ratio,
and $a : n^3a$, or 1 to n^3, the triplicate ratio,
and ſo on.

6. In three continued Proportionals, the difference between the 1ſt and 2d term, is a mean Proportional between the 1ſt term and the ſecond difference of all the terms.

Thus, in the three Propor. $a,\ na,\ n^2a$;

Terms	1ſt difs.	2d dif.
n^2a		
	$n^2a - na$	
na		
	$na - a$	$n^2a - 2na + a$,
a		

then $a : na - a :: na - a : n^2a - 2na + a$.

Or in the numbers 2, 6, 18;

18		
	12	
6		8 the 2d difference;
	4	
2		

then 2, 4, 8 are Proportionals.

7. When four quantities are in proportion, they are alſo in proportion by inverſion, compoſition, diviſion, &c; thus, $a,\ na,\ b,\ nb$ being in proportion, viz,

1	$a : na :: b : nb$; then by	
2. Inverſion	$na : a :: nb : b$;	
3. Alternation	$a : b :: na : nb$;	
4. Compoſition	$a + na : na :: b + nb : nb$;	
5. Converſion	$a + na : a :: b + nb : b$;	
6. Diviſion	$\begin{cases} a - na : a :: b - nb : b; \\ a - na : na :: b - nb : nb. \end{cases}$	

III. *Properties of Harmonical Proportionals.*

1. If three or four numbers in Harmonical Proportion, be either multiplied or divided by any number, the products or quotients will alſo be Harmonical Proportionals.

Thus, 6, 3, 2 being harmon. Propor.
then 12, 6, 4 are alſo harmon. Propor.
and $\frac{6}{2},\ \frac{3}{2},\ \frac{2}{2}$ are alſo harmon. Propor.

2. In the three Harmonical Proportionals $a,\ b,\ c$, when any two of theſe are given, the 3d can be found from the definition of them, viz, that $a : c :: a - b : b - c$; for hence

$b = \dfrac{2ac}{a+c}$ the harmonical mean, and

$c = \dfrac{ab}{2a-b}$ the 3d harmon. to a and b.

3. And of the four Harmonicals, $a,\ b,\ c,\ d$, any three being given, the fourth can be found from the definition of them, viz, that $a : d :: a - b : c - d$; for thence the three $b,\ c,\ d$, will be thus found, viz,

$$b = \frac{2ad - ac}{d};\quad c = \frac{2ad - bd}{a};\quad d = \frac{ac}{2a - b}.$$

4. If there be four numbers diſpoſed in order, as 2, 3, 4, 6, of which one extreme and the two middle terms are in Arithmetical Proportion, and the other extreme and the ſame middle terms are in Harmonical Proportion; then are the four terms in Geometrical Proportion: ſo here

the three 2, 3, 4 are arithmeticals,
and the three 3, 4, 6 are harmonicals,
then the four 2, 3, 4, 6 are geometricals.

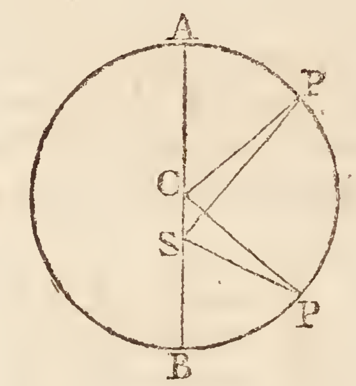
5. If between any two numbers, as 2 and 6, there be interpoſed an arithmetical mean 4, and alſo a harmonical mean 3, the four will then be geometricals, viz, $2 : 3 :: 4 : 6$.

6. Between the three kinds of proportion, there is this remarkable difference; viz, that from any given number there can be raiſed a continued arithmetical ſeries increaſing ad infinitum, but not decreaſing; while the harmonical can be decreaſed ad infinitum, but not increaſed; and the geometrical admits of both.

PROPOSITION, is either ſome truth advanced, and ſhewn to be ſuch by demonſtration; or ſome operation propoſed, and its ſolution ſhewn. In ſhort, it is ſomething propoſed either to be demonſtrated, or to be done or performed. The former is a theorem, and the latter is a problem.

PROSTHAPHERESIS, in Aſtronomy, the difference between the true and mean motion, or between the true and mean place, of a planet, or between the true and equated anomaly; called alſo Equation of the Orbit, or Equation of the Centre, or ſimply the Equation; and it is equal to the angle formed at the planet, and ſubtended by the excentricity of its orbit.

Thus, if S be the ſun, and P the place of a planet in its orbit APB, whoſe centre is C.



Then the mean anomaly is the ∠ ACP,
and the true anomaly the ∠ ASP,
the difference of which is the ∠ CPS,
which is the Proſthapheréſis; which is ſo called, becauſe it is ſometimes to be added to, and ſometimes to be ſubtracted from the mean motion, to give the true one; as is evident from the figure.

PROTRACTING, or PROTRACTION, in Surveying, the act of plotting or laying down the dimenſions taken in the field, by means of a Protractor, &c: Protracting makes one part of ſurveying.

PROTRACTING-*Pin*, a fine pointed pin, or needle, fitted into a handle, uſed to prick off degrees and minutes from the limb of the Protractor.

PROTRACTOR, a mathematical inſtrument, chiefly uſed in ſurveying, for laying down angles upon paper, &c.

The ſimpleſt, and moſt natural Protractor conſiſts of a ſemicircular limb ADB (fig. 7, pl. xix.) commonly of metal, divided into 180°, and ſubtended by a diameter AB; in the middle of which is a ſmall notch C,

called the centre of the Protractor. And for the convenience of reckoning both ways, the degrees are numbered from the left hand towards the right, and from the right hand towards the left.

But this instrument is made much more commodious by transferring the divisions from the circumference to the edge of a ruler, whose side EF is parallel to AB, which is easily done by laying a ruler on the centre C, and over the several divisions on the semicircumference ADB, and marking the intersections of that ruler on the line EF: so that a ruler with these divisions marked on one of its sides as above, and returned down the two ends, and numbered both ways as in the circular Protractor, the fourth or blank side representing the diameter of the circle, is both a more useful form than the circular Protractor, and better adapted for putting into a case.

To make any Angle with the Protractor.—Lay the diameter of the Protractor along the given line which is to be one side of the angle, and its centre at the given angular point; then make a mark opposite the given degree of the angle found on the limb of the instrument, and, removing the Protractor, by a plane ruler laid over that point and the centre, draw a line, which will form the angle sought.

In the same way is any given angle measured, to find the number of degrees it contains.

This Protractor is also very useful in drawing one line perpendicular to another; which is readily done by laying the Protractor across the given line, so that both its centre and the 90th degree on the opposite edge fall upon the line, also one of the edges passing over the given point, by which then let the perpendicular be drawn.

The Improved PROTRACTOR is an instrument much like the former, only furnished with a little more apparatus, by which an angle may be set off to a single minute.

The chief addition is an index attached to the centre, about which it is moveable, so as to play freely and steadily over the limb. Beyond the limb the index is divided, on both edges, into 60 equal parts of the portions of circles, intercepted by two other right lines drawn from the centre, so that each makes an angle of one degree with lines drawn to the assumed points from the centre.

To set off an angle of any number of degrees and minutes with this Protractor, move the index, so that one of the lines drawn on the limb, from one of the forementioned points, may fall upon the number of degrees given; and prick off as many of the equal parts on the proper edge of the index as there are minutes given; then drawing a line from the centre to that point so pricked off, the required angle is thus formed with the given line or diameter of the Protractor.

PROVING *of Gunpowder.* See EPROUVETTE, and GUNPOWDER.

PSEUDO-STELLA, any kind of meteor or phenomenon, appearing in the heavens, and resembling a star.

PTOLEMAIC, or PTOLOMAIC, something relating to Ptolomy; as the Ptolomaic System, the Ptolomaic Sphere, &c. See SYSTEM, SPHERE, &c.

PTOLEMY, or PTOLOMY, (CLAUDIUS), a very celebrated geographer, astronomer, and mathematician, among the Ancients, was born at Pelusium in Egypt, about the 70th year of the Christian era; and died, it has been said, in the 78th year of his age, and in the year of Christ 147. He taught astronomy at Alexandria in Egypt, where he made many astronomical observations, and composed his other works. It is certain that he flourished in the reigns of Marcus Antoninus and Adrian: for it is noted in his Canon, that Antoninus Pius reigned 23 years, which shews that he himself survived him; he also tells us in one place, that he made a great many observations upon the fixed stars at Alexandria, in the second year of Antoninus Pius; and in another, that he observed an eclipse of the moon, in the ninth year of Adrian; from which it is reasonable to conclude that this astronomer's observations upon the heavens were many of them made between the year 125 and 140.

Ptolomy has always been reckoned the prince of astronomers among the Ancients, and in his works has left us an entire body of that science. He has preserved and transmitted to us the observations and principal discoveries of the Ancients, and at the same time augmented and enriched them with his own. He corrected Hipparchus's catalogue of the fixed stars; and formed tables, by which the motions of the sun, moon, and planets, might be calculated and regulated. He was indeed the first who collected the scattered and detached observations of the Ancients, and digested them into a system; which he set forth in his Μεγαλη Συνταξις, sive *Magna Constructio*, divided into 13 books. He adopts and exhibits here the ancient system of the world, which placed the earth in the centre of the universe; and this has been called from him, the Ptolomaic System, to distinguish it from those of Copernicus and Tycho Brahe.

About the year 827 this work was translated by the Arabians into their language, in which it was called *Almagestum*, by order of one of their kings; and from Arabic into Latin, about 1230, by the encouragement of the emperor Frederic the 2d. There were also other versions from the Arabic into Latin; and a manuscript of one, done by Girardus Cremonensis, who flourished about the middle of the 14th century, Fabricius says, is still extant in the library of All Souls College in Oxford. The Greek text of this work began to be read in Europe in the 15th century; and was first published by Simon Grynæus at Basil, 1538, in folio, with the eleven books of commentaries by Theon, who flourished at Alexandria in the reign of the elder Theodosius. In 1541 it was reprinted at Basil, with a Latin version by George Trapezond; and again at the same place in 1551, with the addition of other works of Ptolomy, and Latin versions by Camerarius. We learn from Kepler, that this last edition was used by Tycho.

Of this principal work of the ancient astronomers, it may not be improper to give here a more particular account. In general, it may be observed, that the work is founded upon the hypothesis of the earth's being at rest in the centre of the universe, and that the heavenly bodies, the stars and planets, all move around it in solid orbs, whose motions are all directed by one, which Ptolomy

lomy called the *Primum Mobile*, or Firſt Mover, of which he diſcourſes at large. But, to be more particular, this great work is divided into 13 books.

In the firſt book, Ptolomy ſhews, that the earth is in the centre of thoſe orbs, and of the univerſe itſelf, as he underſtood it: he repreſents the earth as of a ſpherical figure, and but as a point in compariſon of the reſt of the heavenly bodies: he treats concerning the ſeveral circles of the earth, and their diſtances from the equator; as alſo of the right and oblique aſcenſion of the heavenly bodies in a right ſphere.

In the 2d book, he treats of the habitable parts of the earth; of the elevation of the pole in an oblique ſphere, and the various angles which the ſeveral circles make with the horizon, according to the different latitude of places; alſo of the phenomena of the heavenly bodies depending on the ſame.

In the 3d book, he treats of the quantity of the year, and of the unequal motion of the ſun through the zodiac: he here gives the method of computing the mean motion of the ſun, with tables of the ſame; and likewiſe treats of the inequality of days and nights.

In the 4th book, he treats of the lunar motions, and their various phenomena: he gives tables for finding the moon's mean motions, with her latitude and longitude: he diſcourſes largely concerning lunar epicycles; and by comparing the times of a great number of eclipſes, mentioned by Hipparchus, Calippus, and others, he has computed the places of the ſun and moon, according to their mean motions, from the firſt year of Nabonazar, king of Egypt, to his own time.

In the 5th book, he treats of the inſtrument called the Aſtrolabe: he treats alſo of the eccentricity of the lunar orbit, and the inequality of the moon's motion, according to her diſtance from the ſun: he alſo gives tables, and an univerſal canon for the inequality of the lunar motions: he then treats of the different aſpects or phaſes of the moon, and gives a computation of the diameter of the ſun and moon, with the magnitude of the ſun, moon and earth compared together; he ſtates alſo the different meaſures of the diſtance of the ſun and moon, according as they are determined by ancient mathematicians and philoſophers.

In the 6th book, he treats of the conjunctions and oppoſitions of the ſun and moon, with tables for computing the mean time when they happen; of the boundaries of ſolar and lunar eclipſes; of the tables and methods of computing the eclipſes of the ſun and moon, with many other particulars.

In the 7th book, he treats of the fixed ſtars; and ſhews the methods of deſcribing them, in their various conſtellations, on the ſurface of an artificial ſphere or globe: he rectifies the places of the ſtars to his own time, and ſhews how different thoſe places were then, from what they had been in the times of Timocharis, Hipparchus, Ariſtillus, Calippus, and others: he then lays down a catalogue of the ſtars in each of the northern conſtellations, with their latitude, longitude, and magnitudes.

In the 8th book, he gives a like catalogue of the ſtars in the conſtellations of the ſouthern hemiſphere, and in the 12 ſigns or conſtellations of the zodiac. This is the firſt catalogue of the ſtars now extant, and forms the moſt valuable part of Ptolomy's works. He then treats of the galaxy, or milky-way; alſo of the planetary aſpects, with the riſing and ſetting of the ſun, moon, and ſtars.

In the 9th book, he treats of the order of the ſun, moon, and planets, with the periodical revolutions of the five planets; then he gives tables of the mean motions, beginning with the theory of Mercury, and ſhewing its various phenomena with reſpect to the earth.

The 10th book begins with the theory of the planet Venus, treating of its greateſt diſtance from the ſun; of its epicycle, eccentricity, and periodical motions: it then treats of the ſame particulars in the planet Mars.

The 11th book treats of the ſame circumſtances in the theory of the planets Jupiter and Saturn. It alſo corrects all the planetary motions from obſervations made from the time of Nabonazar to his own.

The 12th book treats of the retrogreſſive motion of the ſeveral planets; giving alſo tables of their ſtations, and of the greateſt diſtances of Venus and Mercury from the ſun.

The 13th book treats of the ſeveral hypotheſes of the latitude of the five planets; of the greateſt latitude, or inclination of the orbits of the five planets, which are computed and diſpoſed in tables; of the riſing and ſetting of the planets, with tables of them. Then follows a concluſion or winding up of the whole work.

This great work of Ptolomy will always be valuable on account of the obſervations he gives of the places of the ſtars and planets in former times, and according to ancient philoſophers and aſtronomers that were then extant; but principally on account of the large and curious catalogue of the ſtars, which being compared with their places at preſent, we thence deduce the true quantity of their ſlow progreſſive motion according to the order of the ſigns, or of the preceſſion of the equinoxes.

Another great and important work of Ptolomy was, his *Geography*, in 7 books; in which, with his uſual ſagacity, he ſearches out and marks the ſituation of places according to their latitudes and longitudes; and he was the firſt that did ſo. Though this work muſt needs fall far ſhort of perfection, through the want of neceſſary obſervations, yet it is of conſiderable merit, and has been very uſeful to modern geographers. Cellarius indeed ſuſpects, and he was a very competent judge, that Ptolomy did not uſe all the care and application which the nature of his work required; and his reaſon is, that the author delivers himſelf with the ſame fluency and appearance of certainty, concerning things and places at the remoteſt diſtance, which it was impoſſible he could know any thing of, that he does concerning thoſe which lay the neareſt to him, and fall the moſt under his cognizance. Salmaſius had before made ſome remarks to the ſame purpoſe upon this work of Ptolomy. The Greek text of this work was firſt publiſhed by itſelf at Baſil in 1533, in 4to: afterward with a Latin verſion and notes by Gerard Mercator at Amſterdam, 1605; which laſt edition was reprinted at the ſame place, 1618, in folio, with neat geographical tables, by Bertius.

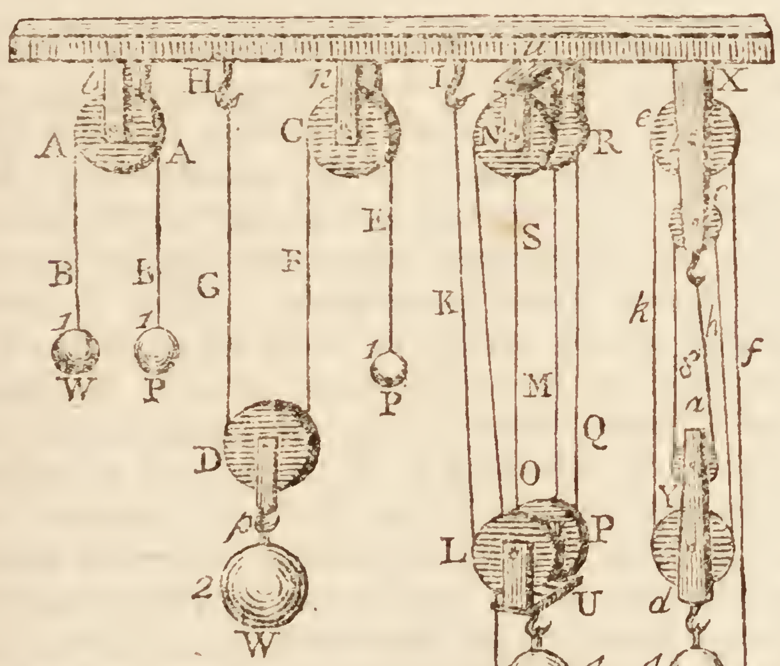
Other

Other works of Ptolomy, though leſs conſiderable than theſe two, are ſtill extant. As, *Libri quatuor de Judiciis Aſtrorum*, upon the firſt two books of which Cardan wrote a commentary.—*Fructus Librorum ſuorum*; a kind of ſupplement to the former work.—*Recenſio Chronologica Regum*: this, with another work of Ptolomy, *De Hypotheſibus Planetarum*, was publiſhed in 1620, 4to, by John Bainbridge, the Savilian profeſſor of Aſtronomy at Oxford: And Scaliger, Petavius, Dodwell, and the other chronological writers, have made great uſe of it.—*Apparentiæ Stellarum Inerrantium*: this was publiſhed at Paris by Petavius, with a Latin verſion, 1630, in folio; but from a mutilated copy, the defects of which have ſince been ſupplied from a perfect one, which Sir Henry Saville had communicated to archbiſhop Uſher, by Fabricius, in the 3d volume of his *Bibliotheca Græca*.—*Elementorum Harmonicorum libri tres*; publiſhed in Greek and Latin, with a commentary by Porphyry the philoſopher, by Dr. Wallis at Oxford, 1682, in 4to; and afterwards reprinted there, and inſerted in the 3d volume of Wallis's works, 1699, in folio.

Mabillon exhibits, in his *German Travels*, an effigy of Ptolomy looking at the ſtars through an optical tube; which effigy, he ſays, he found in a manuſcript of the 13th century, made by Conradus a monk. Hence ſome have fancied, that the uſe of the teleſcope was known to Conradus. But this is only matter of mere conjecture, there being no facts or teſtimonies, nor even probabilities, to ſupport ſuch an opinion.

It is rather likely that the tube was nothing more than a plain open one, employed to ſtrengthen and defend the eye-ſight, when looking at particular ſtars, by excluding adventitious rays from other ſtars and objects; a contrivance which no obſerver of the heavens can ever be ſuppoſed to have been without.

PULLEY, one of the five mechanical powers; conſiſting of a little wheel, being a circular piece of wood or metal, turning on an axis, and having a channel around it, in its edge or circumference, in which a cord ſlides and ſo raiſes up weights.



The Latins call it Trochlea; and the ſeamen, when fitted with a rope, a Tackle. An aſſemblage of ſeveral Pulleys is called a Syſtem of Pulleys, or Polyſpaſton: ſome of which are in a block or caſe, which is fixed; and others in a block which is moveable, and riſes with the weight. The wheel or rundle is called the Sheave or Shiver; the axis on which it turns, the Gudgeon; and the fixed piece of wood or iron, into which it is put, the Block.

Doctrine of the PULLEY.—1. If the equal weights P and W hang by the cord BB upon the pulley A, whoſe block *b* is fixed to the beam HI, they will counterpoiſe each other, juſt in the ſame manner as if the cord were cut in the middle, and its two ends hung upon the hooks fixed in the Pulley at A and A, equally diſtant from the centre.

Hence, a ſingle Pulley, if the lines of direction of the power and the weight be tangents to the periphery, neither aſſiſts nor impedes the power, but only changes its direction. The uſe of the Pulley therefore, is when the vertical direction of a power is to be changed into an horizontal one; or an aſcending direction into a deſcending one; &c. This is found a good proviſion for the ſafety of the workmen employed in drawing with the Pulley. And this change of direction by means of a Pulley has this farther advantage; that if any power can exert more force in one direction than another, we are hence enabled to employ it with its greateſt effect; as for the convenience of a horſe to draw in a horizontal direction, or ſuch like.

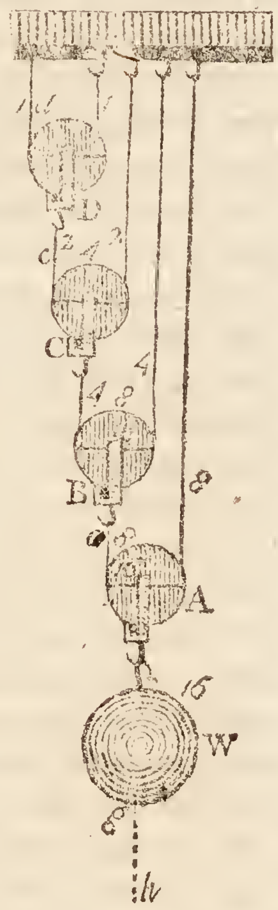
But the great uſe of the Pulley is in combining ſeveral of them together; thus forming what Vitruvius and others call Polyſpaſta; the advantages of which are, that the machine takes up but little room, is eaſily removed, and raiſes a very great weight with a moderate force.

2. When a weight W hangs at the lower end of the moveable block *p* of the Pulley D, and the chord GF goes under the Pulley, it is plain that the part G of the cord bears one half of the weight W, and the part F the other half of it; for they bear the whole between them; therefore whatever holds the upper end of either rope, ſuſtains one half of the weight; and thus the power P, which draws the cord F by means of the cord E, paſſing over the fixed pulley C, will ſuſtain the weight W when its intenſity is only equal to the half of W; that is, in the caſe of one moveable Pulley, the power gained is as 2 to 1, or as the number of ropes G and F to the one rope E.

In like manner, in the caſe of two moveable Pulleys P and L, each of theſe alſo doubles the power, and produces a gain of 4 to 1, or as the number of the ropes Q, M, S, K, ſuſtaining the weight W, to the 1 rope O ſuſtaining the power T; that is, W is to T as 4 to 1. And ſo on, for any number of moveable Pulleys, viz, 3 ſuch Pulleys producing an increaſe of power as 6 to 1; 4 Pulleys, as 8 to 1; &c; each power adding 2 to the number. Alſo the effect is the ſame, when the Pulleys are diſpoſed as in the fixed block X, and the other two as in the moveable block Y; theſe in the lower block giving the ſame advantage to the power, when they riſe all together in one block with the weight.

But if the lower Pulleys do not riſe all together in one block with the weight, but act upon one another, having the weight only faſtened to the loweſt of them, the force

force of the power is ſtill more increaſed, each power doubling the former numbers, the gain of power in this caſe proceeding in the geometrical progreſſion, 1, 2, 4, 8, 16, &c, according to the powers of 2; whereas in the former caſe, the gain was only in arithmetical progreſſion, increaſing by the addition of 2. Thus, a power whoſe intenſity is equal to 8lb applied at *a* will, by means of the lower Pulley A, ſuſtain 16lb; and a power equal to 4lb at *b*, by means of the Pulley, will ſuſtain the power of 8lb acting at *a*, and conſequently the weight of 16lb at W; alſo a third power equal to 2lb at *c*, by means of the Pulley C, will ſuſtain the power of 4lb at *b*; and a 4th power of 1lb at *d*, by means of the Pulley D, will ſuſtain the power 2 at *c*, and conſequently the power 4 at B, and the power 8 at A, and the weight 16 at W.

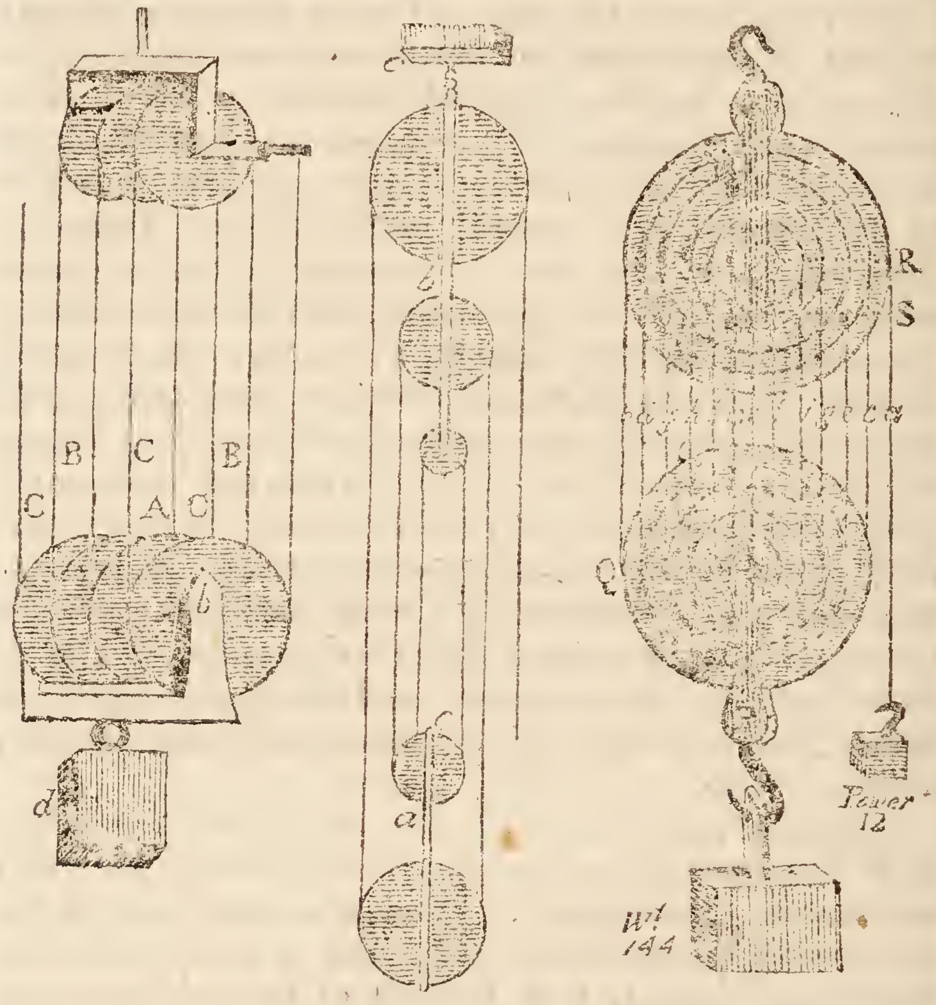


3. It is to be noted however, that, in whatever proportion the power is gained, in that very ſame proportion is the length of time increaſed to produce the ſame effect. For when a power moves a weight by means of ſeveral Pulleys, the ſpace paſſed over by the power is to the ſpace paſſed over by the weight, as the weight is to the power. Hence, the ſmaller a force is that ſuſtains a weight by means of Pulleys, the ſlower is the weight raiſed; ſo that what is ſaved or gained in force, is always ſpent or loſt in time: which is the general property of all the mechanical powers.

The uſual methods of arranging Pulleys in their blocks, may be reduced to two. The firſt conſiſts in placing them one by the ſide of another, upon the ſame pin; the other, in placing them directly under one another, upon ſeparate pins. Each of theſe methods however is liable to inconvenience; and Mr. Smeaton, to avoid the impediments to which theſe combinations are ſubject, propoſes to combine theſe two methods in one. See the Philoſ. Tranſ. vol. 47, pa. 494.

Some inſtances of ſuch combinations of Pulleys are exhibited in the following figures; beſide which, there are alſo other varieties of forms.

A very conſiderable improvement in the conſtruction of Pulleys has been made by Mr. James White, who has obtained a patent for his invention, and of which he gives the following deſcription. The laſt of the three following figures ſhews the machine, conſiſting of two Pulleys Q and R, one fixed and the other moveable. Each of theſe has ſix concentric grooves, capable of having a line put round them, and thus acting like as many different Pulleys, having diameters equal to thoſe of the grooves. Suppoſing then each of the grooves to be a diſtinct Pulley, and that all their diameters were equal, it is evident that if the weight 144 were to be raiſed by pulling at S till the Pulleys touch each other, the firſt Pulley muſt receive the length of line as many times as there are parts of the line hanging between it and the lower Pulley. In the preſent caſe, there are 12 lines, *b*, *d*, *f*, &c, hanging between the



two pulleys, formed by its revolution about the ſix upper lower grooves. Hence as much line muſt paſs over the uppermoſt Pulley as is equal to twelve times the diſtance of the two. But, from an inſpection of the figure, it is plain, that the ſecond Pulley cannot receive the full quantity of line by as much as is equal to the diſtance betwixt it and the firſt. In like manner, the third Pulley receives leſs than the firſt by as much as is the diſtance between the firſt and third; and ſo on to the laſt, which receives only one twelfth of the whole. For this receives its ſhare of line *n* from a fixed point in the upper frame, which gives it nothing; while all the others in the ſame frame receives the line partly by turning to meet it, and partly by the line coming to meet them.

Suppoſing now theſe Pulleys to be equal in ſize, and to move freely as the line determines them, it appears evident, from the nature of the ſyſtem, that the number of their revolutions, and conſequently their velocities, muſt be in proportion to the number of ſuſpending parts that are between the fixed point above mentioned and each Pulley reſpectively. Thus the outermoſt Pulley would go twelve times round in the time that the Pulley under which the part *n* of the line, if equal to it, would revolve only once; and the intermediate times and velocities would be a ſeries of arithmetical proportionals, of which, if the firſt number were 1, the laſt would always be equal to the whole number of terms. Since then the revolutions of equal and diſtinct Pulleys are meaſured by their velocities, and that it is poſſible to find any proportion of velocity, on a ſingle body running on a centre, viz, by finding proportionate diſtances from that centre; it follows, that if the diameters of certain grooves in the ſame ſubſtance be exactly adapted to the above ſeries (the line itſelf being ſuppoſed inelaſtic, and of no magnitude) the neceſſity of

of using several Pulleys in each frame will be obviated, and with that some of the inconveniencies to which the use of the Pulley is liable.

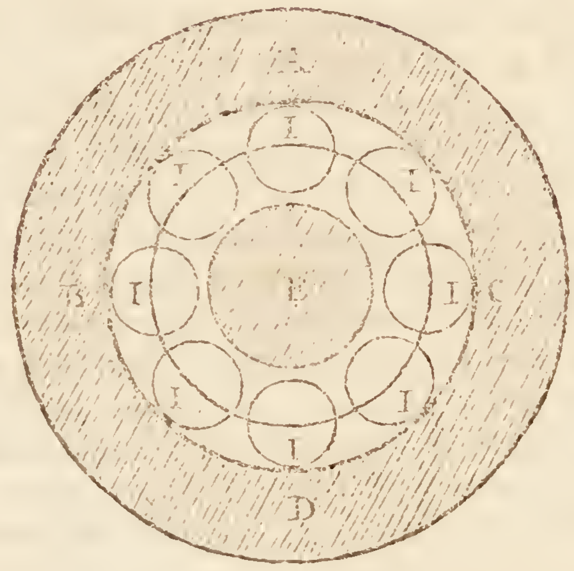
In the figure referred to, the coils of rope by which the weight is supported, are represented by the lines *a*, *b*, *c* &c; *a* is *the line of traction*, commonly called the fall, which passes over and under the proper grooves, until it is fastened to the upper frame just above *n*. In practice, however, the grooves are not arithmetical proportions, nor can they be so: for the diameter of the rope employed must in all cases be deducted from each term; without which the smaller grooves, to which the said diameter bears a larger proportion than to the larger ones, will tend to rise and fall faster than they, and thus introduce worse defects than those which they were intended to obviate.

The principal advantage of this kind of Pulley is, that it destroys lateral friction, and that kind of shaking motion which is so inconvenient in the common Pulley. And lest (says Mr. White) this circumstance should give the idea of weakness, I would observe, that to have pins for the pulleys to run on, is not the only nor perhaps the best method; but that I sometimes use centres fixed to the Pulleys, and revolving on a very short bearing in the side of the frame, by which strength is increased, and friction very much diminished; for to the last moment the motion of the Pulley is perfectly circular: and this very circumstance is the cause of its not wearing out in the centre as soon as it would, assisted by the ever increasing irregularities of a gullied bearing. These Pulleys, when well executed, apply to jacks and other machines of that nature with peculiar advantage, both as to the time of going and their own durability; and it is possible to produce a system of Pulleys of this kind of six or eight parts only, and adapted to the pockets, which, by means of a skain of sewing silk, or a clue of common thread, will raise upwards of an hundred weight.

As a system of Pulleys has no great weight, and lies in a small compass, it is easily carried about, and can be applied for raising weights in a great many cases, where other engines cannot be used. But they are subject to a great deal of friction, on the following accounts; viz, 1st, because the diameters of their axes bear a very considerable proportion to their own diameters; 2d, because in working they are apt to rub against one another, or against the sides of the block; 3dly, because of the stiffness of the rope that goes over and under them. See Ferguson's Mech. pa. 37, 4to.

But the friction of the Pulley is now reduced to nothing as it were, by the ingenious Mr. Garnett's patent friction rollers, which produce a great saving of labour and expence, as well as in the wear of the machine, both when applied to Pulleys and to the axles of wheel-carriages. His general principle is this; between the axle and nave, or centre pin and box, a hollow space is left, to be filled up by solid equal rollers nearly touching each other. These are furnished with axles inserted into a circular ring at each end, by which their relative distances are preserved; and they are kept parallel by means of wires fastened to the rings between the rollers, and which are rivetted to them.

The above contrivance is exhibited in the annexed figure; where ABCD represents a piece of metal to be inserted into the box or nave, of which E is the centre-pin or axle, and 1, 1, 1, &c, rollers of metal having



axes inserted in the brazen circle which passes through their centres; and both circles being rivetted together by means of bolts passing between the rollers from one side of the nave to the other; and thus they are always kept separate and parallel.

PUMP, in Hydraulics, a machine for raising water, and other fluids.

Pumps are probably of very ancient use. Vitruvius ascribes the invention to Ctesebes of Athens, some say of Alexandria, about 120 years before Christ. They are now of various kinds. As the Sucking Pump, the Lifting Pump, the Forcing Pump, Ship Pumps, Chain Pumps, &c. By means of the lifting and forcing Pumps, water may be raised to any height, with a sufficient power, and an adequate apparatus: but by the sucking Pump the water may, by the general pressure of the atmosphere on the surface of the well, be raised only about 33 or 34 feet; though in practice it is seldom applied to the raising it much above 28; because, from the variations observed in the barometer, it appears that the air may sometimes be lighter than 33 feet of water; and whenever that happens, for want of the due counterpoise, this Pump may fail in its performance.

The Common Sucking PUMP.—This consists of a pipe, of wood or metal, open at both ends, having a fixed valve in the lower part of it opening upwards, and a moveable valve or bucket by which the water is drawn or lifted up. This bucket is just the size of the bore of the Pump-pipe, in that part where it works, and leathered round so as to fit it very close, that no air may pass by the sides of it; the valve hole being in the middle of the bucket. The bucket is commonly worked in the upper part of the barrel by a short rod, and another fixed valve placed just below the descent of the bucket: Thus, (fig. 1, pl. 23), AB is the Pump-pipe, C the lower fixed valve, opening upwards, and D is the bucket, or moving valve, also opening upwards.

In working the Pump; draw up the bucket D, by means of the Pump rod, having any sort of a handle fixed to it: this draws up the water that is above it, or if not, the air; in either case the water pushes up the valve C, and enters to supply the void left between C and D, being pushed up by the pressure of the atmosphere on the surface of the water in the well below. Next, the bucket D is pushed down, which shuts the valve

valve C, and prevents the return of the water downwards, which opens the valve D, by which the water afcends above it. And thus, by repeating the ftrokes of the Pump-rod handle, the valves alternately open and fhut, and the water is drawn up at every ftroke, and runs out at the nozle or fpout near the top.

The Lifting Pump differs from the fucking Pump only in the difpofition of its valves and the form of its pifton frame. This kind of Pump is reprefented in fig. 2, pl. 23; where the lower valve D is moveable, being worked up and down with the Pump rod, which lifts the water up, and fo opens the upper valve C, which is fixed, and permits the water to iffue through it, and run out at top. Then as the pifton D defcends, the weight of the water above C fhuts that valve C, and fo prevents its return, till that valve be opened again by another lift of the pifton D. And fo alternately.

The Forcing Pump raifes the water through the fucker, or lower valve C (fig. 3, pl. 23), in the fame manner as the fucking Pump; but as the pifton or plunger D has no valve in it, the water cannot get above it when this is pufhed down again; inftead of which, a fide pipe is inferted between C and D, having a fixed valve at E opening upwards, through which the water is forced out of the Pump by pufhing down the plunger D.

Obfervations on Pumps.—The force required to work a Pump, is equal to the weight of water raifed at each ftroke, or equal to the weight of water filling the cavity of the pipe, and its height equal to the length of the ftroke made by the pifton. Hence if d denote the diameter of the pipe, and l the length of the ftroke, both in inches; then is $\cdot 7854d^2l$ the content of the water raifed at a ftroke, in inches, or $\cdot 0028d^2l$ in ale gallons; and the weight of it is $\frac{d^2l}{220}$ ounces or $\frac{d^2l}{3520}$ lb. But if the handle of the pump be a lever which gains in the power of p to 1, the force of the hand to work the Pump will be only $\frac{d^2l}{3520p}$ lb, or, when p is 5 for inftance, it will be $\frac{d^2l}{17600}$ lb. And all thefe over and above the friction of the moving parts of the Pump.

To the forcing Pump is fometimes adapted an air veffel, which, being compreffed by the water, by its elafticity acts upon the water again, and forces it out to a great diftance, and in a continued ftream, inftead of by jerks or jets. So, Mr. Newfham's water engine, for extinguifhing fire, confifts of two forcing Pumps, which alternately drive water into a clofe veffel of air, by which means the air in it is condenfed, and compreffes the water fo ftrongly, that it rufhes out with great impetuofity and force through a pipe that comes down into it, making a continued uniform ftream.

By means of forcing Pumps, water may be raifed to any height whatever above the level of a river or fpring; and machines may be contrived to work thefe Pumps, either by a running ftream, a fall of water, or by horfes.

Ctefibes's Pump, acts both by fuction and by preffion. Thus, a brafs cylinder ABCD (fig. 5, pl. 23) furnifhed with a valve at L, is placed in the water. In this is fitted the pifton KM, made of green wood, which will not fwell in the water, and is adjufted to the aperture of the cylinder with a covering of leather, but without any valve. Another tube NH is fitted on at H, with a valve I opening upwards.—Now the pifton being raifed, the water opens the valve L, and rifes into the cavity of the cylinder. When the pifton is depreffed again, the valve I is opened, and the water is driven up through the tube HN.

This was the Pump ufed among the Ancients, and that from which both the others have been deduced. Sir Samuel Morland has endeavoured to increafe its force by leffening the friction; which he has done to good effect, fo as to make it work with very little.

There are various kinds of Pumps ufed in fhips, for throwing the water out of the hold, and upon other occafions, as the Chain Pump, &c.

Air-Pump, in Pneumatics, is a machine, by means of which the air is emptied out of veffels, and a kind of vacuum produced in them. For the particulars of which, fee Air-*Pump*.

PUNCHEON, a meafure for liquids, containing $\frac{1}{3}$ of a tun, or a hogfhead and $\frac{1}{3}$, or 84 gallons.

PUNCHINS, or Punchions, in Building, fhort pieces of timber placed to fupport fome confiderable weight.

PUNCTATED *Hyperbola*, in the higher geometry, an hyperbola whofe conjugate oval is infinitely fmall, that is, a point.

PUNCTUM *ex Comparatione*, is either focus, in the ellipfe or hyperbola; fo called by Apollonius, becaufe the rectangle under two abfciffes made at the focus, is equal to one fourth part of what he calls the figure, which is the fquare of the conjugate axis, or the rectangle under the tranfverfe and the parameter.

PURBACH (George), a very eminent mathematician and aftronomer, was born at Purbach, a town upon the confines of Bavaria and Auftria, in 1423, and educated at Vienna. He afterwards vifited the moft celebrated univerfities in Germany, France, and Italy; and found a particular friend and patron in cardinal Cufa at Rome. Returning to Vienna, he was appointed mathematical profeffor, in which office he continued till his death, which happened in 1461, in the 39th year of his age only, to the great lofs of the learned world.

Purbach compofed a great number of pieces, upon mathematical and aftronomical fubjects; and his fame brought many ftudents to Vienna, and among them, the celebrated Regiomontanus, between whom and Purbach there fubfifted the ftricteft friendfhip and union of ftudies till the death of the latter. Thefe two laboured together to improve every branch of learning, by all the means in their power, though aftronomy feems to have been the favourite of both; and had not the immature death of Purbach prevented his further purfuits, there is no doubt but that, by their joint induftry, aftronomy would have been carried to very great perfection. That this is not merely furmife, may be learnt from thofe improvements which Purbach actually did make, to render the ftudy of it more eafy and practicable. His firft effay was, to amend the Latin tranflation of Ptolomy's Almageft, which had been made from the Arabic verfion: this he did, not by the help of the Greek text, for he was unacquainted with that language, but by drawing the moft probable conjectures from a ftrict attention to the fenfe of the author.

He

He then proceeded to other works, and among them, he wrote a tract, which he entitled, *An Introduction to Arithmetic;* then a treatise on *Gnomonics*, or *Dialling*, with tables suited to the difference of climates or latitudes; likewise a small tract concerning the *Altitudes of the Sun*, with a table; also, *Astrolabic Canons*, with a table of the parallels, proportioned to every degree of the equinoctial.

After this, he constructed Solid Spheres, or Celestial Globes, and composed a new table of fixed stars, adding the longitude by which every star, since the time of Ptolomy, had increased. He likewise invented various other instruments, among which was the Gnomon, or Geometrical Square, with canons and a table for the use of it.

He not only collected the various tables of the Primum Mobile, but added new ones. He made very great improvements in Trigonometry, and by introducing the table of Sines, by a decimal division of the radius, he quite changed the appearance of that science: he supposed the radius to be divided into 600000 equal parts, and computed the sines of the arcs, for every ten minutes, in such equal parts of the radius, by the decimal notation, instead of the duodecimal one delivered by the Greeks, and preserved even by the Arabians till our author's time; a project which was completed by his friend Regiomontanus, who computed the sines to every minute of the quadrant, in 10000000th parts of the radius.

Having prepared the tables of the fixed stars, he next undertook to reform those of the planets, and constructed some entirely new ones. Having finished his tables, he wrote a kind of Perpetual Almanac, but chiefly for the moon, answering to the periods of Meton and Calippus; also an Almanac for the Planets, or, as Regiomontanus afterwards called it, an Ephemeris, for many years. But observing there were some planets in the heavens at a great distance from the places where they were described to be in the tables, particularly the sun and moon (the eclipses of which were observed frequently to happen very different from the times predicted), he applied himself to construct new tables, particularly adapted to eclipses; which were long after famous for their exactness. To the same time may be referred his finishing that celebrated work, entitled, *A New Theory of the Planets*, which Regiomontanus afterwards published the first of all the works executed at his new printing-house.

PURE *Hyperbola*, is an Hyperbola without any oval, node, cusp, or conjugate point; which happens through the impossibility of two of its roots.

PURE *Mathematics*, *Proposition*, *Quadratics*, &c. See the several articles.

PURLINES, in Architecture, those pieces of timber that lie across the rafters on the inside, to keep them from sinking in the middle of their length.

PYRAMID, a solid having any plane figure for its base, and its sides triangles whose vertices all meet in a point at the top, called the vertex of the pyramid; the base of each triangle being the sides of the plane base of the Pyramid.—The number of triangles is equal to the number of the sides of the base; and a cone is a round Pyramid, or one having an infinite number of sides.—The Pyramid is also denominated from its base, being triangular when the base is a triangle, quadrangular when a quadrangle, &c.

The *axis* of the Pyramid, is the line drawn from the vertex to the centre of the base. When this axis is perpendicular to the base, the Pyramid is said to be a *right* one; otherwise it is *oblique*.

1. A Pyramid may be conceived to be generated by a line moved about the vertex, and so carried round the perimeter of the base.

2. All Pyramids having equal bases and altitudes, are equal to one another: though the figures of their bases should even be different.

3. Every Pyramid is equal to one-third of the circumscribed prism, or a prism of the same base and altitude; and therefore the solid content of the Pyramid is found by multiplying the base by the perpendicular altitude, and taking $\frac{1}{3}$ of the product.

4. The upright surface of a Pyramid, is found by adding together the areas of all the triangles which form that surface.

5. If a Pyramid be cut by a plane parallel to the base, the section will be a plane figure similar to the base; and these two figures will be in proportion to each other as the squares of their distances from the vertex of the Pyramid.

6. The centre of gravity of a Pyramid is distant from the vertex $\frac{3}{4}$ of the axis.

Frustum of a PYRAMID, is the part left at the bottom when the top is cut off by a plane parallel to the base.

The solid content of the Frustum of a Pyramid is found, by first adding into one sum the areas of the two ends and the mean proportional between them, the 3d part of which sum is a medium section, or is the base of an equal prism of the same altitude; and therefore this medium area or section multiplied by the altitude gives the solid content. So, if A denote the area of one end, a the area of the other end, and h the height; then $\frac{1}{3}(A + a + \sqrt{Aa})$ is the medium area or section, and $\frac{1}{3}(A + a + \sqrt{Aa}) \times h$ is the solid content.

PYRAMIDS *of Egypt*, are very numerous, counting both great and small; but the most remarkable are the three Pyramids of Memphis, or, as they are now called, of Gheisa or Gize. They are square Pyramids, and the dimensions of the greatest of them, are 700 feet on each side of the base, and the oblique height or slant side the same; its base covers, or stands upon, nearly 11 acres of ground. It is thought by some that these Pyramids were designed and used as gnomons, for astronomical purposes; and it is remarkable that their four sides are accurately in the direction of the four cardinal points of the compass, east, west, north, and south.

PYRAMIDAL *Numbers*, are the sums of polygonal numbers, collected after the same manner as the polygonal numbers themselves are found from arithmetical progressions.

These are particularly called First Pyramidals. The sums of First Pyramidals are called Second Pyramidals; and the sums of the 2d are 3d Pyramidals; and so on. Particularly, those arising from triangular numbers, are called Prime Triangular Pyramidals; those arising

from

from pentagonal numbers, are called Prime Pentagonal Pyramidals; and so on.

The numbers	1,	4,	10,	20,	35,	&c,
formed by adding the triangulars	1,	3,	6,	10,	15,	&c,

are usually called simply by the name of Pyramidals; and the general formula for finding them is $n \times \frac{n-1}{2} \times \frac{n-2}{3}$; so the 4th Pyramidal is found by substituting 4 for n; the 5th by substituting 5 for n; &c. See FIGURATE *Numbers*, and POLYGONAL *Numbers*.

PYRAMIDOID, is sometimes used for the parabolic spindle, or the solid formed by the rotation of a semiparabola about its base or greatest ordinate. See PARABOLIC *Spindle*.

PYROMETER, or fire-measurer, a machine for measuring the expansion of solid bodies by heat.

Musschenbroek was the first inventor of this instrument; though it has since received several improvements by other philosophers. He has given a table of the expansions of the different metals, with various degrees of heat. Having prepared cylindric rods of iron, steel, copper, brass, tin, and lead, he exposed them first to a Pyrometer with one flame in the middle; then with two flames; then successively with three, four, and five flames. The effects were as in the following Table, where the degrees of expansion are marked in parts equal to the 12500th part of an inch.

Expansion of	Iron	Steel	Copp.	Brass	Tin	Lead
By 1 flame	80	85	89	110	153	155
By 2 flames placed close together	117	123	115	220		274
By 2 flames at $2\frac{1}{2}$ inches distant	109	94	92	141	219	263
By 3 flames close together	142	168	193	275		
By 4 flames close together	211	270	270	361		
By 5 flames	230	310	310	377		

Tin easily melts when heated by two flames placed close together; and lead with three flames close together, when they burn long.

It hence appears that the expansions of any metal are in a less degree than the number of flames: so two flames give less than a double expansion, three flames less than a triple expansion, and so on, always more and more below the ratio of the number of flames. And the flames placed together cause a greater expansion, than with an interval between them.

For the construction of Musschenbroek's Pyrometer, with alterations and improvements upon it by Desaguliers, see Desag. Exper. Philos. vol. 1, pa. 421; see also Musschenbroek's translation of the Experiments of the Academy del Cimento, printed at Leyden in 1731; and for a Pyrometer of a new construction, by which the expansions of metals in boiling fluids may be examined and compared with Fahrenheit's thermometer, see Musschenb. Introd. ad Philos. Nat. 4to, 1762, vol. 2, pa. 610.

But as it has been observed, that Musschenbroek's Pyrometer was liable to some objections, these have been removed in a good measure by Ellicott, who has given a description of his improved Pyrometer in the Philos. Trans. numb. 443; and the same may be seen in the Abridg. vol. 8, pa. 464. This instrument measures the expansions to the 7200th part of an inch; and by means of it, Mr. Ellicott found, upon a medium, that the expansions of bars of different metals, as nearly of the same dimensions as possible, by the same degree of heat, were as below:

Gold,	Silver,	Brass,	Copper,	Iron,	Steel,	Lead.
73	103	95	89	60	56	149

The great difference between the expansions of iron and brass, has been applied with good success to remove the irregularities in pendulums arising from heat. Philos. Trans. vol. 47, pa. 485.

Mr. Graham used to measure the minute expansions of metal bars, by advancing the point of a micrometer screw, till it sensibly stopped against the end of the bar to be measured. This screw, being small and very lightly hung, was capable of agreement within the 3000 or 4000th part of an inch. And on this general principle Mr. Smeaton contrived his Pyrometer, in which the measures are determined by the contact of a piece of metal with the point of a micrometer-screw. This instrument makes the expansions sensible to the 2345th part of an inch. And when it is used, both the instrument and the bar, to be measured, are immerged in a cistern of water, heated to any degree, up to boiling, by means of lamps placed under the cistern; and the water communicates the same degree of heat to the instrument and bar, and to a mercurial thermometer immerged in it, for ascertaining that degree.

With this Pyrometer Mr. Smeaton made several experiments, which are arranged in a table; and he remarks, that their result agrees very well with the proportions of expansions of several metals given by Mr. Ellicott. The following Table shews how much a foot in length of each metal expands by an increase of heat corresponding to 180 degrees of Fahrenheit's thermometer, or to the difference between the temperatures of freezing and boiling water, expressed in the 10000th part of an inch.

1.	White glass barometer tube	100
2.	Martial regulus of antimony	130
3.	Blistered steel	138
4.	Hard steel	147
5.	Iron	151
6.	Bismuth	167
7.	Copper, hammered	204
8.	Copper 8 parts, mixed with 1 part tin	218
9.	Cast brass	225
10.	Brass 16 parts, with 1 of tin	229
11.	Brass wire	232
12.	Speculum metal	232
13.	Spelter solder, viz 2 parts brass and 1 zinc	247

14. Fine

14. Fine pewter	- - -	274
15. Grain tin	- - -	298
16. Soft folder, viz lead 2 and tin 1	-	301
17. Zinc 8 parts with tin 1, a little hammered		323
18. Lead	- - - -	344
19. Zinc or fpelter	- - -	353
20. Zinc hammered half an inch per foot		373

For a farther account of this inftrument, with its ufe, fee Philof. Tranf. vol. 48, pa. 598.

Mr. Fergufon has conftructed, and defcribed a Pyrometer (Lect. on Mechanics, Suppl. pa. 7, 4to), which makes the expanfion of metals by heat vifible to the 45000th part of an inch. And another plan of a Pyrometer has lately been invented by M. De Luc, in confequence of a hint fuggefted to him by Mr. Ramfden: for an account of which, with the principle of its conftruction and ufe, both in the comparative meafure of the expanfions of bodies by heat, and the meafure of their abfolute expanfion, as well as the experiments made with it, fee M. De Luc's elaborate effay on Pyrometry &c, in the Philof. Tranf. vol. 68, pa. 419—546.

Other very nice and ingenious contrivances, for the meafuring of expanfions by heat, have been made by Mr. Ramfden; which he has fuccefsfully applied in the cafe of the meafuring rods and chains lately employed, by General Roy and Col. Williams, in meafuring the bafe on Hounflow Heath, &c; which determine the expanfions, to great minutenefs, for each degree of the thermometer. See Philof. Tranf. 1785, &c.

PYROPHORUS, the name ufually given to that fubftance by fome called black phofphorus; being a chemical preparation poffeffing the fingular property of kindling fpontaneoufly when expofed to the air; which was accidentally difcovered by M. Homberg, who prepared it of alum and human fæces. Though it has fince been found, by the fon of M. Lemeri, that the fæces are not neceffary to it, but that honey, fugar, flour, and any animal or vegetable matter, may be ufed inftead of the fæces; and M. De Suvigny has fhewn that moft vitriolic falts may be fubftituted for the alum. See Prieftley's Obferv. on Air, vol. 3, Append. p. 386, and vol. 4, Append. p. 479.

PYROTECHNY, the art of fire, or the fcience which teaches the application and management of fire in feveral operations.

Pyrotechny is of two kinds, military and chemical.

Military PYROTECHNY, is the fcience of artificial fire-works, and fire-arms, teaching the ftructure and ufe both of thofe employed in war, as gunpowder, cannon, fhells, carcaffes, mines, fufees, &c; and of thofe made for amufement, as rockets, ftars, ferpents, &c.

Some call Pyrotechny by the name Artillery; though that word is ufually confined to the inftruments employed in war. Others choofe to call it Pyrobology, or rather Pyroballogy, or the art of miffile fires.

Wolfius has reduced Pyrotechny into a kind of mixt mathematical art. Indeed it will not allow of geometrical demonftrations; but he brings it to tolerable rules and reafons; whereas it had formerly been treated by authors at random, and without regard to any reafons at all. See the feveral articles CANNON, GUNPOWDER, ROCKET, SHELL, &c.

Chemical PYROTECHNY, is the art of managing and applying fire in diftillations, calcinations, and other operations of chemiftry.

Some reckon a third kind of Pyrotechny, viz, the art of fufing, refining, and preparing metals.

PYTHAGORAS, one of the greateft philofophers of antiquity, was born about the 47th Olympiad, or 590 years before Chrift. His father's principal refidence was at Samos, but being a travelling merchant, his fon Pythagoras was born at Sidon in Syria; but foon returning home again, our philofopher was brought up at Samos, where he was educated in a manner that was anfwerable to the great hopes that were conceived of him. He was called "the youth with a fine head of hair;" and from the great qualities that foon appeared in him, he was regarded as a good genius fent into the world for the benefit of mankind.

Samos however afforded no philofophers capable of fatisfying his thirft for knowledge; and therefore, at 18 years of age, he refolved to travel in queft of them elfewhere. The fame of Pherecydes drew him firft to the ifland of Syros: from hence he went to Miletus, where he converfed with Thales. He then travelled to Phœnicia, and ftayed fome time at Sidon, the place of his birth; and from hence he paffed into Egypt, where Thales and Solon had been before him.

Having fpent 25 years in Egypt, to acquire all the learning and knowledge he could procure in that country, with the fame view he travelled through Chaldea, and vifited Babylon. Returning after fome time, he went to Crete; and from hence to Sparta, to be inftructed in the laws of Minos and Lycurgus. He then returned to Samos; which, finding under the tyranny of Polycrates, he quitted again, and vifited the feveral countries of Greece. Paffing through Peloponnefus, he ftopped at Phlius, where Leo then reigned; and in his converfation with that prince, he fpoke with fo much eloquence and wifdom, that Leo was at once ravifhed and furprifed.

From Peloponnefus he went into Italy, and paffed fome time at Heraclea, and at Tarentum, but made his chief refidence at Croton; where, after reforming the manners of the citizens by preaching, and eftablifhing the city by wife and prudent counfels, he opened a fchool, to difplay the treafures of wifdom and learning he poffeffed. It is not to be wondered, that he was foon attended by a crowd of difciples, who repaired to him from different parts of Greece and Italy.

He gave his fcholars the rules of the Egyptian priefts, and made them pafs through the aufterities which he himfelf had endured. He at firft enjoined them a five years filence in the fchool, during which they were only to hear; after which, leave was given them to ftart queftions, and to propofe doubts, under the caution however, to fay, "not a little in many words, but much in a few." Having gone through their probation, they were obliged, before they were admitted, to bring all their fortune into the common ftock, which was managed by perfons chofen on purpofe, and called œconomifts, and the whole community had all things in common.

The neceffity of concealing their myfteries induced the Egyptians to make ufe of three forts of ftyles, or ways of expreffing their thoughts; the fimple, the hieroglyphical,

hieroglyphical, and the ſymbolical. In the ſimple, they ſpoke plainly and intelligibly, as in common converſation; in the hieroglyphical, they concealed their thoughts under certain images and characters; and in the ſymbolical, they explained them by ſhort expreſſions, which, under a ſenſe plain and ſimple, included another wholly figurative. Pythagoras borrowed theſe three different ways from the Egyptians, in all the inſtructions he gave; but chiefly imitated the ſymbolical ſtyle, which he thought very proper to inculcate the greateſt and moſt important truths: for a ſymbol, by its double ſenſe, the proper and the figurative, teaches two things at once; and nothing pleaſes the mind more, than the double image it repreſents to our view.

In this manner Pythagoras delivered many excellent things concerning God and the human ſoul, and a great variety of precepts, relating to the conduct of life, political as well as civil; he made alſo ſome conſiderable diſcoveries and advances in the arts and ſciences. Thus, among the works aſcribed to him, there are not only books of phyſic, and books of morality, like that contained in what are câlled his *Golden Verſes*, but treatiſes on politics and theology. All theſe works are loſt: but the vaſtneſs of his mind appears from the wonderful things he performed. He delivered, as antiquity relates, ſeveral cities of Italy and Sicily from the yoke of ſlavery; he appeaſed ſeditions in others; and he ſoftened the manners, and brought to temper the moſt ſavage and unruly ſpirits, of ſeveral people and tyrants. Phalaris, the tyrant of Sicily, it is ſaid, was the only one who could withſtand the remonſtrances of Pythagoras; and he it ſeems was ſo enraged at his diſcourſes, that he ordered him to be put to death. But though the lectures of the philoſopher could make no impreſſion on the tyrant, yet they were ſufficient to reanimate the Sicilians, and to put them upon a bold action. In ſhort, Phalaris was killed the ſame day that he had fixed for the death of the philoſopher.

Pythagoras had a great veneration for marriage; and therefore himſelf married at Croton, a daughter of one of the chief men of that city, by whom he had two ſons and a daughter: one of the ſons ſucceeded his father in the ſchool, and became the maſter of Empedocles: the daughter, named Damo, was diſtinguiſhed both by her learning and her virtues, and wrote an excellent commentary upon Homer. It is related, that Pythagoras had given her ſome of his writings, with expreſs commands not to impart them to any but thoſe of his own family; to which Damo was ſo ſcrupulouſly obedient, that even when ſhe was reduced to extreme poverty, ſhe refuſed a great ſum of money for them.

From the country in which Pythagoras thus ſettled and gave his inſtructions, his ſociety of diſciples was called the Italic ſect of philoſophers, and their reputation continued for ſome ages afterwards, when the Academy and the Lycæum united to obſcure and ſwallow up the Italic ſect. Pythagoras's diſciples regarded the words of their maſter as the oracles of a god; his authority alone, though unſupported by reaſon, paſſed with them for reaſon itſelf: they looked on him as the moſt perfect image of God among men. His houſe was called the temple of Ceres, and his court yard the temple of the Muſes: and when he went into towns, it was ſaid he went thither, "not to teach men, but to heal them."

Pythagoras however was perſecuted by bad men in the laſt years of his life; and ſome ſay he was killed in a tumult raiſed by them againſt him; but according to others, he died a natural death, at 90 years of age, about 497 years before Chriſt.

Beſide the high reſpect and veneration the world has always had for Pythagoras, on account of the excellence of his wiſdom, his morality, his theology, and politics, he was renowned as learned in all the ſciences, and a conſiderable inventor of many things in them; as arithmetic, geometry, aſtronomy, muſic, &c. In arithmetic, the common multiplication table is, to this day, ſtill called Pythagoras's table. In geometry, it is ſaid he invented many theorems, particularly theſe three; 1ſt, Only three polygons, or regular plane figures, can fill up the ſpace about a point, viz, the equilateral triangle, the ſquare, and the hexagon: 2d, The ſum of the three angles of every triangle, is equal to two right angles: 3d, In any right-angled triangle, the ſquare on the longeſt ſide, is equal to both the ſquares on the two ſhorter ſides: for the diſcovery of this laſt theorem, ſome authors ſay he offered to the gods a hecatomb, or a ſacrifice of a hundred oxen; Plutarch however ſays it was only one ox, and even that is queſtioned by Cicero, as inconſiſtent with his doctrine, which forbade bloody ſacrifices: the more accurate therefore ſay, he ſacrificed an ox made of flour, or of clay; and Plutarch even doubts whether ſuch ſacrifice, whatever it was, was made for the ſaid theorem, or for the area of the parabola, which it was ſaid Pythagoras alſo found out.

In aſtronomy his inventions were many and great. It is reported he diſcovered, or maintained the true ſyſtem of the world, which places the ſun in the centre, and makes all the planets revolve about him; from him it is to this day called the old or Pythagorean ſyſtem; and is the ſame as that lately revived by Copernicus. He firſt diſcovered, that Lucifer and Heſperus were but one and the ſame, being the planet Venus, though formerly thought to be two different ſtars. The invention of the obliquity of the zodiac is likewiſe aſcribed to him. He firſt gave to the world the name Κοσμος, *Koſmos*, from the order and beauty of all things comprehended in it; aſſerting that it was made according to muſical proportion: for as he held that the ſun, by him and his followers termed the fiery globe of unity, was ſeated in the midſt of the univerſe, and the earth and planets moving around him, ſo he held that the ſeven planets had an harmonious motion, and their diſtances from the ſun correſponded to the muſical intervals or diviſions of the monochord.

Pythagoras and his followers held the tranſmigration of ſouls, making them ſucceſſively occupy one body after another: on which account they abſtained from fleſh, and lived chiefly on vegetables.

PYTHAGORAS'S *Table*, the ſame as the multiplication-table; which ſee.

PYTHAGOREAN, or PYTHAGORIC *Syſtem*, among the Ancients, was the ſame as the Copernican ſyſtem among the Moderns. In this ſyſtem, the ſun is ſuppoſed at reſt in the centre, with the earth and all the planets revolving about him, each in their orbits. See SYSTEM.

It

It was so called, as having been maintained and cultivated by Pythagoras, and his followers; not that it was invented by him, for it was much older.

PYTHAGOREAN *Theorem*, is that in the 47th proposition of the 1st book of Euclid's Elements; viz, that in a right-angled triangle, the square of the longest side, is equal to the sum of both the squares of the two shorter sides. It has been said that Pythagoras offered a hecatomb, or sacrifice of 100 oxen, to the gods, for inspiring him with the discovery of so remarkable a property.

PYTHAGOREANS, a sect of ancient philosophers, who followed the doctrines of Pythagoras. They were otherwise called the Italic sect, from the circumstance of his having settled in Italy. Out of his school proceeded the greatest philosophers and legislators, Zaleucus, Charondas, Archytas, &c. See the article PYTHAGORAS.

PYXIS *Nautica*, the seaman's compass.

Q.

QUADRAGESIMA, a denomination given to the time of Lent, from its consisting of about 40 days; commencing on Ash Wednesday.

QUADRAGESIMA *Sunday*, is the first Sunday in Lent, or the 1st Sunday after Ash Wednesday.

QUADRANGLE, or QUADRANGULAR *figure*, in Geometry, is a plane figure having four angles; and consequently four sides also.

To the class of Quadrangles belong the square, parallelogram, trapezium, rhombus, and rhomboides.—A square is a regular Quadrangle; a trapezium an irregular one.

QUADRANT, in Geometry, is either the quarter or 4th part of a circle, or the 4th part of its circumference; the arch of which therefore contains 90 degrees.

QUADRANT also denotes a mathematical instrument, of great use in astronomy and navigation, for taking the altitudes of the sun and stars, as also taking angles in surveying, heights-and-distances, &c.

This instrument is variously contrived, and furnished with different apparatus, according to the various uses it is intended for; but they have all this in common, that they consist of the quarter of a circle, whose limb or arch is divided into 90° &c. Some have a plummet suspended from the centre, and are furnished either with plain sights, or a telescope, to look through.

The principal and most useful Quadrants, are the common Surveying Quadrant, the Astronomical Quadrant, Adams's Quadrant, Cole's Quadrant, Collins's or Sutton's Quadrant, Davis's Quadrant, Gunter's Quadrant, Hadley's Quadrant, the Horodictical Quadrant, and the Sinical Quadrant, &c. Of these in their order.

1. *The Common, or Surveying* QUADRANT.—This instrument ABC, fig. 1, pl. 24, is made of brass, or wood, &c; the limb or arch of which BC is divided into 90°, and each of these farther divided into as many equal parts as the space will allow, either diagonally or otherwise. On one of the radii AC, are fitted two moveable sights; and to the centre is sometimes also annexed a label, or moveable index AD, bearing two other sights; but instead of these last sights, there is sometimes fitted a telescope. Also from the centre hangs a thread with a plummet; and on the under side or face of the instrument is fitted a ball and socket, by means of which it may be put into any position. The general use of it is for taking angles in a vertical plane, comprehended under right lines going from the centre of the instrument, one of which is horizontal, and the other is directed to some visible point. But besides the parts above described, there is often added on the face, near the centre, a kind of compartment EE, called a Quadrat, or Geometrical Square, which is a kind of separate instrument, and is particularly useful in Altimetry and Longimetry, or Heights-and-Distances.

This Quadrant may be used in different situations; in each of them, the plane of the instrument must be set parallel to that of the eye and the objects whose angular distance is to be taken. Thus, for observing heights or depths, its plane must be disposed vertically, or perpendicular to the horizon; but to take horizontal angles or distances, its plane must be disposed parallel to the horizon.

Again, heights and distances may be taken two ways, viz, by means of the fixed sights and plummet, or by the label; as also, either by the degrees on the limb, or by the Quadrat. Thus, fig. 2 pl. 24 shews the manner of taking an angle of elevation with this Quadrant; the eye is applied at C, and the instrument turned vertically about the centre A, till the object R be seen through the sights on the radius AC; then the angle of elevation RAH, made with the horizontal line KAH, is equal to the angle BAD, made by the plumb line and the other radius of the Quadrant, and the quantity of it is shewn by the degrees in the arch BD cut off by the plumb line AD.

See the use of the instrument in my Mensuration, under the section of Heights-and-Distances.

2. *The Astronomical* QUADRANT, is a large one, usually

ally made of brafs or iron bars; having its limb EF (fig. 3 pl. 24) nicely divided, either diagonally or otherwife, into degrees, minutes, and feconds, if room will permit, and furnifhed either with two pair of plain fights or two telefcopes, one on the fide of the Quadrant at AB, and the other CD moveable about the centre by means of the fcrew G. The dented wheels I and H ferve to direct the inftrument to any object or phenomenon.

The application of this ufeful inftrument, in taking obfervations of the fun, planets, and fixed ftars, is obvious; for being turned horizontally upon its axis, by means of the telefcope AB, till the object is feen through the moveable telefcope, then the degrees &c cut by the index, give the altitude &c required.

3. *Cole's* Quadrant, is a very ufeful inftrument, invented by Mr. Benjamin Cole. It confifts of fix parts, viz, the ftaff AB (fig. 11 pl. 24); the quadrantal arch DE; three vanes A, B, C; and the vernier FG. The ftaff is a bar of wood about 2 feet long, an inch and a quarter broad, and of a fufficient thicknefs to prevent it from bending or warping. The quadrantal arch is alfo of wood; and is divided into degrees and 3d parts of degrees, to a radius of about 9 inches; and to its extremities are fitted two radii, which meet in the centre of the Quadrant by a pin, about which it eafily moves. The fight-vane A is a thin piece of brafs, near two inches in height, and one broad, fet perpendicularly on the end of the ftaff A, by means of two fcrews paffing through its foot. In the middle of this vane is drilled a fmall hole, through which the coincidence or meeting of the horizon and folar fpot is to be viewed. The horizon-vane B is about an inch broad, and 2 inches and a half high, having a flit cut through it of near an inch long, and a quarter of an inch broad; this vane is fixed in the centre-pin of the inftrument, in a perpendicular pofition, by means of two fcrews paffing through its foot, by which its pofition with refpect to the fight-vane is always the fame, their angle of inclination being equal to 45 degrees. The fhade-vane C is compofed of two brafs plates. The one, which ferves as an arm, is about $4\frac{1}{2}$ inches long, and $\frac{3}{4}$ of an inch broad, being pinned at one end to the upper limb of the Quadrant by a fcrew, about which it has a fmall motion; the other end lies in the arch, and the lower edge of the arm is directed to the middle of the centre-pin: the other plate, which is properly the vane, is about 2 inches long, being fixed perpendicularly to the other plate, at about half an inch diftance from that end next the arch; this vane may be ufed either by its fhade, or by the folar fpot caft by a convex lens placed in it. And becaufe the wood-work is often fubject to warp or twift, therefore this vane may be rectified by means of a fcrew, fo that the warping of the inftrument may occafion no error in the obfervation, which is performed in the following manner: Set the line G on the vernier againft a degree on the upper limb of the Quadrant, and turn the fcrew on the backfide of the limb forward or backward, till the hole in the fight-vane, the centre of the glafs, and the funk fpot in the horizon-vane, lie in a right line.

To find the Sun's Altitude by this inftrument. Turn your back to the fun, holding the ftaff of the inftrument with the right hand, fo that it be in a vertical plane paffing through the fun; apply one eye to the fight-vane, looking through that and the horizon-vane till the horizon be feen; with the left hand flide the quadrantal arch upwards, till the folar fpot or fhade, caft by the fhade-vane, fall directly upon the fpot or flit in the horizon-vane; then will that part of the quadrantal arch, which is raifed above G or S (according as the obfervation refpects either the folar fpot or fhade) fhew the altitude of the fun at that time. But for the meridian altitude, the obfervation muft be continued, and as the fun approaches the meridian, the fea will appear through the horizon-vane, which completes the obfervation; and the degrees and minutes, counted as before, will give the fun's meridian altitude: or the degrees counted from the lower limb upwards will give the zenith diftance.

4. *Adams's* Quadrant, differs only from Cole's Quadrant, juft defcribed, in having an horizontal vane, with the upper part of the limb lengthened; fo that the glafs, which cafts the folar fpot on the horizon-vane, is at the fame diftance from the horizon-vane as the fight-vane at the end of the index.

5. *Collins's* or *Sutton's* Quadrant, (fig. 8 pl. 24) is a ftereographic projection of one quarter of the fphere between the tropics, upon the plane of the ecliptic, the eye being in its north pole; and fitted to the latitude of London. The lines running from right to left, are parallels of altitude; and thofe croffing them are azimuths. The fmaller of the two circles, bounding the projection, is one quarter of the tropic of Capricorn; and the greater is a quarter of the tropic of Cancer. The two ecliptics are drawn from a point on the left edge of the Quadrant, with the characters of the figns upon them; and the two horizons are drawn from the fame point. The limb is divided both into degrees and time; and by having the fun's altitude, the hour of the day may here be found to a minute. The quadrantal arches next the centre contain the calendar of months; and under them, in another arch, is the fun's declination. On the projection are placed feveral of the moft remarkable fixed ftars between the tropics; and the next below the projection is the Quadrant and line of fhadows.

To find the Time of the Sun's Rifing or Setting, his Amplitude, his Azimuth, Hour of the Day, &c. by this Quadrant. Lay the thread on the day and the month, and bring the bead to the proper ecliptic, either of fummer or winter, according to the feafon, which is called *rectifying*; then by moving the thread bring the bead to the horizon, in which cafe the thread will cut the limb in the point of the time of the fun's rifing or fetting before or after 6: and at the fame time the bead will cut the horizon in the degrees of the fun's amplitude.—Again, obferving the fun's altitude with the Quadrant, and fuppofing it found to be 45° on the 5th of May, lay the thread over the 5th of May; then bring the bead to the fummer ecliptic, and carry it to the parallel of altitude 45°; in which cafe the thread will cut the limb at 55° 15′, and the hour will be feen among the hour-lines to be either 41m. paft 9 in the morning, or 19m. paft 2 in the afternoon.——Laftly, the bead fhews among the azimuths the fun's diftance from the fouth 50° 41′.

7

But

But note, that if the ſun's altitude be leſs than what it is at 6 o'clock, the operation muſt be performed among thoſe parallels above the upper horizon; the bead being rectified to the winter ecliptic.

6. *Davis's* QUADRANT, the ſame as the BACK-STAFF; which ſee.

7. *Gunner's* QUADRANT, (fig. 6 pl. 24), ſometimes called the *Gunner's Square*, is uſed for elevating and pointing cannon, mortars, &c. and conſiſts of two branches either of wood or braſs, between which is a quadrantal arch divided into 90°, and furniſhed with a thread and plummet.

The uſe of this inſtrument is very eaſy; for if the longer branch, or bar, be placed in the mouth of the piece, and it be elevated till the plummet cut the degree neceſſary to hit a propoſed object, the thing is done.

Sometimes on the ſides of the longer bar, are noted the diviſion of diameters and weights of iron balls, as alſo the bores of pieces.

8. *Gunter's* QUADRANT, ſo called from its inventor Edmund Gunter, (fig. 4 pl. 24) beſide the apparatus of other Quadrants, has a ſtereographic projection of the ſphere on the plane of the equinoctial; and alſo a calendar of the months, next to the diviſions of the limb; by which, beſide the common purpoſes of other Quadrants, ſeveral uſeful queſtions in aſtronomy, &c, are eaſily reſolved.

Uſe of Gunter's Quadrant. — 1. To find the ſun's meridian altitude for any given day, or converſely the day of the year anſwering to any given meridian altitude. Lay the thread to the day of the month in the ſcale next the limb; then the degree it cuts in the limb is the ſun's meridian altitude. And, contrariwiſe, the thread being ſet to the meridian altitude, it ſhews the day of the month.

2. To find the hour of the day. Having put the bead, which ſlides on the thread, to the ſun's place in the ecliptic, obſerve the ſun's altitude by the Quadrant; then if the bead be laid over the ſame in the limb, the bead will fall upon the hour required. On the contrary, laying the bead on a given hour, having firſt rectified or ſet it to the ſun's place, the degree cut by the thread on the limb gives the altitude.

Note, the bead may be rectified otherwiſe, by bringing the thread to the day of the month, and the bead to the hour-line of 12.

3. To find the ſun's declination from his place given; and the contrary. Bring the bead to the ſun's place in the ecliptic, and move the thread to the line of declination E T, ſo ſhall the bead cut the degree of declination required. On the contrary, the bead being adjuſted to a given declination, and the thread moved to the ecliptic, the bead will cut the ſun's place.

4. The ſun's place being given, to find the right aſcenſion; or contrariwiſe. Lay the thread on the ſun's place in the ecliptic, and the degree it cuts on the limb is the right aſcenſion ſought. And the converſe.

5. The ſun's altitude being given, to find his azimuth; and contrariwiſe. Rectify the bead for the time, as in the ſecond article, and obſerve the ſun's altitude; bring the thread to the complement of that altitude; then the bead will give the azimuth ſought, among the azimuth-lines.

9. *Hadley's* QUADRANT, (fig. 7 pl. 24) ſo called from its inventor John Hadley, Eſq, is now univerſally uſed as the beſt of any for nautical and other obſervations.

It ſeems the firſt idea of this excellent inſtrument was ſuggeſted by Dr. Hooke; for Dr. Sprat, in his Hiſtory of the Royal Society, pa. 246, mentions the invention of a new inſtrument for taking angles by reflection, by which means the eye at once ſees the two objects both as touching the ſame point, though diſtant almoſt to a ſemicircle; which is of great uſe for making exact obſervations at ſea. This inſtrument is deſcribed and illuſtrated by a figure in Hooke's Poſthumous works, pa. 503. But as it admitted of only one reflection, it would not anſwer the purpoſe. The matter however was at laſt effected by Sir Iſaac Newton, who communicated to Dr. Halley a paper of his own writing, containing the deſcription of an inſtrument with two reflections, which ſoon after the doctor's death was found among his papers by Mr. Jones, by whom it was communicated to the Royal Society, and it was publiſhed in the Philoſ. Tranſ. for the year 1742. See alſo the Abridg. vol. 8, pa. 129. How it happened that Dr. Halley never mentioned this in his lifetime, is hard to ſay; but it is very extraordinary; more eſpecially as Mr. Hadley had deſcribed, in the Tranſac. for 1731, his inſtrument, which is conſtructed on the ſame principles. See alſo Abr. vol. 6, pa. 139. Mr. Hadley, who was well acquainted with Sir Iſaac Newton, might have heard him ſay, that Dr. Hooke's propoſal could be effected by means of a double reflection; and perhaps in conſequence of this hint, he might apply himſelf, without any previous knowledge of what Newton had actually done, to the conſtruction of his inſtrument. Mr. Godfrey too, of Pennſylvania, had recourſe to a ſimilar expedient; for which reaſon ſome gentlemen of that colony have aſcribed the invention of this excellent inſtrument to him. The truth may probably be, that each of theſe gentlemen diſcovered the method independent of one another. See Abr. Philoſ. Tranſ. vol. 8, pa. 366; alſo Tranſ. of the American Society, vol. 1, pa. 21 Appendix.

This inſtrument conſiſts of the following particulars: 1. An octant, or the 8th part of a circle, ABC. 2. An index D. 3. The ſpeculum E. 4. Two horizontal glaſſes, F, G. 5. Two ſcreens, K and K. 6. Two ſight-vanes, H and I.

The octant conſiſts of two radii, AB, AC, ſtrengthened by the braces L, M, and the arch BC; which, though containing only 45°, is nevertheleſs divided into 90 primary diviſions, each of which ſtands for degrees, and are numbered 0, 10, 20, 30, &c, to 90; beginning at each end of the arch for the convenience of numbering both ways, either for altitudes or zenith diſtances: alſo each degree is ſubdivided into minutes, by means of a vernier. But the number of theſe diviſions varies with the ſize of the inſtrument.

The index D, is a flat bar, moveable about the centre of the inſtrument; and that part of it which ſlides over the graduated arch, BC, is open in the middle, with Vernier's ſcale on the lower part of it; and

and underneath is a fcrew, ferving to faften the index againft any divifion.

The fpeculum E is a piece of flat glafs, quickfilvered on one fide, fet in a brafs box, and placed perpendicular to the plane of the inftrument, the middle part of the former coinciding with the centre of the latter: and becaufe the fpeculum is fixed to the index, the pofition of it will be altered by the moving of the index along the arch. The rays of an obferved object are received on the fpeculum, and from thence reflected on one of the horizon glaffes, F or G; which are two fmall pieces of looking glafs placed on one of the limbs, their faces being turned obliquely to the fpeculum, from which they receive the reflected rays of objects. This glafs F has only its lower part filvered, and fet in brafs-work; the upper part being left tranfparent to view the horizon. The glafs G has in its middle a tranfparent flit, through which the horizon is to be feen. And becaufe the warping of the materials, and other accidents, may diftend them from their true fituation, there are three fcrews paffing through their feet, by which they may be eafily replaced.

The fcreens are two pieces of coloured glafs, fet in two fquare brafs frames K and K, which ferve as fcreens to take off the glare of the fun's rays, which would otherwife be too ftrong for the eye; the one is tinged much deeper than the other; and as they both move on the fame centre, they may be both or either of them ufed: in the fituation they appear in the figure, they ferve for the horizon-glafs F; but when they are wanted for the horizon-glafs G, they muft be taken from their prefent fituation, and placed on the Quadrant above G.

The fight-vanes are two pins, H and I, ftanding perpendicularly to the plane of the inftrument: that at H has one hole in it, oppofite to the tranfparent flit in the horizon-glafs G; the other, at I, has two holes in it, the one oppofite to the middle of the tranfparent part of the horizon-glafs F, and the other rather lower than the quick-filvered part: this vane has a piece of brafs on the back of it, which moves round a centre, and ferves to cover either of the holes.

Of the Obfervations.—There are two forts of obfervations to be made with this inftrument: the one is when the back of the obferver is turned towards the object, and therefore called the *back obfervation*; the other when the face of the obferver is turned towards the object, which is called the *fore-obfervation.*

To Rectify the Inftrument for the Fore-obfervation: Slacken the fcrew in the middle of the handle behind the glafs F; bring the index clofe to the button *h*; hold the inftrument in a vertical pofition, with the arch downwards; look through the right-hand hole in the vane I, and through the tranfparent part of the glafs F, for the horizon; and if it lie in the fame right line with the image of the horizon feen on the filvered part, the glafs F is rightly adjufted; but if the two horizontal lines difagree, turn the fcrew which is at the end of the handle backward or forward, till thofe lines coincide; then faften the middle fcrew of the handle, and the glafs is rightly adjufted.

To take the Sun's Altitude by the Fore-obfervation. Having fixed the fcreens above the horizon-glafs F, and fuited them proportionally to the ftrength of the fun's rays, turn your face towards the fun, holding the inftrument with your right hand, by the braces L and M, in a vertical pofition, with the arch downward; put your eye clofe to the right-hand hole in the vane I, and view the horizon through the tranfparent part of the horizon-glafs F, at the fame time moving the index D with the left hand, till the reflex folar fpot coincides with the line of the horizon; then the degrees counted from C, or that end next your body, will give the fun's altitude at that time, obferving to add or fubtract 16 minutes according as the upper or lower edge of the fun's reflex image is made ufe of.

But to get the fun's meridian altitude, which is the thing wanted for finding the latitude; the obfervations muft be continued; and as the fun approaches the meridian the index D muft be continually moved towards B, to maintain the coincidence between the reflex folar fpot and the horizon; and confequently as long as this motion can maintain the fame coincidence, the obfervation muft be continued, till the fun has reached the meridian, and begins to defcend, when the coincidence will require a retrograde motion of the index, or towards C; and then the obfervation is finifhed, and the degrees counted as before will give the fun's meridian altitude, or thofe from B will give the zenith diftance; obferving to add the femi-diameter, or 16′, when his lower edge is brought to the horizon; or to fubtract 16′, when the horizon and upper edge coincide.

To take the Altitude of a Star by the Fore-obfervation. Through the vane H, and the tranfparent flit in the glafs G, look directly to the ftar; and at the fame time move the index, till the image of the horizon behind you, being reflected by the great fpeculum, be feen in the filvered part of G, and meet the ftar; then will the index fhew the degrees of the ftar's altitude.

To Rectify the Inftrument for the Back-obfervation. Slacken the fcrew in the middle of the handle, behind the glafs G; turn the button *h* on one fide, and bring the index as many degrees before o as is equal to double the dip of the horizon at your height above the water; hold the inftrument vertical, with the arch downward; look through the hole of the vane H; and if the horizon, feen through the tranfparent flit in the glafs G, coincide with the image of the horizon feen in the filvered part of the fame glafs, then the glafs G is in its proper pofition; but if not, fet it by the handle, and faften the fcrew as before.

To take the Sun's Altitude by the Back-obfervation. Put the ftem of the fcreens K and K into the hole *r*, and in proportion to the ftrength or faintnefs of the fun's rays, let either one or both or neither of the frames of thofe glaffes be turned clofe to the face of the limb; hold the inftrument in a vertical pofition, with the arch downward, by the braces L and M, with the left hand; turn your back to the fun, and put one eye clofe to the hole in the vane H, obferving the horizon through the tranfparent flit in the horizon glafs G; with the right hand move the index D, till the reflected image of the fun be feen in the filvered part of the glafs G, and in a right line with the horizon; fwing your body to and fro, and if the obfervation be well made, the fun's image will be obferved to brufh the horizon, and the degrees reckoned from C, or that part of the arch fartheft from your

body, will give the sun's altitude at the time of observation; observing to add 16′ or the sun's semidiameter if the sun's upper edge be used, and subtract the same for the lower edge.

The directions just given, for taking altitudes at sea, would be sufficient, but for two corrections that are necessary to be made before the altitude can be accurately determined; viz, one on account of the observer's eye being raised above the level of the sea, and the other on account of the refraction of the atmosphere, especially in small altitudes.

The following tables therefore shew the corrections to be made on both these accounts.

TABLE I. Dip of the Horizon of the Sea.		TABLE II. Refractions of the Stars &c in Altitude.			
Height of the Eye.	Dip of the Horizon.	Appar. Altitude in Deg.	Refraction.	Appar. Altitude in Deg.	Refraction.
Feet.	′ ″	°	′ ″	°	′ ″
1	0 57	0	33 0	11	4 47
2	1 21	$\frac{1}{4}$	30 35	12	4 23
3	1 39	$\frac{1}{2}$	28 22	15	3 30
5	2 8	1	24 29	20	2 35
10	3 1	2	18 35	25	2 2
15	3 42	3	14 36	30	1 38
20	4 16	4	11 51	35	1 21
25	4 46	5	9 54	40	1 8
30	5 14	6	8 29	45	0 57
35	5 39	7	7 20	50	0 48
40	6 2	8	6 29	60	0 33
45	6 24	9	5 48	70	0 21
50	6 44	10	5 15	80	0 10

General Rules for these Corrections.

1. In the fore-observations, add the sum of both corrections to the observed zenith distance, for the true zenith distance: or subtract the said sum from the observed altitude, for the true one. 2. In the back-observation, add the dip and subtract the refraction, for altitudes; and for zenith distances, do the contrary, viz, subtract the dip, and add the refraction.

Example. By a back observation, the altitude of the sun's lower edge was found by Hadley's Quadrant to be 25° 12′; the eye being 30 feet above the horizon. By the tables, the dip on 30 feet is 5′ 14″, and the refraction on 25° 12′ is 2′ 1″. Hence

Appar. alt. lower limb	25°	12′	0″
Sun's semidiameter, sub.	0	16	0
Appar. alt. of centre	24	56	0
Dip. of horizon, add	0	5	14
	25	1	14
Refraction, subtract	0	2	1
True alt. of centre	24	59	13

In the case of the moon, besides the two corrections above, another is to be made for her parallaxes. But for all these particulars, see the Requisite Tables for the Nautical Almanac, also Robertson's Navigation, vol. 2, pa. 340 &c, edit. 1780.

10. *Horodictical* QUADRANT, a pretty commodious instrument, and is so called from its use in telling the hour of the day. Its construction is as follows. From the centre of the Quadrant C, (fig. 5 pl. 24), whose limb AB is divided into 90°, describe seven concentric circles at any intervals; and to these add the signs of the zodiac, in the order represented in the figure. Then, applying a ruler to the centre C and the limb A B, mark upon the several parallels the degrees corresponding to the altitude of the sun, when in them, for the given hours; connect the points belonging to the same hour with a curve line, to which add the number of the hour. To the radius CA fit a couple of sights, and to the centre of the Quadrant C tie a thread with a plummet, and upon the thread a bead to slide.

Now if the bead be brought to the parallel in which the sun is, and the Quadrant be directed to the sun, till a visual ray pass through the sights, the bead will shew the hour. For the plummet, in this situation, cuts all the parallels in the degrees corresponding to the sun's altitude. And since the bead is in the parallel which the sun describes, and because hour-lines pass through the degrees of altitude to which the sun is elevated every hour, therefore the bead must shew the present hour.

11. *Sinical* QUADRANT, is one of some use in Navigation. It consists of several concentric quadrantal arches, divided into 8 equal parts by means of radii, with parallel right lines crossing each other at right angles. Now any one of the arches, as BC, (fig. 10 pl. 24) may represent a Quadrant of any great circle of the sphere, but is chiefly used for the horizon or meridian. If then BC be taken for a Quadrant of the horizon, either of the sides, as AB, may represent the meridian; and the other side, AC, will represent a parallel, or line of east-and-west; all the other lines, parallel to AB, will be also meridians; and all those parallel to AC, east-and-west lines, or parallels. Again, the eight spaces into which the arches are divided by the radii, represent the eight points of the compass in a quarter of the horizon; each containing 11° 15′. The arch BC is likewise divided into 90°, and each degree subdivided into 12′, diagonalwise. To the centre is fixed a thread, which, being laid over any degree of the Quadrant, serves to divide the horizon.

If the sinical Quadrant be taken for a fourth part of the meridian, one side of it, AB, may be taken for the common radius of the meridian and equator; and then the other, AC, will be half the axis of the world. The degrees of the circumference, BC, will represent degrees of latitude; and the parallels to the side AB, assumed from every point of latitude to the axis AC, will be radii of the parallels of latitude, as likewise the cosine of those latitudes.

Hence, suppose it be required to find the degrees of longitude contained in 83 of the lesser leagues in the parallel of 48°: lay the thread over 48° of latitude on the circumference, and count thence the 83 leagues on AB, beginning at A; this will terminate in H, allow-

ing every small interval four leagues. Then tracing out the parallel HE, from the point H to the thread; the part AE of the thread shews that 125 greater or equinoctial leagues make 6° 15′; and therefore that the 83 lesser leagues AH, which make the difference of longitude of the course, and are equal to the radius of the parallel HE, make 6° 15′ of the said parallel.

When the ship sails upon an oblique course, such course, beside the north and south greater leagues, gives lesser leagues easterly and westerly, to be reduced to degrees of longitude of the equator. But these leagues being made neither on the parallel of departure, nor on that of arrival, but in all the intermediate ones, there must be found a mean proportional parallel between them. To find this, there is on the instrument a scale of cross latitudes. Suppose then it were required to find a mean parallel between the parallels of 40° and 60°; take with the compasses the middle between the 40th and 60th degree on the scale: this middle point will terminate against the 51st degree, which is the mean parallel sought.

The chief use of the sinical Quadrant, is to form upon it triangles similar to those made by a ship's way with the meridians and parallels; the sides of which triangles are measured by the equal intervals between the concentric Quadrants and the lines N and S, E and W: and every 5th line and arch is made deeper than the rest. Now suppose a ship has sailed 150 leagues north-east-by-north, or making an angle of 33° 45′ with the north part of the meridian: here are given the course and distance sailed, by which a triangle may be formed on the instrument similar to that made by the ship's course; and hence the unknown parts of the triangle may be found. Thus, supposing the centre A to represent the place of departure; count, by means of the concentric circles along the point the ship sailed on, viz, AD, 150 leagues: then in the triangle AED, similar to that of the ship's course, find AE = difference of latitude, and DE = difference of longitude, which must be reduced according to the parallel of latitude come to.

Sutton's QUADRANT. See *Collins's* QUADRANT.

12. QUADRANT *of Altitude*, (fig. 9 pl. 24) is an appendix to the artificial globe, consisting of a thin slip of brass, the length of a quarter part of one of the great circles of the globe, and graduated. At the end, where the division terminates, is a nut riveted on, and furnished with a screw, by means of which the instrument is fitted on the meridian, and moveable round upon the rivet to all points of the horizon, as represented in the figure referred to.—Its use is to serve as a scale in measuring of altitudes, amplitudes, azimuths, &c.

QUADRANTAL *Triangle*, is a spherical triangle, which has one side equal to a quadrant or quarter part of a circle.

QUADRAT, called also *Geometrical Square*, and *Line of Shadows*: it is often an additional member on the face of Gunter's and Sutton's quadrants; and is chiefly useful in taking heights or depths. See my Mensuration, the chap. on Altimetry and Longimetry, or Heights and Distances.

QUADRAT, in Astrology, is the same as quartile, being an aspect of the heavenly bodies when they are distant from each other a quadrant, or 90°, or 3 signs, and is thus marked □.

QUADRATIC *Equations*, in Algebra, are those in which the unknown quantity is of two dimensions, or raised to the 2d power. See EQUATION.

Quadratic equations are either simple, or affected, that is compound.

A Simple QUADRATIC *equation*, is that which contains the 2d power only of the unknown quantity, without any other power of it: as $x^2 = 25$, or $y^2 = ab$. And in this case, the value of the unknown quantity is found by barely extracting the square root on both sides of the equation: so in the equations above, it will be $x = \pm 5$, and $y = \pm \sqrt{ab}$; where the sign of the root of the known quantity is to be taken either plus or minus, for either of these may be considered as the sign of the value of the root x, since either of these, when squared, make the same square, $\overline{+5}|^2 = 25$, and $\overline{-5}|^2 = 25$ also; and hence the root of every quadratic or square, has two values.

Compound or *Affected* QUADRATICS, are those which contain both the 1st and 2d powers of the unknown quantity; as $x^2 + ax = b$, or $x^{2n} - ax^n = \pm b$, where n may be of any value, and then x^n is to be considered as the root or unknown quantity.

Affected quadratics are usually distinguished into three forms, according to the signs of the terms of the equation:

Thus, 1st form, $x^2 + ax = b$,
2d form, $x^2 - ax = b$,
3d form, $x^2 - ax = -b$.

But the method of extracting the root, or finding the value of the unknown quantity x, is the same in all of them. And that method is usually performed by what is called completing the square, which is done by taking half the coefficient of the 2d term or single power of the unknown quantity, then squaring it, and adding that square to both sides of the equation, which makes the unknown side a complete square. Thus, in the equation $x^2 + ax = b$, the coefficient of the 2d term being a, its half is $\frac{1}{2}a$, the square of which is $\frac{1}{4}a^2$, and this added to both sides of the equation, it becomes $x^2 + ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 + b$, the former side of which is now a complete square. The square being thus completed, its root is next to be extracted; in order to which, it is to be observed that the root on the unknown side consists of two terms, the one of which is always x the square root of the first term of the equation, and the other part is $\frac{1}{2}a$ or half the coefficient of the 2d term: thus then the root of $x^2 + ax + \frac{1}{4}a^2$ the first side of the completed equation being $x + \frac{1}{2}a$, and the root of the other side $\frac{1}{4}a^2 + b$ being $\pm \sqrt{a^2 + b}$, it follows that $x + \frac{1}{2}a = \pm \sqrt{\frac{1}{4}a^2 + b}$, and hence, by transposing $\frac{1}{2}a$, it is $x = -\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b}$, the two values of x, or roots of the given equation $x^2 + a = b$. And thus is found the root, or value of x, in the three forms of equations above mentioned: thus,

1st form $x = -\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b}$,

2d form $x = +\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b}$,

3d form $x = +\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b}$.

Where

Where it is obſervable that, becauſe of the double ſign $\pm$, every form has two roots: in the 1ſt and 2d forms thoſe roots are the one poſitive and the other negative, the poſitive root being the leſs of the two in the 1ſt form, but the greater in the 2d form; and in the 3d form the roots are both poſitive. Again, the two roots of the 1ſt and 2d forms, are always both of them real; but in the 3d form, the two roots are either both real or both imaginary, viz, both real when $\frac{1}{4}a^2$ is greater than b, or both imaginary when $\frac{1}{4}a^2$ is leſs than b, becauſe in this caſe $\frac{1}{4}a^2 - b$ will be a negative quantity, the root of which is impoſſible, or an imaginary quantity.

Example of the 1ſt form, let $x^2 + 6x = 7$. Here then $a = 6$, and $b = 7$; then $x = -\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b} = -3 \pm \sqrt{16} = -3 \pm 4 = +1$ or -7.

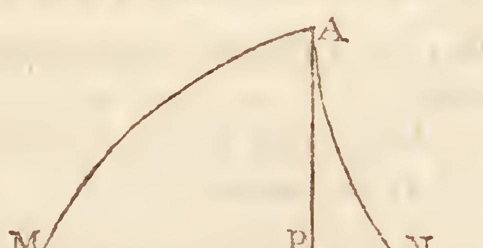
Example of the 2d form, let $x^2 - 6x = 7$. Here alſo $a = 6$, and $b = 7$; then $x = +\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b} = +3 \pm \sqrt{16} = +3 \pm 4 = +7$ or -1; the ſame two roots as before, with the ſigns changed.

Example of the 3d form, let $x^2 - 6x = -7$. Here again $a = 6$, and $b = 7$; then $x = +\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b} = +3 \pm \sqrt{2}$, the two roots both real.

But if $x^2 - 6x = -11$; then $a = 6$, and $b = 11$, which gives x or $+\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 - b} = +3 \pm \sqrt{-2}$, the two roots both imaginary.

All equations whatever that have only two different powers of the unknown quantity, of which the index of the one is juſt double to that of the other, are reſolved like Quadratics, by completing the ſquare. Thus, the equation $x^4 + ax^2 = b$, by completing the ſquare becomes $x^4 + ax^2 + \frac{1}{4}a^2 = \frac{1}{4}a^2 + b$; whence, extracting the root on both ſides, $x^2 + \frac{1}{2}a = \pm\sqrt{\frac{1}{4}a^2 + b}$; therefore $x^2 = -\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b}$, and conſequently $x = \pm\sqrt{-\frac{1}{2}a \pm \sqrt{\frac{1}{4}a^2 + b}}$, where the root x has four values, becauſe the given equation $x^4 + ax^2 = b$ riſes to the 4th power. See Equation.

QUADRATRIX, or Quadratix, in Geometry, is a mechanical line, by means of which, right lines are found equal to the circumference of circles, or other curves, and of the parts of the ſame. Or, more accurately, the *Quadratrix of a curve*, is a tranſcendental curve deſcribed on the ſame axis, the ordinates of which being given, the quadrature of the correſpondent parts in the other curve is likewiſe given. See Curve.—Thus, for example, the curve AND may be



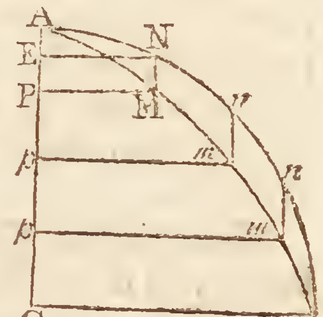
called the Quadratrix of the parabola AMC, when the area APMA bears ſome ſuch relation as the following to the abſciſs AP or ordinate PN, viz,

when APM $=$ PN^2,
or APM $=$ AP $\times$ PN,
or APM $= a \times$ PN,

where a is ſome given conſtant quantity.

The moſt diſtinguiſhed of theſe Quadratices are, thoſe of Dinoſtrates and of Tſchirnhauſen for the circle, and that of Mr. Perks for the hyperbola.

Quadratrix *of Dinoſtrates*, is a curve AMD, by which the quadrature of the circle is effected, though not geometrically, but only mechanically. It is ſo called from its inventor Dinoſtrates; and the geneſis or deſcription of it is as follows: Divide the quadrantal arc ANB into any number of equal parts, in the points N, n, n, &c; and alſo the radius AC into the ſame number of parts at the points P, p, p, &c. To the points of N, n, n, &c, draw the radii CN, Cn, &c; and from the points P, p, &c, the parallels to CB, as PM, Pm, &c: then through all the points of interſection draw the curve AMmD, and it will be the Quadratrix of Dinoſtrates.



Or the ſame curve may be conceived as deſcribed by a continued motion, thus: Conceive a radius CN to revolve with a uniform motion about the centre C, from the poſition AC to the poſition BC; and at the ſame time a ruler PM always moving uniformly parallel towards CB; the two uniform motions being ſo regulated that the radius and the ruler ſhall arrive at the poſition BC at the ſame time. For thus the continual interſection M, m, &c. of the revolving radius, and moving ruler, will deſcribe the Quadratrix AMm &c. Hence,

1. *For the Equation of the Quadratrix:* Since, from the relation of the uniform motions, it is always, AB : AN : : AC : AP; therefore if AB $= a$, AC $= r$, AP $= x$, and AN $= z$, it will be $a : z : : r : x$, or $ax = rz$, which is the equation of the curve.

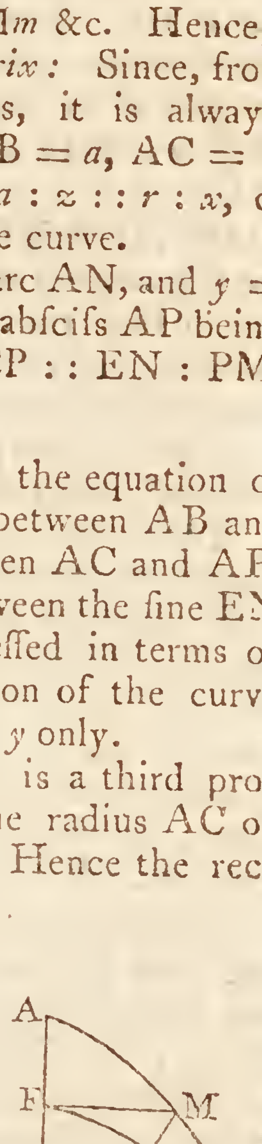
Or, if s denote the ſine NE of the arc AN, and $y =$ PM the ordinate of the curve AM, its abſciſs AP being x; then, by ſimilar triangles, CE : CP : : EN : PM, that is, $\sqrt{r^2 - s^2} : r - x : : s : y$,

and hence $y\sqrt{r^2 - s^2} = r - x \,.\, s$, the equation of the curve. And when the relation between AB and AN is given, in terms of that between AC and AP, hence will be expreſſed the relation between the ſine EN and the radius CB, or s will be expreſſed in terms of r and x; and conſequently the equation of the curve will be expreſſed in terms of r, x, and y only.

2. The baſe of the Quadratrix CD is a third proportional to the quadrant AB and the radius AC or CB; i. e. CD : CB : : CB : AB. Hence the rectification and quadrature of the circle.

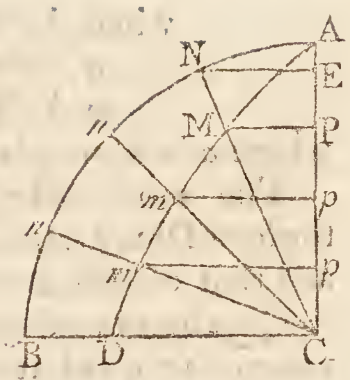
3. A quadrantal arc DF deſcribed with the centre C and radius CD, will be equal in length to the radius CA or CB.

4. CDF being a quadrant inſcribed in the Quadratrix AMD, if the baſe CD be $= 1$, and the circular arc DG $= x$; then in the area



$$CFMD = x - \frac{1}{9}x^3 - \frac{1}{225}x^5 - \frac{2}{6615}x^7$$

&c. See Quadrature.



QUADRATRIX *of Tſchirnhauſen*, is a tranſcendental curve AM*m*B by which the quadrature of the circle is likewiſe effected. This was invented by M. Tſchirnhauſen, and its geneſis, in imitation of that of Dinoſtrates, is as follows: Divide the quadrant ANC, and the radius AC, each into equal parts, as before; and from the points P, *p*, &c, draw the lines PM, *pm*, &c, parallel to CB; alſo from the points N, *n*, &c, the lines NM, *nm*, &c, parallel to the other radius AC; ſo ſhall all the interſections M, *m*, &c, be in the curve of the Quadratrix AM*m*B.

Now for the Equation of this Quadratrix; it is, as before, AB : AN : : AC : AP,

or $a : z : : r : x$, or $ax = rz$.

Or, becauſe here $y = PM = EN = s$; therefore s, as before, expreſſed in terms of r and x, gives the equation of this Quadratrix in terms of r, x, and y, and that in a ſimpler form than the other. Thus, from the nature of the circle and the conſtruction of the Quadratrix, it is

$$y \text{ or } s = x + \frac{r^2 - x^2}{2 \cdot 3r^2}A + \frac{3^2r^2 - x^2}{4 \cdot 5r^2}B + \frac{5^2r^2 - x^2}{6 \cdot 7r^2}C \;\&c,$$

where A, B, C, &c, are the preceding terms; which is the equation of the curve or Quadratrix of Tſchirnhauſen.

By either Quadratix, it is evident that an arc or angle is eaſily divided into three, or any other number of equal parts; viz, by dividing the correſponding radius, or part of it, into the ſame number of equal parts: for AN is always the ſame part of AB, that AP is of AC.

QUADRATURE, in Aſtronomy, that aſpect or poſition of the moon when ſhe is 90° diſtant from the ſun. Or, the Quadratures or quarters are the two middle points of the moon's orbit between the points of conjunction and oppoſition, viz, the points of the 1ſt and 3d quarters; at which times the moon's face ſhews half full, being dichotomized or biſected.

The moon's orbit is more convex in the Quadratures than in the ſyzygies, and the greater axis of her orbit paſſes through the Quadratures, at which points alſo ſhe is moſt diſtant from the earth.—In the Quadratures, and within 35° of them, the apſes of the moon go backwards, or move in antecedentia; but in the ſyzygies the contrary.—When the nodes are in the Quadratures, the inclination of the moon's orbit is greateſt, but leaſt when they are in the ſyzygies.

QUADRATURE *Lines*, or *Lines of* QUADRATURE, are two lines often placed on Gunter's ſector. They are marked with the letter Q, and the figures 5, 6, 7, 8, 9, 10; of which Q denotes the ſide of a ſquare, and the figures denote the ſides of polygons of 5, 6, 7, &c ſides. Alſo S denotes the ſemidiameter of a circle, and 90 a line equal to the quadrant or 90° in circumference.

QUADRATURE, in Geometry, is the ſquaring of a figure, or reducing it to an equal ſquare, or finding a ſquare equal to the area of it.

The Quadrature of rectilineal figures falls under common geometry, or menſuration; as amounting to no more than the finding their areas, or ſuperficies; which are in effect their ſquares: which was fully effected by Euclid.

The QUADRATURE *of Curves*, that is, the meaſuring of their areas, or the finding a rectilineal ſpace equal to a propoſed curvilineal one, is a matter of much deeper ſpeculation; and makes a part of the ſublime or higher geometry. The lunes of Hypocrates are the firſt curves that were ſquared, as far as we know of. The circle was attempted by Euclid and others before him: he ſhewed indeed the proportion of one circle to another, and gave a good method of approximating to the area of the circle, by deſcribing a polygon between any two concentric circles, however near their circumferences might be to each other. At this time the conic ſections were admitted in geometry, and Archimedes, perfectly, for the firſt time, ſquared the parabola, and he determined the relations of ſpheres, ſpheroids, and conoids, to cylinders and cones; and by purſuing the method of exhauſtions, or by means of inſcribed and circumſcribed polygons, he approximated to the periphery and area of the circle; ſhewing that the diameter is to the circumference nearly as 7 to 22, and the area of the circle to the ſquare of the diameter as 11 to 14 nearly. Archimedes likewiſe determined the relation between the circle and ellipſe, as well as that of their ſimilar parts. It is probable too that he attempted the hyperbola; but it is not likely that he met with any ſucceſs, ſince approximations to its area are all that can be given by the various methods that have ſince been invented. Beſide theſe figures, he left a treatiſe on a ſpiral curve; in which he determined the relation of its area to that of the circumſcribed circle; as alſo the relation of their ſectors.

Several other eminent men among the Ancients wrote upon this ſubject, both before and after Euclid and Archimedes; but their attempts were uſually confined to particular parts of it, and made according to methods not eſſentially different from theirs. Among theſe are to be reckoned Thales, Anaxagoras, Pythagoras, Bryſon, Antiphon, Hypocrates of Chios, Plato, Apollonius, Philo, and Ptolomy; moſt of whom wrote upon the Quadrature of the circle; and thoſe after Archimedes, by his method, uſually extended the approximation to a higher degree of accuracy.

Many of the Moderns have alſo proſecuted the ſame problem of the Quadrature of the circle, after the ſame methods, to ſtill greater lengths; ſuch are Vieta, and Metius; whoſe ratio between the diameter and the circumference, is that of 113 to 355, which is within about $\frac{3}{10000000}$ of the true ratio; but above all, Ludolph van Collen, or a Ceulen, who, with an amazing degree of induſtry and patience, by the ſame methods, extended the ratio to 36 places of figures, making the ratio to be that of

1 to 3·14159,26535,89793,23846,26433,83279,50288 + or 9 —. And the ſame was repeated and confirmed by his editor Snellius. See DIAMETER, and CIRCLE; alſo the Preface to my Menſuration.

Though the Quadrature, eſpecially of the circle, be a thing which many of the principal mathematicians, among the Ancients, were very ſolicitous about; yet nothing of this kind has been done ſo conſiderable, as about

about and ſince the middle of the laſt century; when, for example, in the year 1657, Sir Paul Neil, Lord Brouncker, and Sir Chriſtopher Wren geometrically demonſtrated the equality of ſome curvilineal ſpaces to rectilineal ones. Soon after this, other perſons did the like in other curves; and not long afterwards the thing was brought under an analytical calculus, the firſt ſpecimen of which ever publiſhed, was given by Mercator in 1638, in a demonſtration of Lord Brouncker's Quadrature of the hyperbola, by Dr. Wallis's method of reducing an algebraical fraction into an infinite ſeries by diviſion.

Though, by the way, it appears that Sir Iſaac Newton had diſcovered a method of attaining the area of all quadrable curves analytically, by his Method of Fluxions, before the year 1668. See his *Fluxions*, alſo his *Analyſis per Æquationes Numero Terminorum Infinitas*, and his *Introductio ad Quadraturam Curvarum;* where the Quadratures of Curves are given by general methods.

It is conteſted, between Mr. Huygens and Sir Chriſtopher Wren, which of the two firſt found out the Quadrature of any determinate cycloidal ſpace. Mr. Leibnitz afterwards diſcovered that of another ſpace; and Mr. Bernouili, in 1699, found out the Quadrature of an infinity of cycloidal ſpaces, both ſegments and ſectors &c.

As to the Quadrature of the Circle in particular, or the finding a ſquare equal to a given circle, it is a problem that has employed the mathematicians of all ages, but ſtill without the deſired ſucceſs. This depends on the ratio of the diameter to the circumference, which has never yet been determined in preciſe numbers. Many perſons have approached very near this ratio; for which ſee CIRCLE.

Strict geometry here failing, mathematicians have had recourſe to other means, and particularly to a ſort of curves called quadratices: but theſe being mechanical curves, inſtead of geometrical ones, or rather tranſcendental inſtead of algebraical ones, the problem cannot fairly be effected by them.

Hence recourſe has been had to analytics. And the problem has been attempted by three kinds of algebraical calculations. The firſt of theſe gives a kind of tranſcendental Quadratures, by equations of indefinite degrees. The ſecond by vulgar numbers, though irrationally ſuch; or by the roots of common equations, which for the general Quadrature is impoſſible. The third by means of certain ſeries, exhibiting the quantity of a circle by a progreſſion of terms. See SERIES.

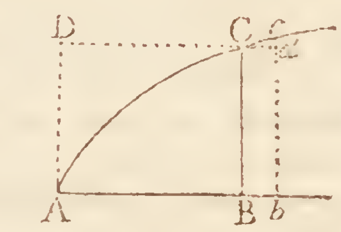
Thus, for example, the diameter of a circle being 1, it has been found that the quadrant, or one-fourth of the circumference, is equal to $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c, making an infinite ſeries of fractions, whoſe common numerator is 1, and denominators the natural ſeries of odd numbers; and all theſe terms alternately will be too great, and too little. This ſeries was diſcovered by Leibnitz and Gregory. And the ſame ſeries is alſo the area of the circle.

If the ſum of this ſeries could be found, it would give the Quadrature of the circle: but this is not yet done; nor is it at all probable that it ever will be done; though the impoſſibility has never yet been demonſtrated.

To this it may be added, that as the ſame magnitude may be expreſſed by ſeveral different ſeries, poſſibly the circumference of the circle may be expreſſed by ſome other ſeries, whoſe ſum may be found. And there are many other ſeries, by which the quadrant, or area, to the diameter, has been expreſſed; though it has never been found that any one of them is actually ſummable. Such as this ſeries, $1 - \frac{1}{6} - \frac{1}{40} - \frac{1}{112}$ &c, invented by Newton; with innumerable others.

But though a definite Quadrature of the whole circle was never yet given, nor of any aliquot part of it; yet certain other portions of it have been ſquared. The firſt partial Quadrature was given by Hippocrates of Chios; who ſquared a portion called, from its figure, the *lune*, or *lunule;* but this Quadrature has no dependence on that of the circle. And ſome modern geometricians have found out the Quadrature of any portion of the lune taken at pleaſure, independently of the Quadrature of the circle; though ſtill ſubject to a certain reſtriction, which prevents the Quadrature from being perfect, and what the geometricians call abſolute and indefinite. See LUNE. And for the Quadrature of the different kinds of curves, ſee their ſeveral particular names.

QUADRATURES *by Fluxions.*—The moſt general method of Quadratures yet diſcovered, is that of Newton, by means of Fluxions, and is as follows. AC being any curve to be ſquared, AB an abſciſs, and BC an ordinate perpendicular to it, alſo bc another ordinate indefinitely near to the former. Putting AB $= x$, and BC $= y$;



then is Bb $= \dot{x}$ the fluxion of the abſciſs, and $y\dot{x} =$ Cb the fluxion of the area ABC ſought. Now let the value of the ordinate y be found in terms of the abſciſs x, or in a function of the abſciſs, and let that function be called X, that is $y =$ X; then ſubſtituting X for y in $y\dot{x}$, gives X$\dot{x}$ the fluxion of the area; and the fluent of this, being taken, gives the area or Quadrature of ABC as required, for any curve, whatever its nature may be.

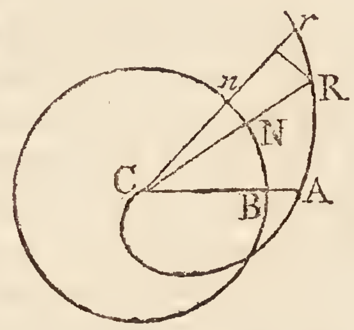
Ex. Suppoſe for example, AC to be a common parabola; then its equation is $px = y^2$, where p is the parameter; which gives $y = \sqrt{px}$, the value of y in a function of x, and is what is called X above; hence then $y\dot{x} = \dot{x}\sqrt{px} = p^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}$ is the fluxion of the area; and the fluent of this is $\frac{2}{3}p^{\frac{1}{2}}x^{\frac{3}{2}} = \frac{2}{3}x\sqrt{px} = \frac{2}{3}xy = \frac{2}{3}$ of the circumſcribing rectangle BD; which therefore is the Quadrature of the parabola.

Again, if AC be a circle whoſe diameter is d; then its equation is $y^2 = dx - x^2$, which gives $y = \sqrt{dx - x^2}$, and the fluxion of the area $y\dot{x} = \dot{x}\sqrt{dx - x^2}$. But as the fluent of this cannot be found in finite terms, the quantity $\sqrt{dx - x^2}$ is thrown into a ſeries, and then the fluxion of the area is

is $y\dot{x} = \dot{x}\sqrt{dx - x^2} = \dot{x}\sqrt{dx} \times (1 - \frac{x}{2d} - \frac{x^2}{2.4d^2} - \frac{1.3x^3}{2.4.6d^3}$ &c) ; and the fluent of this gives

$$x\sqrt{dx} \times (\frac{2}{3} - \frac{1}{5}.\frac{x}{d} - \frac{1}{4.7}.\frac{x^2}{d^2} - \frac{1.3}{4.6.9}.\frac{x^3}{d^3}\text{\&c})$$

for the general expreſſion of the area ABC. Now when the ſpace becomes a ſemicircle, x becomes $= d$, and then the ſeries above becomes $d^2.(\frac{2}{3} - \frac{1}{5} - \frac{1}{4.7} - \frac{1.3}{4.6.9}$ &c) for the area of the ſemicircle whoſe diameter is d.



In ſpirals CAR, or curves referred to a centre C; put $y =$ any radius CR, $x =$ BN the arc of a circle deſcribed about the centre C, at any diſtance CB $= a$, and Cnr another ray indefinitely near CNR: then $\frac{1}{2}$ CN . Nn = $\frac{1}{2}a\dot{x}$ = CNn, and by ſim. fig. CN^2 : CR^2 or $a^2 : y^2$ :: CNn : $\frac{y^2\dot{x}}{2a}$ = CRr the fluxion of the area deſcribed by the revolving ray CR ; then the fluent of this, for any particular caſe, will be the Quadrature of the ſpiral. So if, for inſtance, it be Archimedes's ſpiral, in which $x : y$ in a conſtant ratio ſuppoſe as $m : n$, or $my = nx$, and $y^2 = \frac{n^2x^2}{m^2}$; hence then CR$r = \frac{y^2\dot{x}}{2a} = \frac{n^2x^2\dot{x}}{2am^2}$ the fluxion of the area; the fluent of which is $\frac{n^2x^3}{6am^2} = \frac{xy^2}{6a}$ the general Quadrature of the ſpiral of Archimedes.

QUADRILATERAL, or Quadrilateral *Figure*, is a figure comprehended by four right lines; and having conſequently alſo four angles, for which reaſon it is otherwiſe called a quadrangle.

The general term Quadrilateral comprehends theſe ſeveral particular ſpecies or figures, viz, the ſquare, parallelogram, rectangle, rhombus, rhomboides, and trapezium.

If the oppoſite ſides be parallel, the Quadrilateral is a parallelogram. If the parallelogram have its angles right ones, it is a rectangle; if oblique, it is an oblique one. The rectangle having all its ſides equal, becomes a ſquare; and the oblique parallelogram having all its ſides equal, is a rhombus, but if only the oppoſites be equal, it is a rhomboides. All other forms of the Quadrilateral, are trapeziums, including all the irregular ſhapes of it.

The ſum of all the four angles of any Quadrilateral, is equal to 4 right angles. Alſo, the two oppoſite angles of a Quadrilateral inſcribed in a circle taken together, are equal to two right angles. And in this caſe the rectangle of the two diagonals, is equal to the ſum of the two rectangles of the oppoſite ſides. For the properties of the particular ſpecies of Quadrilaterals, ſee their reſpective names, Square, Rectangle, Parallelogram, Rhombus, Rhomboides, and Trapezium.

QUADRIPARTITION, is the dividing by 4, or into four equal parts.—Hence *quadripartite*, &c, the 4th part, or ſomething parted into four.

QUADRUPLE, is four-fold, or ſomething taken four times, or multiplied by 4; and ſo is the converſe of Quadripartition.

QUALITY, denotes generally the property or affection of ſome being, by which it affects our ſenſes in a certain way, &c.

Senſible Qualities are ſuch as are the more immediate object of the ſenſes: as figure, taſte, colour, ſmell, hardneſs, &c.

Occult Qualities, among the Ancients, were ſuch as did not admit of a rational ſolution in their way.

Dr. Keil demonſtrates, that every Quality which is propagated in orbem, ſuch as light, heat, cold, odour, &c, has its efficacy or intenſity either increaſed, or decreaſed, in a duplicate ratio of the diſtances from the centre of radiation inverſely. So at double the diſtance from the earth's centre, or from a luminous or hot body, the weight or light or heat, is but a 4th part; and at 3 times the diſtance, they are 9 times leſs, or a 9th part, &c.

Sir Iſaac Newton lays it down as one of the rules of philoſophizing, that thoſe Qualities of bodies that are incapable of being intended and remitted, and which are found to obtain in all bodies upon which the experiment could ever be tried, are to be eſteemed univerſal Qualities of all bodies.

Quality *of Curvature*, in the higher geometry, is uſed to ſignify its form, as it is more or leſs inequable, or as it is varied more or leſs in its progreſs through different parts of the curve. Newton's Method of Fluxions, pa. 75; and Maclaurin's Fluxions, art. 369.

QUANTITY, denotes any thing capable of eſtimation, or menſuration; or which being compared with another thing of the ſame kind, may be ſaid to be either greater or leſs, equal or unequal to it.

Mathematics is the doctrine or ſcience of Quantity.

Phyſical or *Natural* Quantity, is of two kinds: 1ſt, that which nature exhibits in matter, and its extenſion; and 2dly, in the powers and properties of natural bodies; as gravity, motion, light, heat, cold, denſity, &c.

Quantity is popularly diſtinguiſhed into continued and diſcrete.

Continued Quantity, is when the parts are connected together, and is commonly called magnitude; which is the object of geometry.

Diſcrete Quantity, is when the parts, of which it conſiſts, exiſt diſtinctly, and unconnected; which makes what is called multitude or number, the object of arithmetic.

The notion of continued Quantity, and its difference from diſcrete, appears to ſome without foundation. Mr. Machin conſiders all mathematical Quantity, or that for which any ſymbol is put, as nothing elſe but number, with regard to ſome meaſure, which is conſidered as 1; for that we know nothing preciſely how much any thing is, but by means of number. The notion of continued Quantity, without regard to ſome meaſure, is indiſtinct and confuſed; and though ſome ſpecies of ſuch Quantity, conſidered phyſically, may be deſcribed by motion, as lines by the motion of points,

points, and furfaces by the motion of lines; yet the magnitudes, or mathematical Quantities, are not made by the motion, but by numbering according to a meafure. Philof. Tranf. numb. 447, pa. 228.

QUANTITY *of Action*. See ACTION.

QUANTITY *of Curvature* at any point of a curve is determined by the circle of curvature at that point, and is reciprocally proportional to the radius of curvature.

QUANTITY *of Matter* in any body, is its meafure arifing from the joint confideration of its magnitude and denfity, being expreffed by, or proportional to the product of the two. So,

if M and m denote the magnitude of two bodies,
and D and d their denfities;
then DM and dm will be as their Quantities of matter.

The Quantity of matter of a body is beft difcovered by its abfolute weight, to which it is always proportional, and by which it is meafured.

QUANTITY *of Motion*, or the *Momentum*, of any body, is its meafure arifing from the joint confideration of its Quantity, and the velocity with which it moves. So,

if q denote the Quantity of matter,
and v the velocity of any body;
then qv will be its quantity of motion.

QUANTITIES, in Algebra, are the expreffions of indefinite numbers, that are ufually reprefented by letters. Quantities are properly the fubject of Algebra; which is wholly converfant in the computation of fuch Quantities.

Algebraic Quantities are either *given* and *known*, or elfe they are *unknown* and *fought*. The given or known Quantities are reprefented by the firft letters of the alphabet, as a, b, c, d, e, &c, and the unknown or required Quantities, by the laft letters, as z, y, x, w, &c.

Again, Algebraic Quantities are either pofitive or negative.

A pofitive or affirmative Quantity, is one that is to be added, and has the fign + or plus prefixed, or underftood; as ab or $+ ab$. And a negative or privative Quantity, is one that is to be fubtracted, and has the fign − or minus prefixed; as $- ab$.

QUART, a meafure of capacity, being the quarter or 4th part of fome other meafure. The Englifh Quart is the 4th part of the gallon, and contains two pints. The Roman Quart, or quartarius, was the 4th part of their congius. The French, befides their Quart or pot of 2 pints, have various other Quarts, diftinguifhed by the whole of which they are Quarters; as Quart de muid, and Quart de boiffeau.

QUARTER, the 4th part of a whole, or one part of an integer, which is divided into four equal portions.

QUARTER, in weights, is the 4th part of the quintal, or hundred weight; and fo contains 28 pounds.

QUARTER is alfo a dry meafure, containing of corn 8 bufhels ftriked; and of coals the 4th part of a chaldron.

Quarter, in Aftronomy, the moon's period, or lunation, is divided into 4 ftages or Quarters; each containing between 7 and 8 days. The firft Quarter is from the new moon to the quadrature; the fecond is from thence to the full moon, and fo on.

QUARTER, in Navigation, is the Quarter or 4th part of a point, wind, or rhumb; or of the diftance between two points &c. The Quarter contains an arch of $2^\circ 48' 45''$, being the 4th part of $11^\circ 15'$, which is one point.

QUARTER *Round*, in Architecture, is a term ufed by the workmen for any projecting moulding, whofe contour is a Quarter of a circle, or nearly fo.

QUARTILE, an afpect of the planets when they are at the diftance of 3 figns or 90° from each other: and is denoted by the character □.

QUEUE D'ARONDE, or *Swallow's Tail*, in Fortification, is a detached or outwork, whofe fides fpread or open towards the campaign, or draw narrower and clofer towards the gorge. Of this kind are either fingle or double tenailles, and fome horn-works, whofe fides are not parallel, but are narrow at the gorge, and open at the head, like the figure of a fwallow's tail.

On the contrary, when the fides are lefs than the gorge, the work is called *contre Queue d'aronde*.

QUEUE *d'aronde*, in Carpentry, a method of jointing, called alfo dove-tailing.

QUINCUNX, denotes $\frac{5}{12}$ths of any thing. So 10 is quincunx of 24, being $\frac{5}{12}$ of it.

QUINCUNX, in Aftronomy, is that pofition, or afpect, of the planets, when diftant from each other by $\frac{5}{12}$ths of the whole circle, or 5 figns out of the 12, that is 150 degrees. The Quincunx is marked Q, or V*c*.

QUINDECAGON, is a plane figure of 15 angles, and confequently the fame number of fides. When thofe are all equal, it is a regular Quindecagon, otherwife not.

Euclid fhews how to infcribe this figure in a circle, prop. 16, lib. 4. And the fide of a regular Quindecagon, fo infcribed, is equal in power to the half difference between the fide of the equilateral triangle, and the fide of the pentagon; and alfo to the difference of the perpendiculars let fall on both fides, taken together.

QUINQUAGESIMA-*Sunday*, is the fame as Shrove-Sunday, and is fo called as being about the 50th day before Eafter, being indeed the 7th Sunday before it. Anciently the term Quinquagefima was ufed for Whitfunday, and for the 50 days between Eafter and Whitfunday; but to diftinguifh this Quinquagefima from that before Eafter, it was called the pafchal Quinquagefima.

QUINQUEANGLED, or Quinqueangular, confifting of 5 angles.

QUINTAL, the weight of a hundred pounds, in moft countries; but in England it is the hundred weight, or 112 pounds. Quintal was alfo formerly ufed for a weight of lead, iron, or other common metal, ufually equal to a hundred pounds, at 6 fcore to the hundred.

QUINTILE, in Aftronomy, an afpect of the planets when they are diftant the 5th part of the zodiac, or 72 degrees; and is marked thus, C, or O.

QUINTUPLE, is five-fold, or five times as much as another thing.

QUOIN, in Architecture, an angle or corner of ftone or brick walls. When thefe ftand out beyond the reft of the wall, their edges being chamferred off, they are called *ruftic Quoins*.

QUOIN, in Artillery, is a loofe wedge of wood, which is

is put in below the breech of a cannon, to raise or depress it more or less.

QUOTIENT, in Arithmetic, is the result of the operation of division, or the number that arises by dividing the dividend by the divisor, shewing how often the latter is contained in the former. Thus the Quotient of 12 divided by 3 is 4; which is usually thus disposed, or expressed,

3) 12 (4 the quotient,

or thus $12 \div 3 = 4$ the Quotient, or thus $\frac{12}{3}$ like a vulgar fraction; all these meaning the same thing.—In division, as the divisor is to the dividend, so is unity or 1 to the Quotient; thus $3 : 12 :: 1 : 4$ the Quotient.

R.

RAD RAF

RADIANT *Point*, or Radiating *Point*, is any point from whence rays proceed.

Every Radiant point diffuses innumerable rays all around: but those rays only are visible from which right lines can be drawn to the pupil of the eye; because the rays are all in right lines. All the rays proceeding from the same Radiant continually diverge; but the crystalline collects or reunites them again.

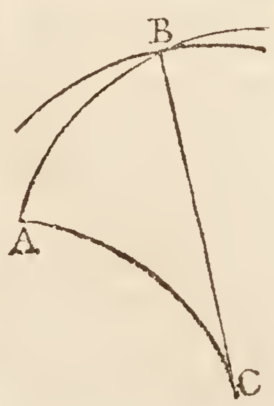
RADIATION, is the casting or shooting forth of rays of light as from a centre.—Every visible body is a radiating body; it being only by means of its rays that it affects the eye.—The surface of a radiating or visible body, may be conceived as consisting of radiant points.

RADICAL *Sign*, in Algebra, the sign or character denoting the root of a quantity; and is this $\surd$. So $\sqrt{2}$ is the square root of 2, and $\sqrt[3]{2}$ is the cube root of 2, &c.

RADIOMETER, a name which some writers give to the Radius Astronomicus, or Jacob's Staff. See Fore-Staff.

RADIUS, in Geometry, the semidiameter of a circle; or a right line drawn from the centre to the circumference.—It is implied in the definition of a circle, and it is apparent from its construction, that all the radii of the same circle are equal.—The Radius is sometimes called, in Trigonometry, the Sinus Totus, or whole sine.

Radius, in the Higher Geometry. Radius *of the Evoluta*, Radius *Osculi*, called also the *Radius of concavity*, and the *Radius of curvature*, is the right line CB, representing a thread, by whose evolution from off the curve AC, upon which it was wound, the curve AB is formed. Or it is the Radius of a circle having the same curvature, in a given point of the curve at B, with that of the curve in that point. See Curvature and Evolute, where the method of finding this Radius may be seen.



Radius *Astronomicus*, an instrument usually called Jacob's Staff, the Cross-staff, or Fore-staff.

Radius, in Mechanics, is applied to the spokes of a wheel; because issuing like rays from its centre.

Radius, in Optics. See Ray.

Radius *Vector*, is used for a right line drawn from the centre of force in any curve in which a body is supposed to move by a centripetal force, to that point of the curve where the body is supposed to be.

RADIX, or *Root*, is a certain finite expression or function, which, being evolved or expanded according to the rules proper to its form, shall produce a series. That finite expression, or Radix, is also the value of the infinite series. So $\frac{1}{3}$ is the radix of ·3333 &c, because $\frac{1}{3}$ being evolved or expanded, by dividing 1 by 3, gives the infinite series ·3333 &c. In like manner, the Radix

of $1 - r + r^2 - r^3 + r^4$ &c is $\frac{1}{1 + r}$,

of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16}$ &c is $\frac{1}{1 + \frac{1}{2}}$,

of $1 - 1 + 1 - 1 + 1$ &c is $\frac{1}{1 + 1}$,

of $1 - 2 + 4 - 8 + 16$ &c is $\frac{1}{1 + 2}$,

of $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$ &c is $\frac{1}{2 + 1}$,

of $1 + x + x^2 + x^3 + x^4$ &c is $\frac{1}{1 + x}$,

of $1 + 2x + 3x^2 + 4x^3$ &c is $\frac{1}{(1-x)^2}$,

of $1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{5x^6}{16}$ &c is $\sqrt{\frac{1}{1 - x^2}}$.

See my Tracts, vol. 1, pa. 9, and 31, &c.

RAFTERS, in Architecture, are pieces of timber which stand by pairs on the raising-piece, or wall plate, and meet in an angle at the top, forming the roof of a building.

RAIN, water that defcends from the atmofphere in the form of drops of a confiderable fize. Rain is apparently a precipitated cloud; as clouds are nothing but vapours raifed from moifture, waters, &c. By this circumftance it is diftinguifhed from dew and fog: in the former of which the drops are fo fmall that they are quite invifible; and in the latter, though their fize be larger, they feem to have very little more fpecific gravity than the atmofphere itfelf, and may therefore be reckoned hollow fpherules rather than drops.

It is univerfally agreed, that Rain is produced by the water previoufly abforbed by the heat of the fun, or otherwife, from the terraqueous globe, into the atmofphere, as vapours, or veficulæ. Thefe veficulæ, being fpecifically lighter than the atmofphere, are buoyed up by it, till they arrive at a region where the air is in a juft balance with them; and there they float, till by fome new agent they are converted into clouds, and thence either into Rain, fnow, hail, mift, or the like.

But the agent in this formation of the clouds into Rain, and even of the vapours into clouds, has been much controverted. Moft philofophers will have it, that the cold, which conftantly occupies the fuperior regions of the air, chills and condenfes the veficulæ, at their arrival from a warmer quarter; congregates them together, and occafions feveral of them to coalefce into little maffes: and thus their quantity of matter increafing in a higher proportion than their furface, they become an overload to the thin air, and fo defcend in Rain.

Dr. Derham accounts for the precipitation, hence; that the veficulæ being full of air, when they meet with a colder air than that they contain, this is contracted into a lefs fpace: and confequently the watry fhell or cafe becomes thicker, fo as to become heavier than the air, &c.

But this feparation cannot be afcribed to cold, fince Rain often takes place in very warm weather. And though we fhould fuppofe the condenfation owing to the cold of the higher regions, yet there is a remarkable fact which will not allow us to have recourfe to this fuppofition: for it is certain that the drops of Rain increafe in fize confiderably as they defcend. On the top of a hill for inftance, they will be fmall and inconfiderable, forming only a drizzling fhower; but half way down the hill it is much more confiderable; and at the bottom the drops will be very large, defcending in an impetuous Rain. Which fhews that the atmofphere condenfes the vapours as well where it is warm as where it is cold.

Others allow the cold only a part in the action, and bring in the winds as fharers with it: alledging, that a wind blowing againft a cloud will drive its veficulæ upon one another, by which means feveral of them, coalefcing as before, will be enabled to defcend; and that the effect will be ftill more confiderable, if two oppofite winds blow together towards the fame place: they add, that clouds already formed, happening to be aggregated by frefh acceffions of vapour continually afcending, may thence be enabled to defcend.

Yet the grand caufe, according to Rohault, is ftill behind. That author conceives it to be the heat of the air, which, after continuing for fome time near the earth, is at length carried up on high by a wind, and there thawing the fnowy villi or flocks of the half frozen veficulæ, it reduces them into drops; which, coalefcing, defcend, and have their diffolution perfected in their progrefs through the lower and warmer ftages of the atmofphere.

Others, as Dr. Clarke, &c, afcribe this defcent of the clouds rather to an alteration of the atmofphere than of the veficulæ; and fuppofe it to arife from a diminution of the fpring or elaftic force of the air. This elafticity, which depends chiefly or wholly on the dry terrene exhalations, being weakened, the atmofphere finks under its burden; and the clouds fall, on the common principle of precipitation.

Now the fmall veficulæ, by thefe or any other caufes, being once upon the defcent, will continue to defcend notwithftanding the increafe of refiftance they every moment meet with in their progrefs through ftill denfer and denfer parts of the atmofphere. For as they all tend toward the fame point, viz, the centre of the earth, the farther they fall, the more coalitions will they make; and the more coalitions, the more matter will there be under the fame furface; the furface only increafing as the fquares, but the folidity as the cubes of the diameters: and the more matter under the fame furface, the lefs friction or refiftance there will be to the fame matter.

Thus then, if the caufes of rain happen to act early enough to precipitate the afcending veficulæ, before they are arrived at any confiderable height, the coalitions being few in fo fhort a defcent, the drops will be proportionably fmall; thus forming what is called dew. If the vapours prove more copious, and rife a little higher, there is produced a mift or fog. A little higher ftill, and they produce a fmall rain, &c. If they neither meet with cold nor wind enough to condenfe or diffipate them; they form a heavy, thick, dark fky, which lafts fometimes feveral days, or even weeks.

But later writers on this part of philofophical fcience have, with greater fhew of truth, confidered Rain as an electrical phenomenon. Signior Beccaria reckons Rain, hail, and fnow, among the effects of a moderate electricity in the atmofphere. Clouds that bring Rain, he thinks are produced in the fame manner as thunder clouds, only by a moderate electricity. He defcribes them at large; and the refemblance which all their phenomena bear to thofe of thunder clouds, is very ftriking. He notes feveral circumftances attending Rain without lightning, which render it probable that it is produced by the fame caufe as when it is accompanied with lightning. Light has been feen among the clouds by night in rainy weather; and even by day rainy clouds are fometimes feen to have a brightnefs evidently independent of the fun. The uniformity with which the clouds are fpread, and with which the Rain falls, he thinks are evidences of an uniform caufe like that of electricity. The intenfity alfo of electricity in his apparatus, ufually correfponded very nearly to the quantity of Rain that fell in the fame time. Sometimes all the phenomena of thunder, lightning, hail, Rain, fnow, and wind, have been obferved at one time; which fhews the connection they all have with fome common caufe. Signior Beccaria therefore fuppofes that, previous to Rain, a quantity of electric

matter escapes out of the earth, in some place where there is a redundancy of it; and in its ascent to the higher regions of the air, collects and conducts into its path a great quantity of vapours. The same cause that collects, will condense them more and more; till, in the places of the nearest intervals, they come almost into contact, so as to form small drops; which, uniting with others as they fall, come down in the form of Rain. The Rain will be heavier in proportion as the electricity is more vigorous, and the cloud approaches more nearly to a thunder cloud: &c. See *Lettere dell Elettricismo*; and Priestley's Hist. &c of Electricity, vol. 1, pa. 427, &c, 8vo. And for farther accounts of the phenomena of Rain &c, see BAROMETER, EVAPORATION, OMBROMETER, PLUVIAMETER, VAPOUR, &c. See also the Theory of Rain, by Dr. James Hutton, art. 2 vol. 1 of Transactions of the Royal Society of Edinburgh.

Quantity of RAIN. As to the general quantity of Rain that falls, with its proportion in several places at the same time, and in the same place at different times, there are many observations, journals, &c, in the Philos. Trans. the Memoirs of the French Academy, &c. And upon measuring the rain that falls annually, its depth, on a medium, is found as in the following table:

Mean Annual Depth of Rain for several Places.

At	*Observed by*	*Insh.*
Townley, in Lancashire	Mr. Townley - -	$42\frac{1}{2}$
Upminster, in Essex	Dr. Derham - -	$19\frac{1}{4}$
Zurich, Swisserland -	Dr. Scheuchzer - -	$32\frac{1}{4}$
Pisa, in Italy - -	Dr. Mich. Ang. Tilli	$43\frac{1}{4}$
Paris, in France -	M. De la Hire - -	19
Lisle, Flanders - -	M. De Vauban - -	24

Quantity of Rain fallen in several Years at Paris and Upminster.

At Paris.	*Years.*	*At Upminster.*
Inches 21·37	1700	19·03 Inches
27·77	1701	18·69
17·45	1702	20·38
18·51	1703	23·99
21·20	1704	15·80
14·82	1705	16·93
20·19	Mediums	19·14

Medium Quantity of Rain at London, for several Years, from the Philos. Trans.

Viz, in	1774 - - - -	26·328 inches.
	1775 - - - -	24·083
	1776 - - - -	20·354
	1777 - - - -	25·371
	1778 - - - -	20·772
	1779 - - - -	26·785
	1780 - - - -	17·313
Medium of these 7 years		23·001

See also Philos. Trans. Abr. vol. 4, pt. 2, pa. 81, &c; and vol. 10 in many places; also the Meteorological Journal of the Royal Society, published annually in the Philos. Trans. and the article PLUVIAMETER or OMBROMETER.

It is reasonably to be expected, and all experience shews, that the most Rain falls in places near the sea coast, and less and less as the places are situated more inland. Some differences also arise from the circumstances of hills, valleys, &c. So when the quantity of Rain fallen in one year at London, is 20 inches, that on the western coast of England will often be twice as much, or 40 inches, or more. Those winds also bring most Rain, that blow from the quarter in which is the most and nearest sea; as our west and south-west winds.

It is also found, by the pluviameter or Rain-gage, that, in any one place, the more Rain is collected in the instrument, as it is placed nearer the ground; without any appearance of a difference, between two places, on account of their difference of level above the sea, provided the instrument is but as far from the ground at the one place, as it is from the ground at the other. These effects are remarked in the Philos. Trans. for 1769 and 1771, the former by Dr. Heberden, and the latter by Mr. Daines Barrington. Dr. Heberden says, "A comparison having been made between the quantity of Rain, which fell in two places in London, about a mile distant from one another, it was found, that the Rain in one of them constantly exceeded that in the other, not only every month, but almost every time that it rained. The apparatus used in each of them was very exact, and both made by the same artist; and upon examining every probable cause, this unexpected variation did not appear to be owing to any mistake, but to the constant effect of some circumstance, which not being supposed to be of any moment, had never been attended to. The Rain-gage in one of these places was fixed so high, as to rise above all the neighbouring chimnies; the other was considerably below them; and there appeared reason to believe, that the difference of the quantity of Rain in these two places was owing to this difference in the placing of the vessel in which it was received. A funnel was therefore placed above the highest chimnies, and another upon the ground of the garden belonging to the same house, and there was found the same difference between these two, though placed so near one another, which there had been between them, when placed at similar heights in different parts of the town. After this fact was sufficiently ascertained, it was thought proper to try whether the difference would be greater at a much greater height; and a Rain-gage was therefore placed upon the square part of the roof of Westminster Abbey. Here the quantity of Rain was observed for a twelvemonth, the Rain being measured at the end of every month, and care being taken that none should evaporate by passing a very long tube of the funnel into a bottle through a cork, to which it was exactly fitted. The tube went down very near to the bottom of the bottle, and therefore the Rain which fell into it would soon rise above the end of the tube, so that the water was no where open to the air except

for

for the small space of the area of the tube: and by trial it was found that there was no sensible evaporation through the tube thus fitted up.

The following table shews the result of these observations.

From July the 7th 1766, to July the 7th 1767, there fell in a Rain-gage, fixed

1766.	Below the top of a house.	Upon the top of a house.	Upon Westminster Abbey.
	Inches.	*Inches.*	*Inches.*
From the 7th to the end of July	3·591	3·210	2·311
August	0·558	0·479	0·508
September	0·421	0·344	
October	2·364	2·061	1·416
November	1·079	0·842	0·632
December	1·612	1·258	0·994
1767, January	2·071	1·455	1·035
February	2·864	2·494	1·335
March	1·807	1·303	0·587
April	1·437	1·213	0·994
May	2·432	1·745	1·142
June	1·997	1·426	1·145
July 7	0·395	0·309	
	22·608	18·139	12·099

By this table it appears, that there fell below the top of a house above a fifth part more Rain, than what fell in the same space above the top of the same house; and that there fell upon Westminster Abbey not much above one half of what was found to fall in the same space below the tops of the houses. This experiment has been repeated in other places with the same result. What may be the cause of this extraordinary difference, has not yet been discovered; but it may be useful to give notice of it, in order to prevent that error, which would frequently be committed in comparing the Rain of two places without attending to this circumstance."

Such were the observations of Dr. Heberden on first announcing this circumstance, viz, of different quantities of Rain falling at different heights above the ground. Two years afterward, Daines Barrington Esq. made the following experiments and observations, to shew that this effect, with respect to different places, respected only the several heights of the instrument above the ground at those places, without regard to any real difference of level in the ground at those places.

Mr. Barrington caused two other Rain-gages, exactly like those of Dr. Heberden, to be placed, the one upon mount Rennig, in Wales, and the other on the plane below, at about half a mile's distance, the perpendicular height of the mountain being 450 yards, or 1350 feet; each gage being at the same height above the surface of the ground at the two stations.

The results of the Experiment are as below:

1770.	Bottom of the mountain.	Top of the mountain.
	Inches.	*Inches.*
From July 6 to 16	0·709	0·648
July 16 to 29	2·185	2·124
July 29 to Aug. 10.	0·610	0·656
Sept. 9 both bottles had run over.		
Sept. 9 to 30	3·234	2·464
Oct. 17. both bottles had run over.		
Oct. 17 to 22	0·747	0·885
Oct. 22 to 29	1·281	1·388
Nov. 20 both bottles were broken by the frost	8·766	8·165

"The inference to be drawn from these experiments, Mr. Barrington observes, seems to be, that the increase of the quantity of Rain depends upon its nearer approximation to the earth, and scarcely at all upon the height of places, provided the Rain-gages are fixed at about the same distance from the ground.

"Possibly also a much controverted point between the inhabitants of mountains and plains may receive a solution from these experiments; as in an *adjacent valley, at least*, very nearly the same quantity of Rain appears to fall within the same period of time as upon the neighbouring mountains."

Dr. Heberden also adds the following note. "It may not be improper to subjoin to the foregoing account, that, in places where it was first observed, a different quantity of Rain would be collected, according as the Rain-gages were placed above or below the tops of the neighbouring buildings; the Rain-gage below the top of the house, into which the greater quantity of Rain had for several years been found to fall, was above 15 feet above the level of the other Rain-gage, which in another part of London was placed above the top of the house, and into which the lesser quantity always fell. This difference therefore does not, as Mr. Barrington justly remarks, depend upon the greater quantity of atmosphere, through which the Rain descends: though this has been supposed by some, who have thence concluded that this appearance might readily be solved by the accumulation of more drops, in a descent through a great depth of atmosphere."

RAINBOW, *Iris*, or simply the *Bow*, is a meteor in form of a party-coloured arch, or semicircle, exhibited in a rainy sky, opposite to the sun, by the refraction and reflection of his rays in the drops of falling rain. There is also a secondary, or fainter bow, usually seen investing the former at some distance. Among naturalists, we also read of lunar Rainbows, marine Rainbows, &c.

The Rainbow, Sir Isaac Newton observes, never appears but where it rains in the sunshine; and it may be represented artificially, by contriving water to fall

 in

in ſmall drops, like rain, through which the ſun ſhining, exhibits a bow to a ſpectator placed between the ſun and the drops, eſpecially if there be diſpoſed beyond the drops ſome dark body, as a black cloth, or ſuch like.

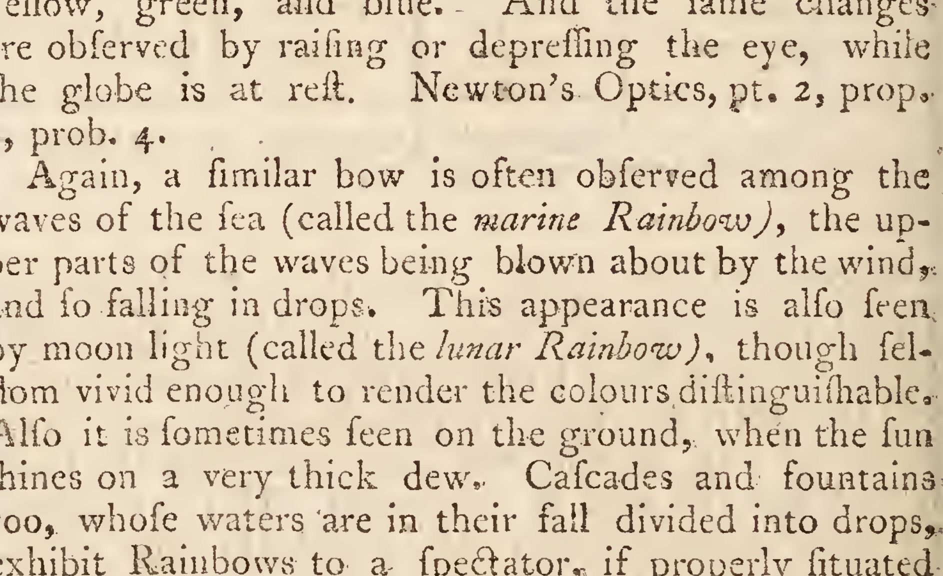
Some of the ancients, as appears by Ariſtotle's tract on Meteors, knew that the Rainbow was cauſed by the refraction of the ſun's light in drops of falling rain. Long afterwards, one Fletcher of Breſlaw, in a treatiſe which he publiſhed in 1571, endeavoured more particularly to account for the colours of the Rainbow by means of a double refraction, and one reflection. But he imagined that a ray of light, after entering a drop of rain, and ſuffering a refraction, both at its entrance and exit, was afterwards reflected from another drop, before it reached the eye of the ſpectator. It ſeems he overlooked the reflection at the farther ſide of the drop, or elſe he imagined that all the bendings of the light within the drop would not make a ſufficient curvature, to bring the ray of the ſun to the eye of the ſpectator. But Antonio de Dominis, biſhop of Spalato, about the year 1590, whoſe treatiſe *De Radiis Viſûs et Lucis* was publiſhed in 1611 by J. Bartolus, firſt advanced, that the double refraction of Fletcher, with an intervening reflection, was ſufficient to produce the colours of the Rainbow, and alſo to bring the rays that formed them to the eye of the ſpectator, without any ſubſequent reflection. He diſtinctly deſcribes the progreſs of a ray of light entering the upper part of the drop, where it ſuffers one refraction, and after being by that thrown upon the back part of the inner ſurface, is from thence reflected to the lower part of the drop; at which place undergoing a ſecond refraction, it is thereby bent ſo as to come directly to the eye. To verify this hypotheſis, he procured a ſmall globe of ſolid glaſs, and viewing it when it was expoſed to the rays of the ſun, in the ſame manner in which he had ſuppoſed the drops of rain were ſituated with reſpect to them, he actually obſerved the ſame colours which he had ſeen in the true Rainbow, and in the ſame order. Thus this author ſhewed how the interior bow is formed in round drops of rain, viz, by two refractions of the ſun's rays and one reflection between them; and he likewiſe ſhewed that the exterior bow is formed by two refractions and two ſorts of reflections between them in each drop of water.

The theory of A. de Dominis was adopted, and in ſome degree improved with reſpect to the exterior bow, by Des Cartes, in his treatiſe on Meteors; and indeed he was the firſt who, by applying mathematics to the inveſtigation of this ſurpriſing appearance, ever gave a tolerable theory of the Rainbow. Philoſophers were however ſtill at a loſs when they endeavoured to aſſign reaſons for all the particular colours, and for the order of them. Indeed nothing but the doctrine of the different refrangibility of the rays of light, a diſcovery which was reſerved for the great Newton, could furniſh a complete ſolution of this difficulty.

Dr. Barrow, in his Lectiones Opticæ, at Lect. 12, n. 14, ſays, that a friend of his (by whom we are to underſtand Mr. Newton) communicated to him a way of determining the angle of the Rainbow, which was hinted to Newton by Sluſius, without making a table of the refractions, as Des Cartes did. The doctor ſhews the method; as alſo ſeveral other matters, at n. 14, 15, 16, relating to the Rainbow, worthy the genius of thoſe two eminent men. But the ſubject was given more perfectly by Newton afterwards, viz, in his Optics, prop. 9; where he makes the breadth of the interior bow to be nearly 2° 15′, that of the exterior 3° 40′, their diſtance 8° 25′, the greateſt ſemidiameter of the interior bow 42° 17′, and the leaſt of the exterior 50° 42′, when their colours appear ſtrong and perfect.

The doctrine of the Rainbow may be illuſtrated and confirmed by experiment in ſeveral different ways. Thus, by hanging up a glaſs globe, full of water, in the ſun-ſhine, and viewing it in ſuch a poſture that the rays which come from the globe to the eye, may include an angle either of 42° or 50° with the ſun's rays; for ex. if the angle be about 42°, the ſpectator will ſee a full red colour in that ſide of the globe oppoſite to the ſun. And by varying the poſition ſo as to make that angle gradually leſs, the other colours, yellow, green, and blue, will appear ſucceſſively, in the ſame ſide of the globe, and that very bright. But if the angle be made about 50°, ſuppoſe by raiſing the globe, there will appear a red colour in that ſide of the globe toward the ſun, though ſomewhat faint; and if the angle be made greater, as by raiſing the globe ſtill higher, this red will change ſucceſſively to the other colours, yellow, green, and blue. And the ſame changes are obſerved by raiſing or depreſſing the eye, while the globe is at reſt. Newton's Optics, pt. 2, prop. 9, prob. 4.

Again, a ſimilar bow is often obſerved among the waves of the ſea (called the *marine Rainbow)*, the upper parts of the waves being blown about by the wind, and ſo falling in drops. This appearance is alſo ſeen by moon light (called the *lunar Rainbow)*, though ſeldom vivid enough to render the colours diſtinguiſhable. Alſo it is ſometimes ſeen on the ground, when the ſun ſhines on a very thick dew. Caſcades and fountains too, whoſe waters are in their fall divided into drops, exhibit Rainbows to a ſpectator, if properly ſituated during the time of the ſun's ſhining; and even water blown violently out of the mouth of an obſerver, ſtanding with his back to the ſun, never fails to produce the ſame phenomenon. The artificial Rainbow may even be produced by candle light on the water which is ejected by a ſmall fountain or jet d'eau. All theſe are of the ſame nature, and they depend upon the ſame cauſes; ſome account of which is as follows.



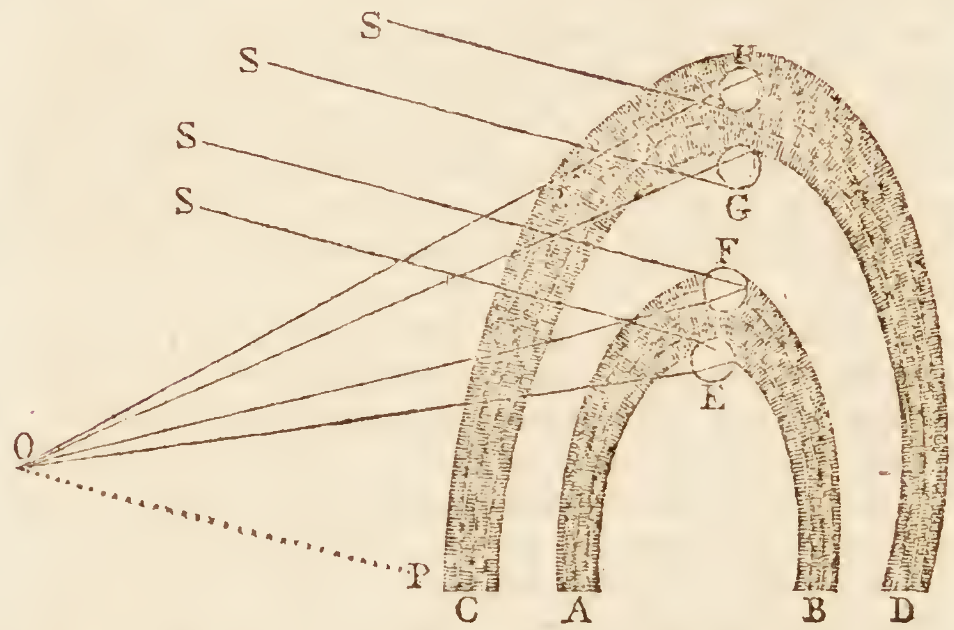
Let

Let the circle WQGB repreſent a drop of water, or a globe, upon which a beam of parallel light falls, of which let TB repreſent a ray falling perpendicularly at B, and which conſequently either paſſes through without refraction, or is reflected directly back from Q. Suppoſe another ray IK, incident at K, at a diſtance from B, and it will be refracted according to a certain ratio of the ſines of incidence and refraction to each other, which in rain water is as 529 to 396, to a point L, whence it will be in part tranſmitted in the direction L Z, and in part reflected to M, where it will again in part be reflected, and in part tranſmitted in the direction MP, being inclined to the line deſcribed by the incident ray in the angle IOP. Another ray AN, ſtill farther from B, and conſequently incident under a greater angle, will be refracted to a point F, ſtill farther from Q, whence it will be in part reflected to G, from which place it will in part emerge, forming an angle AXR with the incident AN, greater than that which was formed between the ray MP and its incident ray. And thus, while the angle of incidence, or diſtance of the point of incidence from B, increaſes, the diſtance between the point of reflection and Q, and the angle formed between the incident and emergent reflected rays, will alſo increaſe; that is, as far as it depends on the diſtance from B: but as the refraction of the ray tends to carry the point of reflection towards Q, and to diminiſh the angle formed between the incident and emergent reflected ray, and that the more the greater the diſtance of the point of incidence from B, there will be a certain point of incidence between B and W, with which the greateſt poſſible diſtance between the point of reflection and Q, and the greateſt poſſible angle between the incident and emergent reflected ray, will correſpond. So that a ray incident nearer to B ſhall, at its emergence after reflection, form a leſs angle with the incident, by reaſon of its more direct reflection from a point nearer to Q; and a ray incident nearer to W, ſhall at its emergence form a leſs angle with the incident, by reaſon of the greater quantity of the angles of refraction at its incidence and emergence. The rays which fall for a conſiderable ſpace in the vicinity of that point of incidence with which the greateſt angle of emergence correſponds, will, after emerging, form an angle with the incident rays differing inſenſibly from that greateſt angle, and conſequently will proceed nearly parallel to each other; and thoſe rays which fall at a diſtance from that point will emerge at various angles, and conſequently will diverge. Now, to a ſpectator, whoſe back is turned towards the radiant body, and whoſe eye is at a conſiderable diſtance from the globe or drop, the divergent light will be ſcarcely, if at all, perceptible; but if the globe be ſo ſituated, that thoſe rays that emerge parallel to each other, or at the greateſt poſſible angle with the incident, may arrive at the eye of the ſpectator, he will, by means of thoſe rays, behold it nearly with the ſame ſplendour at any diſtance.

In like manner, thoſe rays which fall parallel on a globe, and are emitted after two reflections, ſuppoſe at the points F and G, will emerge at H parallel to each other, when the angle they make with the incident AN is the leaſt poſſible; and the globe muſt be ſeen very reſplendent when its poſition is ſuch, that thoſe parallel rays fall on the eye of the ſpectator.

The quantities of theſe angles are determined by calculation, the proportion of the ſines of incidence and refraction to each other being known. And this proportion being different in rays which produce different colours, the angles muſt vary in each. Thus it is found, that the greateſt angle in rain water for the leaſt refrangible, or red rays, emitted parallel after one reflection, is 42° 2′, and for the moſt refrangible or violet rays, emitted parallel after one reflection, 40° 17′; likewiſe, after two reflections, the leaſt refrangible, or red rays, will be emitted nearly parallel under an angle of 50° 57′, and the moſt refrangible, or violet, under an angle of 54° 7′; and the intermediate colours will be emitted nearly parallel at intermediate angles.

Suppoſe now, that O is the ſpectator's eye, and OP a line drawn parallel to the ſun's rays, SE, SF, SG, and SH;



and let POE, POF, POG, POH be angles of 40° 17′, 42° 2′, 50° 57′, and 54° 7′ reſpectively; then theſe angles turned about their common ſide OP, will with their other ſides OE, OF, OG, OH deſcribe the verges of the two Rainbows, as in the figure. For, if E, F, G, H be drops placed any where in the conical ſuperficies deſcribed by OE, OF, OG, OH, and be illuminated by the ſun's rays SE, SF, SG, SH; the angle SEO being equal to the angle POE, or 40° 17′, will be the greateſt angle in which the moſt refrangible rays can, after one reflection, be refracted to the eye, and therefore all the drops in the line OE muſt ſend the moſt refrangible rays moſt copiouſly to the eye, and ſo ſtrike the ſenſe with the deepeſt violet colour in that region. In like manner, the angle SFO being equal to the angle POF, or 42° 2′, will be the greateſt in which the leaſt refrangible rays after one reflection can emerge out of the drops, and therefore thoſe rays muſt come moſt copiouſly to the eye from the drops in the line OF, and ſtrike the ſenſe with the deepeſt red colour in that region. And, by the ſame argument, the rays which have the intermediate degrees of refrangibility will come moſt copiouſly from drops between E and F, and ſtrike the ſenſes with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progreſs

progrefs from E to F, or from the infide of the bow to the outfide, in this order, violet, indigo, blue, green, yellow, orange, red. But the violet, by the mixture of the white light of the clouds, will appear faint, and inclined to purple.

Again, the angle SGO being equal to the angle POG, or 50° 57′, will be the leaft angle in which the leaft refrangible rays can, after two reflections, emerge out of the drops, and therefore the leaft refrangible rays muft come moft copioufly to the eye from the drops in the line OG, and ftrike the fenfe with the deepeft red in that region. And the angle SHO being equal to the angle POH, or 54° 7′, will be the leaft angle in which the moft refrangible rays, after two reflections, can emerge out of the drops, and therefore thofe rays muft come moft copioufly to the eye from the drops in the line OH, and ftrike the fenfe with the deepeft violet in that region. And, by the fame argument, the drops in the regions between G and H will ftrike the fenfe with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progrefs from G to H, or from the infide of the bow to the outfide, in this order, red, orange, yellow, green, blue, indigo, and violet. And fince the four lines OE, OF, OG, OH may be fituated any where in the above-mentioned conical fuperficies, what is faid of the drops and colours in thefe lines, is to be underftood of the drops and colours every where in thofe fuperficies.

Thus there will be made two bows of colours, an interior and ftronger, by one reflection in the drops, and an exterior and fainter by two; for the light becomes fainter by every reflection; and their colours will lie in a contrary order to each other, the red of both bows bordering upon the fpace GF, which is between the bows. The breadth of the interior bow, EOF, meafured acrofs the colours, will be 1° 15′, and the breadth of the exterior GOH, will be 3° 10′, alfo the diftance between them GOF, will be 8° 55′, the greateft femidiameter of the innermoft, that is, the angle POF, being 42° 2′, and the leaft femidiameter of the outermoft POG being 50° 57′. Thefe are the meafures of the bows as they would be, were the fun but a point; but by the breadth of his body, the breadth of the bows will be increafed by half a degree, and their diftance diminifhed by as much; fo that the breadth of the inner bow will be 2° 15′, that of the outer 3° 40′, their diftance 8° 25′; the greateft femidiameter of the interior bow 42° 17′, and the leaft of the exterior 50° 42′. And fuch are the dimenfions of the bows in the heavens found to be, very nearly, when their colours appear ftrong and perfect.

The light which comes through drops of rain by two refractions without any reflection, ought to appear ftrongeft at the diftance of about 26 degrees from the fun, and to decay gradually both ways as the diftance from the fun increafes and decreafes. And the fame is to be underftood of light tranfmitted through fpherical hailftones. If the hail be a little flatted, as it often is, the light tranfmitted may grow fo ftrong at a little lefs diftance than that of 26°, as to form a halo about the fun and moon; which halo, when the ftones are duly figured, may be coloured, and then it muft be red within, by the leaft refrangible rays, and blue without, by the moft refrangible ones.

The light which paffes through a drop of rain after two refractions, and three or more reflections, is fcarce ftrong enough to caufe a fenfible bow.

As to the dimenfion of the Rainbow, Des Cartes firft determined its diameter by a tentative and indirect method; laying it down, that the magnitude of the bow depends on the degree of refraction of the fluid; and affuming the ratio of the fine of incidence to that of refraction, to be in water as 250 to 187. But Dr. Halley, in the Philof. Tranf. number 267, gave a fimple direct method of determining the diameter of the Rainbow from the ratio of the refraction of the fluid being given; or, vice verfa, the diameter of the Rainbow being given, to determine the refractive power of the fluid. And Dr. Halley's principles and conftruction were farther explained by Dr. Morgan, bifhop of Ely, in his Differtation on the Rainbow, among the notes upon Rohault's Syftem of Philofophy, part 3, chap. 17.

From the theory of the Rainbow, all the particular phenomena of it are eafily deducible. Hence we fee, 1ft, Why the iris is always of the fame breadth; becaufe the intermediate degrees of refrangibility of the rays between red and violet, which are its extreme colours, are always the fame.

2dly, Why the bow fhifts its fituation as the eye does; and, as the popular phrafe has it, flies from thofe who follow it, and follows thofe that fly from it; the coloured drops being difpofed under a certain angle, about the axis of vifion, which is different in different places: whence alfo it follows, that every different fpectator fees a different bow.

3dly, Why the bow is fometimes a larger portion of a circle, fometimes a lefs: its magnitude depending on the greater or lefs part of the furface of the cone, above the furface of the earth, at the time of its appearance; and the higher the fun, always the lefs the Rainbow.

4thly, Why the bow never appears when the fun is above a certain altitude; the furface of the cone, in which it fhould be feen, being loft in the ground at a little diftance from the eye, when the fun is above 42° high.

5thly, Why the bow never appears greater than a femicircle, on a plane; fince, be the fun never fo low, and even in the horizon, the centre of the bow is ftill in the line of afpect; which in this cafe runs along the earth, and is not at all raifed above the furface. Indeed if the fpectator be placed on a very confiderable eminence, and the fun in the horizon, the line of afpect, in which the centre of the bow is, will be confiderably raifed above the horizon. And if the eminence be very high, and the rain near, it is poffible the bow may be an entire circle.

6thly, How the bow may chance to appear inverted, or the concave fide turned upwards; viz, a cloud happening to intercept the rays, and prevent their fhining on the upper part of the arch: in which cafe, only the lower part appearing, the bow will feem as if turned upfide down; which has probably been the cafe in feveral prodigies of this kind, related by authors.

Lunar

Lunar RAINBOW. The moon fometimes alfo exhibits the phenomenon of an iris, by the refraction of her rays in the drops of rain in the night-time.

Ariftotle fays, he was the firft that ever obferved it; and adds, it is never feen but at the time of the full moon; her light at other times being too faint to affect the fight after two refractions and one reflection.

The lunar iris has all the colours of the folar, very diftinct and pleafant; only fainter, both from the different intenfity of the rays, and the different difpofition of the medium.

Marine RAINBOW. This is a phenomenon fometimes obferved in a much agitated fea; when the wind, fweeping part of the tops of the waves, carries them aloft; fo that the fun's rays, falling upon them, are refracted, &c, as in a common fhower, and there paint the colours of the bow. Thefe bows are lefs diftinguifhable and bright than the common bow: but then they exceed as to numbers, there being fometimes 20 or 30 feen together. They appear at noon day, and in a pofition oppofite to that of the common bow, the concave fide being turned upwards, as indeed it ought to be.

RAIN-GAGE, an inftrument for meafuring the quantity of rain that falls. It is the fame as OMBROMETER, or PLUVIAMETER, which fee.

RAKED *Table*, or RAKING *Table*, in Architecture, a member hollowed in the fquare of a pedeftal, or elfewhere.

RAM, in Aftronomy. See ARIES.

RAM, *battering*. See BATTERING *Ram*.

RAMS-HORNS, in Fortification, a name given by Belidor to the Tenailles.

RAMPART, or RAMPIER, in Fortification, a maffy bank or elevation of earth around a place, to cover it from the direct fire of an enemy, and of fufficient thicknefs to refift the efforts of their cannon for many days. It is formed into baftions, curtains, &c.

Upon the Rampart the foldiers continually keep guard, and the pieces of artillery are planted for defence. Alfo, to fhelter the men from the enemy's fhot, the outfide of the Rampart is built higher than the reft, i. e. a parapet is raifed upon it with a platform. It is encompaffed with a moat or ditch, out of which is dug the earth that forms the Rampart, which is raifed floping, that the earth may not flip down, and having a berme at bottom, or is otherwife fortified, being lined with a facing of brick or ftone.

The height of the Rampart need not be more than 3 fathoms, this being fufficient to cover the houfes from the battery of the cannon; neither need its thicknefs be more than 10 or 12, unlefs more earth come out of the ditch than can otherwife be beftowed.

The Ramparts of halfmoons are the better for being low, that the fmall fire of the defendants may the better reach the bottom of the ditch; but yet they muft be fo high as not to be commanded by the covert-way.

RAMPART is alfo ufed, in civil architecture, for the void fpace left between the wall of a city and the houfes. This is what the Romans called Pomœrium, where it was forbidden to build, and where they planted rows of trees for the people to walk and amufe themfelves under.

RAMUS (PETER), a celebrated French mathematician and philofopher, was born in 1515, in a village of Vermandois in Picardy. He was defcended of a good family, which had been reduced to extreme poverty by the wars and other misfortunes. His own life too, fays Bayle, was the fport of fortune. In his infancy he was twice attacked by the plague. At 8 years of age, a thirft for learning urged him to go to Paris; but he was foon forced by poverty to leave that city. He returned to it again as foon as he could; but, being unable to fupport himfelf, he left it a fecond time: yet his paffion for ftudy was fo violent, that notwithftanding his bad fuccefs in the two former vifits, he ventured upon a third. He was maintained there fome months by one of his uncles; after which he was obliged to become a fervant in the college of Navarre. Here he fpent the day in waiting upon his mafters, and the greateft part of the night in ftudy.

After having finifhed claffical learning and rhetoric, he went through a courfe of philofophy, which took him up three years and a half in the fchools. The thefis, which he made for his mafter of arts degree, offended every one; for he maintained in it, that all that Ariftotle had advanced was falfe; and he gave very good anfwers to the objections of the profeffors. This fuccefs encouraged him to examine the doctrine of Ariftotle more clofely, and to combat it vigoroufly: but he confined himfelf chiefly to his logic. The two firft books he publifhed, the one entitled, *Inftitutiones Dialecticæ*, the other *Ariftotelicæ Animadverfiones*, occafioned great difturbances in the univerfity of Paris. The profeffors there, who were adorers of Ariftotle, ought to have refuted Ramus's books, if they could, by writings and lectures: but inftead of confining themfelves within the juft bounds of academical wars, they profecuted this anti-peripatetic before the civil magiftrate, as a man who was going to fap the foundations of religion. They raifed fuch clamours, that the caufe was carried before the parliament of Paris: but, perceiving that it would be examined equably, his enemies by their intrigues took it from that tribunal, to bring it before the king's council, in 1543. The king ordered, that Ramus and Anthony Govea, who was his principal adverfary, fhould choofe two judges each, to pronounce on the controverfy, after they fhould have ended their difputation; while he himfelf appointed a deputy. Ramus appeared before the five judges, though three of them were his declared enemies. The difpute lafted two days, and Govea had all the advantages he could defire; Ramus's books being prohibited in all parts of the kingdom, and their author fentenced not to teach philofophy any longer; upon which his enemies triumphed in the moft indecent manner.

The year after, the plague made great havoc in Paris, and forced moft of the ftudents in the college of Prefle to quit it; but Ramus, being prevailed upon to teach in it, foon drew together a great number of auditors. The Sorbonne attempted in vain to drive him from that college; for he held the headfhip of that houfe

houſe by arrêt of parliament. Through the patronage and protection of the cardinal of Lorrain, he obtained from Henry the 2d, in 1547, the liberty of ſpeaking and writing, and the regal profeſſorſhip of philoſophy and eloquence in 1551. The parliament of Paris had, before this, maintained him in the liberty of joining philoſophical lectures to thoſe of eloquence; and this arrêt or decree had put an end to ſeveral proſecutions, which Ramus and his pupils had ſuffered. As ſoon as he was made regius profeſſor, he was fired with a new zeal for improving the ſciences, notwithſtanding the hatred of his enemies, who were never at reſt.

Ramus bore at that time a part in a very ſingular affair. About the year 1550, the royal profeſſors corrected among other abuſes, that which had crept into the pronunciation of the Latin tongue. Some of the clergy followed this regulation; but the Sorbonniſts were much offended at it as an innovation, and defended the old pronunciation with great zeal. Things at length were carried ſo far, that a miniſter, who had a good living, was very ill treated by them; and cauſed to be ejected from his benefice for having pronounced *quiſquis, quanquam*, according to the new way, inſtead of *kiſkis, kankam*, according to the old. The miniſter applied to the parliament; and the royal profeſſors, with Ramus among them, fearing he would fall a victim to the credit and authority of the faculty of divines, for preſuming to pronounce the Latin tongue according to their regulations, thought it incumbent on them to aſſiſt him. Accordingly, they went to the court of juſtice, and repreſented in ſuch ſtrong terms the indignity of the proſecution, that the miniſter was cleared, and every perſon had the liberty of pronouncing as he pleaſed.

Ramus was bred up in the Catholic religion, but afterwards deſerted it. He began to diſcover his new principles by removing the images from the chapel of his college of Preſle, in 1552. Hereupon ſuch a perſecution was raiſed againſt him by the Religioniſts, as well as Ariſtotelians, that he was driven out of his profeſſorſhip, and obliged to conceal himſelf. For that purpoſe, with the king's leave he went to Fontainbleau; where, by the help of books in the king's library, he proſecuted geometrical and aſtronomical ſtudies. As ſoon as his enemies found out his retreat, they renewed their perſecutions; and he was forced to conceal himſelf in ſeveral other places. In the mean time, his curious and excellent collection of books in the college of Preſle was plundered: but after a peace was concluded in 1563, between Charles the 9th and the Proteſtants, he again took poſſeſſion of his employment, maintained himſelf in it with vigour, and was particularly zealous in promoting the ſtudy of the mathematics.

This continued till the ſecond civil war in 1567, when he was forced to leave Paris, and ſhelter himſelf among the Hugonots, in whoſe army he was at the battle of St. Denys. Peace having been concluded ſome months after, he was reſtored to his profeſſorſhip; but, foreſeeing that the war would ſoon break out again, he did not care to venture himſelf in a freſh ſtorm, and therefore obtained the king's leave to viſit the univerſities of Germany. He accordingly undertook this journey in 1568, and received great honours wherever he came. He returned to France, after the third war in 1571; and loſt his life miſerably, in the maſſacre of St. Bartholomew's day, 1572, at 57 years of age. It is ſaid, that he was concealed in a granary during the tumult; but diſcovered and dragged out by ſome peripatetic doctors who hated him; theſe, after ſtripping him of all his money under pretence of preſerving his life, gave him up to the aſſaſſins, who, after cutting his throat and giving him many wounds, threw him out of the window; and his bowels guſhing out in the fall, ſome Ariſtotelian ſcholars, encouraged by their maſters, ſpread them about the ſtreets; then dragged his body in a moſt ignominious manner, and threw it into the river.

Ramus was a great orator, a man of univerſal learning, and endowed with very fine qualities. He was ſober, temperate, and chaſte. He ate but little, and that of boiled meat; and drank no wine till the latter part of his life, when it was preſcribed by the phyſicians. He lay upon ſtraw; roſe early, and ſtudied hard all day; and led a ſingle life with the utmoſt purity. He was zealous for the proteſtant religion, but at the ſame time a little obſtinate, and given to contradiction. The proteſtant miniſters did not love him much, for he made himſelf a kind of head of a party, to change the diſcipline of the proteſtant churches: his deſign was to introduce a democratical government in the church, but this deſign was traverſed, and defeated in a national ſynod. His ſect flouriſhed however for ſome time afterwards, ſpreading pretty much in Scotland and England, and ſtill more in Germany.

He publiſhed a great many books; but mathematics was chiefly obliged to him. Of this kind, his writings were principally theſe following:

1. *Scholarum Mathematicarum libri* 31.
2. *Arithmeticæ libri duo.—Algebræ libri duo.—Geometriæ libri* 27.

Theſe were greatly enlarged and explained by Schoner, and publiſhed in 2 volumes 4to. There were ſeveral editions of them; mine is that of 1627, at Frankfort.—The Geometry, which is chiefly practical, was tranſlated into Engliſh by William Bedwell, and publiſhed in 4to, at London, 1636.

RANDOM-Shot, is a ſhot diſcharged with the axis of the gun elevated above the horizontal or point-blank direction.

Random, of a ſhot, alſo ſometimes means the range of it, or the diſtance to which it goes at the firſt graze, or where it ſtrikes the ground. See Range.

RANGE, in Gunnery, ſometimes means the path a ſhot flies in. But more uſually,

Range now means the diſtance to which the ſhot flies when it ſtrikes the ground or other object, called alſo the amplitude of the ſhot. But Range is the term in preſent uſe.

Were it not for the reſiſtance of the air, the greateſt Range, on a horizontal plane, would be when the ſhot is diſcharged at an angle of 45° above the horizon; and all other Ranges would be the leſs, the more the angle of elevation is above or below 45°; but ſo as that at equal diſtances above and below 45°, the two Ranges are equal to each other. But, on account of the reſiſtance of the air, the Ranges are altered; and that in different proportions, both for the different ſizes

ſizes of the ſhot, and their different velocities: ſo that the greateſt Range, in practice, always lies below the elevation of 45°, and the more below it as the ſhot is ſmaller, and as its velocity is greater; ſo as that the ſmalleſt balls, diſcharged with the greateſt velocity in practice, ranges the fartheſt with an elevation of 30° or under, while the largeſt ſhot, with very ſmall velocities, range fartheſt with nearly 45° elevation; and at all the intermediate degrees in the other caſes. See PROJECTILES.

RARE, in Phyſics, is the quality of a body that is very porous, whoſe parts are at a great diſtance from one another, and which contains but little matter under a great magnitude. In which ſenſe Rare ſtands oppoſed to denſe.

The corpuſcular philoſophers, viz, the Epicureans, Gaſſendiſts, Newtonians, &c, aſſert that bodies are rarer, ſome than others, in virtue of a greater quantity of pores, or of vacuity lying between their parts or particles. The Carteſians hold, that a greater rarity only conſiſts in a greater quantity of materia ſubtilis contained in the pores. And laſtly, the Peripatetics contend, that rarity is a new quality ſuperinduced upon a body, without any dependence on either vacuity or ſubtile matter.

RAREFACTION, in Phyſics, the rendering a body rarer, that is bringing it to expand or occupy more room or ſpace, without the acceſſion of new matter: and it is oppoſed to condenſation. The more accurate writers reſtrict the term Rarefaction to that kind of expanſion which is effected by means of heat: and the expanſion from other cauſes they term *dilatation*; if indeed there be other cauſes; for though ſome philoſophers have attributed it to the action of a repulſive principle in the matter itſelf; yet from the many diſcoveries concerning the nature and properties of the electric fluid and fire, there is great reaſon to believe that this repulſive principle is no other than elementary fire.

The Carteſians deny any ſuch thing as abſolute Rarefaction: extenſion, according to them, conſtituting the eſſence of matter, they are obliged to hold all extenſion equally full. Hence they make Rarefaction to be no other than an acceſſion of freſh, ſubtile, and inſenſible matter, which, entering the parts of bodies, ſenſibly diſtends them.

It is by Rarefaction that gunpowder has its effect; and to the ſame principle alſo we owe eolipiles, thermometers, &c. As to the air, the degree to which it is rarefiable exceeds all imagination, experience having ſhewn it to be far above 10,000 times more than the uſual ſtate of the atmoſphere; and as it is found to be about 1000 times denſer in gunpowder than the atmoſphere, it follows that experience has found it differ by about 10 millions of times. Perhaps indeed its degree of expanſion is abſolutely beyond all limits.

Such immenſe Rarefaction, Newton obſerves, is inconceivable on any other principle than that of a repelling force inherent in the air, by which its particles mutually fly from one another. This repelling force, he obſerves, is much more conſiderable in air than in other bodies, as being generated from the moſt fixed bodies, and that with much difficulty, and ſcarce without fermentation; thoſe particles being always found to fly from each other with the greateſt force, which, when in contact, cohere the moſt firmly together. See AIR.

Upon the Rarefaction of the air is founded the uſeful method of meaſuring altitudes by the barometer, in all the caſes of which, the rarity of the air is found to be inverſely as the force that compreſſes it, or inverſely as the weight of all the air above it at any place.

RARITY, thinneſs, ſubtlety, or the contrary to denſity.

RATCH, or RASH, in Clock-Work, a ſort of wheel having 12 fangs, which ſerve to lift up the detents every hour, to make the clock ſtrike.

RATCHETS, in a Watch, are the ſmall teeth at the bottom of the fuſee, or barrel, that ſtop it in winding up.

RATIO, according to Euclid, is the habitude or relation of two magnitudes of the ſame kind in reſpect of quantity. So the ratio of 2 to 1 is double, that of 3 to 1 triple, &c. Several mathematicians have found fault with Euclid's definition of a Ratio, and others have as much defended it, eſpecially Dr. Barrow, in his Mathematical Lectures, with great ſkill and learning.

Ratio is ſometimes confounded with proportion, but very improperly, as being quite different things; for proportion is the ſimilitude or equality or identity of two Ratios. So the Ratio of 6 to 2 is the ſame as that of 3 to 1, and the Ratio of 15 to 5 is that of 3 to 1 alſo; and therefore the Ratio of 6 to 2 is ſimilar or equal or the ſame with that of 15 to 5, which conſtitutes proportion, which is thus expreſſed, 6 is to 2 as 15 to 5, or thus $6 : 2 :: 15 : 5$, which means the ſame thing. So that Ratio exiſts between two terms, but proportion between two Ratios or four terms.

The two quantities that are compared, are called the *terms* of the Ratio, as 6 and 2; the firſt of theſe 6 being called the *antecedent*, and the latter 2 the *conſequent*. Alſo the *index* or *exponent* of the Ratio, is the quotient of the two terms: ſo the index of the Ratio of 6 to 2 is $\frac{6}{2}$ or 3, and which is therefore called a *triple Ratio*.

Wolfius diſtinguiſhes Ratios into *rational* and *irrational*.

Rational RATIO is that which can be expreſſed between two rational numbers; as the Ratio of 6 to 2, or of $6\sqrt{3}$ to $2\sqrt{3}$, 3 to 1. And

Irrational RATIO is that which cannot be expreſſed by that of one rational number to another; as the Ratio of $\sqrt{6}$ to $\sqrt{2}$, or of $\sqrt{3}$ to root $\sqrt{1}$, that is $\sqrt{3}$ to 1, which cannot be expreſſed in rational numbers.

When the two terms of a Ratio are equal, the Ratio is ſaid to be that of *equality*; as of 3 to 3, whoſe index is 1, denoting the ſingle or equal Ratio. But when the terms are not equal, as of 6 to 2, it is a *Ratio of inequality*.

Farther, when the antecedent is the greater term, as in 6 to 2, it is ſaid to be the *Ratio of greater inequality*: but when the antecedent is the leſs term, as in the Ratio of 2 to 6, it is ſaid to be *the Ratio of leſs*

leſs inequality. In the former caſe, if the leſs term be an aliquot part of the greater, the Ratio of greater inequality is ſaid to be *multiplex* or *multiple;* and the Ratio of the leſs inequality, *ſub-multiple*. Particularly, in the firſt caſe, if the exponent of the Ratio be 2, as in 6 to 3, the Ratio is called *duple* or *double;* if 3, as in 6 to 2, it is *triple;* and ſo on. In the ſecond caſe, if the Ratio be $\frac{1}{2}$, as in 3 to 6, the Ratio is called *ſubduple;* if $\frac{1}{3}$, as in 2 to 6, it is *ſubtriple;* and ſo on.

If the greater term contain the leſs once, and one aliquot part of the ſame over; the Ratio of the greater inequality is called *ſuperparticular*, and the Ratio of the leſs *ſubſuperparticular*. Particularly, in the firſt caſe, if the exponent be $\frac{3}{2}$ or $1\frac{1}{2}$, it is called *ſeſquialterate;* if $\frac{4}{3}$ or $1\frac{1}{3}$, *ſeſquitertial;* &c. In the other caſe, if the exponent be $\frac{7}{3}$, the Ratio is called *ſubſeſquialterate;* if $\frac{3}{4}$, it is *ſubſeſquitertial*.

When the greater term contains the leſs once and ſeveral aliquot parts over, the Ratio of the greater inequality is called *ſuperpartiens*, and that of the leſs inequality is *ſubſuperpartiens*. Particularly, in the former caſe, if the exponent be $\frac{5}{3}$ or $1\frac{2}{3}$, the Ratio is called *ſuperbipartiens tertias;* if the exponent be $\frac{7}{4}$ or $1\frac{3}{4}$, *ſupertripartiens quartas;* if $\frac{11}{7}$ or $1\frac{4}{7}$, *ſuperquadripartiens ſeptimas;* &c. In the latter caſe, if the exponent be the reciprocals of the former, or $\frac{3}{5}$, the Ratio is called *ſubſuperbipartiens tertias;* if $\frac{4}{7}$, *ſubſupertripartiens quartas;* if $\frac{7}{11}$, *ſubſuperquadripartiens ſeptimas;* &c.

When the greater term contains the leſs ſeveral times, and ſome one part over; the ratio of the greater inequality is called *multiplex ſuperparticular;* and the Ratio of the leſs inequality is called *ſubmultiplex ſubſuperparticular*. Particularly, in the former caſe, if the exponent be $\frac{5}{2}$ or $2\frac{1}{2}$, the ratio is called *dupla ſeſquialtera;* if $\frac{13}{4}$ or $3\frac{1}{4}$, *tripla ſeſquiquarta*, &c. In the latter caſe, if the exponent be $\frac{2}{5}$, the Ratio is called *ſubdupla ſubſeſquialtera;* if $\frac{4}{13}$, *ſubtripla ſubſeſquiquarta*, &c. Laſtly, when the greater term contains the leſs ſeveral times, and ſeveral aliquot parts over; the Ratio of the greater inequality is called *multiplex ſuperpartiens;* that of the leſs inequality, *ſubmultiplex ſubſuperpartiens*. Particularly, in the former caſe, if the exponent be $\frac{8}{3}$ or $2\frac{2}{3}$, the Ratio is called *dupla ſuperbipartiens tertias;* if $\frac{25}{7}$ or $3\frac{4}{7}$, *tripla ſuperbiquadripartiens ſeptimas*, &c. In the latter caſe, if the exponent be $\frac{3}{8}$, the Ratio is called *ſubdupla ſubſuperbipartiens tertias;* if $\frac{7}{25}$, *ſubtripla ſubſuperquadripartiens ſeptimas;* &c.

Theſe are the various denominations of rational Ratios, names which are very neceſſary to the reading of the ancient authors; though they occur but rarely among the modern writers, who uſe inſtead of them the ſmalleſt numeral terms of the Ratios; ſuch 2 to 1 for duple, and 3 to 2 for ſeſquialterate, &c.

Compound RATIO, is that which is made up of two or more other Ratios, viz, by multiplying the exponents together, and ſo producing the compound Ratio of the product of all the antecedents to the product of all the conſequents.

Thus the compound Ratio of 5 to 3,
and 7 to 4,
is the Ratio of - - - - 35 to 12.

Particularly, if a Ratio be compounded of two equal Ratios, it is called the *duplicate Ratio;* if of three equal Ratios, the *triplicate Ratio;* if of four equal Ratios, the *quadruplicate Ratio;* and ſo on, according to the powers of the exponents, for all *multiplicate Ratios*. So the ſeveral multiplicate Ratios of

the ſimple Ratio of - 3 to 2, are thus, viz.
the duplicate Ratio - 9 : 4,
the triplicate Ratio - 27 : 8,
the quadruplicate Ratio 81 : 16, &c.

Properties of RATIOS. Some of the more remarkable properties of Ratios are as follow:

1. The like multiples, or the like parts, of the terms of a Ratio, have the ſame Ratio as the terms themſelves. So $a : b$, and $na : nb$, and $\frac{a}{n} : \frac{b}{n}$ are all the ſame Ratio.

2. If to, or from, the terms of any Ratio, be added or ſubtracted either their like parts, or their like multiples, the ſums or remainders will ſtill have the ſame Ratio. So $a : b$, and $a \pm na : b \pm nb$, and $a \pm \frac{a}{n} : b \pm \frac{b}{n}$ are all the ſame Ratio.

3. When there are ſeveral quantities in the ſame continued Ratio, a, b, c, d, e, &c. whatever Ratio the firſt has to the 2d,

the 1ſt to the 3d has the duplicate of that Ratio,
the 1ſt to the 4th has the triplicate of that Ratio,
the 1ſt to the 5th has the quadruplicate of it,

and ſo on. Thus, the terms of the continued Ratio being $1, r, r^2, r^3, r^4, r^5$, &c, where each term has to the following one the Ratio of 1 to r, the Ratio of the 1ſt to the 2d; then $1 : r^2$ is the duplicate, $1 : r^3$ the triplicate, $1 : r^4$ the quadruplicate, and ſo on, according to the powers of $1 : r$.

For other properties ſee PROPORTION.

To approximate to a RATIO *in ſmaller Terms*.—Dr. Wallis, in a ſmall tract at the end of Horrox's works, treats of the nature and ſolution of this problem, but in a very tedious way; and he has proſecuted the ſame to a great length in his Algebra, chap. 10 and 11, where he particularly applies it to the Ratio of the diameter of a circle to its circumference. Mr. Huygens too has given a ſolution, with the reaſons of it, in a much ſhorter and more natural way, in his Deſcrip. Autom. Planet. Opera Reliqua, vol 1, pa. 174.

So alſo has Mr. Cotes, at the beginning of his Harmon. Menſurarum. And ſeveral other perſons have done the ſame thing, by the ſame or ſimilar methods. The problem is very uſeful, for expreſſing a Ratio in ſmall numbers, that ſhall be near enough in practice, to any given Ratio in large numbers, ſuch as that of the diameter of a circle to its circumference. The principle of all theſe methods, conſiſts in reducing the terms of the propoſed Ratio into a ſeries of what are called continued fractions, by dividing the greater term by the leſs, and the leſs by the remainder, and ſo on, always the laſt diviſor by the laſt remainder, after the manner of finding the greateſt common meaſure of the two terms; then connecting all the quotients &c. together in a ſeries of continued fractions; and laſtly collecting gradually theſe fractions together one after another. So if $\frac{b}{a}$ be any fraction, or exponent of any Ratio; then dividing thus,

$a)\ b\ (c$

$a)\ b\ (c$
$\quad d)\ a\ (e$
$\quad\quad f)\ d\ (g$
$\quad\quad\quad h)\ f\ (i$
$\quad\quad\quad\quad k)\ h\ (l$
$\quad\quad\quad\quad\quad$ &c.

gives c, e, g, i, &c, for the ſeveral quotients, and theſe, formed in the uſual way, give the approximate value of the given Ratio in a ſeries of continued fractions; thus,

$$\frac{b}{a} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{1}{i} + \text{\&c.}}}$$

Then collecting the terms of this ſeries, one after another, ſo many values of $\frac{b}{a}$ are obtained, always nearer and nearer; the firſt value being c or $\frac{c}{1}$, the next

$$c + \frac{1}{e} = \frac{ce+1}{e} = \frac{A}{B},$$

the 3d value $c + \cfrac{1}{e + \cfrac{1}{g}} = c + \cfrac{1}{\cfrac{ge+1}{g}} = c + \frac{g}{ge+1} =$

$$\frac{cge + c + g}{ge+1} = \frac{(ce+1)g + c}{ge+1} = \frac{Ag+c}{Bg+1} = \frac{C}{D};$$

in like manner,

the 4th value is $\frac{Ci + A}{Di + B} = \frac{E}{F}$;

the 5th value is $\frac{El + C}{Fl + D} = \frac{G}{H}$; &c.

From whence comes this general rule: Having found any two of theſe values, multiply the terms of the latter of them by the next quotient, and to the two products add the correſponding terms of the former value, and the ſums will be the terms of the next value, &c.

For example, let it be required to find a ſeries of Ratios in leſſer numbers, conſtantly approaching to the Ratio of 100000 to 314159, or nearly the Ratio of the diameter of a circle to its circumference. Here firſt dividing, thus,

$100000)\ 314159\ (3 = c$
$\quad d = 14159)\ 100000\ (7 = e$
$\quad\quad f = 887)\ 14159\ (15 = g$
$\quad\quad\quad h = 854)\ 887\ (1 = i$, &c.
$\quad\quad\quad\quad$ &c.

there are obtained the quotients 3, 7, 15, 1, 25, 1, 7, 4.

Hence 3 or $\frac{3}{1} = c$, the 1ſt value;

$$\frac{ce+1}{e} = \frac{3.7+1}{1.7} = \frac{22}{7} = \frac{A}{B}, \text{ the 2d value;}$$

$$\frac{Ag+c}{Bg+1} = \frac{22.15+3}{7.15+1} = \frac{333}{106} = \frac{C}{D}, \text{ the 3d value;}$$

$$\frac{Ci+A}{Di+B} = \frac{333.1+22}{106.1+7} = \frac{355}{113} = \frac{E}{F}, \text{ the 4th value;}$$

and ſo on; where the ſucceſſive continual approximating values of the propoſed Ratio are $\frac{3}{1}$, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, &c; the 2d of theſe, viz. $\frac{22}{7}$, being the approximation of Archimedes; and the 4th, viz $\frac{355}{133}$, is that of Metius, which is very near the truth, being equal
to 3·1415929,
the more accurate Ratio being - - 3·1415927.

The doctrine of Ratios and Proportions, as delivered by Euclid, in the fifth book of his Elements, is conſidered by moſt perſons as very obſcure and objectionable, particularly the definition of proportionality; and ſeveral ingenious gentlemen have endeavoured to elucidate that ſubject. Among theſe, the Rev. Mr. Abram Robertſon, of Chriſt Church College, Oxford, lecturer in geometry in that univerſity, printed a neat little paper there in 1789, for the uſe of his claſſes, being a demonſtration of that definition, in 7 propoſitions, the ſubſtance of which is as follows. He firſt premiſes this advertiſement:

"As demonſtrations depending upon proportionality pervade every branch of mathematical ſcience, it is a matter of the higheſt importance to eſtabliſh it upon clear and indiſputable principles. Moſt mathematicians, both ancient and modern, have been of opinion that Euclid has fallen ſhort of his uſual perſpicuity in this particular. Some have queſtioned the truth of the definition upon which he has founded it, and, almoſt all who have admitted its truth and validity have objected to it *as a definition*. The author of the following propoſitions ranks himſelf amongſt objectors of the laſt mentioned deſcription. He thinks that Euclid muſt have founded the definition in queſtion upon the reaſoning contained in the firſt ſix demonſtrations here given, or upon a ſimilar train of thinking; and in his opinion *a definition* ought to be as ſimple, or as free from a multiplicity of conditions, as the ſubject will admit."

He then lays down theſe four definitions:

"1. Ratio is the relation which one magnitude has to another, of the ſame kind, with reſpect to quantity."

"2. If the firſt of four magnitudes be exactly as great when compared to the ſecond, as the third is when compared to the fourth, the firſt is ſaid to have to the ſecond the ſame Ratio that the third has to the fourth."

"3. If the firſt of four magnitudes be greater, when compared to the ſecond, than the third is when compared to the fourth, the firſt is ſaid to have to the ſecond a greater Ratio than the third has to the fourth."

"4. If the firſt of four magnitudes be leſs, when compared to the ſecond, than the third is when compared to the fourth, the firſt is ſaid to have to the ſecond a leſs Ratio than the third has to the fourth."

Mr. Robertſon then delivers the propoſitions, which are the following:

"*Prop.* 1. If the firſt of four magnitudes have to the ſecond, the ſame Ratio which the third has to the fourth; then,

then, if the firſt be equal to the ſecond, the third is equal to the fourth; if greater, greater; if leſs, leſs."

"*Prop.* 2. If the firſt of four magnitudes be to the ſecond as the third to the fourth, and if any equimultiples whatever of the firſt and third be taken, and alſo any equimultiples of the ſecond and fourth; the multiple of the firſt will be to the multiple of the ſecond as the multiple of the third to the multiple of the fourth."

"*Prop.* 3. If the firſt of four magnitudes be to the ſecond as the third to the fourth, and if any like aliquot parts whatever be taken of the firſt and third, and any like aliquot parts whatever of the ſecond and fourth, the part of the firſt will be to the part of the ſecond as the part of the third to the part of the fourth."

"*Prop.* 4. If the firſt of four magnitudes be to the ſecond as the third to the fourth, and if any equimultiples whatever be taken of the firſt and third, and any whatever of the ſecond and fourth; if the multiple of the firſt be equal to the multiple of the ſecond, the multiple of the third will be equal to the multiple of the fourth; if greater, greater; if leſs, leſs."

"*Prop.* 5. If the firſt of four magnitudes be to the ſecond as the third is to a magnitude leſs than the fourth, then it is poſſible to take certain equimultiples of the firſt and third, and certain equimultiples of the ſecond and fourth, ſuch, that the multiple of the firſt ſhall be greater than the multiple of the ſecond, but the multiple of the third not greater than the multiple of the fourth."

"*Prop.* 6. If the firſt of four magnitudes be to the ſecond as the third is to a magnitude greater than the fourth, then certain equimultiples can be taken of the firſt and third, and certain equimultiples of the ſecond and fourth, ſuch, that the multiple of the firſt ſhall be leſs than the multiple of the ſecond, but the multiple of the third not leſs than the multiple of the fourth."

"*Prop.* 7. If any equimultiples whatever be taken of the firſt and third of four magnitudes, and any equimultiples whatever of the ſecond and fourth; and if when the multiple of the firſt is leſs than that of the ſecond, the multiple of the third is alſo leſs than that of the fourth; or if when the multiple of the firſt is equal to that of the ſecond, the multiple of the third is alſo equal to that of the fourth; or if when the multiple of the firſt is greater than that of the ſecond, the multiple of the third is alſo greater than that of the fourth: then, the firſt of the four magnitudes ſhall be to the ſecond as the third to the fourth."

And all theſe propoſitions Mr. Robertſon demonſtrates by ſtrict mathematical reaſoning.

RATIONAL, in Arithmetic &c, the quality of numbers, fractions, quantities, &c, when they can be expreſſed by common numbers; in contradiſtinction to irrational or ſurd ones, which cannot be expreſſed in common numbers. Suppoſe any quantity to be 1; there are infinite other quantities, ſome of which are commenſurable to it, either ſimply, or in power: theſe Euclid calls *Rational quantities*. The reſt, that are incommenſurable to 1, he calls *irrational quantities*, or *ſurds*.

RATIONAL *Horizon*, or *True Horizon*, is that whoſe plane is conceived to paſs through the centre of the earth; and which therefore divides the globe into two equal portions or hemiſpheres. It is called the Rational horizon, becauſe only conceived by the underſtanding; in oppoſition to the ſenſible or apparent horizon, or that which is viſible to the eye.

RAVELIN, in Fortification, was anciently a flat baſtion, placed in the middle of a curtain. But

RAVELIN is now a detached work, compoſed only of two faces, which form a ſalient angle uſually without flanks. Being a triangular work reſembling the point of a Baſtion with the flanks cut off. It raiſed before the curtain, on the counterſcarf of the place; and ſerving to cover it and the adjacent flanks from the direct fire of an enemy. It is alſo uſed to cover a bridge or a gate, and is always placed without the moat.

There are alſo double Ravelins, which ſerve to defend each other; being ſo called when they are joined by a curtain.

What the engineers call a Ravelin, the men uſually call a demilune, or halfmoon.

RAY, in Geometry, the ſame as RADIUS.

RAY, in Optics, a beam or line of light, propagated from a radiant point, through any medium.

If the parts of a Ray of light lie all in a ſtraight line between the radiant point and the eye, the Ray is ſaid to be *direct*: the laws and properties of which make the ſubject of Optics.—If any of them be turned out of that direction, or bent in their paſſage, the Ray is ſaid to be *refracted*.—If it ſtrike on the ſurface of any body, and be thrown off again, it is ſaid to be *reflected*.—In each caſe, the Ray, as it falls either directly on the eye, or on the point of reflection, or of refraction, is ſaid to be *incident*.

Again, if ſeveral Rays be propagated from the radiant object equidiſtantly from one another, they are called *parallel* Rays. If they come inclining towards each other, they are called *converging* Rays. And if they go continually receding from each other, they are called *diverging* Rays.

It is from the different circumſtances of Rays, that the ſeveral kinds of bodies are diſtinguiſhed in Optics. A body, for example, that diffuſes its own light, or emits Rays of its own, is called a *radiating* or *lucid* or *luminous* body. If it only reflect Rays which it receives from another, it is called an *illuminated* body. If it only tranſmit Rays, it is called a *tranſparent* or *tranſlucent* body. If it intercept the Rays, or refuſe them paſſage, it is called an *opaque* body.

It is by means of Rays reflected from the ſeveral points of illuminated objects to the eye, that they become viſible, and that viſion is performed; whence ſuch Rays are called *viſual* Rays.

The Rays of light are not homogeneous, or ſimilar, but differ in all the properties we know of; viz, refrangibility, reflexibility, and colour. It is probably from the different refrangibility that the other differences have their riſe; at leaſt it appears that thoſe Rays which agree or differ in this, do ſo in all the reſt. It is not however to be underſtood that the property or effect called colour, exiſts in the Rays of light themſelves; but from the different ſenſations the differently diſpoſed Rays excite in us, we call them *red Rays*, *yellow Rays*, &c. Each beam of light however, as it comes from the ſun, ſeems to be compounded of all the ſorts of Rays mixed together; and it is only by ſplitting or ſeparating the parts of it, that theſe different ſorts become obſervable; and this is done by tranſmitting the

beam

beam through a glaſs priſm, which refracting it in the paſſage, and the parts that excite the different colours having different degrees of refrangibility, they are thus ſeparated from one another, and exhibited each apart, and appearing of the different colours.

Beſide refrangibility, and the other properties of the Rays of light already aſcertained by obſervation and experiment, Sir I. Newton ſuſpects they may have many more; particularly a power of being inflected or bent by the action of diſtant bodies; and thoſe Rays which differ in refrangibility, he conceives likewiſe to differ in flexibility.

Theſe Rays he ſuſpects may be very ſmall bodies emitted from ſhining ſubſtances. Such bodies may have all the conditions of light: and there is that action and reaction between tranſparent bodies and light, which very much reſembles the attractive force between other bodies. Nothing more is required for the production of all the various colours, and all the degrees of refrangibility, but that the Rays of light be bodies of different ſizes; the leaſt of which may make violet the weakeſt and darkeſt of the colours, and be the moſt eaſily diverted by refracting ſurfaces from its rectilinear courſe; and the reſt, as they are larger and larger, may make the ſtronger and more lucid colours, blue, green, yellow, and red. See COLOUR, LIGHT, REFRACTION, REFLECTION, INFLECTION, CONVERGING, DIVERGING, &c, &c.

Reflected RAYS, thoſe Rays of light which are reflected, or thrown back again, from the ſurfaces of bodies upon which they ſtrike. It is found that, in all the Rays of light, the angle of reflection is equal to the angle of incidence.

Refracted RAYS, are thoſe Rays of light, which are bent or broken, in paſſing out of one medium into another.

Pencil of RAYS, a number of Rays iſſued from a point of an object, and diverging in the form of a cone.

Principal RAY, in Perſpective, is the perpendicular diſtance between the eye and the vertical plane or table, as ſome call it.

RAY *of Curvature*. See *Radius of* CURVATURE.

REAUMUR (RENE-ANTOINE-FERCHAULT, Sieur de), a reſpectable French philoſopher, was born at Rochelle in 1683. After the uſual courſe of ſchool education, he was ſent to Poitiers to ſtudy philoſophy, and, in 1699, to Bourges to ſtudy the law, the profeſſion for which he was intended. But philoſophy and mathematics having very early been his favourite purſuits, he quitted the law, and repaired to Paris in 1703, to purſue thoſe ſciences to the beſt advantage; and here his character procured him a ſeat in the Academy in the year 1708; which he held till the time of his death, which happened the 18th of November 1757, at 74 years of age.

Reaumur ſoon juſtified the choice that was made of him by the Academy. He made innumerable obſervations, and wrote a great multitude of pieces upon the various branches of natural philoſophy. His Hiſtory of Inſects, in 6 vols. quarto, at Paris, is his principal work. Another edition was printed in Holland, in 12 vols. 12mo. He made alſo great and uſeful diſcoveries concerning iron; ſhewing how to change common wrought iron into ſteel, how to ſoften caſt iron, and to make works in caſt iron as fine as in wrought iron. His labours and diſcoveries concerning iron were rewarded by the duke of Orleans, regent of the kingdom, by a penſion of 12 thouſand livres, equal to about 500l. Sterling; which however he would not accept but on condition of its being put under the name of the Academy, who might enjoy it after his death. It was owing to Reaumur's endeavours that there were eſtabliſhed in France manufactures of tin-plates, of porcelain in imitation of china-ware, &c. They owe to him alſo a new thermometer, which bears his name, and is pretty generally uſed on the continent, while that of Fahrenheit is uſed in England, and ſome few other places. Reaumur's thermometer is a ſpirit one, having the freezing point at 0, and the boiling point at 80.

Reaumur is eſteemed as an exact and clear writer; and there is an elegance in his ſtyle and manner, which is not commonly found among thoſe who have made only the ſciences their ſtudy. He is repreſented alſo as a man of a moſt amiable diſpoſition, and with qualities to make him beloved as well as admired. He left a great variety of papers and natural curioſities, which he bequeathed to the Academy of Sciences.

The works publiſhed by him, are the following.

1. The Art of changing Forged Iron into Steel; of Softening Caſt Iron; and of making works of Caſt Iron, as fine as of Wrought Iron. Paris, 1722, 1 vol. in 4to.

2. Natural Hiſtory of Inſects, 6 vols. in 4to.

His memoirs printed in the volumes of the Academy of Sciences, are very numerous, amounting to upwards of a hundred, and on various ſubjects, from the year 1708 to 1763, ſeveral papers in almoſt every volume.

RECEIVER, *of an Air Pump*, is part of its apparatus; being a glaſs veſſel placed on the top of the plate, out of which the air is to be exhauſted.

RECEPTION, in Aſtrology, is a dignity befalling two planets when they exchange houſes: for example, when the ſun arrives in Cancer, the houſe of the moon; and the moon, in her turn, arrives in the ſun's houſe.—The ſame term is alſo uſed when two planets exchange exaltation.

RECESSION *of the Equinoxes*. See PRECESSION *of the Equinoxes*.

RECIPROCAL, in Arithmetic, &c, is the quotient ariſing by dividing 1 by any number or quantity. So, the Reciprocal of 2 is $\frac{1}{2}$; of 3 is $\frac{1}{3}$, and of a is $\frac{1}{a}$, &c. Hence, the Reciprocal of a vulgar fraction is found, by barely making the numerator and the denominator mutually change places: ſo the Reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2; of $\frac{2}{3}$, is $\frac{3}{2}$; of $\frac{a}{b}$, is $\frac{b}{a}$, &c. Hence alſo, any quantity being multiplied by its Reciprocal, the product is always equal to unity or 1: ſo $\frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$, and $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$, and $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ab} = 1$.

TABLE

Table of Reciprocals.

No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.
1	1	61	0163934	121	0082645	181	0055249	241	0041494	301	0033223
2	5	62	0161290	122	0081967	182	0054945	242	0041322	302	0033113
3	3333333	63	0158730	123	0081300	183	0054645	243	0041152	303	0033003
4	25	64	015625	124	0080645	184	0054348	244	0040984	304	0032895
5	2	65	0153846	125	008	185	0054054	245	0040816	305	0032787
6	1666666	66	0151515	126	0079365	186	0053763	246	004065	306	0032680
7	1428571	67	0149254	127	0078740	187	0053476	247	0040486	307	0032573
8	125	68	0147059	128	0078125	188	0053191	248	0040323	308	0032468
9	1111111	69	0144928	129	0077519	189	0052910	249	0040161	309	0032362
10	1	70	0142857	130	0076923	190	0052632	250	004	310	0032258
11	0909090	71	0140845	131	0076336	191	0052356	251	0039841	311	0032154
12	0833333	72	0138888	132	0075757	192	0052083	252	0039683	312	0032051
13	0769230	73	0136986	133	0075188	193	0051813	253	0039526	313	0031949
14	0714285	74	0135135	134	0074627	194	0051546	254	0039370	314	0031847
15	0666666	75	0133333	135	0074074	195	0051282	255	0039216	315	0031746
16	0625	76	0131579	136	0073529	196	0051020	256	0039063	316	0031646
17	0588235	77	0129870	137	0072993	197	0050761	257	0038911	317	0031546
18	0555555	78	0128205	138	0072464	198	0050505	258	0038760	318	0031447
19	0526316	79	0126582	139	0071942	199	0050251	259	0038610	319	0031348
20	05	80	0125	140	0071429	200	005	260	0038462	320	003125
21	0476190	81	0123457	141	0070922	201	0049751	261	0038314	321	0031153
22	0454545	82	0121950	142	0070423	202	0049504	262	0038168	322	0031056
23	0434783	83	0120482	143	0069930	203	0049261	263	0038023	323	0030960
24	0416666	84	0119048	144	0069444	204	0049020	264	0037878	324	0030846
25	04	85	0117647	145	0068966	205	0048750	265	0037736	325	0030769
26	0384615	86	0116279	146	0068493	206	0048544	266	0037594	326	0030675
27	0370370	87	0114943	147	0068027	207	0048309	267	0037453	327	0030581
28	0357143	88	0113636	148	0067567	208	0048077	268	0037313	328	0030488
29	0344828	89	0112360	149	0067114	209	0047847	269	0037175	329	0030395
30	0333333	90	0111111	150	0066666	210	0047619	270	0037037	330	0030303
31	0322581	91	0109890	151	0066225	211	0047393	271	0036900	331	0030211
32	03125	92	0108696	152	0065789	212	0047170	272	0036765	332	0030120
33	0303030	93	0107527	153	0065359	213	0046948	273	0036630	333	0030030
34	0294118	94	0106383	154	0064935	214	0046729	274	0036496	334	0029940
35	0285714	95	0105263	155	0064516	215	0046512	275	0036363	335	0029851
36	0277777	96	0104166	156	0064103	216	0046296	276	0036232	336	0029762
37	0270270	97	0103093	157	0063694	217	0046083	277	0036101	337	0029674
38	0263158	98	0102041	158	0063291	218	0045872	278	0035971	338	0029586
39	0256410	99	0101010	159	0062893	219	0045662	279	0035842	339	0029499
40	025	100	01	160	00625	220	0045454	280	0035714	340	0029412
41	0243902	101	0099009	161	0062112	221	0045249	281	0035587	341	0029326
42	0238095	102	0098039	162	0061728	222	0045045	282	0035461	342	0029240
43	0232558	103	0097087	163	0061350	223	0044843	283	0035336	343	0029155
44	0227272	104	0096154	164	0060975	224	0044643	284	0035211	344	0029070
45	0222222	105	0095238	165	0060606	225	0044444	285	0035088	345	0028986
46	0217391	106	0094340	166	0060241	226	0044248	286	0034965	346	0028902
47	0212766	107	0093458	167	0059880	227	0044053	287	0034843	347	0028818
48	0208333	108	0092592	168	0059524	228	0043860	288	0034722	348	0028736
49	0204082	109	0091743	169	0059172	229	0043668	289	0034602	349	0028653
50	02	110	0090909	170	0058824	230	0043478	290	0034483	350	0028571
51	0196078	111	0090090	171	0058480	231	0043290	291	0034364	351	0028490
52	0192308	112	0089286	172	0058141	232	0043103	292	0034246	352	0028409
53	0188679	113	0088496	173	0057803	233	0042918	293	0034130	353	0028329
54	0185185	114	0087719	174	0057471	234	0042735	294	0034014	354	0028248
55	0181818	115	0086957	175	0057143	235	0042553	295	0033898	355	0028169
56	0178571	116	0086207	176	0056818	236	0042373	296	0033783	356	0028070
57	0175439	117	0085470	177	0056497	237	0042194	297	0033670	357	0028011
58	0172414	118	0084745	178	0056180	238	0042017	298	0033557	358	0027933
59	0169490	119	0084034	179	0055866	239	0041841	299	0033415	359	0027855
60	0166666	120	0083333	180	0055555	240	0041666	300	0033333	360	0027777

Table of Reciprocals.

No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip	No.	Recip.
361	0027701	421	0023753	481	0020790	541	0018484	601	0016639	661	0015129
362	0027624	422	0023697	482	0020747	542	0018450	602	0016611	662	0015106
363	0027548	423	0023641	483	0020704	543	0018416	603	0016584	663	0015083
364	0027473	424	0023585	484	0020661	544	0018382	604	0016556	664	0015060
365	0027397	425	0023529	485	0020619	545	0018349	605	0016529	665	0015038
366	0027322	426	0023474	486	0020576	546	0018325	606	0016501	666	0015015
367	0027248	427	0023419	487	0020534	547	0018282	607	0016474	667	0014993
368	0027174	428	0023364	488	0020492	548	0018248	608	0016447	668	0014970
369	0027100	429	0023310	489	0020450	549	0018215	609	0016420	669	0014948
370	0027027	430	0023256	490	0020408	550	0018181	610	0016393	670	0014925
371	0026954	431	0023202	491	0020367	551	0018149	611	0016367	671	0014903
372	0026882	432	0023148	492	0020325	552	0018116	612	0016340	672	0014881
373	0026810	433	0023095	493	0020284	553	0018083	613	0016313	673	0014859
374	0026738	434	0023042	494	0020243	554	0018051	614	0016287	674	0014837
375	0026666	435	0022989	495	0020202	555	0018018	615	0016260	675	0014814
376	0026596	436	0022936	496	0020162	556	0017986	616	0016234	676	0014793
377	0026525	437	0022883	497	0020121	557	0017953	617	0016207	677	0014771
378	0026455	438	0022831	498	0020080	558	0017921	618	0016181	678	0014749
379	0026385	439	0022779	499	0020040	559	0017889	619	0016155	679	0014728
380	0026316	440	0022727	500	002	560	0017857	620	0016129	680	0014706
381	0026247	441	0022676	501	0019960	561	0017825	621	0016103	681	0014684
382	0026178	442	0022624	502	0019920	562	0017794	622	0016077	682	0014663
383	0026110	443	0022573	503	0019881	563	0017762	623	0016051	683	0014641
384	0026042	444	0022522	504	0019841	564	0017730	624	0016026	684	0014620
385	0025974	445	0022472	505	0019801	565	0017699	625	0016	685	0014599
386	0025907	446	0022422	506	0019763	566	0017668	626	0015974	686	0014577
387	0025840	447	0022371	507	0019724	567	0017637	627	0015949	687	0014556
388	0025773	448	0022321	508	0019685	568	0017606	628	0015924	688	0014535
389	0025707	449	0022272	509	0019646	569	0017575	629	0015898	689	0014514
390	0025641	450	0022222	510	0019608	570	0017544	630	0015873	690	0014493
391	0025575	451	0022173	511	0019569	571	0017513	631	0015848	691	0014472
392	0025510	452	0022124	512	0019531	572	0017483	632	0015823	692	0014451
393	0025445	453	0022075	513	0019493	573	0017452	633	0015798	693	0014430
394	0025381	454	0022026	514	0019455	574	0017422	634	0015773	694	0014409
395	0025316	455	0021978	515	0019417	575	0017391	635	0015748	695	0014388
396	0025252	456	0021930	516	0019380	576	0017361	636	0015723	696	0014368
397	0025189	457	0021882	517	0019342	577	0017331	637	0015699	697	0014347
398	0025126	458	0021834	518	0019305	578	0017301	638	0015674	698	0014327
399	0025063	459	0021786	519	0019268	579	0017271	639	0015649	699	0014306
400	0025	460	0021739	520	0019231	580	0017241	640	0015625	700	0014286
401	0024938	461	0021692	521	0019194	581	0017212	641	0015601	701	0014265
402	0024876	462	0021645	522	0019157	582	0017182	642	0015576	702	0014245
403	0024814	463	0021598	523	0019120	583	0017153	643	0015552	703	0014225
404	0024752	464	0021552	524	0019084	584	0017123	644	0015528	704	0014205
405	0024691	465	0021505	525	0019048	585	0017094	645	0015504	705	0014184
406	0024631	466	0021459	326	0019011	586	0017065	646	0015480	706	0014164
407	0024570	467	0021413	527	0018975	587	0017036	647	0015456	707	0014144
408	0024510	468	0021368	528	0018939	588	0017007	648	0015432	708	0014124
409	0024450	469	0021322	529	0018904	589	0016978	649	0015408	709	0014104
410	0024390	470	0021277	530	0018868	590	0016949	650	0015385	710	0014085
411	0024331	471	0021231	531	0018832	591	0016920	651	0015361	711	0014065
412	0024272	472	0021187	532	0018797	592	0016891	652	0015337	712	0014045
413	0024213	473	0021142	533	0018762	593	0016863	653	0015314	713	0014025
414	0024155	474	0021097	534	0018727	594	0016835	654	0015291	714	0014006
415	0024096	475	0021053	535	0018692	595	0016807	655	0015267	715	0013986
416	0024038	476	0021008	536	0018657	596	0016779	656	0015244	716	0013966
417	0023981	477	0020964	537	0018622	597	0016750	657	0015221	717	0013947
418	0023923	478	0020921	538	0018587	598	0016722	658	0015198	718	0013928
419	0023866	479	0020877	539	0018553	599	0016694	659	0015175	719	0013908
420	0023810	480	0020833	540	0018518	600	0016666	660	0015151	720	0013888

Table of Reciprocals.

No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.	No.	Recip.
721	0013870	768	0013021	815	0012270	862	0011601	909	0011001	956	0010460
722	0013850	769	0013004	816	0012255	863	0011587	910	0010989	957	0010449
723	0013831	770	0012987	817	0012240	864	0011574	911	0010977	958	0010438
724	0013812	771	0012970	818	0012225	865	0011561	912	0010965	959	0010428
725	0013793	772	0012953	819	0012210	866	0011547	913	0010953	960	0010416
726	0013774	773	0012937	820	0012195	867	0011534	914	0010941	961	0010406
727	0013755	774	0012920	821	0012180	868	0011521	915	0010929	962	0010395
728	0013736	775	0012903	822	0012165	869	0011507	916	0010917	963	0010384
729	0013717	776	0012887	823	0012151	870	0011494	917	0010905	964	0010373
730	0013699	777	0012870	824	0012136	871	0011481	918	0010893	965	0010363
731	0013680	778	0012853	825	0012121	872	0011468	919	0010881	966	0010352
732	0013661	779	0012837	826	0012106	873	0011455	920	0010870	967	0010341
733	0013643	780	0012821	827	0012092	874	0011442	921	0010858	968	0010331
734	0013624	781	0012804	828	0012077	875	0011429	922	0010846	969	0010320
735	0013605	782	0012788	829	0012063	876	0011416	923	0010834	970	0010309
736	0013587	783	0012771	830	0012048	877	0011403	924	0010823	971	0010299
737	0013569	784	0012755	831	0012034	878	0011390	925	0010810	972	0010288
738	0013550	785	0012739	832	0012019	879	0011377	926	0010799	973	0010277
739	0013532	786	0012723	833	0012005	880	0011363	927	0010787	974	0010267
740	0013513	787	0012706	834	0011990	881	0011351	928	0010776	975	0010256
741	0013495	788	0012690	835	0011976	882	0011338	929	0010764	976	0010246
742	0013477	789	0012674	836	0011962	883	0011325	930	0010753	977	0010235
743	0013459	790	0012658	837	0011947	884	0011312	931	0010741	978	0010225
744	0013441	791	0012642	838	0011933	885	0011299	932	0010730	979	0010215
745	0013423	792	0012626	839	0011919	886	0011287	933	0010718	980	0010204
746	0013405	793	0012610	840	0011905	887	0011274	934	0010707	981	0010194
747	0013387	794	0012594	841	0011891	888	0011261	935	0010695	982	0010183
748	0013369	795	0012579	842	0011876	889	0011249	936	0010684	983	0010173
749	0013351	796	0012563	843	0011862	890	0011236	937	0010672	984	0010163
750	0013333	797	0012547	844	0011848	891	0011223	938	0010661	985	0010152
751	0013316	798	0012531	845	0011834	892	0011211	939	0010650	986	0010142
752	0013298	799	0012516	846	0011820	893	0011198	940	0010638	987	0010132
753	0013280	800	00125	847	0011806	894	0011186	941	0010627	988	0010121
754	0013263	801	0012484	848	0011792	895	0011173	942	0010616	989	0010111
755	0013245	802	0012469	849	0011779	896	0011161	943	0010604	990	0010101
756	0013228	803	0012453	850	0011765	897	0011148	944	0010593	991	0010091
757	0013210	804	0012438	851	0011751	898	0011136	945	0010582	992	0010081
758	0013193	805	0012422	852	0011737	899	0011123	946	0010571	993	0010070
759	0013175	806	0012407	853	0011723	900	0011111	947	0010560	994	0010060
760	0013158	807	0012392	854	0011710	901	0011099	948	0010549	995	0010050
761	0013141	808	0012376	855	0011696	902	0011086	949	0010537	996	0010040
762	0013123	809	0012361	856	0011682	903	0011074	950	0010526	997	0010030
763	0013106	810	0012346	857	0011669	904	0011062	951	0010515	998	0010020
764	0013089	811	0012330	858	0011655	905	0011050	952	0010504	999	0010010
765	0013072	812	0012315	859	0011641	906	0011038	953	0010493	1000	001
766	0013055	813	0012300	860	0011628	907	0011025	954	0010482		
767	0013038	814	0012285	861	0011614	908	0011013	955	0010471		

Of the preceding Table, the use is evidently to shorten arithmetical calculations, and will appear eminently great to those mathematicians and others who are frequently concerned in such kinds of computations. The structure of the Table is evident; the first column contains the natural series of numbers from 1 to 1000, the 2d the Reciprocals. These Reciprocals (which are no other than the decimal values of the quotients resulting from the division of unity or 1 by each of the several numbers from 1 to 1000) are not only useful in shewing by inspection the quotient when the dividend is unity, but are also applied with much advantage in turning many divisions into multiplications, which are much easier performed, and are done by multiplying the Reciprocal of the divisor (as found in the Table) by the dividend, for the quotient; they will also apply to good purpose in summing the terms of many converging series.

The Reciprocals are carried on to 7 places of decimals (for the column of Reciprocals must be accounted all decimal figures, although they have not the decimal point placed before them, which is omitted to save room), each being set down to the nearest figure in the last place, that is, when the next figure beyond the last set down in the Table came out a 5 or more, the last figure

figure was increaſed by 1, otherwiſe not; excepting in the repetends which occurred among the Reciprocals, where the real laſt figure is always ſet down; the Reciprocals, which in the Table conſiſt of leſs than ſeven figures, are thoſe which terminate, and are complete within that number; ſuch as ·5 the Reciprocal of 2, ·25 the Reciprocal of 4, &c.

Reciprocal *Figures*, in Geometry, are ſuch as have the antecedents and conſequents of the ſame ratio in both figures. So, in the two rectangles BE and BD, if AB : DC :: BC : AE, then thoſe rectangles are reciprocal figures; and are alſo equal.

Reciprocal *Proportion*, is when, in four quantities, the two latter terms have the Reciprocal ratio of the two former, or are proportional to the Reciprocals of them. Thus, 24, 15, 5, 8 form a Reciprocal proportion, becauſe

$$\frac{1}{24} : \frac{1}{15} :: 5 : 8, \text{ or } 15 : 24 :: 5 : 8.$$

Reciprocal *Ratio*, of any quantity, is the ratio of the Reciprocal of the quantity.

RECIPROCALLY. One quantity is Reciprocally as another, when the one is greater in proportion as the other is leſs; or when the one is proportional to the Reciprocal of the other. So a is Reciprocally as b, when a is always proportional to $\frac{1}{b}$. Like as in the mechanic powers, to perform any effect, the leſs the power is, the greater muſt be the time of performing it; or, as it is ſaid, what is gained in power, is loſt in time. So that, if p denote any power or agent, and t the time of its performing any given ſervice; then p is as $\frac{1}{t}$, and t is as $\frac{1}{p}$; that is, p and t are Reciprocally proportionals to each other.

RECKONING, in Navigation, is the eſtimating the quantity of a ſhip's way; or of the courſe and diſtance run. Or, more generally, a ſhip's Reckoning is that account, by which it may at any time be known where the ſhip is, and conſequently on what courſe or courſes ſhe muſt ſteer to gain her intended port. The Reckoning is uſually performed by keeping an account of the courſes ſteered, and the diſtance run, with any accidental circumſtances that occur. The courſes ſteered are obſerved by the compaſs; and the diſtances run are eſtimated from the rate of running, and the time run upon each courſe. The rate of running is meaſured by the log, from time to time; which however is liable to great irregularities. Anciently Vitruvius, for meaſuring the rate of ſailing, adviſed an axis to be paſſed through the ſides of the ſhip, with two large heads protending out of the ſhip, including wheels touching the water, by the revolution of which the ſpace paſſed over in a given time is meaſured. And the ſame has been ſince recommended by Snellius.

Reckoning, *Dead*. See Dead *Reckoning*.

RECLINATION *of a Plane*, in Dialling, is the angular quantity which a dial-plane leans backwards, from an exactly upright or vertical plane, or from the zenith.

RECLINER, or Reclining *Dial*, is a dial whoſe plane reclines from the perpendicular, that is, leans backwards, or from you, when you ſtand before it.

Recliner, *Declining*, or *Declining* Reclining *Dial*, is one which neither ſtands perpendicularly, nor oppoſite to one of the cardinal points.

RECOIL, or Rebound, the reſilition, or flying backward, of a body, eſpecially a fire-arm. This is the motion by which, upon exploſion, it ſtarts or flies backwards; and the cauſe of it is the reſiſtance of the ball and the impelling force of the powder, which acts equally on the gun and on the ball. It has been commonly ſaid by authors, that the momentum of the ball is equal to that of the gun with its carriage together; but this is a miſtake; for the latter momentum is nearly equal to that of the ball and half the weight of the powder together, moving with the velocity of the ball. So that, if the gun, and the ball with half the powder, were of equal weight, the piece would recoil with the ſame velocity as the ball is diſcharged. But the heavier any body is, the leſs will its velocity be, to have the ſame momentum, or force; and therefore ſo many times as the cannon and carriage is heavier than the ball and half the powder, juſt as many times will the velocity of the ball be greater than that of the gun; and in the ſame ratio nearly is the length of the barrel before the charge, to the quantity the gun Recoils in the time the ball is paſſing along the bore of the gun. So, if a 24 pounder of 10 feet long be 6400lb weight, and charged with 6lb of powder; then, when the ball quits the piece, the gun will have Recoiled $\frac{28}{6400} \times 10 = \frac{7}{160}$ of a foot, or nearly half an inch.

RECORDE (Robert), a learned phyſician and mathematician, was born of a good family in Wales, and flouriſhed in the reigns of Henry the 8th, Edward the 6th, and Mary. There is no account of the exact time of his birth, though it muſt have been early in the 16th century, as he was entered of the univerſity of Oxford about the year 1525, where he was elected fellow of Allſouls college in 1531. Making phyſic his profeſſion, he went to Cambridge, where he was honoured with the degree of doctor in that faculty, in 1545, and highly eſteemed by all that knew him for his great knowledge in ſeveral arts and ſciences. He afterwards returned to Oxford, where, as he had done before he went to Cambridge, he publicly taught arithmetic, and other branches of the mathematics, with great applauſe. It ſeems he afterwards repaired to London, and it has been ſaid he was phyſician to Edward the 6th and Mary, to which princes he dedicates ſome of his books; and yet he ended his days in the King's-bench priſon, Southwark, where he was confined for debt, in the year 1558, at a very immature age.

Recorde publiſhed ſeveral mathematical books, which are moſtly in dialogue, between the maſter and ſcholar. They are as follow:

1. *The Pathway to Knowledge*, containing the firſt Principles of Geometrie, as they may moſte aptly be applied unto practiſe, bothe for uſe of Inſtrumentes Geometricall and Aſtronomicall, and alſo for Projection of Plattes much neceſſary for all ſortes of men. Lond. 4to, 1551.

2. *The Ground of Arts*, teaching the perfect worke and practice of Arithmeticke, both in whole numbers and

and fractions, after a more easie and exact forme then in former time hath beene set forth, 8vo, 1552.—This work went through many editions, and was corrected and augmented by several other persons; as first by the famous Dr. John Dee; then by John Mellis, a schoolmaster, 1590; next by Robert Norton; then by Robert Hartwell, practitioner in mathematics, in London; and lastly by R. C. and printed in 8vo, 1623.

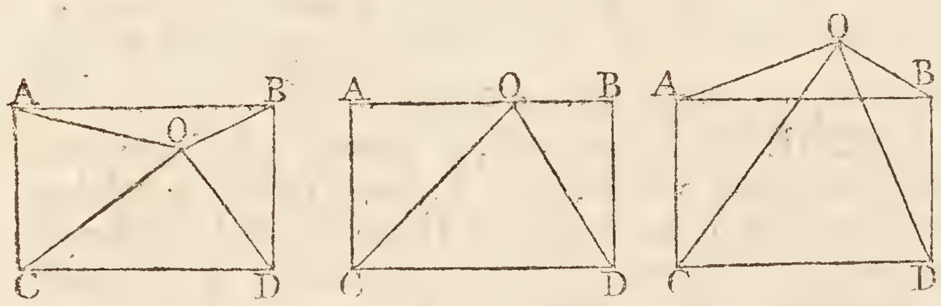
3. *The Castle of Knowledge*, containing the Explication of the Sphere bothe Celestiall and Materiall, and divers other things incident thereto. With sundry pleasaunt proofes and certaine newe demonstrations not written before in any vulgare woorkes. Lond. folio, 1556.

4. *The Whetstone of Witte*, which is the seconde part of Arithmetike: containing the Extraction of Rootes: the Cossike Practise, with the rules of Equation: and the woorkes of Surde Nombers. Lond. 4to, 1557.—For an analysis of this work on Algebra, with an account of what is new in it, see pa. 79 of vol. 1, under the article ALGEBRA.

Wood says he wrote also several pieces on physic, anatomy, politics, and divinity; but I know not whether they were ever published. And Sherburne says that he published *Cosmographiæ Isagogen*; also that he wrote a book, *De Arte faciendi Horologium*; and another, *De Usu Globorum, & de Statu Temporum*; which I have never seen.

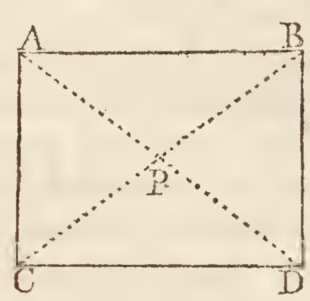
RECTANGLE, in Geometry, is a right-angled parallelogram, or a right-angled quadrilateral figure.

If from any point O, lines be drawn to all the four



angles of a Rectangle; then the sum of the squares of the lines drawn to the opposite corners will be equal, in whatever part of the plane the point O is situated; viz, $OA^2 + OD^2 = OB^2 + OC^2$. For other properties of the Rectangle, see PARALLELOGRAM; for the Rectangle being a species of the parallelogram, whatever properties belong to the latter, must equally hold in the former.

For the Area of a RECTANGLE. Multiply the length by the breadth or height.—*Otherwise*; Multiply the product of the two diagonals by half the sine of their angle at the intersection.



That is, $AB \times AC$, or $AD \times BC \times \frac{1}{2}$ sin. $\angle P =$ area. A Rectangle, as of two lines AB and AC, is thus denoted, $AB \times AC$, or $AB . AC$; or else thus expressed, the Rectangle of, or under, AB and AC.

RECTANGLE, in Arithmetic, is the same with product or factum. So the Rectangle of 3 and 4, is 3×4 or 12; and of a and b is $a \times b$ or ab.

RECTANGLED, RIGHT-ANGLED, or RECTANGULAR, is applied to figures and solids that have at least one right angle, if not more. So a Right-angled triangle, has one right angle; a Right-angled parallelogram is a rectangle, and has four right angles. Such also are squares, cubes, parallelopipedons.

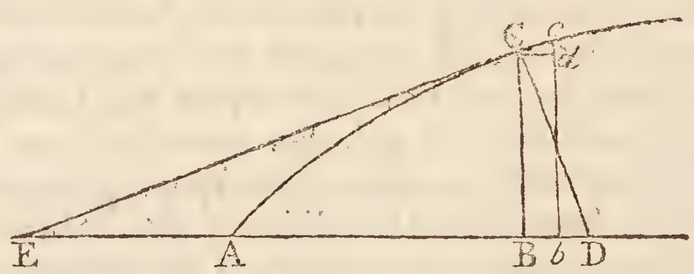
Solids are also said to be Rectangular with respect to their situation, viz, when their axis is perpendicular to their base; as right cones, pyramids, cylinders, &c.

The Ancients used the phrase *Rectangular section of a cone*, to denote a parabola; that conic section, before Apollonius, being only considered in a cone having its vertex a right-angle. And hence it was, that Archimedes entitled his book of the quadrature of the parabola, by the name of *Rectanguli Coni Sectio*.

RECTIFICATION, in Geometry, is the finding of a right line equal to a curve. The Rectification of curves is a branch of the higher geometry, a branch in which the use of the inverse method of Fluxions is especially useful. This is a problem to which all mathematicians, both ancient and modern, have paid the greatest attention, and particularly as to the Rectification of the circle, or finding the length of the circumference, or a right line equal to it; but hitherto without the perfect effect: upon this also depends the quadrature of the circle, since it is demonstrated that the area of a circle is equal to a right-angled triangle, of which one of the sides about the right angle is the radius, and the other equal to the circumference: but it is much to be feared that neither the one nor the other will ever be accomplished. Innumerable approximations however have been made, from Archimedes, down to the mathematicians of the present day. See CIRCLE and CIRCUMFERENCE.

The first person who gave the Rectification of any curve, was Mr. Neal, son of Sir Paul Neal, as we find at the end of Dr. Wallis's treatise on the Cissoid; where he says, that Mr. Neal's Rectification of the curve of the semicubical parabola, was published in July or August, 1657. Two years after, viz in 1659, Van Hauraet, in Holland, also gave the Rectification of the same curve; as may be seen in Schooten's Commentary on Des Cartes's Geometry.

The most comprehensive method of Rectification of curves, is by the inverse method of fluxions, which is thus: Let ACc be any curve line, AB an abscis, and



BC a perpendicular ordinate; also bc another ordinate indefinitely near to BC; and Cd drawn parallel to the abscis AB. Put the abscis $AB = x$, the ordinate $BC = y$, and the curve $AC = z$: then is $Cd = Bb = \dot{x}$ the fluxion of the abscis AB, and $cd = \dot{y}$ the fluxion of the ordinate BC, also $Cc = \dot{z}$ the fluxion of the curve AB. Hence because Ccd may be considered as a plane right-angled triangle, $Cc^2 = Cd^2 + cd^2$, or $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$; and therefore $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2}$; which is the fluxion of the length of any curve; and consequently, out of this equation expelling either $\dot{x}$ or $\dot{y}$, by means of the particular equation expressing the nature of the curve in question, the fluents of the resulting equation, being then taken, will give the length of the curve, in finite terms when it is rectifiable,

rectifiable, otherwise in an infinite series, or in a logarithmic or exponential &c expression, or by means of some other curve, &c.

Ex. 1. *To rectify the common parabola.*—In this case, the equation of the curve is $2ax = y^2$, where a is half the parameter. The fluxion of this equation is $2a\dot{x} = 2y\dot{y}$, and hence $\dot{x}^2 = \frac{y^2\dot{y}^2}{a^2}$; this being substituted in the general equation $\dot{z} = \sqrt{\dot{x}^2 + \dot{y}^2}$, it becomes $\dot{z} = \frac{\dot{y}\sqrt{aa + yy}}{a}$; the correct fluents of which give

$$z = \frac{y\sqrt{aa + yy}}{2a} + \tfrac{1}{2}a \times \text{hyp. log. of } \frac{y + \sqrt{aa + yy}}{a},$$

which is the length of the curve AC, when it is a parabola.

And the same might be expressed by an infinite series, by expanding the quantity $\sqrt{aa + yy}$. See my Mensuration, pa. 361, 2d edit.

Ex. 2. *To rectify the Circle.*—The equation of the circle may be expressed either in terms of the sine, or versed sine, or tangent, or secant, &c, and the radius. Let therefore the radius of the circle be DA or DC $= r$, the versed sine AB $= x$, the right sine BC $= y$, the tangent CE $= t$, and the secant DE $= s$; then, by the nature of the circle, we have these equations, $y^2 = 2rx - x^2 = \frac{r^2t^2}{r^2 + t^2} = \frac{s^2 - r^2}{s^2}r^2$; and by means of the fluxions of these equations, with the general equation $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$, are obtained the following fluxional forms for the fluxion of the curve, the fluent of any one of which will be the curve itself, viz,

$$\dot{z} = \frac{r\dot{x}}{\sqrt{2rx - xx}} = \frac{r\dot{y}}{\sqrt{rr - yy}} = \frac{r^2\dot{t}}{r^2 + t^2} = \frac{r^2\dot{s}}{\sqrt{s^2 - r^2}}.$$

Hence the value of the curve, from the fluent of each of these, gives the four following forms, in series, viz, the curve, putting $d = 2r$ the diameter, is z

$$= (1 + \frac{x}{2.3d} + \frac{3x^2}{2.4.5d^2} + \frac{3.5x^3}{2.4.6.7d^3} \text{ &c}) \sqrt{dx},$$

$$= (1 + \frac{y^2}{2.3r^2} + \frac{3y^4}{2.4.5r^4} + \frac{3.5y^6}{2.4.6.7r^6} \text{ &c}) y,$$

$$= (1 - \frac{t^2}{3r^2} + \frac{t^4}{5r^4} - \frac{t^6}{7r^6} + \frac{t^8}{9r^8} \quad \text{&c}) t,$$

$$= (\frac{s - r}{s} + \frac{s^3 - r^3}{2.3s^3} + \frac{3(s^5 - r^5)}{2.4.5s^5} \quad \text{&c}) r.$$

See my Mensur. 2d edit. pa. 118 &c, also most treatises on Fluxions.

It is evident that the simplest of these series is the third, or that which is expressed in terms of the tangent. It will therefore be the properest form to calculate an example by in numbers. And for this purpose it will be convenient to assume some arc whose tangent, or at least its square, is known to be some small finite number. Now the arc of 45° it is known has its tangent equal to the radius; and therefore, taking the radius $r = 1$, and consequently the tangent of 45° or $t = 1$ also, in this case the arc of 45° to the radius 1, or the quadrant to the diameter 1, will be $= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c. But as this series converges very slowly, some smaller arch must be taken, that the series may converge faster; such as the arc of 30°, whose tangent is $= \sqrt{\frac{1}{3}} = \cdot 5773502$, or its square $t^2 = \frac{1}{3}$; and hence, after the first term, the succeeding terms will be found by dividing always by 3, and these quotients divided by the absolute numbers 3, 5, 7, 9, &c; and lastly adding every other term together into two sums, the one the sum of the positive terms, and the other the sum of the negative ones, then lastly the one sum taken from the other leaves the length of the arc of 30°, which is the 12th part of the whole circumference when the radius is 1, or the 6th part when the diameter is 1, and consequently 6 times that arc will be the length of the whole circumference to the diameter 1; therefore multiply the 1st term $\sqrt{\frac{1}{3}}$ by 6, and the product is $\sqrt{\frac{36}{3}}$ or $\sqrt{12} = 3\cdot 4641016$; hence the operation will be conveniently made as follows:

		+ Terms.	− Terms.
1)	3·4641016	(3·4641016	
3)	1·1547005	(	0·3849002
5)	3849002	(769800	
7)	1283001	(	183286
9)	427667	(47519	
11)	142556	(	12960
13)	47519	(3655	
15)	15840	(	1056
17)	5280	(311	
19)	1760	(	93
21)	587	(28	
23)	196	(	8
25)	65	(3	
27)	22	(	1
		+ 3·5462332	−0·4046406
		− 0·4046406	
		3·1415926 the circumference.	

Various other series for the Rectification of the circle may be seen in different parts of my Mensuration, as at pa. 121, 122, 137, 138, 422, &c. See also my paper on this subject in the Philos. Trans. vol. 66, pa. 476.

RECTIFIER, in Navigation, is an instrument used for determining the variation of the compass, in order to rectify the ship's course. It consists of two circles, either laid upon, or let into one another, and so fastened together in their centres that they represent two compasses, the one fixed, and the other moveable. Each is divided into 32 points of the compass, and 360°, and numbered both ways, from the north and the south, ending at the east and west in 90°. The fixed compass represents the horizon, in which the north, and all the other points, are liable to variation. In the centre of the

the moveable compaſs is faſtened a ſilk thread, long enough to reach the outſide of the fixed compaſs: but when the inſtrument is made of wood, an index is uſed inſtead of the thread.

RECTIFYING *of Curves*. See RECTIFICATION.

RECTIFYING *of the Globe or Sphere*, is a previous adjuſtment of it, to prepare it for the ſolution of problems. This uſually conſiſts in placing it in the ſame poſition as the true ſphere of the world has at ſome certain time propoſed; which is done firſt by elevating the pole above the horizon as much as the latitude of the place is, then bringing the ſun's place for the given day, found in the ecliptic, to the graduated ſide of the braſs or general meridian, next move the hour-index to the upper hour of 12, ſo ſhall the globe be Rectified for noon of that day; and if the globe be turned about till the hour-index point at any propoſed hour, then is the globe in the real poſition of the earth at that time, if the whole globe be ſet in the north and ſouth poſition by means of the compaſs.

RECTILINEAL, RECTILINEAR, or *Right-lined*, is the quality or nature of figures that are bounded by right lines, or formed by right lines.

RECURRING *Series*, is a ſeries conſtituted in ſuch a manner, that having taken at pleaſure any number of its terms, each following term ſhall be related to the ſame number of preceding terms according to a conſtant law of relation. See *Recurring* SERIES.

RED, in Phyſics, or Optics, one of the ſimple or primary colours of natural bodies, or rather of the rays of light.—The Red rays are the leaſt refrangible of all the rays of light. And hence, as Newton ſuppoſes the different degrees of refrangibility to ariſe from the different magnitudes of the luminous particles of which the rays conſiſt; therefore the Red rays, or Red light, is concluded to be that which conſiſts of the largeſt particles. See COLOUR and LIGHT.

Authors diſtinguiſh three general kinds of Red: one bordering on the blue, as colombine, or dove-colour, purple, and crimſon; another bordering on yellow, as flame-colour and orange; and between theſe extremes is a medium, which is that which is properly called Red.

REDANS, or REDANT, or REDENS, in Fortification, is a kind of work indented like the teeth of a ſaw, with ſalient and re-entering angles; to the end that one part may flank or defend another. It is called alſo *ſaw work*, and *indented work*.

Redans are often uſed in fortifying of walls, where it is not neceſſary to be at the expence of building baſtions; as when they ſtand on the ſide of a river, or a marſh, or the ſea, &c. But the fault of ſuch fortification is, that the beſiegers from one battery may ruin both the ſides of the tenaille or front of a place, and make an aſſault without fear of being enfiladed, ſince the defences are ruined.

The parapet of the corridor alſo is frequently Redented, or carried on by the way of Redans.

REDINTEGRATION, is the taking or finding the integral or fluent again, from the fluxion. See FLUXION and FLUENT.

REDOUBT, or REDOUTE, in Fortification, a ſmall fort, without any defence but in front, uſed in trenches, lines of circumvallation, contravallation, and approach; as alſo for the lodging of corps de garde, and to defend paſſages.

A Detached REDOUBT, is a kind of work reſembling a ravelin, with flanks, placed beyond the glacis.—It is made to occupy ſome ſpot of ground which might be advantageous to the beſiegers; and alſo to oblige the enemy to open his trenches farther off than he would otherwiſe do.

REDUCING *Scale*, or SURVEYING *Scale*, is a broad, thin ſlip of box, or ivory, having ſeveral lines and ſcales of equal parts upon it; uſed by ſurveyors for turning chains and links into roods and acres, by inſpection. They uſe it alſo to reduce maps and draughts from one dimenſion to another.

REDUCTION, in general, is the bringing or changing ſome thing to a different form, ſtate, or denomination.

REDUCTION, in *Arithmetic*, is commonly underſtood of the changing of money, weights, or meaſures, to other denominations, of the ſame value; and it is of two kinds, *Reduction Deſcending*, which is the changing a number to its equivalent value in a lower denomination; as pounds into ſhillings or pence: and *Reduction Aſcending*, which is the changing numbers to higher denominations; as pence to ſhillings or pounds.

RULE. *To perform Reduction*; conſider how many of the leſs denomination make one of the greater, as how many pence make a ſhilling, or how many ſhillings make a pound; and multiply by that number when the Reduction is deſcending, but divide by it when it is aſcending. So to reduce 23l. into pence; and converſely thoſe pence into pounds; multiply or divide by 12 and 20, as here below.

$$\begin{array}{r} 23\ l. \\ 20 \\ \hline 460\ sh. \\ 12 \\ \hline 5520\ d. \end{array} \qquad\qquad \begin{array}{l} 12\ )\ 5520\ d. \\ \\ 20\ )\ 460\ sh. \\ \qquad\ \ 23\ l. \end{array}$$

REDUCTION *of Fractions*. See FRACTION, and DECIMAL.

REDUCTION *of Equations*, in Algebra. See EQUATION.

REDUCTION *of Curves*. See CURVE.

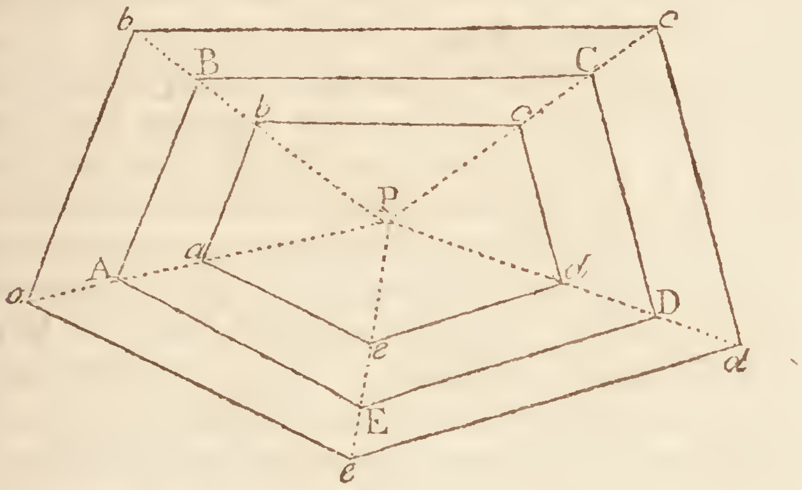
REDUCTION *of a Figure, Deſign, or Draught*, is the making a copy of it, either larger or ſmaller than the original, but ſtill preſerving the form and proportion.

Figures and plans are reduced, and copied, in various ways; as by the Pentagraph, and Proportional compaſſes. See PENTAGRAPH, and *Proportional* COMPASSES. The beſt of the other methods of reducing are as below.

To reduce a Simple Rectilinear Figure by Lines.

Pitch upon a point P any where about the given figure ABCDE, either within it, or without it, or in one ſide or angle; but near the middle is beſt. From that point P draw lines through all the angles; upon one of

of which take Pa to PA in the propofed proportion of the fcales, or linear dimenfions; then draw ab parallel to AB, bc to BC, &c; fo fhall $abcde$ be the reduced figure fought, either greater or fmaller than the original.

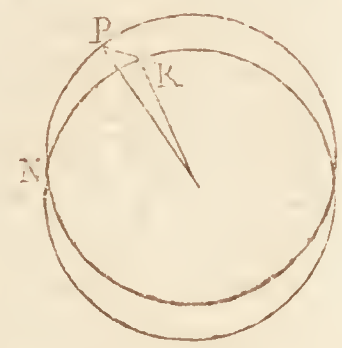


To Reduce a Figure by a Scale.—Meafure all the fides, and diagonals, of the figure, as ABCDE, by a fcale; and lay down the fame meafures refpectively, from another fcale, in the proportion required.

To Reduce a Map, Defign, or Figure, by Squares.—Divide the original into a number of little fquares; and divide a frefh paper, of the dimenfions required, into the fame number of other fquares, either greater or fmaller as required. This done, in every fquare of the fecond figure, draw what is found in the correfponding fquare of the firft or original figure.

The crofs lines forming thefe fquares, may be drawn with a pencil, and thefe rubbed out again after the work is finifhed. But a more ready and convenient way, efpecially when fuch Reductions are often wanted, would be to keep always at hand frames of fquares ready made, of feveral fizes; for by only juft laying them down upon the papers, the correfponding parts may be readily copied. Thefe frames may be made of four ftiff or inflexible bars, ftrung acrofs with horfe hairs, or fine catgut.

REDUCTION *to the Ecliptic*, in Aftronomy, is the difference between the argument of latitude, as NP, and an arc of the ecliptic NR, intercepted between the place of a planet, and the node.—To find this Reduction, or difference; in the right-angled fpherical triangle NPR, are given the angle of inclination, and the argument of latitude NP; to find NR; then the difference between NP and NR is the Reduction fought.



REDUNDANT *Hyperbola*, is a curve of the higher kind, fo called becaufe it exceeds the conical hyperbola in the number of legs; being a triple hyperbola, with 6 hyperbolic legs. See Newton's Enum. Lin. tertii Ordinis, nomina formarum, &c.

RE-ENTERING *Angle*, in Fortification, is an angle whofe point is turned inwards, or towards the place.

REFLECTED *Ray*, or *Vifion*, is that which is made by the reflection of light, or by light firft received upon the furface of fome body, and thence reflected again. See RAY, VISION, and REFLECTION.

REFLECTING, or REFLEXIVE, *Dial*, is a kind of dial which fhews the hour by means of a thin piece of looking-glafs plate, duly placed to throw the fun's rays to the top of a cieling, on which the hour-lines are drawn.

REFLECTION, or REFLEXION, in Mechanics, is the return, or regreffive motion of a moveable body, occafioned by the refiftance of another body, which hinders it from purfuing its former courfe of direction.

Reflection is conceived, by the lateft and beft authors, as a motion peculiar to elaftic bodies, by which, after ftriking on others which they cannot remove, they recede, or turn back, or afide, by their elaftic power.

On this principle it is afferted, that there may be, and is, a period of reft between the incidence and the reflection; fince the reflected motion is not a continuation of the other, but a new motion, arifing from a new caufe or principle, viz, the power of elafticity.

It is one of the great laws of Reflection, that the angle of incidence is equal to the angle of Reflection; i. e. that the angle which the direction of motion of a ftriking body makes with the furface of the body ftruck, is equal to the angle made between the fame furface and the direction of motion after the ftroke. See INCIDENCE and PERCUSSION.

REFLECTION *of the Rays of Light*, like that of other bodies, is their motion after being reflected from the furfaces of bodies.

The Reflection of the rays of light from the furfaces of bodies, is the means by which thofe bodies become vifible. And the difpofition of bodies to reflect this or that kind of rays moft copioufly, is the caufe of their being of this or that colour. Alfo, the Reflection of light, from the furfaces of mirrors, makes the fubject of catoptrics.

The Reflection of light, Newton has fhewn, is not effected by the rays ftriking on the very parts of the bodies; but by fome power of the body equally diffufed throughout its whole furface, by which it acts upon the ray, attracting or repelling it without any real immediate contact. This power he alfo fhews is the fame by which, in other circumftances, the rays are refracted; and by which they are at firft emitted from the lucid body.

Dr. Prieftley fays, it is not more probable, that the rays of light are tranfmitted from the fun, with an uniform difpofition to be reflected or refracted, according to the circumftances of the bodies on which they impinge; and that the tranfmiffion of fome of the rays, apparently under the fame circumftances, with others that are reflected, is owing to the minute vibrations of the fmall parts of the furfaces of the mediums through which the rays pafs; vibrations that are independent of action and reaction between the bodies and the particles of light at the time of their impinging, though probably excited by the action of preceding rays. Hift. of Light and Colours, pa. 309.

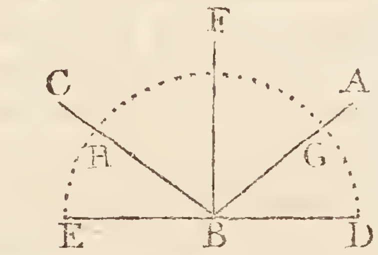
Newton concludes his account of the Reflection of light with obferving, that if light be reflected not by impinging on the folid parts of bodies, but by fome other principle, it is probable that as many of its

rays

rays as impinge on the folid parts of bodies are not reflected, but ftifled and loft in the bodies. Otherwife, he fays, we muft fuppofe two kinds of Reflection; for fhould all the rays be reflected which impinge on the internal parts of clear water or cryftal, thofe fubftances would rather have a cloudy colour, than a clear tranfparency. To make bodies look black, it is neceffary that many rays be ftopped, retained and loft in them; and it does not feem probable that any rays can be ftopped and ftifled in them, which do not impinge on their parts: and hence, he fays, we may underftand, that bodies are much more rare and porous than is commonly believed. However, M. Bouguer difputes the fact of light being ftifled or loft by impinging on the folid parts of bodies.

Reflection, in Catoptrics, is the return of a ray of light from the polifhed furface of a fpeculum or mirror, as driven thence by fome power refiding in it.

The ray thus returned is called a *reflex* or *reflected ray*, or a *ray of Reflection*; and the point of the fpeculum where the ray commences, is called the *point of Reflection*. Thus, the ray AB, proceeding from the radiant A, and ftriking on the point of the fpeculum B, being returned thence to C, BC reprefents the reflected ray, and B the point of Reflection; in refpect of which, AB reprefents the incident ray, or ray of incidence, and B the point of incidence; alfo the angle CBE is the angle of Reflection, and ABD the angle of incidence; where DE is the reflecting furface, or at leaft a tangent to it at the point B. Though fome count the angle of incidence and of Reflection from the perpendicular BF.



General Laws of Reflection.—I. *When a ray of light is reflected from a fpeculum of any form, the angle of incidence is always equal to the angle of Reflection.* This law obtains in the percuffions of all kinds of bodies; and confequently muft do fo in thofe of light; and the proof of it may be feen at the article Incidence.

This law is confirmed alfo by experiments on all bodies; and on the rays of light in this manner: A ray from the fun falling on a mirror, in a dark room, through a fmall hole, you will have the pleafure to fee it rebound, fo as to make the angle of Reflection equal to the angle of incidence. And the fame may be fhewn in various other ways: thus ex. gr. placing a femicircle DFE on a mirror DE, its centre on B, and its limb or plane perpendicular to the fpeculum; and affuming equal arcs DG and EH; place an object in A, and the eye in C: then will the object be feen by a ray reflected from the point B. But by covering B, the object will ceafe to be feen.

II. *Every point of a fpeculum reflects rays falling on it, from every part of an object.*

III. *If the eye C and the radiant point A change places, the point will continue to radiate upon the eye, in the fame courfe or path as before.*

IV. *The plane of Reflection is perpendicular to the furface of the fpeculum; and it paffes through the centre in fpherical fpecula.*

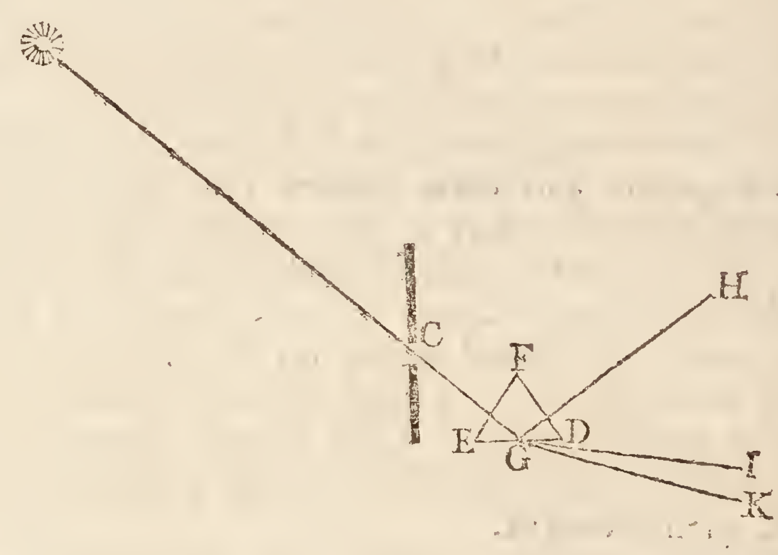
Reflection *of the Moon*, is a term ufed by fome authors for what is otherwife called *her variation*; being the 3d inequality in her motion, by which her true place out of the quadratures differs from her place twice equated.

Reflection is alfo ufed in the Copernican fyftem, for the diftance of the pole from the horizon of the difc; which is the fame thing as the fun's declination in the Ptolomaic fyftem.

REFLECTOIRE Curve. See *Reflectoire* Curve.

REFLEXIBILITY *of the rays of light*, is that property by which they are difpofed to be reflected. Or, it is their difpofition to be turned back into the fame medium, from any other medium on whofe furface they fall. Hence thofe rays are faid to be more or lefs reflexible, which are returned back more or lefs eafily under the fame incidence. Thus, if light pafs out of glafs into air, and by being inclined more and more to the common furface of the glafs and air, begins at length to be totally reflected by that furface, thofe forts of rays which at like incidences are reflected moft copioufly, or the rays which by being inclined begin fooneft to be totally reflected, are the moft reflexible rays.

That rays of light are of different colours, and endued with different degrees of reflexibility, was firft difcovered by Sir I. Newton; and it is fhewn by the following experiment. Applying a prifm DFE to



the aperture G of a darkened room in fuch manner that the light be reflected from the bafe in G; the violet rays are feen firft reflected into HG; the other rays continuing ftill refracted to I and K. After the violet, the blue are all reflected; then the green, &c.——Hence it appears, that the differently coloured rays differ in degree of Reflexibility. And from other experiments it appears, that thofe rays which are moft reflexible, are alfo moft refrangible.

REFLUX *of the Sea*, is the ebbing of the water, or its return from the fhore; being fo called, becaufe it is the oppofite motion to the flood or flux. See Tide.

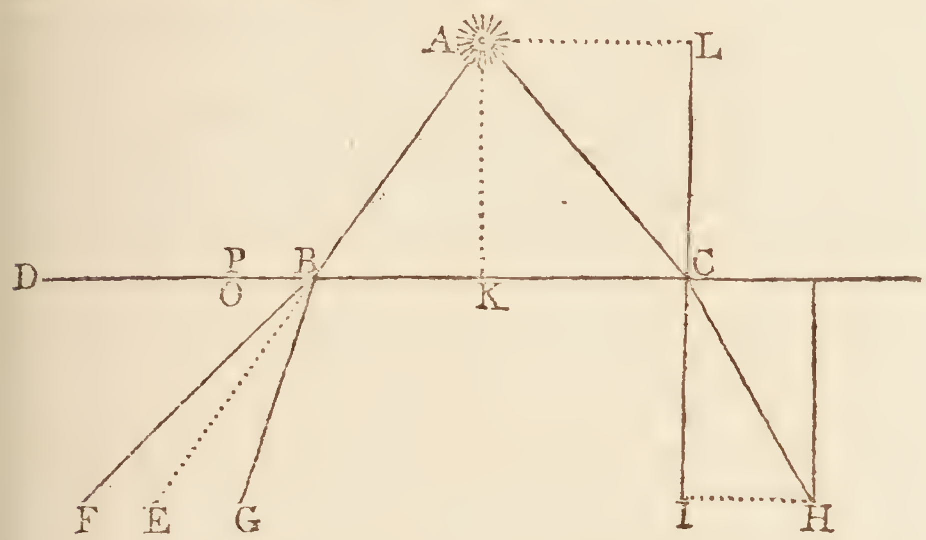
REFRACTED *Angle*, or *Angle of Refraction*, in Optics, is the angle which the refracted ray makes with the refracting furface; or fometimes it denotes the complement of that, or the angle it makes with the perpendicular to the faid furface.

Refracted *Dials*, or *Refracting Dials*, are fuch as fhew the hour by means of fome refracting tranfparent fluid.

Refracted *Ray*, or *Ray of* Refraction, is a ray

ray after it is broken or bent, at the common surface of two different mediums, where it passes from the one into the other. See RAY, and REFRACTION.

REFRACTION, in Mechanics, is the deviation of a moving body from its direct course, by reason of the different density of the medium it moves in; or a flexion and change of determination, occasioned by a body's passing obliquely out of one medium into another of a different density.



Thus a ball A, moving in the air in the line AB, and falling obliquely on the surface of the water CD, does not proceed straight in the same direction, as to E, but deviates or is deflected to F. Again, if the ball move in water in the line AB, and fall obliquely on a surface of air CD; it will in this case also deviate from the same continued direction BE, but now the contrary way, and will go to G, on the other side of it. Now the deflection in either case is called the *Refraction*, the Refraction being towards the denser surface BD in the former case, but from it in the latter.

These Refractions are supposed to arise from hence; that the ball arriving at B, in the first case finds more resistance or opposition on the one side O, or from the side of the water, than it did from the side P, or that of the air; and in the latter more resistance from the side P, which is now the side of the water, than the side O, which is that of the air. And so for any other different media: a visible instance of which is often perceived in the falling of shot or shells into the earth, as clay &c, when the perforation is found to rise a little upwards, toward the surface. However another reason is assigned for the Refraction of the rays of light, whose Refractions lie the contrary way to those above, as will be seen in what follows, viz, that water by its greater attraction accelerates the motion of the rays of light more than air does.

REFRACTION *of Light*, in Optics, is an inflection or deviation of the rays from their rectilinear course on passing obliquely out of one medium into another, of a different density.

That a body may be refracted, it is necessary that it should fall obliquely on the second medium: in perpendicular incidence there is no Refraction. Yet Voscius and Snellius imagined they had observed a perpendicular ray of light undergo a Refraction; a perpendicular object appearing in the water nearer than it really was: but this was attributing that to a Refraction of the perpendicular rays, which was owing to the divergency of the oblique rays after refraction, from a nearer point. Yet there is a manifest Refraction even of perpendicular rays found in island crystal.

Rohault adds, that though an oblique incidence be necessary in all other mediums we know of, yet the obliquity must not exceed a certain degree; if it do, the body will not penetrate the medium, but will be reflected instead of being refracted. Thus, cannon-balls, in sea engagements, falling very obliquely on the surface of the water, are observed to bound or rise from it, and to sweep the men from off the enemy's decks. And the same thing happens to the little stones with which children make their ducks and drakes along the surface of the water.

The ancients confounded Refraction with Reflection; and it was Newton who first taught the true difference between them. He shews however that there is a good deal of analogy between them, and particularly in the case of light.

The laws of Refraction of the rays of light in mediums differently terminated, i. e. whose surfaces are plane, concave, and convex, make the subject of Dioptrics.—By Refraction it is, that convex glasses, or lenses, collect the rays, magnify objects, burn, &c; and hence the foundation of microscopes, telescopes, &c.—And by Refraction it is, that all remote objects are seen out of their real places; particularly, that the heavenly bodies are apparently higher than they are in reality. The Refraction of the air has many times so uncertain an influence on the places of celestial objects, near the horizon, that wherever Refraction is concerned, the conclusions deduced from observations that are much affected by it, will always remain doubtful, and sometimes too precarious to be relied on. See Dr. Bradley in Philos. Trans. number 485.

As to the cause of Refraction, it does not appear that any person before Des Cartes attempted to explain it; this he undertook to do by the resolution of forces, on the principles of mechanics; in consequence of which, he was obliged to suppose that light passes with more ease through a dense medium than a rare one: thus, the ray AC falling obliquely on a denser medium at C is supposed to be acted on by two forces, one of them impelling it in the direction AL, and the other in AK, which alone can be affected by the change of medium: and since, after the ray has entered the denser medium, it approaches the perpendicular CI, it is plain that this force must have received an increase, whilst the other continued the same.

The first person who questioned the truth of this explanation of the cause of Refraction, was Fermat; he asserted, contrary to Des Cartes, that light suffers greater resistance in water than in air, and greater in glass than in water; and he maintained that the resistance of different mediums, with respect to light, is in proportion to their densities. Leibnitz also adopted the same general idea; and they reasoned upon the subject in the following manner. Nature, say they, accomplishes her ends by the shortest methods; and therefore light ought to pass from one point to another, either by the shortest course, or by that in which the least time is required. But it is plain that the path in which light passes, when it falls obliquely upon a denser

ſer medium, is not the moſt direct or the ſhorteſt; and therefore it muſt be that in which the leaſt time is ſpent. And whereas it is demonſtrable, that light falling obliquely upon a denſer medium (in order to take up the leaſt time poſſible, in paſſing from a point in one medium to a point in the other) muſt be refracted in ſuch a manner, that the ſine of the angles of incidence and Refraction muſt be to one another, as the different facilities with which light is tranſmitted in thoſe mediums; it follows that, ſince light approaches the perpendicular when it paſſes obliquely from air into water, the facility with which water ſuffers light to paſs through it, is leſs than that of the air; ſo that the light meets with greater reſiſtance in water than in air.

This method of arguing from final cauſes could not ſatisfy philoſophers. Dr. Smith obſerves, that it agrees only to the caſe of Refraction at a plane ſurface; and that the hypotheſis is altogether arbitrary.

Dechales, in explaining the law of Refraction, ſuppoſes that every ray of light is compoſed of ſeveral ſmaller rays, which adhere to one another; and that they are refracted towards the perpendicular, in paſſing into a denſer medium, becauſe one part of the ray meets with more reſiſtance than another part; ſo that the former traverſes a ſmaller ſpace than the latter; in conſequence of which the ray muſt neceſſarily bend a little towards the perpendicular. This hypotheſis was adopted by the celebrated Dr. Barrow, and indeed ſome ſay, he was the author of it. On this hypotheſis it is plain, that mediums of a greater refractive power, muſt give a greater reſiſtance to the paſſage of the rays of light, than mediums of a leſs refractive power; which is contrary to fact.

The Bernoullis, both father and ſon, have attempted to explain the cauſe of Refraction on mechanical principles; the former on the equilibrium of forces, and the latter on the ſame principles with the ſuppoſition of etherial vortices: but neither of theſe hypotheſes have gained much credit.

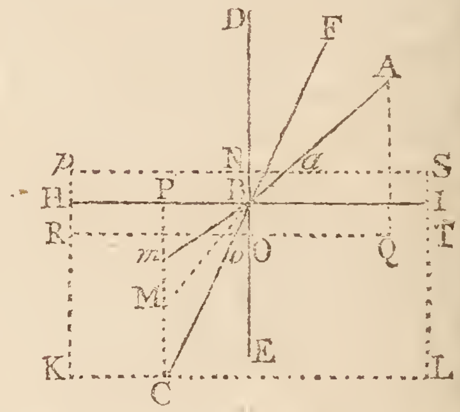
M. Mairan ſuppoſes a ſubtle fluid, filling the pores of all bodies, and extending, like an atmoſphere, to a ſmall diſtance beyond their ſurfaces; and then he ſuppoſes that the Refraction of light is nothing more than a neceſſary and mechanical effect of the incidence of a ſmall body in thoſe circumſtances. There is more, he ſays, of the refracting fluid, in water than in air, more in glaſs than in water, and in general more in a denſe medium than in one that is rarer.

Maupertuis ſuppoſes that the courſe which every ray takes, in paſſing out of one medium into another, is that which requires the leaſt quantity of action, which depends upon the velocity of the body and the ſpace it paſſes over; ſo that it is in proportion to the ſum of the products ariſing from the ſpaces multiplied by the velocities with which bodies paſs over them. From this principle he deduces the neceſſity of the ſine of the angle of incidence being in a conſtant proportion to that of Refraction; and alſo all the other laws relating to the propagation and reflection of light.

Dr. Smith (in his Optics, Remarks, p. 70) obſerves, that all other theories for explaining the reflection and Refraction of light, except that of Newton, ſuppoſe that it ſtrikes upon bodies and is reſiſted by them; which has never been proved by any deduction from experience. On the contrary, it appears by various conſiderations, and might be ſhewn by the obſervations of Mr. Molyneux and Dr. Bradley on the parallax of the fixed ſtars, that their rays are not at all impeded by the rapid motion of the earth's atmoſphere, nor by the object glaſs of the teleſcope, through which they paſs. And by Newton's theory of Refraction, which is grounded on experience only, it appears that light is ſo far from being reſiſted and retarded by Refraction into any denſe medium, that it is ſwifter there than in vacuo in the ratio of the ſine of incidence in vacuo to the ſine of Refraction into the denſe medium. Prieſtley's Hiſt. of Light, &c, p. 102 and 333.

Newton ſhews that the Refraction of light is not performed by the rays falling on the very ſurface of bodies; but that it is effected, without any contact, by the action of ſome power belonging to bodies, and extending to a certain diſtance beyond their ſurfaces; by which ſame power, acting in other circumſtances, they are alſo emitted and reflected.

The manner in which Refraction is performed by mere attraction, without contact, may be thus accounted for: Suppoſe HI the boundary of two mediums, N and O; the firſt the rarer, ex. gr. air; the ſecond the denſer, ex. gr. glaſs; the attraction of the mediums here will be as their denſities. Suppoſe p S to be the diſtance to which the attracting force of the denſer medium exerts itſelf within the rarer. Now



let a ray of light Aa fall obliquely on the ſurface which ſeparates the mediums, or rather on the ſurface pS, where the action of the ſecond and more reſiſting medium commences: as the ray arrives at a, it will begin to be turned out of its rectilinear courſe by a ſuperior force, with which it is attracted by the medium O, more than by the medium N; hence the ray is bent out of its right line in every point of its paſſage between pS and RT, within which diſtance the attraction acts; and therefore between theſe lines it deſcribes a curve aBb; but beyond RT, being out of the ſphere of attraction of the medium N, it will proceed uniformly in a right line, according to the direction of the curve in the point b.

Again, ſuppoſe N the denſer and more attracting medium, O the rarer, and HI the boundary as before; and let RT be the diſtance to which the denſer medium exerts its attractive force within the rarer: even when the ray has paſſed the point B, it will be within the ſphere of the ſuperior attraction of the denſer medium; but that attraction acting in lines perpendicular to its ſurface, the ray will be continually drawn from its ſtraight courſe BM perpendicularly towards HI: thus, having two forces or directions, it will have a compound motion, by which, inſtead of BM, it will deſcribe Bm, which Bm will in ſtrictneſs be a curve. Laſtly, after it has arrived at m, being out of the influence of the medium N, it will perſiſt uniformly, in a right line, in the direction in which the extremity of the

the curve leaves it.—Thus we see how Refraction is performed, both towards the perpendicular DE, and from it.

REFRACTION *in Dioptrics*, is the inflexion or bending of the rays of light, in passing the surfaces of glasses, lenses, and other transparent bodies of different densities. Thus, a ray, as AB, falling obliquely from the radiant A, upon a point B, in a diaphanous surface HI, rarer or denser than the medium along which it was propagated from the radiant, has its direction there altered by the action of the new medium; and instead of proceeding to M, it deviates, as for ex. to C.

This deviation is called the *Refraction of the ray*; BC the *refracted ray*, or *line of Refraction*; and B the *point of Refraction*.—The line AB is also called the *line of incidence*; and in respect of it, B is also called the *point of incidence*. The plane in which both the incident and refracted ray are found, is called the *plane of Refraction*; also a right line BE drawn in the refracting medium perpendicular to the refracting surface at the point of Refraction B, is called the *axis of Refraction*; and its continuation DB along the medium through which the ray falls, is called the *axis of incidence*.—Farther, the angle ABI, made by the incident ray and the refracting surface, is usually called the *angle of incidence*; and the angle ABD, between the incident ray and the axis of incidence, is the *angle of inclination*. Moreover, the angle MBC, between the refracted and incident rays, is called the *angle of Refraction*; and the angle CBE, between the refracted ray and the axis of Refraction, is the *refracted angle*. But it is also very common to call the angles ABD and CBE made by the perpendicular with the incident and refracted rays, the *angles of incidence and Refraction*.

General Laws of REFRACTION.—I. *A ray of light in its passage out of a rarer medium into a denser*, ex. gr. *out of air into water or into glass, is refracted towards the perpendicular*, i. e. *towards the axis of Refraction*. Hence, the refracted angle is less than the angle of inclination; and the angle of Refraction less than that of incidence; as they would be equal were the ray to proceed straight from A to M.

II. *The ratio of the sines of the angles* ABD, CBE, *made by the perpendicular with the incident and refracted rays, is a constant and fixed ratio*; whatever be the obliquity of the incident ray, the mediums remaining. Thus, the Refraction out of air, into water, is nearly as 4 to 3, and into glass it is nearly as 3 to 2. As to air in particular, it is shewn by Newton, that a ray of light, in traversing quite through the atmosphere, is refracted the same as it would be, were it to pass with the same obliquity out of a vacuum immediately into air of equal density with that in the lowest part of the atmosphere.

The true law of Refraction was first discovered by Willebrord Snell, professor of Mathematics at Leyden; who found by experiment that the cosecants of the angles of incidence and Refraction, are always in the same ratio. It was commonly attributed however to Des Cartes; who, having seen it in a MS. of Snell's, first published it in his Dioptrics, without naming Snellius, as Huygens asserts; Des Cartes having only altered the form of the law, from the ratio of the cosecants, to that of the sines, which is the same thing.

It is to be observed however, that as the rays of light are not all of the same degree of refrangibility, this constant ratio must be different in different kinds: so that the ratio mentioned by authors, is to be understood of rays of the mean refrangibility, i. e. of green rays. The difference of Refraction between the least and most refrangible rays, that is, between violet and red rays, Newton shews, is about the $\frac{1}{55}$ of the whole Refraction of the mean refrangible; which difference, he allows, is so small, that it seldom needs to be regarded.

Different transparent substances have indeed very different degrees of Refraction, and those not according to any regular law; as appears by many experiments of Newton, Euler, Hawksbee, &c. See Newton's Optics, 3d edit. pa. 247; Hawksbee's Experim. pa. 292; Act. Berlin. 1762, pa. 302; Priestley's Hist. of Light &c, pa. 479.

Whence the different refractive powers in different fluids arise, has not been determined. Newton shews, that in many bodies, as glass, crystal, selenites, pseudo-topaz, &c, the refractive power is indeed proportionable to their densities; whilst in sulphureous bodies, as camphor, linseed, and olive oil, amber, spirit of turpentine, &c, the power is two or three times greater than in other bodies of equal density; and yet even these have the refractive power with respect to each other, nearly as their densities. Water has a refractive power in a medium degree between those two kinds of substances; whilst salts and vitriols have refractive powers in a middle degree between those of earthy substances and water, and accordingly are composed of those two sorts of matter. Spirit of wine has a refractive power in a middle degree between those of water and oily substances; and accordingly it seems to be composed of both, united by fermentation. It appears therefore, that all bodies seem to have their refractive powers nearly proportional to their densities, excepting so far as they partake more or less of sulphureous oily particles, by which those powers are altered.

Newton suspected that different degrees of heat might have some effect on the refractive power of bodies; but his method of determining the general Refraction was not sufficiently accurate to ascertain this circumstance. Euler's method however was well adapted to this purpose: from his experiments he infers, that the focal distance of a single lens of glass diminishes with the heat communicated to it; which dimination is owing to a change in the refractive power of the glass itself, which is probably increased by heat, and diminished by cold, as well probably as that of all other translucent substances.

From the law above laid down it follows, that one angle of inclination, and its corresponding refracted angle, being found by observation, the refracted angles corresponding to the several other angles of inclination are thence easily computed. Now, Zahnius and Kircher have found, that if the angle of inclination be 70°, the refracted angle, out of air into glass, will be 38° 50′; on which principle Zahnius has constructed a table of those Refractions for the several degrees of the

angle of inclination; a ſpecimen of which here follows:

Angle of Inclination.	Refracted Angle.			Angle of Refraction.		
°	°	′	″	°	′	″
1	0	40	5	0	19	55
2	1	20	6	0	39	54
3	2	0	4	0	59	56
4	2	40	5	1	19	55
5	3	20	3	1	39	57
10	6	39	16	3	20	44
20	13	11	35	6	48	25
30	19	29	29	10	30	31
45	28	9	19	16	50	41
90	41	51	40	48	8	20

Hence it appears, that if the angle of inclination be leſs than 20°, the angle of Refraction out of air into glaſs is almoſt $\frac{1}{3}$ of the angle of inclination; and therefore a ray is refracted to the axis of Refraction by almoſt a third part of the quantity of its angle of inclination. And on this principle it is that Kepler, and moſt other dioptrical writers, demonſtrate the Refractions in glaſſes; though in eſtimating the law of theſe Refractions he followed the example of Alhazen and Vitello, and ſought to diſcover it in the proportion of the angles, and not in that of the ſines, or coſecants, as diſcovered by Snellius, as mentioned above.

The refractive powers of ſeveral ſubſtances, as determined by different philoſophers, may be ſeen in the following tables; in which the ray is ſuppoſed to paſs out of air into each of the ſubſtances, and the annexed numbers ſhew the proportion to unity or 1, between the ſines of the angles of incidence and Refraction.

1. By Sir Iſaac Newton's Obſervations.

Air	0·9997
Rain water	1·3358
Spirit of wine	1·3698
Oil of vitriol	1·4285
Alum	1·4577
Oil olive	1.4666
Borax	1·4667
Gum Arabic	1·4771
Linſeed oil	1·4814
Selenites	1·4878
Camphor	1·5000
Dantzick vitriol	1·5000
Nitre	1·5238
Sal gem	1·5455
Glaſs	1·5500
Amber	1·5556
Rock cryſtal	1·5620
Spirit of turpentine	1·5625
A yellow pſeudo-topaz	1·6429
Iſland cryſtal	1·6666
Glaſs of antimony	1·8889
A Diamond	2·4390

2. By Mr. Hawkſbee.

Water	1·3359
Spirit of honey	1·3359
Oil of amber	1·3377
Human urine	1·3419
White of an egg	1·3511
French brandy	1·3625
Spirit of wine	1·3721
Diſtilled vinegar	1·3721
Gum ammoniac	1·3723
Aqua regia	1·3898
Aqua fortis	1·4044
Spirit of nitre	1·4076
Cryſtalline humour of an ox's eye	1·4635
Oil of vitriol	1·4262
Oil of turpentine	1·4833
Oil of amber	1·5010
Oil of cloves	1·5136
Oil of cinnamon	1·5340

3. By Mr. Euler, junior.

Rain or diſtilled water	1·3358
Well water	1·3362
Diſtilled vinegar	1·3442
French wine	1·3458
A ſolution of gum arabic	1·3467
French brandy	1·3600
Ditto a ſtronger kind	1·3618
Spirit of wine rectified	1.3683
Ditto more highly rectified	1·3706
White of an egg	1·3685
Spirit of nitre	1·4025
Oil of Provence	1·4651
Oil of turpentine	1·4822

III. *When a ray paſſes out of a denſer medium into a rarer, it is refracted from the perpendicular, or from the axis of Refraction.*

This is exactly the reverſe of the 2d law, and the quantity of Refraction is equal in both caſes, or both forwards and backwards; ſo that a ray would take the ſame courſe back, by which another paſſed forward, viz, if a ray would paſs from A by B to C, another would paſs from C by B to A. Hence, in this caſe, the angle of Refraction is greater than the angle of inclination. Hence alſo, if the angle of inclination be leſs than 30°, MBC is nearly equal to $\frac{1}{3}$ of MBE; therefore MBC is $\frac{1}{2}$ of CBE; conſequently, if the Refraction be out of glaſs into air, and the angle of inclination leſs than 30°, the ray is refracted from the axis of Refraction by almoſt the half of the angle of inclination. And this is the other dioptrical principle uſed by moſt authors after Kepler, to demonſtrate the Refractions of glaſſes.

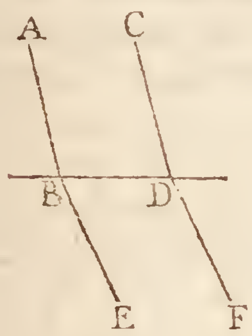
If the Refraction be out of air into glaſs, the ratio of the ſines of inclination and Refraction is as 3 to 2, or more accurately as 17 to 11; if out of air into water as 4 to 3; therefore if the courſe be the contrary way, viz, out of glaſs or water into air, the ratio of the ſines will be, in the former caſe as 2 to 3 or 11 to 17, and in the latter as 3 to 4. So that, if the Refraction be from water or glaſs into air, and the angle of incidence

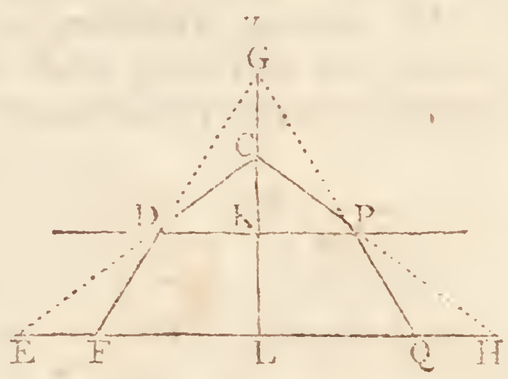
dence or inclination be greater than about $48\frac{1}{2}$ degrees in water, or greater than about 40° in glaſs, the ray will not be refracted into air; but will be reflected into a line which makes the angle of reflection equal to the angle of incidence; becauſe the ſines of $48\frac{1}{2}$ and 40° are to the radius, as 3 to 4, and as 11 to 17 nearly; and therefore when the ſine has a greater proportion to the radius than as above, the ray will not be refracted.

IV. *A ray falling on a curve ſurface, whether concave or convex, is refracted after the ſame manner as if it fell on a plane which is a tangent to the curve in the point of incidence.* Becauſe the curve and its tangent have the point of contact common to both, where the ray is refracted.

Laws of REFRACTION *in Plane Surfaces.*

1. If parallel rays, AB and CD, be refracted out of one tranſparent medium into another of a different denſity, they will continue parallel after Refraction, as BE and DF. Hence a glaſs that is plane on both ſides, being turned either directly or obliquely to the ſun, &c, the light paſſing through it will be propagated in the ſame manner as if the glaſs were away.

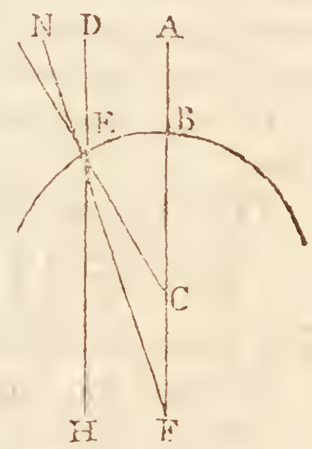




2. If two rays CD and CP, proceeding from the ſame radiant C, and falling on a plane ſurface of a different denſity, ſo that the points of Refraction D and P be equally diſtant from the perpendicular of incidence GK, the refracted rays DF and PQ have the ſame virtual focus, or the ſame point of diſperſion G.—Hence, when refracted rays, falling on the eye placed out of the perpendicular of incidence, are either equally diſtant from the perpendicular, or very near each other, they will flow upon the eye as if they came to it from the point G; conſequently the point C will be ſeen by the refracted rays as in G. And hence alſo, if the eye be placed in a denſe medium, objects in a rarer will appear more remote than they are; and the place of the image, in any caſe, may be determined from the ratio of Refraction: Thus, to fiſhes ſwimming under water, objects out of the water muſt appear farther diſtant than in reality they are. But, on the contrary, if the eye at E be placed in a rarer medium, then an object G placed in a denſer, appears, at C, nearer than it is; and the place of the image may be determined in any given caſe by the ratio of Refraction: and thus the bottom of a veſſel full of water is raiſed by Refraction a third part of its depth, with reſpect to an eye placed perpendicularly over the refracting ſurface; and thus alſo fiſhes and other bodies, under water, appear nearer than they really are.

3. If the eye be placed in a rarer medium; then an object ſeen in a denſer, by a ray refracted in a plane ſurface, will appear larger than it really is. But if the eye be in a denſer medium, and the object in a rarer, the object will appear leſs than it is. And, in each caſe, the apparent magnitude FQ is to the real one EH, as the rectangle CK · GL to GK · CL, or in the compound ratio of the diſtance CK of the point to which the rays tend before Refraction, from the refracting ſurface DP, to the diſtance GK of the eye from the ſame, and of the diſtance GL of the object EH from the eye, to its diſtance CL from the point to which the rays tend before Refraction.—Hence, if the object be very remote, CL will be phyſically equal to GL; and then the real magnitude EL is to the apparent magnitude FL, as GK to CK, or as the diſtance of the eye G from the refracting plane, to the diſtance of the point of convergence F from the ſame plane. And hence alſo, objects under water, to an eye in the air, appear larger than they are; and to fiſhes under water, objects in the air appear leſs than they are.

Laws of REFRACTION *in Spherical Surfaces, both concave and convex.*



1. A ray of light DE, parallel to the axis, after a ſingle refraction at E, meets the axis in the point F, beyond the centre C.

2. Alſo in that caſe, the ſemidiameter CB or CE will be to the refracted ray EF, as the ſine of the angle of refraction to the ſine of the angle of inclination BCE. But the diſtance of the focus, or point of concurrence from the centre, CF, is to the refracted ray EF, as the ſine of the refracted angle to the ſine of the angle of inclination.

3. Hence alſo, in this caſe, the diſtance BF of the focus from the refracting ſurface, muſt be to CF its diſtance from the centre, in a ratio greater than that of the ſine of the angle of inclination to the ſine of the refracted angle. But thoſe ratios will be nearly equal when the rays are very near the axis, and the angle of inclination BCE is only of a few degrees. And when the Refraction is out of air into glaſs, then

For rays near the axis,	For more diſtant rays,
BF : FC :: 3 : 2,	BF : FC > 3 : 2,
BC : BF :: 1 : 3.	BC : BF < 1 : 3.

But if the Refraction be out of air into water, then

For rays near the axis,	For more diſtant rays,
BF : FC :: 4 : 3,	BF : FC > 4 : 3,
BC : BF :: 1 : 4,	BC : BF < 1 : 4.

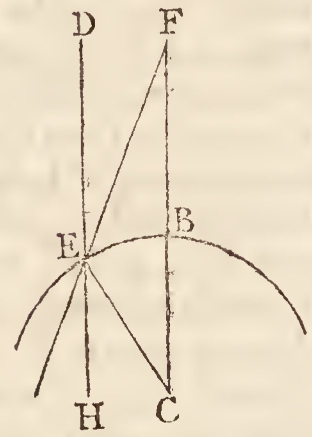
Hence, as the ſun's rays are parallel as to ſenſe, if they fall on the ſurface of a ſolid glaſs ſphere, or of a ſphere full of water, they will not meet the axis within the ſphere: ſo that Vitello was miſtaken when he imagined that the ſun's rays, falling on the ſurface of a cryſtalline ſphere, were refracted to the centre.

4. If a ray HE fall parallel to the axis FA, out of a rarer medium, on the concave ſpherical ſurface BE of a denſer one; the refracted ray EN will diverge from the point of the axis F, ſo that FE will be to FC, in the ratio of the ſine of the angle of inclination, to the ſine of the refracted angle. Conſequently FB to FC is in a greater ratio than that; unleſs when the rays are very near the axis, and the angle BCE is very ſmall,

 for

for then FB will be to FC nearly in that ratio. And hence, in the caſes of Refraction out of air into water or glaſs, the ratios of BC, BF and CF, will be the ſame as ſpecified in the laſt article.

5. If a ray DE, parallel to the axis FC, paſs out of a denſer into a rarer ſpherical convex medium, it will diverge from the axis after Refraction; and the diſtance FC of the point of diſperſion, or of the virtual focus F, from the centre of the ſphere, will be to its ſemidiameter CE or CB, as the ſine of the refracted angle is to the ſine of the angle of Refraction; but to the portion of the refracted ray drawn back, FE, it will be in the ratio of the ſine of the refracted angle to the ſine of the angle of inclination. Conſequently FC will be to FB, in a greater ratio than this laſt one: unleſs when the rays DE fall very near the axis FC, for then FC to FB will be very nearly in that ratio.



Hence, when the Refraction is out of glaſs into air; then,

For rays near the axis,	For more diſtant rays,
FC : FB :: 3 : 2,	FC : FB > 3 : 2,
BC : BF :: 1 : 2.	BC : BF > 1 : 2,

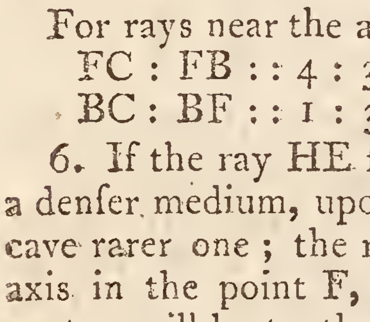
But when the Refraction is out of water into air; then,

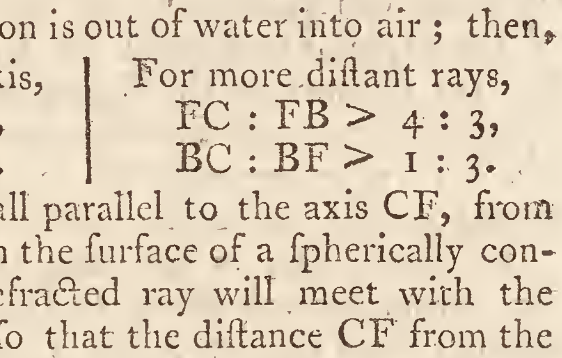
For rays near the axis,	For more diſtant rays,
FC : FB :: 4 : 3,	FC : FB > 4 : 3,
BC : BF :: 1 : 3.	BC : BF > 1 : 3.

6. If the ray HE fall parallel to the axis CF, from a denſer medium, upon the ſurface of a ſpherically concave rarer one; the refracted ray will meet with the axis in the point F, ſo that the diſtance CF from the centre, will be to the refracted ray FE, as the ſine of the refracted angle, to the ſine of the angle of inclination. Conſequently FC will be to FB, in a greater ratio than that above mentioned: unleſs when the rays are very near the axis, for then FC is to FB very nearly in that ratio; and the three FB, FC, BC are, in the caſes of air, water and glaſs, in the numeral ratios as ſpecified at the end of the laſt article. See Wolfius, Elem. Matheſ. tom. 3 p. 179 &c.

Refraction *in a Glaſs Priſm.*

ABC being the tranſverſe ſection of a priſm; if a ray of light DE fall obliquely upon it out of the air; inſtead of proceeding ſtraight on to F, being refracted





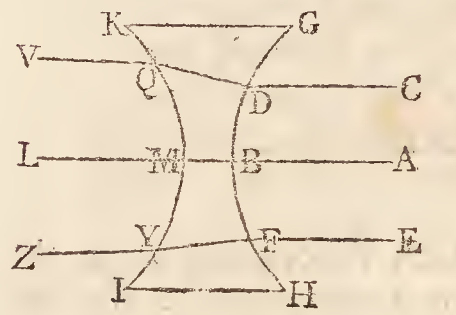
towards the perpendicular IE, it will decline to G. Again, ſince the ray EG, paſſing out of glaſs into air, falls obliquely on BC, it will be refracted to M, ſo as to recede from the perpendicular GO. And hence ariſe the various phenomena of the priſm. See Colour.

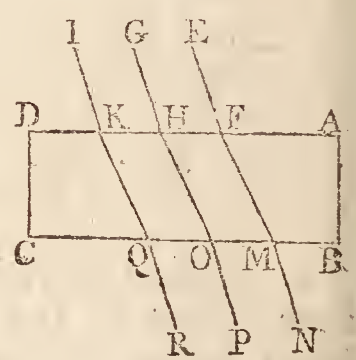
Refraction *in a Convex Lens.*

If parallel rays, AB, CD, EF, fall on the ſurface of a convex lens XBZ (the laſt fig. above); the perpendicular ray AB will paſs unrefracted to K, where emerging, as before, perpendicularly, into air, it will proceed ſtraight on to G. But the rays CD and EF, falling obliquely out of air into glaſs, at D and F, will be refracted towards the axis of Refraction, or towards the perpendiculars at D and F, and ſo decline to Q and P; where emerging again obliquely out of the glaſs into the ſurface of the air, they will be refracted from the perpendicular, and proceed in the directions QG and PG, meeting in G. And thus alſo will all the other rays be refracted ſo as to meet the reſt near the place G. See Focus and Lens.—Hence the great property of convex glaſſes; viz, that they collect parallel rays, or make them converge into a point.

Refraction *in a Concave Lens.*

Parallel rays AB, CD, EF, falling on a concave lens GBHIMK, the ray AB falling perpendicularly on the glaſs at B, will paſs unrefracted to M; where, being ſtill perpendicular, it will paſs into the air to L, with-





out Refraction. But the ray CD, falling obliquely on the ſurface of the glaſs, will be refracted towards the perpendicular at D, and proceed to Q; where again falling obliquely out of the glaſs upon the ſurface of air, it will be refracted from the perpendicular at Q, and proceed to V. After the ſame manner the ray EF is firſt refracted to Y, and thence to Z.—Hence the great property of concave glaſſes; viz, that they diſperſe parallel rays, or make them diverge. See Lens.

Refraction *in a Plane Glaſs.*

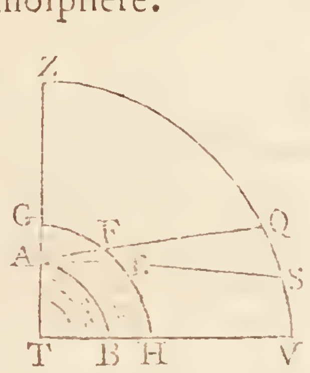
If parallel rays EF, GH, IK, (the laſt fig. above) fall obliquely on a plane glaſs ABCD; the obliquity being the ſame in all, by reaſon of their paralleliſm, they will be all equally refracted towards the perpendicular; and accordingly, being ſtill parallel at M, O, and Q, they will paſs out into the air equally refracted again from the perpendicular, and ſtill parallel. Thus will the rays EF, GH, and IK, at their entering the glaſs, be inflected towards the right; and in their going out as much inflected to the left; ſo that the firſt Refraction is here undone by the ſecond, thereby cauſing the rays on their emerging from the glaſs, to be parallel to their firſt direction before they entered it; though not ſo as that the object is ſeen in its true place; for the ray RQ, being produced back again, will not coincide with the ray IK, but will fall to the left of it; and this the more as the glaſs is thicker; however, as to

to the colour, the ſecond Refraction does really undo the firſt. See Colour.

Refraction *in Aſtronomy*, or Refraction *of the Stars*, is an inflexion of the rays of thoſe luminaries, in paſſing through our atmoſphere; by which the apparent altitudes of the heavenly bodies are increaſed.

This Refraction ariſes from hence, that the atmoſphere is unequally denſe in different ſtages or regions; rareſt of all at the top, and denſeſt of all at the bottom; which inequality in the ſame medium, makes it equivalent to ſeveral unequal mediums, by which the courſe of the ray of light is continually bent into a continued curve line. See Atmosphere.——And Sir Iſaac Newton has ſhewn, that a ray of light, in paſſing from the higheſt and rareſt part of the atmoſphere, down to the loweſt and denſeſt, undergoes the ſame quantity of Refraction that it would do in paſſing immediately, at the ſame obliquity, out of a vacuum into air of equal denſity with that in the loweſt part of the atmoſphere.

The effect of this Refraction may be thus conceived. Suppoſe ZV a quadrant of a vertical circle deſcribed from the centre of the earth T, under which is AB a quadrant of a circle on the ſurface of the earth, and GH a quadrant of the ſurface of the atmoſphere. Then ſuppoſe SE a ray of light emitted by a ſtar at S, and falling on the atmoſphere at E: this ray coming out of the ethereal medium, which is much rarer than our air, or perhaps out of a perfect vacuum, and falling on the ſurface of the atmoſphere, will be refracted towards the perpendicular, or inclined down more towards the earth; and ſince the upper air is again rarer than that near the earth, and grows ſtill denſer as it approaches the earth's ſurface, the ray in its progreſs will be continually refracted, ſo as to arrive at the eye in the curve line EA. Then ſuppoſing the right line AF to be a tangent to the arch at A, the ray will enter the eye at A in the direction of AF; and therefore the ſtar will appear in the heavens at Q, inſtead of S, higher or nearer the zenith than the ſtar really is.



Hence ariſe the phenomena of the crepuſculum or twilight; and hence alſo it is that the moon is ſometimes ſeen eclipſed, when ſhe is really below the horizon, and the ſun above it.

That there is a real Refraction of the ſtars &c, is deduced not only from phyſical conſiderations, and from arguments a priori, and a ſimilitudine, but alſo from preciſe aſtronomical obſervation: for there are numberleſs obſervations by which it appears that the ſun, moon, and ſtars riſe much ſooner, and appear higher, than they ſhould do according to aſtronomical calculations. Hence it is argued, that as light is propagated in right lines, no rays could reach the eye from a luminary below the horizon, unleſs they were deflected out of their courſe, at their entrance into the atmoſphere; and therefore it appears that the rays are refracted in paſſing through the atmoſphere.

Hence the ſtars appear higher by Refraction than they really are; ſo that to bring the obſerved or apparent altitudes to the true ones, the quantity of Refraction muſt be ſubtracted. And hence, the ancients, as they were not acquainted with this Refraction, reckoned their altitudes too great, ſo that it is no wonder they ſometimes committed conſiderable errors. Hence alſo, Refraction lengthens the day, and ſhortens the night, by making the ſun appear above the horizon a little before his riſing, and a little after his ſetting. Refraction alſo makes the moon and ſtars appear to riſe ſooner and ſet later than they really do. The apparent diameter of the ſun or moon is about 32′; the horizontal refraction is about 33′; whence the ſun and moon appear *wholly* above the horizon when they are entirely below it. Alſo, from obſervations it appears that the Refractions are greater nearer the pole than at leſſer latitudes, cauſing the ſun to appear ſome days above the horizon, when he is really below it; doubtleſs from the greater denſity of the atmoſphere, and the greater obliquity of the incidence.

Stars in the zenith are not ſubject to any Refraction: thoſe in the horizon have the greateſt of all: from the horizon, the Refraction continually decreaſes to the zenith. All which follows from hence, that in the firſt caſe, the rays are perpendicular to the medium; in the ſecond, their obliquity is the greateſt, and they paſs through the largeſt ſpace of the lower and denſer part of the air, and through the thickeſt vapours; and in the third, the obliquity is continually decreaſing.

The air is condenſed, and conſequently Refraction is increaſed, by cold; for which reaſon it is greater in cold countries than in hot ones. It is alſo greater in cold weather than in hot, in the ſame country; and the morning Refraction is greater than that of the evening, becauſe the air is rarefied by the heat of the ſun in the day, and condenſed by the coldneſs of the night. Refraction is alſo ſubject to ſome ſmall variation at the ſame time of the day in the fineſt weather.

At the ſame altitudes, the ſun, moon, and ſtars all undergo the ſame Refraction: for at equal altitudes the incident rays have the ſame inclinations; and the ſines of the refracted angles are as the ſines of the angles of inclination, &c.

Indeed Tycho Brahe, who firſt deduced the Refractions of the ſun, moon, and ſtars, from obſervation, and whoſe table of the Refraction of the ſtars is not much different from thoſe of Flamſteed and Newton, except near the horizon, makes the ſolar Refractions about 4′ greater than thoſe of the fixed ſtars; and the lunar Refractions alſo ſometimes greater than thoſe of the ſtars, and ſometimes leſs. But the theory of Refractions, found out by Snellius, was not fully underſtood in his time.

The horizontal Refraction, being the greateſt, is the cauſe that the ſun and moon appear of an oval form at their riſing and ſetting; for the lower edge of each being more refracted than the upper edge, the perpendicular diameter is ſhortened, and the under edge appears more flatted alſo.—Hence alſo, if we take with an inſtrument the diſtance of two ſtars when they are in the ſame vertical and near the horizon, we ſhall find it conſiderably leſs than if we meaſure it when they are both at ſuch a height as to ſuffer little or no Refraction; becauſe the lower ſtar is more elevated than the higher. There is alſo another alteration made by Refraction in the apparent diſtance of ſtars: when two ſtars are in the ſame almicantar, or parallel of declination, their apparent

parent diſtance is leſs than the true; for ſince Refraction makes each of them higher in the azimuth or vertical in which they appear, it muſt bring them into parts of the vertical where they come nearer to each other; becauſe all vertical circles converge and meet in the zenith. This contraction of diſtance, according to Dr. Halley (Philoſ. Tranſ. numb. 368) is at the rate of at leaſt one ſecond in a degree; ſo that, if the diſtance between two ſtars in a poſition parallel to the horizon meaſure 30°, it is at moſt to be reckoned only 29° 59′ 30″.

The quantity of the Refraction at every altitude, from the horizon, where it is greateſt, to the zenith where it is nothing, has been determined by obſervation, by many aſtronomers; thoſe of Dr. Bradley and Mr. Mayer are eſteemed the moſt correct of any, being nearly alike, and are now uſed by moſt aſtronomers. Doctor Bradley, from his obſervations, deduced this very ſimple and general rule for the Refraction r at any altitude a whatever; viz,

as rad. 1 : cotang. $\overline{a + 3r}$ :: 57″ : r'' the Refraction in ſeconds.

This rule, of Dr. Bradley's, is adapted to theſe ſtates of the barometer and thermometer, viz,

either 29·6 inc. barom. and 50° thermometer,
or 30 — barom. and 55 thermometer,

for both which ſtates it anſwers equally the ſame. But for any other ſtates of the barometer and thermometer, the Refraction above-found is to be corrected in this manner; viz, if b denote any other height of the barometer in inches, and t the degrees of the thermometer, r being the Refraction uncorrected, as found in the manner above. Then

as $29{\cdot}6 : b :: r :$ R the Refraction corrected on account of the barometer,

and $400 : 450\,t :: \mathrm{R} :$ the Refraction corrected both on account of the barometer and thermometer; which final corrected Refraction is therefore $= \frac{450 - t}{11840} br$.

Or, to correct the ſame Refraction r by means of the latter ſtate, viz, barom. 30 and therm. 55, it will be

as $30 : b :: r : \mathrm{R} = \frac{br}{30}$,

and $400 : 455 - t :: \mathrm{R} : \frac{455 - t}{400}\mathrm{R} = \frac{455 - t}{12000} br$ the correct Refraction.

From Dr. Bradley's rule, $r = 57'' \times \cot.\overline{a + 3r}$, the following Table of the mean aſtronom. Refrac. is computed.

Mean Aſtronomical Refractions in Altitude.

Apparent Altitude.		Refraction.		Apparent Altitude.		Refraction.		Apparent Altitude.		Refraction.		Apparent Altitude.		Refraction.		Apparent Altitude.	Refraction.
0°	0′	33′	0″	3°	0′	14′	36″	8°	30′	6′	8″	20°	0′	2′	35″	54°	41″
0	5	32	10	3	5	14	20	8	40	6	1	20	30	2	31	55	40
0	10	31	22	3	10	14	4	8	50	5	55	21	0	2	27	56	38
0	15	30	35	3	15	13	49	9	0	5	48	21	30	2	24	57	37
0	20	29	50	3	20	13	34	9	10	5	42	22	0	2	20	58	35
0	25	29	6	3	25	13	20	9	20	5	36	23	-	2	14	59	34
0	30	28	22	3	30	13	6	9	30	5	31	24	-	2	7	60	33
0	35	27	41	3	40	12	40	9	40	5	25	25	-	2	2	61	31
0	40	27	0	3	50	12	15	9	50	5	20	26	-	1	56	62	30
0	45	26	20	4	0	11	51	10	0	5	15	27	-	1	51	63	29
0	50	25	42	4	10	11	29	10	15	5	7	28	-	1	47	64	28
0	55	25	5	4	20	11	8	10	30	5	0	29	-	1	42	65	26
1	0	24	29	4	30	10	48	10	45	4	53	30	-	1	38	66	25
1	5	23	54	4	40	10	29	11	0	4	47	31	-	1	35	67	24
1	10	23	20	4	50	10	11	11	15	4	40	32	-	1	31	68	23
1	15	22	47	5	0	9	54	11	30	4	34	33	-	1	28	69	22
1	20	22	15	5	10	9	38	11	45	4	29	34	-	1	24	70	21
1	25	21	44	5	20	9	23	12	0	4	23	35	-	1	21	71	19
1	30	21	15	5	30	9	8	12	20	4	16	36	-	1	18	72	18
1	35	20	46	5	40	8	54	12	40	4	9	37	-	1	16	73	17
1	40	20	18	5	50	8	41	13	0	4	3	38	-	1	13	74	16
1	45	19	51	6	0	8	28	13	20	3	57	39	-	1	10	75	15
1	50	19	25	6	10	8	15	13	40	3	51	40	-	1	8	76	14
1	55	19	0	6	20	8	3	14	0	3	45	41	-	1	5	77	13
2	0	18	35	6	30	7	51	14	20	3	40	42	-	1	3	78	12
2	5	18	11	6	40	7	40	14	40	3	35	43	-	1	1	79	11
2	10	17	48	6	50	7	30	15	0	3	30	44	-	0	59	80	10
2	15	17	26	7	0	7	20	15	30	3	24	45	-	-	57	81	9
2	20	17	4	7	10	7	11	16	0	3	17	46	-	-	55	82	8
2	25	16	44	7	20	7	2	16	30	3	10	47	-	-	53	83	7
2	30	16	24	7	30	6	53	17	0	3	4	48	-	-	51	84	6
2	35	16	4	7	40	6	45	17	30	2	59	49	-	-	49	85	5
2	40	15	45	7	50	6	37	18	0	2	54	50	-	-	48	86	4
2	45	15	27	8	0	6	29	18	30	2	49	51	-	-	46	87	3
2	50	15	9	8	10	6	22	19	0	2	45	52	-	-	44	88	2
2	55	14	52	8	20	6	15	19	30	2	39	53	-	-	43	89	1

Mr. Mayer fays his rule was deduced from theory, and, when reduced from French meafure and Reaumur's thermometer, to Englifh meafure and Fahrenheit's thermometer, it is this,

$$r = \frac{74\cdot4b \times \text{cof. } a}{(1+\cdot00248t)^{\frac{3}{2}}}\left(\sqrt{1+\frac{17\cdot14 \text{ fin. } a}{1+\cdot00248t}} - \frac{17\cdot14 \text{ fin. } a}{(1+\cdot00248t)^{\frac{1}{2}}}\right)$$

or $r = \frac{74\cdot4b \times \text{cof. } a \times \text{tang. } \frac{1}{2}A}{(1+\cdot00248t)^{\frac{3}{2}}}$ the Refraction in feconds, corrected for both barometer and thermometer: where the letters denote the fame things as before, except A, which denotes the angle whofe tangent is $\frac{\sqrt{1+\cdot00248t}}{17\cdot14 \text{ fin. } a}$.

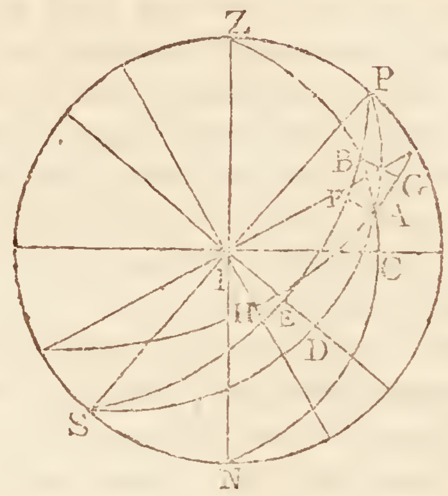
Mr. Simpfon too (Differt. pa. 46 &c) has ingenioufly determined by theory the aftronomical Refractions, from which he brings out this rule, viz, As 1 to ·9986 or as radius to fine of 86° 58′ 30″, fo is the fine of any given zenith diftance, to the fine of an arc; then $\frac{2}{11}$ of the difference between this arc and the zenith diftance, is the Refraction fought for that zenith diftance. And by this rule Mr. Simpfon computed a Table of the mean Refractions, which are not much different from thofe of Dr. Bradley and Mr. Mayer, and are as in the following Table.

Mr. Simpfon's Table of Mean Refractions.

Apparent Altitude.	Refraction.	Apparent Altitude.	Refraction.	Apparent Altitude.	Refraction.
0°	33′ 0″	17°	2′ 50″	38°	1′ 7″
1	23 50	18	2 40	40	1 2
2	17 43	19	2 31	42	0 58
3	13 44	20	2 23	44	0 54
4	11 5	21	2 16	46	0 50
5	9 10	22	2 9	48	0 47
6	7 49	23	2 3	50	0 44
7	6 48	24	1 57	52	0 41
8	5 59	25	1 52	54	0 38
9	5 21	26	1 47	56	0 35
10	4 50	27	1 42	58	0 32
11	4 24	28	1 38	60	0 30
12	4 2	29	1 34	65	0 24
13	3 43	30	1 30	70	0 19
14	3 27	32	1 23	75	0 14
15	3 13	34	1 17	80	0 9
16	3 1	36	1 12	85	0 4½

It is evident that all obferved altitudes of the heavenly bodies ought to be diminifhed by the numbers taken out of the foregoing Table. It is alfo evident that the Refraction diminifhes the right and oblique afcenfions of a ftar, and increafes the defcenfions: it increafes the northern declination and latitude, but decreafes the fouthern: in the eaftern part of the heavens it diminifhes the longitude of a ftar, but in the weftern part of the heavens it increafes the fame.

REFRACTION *of Altitude*, is an arc of a vertical circle, as AB, by which the altitude of a ftar AC is increafed by the Refraction.



REFRACTION *of Afcenfion and Defcenfion*, is an arc DE of the equator, by which the afcenfion and defcenfion of a ftar, whether right or oblique, is increafed or diminifhed by the Refraction.

REFRACTION *of Declination*, is an arc BF of a circle of declination, by which the declination of a ftar DA or EF is increafed or diminifhed by Refraction.

REFRACTION *of Latitude* is an arc AG of a circle of latitude, by which the latitude of a ftar AH is increafed or diminifhed by the Refraction.

REFRACTION *of Longitude* is an arc IH of the ecliptic, by which the longitude of a ftar is increafed or diminifhed by means of the Refraction.

Terreftrial REFRACTION, is that by which terreftrial objects appear to be raifed higher than they really are, in obferving their altitudes. The quantity of this Refraction is eftimated by Dr. Mafkelyne at one-tenth of the diftance of the object obferved, expreffed in degrees of a great circle. So, if the diftance be 10000 fathoms, its 10th part 1000 fathoms, is the 60th part of a degree of a great circle on the earth, or 1′, which therefore is the Refraction in the altitude of the object at that diftance. (Requifite Tables, 1766, pa. 134).

But M. Le Gendre is induced, he fays, by feveral experiments, to allow only $\frac{1}{14}$th part of the diftance for the Refraction in altitude. So that, upon the diftance of 10000 fathoms, the 14th part of which is 714 fathoms, he allows only 44″ of terreftrial Refraction, fo many being contained in the 714 fathoms. See his Memoir concerning the Trigonometrical operations, &c.

Again, M. de Lambre, an ingenious French aftronomer, makes the quantity of the Terreftrial Refraction to be the 11th part of the arch of diftance. But the Englifh meafurers, Col. Edw. Williams, Capt. Mudge, and Mr. Dalby, from a multitude of exact obfervations made by them, determine the quantity of the medium Refraction to be the 12th part of the faid diftance.

The quantity of this Refraction, however, is found to vary confiderably, with the different ftates of the weather and atmofphere, from the 15th part of the diftance, to the 9th part of the fame; the medium of which is the 12th part, as above mentioned.

Some whimfical effects of this Refraction are alfo related, arifing from peculiar fituations and circumftances. Thus, it is faid, any perfon ftanding by the fide of the river Thames at Greenwich, when it is high-water

water there, he can fee the cattle grazing on the Ifle of Dogs, which is the marfhy meadow on the other fide of the river at that place; but when it is low-water there, he cannot fee any thing of them, as they are hid from his view by the land wall or bank on the other fide, which is raifed higher than the marfh, to keep out the waters of the river. This curious effect is probably owing to the moift and denfe vapours, juft above and rifing from the furface of the water, being raifed higher or lifted up with the furface of the water at the time of high tide, through which the rays pafs, and are the more refracted.

Again, a fimilar inftance is related in a letter to me, from an ingenious friend, Mr. Abr. Crocker of Frome in Somerfetfhire, dated January 12, 1795. "My Devonfhire friend," fays he, (whofe feat is in the vicinity of the town of Modbury, 12 miles in a geographical line from Maker tower near Plymouth) "being on a pleafure fpot in his garden, on the 4th of December 1793, with fome friends, viewing the furrounding country, with an achromatic telefcope, defcried an object like a perpendicular pole ftanding up in the chafm of a hedge which bounded their view at about 9 miles diftance; which, from its direction, was conjectured to be the flagftaff on Maker tower.—Directing the glafs, on the morning of the next day, to the fame part of the horizon, a flag was perceived on the pole; which corroborated the conjecture of the preceding day. This day's view alfo difcovered the pinnacles and part of the fhaft of the tower.—Viewing the fame fpot at 8 in the morning on the 9th of January 1794, the whole tower and part of the roof of the church, with other remote objects not before noticed, became vifible.

"It is neceffary to give you the ftate of the weather there, on thofe days.

1793.	Barometer.	Thermom.	Wind	
Dec. 4	29·93, rifing	36·0	N. E.	Froftymorning, a mift over the land below.
5	29·97, rifing	35·2	W.	Ditto.
1794.				
Jan. 9	30·01, falling	29·8	W.	Hardwhitefroft, a fog over the lowlands; clear in thefurrounding country.

"The fingularity of this phenomenon has occafioned repeated obfervations on it; from all which it appears that the fummer feafon, and wet windy weather, are unfavourable to this refracted elevation; but that calm frofty weather, with the abfence of the fun, are favourable to it.

"From hence a queftion arifes; what is the principal or moft general caufe of atmofpheric Refraction, which produces fuch extraordinary appearances?"

The following is alfo a copy of a letter to Mr. Crocker on this curious phenomenon, from his friend above mentioned, viz, Mr. John Andrews, of Traine, near Modbury, dated the 1ft of February 1795.

"My good Friend,

"Finding, by your favour of laft Sunday, the proceedings which are going on in refpect to my obfervations on the phenomenon of *Looming*, I have thought it neceffary to beftow about half this day in preparing, what I am obliged to call, in my way, *drawings*, illuftrative of thofe obfervations.—I have endeavoured to diftinguifh, by different tints and fhades, the grounds which lie nearer or more remote; but this will perhaps be better explained by the letters of reference, which I have inferted as they may be ferviceable in future correfpondence.—I believe the drawings, rough as they are, give a tolerably exact reprefentation of the fcenes: they may be properly copied to fend to London by one of your ingenious fons.—I have been attentive in my obfervations, or rather in looking out for obfervations, during the late hard frofts, which you will be furprifed to learn, have (except on one or two days) been very unpropitious to the phenomenon; but they have compenfated for that difappointment, by a difcovery, that a dry froft, though ever fo intenfe, has no tendency to produce it. A hoar froft, or that kind of dewy vapour which, in a fufficient degree of cold, occafions a hoar froft, appears effentially neceffary. This took place pretty favourably on the 6th of January, when the elevation was equal to that reprefented in the third drawing (*fee plate* 25, *fig.* 3), much like what it was on the 9th of January 1794, and confirmed me as to the certainty of fome peculiar appearances, hinted at in my letter of the 14th of that month, but not there defcribed. What I allude to, was a fluctuating appearance of two horizons, one above the other, with a complete vacancy between them, exactly like what may be often obferved looking through an uneven pane of glafs. Divers inftances of this were feen by my brother and myfelf on the 6th of laft January; continually varying and intermitting, but not rapidly, fo that they were capable of diftinct obfervation.—Till that day I had formed, as I thought, a plaufible theory, to account for, as well this latter, as all the other phenomena; but now, unlefs my imagination deceives me, I am left in impenetrable darknefs. The vacant line of feparation, you will take notice, would often increafe fo much in breadth, as to efface entirely the upper of the two horizons; forming then a kind of dent or gap in the remaining horizon, which horizon at the places contiguous to the extremities of the vacancy, feemed of the fame height as the upper horizon was, before effaced. This vacancy was feveral times feen to approach and take in the tower, and immediately to admit a view of the whole or moft part of its body (like that in the third drawing) which was not the cafe before: exactly, to all appearance, as if it had opened a gap for that purpofe in the intercepted ground.—It remains therefore to be determined by future obfervations, whether the feparation is effected by an elevation of the upper, or depreffion of the lower horizon; and if the latter, why the vacancy does not caufe the tower to difappear, as well as the intervening ground?—As an opportunity for this purpofe may not foon occur, I hope you will not wait for it, in your communications to him who is, Dear Sir, yours very truly,

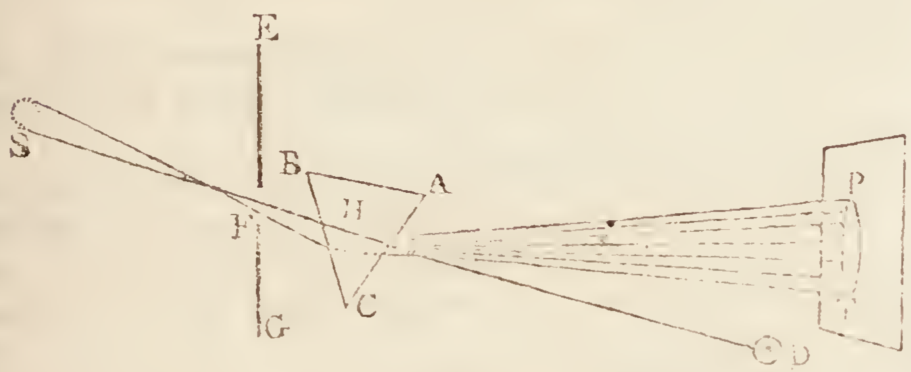
John Andrews."

See the reprefentations in plate 25, of the appearances, in three different ftates of the atmofphere, with the explanations of them.

REFRAN-

REFRANGIBILITY *of Light*, the diſpoſition of the rays to be refracted. And a greater or leſs Refrangibility, is a diſpoſition to be more or leſs refracted, in paſſing at equal angles of incidence into the ſame medium.

That the rays of light are differently refrangible, is the foundation of Newton's whole theory of light and colours; and the truth and circumſtances of the principle he evinced from ſuch experiments as the following.



Let EG repreſent the window-ſhutter of a dark room, and F a hole in it, through which the light paſſes, from the luminous object S, to the glaſs priſm ABC within the room, which refracts it towards the oppoſite ſide, or a ſcreen, at PT, where it appears of an oblong form; its length being about five times the breadth, and exhibiting the various colours of the rainbow; whereas without the interpoſition of the priſm, the ray of light would have proceeded on in its firſt direction to D. Hence then it follows,

1. That the rays of light are refrangible. This appears by the ray being refracted from its original direction SHD, into another one, HP or HT, by paſſing through a different medium.

2. That the ray SFH is a compound one, which, by means of the priſm, is decompounded or ſeparated into its parts, HP, HT, &c, which it hence appears are all endued with different degrees of Refrangibility, as they are tranſmitted to all the intermediate points from T to P, and there painting all the different colours.

From this, and a great variety of other experiments, Newton proved, that the blue rays are more refracted than the red ones, and that there is likewiſe unequal refraction in the intermediate rays; and upon the whole it appears that the ſun's rays have not all the ſame Refrangibility, and conſequently are not of the ſame nature. It is alſo obſerved that thoſe rays which are moſt refrangible, are alſo moſt reflexible. See REFLEXIBILITY; alſo Newton's Optics, pa. 22 &c, 3d edit.

The difference between Refrangibility and reflexibility was firſt diſcovered by Sir Iſaac Newton, in 1671-2, and communicated to the Royal Society, in a letter dated Feb. 6 of that year, which was publiſhed in the Philoſ. Tranſ. numb. 80, pa. 3075; and from that time it was vindicated by him, from the objections of ſeveral authors; particularly Pardies, Mariotte, Linus or Lin, and other gentlemen of the Engliſh college at Liege; and at length it was more fully laid down, illuſtrated, and confirmed, by a great variety of experiments, in his excellent treatiſe on Optics.

But farther, as not only theſe colours of light produced by refraction in a priſm, but alſo thoſe reflected from opaque bodies, have their different degrees of Refrangibility and reflexibility; and as a white light ariſes from a mixture of the ſeveral coloured rays together, the ſame great author concluded that all homogeneous light has its proper colour, correſponding to its degree of Refrangibility, and not capable of being changed by any reflexions, or any refractions; that the ſun's light is compoſed of all the primary colours; and that all compound colours ariſe from the mixture of the primary ones, &c.

The different degrees of Refrangibility, he conjectures to ariſe from the different magnitude of the particles compoſing the different rays. Thus, the moſt refrangible rays, that is the red ones, he ſuppoſes may conſiſt of the largeſt particles; the leaſt refrangible, i. e. the violet rays, of the ſmalleſt particles; and the intermediate rays, yellow, green, and blue, of particles of intermediate ſizes. See COLOUR.

For the method of correcting the effect of the different Refrangibility of the rays of light in glaſſes, ſee ABERRATION and TELESCOPE.

REGEL, or RIGEL, a fixed ſtar of the firſt magnitude, in the left foot of Orion.

REGIOMONTANUS. See *John* MULLER.

REGION, of the Air or Atmoſphere. Authors divide the atmoſphere into three ſtages, called the upper, middle, and lower Regions.—The loweſt Region is that in which we breathe, and is bounded by the reflexion of the ſun's rays, that is, by the height to which they rebound from the earth.—The middle Region is that in which the clouds reſide, and where meteors are formed, &c; extending from the extremity of the loweſt, to the tops of the higheſt mountains.—The upper Region commences from the tops of the mountains, and reaches to the utmoſt limits of the atmoſphere. In this Region there probably reigns a perpetual equable calmneſs, clearneſs, and ſerenity.

Elementary REGION, according to the Ariſtotelians, is a ſphere terminated by the concavity of the moon's orb, comprehending the earth's atmoſphere.

Ethereal REGION, is the whole extent of the univerſe, compriſing all the heavens with the orbs of the fixed ſtars and other celeſtial bodies.

REGION, in Geography, a country or particular diviſion of the earth, or a tract of land inhabited by people of the ſame nation.

REGIONS *of the Moon*. Modern aſtronomers divide the moon into ſeveral Regions, or provinces, to each of which they give its proper name.

REGIONS *of the Sea*, are the two parts into which the whole depth of the ſea is conceived to be divided. The upper of theſe extends from the ſurface of the water, down as low as the rays of the ſun can pierce, and extend their influence; and the lower Region extends from thence to the bottom of the ſea.

Subterranean REGIONS. Theſe are three, into which the earth is divided, at different depths below the ſurface, according to different degrees of cold or warmth; and it is imagined that the 2d or middlemoſt of theſe Regions is the coldeſt of the three.

REGIS (PETER SYLVAIN), a French philoſopher, and great propagator of Carteſianiſm, was born in Agenois 1632.

He ſtudied the languages and philoſophy under the Jeſuits

Jesuits at Cahors, and afterwards divinity in the university of that town, being designed for the church. His progress in learning was so uncommon, that at the end of four years he was offered a doctor's degree without the usual charges; but he did not think it became him till he should study also in the Sorbonne at Paris. He accordingly repaired to the capital for that purpose; but he soon became disgusted with theology; and, as the philosophy of Des Cartes began at that time to make a noise through the lectures of Rohault, he conceived a taste for it, and gave himself up entirely to it.

Having, by attending those lectures, and by close study, become an adept in that philosophy, he went to Toulouse in 1665, where he set up lectures in it himself. Having a clear and fluent manner, and a happy way of explaining his subject, he drew all sorts of people to his discourses; the magistrates, the literati, the ecclesiastics, and the very women, who all now affected to renounce the ancient philosophy.

In 1671, he received at Montpellier the same applauses for his lectures as at Toulouse. Finally, in 1680 he returned to Paris; where the concourse about him was such, that the sticklers for Peripateticism began to be alarmed. These applying to the archbishop of Paris, he thought it expedient, in the name of the king, to put a stop to the lectures; which accordingly were discontinued for several months. Afterwards his whole life was spent in propagating the new philosophy, both by lectures, and by publishing books. In defence of his system, he had disputes with Huet, Du Hamel, Malbranche, and others. His works, though abounding with ingenuity and learning, have been neglected in consequence of the great discoveries and advancement in philosophic knowledge that has been since made.—He was chosen a member of the Academy of Sciences in 1699; and died in 1707, at 75 years of age.

His works, which he published, are,

1. *A System of Philosophy;* containing Logic, Metaphysics, and Morals; in 1690, 3 vols in 4to. being a compilation of the different ideas of Des Cartes.—It was reprinted the year after at Amsterdam, with the addition of a Discourse upon Ancient and Modern Philosophy.

2. *The Use of Reason and of Faith.*

3. An Answer to Huet's Censures of the Cartesian Philosophy; and an Answer to Du Hamel's Critical Reflections.

4. Some pieces against Malbranche, to shew that the apparent magnitude of an object depends solely on the magnitude of its image, traced on the retina.

5. A small piece upon the question, Whether Pleasure makes our present happiness?

REGRESSION, or RETROGRADATION *of Curves,* &c. See RETROGRADATION.

REGULAR *Figure,* in Geometry, is a figure that is both equilateral and equiangular, or having all its sides and angles equal to one another.

For the dimensions, properties, &c, of regular figures, see POLYGON.

REGULAR *Body,* called also *Platonic Body,* is a body or solid comprehended by like, equal, and regular plane figures, and whose solid angles are all equal.

The plane figures by which the solid is contained, are the faces of the solid. And the sides of the plane figures are the edges, or linear sides of the solid.

There are only five Regular Solids, viz,

The tetraedron, or regular triangular pyramid, having 4 triangular faces;

The hexaedron, or cube, having 6 square faces;

The octaedron, having 8 triangular faces;

The dodecaedron, having 12 pentagonal faces;

The icosaedron, having 20 triangular faces.

Besides these five, there can be no other Regular Bodies in nature.

PROB. 1. *To construct or form the Regular Solids.*—See the method of describing these figures under the article BODY.

2. *To find either the Surface or the Solid Content of any of the Regular Bodies.*—Multiply the proper tabular area or surface (taken from the following Table) by the square of the linear edge of the solid, for the superficies. And

Multiply the tabular solidity, in the last column of the Table, by the cube of the linear edge, for the solid content.

Surfaces and Solidities of Regular Bodies, the side being unity or 1.

No. of sides.	Name.	Surface.	Solidity.
4	Tetraedron	1·7320508	0·1178513
6	Hexaedron	6·0000000	1·0000000
8	Octaedron	3·4641016	0·4714045
12	Dodecaedron	20·6457788	7·6631189
20	Icosaedron	8·6602540	2·1816950

3. The Diameter of a Sphere being given, to find the side of any of the Platonic bodies, that may be either inscribed in the sphere, or circumscribed about the sphere, or that is equal to the sphere.

Multiply the given diameter of the sphere by the proper or corresponding number, in the following Table, answering to the thing sought, and the product will be the side of the Platonic body required.

The diam. of a sphere being 1, the side of a	That may be inscribed in the sphere, is	That may be circumscribed about the sphere, is	That is equal to the sphere, is
Tetraedron	0·816497	2·44948	1·64417
Hexaedron	0·577350	1·00000	0·88610
Octaedron	0·707107	1·22474	1·03576
Dodecaedron	0·525731	0·66158	0·62153
Icosaedron	0·356822	0·44903	0·40883

4. The side of any of the five Platonic bodies being given, to find the diameter of a sphere, that may either be inscribed in that body, or circumscribed about it, or that is equal to it.—As the respective number in the Table above, under the title, *inscribed, circumscribed,* or *equal,* is to 1, so is the side of the given Platonic body,

body, to the diameter of its inscribed, circumscribed, or equal sphere.

5. The side of any one of the five Platonic bodies being given; to find the side of any of the other four bodies, that may be equal in solidity to that of the given body.—As the number under the title *equal* in the last column of the table above, against the given Platonic body, is to the number under the same title, against the body whose side is sought, so is the side of the given Platonic body, to the side of the body sought.

See demonstrations of many other properties of the Platonic bodies, in my Mensuration, part 3 sect. 2 pa. 249, &c, 2d edition.

REGULAR *Curve.* See CURVE.

REGULATOR *of a Watch*, is a small spring belonging to the balance, serving to adjust the going, and to make it go either faster or slower.

REGULUS, in Astronomy, a star of the first magnitude, in the constellation Leo; called also, from its situation, *Cor Leonis*, or the *Lion's Heart*; by the Arabs, *Alhabor*; and by the Chaldeans, *Kalbeleced*, or *Karbeleceid*; from an opinion of its influencing the affairs of the heavens; as Theon observes.

The longitude of Regulus, as fixed by Flamsteed, is 25° 31′ 21″, and its latitude 0° 26′ 38″ north. See LEO.

REINFORCE, in Gunnery, is that part of a gun next the breech, which is made stronger to resist the force of the powder. There are usually two Reinforces in each piece, called the first and second Reinforce. The second is somewhat smaller than the first, because the inflamed powder in that part is less strong.

REINFORCE *Rings* of a cannon, are flat mouldings, like iron hoops, placed at the breech end of the first and second Reinforce, projecting beyond the rest of the metal about a quarter of an inch.

REINHOLD (ERASMUS), an eminent astronomer and mathematician, was born at Salfeldt in Thuringia, a province in Upper Saxony, the 11th of October 1511. He studied mathematics under James Milichi at Wittemberg, in which university he afterwards became professor of those sciences, which he taught with great applause. After writing a number of useful and learned works, he died the 19th of February 1553, at 42 years of age only. His writings are chiefly the following:

1. *Theoriæ novæ Planetarum G. Purbachii*, augmented and illustrated wirh diagrams and Scholia in 8vo, 1542; and again in 1580.—In this work, among other things worthy of notice, he teaches (pa. 75 and 76) that the centre of the lunar epicycle describes an *oval figure* in each monthly period, and that the orbit of Mercury is also of the same oval figure.

2. *Ptolomy's Almagest*, the first book, in Greek, with a Latin version, and Scholia, explaining the more obscure passages; in 8vo, 1549.—At the end of pa. 123 he promises an edition of Theon's Commentaries, which are very useful for understanding Ptolomy's meaning; but his immature death prevented Reinhold from giving this and other works which he had projected.

3. *Prutenicæ Tabulæ Cœlestium Motuum*, in 4to, 1551; again in 1571; and also in 1585.—Reinhold spent seven years labour upon this work, in which he was assisted by the munificence of Albert, duke of Prussia, from whence the tables had their name. Reinhold compared the observations of Copernicus with those of Ptolomy and Hipparchus, from whence he constructed these new tables, the uses of which he has fully explained in a great number of precepts and canons, forming a complete introduction to practical astronomy.

4. *Primus liber Tabularum Directionum*; to which are added, the *Canon Fœcundus*, or Table of Tangents, to every minute of the quadrant; and New Tables of Climates, Parallels and Shadows, with an Appendix containing the second Book of the Canon of Directions; in 4to, 1554.—Reinhold here supplies what was omitted by Regiomontanus in his Table of Directions, &c; shewing the finding of the sines, and the construction of the tangents, the sines being found to every minute of the quadrant, to the radius 10,000,000; and he produced the Oblique Ascensions from 60 degrees to the end of the quadrant. He teaches also the use of these tables in the solution of spherical problems.

Reinhold prepared likewise an edition of many other works, which are enumerated in the Emperor's Privilege, prefixed to the Prutenic Tables. Namely, Ephemerides for several years to come, computed from the new tables. Tables of the Rising and Setting of several Fixed Stars, for many different climates and times. The illustration and establishment of Chronology, by the eclipses of the luminaries, and the great conjunctions of the planets, and by the appearance of comets, &c. The Ecclesiastical Calendar. The History of Years, or Astronomical Calendar. *Isagoge Spherica*, or Elements of the Doctrine of the Primum Mobile. *Hypotyposes Orbium Cœlestium*, or the Theory of Planets. Construction of a New Quadrant. The Doctrine of Plane and Spherical Triangles. Commentaries on the work of Copernicus. Also Commentaries on the 15 books of Euclid, on Ptolomy's Geography, and on the Optics of Alhazen the Arabian.—Reinhold also made Astronomical Observations, but with a wooden quadrant, which observations were seen by Tycho Brahe when he passed through Wittemberg in the year 1575, who wondered that so great a cultivator of astronomy was not furnished with better instruments.

Reinhold left a son, named also Erasmus after himself, an eminent mathematician and physician at Salfeldt. He wrote a small work in the German language, on Subterranean Geometry, printed in 4to at Erfurt 1575.—He wrote also concerning the New Star which appeared in Cassiopeia in the year 1572; with an Astrological Prognostication, published in 1574, in the German language.

RELAIS, in Fortification, a French term, the same with berme.

RELATION, in Mathematics, is the habitude or respect of quantities of the same kind to each other, with regard to their magnitude; more usually called *ratio*.—And the equality, identity, or sameness of two such Relations, is called proportion.

RELATION, *Inharmonical*, in Musical Composition,

is that whoſe extremes form a falſe or unnatural interval, incapable of being ſung.—This is otherwiſe called a *falſe Relation*, and ſtands oppoſed to a juſt or true one.

RELATIVE *Gravity*, *Levity*, *Motion*, *Neceſſity*, *Place*, *Space*, *Time*, *Velocity*, *&c.* See the ſeveral ſubſtantives.

RELIEVO, in Architecture, denotes the ſally or projecture of any ornament.

REMAINDER, is the difference between two quantities, or that which is left after ſubtracting one from the other.

RENDERING, in Building. See PARGETING.

REPELLING *Power*, in Phyſics, is a certain power or faculty, reſiding in the minute particles of natural bodies, by which, under certain circumſtances, they mutually fly from each other. This is the reverſe or oppoſite of the attractive power. Newton ſhews, from obſervation, that ſuch a force does really exiſt; and he argues, that as in algebra, where poſitive quantities ceaſe, there negative ones begin; ſo in phyſics, where the attractive force ceaſes, there a Repelling force muſt begin.

As the Repelling power ſeems to ariſe from the ſame principle as the attractive, only exerciſed under different circumſtances, it is governed by the ſame laws. Now the attractive power we find is ſtronger in ſmall bodies, than in great ones, in proportion to the maſſes; therefore the Repelling is ſo too. But the rays of light are the moſt minute bodies we know of; and therefore their Repelling force muſt be the greateſt. It is computed by Newton, that the attractive force of the rays of light is above 1000000000000000, or one thouſand million of millions of times ſtronger than the force of gravity on the ſurface of the earth: hence ariſes that inconceivable velocity with which light muſt move to reach from the ſun to the earth in little more than 7 minutes of time. For the rays emitted from the body of the ſun, by the vibrating motion of its parts, are no ſooner got without the ſphere of attraction of the ſun, than they come within the action of the Repelling power.

The elaſticity or ſpringineſs of bodies, or that property by which, after having their figure altered by an external force, they return to their former ſhape again, follows from the Repelling power. See REPULSION.

REPERCUSSION. See REFLECTION.

REPETEND, in Arithmetic, denotes that part of an infinite decimal fraction, which is continually repeated ad infinitum. Thus in the numbers 2·13 13 13 &c. the figures 13 are the Repetend, and marked thus $\dot{1}\dot{3}$.

Theſe Repetends chiefly ariſe in the reduction of vulgar fractions to decimals. Thus, $\frac{1}{3} = 0·333$ &c $= 0·\dot{3}$; and $\frac{1}{6} = 01666$ &c $= 0·1\dot{6}$; and $\frac{1}{7} = 0·142857\ 142857$ &c $= 0·\dot{1}4285\dot{7}$. Where it is to be obſerved, that a point is ſet over the figure of a ſingle Repetend, and a point over the firſt and laſt figure when there are ſeveral that repeat.

Repetends are either *ſingle* or *compound*.

A *Single* REPETEND is that in which only one figure repeats; as $0·\dot{3}$, or $0·\dot{6}$, &c.

A *Compound* REPETEND, is that in which two or more figures are repeated; as $·\dot{1}\dot{3}$, or $·\dot{2}1\dot{5}$, or $·\dot{1}4285\dot{7}$, &c.

Similar REPETENDS are ſuch as begin at the ſame place, and conſiſt of the ſame number of figures: as $·\dot{3}$ and $·\dot{6}$, or $1·\dot{3}4\dot{1}$ and $2·\dot{1}5\dot{6}$.

Diſſimilar REPETENDS begin at different places, and conſiſt of an unequal number of figures.

To find the finite Value of any Repetend, or to reduce it to a Vulgar Fraction. Take the given repeating figure or figures for the numerator; and for the denominator, take as many 9's as there are recurring figures or places in the given Repetend.

$$\text{So } ·\dot{3} = \frac{3}{9} = \frac{1}{3}; \text{ and } ·0\dot{5} = ·0\frac{5}{9} = \frac{5}{90} = \frac{1}{18};$$

$$\text{and } ·\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333}; \text{ and } 2·\dot{6}\dot{3} = 2\frac{63}{99} = 2\frac{7}{11};$$

$$\text{and } ·\dot{0}59440\dot{5} = \frac{594405}{9999990} = \frac{17}{280};$$

$$\text{and } ·\dot{7}6923\dot{0} = \frac{769230}{999999} = \frac{10}{13}.$$

Hence it follows, that every ſuch infinite Repetend has a certain determinate and finite value, or can be expreſſed by a terminate vulgar fraction. And conſequently, that an infinite decimal which does not repeat or circulate, cannot be completely expreſſed by a finite vulgar fraction.

It may farther be obſerved, that if the numerator of a vulgar fraction be 1, and the denominator any prime number, except 2 and 5, the decimal which ſhall be equal to that vulgar fraction, will always be a Repetend, beginning at the firſt place of decimals; and this Repetend muſt neceſſarily be a ſubmultiple, or an aliquot part of a number expreſſed by as many 9's as the Repetend has figures; that is, if the Repetend have ſix figures, it will be a ſubmultiple of 999999; if four figures, a ſubmultiple of 9999 &c. From whence it follows, that if any prime number be called p, the ſeries 9999 &c, produced as far as is neceſſary, will always be diviſible by p, and the quotient will be the Repetend of the decimal fraction $= \frac{1}{p}$.

RESIDUAL *Figure*, in Geometry, the figure remaining after ſubtracting a leſs from a greater.

RESIDUAL *Root*, is a root compoſed of two parts or members, only connected together with the ſign − or minus. Thus, $a - b$, or $5 - 3$, is a reſidual root; and is ſo called, becauſe its true value is no more than the reſidue, or difference between the parts a and b, or 5 and 3, which in this caſe is 2.

RESIDUUM *of a Charge*, in Electricity, firſt diſcovered by Mr. Gralath, in Germany, in 1746, is that part of the charge that lay on the uncoated part of a Leyden phial, which does not part with all its electricity at once; ſo that it is afterwards gradually diffuſed to the coating.

RESISTANCE, or RESISTING *Force*, in Phyſics, any power which acts in oppoſition to another, ſo as to deſtroy or diminiſh its effect.

There

There are various kinds of Resistance, arising from the various natures and properties of the resisting bodies, and governed by various laws: as, the Resistance of solids, the Resistance of fluids, the Resistance of the air, &c. Of each of these in their order, as below.

Resistance *of Solids*, in Mechanics, is the force with which the quiescent parts of solid bodies oppose the motion of others contiguous to them.

Of these, there are two kinds. The first where the resisting and the resisted parts, i. e. the moving and quiescent bodies, are only contiguous, and do not cohere; constituting separate bodies or masses. This Resistance is what Leibnitz calls *Resistance of the surface*, but which is more properly called *friction*: for the laws of which, see the article Friction.

The second case of Resistance, is where the resisting and resisted parts are not only contiguous, but cohere, being parts of the same continued body or mass. This Resistance was first considered by Galileo, and may properly be called *renitency*.

As to what regards the Resistance of bodies when struck by others in motion, see Percussion, and Collision.

Theory of the Resistance of the Fibres of Solid Bodies.—To conceive an idea of this Resistance, or renitency of the parts, suppose a cylindrical body suspended vertically by one end. Here all its parts, being heavy, tend downwards, and endeavour to separate the two contiguous planes or surfaces where the body is the weakest; but all the parts of them resist this separation by the force with which they cohere, or are bound together. Here then are two opposite powers; viz, the weight of the cylinder, which tends to break it; and the force of cohesion of the parts, which resists the fracture.

If now the base of the cylinder be increased, without increasing its length; it is evident that both the Resistance and the weight will be increased in the same ratio as the base; and hence it appears that all cylinders of the same matter and length, whatever their bases be, have an equal Resistance, when vertically suspended.

But if the length of the cylinder be increased, without increasing its base, its weight is increased, while the Resistance or strength continues unaltered; consequently the lengthening has the effect of weakening it, or increases its tendency to break.

Hence to find the greatest length a cylinder of any matter may have, when it just breaks with the addition of another given weight, we need only take any cylinder of the same matter, and fasten to it the least weight that is just sufficient to break it; and then consider how much it must be lengthened, so that the weight of the part added, together with the given weight, may be just equal to that weight, and the thing is done. Thus, let l denote the first length of the cylinder, c its weight, g the given weight the lengthened cylinder is to bear, and w the least weight that breaks the cylinder l, also x the length sought;

then as $l : x :: c : \frac{cx}{l} =$ the weight of the longest cylinder sought; and this, together with the given weight g, must be equal to c together with the weight w; hence then

$\frac{cx}{l} + g = c + w$; therefore $x = \frac{c + w - g}{c} l =$ the whole length of the cylinder sought. If the cylinder must just break with its own weight, then is $g = 0$, and in that case $x = \frac{c + w}{c} l$ is the whole length that just breaks by its own weight. By this means Galileo found that a copper wire, and of consequence any other cylinder of copper, might be extended to 4801 braccios or fathoms of 6 feet each.

If the cylinder be fixed by one end into a wall, with the axis horizontally; the force to break it, and its Resistance to fracture, will here be both different; as both the weight to cause the fracture, and the Resistance of the fibres to oppose it, are combined with the effects of the lever; for the weight to cause the fracture, whether of the weight of the beam alone, or combined with an additional weight hung to it, is to be supposed collected into the centre of gravity, where it is considered as acting by a lever equal to the distance of that centre beyond the face of the wall where the cylinder or other prism is fixed; and then the product of the said whole weight and distance, will be the momentum or force to break the prism. Again, the Resistance of the fibres may be supposed collected into the centre of the transverse section, and all acting there at the end of a lever equal to the vertical semidiameter of the section, the lowest point of that diameter being immoveable, and about which the whole diameter turns when the prism breaks; and hence the product of the adhesive force of the fibres multiplied by the said semidiameter, will be the momentum of Resistance, and must be equal to the former momentum when the prism just breaks.

Hence, to find the length a prism will bear, fixed so horizontally, before it breaks, either by its own weight, or by the addition of any adventitious weight; take any length of such a prism, and load it with weights till it just break. Then, put

$l =$ the length of this prism,
$c =$ its weight,
$w =$ the weight that breaks it,
$a =$ distance of weight w,
$g =$ any given weight to be borne,
$d =$ its distance,
$x =$ the length required to break.

Then $l : x :: c : \frac{cx}{l}$ the weight of the prism x,

and $\frac{cx}{l} \times \frac{1}{2} x = \frac{cx^2}{2l} =$ its momentum; also $dg =$ the momentum of the weight g; therefore $\frac{cx^2}{2l} + dg$ is the momentum of the prism x and its added weight. In like manner $\frac{1}{2}cl + aw$ is that of the former or short prism and the weight that brake it; consequently $\frac{cx^2}{2l} + dg = \frac{1}{2}cl + aw$, and $x = \sqrt{\frac{aw + \frac{1}{2}cl - dg}{c} \times 2l}$ is the length sought, that just

breaks

breaks with the weight g at the diftance d. If this weight g be nothing, then $x = \sqrt{\frac{aw + \frac{1}{2}cl}{c}} \times 2l$ is the length of the prifm that juft breaks with its own weight.

If two prifms of the fame matter, having their bafes and lengths in the fame proportion, be fufpended horizontally; it is evident that the greater has more weight than the leffer, both on account of its length, and of its bafe; but it has lefs Refiftance on account of its length, confidered as a longer arm of a lever, and has only more Refiftance on account of its bafe; therefore it exceeds the leffer in its momentum more than it does in its Refiftance, and confequently it muft break more eafily.

Hence appears the reafon why, in making fmall machines and models, people are apt to be miftaken as to the Refiftance and ftrength of certain horizontal pieces, when they come to execute their defigns in large, by obferving the fame proportions as in the fmall.

When the prifm, fixed vertically, is juft about to break, there is an equilibrium between its pofitive and relative weight; and confequently thofe two oppofite powers are to each other reciprocally as the arms of the lever to which they are applied, that is, as half the diameter to half the axis of the prifm. On the other hand, the Refiftance of a body is always equal to the greateft weight which it will juft fuftain in a vertical pofition, that is, to its abfolute weight. Therefore, fubftituting the abfolute weight for the Refiftance, it appears, that the abfolute weight of a body, fufpended horizontally, is to its relative weight, as the diftance of its centre of gravity from the fixed point or axis of motion, is to the diftance of the centre of gravity of its bafe from the fame.

The difcovery of this important truth, at leaft of an equivalent to it, and to which this is reducible, we owe to Galileo. On this fyftem of Refiftance of that author, Mariotte made an ingenious remark, which gave birth to a new fyftem. Galileo fuppofes that where the body breaks, all the fibres break at once; fo that the body always refifts with its whole abfolute force, or the whole force that all its fibres have in the place where it breaks. But Mariotte, finding that all bodies, even glafs itfelf, bend before they break, fhews that fibres are to be confidered as fo many little bent fprings, which never exert their whole force, till ftretched to a certain point, and never break till entirely unbent. Hence thofe neareft the fulcrum of the lever, or loweft point of the fracture, are ftretched lefs than thofe farther off, and confequently employ a lefs part of their force, and break later.

This confideration only takes place in the horizontal fituation of the body: in the vertical, the fibres of the bafe all break at once; fo that the abfolute weight of the body muft exceed the united Refiftance of all its fibres; a greater weight is therefore required here than in the horizontal fituation, that is, a greater weight is required to overcome their united Refiftance, than to overcome their feveral Refiftances one after another.

Varignon has improved on the fyftem of Mariotte, and fhewn that to Galileo's fyftem, it adds the confideration of the centre of percuffion. In each fyftem, the fection, where the body breaks, moves on the axis of equilibrium, or line at the lower extremity of the fame fection; but in the fecond, the fibres of this fection are continually ftretching more and more, and that in the fame ratio, as they are fituated farther and farther from the axis of equilibrium, and confequently are ftill exerting a greater and greater part of their whole force.

Thefe unequal extenfions, like all other forces, muft have fome common centre where they are united, making equal efforts on each fide of it; and as they are precifely in the fame proportion as the velocities which the feveral points of a rod moved circularly would have to one another, the centre of extenfion of the fection where the body breaks, muft be the fame as its centre of percuffion. Galileo's hypothefis, where fibres ftretch equally, and break all at once, correfponds to the cafe of a rod moving parallel to itfelf, where the centre of extenfion or percuffion does not appear, as being confounded with the centre of gravity.

Hence it follows, that the Refiftance of bodies in Mariotte's fyftem, is to that in Galileo's, as the diftance of the centre of percuffion, taken on the vertical diameter of the fracture, is to the whole of that diameter. Hence alfo, the Refiftance being lefs than what Galileo imagined, the relative weight muft alfo be lefs, and in the ratio juft mentioned. So that, after conceiving the relative weight of a body, and its Refiftance equal to its abfolute weight, as two contrary powers applied to the two arms of a lever, in the hypothefis of Galileo, there needs nothing to change it into that of Mariotte, but to imagine that the Refiftance, or the abfolute weight, is become lefs, in the ratio above mentioned, every thing elfe remaining the fame.

One of the moft curious, and perhaps the moft ufeful queftions in this refearch, is to find what figure a body muft have, that its Refiftance may be equal or proportional in every part to the force tending to break it. Now to this end, it is neceffary, fome part of it being conceived as cut off by a plane parallel to the fracture, that the momentum of the part retrenched be to its Refiftance, in the fame ratio as the momentum of the whole is to its Refiftance; thefe four powers acting by arms of levers peculiar to themfelves, and are proportional in the whole, and in each part, of a folid of equal Refiftance. From this proportion, Varignon eafily deduces two folids, which fhall refift equally in all their parts, or be no more liable to break in one part than in another: Galileo had found one before. That difcovered by Varignon is in the form of a trumpet, and is to be fixed into a wall at its greater end; fo that its magnitude or weight is always diminifhed in proportion as its length, or the arm of the lever by which its weight acts, is increafed. It is remarkable that, howfoever different the two fyftems may be, the folids of equal Refiftance are the fame in both.

For the Refiftance of a folid fupported at each end, as of a beam between two walls, fee Beam.

Resistance *of Fluids*, is the force with which bodies,

bodies, moving in fluid mediums, are impeded and retarded in their motion.

A body moving in a fluid is refifted from two caufes. The firft of thefe is the cohefion of the parts of the fluid. For a body, in its motion, feparating the parts of a fluid, muft overcome the force with which thofe parts cohere. The fecond is the inertia, or inactivity of matter, by which a certain force is required to move the particles from their places, in order to let the body pafs.

The retardation from the firft caufe is always the fame in the fame fpace, whatever the velocity be, the body remaining the fame; that is, the Refiftance is as the fpace run through, in the fame time: but the velocity is alfo in the fame ratio of the fpace run over in the fame time: and therefore the Refiftance, from this caufe, is as the velocity itfelf.

The Refiftance from the fecond caufe, when a body moves through the fame fluid with different velocities, is as the fquare of the velocity. For, firft the Refiftance increafes according to the number of particles or quantity of the fluid ftruck in the fame time; which number muft be as the fpace run through in that time, that is, as the velocity: but the Refiftance alfo increafes in proportion to the force with which the body ftrikes againft every part; which force is alfo as the velocity of the body, fo as to be double with a double velocity, and triple with a triple one, &c: therefore, on both thefe accounts, the Refiftance is as the velocity multiplied by the velocity, or as the fquare of the velocity. Upon the whole therefore, on account of both caufes, viz, the tenacity and inertia of the fluid, the body is refifted partly as the velocity and partly as the fquare of the velocity.

But when the fame body moves through different fluids with the fame velocity, the Refiftance from the fecond caufe follows the proportion of the matter to be removed in the fame time, which is as the denfity of the fluid.

Hence therefore, if d denote the denfity of the fluid,
v the velocity of the body,
and a and b conftant coefficients:
then $adv^2 + bv$ will be proportional to the whole Refiftance to the fame body, moving with different velocities, in the fame direction, through fluids of different denfities, but of the fame tenacity.

But, to take in the confideration of different tenacities of fluids; if t denote the tenacity, or the cohefion of the parts of the fluid, then $adv^2 + btv$ will be as the faid whole Refiftance.

Indeed the quantity of Refiftance from the cohefion of the parts of fluids, except in glutinous ones, is very fmall in refpect of the other Refiftance; and it alfo increafes in a much lower degree, being only as the velocity, while the other increafes as the fquare of the velocity, and rather more. Hence then the term btv is very fmall in refpect of the other term adv^2; and confequently the Refiftance is nearly as this latter term; or nearly as the fquare of the velocity. Which rule has been employed by moft authors, and is very near the truth in flow motions; but in very rapid ones, it differs confiderably from the truth, as we fhall perceive below; not indeed from the omiffion of the fmall term ctv, due to the cohefion, but from the want of the full counter preffure on the hinder part of the body, a vacuum, either perfect or partial, being left behind the body in its motion; and alfo perhaps to fome compreffion or accumulation of the fluid againft the fore part of the body. Hence,

To conceive the Refiftance of fluids to a body moving in them, we muft diftinguifh between thofe fluids which, being greatly compreffed by fome incumbent weight, always clofe up the fpace behind the body in motion, without leaving any vacuity there; and thofe fluids which, not being much compreffed, do not quickly fill up the fpace quitted by the body in motion, but leave a kind of vacuum behind it. Thefe differences, in the refifting fluids, will occafion very remarkable varieties in the laws of their Refiftance, and are abfolutely neceffary to be confidered in the determination of the action of the air on fhot and fhells; for the air partakes of both thefe affections, according to the different velocities of the projected body.

In treating of thefe Refiftances too, the fluids may be confidered either as continued or difcontinued, that is, having their particles contiguous or elfe as feparated and unconnected; and alfo either as elaftic or non-elaftic. If a fluid were fo conftituted, that all the particles compofing it were at fome diftance from each other, and having no action between them, then the Refiftance of a body moving in it would be eafily computed, from the quantity of motion communicated to thofe particles; for inftance, if a cylinder moved in fuch a fluid in the direction of its axis, it would communicate to the particles it met with, a velocity equal to its own, and in its own direction, when neither the cylinder nor the parts of the fluid are elaftic: whence, if the velocity and diameter of the cylinder be known, and alfo the denfity of the fluid, there would thence be determined the quantity of motion communicated to the fluid, which (as action and reaction are equal) is the fame with the quantity loft by the cylinder, and confequently the Refiftance would thus be afcertained.

In this kind of difcontinued fluid, the particles being detached from each other, every one of them can purfue its own motion in any direction, at leaft for fome time, independent of the neighbouring ones; fo that, inftead of a cylinder moving in the direction of its axis, if a body with a furface oblique to its direction be fuppofed to move in fuch a fluid, the motion which the parts of the fluid will hence acquire, will not be in the direction of the refifted body, but perpendicular to its oblique furface; whence the Refiftance to fuch a body will not be eftimated from the whole motion communicated to the particles of the fluid, but from that part of it only which is in the direction of the refifted body. In fluids then, where the parts are thus difcontinued from each other, the different obliquities of that furface which goes foremoft, will occafion confiderable changes in the Refiftance; although the tranfverfe fection of the folid fhould in all cafes be the fame: And Newton has particularly determined that, in a fluid thus conftituted, the Refiftance of a globe is but half the Refiftance of a cylinder of the fame diameter, moving, in the direction of its axis, with the fame velocity.

But though the hypothefis of a fluid thus conftituted be

be of great use in explaining the nature of Resistances, yet we know of no such fluid existing in nature; and the fluids with which we are conversant being so formed, that their particles either lie contiguous to each other, or at least act on each other in the same manner as if they did: consequently, in these fluids, no one particle that is contiguous to the resisted body, can be moved, without moving at the same time a great number of others, some of which will be distant from it; and the motion thus communicated to a mass of the fluid, will not be in any one determined direction, but different in all the particles, according to the different positions in which they lie in contact with those from which they receive their impulse; whence, great numbers of the particles being diverted into oblique directions, the Resistance of the moving body, which will depend on the quantity of motion communicated to the fluid in its own direction, will be different in quantity from what it would be in the foregoing supposition, and its estimation becomes much more complicated and operose.

If the fluid be compressed by the incumbent weight of its upper parts (as all fluids are with us, except at their very surface), and if the velocity of the moving body be much less than that with which the parts of the fluid would rush into a void space, in consequence of their compression; it is evident, that in this case the space left by the moving body will be instantaneously filled up by the fluid; and the parts of the fluid against which the foremost part of the body presses in its motion, will, instead of being impelled forwards in the direction of the body, in some measure circulate towards the hinder part of the body, in order to restore the equilibrium, which the constant influx of the fluid behind the body would otherwise destroy; whence the progressive motion of the fluid, and consequently the Resistance of the body, which depends upon it, would in this instance be much less, than in the hypothesis where each particle is supposed to acquire, from the stroke of the resisting body, a velocity equal to that with which the body moved, and in the same direction. Newton has determined, that the Resistance of a cylinder, moving in the direction of its axis, in such a compressed fluid as we have here treated of, is but one-fourth part of the Resistance to the same cylinder, if it moved with the same velocity in a fluid constituted in the manner described in the first hypothesis, each fluid being supposed of the same density.

But again, it is not only in the quantity of their Resistance that these fluids differ, but also in the different manner in which they act upon solids of different forms moving in them. In the discontinued fluid, first described, the obliquity of the foremost surface of the moving body would diminish the Resistance; but the same thing does not hold true in compressed fluids, at least not in any considerable degree; for the chief Resistance in compressed fluids arises from the greater or less facility with which the fluid, impelled by the fore part of the body, can circulate towards its hinder part; and this being little, if at all, affected by the form of the moving body, whether it be cylindrical, conical, or spherical, it follows, that while the transverse section of the body is the same, and consequently the quantity of impelled fluid also, the change of figure in the body will scarcely affect the quantity of its Resistance.

And this case, viz, the Resistance of a compressed fluid to a solid, moving in it with a velocity much less than what the parts of the fluid would acquire from their compression, has been very fully considered by Newton, who has ascertained the quantity of such a Resistance, according to the different magnitudes of the moving body, and the density of the fluid. But he expressly informs us that the rules he has laid down, are not generally true, but only upon a supposition that the compression of the fluid be increased in the greater velocities of the moving body: however, some unskilful writers, who have followed him, overlooking this caution, have applied his determination to bodies moving with all sorts of velocities, without attending to the different compressions of the fluids they are resisted by; and by this means they have accounted the Resistance, for instance, of the air to musket and cannon shot, to be but about one-third part of what it is found to be by experience.

It is indeed evident that the resisting power of the medium must be increased, when the resisted body moves so fast that the fluid cannot instantaneously press in behind it, and fill the deserted space; for when this happens, the body will be deprived of the pressure of the fluid behind it; which in some measure balanced its Resistance, or at least the fore pressure, and must support on its fore part the whole weight of a column of the fluid, over and above the motion it gives to the parts of the same; and besides, the motion in the particles driven before the body, is less affected in this case by the compression of the fluid, and consequently they are less deflected from the direction in which they are impelled by the resisted surface; whence it happens that this species of Resistance approaches more and more to that described in the first hypothesis, where each particle of the fluid being unconnected with the neighbouring ones, pursued its own motion, in its own direction, without being interrupted or deflected by their contiguity; and therefore, as the Resistance of a discontinued fluid to a cylinder, moving in the direction of its axis, is 4 times greater than the Resistance of a fluid sufficiently compressed of the same density, it follows that the Resistance of a fluid, when a vacuity is left behind the moving body, may be near 4 times greater than that of the same fluid, when no such vacuity is formed; for when a void space is thus left, the Resistance approaches in its nature to that of a discontinued fluid.

This then may probably be the case in a cylinder moving in the same compressed fluid, according to the different degrees of its velocity; so that if it set out with a great velocity, and moves in the fluid till that velocity be much diminished, the resisting power of the medium may be near 4 times greater in the beginning of its motion than in the end.

In a globe, the difference will not be so great, because, on account of its oblique surface, its Resistance in a discontinued medium is but about twice as much as in one properly compressed; for its oblique surface diminishes its Resistance in one case, and not in the other: however, as the compression of the medium, even

even when a vacuity is left behind the moving body, may yet confine the oblique motion of the parts of the fluid, which are driven before the body, and as in an elaſtic fl id, ſuch as our air is, there will be ſome degree of condenſation in thoſe parts; it is highly probable that the Reſiſtance of a globe, moving in a compreſſed fluid with a very great velocity, may greatly exceed the proportion of the Reſiſtance to ſlow motions.

And as this increaſe of the reſiſting power of the medium will take place, when the velocity of the moving body is ſo great, that a perfect vacuum is left behind it, ſo ſome degree of augmentation will be ſenſible in velocities much ſhort of this; for even when, by the compreſſion of the fluid, the ſpace left behind the body is inſtantaneouſly filled up; yet, if the velocity with which the parts of the fluid ruſh in behind, is not much greater than that with which the body moves, the ſame reaſons that have been urged above, in the caſe of an abſolute vacuity, will hold in a leſs degree in this inſtance; and therefore it is not to be ſuppoſed that, in the increaſed Reſiſtance which has been hitherto treated of, it immediately vaniſhes when the compreſſion of the fluid is juſt ſufficient to prevent a vacuum behind the reſiſted body; but we muſt conſider it as diminiſhing only according as the velocity, with which the parts of the fluid follow the body, exceeds that with which the body moves.

Hence then it may be concluded, that if a globe ſets out in a reſiſting medium, with a velocity much exceeding that with which the particles of the medium would ruſh into a void ſpace, in conſequence of their compreſſion, ſo that a vacuum is neceſſarily left behind the globe in its motion; the Reſiſtance of this medium to the globe will be much greater, in proportion to its velocity, than what we are ſure, from Sir I. Newton, would take place in a ſlower motion. We may farther conclude, that the reſiſting power of the medium will gradually diminiſh as the velocity of the globe decreaſes, till at laſt, when it moves with velocities which bear but a ſmall proportion to that with which the particles of the medium follow it, the Reſiſtance becomes the ſame with what is aſſigned by Newton in the caſe of a compreſſed fluid.

And from this determination may be ſeen, how falſe that poſition is, which aſſerts the Reſiſtance of any medium to be always in the duplicate ratio of the velocity of the reſiſted body; for it plainly appears, by what has been ſaid, that this can only be conſidered as nearly true in ſmall variations of velocity, and can never be applied in comparing together the Reſiſtances to all velocities whatever, without incurring the moſt enormous errors. See Robins's Gunnery, chap. 2 prop. 1, and my Select Exerciſes pa. 225 &c. See alſo the articles RESISTANCE *of the Air*, PROJECTILE, and GUNNERY.

Reſiſtance and retardation are uſed indifferently for each other, as being both in the ſame proportion, and the ſame Reſiſtance always generating the ſame retardation. But with regard to different bodies, the ſame Reſiſtance frequently generates different retardations; the Reſiſtance being as the quantity of motion, and the retardation that of the celerity. For the difference and meaſure of the two, ſee RETARDATION.

The retardations from this Reſiſtance may be compared together, by comparing the Reſiſtance with the gravity or quantity of matter. It is demonſtrated that the Reſiſtance of a cylinder, which moves in the direction of its axis, is equal to the weight of a column of the fluid, whoſe baſe is equal to that of the cylinder, and its altitude equal to the height through which a body muſt fall in vacuo, by the force of gravity, to acquire the velocity of the moving body. So that, if a denote the area of the face or end of the cylinder, or other priſm, v its velocity, and n the ſpecific gravity of the fluid; then, the altitude due to the velocity v being $\frac{v^2}{4g}$, the whole Reſiſtance, or motive force m, will be $a \times n \times \frac{v^2}{4g} = \frac{anv^2}{4g}$; the quantity g being $= 16\frac{1}{12}$ feet, or the ſpace a body falls, in vacuo, in the firſt ſecond of time. And the Reſiſtance to a globe of the ſame diameter would be the half of this.—Let a ball, for inſtance, of 3 inches diameter, be moved in water with a celerity of 16 feet per ſecond of time: now from experiments on pendulums, and on falling bodies, it has been found, that this is the celerity which a body acquires in falling from the height of 4 feet; therefore the weight of a cylinder of water of 3 inches diameter, and 4 feet high, that is a weight of about 12 lb 4 oz, is equal to the Reſiſtance of the cylinder; and conſequently the half of it, or 6 lb 2 oz is that of the ball. Or, the formula

$$\frac{anv^2}{4g} \text{ gives } \frac{\cdot 7854 \times 9 \times 1000 \times 16 \times 16}{144 \times 4 \times 16} = 196 \text{ oz,}$$

or 12 lb 4 oz, for the Reſiſtance of the cylinder, or 6 lb 2 oz for that of the ball, the ſame as before.

Let now the Reſiſtance, ſo diſcovered, be divided by the weight of the body, and the quotient will ſhew the ratio of the retardation to the force of gravity. So if the ſaid ball, of 3 inches diameter, be of caſt iron, it will weigh nearly 61 ounces, or $3\frac{4}{5}$ lb; and the Reſiſtance being 6 lb 2 oz, or 98 ounces; therefore, the Reſiſtance being to the gravity as 98 to 61, the retardation, or retarding force, will be $\frac{98}{61}$ or $1\frac{3}{5}$, the force of gravity being 1. Or thus; becauſe a the area of a great circle of the ball, is $= pd^2$, where d is the diameter, and $p = \cdot 7854$, therefore the Reſiſtance to the ball is $m = \frac{pnd^2v^2}{8g}$; and becauſe its ſolid content is $w = \frac{2}{3}pd^3$, and its weight $\frac{2}{3}Npd^3$, where N denotes its ſpecific gravity; therefore, dividing the Reſiſtance or motive force m by the weight w, gives $\frac{m}{w} = \frac{3nv^2}{16Ndg} = f$ the retardation, or retarding force, that of gravity being 1; which is therefore as the ſquare of the velocity directly, and as the diameter inverſely; and this is the reaſon why a large ball overcomes the Reſiſtance better than a ſmall one, of the ſame denſity. See my Select Exerciſes, pa. 225 &c.

RESISTANCE *of Fluid Mediums to the Motion of Falling Bodies*.—A body freely deſcending in a fluid, is accelerated by the relative gravity of the body, (that is, the difference between its own abſolute gravity and that of a like bulk of the fluid), which continually acts upon it, yet not equably, as in a vacuum: the Reſiſtance of the fluid occaſions a retardation, or diminution of

of acceleration, which diminution increafes with the velocity of the body. Hence it happens, that there is a certain velocity, which is the greateft that a body can acquire by falling; for if its velocity be fuch, that the Refiftance arifing from it becomes equal to the relative weight of the body, its motion can be no longer accelerated; for the motion here continually generated by the relative gravity, will be deftroyed by the Refiftance, or the force of Refiftance is equal to the relative gravity, and the body forced to go on equably: for after the velocity is arrived at fuch a degree, that the refifting force is equal to the weight that urges it, it will increafe no longer, and the globe muft afterward continue to defcend with that velocity uniformly. A body continually comes nearer and nearer to this greateft celerity, but can never attain accurately to it. Now, N and n being the fpecific gravities of the globe and fluid, $N - n$ will be the relative gravity of the globe in the fluid, and therefore $w = \frac{2}{3}pd^3(N-n)$ is the weight by which it is urged downward; alfo $m = \frac{pnd^2v^2}{8g}$ is the Refiftance, as above; therefore thefe two muft be equal when the velocity can be no farther increafed, or $m = w$, that is $\frac{pnd^2v^2}{8g} = \frac{2}{3}pd^3$ $(N-n)$, or $nv^2 = \frac{16}{3}dg\,(N-n)$; and hence $v = \sqrt{\frac{16}{3}dg \times \frac{N-n}{n}}$ is the faid uniform or greateft velocity to which the body may attain; which is evidently the greater in the fubduplicate proportion of v the diameter of the ball. But v is always $= \sqrt{4gfs}$, the velocity generated by any accelerative force f in defcribing the fpace s; which being compared with the former, it gives $s = \frac{4}{3}d$, when f is $= \frac{N-n}{n}$; that is, the greateft velocity is that which is generated by the accelerating force $\frac{N-n}{n}$ in paffing over the fpace $\frac{4}{3}d$ or $\frac{4}{3}$ of the diameter of the ball, or it is equal to the velocity generated by gravity in defcribing the fpace $\frac{N-n}{n} \times \frac{4}{3}d$. For ex. if the ball be of lead, which is about $11\frac{1}{4}$ times the denfity of water; then $N = 11\frac{1}{4}$, $n = 1$, $N - n = \frac{N-n}{n} = 10\frac{1}{4} = \frac{41}{4}$, and $\frac{N-n}{n} \times \frac{4}{3}d = \frac{41}{3}d = 13\frac{2}{3}d$; that is, the uniform or greateft velocity of a ball of lead, defcending in water, is equal to that which a heavy body acquired by falling in vacuo through a fpace equal to $13\frac{2}{3}$ of the diameter of the ball, which velocity is $v = 2\sqrt{\frac{4}{3}dg \times \frac{N-n}{n}} = 2\sqrt{13\frac{2}{3}dg} = 8\sqrt{13\frac{2}{3}d}$ nearly, or 8 times the root of the fame fpace.

Hence it appears, how foon fmall bodies come to their greateft or uniform velocity in defcending in a fluid, as water, and how very fmall that velocity is: which explains the reafon of the flow precipitation of mud, and fmall particles, in water, as alfo why, in precipitations, the larger and grofs particles defcend fooneft, and the loweft.

Farther, where $N = n$, or the denfity of the fluid is equal to that of the body, then $N - n = 0$, confequently the velocity and diftance defcended are each nothing, and the body will juft float in any part of the fluid.

Moreover, when the body is lighter than the fluid, then N is lefs than n, and $N - n$ becomes a negative quantity, or the force and motion tend the contrary way, that is, the ball will afcend up towards the top of the fluid by a motive force which is as $n - N$. In this cafe then, the body afcending by the action of the fluid, is moved exactly by the fame laws as a heavier body falling in the fluid. Wherever the body is placed, it is fuftained by the fluid, and carried up with a force equal to the difference of the weight of a quantity of the fluid of the fame bulk as the body, from the weight of the body; there is therefore a force which continually acts equably upon the body; by which not only the action of gravity of the body is counteracted, fo as that it is not to be confidered in this cafe; but the body is alfo carried upwards by a motion equably accelerated, in the fame manner as a body heavier than a fluid defcends by its relative gravity: but the equability of acceleration is deftroyed in the fame manner by the Refiftance, in the afcent of a body lighter than the fluid, as it is deftroyed in the defcent of a body that is heavier.

For the circumftances of the correfpondent velocity, fpace, and time, &c, of a body moving in a fluid in which it is projected with a given velocity, or defcending by its own weight, &c, fee my Select Exercifes, prop. 29, 30, 31, and 32, pag. 221 &c.

Resistance *of the Air*, in Pneumatics, is the force with which the motion of bodies, particularly of projectiles, is retarded by the oppofition of the air or atmofphere. See Gunnery, Projectiles, &c.

The air being a fluid, the general laws of the Refiftance of fluids obtain in it; fubject only to fome variations and irregularities from the different degrees of denfity in the different ftations or regions of the atmofphere.

The Refiftance of the air is chiefly of ufe in military projectiles, in order to allow for the differences caufed in their flight and range by it. Before the time of Mr. Robins, it was thought that this Refiftance to the motion of fuch heavy bodies as iron balls and fhells, was too inconfiderable to be regarded, and that the rules and conclufions derived from the common parabolic theory, were fufficiently exact for the common practice of gunnery. But that gentleman fhewed, in his New Principles of Gunnery, that, fo far from being inconfiderable, it is in reality enormoufly great, and by no means to be rejected without incurring the groffeft errors; fo much fo, that balls or fhells which range, at the moft, in the air, to the diftance of two or three miles, would in a vacuum range to 20 or 30 miles, or more. To determine the quantity of this Refiftance, in the cafe of different velocities, Mr. Robins difcharged mufket balls, with various degrees of known velocity, againft his ballistic pendulums, placed at feveral different diftances, and fo difcovered by experiment the quantity of velocity loft, when paffing through thofe diftances

or ſpaces of air, with the ſeveral known degrees of celerity. For having thus known, the velocity loſt or deſtroyed, in paſſing over a certain ſpace, in a certain time, (which time is very nearly equal to the quotient of the ſpace divided by the medium velocity between the greateſt and leaſt, or between the velocity at the mouth of the gun and that at the pendulum): that is, knowing the velocity v, the ſpace s, and time t, the reſiſting force is thence eaſily known, being equal to $\frac{vb}{2gt}$ or $\frac{vVb}{2gs}$, where b denotes the weight of the ball, and V the medium velocity above-mentioned. The balls employed upon this occaſion by Mr. Robins, were leaden ones, of $\frac{1}{12}$ of a pound weight, and $\frac{3}{4}$ of an inch diameter; and to the medium velocity of

1600 feet the Reſiſtance was 11 lb,
1065 feet - - - - - - - it was $2\frac{4}{5}$;

but by the theory of Newton, before laid down, the former of theſe ſhould be only $4\frac{1}{2}$ lb, and the latter 2 lb: ſo that, in the former caſe the real Reſiſtance is more than double of that by the theory, being increaſed as 9 to 22; and in the leſſer velocity the increaſe is from 2 to $2\frac{4}{5}$, or as 5 to 7 only.

Mr. Robins alſo invented another machine, having a whirling or circular motion, by which he meaſured the Reſiſtances to larger bodies, though with much ſmaller velocities: it is deſcribed, and a figure of it given, near the end of the 1ſt vol. of his works.

That this reſiſting power of the air to ſwift motions is very ſenſibly increaſed beyond what Newton's theory for ſlow motions makes it, ſeems hence to be evident. By other experiments it appears that the Reſiſtance is very ſenſibly increaſed, even in the velocity of 400 feet. However, this increaſed power of Reſiſtance diminiſhes as the velocity of the reſiſted body diminiſhes, till at length, when the motion is ſufficiently abated, the actual Reſiſtance coincides with that ſuppoſed in the theory nearly. For theſe varying Reſiſtances Mr. Robins has given a rule, extending to 1670 feet velocity.

Mr. Euler has ſhewn, that the common doctrine of Reſiſtance anſwers pretty well when the motion is not very ſwift, but in ſwift motions it gives the Reſiſtance leſs than it ought to be, on two accounts. 1. Becauſe in quick motions, the air does not fill up the ſpace behind the body faſt enough to preſs on the hinder parts, to counterbalance the weight of the atmoſphere on the fore part. 2. The denſity of the air before the ball being increaſed by the quick motion, will preſs more ſtrongly on the fore part, and ſo will reſiſt more than lighter air in its natural ſtate. He has ſhewn that Mr. Robins has reſtrained his rule to velocities not exceeding 1670 feet per ſecond; whereas had he extended it to greater velocities, the reſult muſt have been erroneous; and he gives another formula himſelf, and deduces concluſions differing from thoſe of Mr. Robins. See his Principles of Gunnery inveſtigated, tranſlated by Brown in 1777, pa. 224 &c.

Mr. Robins having proved that, in very great changes of velocity, the Reſiſtance does not accurately follow the duplicate ratio of the velocity, lays down two poſitions, which he thought might be of ſome ſervice in the practice of artillery, till a more complete and accurate theory of Reſiſtance, and the changes of its augmentation, may be obtained. The firſt of theſe is, that till the velocity of the projectile ſurpaſs 1100 or 1200 feet in a ſecond, the Reſiſtance may be eſteemed to be in the duplicate ratio of the velocity: and the ſecond is, that when the velocity exceeds 1100 or 1200 feet, then the abſolute quantity of the Reſiſtance will be near 3 times as great as it ſhould be by a compariſon with the ſmaller velocities. Upon theſe principles he proceeds in approximating to the actual ranges of pieces with ſmall angles of elevation, viz, ſuch as do not exceed 8° or 10°, which he ſets down in a table, compared with their correſponding potential ranges. See his Mathematical Tracts, vol. 1 pa. 179 &c. But we ſhall ſee preſently that theſe poſitions are both without foundation; that there is no ſuch thing as a ſudden or abrupt change in the law of Reſiſtance, from the ſquare of the velocity to one that gives a quantity three times as much; but that the change is ſlow and gradual, continually from the ſmalleſt to the higheſt velocities; and that the increaſed real Reſiſtance no where riſes higher than to about double of that which Newton's theory gives it.

Mr. Glenie, in his Hiſtory of Gunnery, 1776, pa. 49, obſerves, in conſequence of ſome experiments with a rifled piece, properly fitted for experimental purpoſes, that the Reſiſtance of the air to a velocity ſomewhat leſs than that mentioned in the firſt of the above propoſitions, is conſiderably greater than in the duplicate ratio of the velocity; and that, to a celerity ſomewhat greater than that ſtated in the ſecond, the Reſiſtance is conſiderably leſs than that which is treble the Reſiſtance in the ſaid ratio. Some of Robins's own experiments ſeem neceſſarily to make it ſo; ſince, to a velocity no quicker than 400 feet in a ſecond, he found the Reſiſtance to be ſomewhat greater than in that ratio. But the true value of the ratio, and other circumſtances of this Reſiſtance, will more fully appear from what follows.

The ſubject of the Reſiſtance of the air, as begun by Robins, has been proſecuted by myſelf, to a very great extent and variety, both with the whirling machine, and with cannon balls of all ſizes, from 1 lb to 6 lb weight, as well as with figures of many other different ſhapes, both on the fore part and hind part of them, and with planes ſet at all varieties of angles of inclination to the path or motion of the ſame; from all which I have obtained the real Reſiſtance to bodies for all velocities, from 1 up to 2000 feet per ſecond; together with the law of the Reſiſtance to the ſame body for all different velocities, and for different ſizes with the ſame velocity, and alſo for all angles of inclination; a full account of which would make a book of itſelf, and muſt be reſerved for ſome other occaſion. In the mean time, ſome general tables of concluſions may be taken as below.

TABLE I. *Resistances of different Bodies.*

Veloc. per Sec.	Small Hemif.	Large Hemif.		Cone		Cylinder	Whole globe	Resif. as the power of the veloc.
	flat side	flat side	round side	vertex	base			
feet	oz	oz	oz	oz	oz	oz	oz	
3	·028	·051	·020	·028	·064	·050	·027	
4	·048	·096	·039	·048	·109	·090	·047	
5	·072	·148	·063	·071	·162	·143	·068	
6	·103	·211	·092	·098	·225	·205	·094	
7	·141	·284	·123	·129	·298	·278	·125	
8	·184	·368	·160	·168	·382	·360	·162	
9	·233	·464	·199	·211	·478	·456	·205	
10	·287	·573	·242	·260	·587	·565	·255	
11	·349	·698	·292	·315	·712	·688	·310	2·052
12	·418	·836	·347	·376	·850	·826	·370	2·042
13	·492	·988	·409	·440	1·000	·979	·435	2·036
14	·573	1·154	·478	·512	1·166	1·145	·505	2·031
15	·661	1·336	·552	·589	1·346	1·327	·581	2·031
16	·754	1·538	·634	·673	1·546	1·526	·663	2·033
17	·853	1·757	·722	·762	1·763	1·745	·752	2·038
18	·959	1·998	·818	·858	2·002	1·986	·848	2·044
19	1·073	2·258	·922	·959	2·260	2·246	·949	2·047
20	1·196	2·542	1·033	1·069	2·540	2·528	1·057	2·051
Mean propor. Nos.	140	288	119	126	291	285	124	2·040
1	2	3	4	5	6	7	8	9

In this Table are contained the Resistances to several forms of bodies, when moved with several degrees of velocity, from 3 feet per second to 20. The names of the bodies are at the tops of the columns, as also which end went foremost through the air; the different velocities are in the first column, and the Resistances on the same line, in their several columns, in avoirdupois ounces and decimal parts. So on the first line are contained the Resistances when the bodies move with a velocity of 3 feet in a second, viz, in the 2d column for the small hemisphere, of $4\frac{3}{4}$ inches diameter, its Resistance ·028 oz when the flat side went foremost; in the 3d and 4th columns the Resistances to a larger hemisphere, first with the flat side, and next the round side foremost, the diameter of this, as well as all the following figures being $6\frac{3}{8}$ inches, and therefore the area of the great circle = 32 sq. inches, or $\frac{2}{9}$ of a sq. foot; then in the 5th and 6th columns are the Resistances to a cone, first its vertex and then its base foremost, the altitude of the cone being $6\frac{5}{8}$ inches, the same as the diameter of its base; in the 7th column the Resistance to the end of the cylinder, and in the 8th that against the whole globe or sphere. All the numbers shew the real weights which are equal to the Resistances; and at the bottoms of the columns are placed proportional numbers, which shew the mean proportions of the Resistances of all the figures to one another, with any velocity. Lastly, in the 9th column are placed the exponents of the power of the velocity which the Resistances in the 8th column bear to each other, viz, which that of the 10 feet velocity bears to each of the following ones, the medium of all of them being as the 2·04 power of the velocity, that is, very little above the square or second power of the velocity, so far as the velocities in this Table extend.

From this Table the following inferences are easily deduced.

1. That the Resistance is nearly in the same proportion as the surfaces; a small increase only taking place in the greater surfaces, and for the greater velocities. Thus, by comparing together the numbers in the 2d and 3d columns, for the bases of the two hemispheres, the areas of which bases are in the proportion of $17\frac{3}{4}$ to 32, or 5 to 9 very nearly, it appears that the numbers in those two columns, expressing the Resistances, are nearly as 1 to 2 or 5 to 10, as far as the velocity of 12 feet; but after that, the Resistances on the greater surface increase gradually more and more above that proportion.

2. The Resistance to the same surface, with different velocities, is, in these slow motions, nearly as the square of the velocity; but gradually increases more and more above that proportion as the velocity increases. This is manifest from all the columns; and the index of the power of the velocity is set down in the 9th column, for the Resistances in the 8th, the medium being 2·04; by which it appears that the Resistance to the same body is, in these slow motions, as the 2·04 power of the velocity, or nearly as the square of it.

3. The round ends, and sharp ends, of solids, suffer less Resistance than the flat or plane ends, of the same diameter; but the sharper end has not always the less Resistance. Thus, the cylinder, and the flat ends of the hemisphere and cone, have more Resistance, than the round or sharp ends of the same; but the round side of the hemisphere has less Resistance than the sharper end of the cone.

4. The Resistance on the base of the hemisphere, is to that on the round, or whole sphere, as $2\frac{1}{2}$ to 1, instead of 2 to 1, as the theory gives that relation. Also the experimented Resistance, on each of these, is nearly $\frac{1}{4}$ more than the quantity assigned by the theory.

5. The Resistance on the base of the cone, is to that on the vertex, nearly as $2\frac{3}{10}$ to 1; and in the same ratio is radius to the sine of the angle of inclination of the side of the cone to its path or axis. So that, in this instance, the Resistance is directly as the sine of the angle of incidence, the transverse section being the same.

6. When the hinder parts of bodies are of different forms, the Resistances are different, though the foreparts be exactly alike and equal; owing probably to the different pressures of the air on the hinder parts. Thus, the Resistance to the fore part of the cylinder, is less than on the equal flat surface of the cone, or of the hemisphere; because the hinder part of the cylinder is more pressed or pushed, by the following air than those of the other two figures; also, for the same reason, the base of the hemisphere suffers a less Resistance than that of the cone, and the round side of the hemisphere less than the whole sphere.

TABLE II. *Resistances both by Experiment and Theory, to a Globe of 1·965 Inches Diameter.*

Veloc. per sec. in feet.	Resist. by Exper. oz.	Resist. by Theory. oz.	Ratio of Exper. to Theory.	Resist. as the power of the veloc.
5	0·006	0·005	1·20	
10	0·024½	0·020	1·23	
15	0·055	0·044	1·25	
20	0·100	0·079	1·27	
25	0·157	0·123	1·28	2·022
30	0·23	0·177	1·30	2·055
40	0·42	0·314	1·33	2·068
50	0·67	0·491	1·36	2·075
100	2·72	1·964	1·38	2·059
200	11	7·9	1·40	2·041
300	25	18·7	1·41	2·039
400	45	31·4	1·43	2·039
500	72	49	1·47	2·044
600	107	71	1·51	2·051
700	151	96	1·57	2·059
800	205	126	1·63	2·067
900	271	159	1·70	2·077
1000	350	196	1·78	2·086
1100	442	238	1·86	2·095
1200	546	283	1·90	2·102
1300	661	332	1·99	2·107
1400	785	385	2·04	2·111
1500	916	442	2·07	2·113
1600	1051	503	2·09	2·113
1700	1186	568	2·08	2·111
1800	1319	636	2·07	2·108
1900	1447	709	2·04	2·104
2000	1569	786	2·00	2·098

In the first column of this Table are contained the several velocities, gradually from 0 up to the great velocity of 2000 feet per second, with which a ball or globe moved. In the 2d column are the experimented Resistances, in averdupois ounces. In the 3d column are the correspondent Resistances, as computed by the foregoing theory. In the 4th column are the ratios of these two Resistances, or the quotients of the former divided by the latter. And in the 5th or last, the indexes of the power of the velocity which is proportional to the experimented Resistance; which are found by comparing the Resistance of 20 feet velocity with each of the following ones.

From the 2d, 3d and 4th columns it appears, that at the beginning of the motion, the experimented Resistance is nearly equal to that computed by theory; but that, as the velocity increases, the experimented Resistance gradually exceeds the other more and more, till at the velocity of 1300 feet the former becomes just double the latter; after which the difference increases a little farther, till at the velocity of 1600 or 1700, where that excess is the greatest, and is rather less than $2\frac{1}{10}$; after this, the difference decreases gradually as the velocity increases, and at the velocity of 2000, the former Resistance again becomes just double the latter.

From the last column it appears that, near the beginning, or in slow motions, the Resistances are nearly as the square of the velocities; but that the ratio gradually increases, with some small variation, till at the velocity of 1500 or 1600 feet it becomes as the $2\frac{1}{9}$ power of the velocity nearly, which is its highest ascent; and after that it gradually decreases again, as the velocity goes higher. And similar conclusions have also been derived from experiments with larger balls or globes.

And hence we perceive that Mr. Robins's positions are erroneous on two accounts, viz, both in stating that the Resistance changes suddenly, or all at once, from being as the square of the velocity, so as then to become as some higher and constant power; and also when he states the Resistance as rising to the height of 3 times that which is given by the theory: since the ratio of the Resistance both increases gradually from the beginning, and yet never ascends higher than $2\frac{9}{100}$ of the theory.

TABLE III. *Resistance to a Plane, set at various Angles of Inclination to its Path.*

Angle with the Path.	Experim. Resistances. oz.	Resist. by this Formula. $\cdot 84 s^{1\cdot 842c}$	Sines of the Angles to Radius ·840.
0°	·000	·000	·000
5	·015	·009	·073
10	·044	·035	·146
15	·082	·076	·217
20	·133	·131	·287
25	·200	·199	·355
30	·278	·278	·420
35	·362	·363	·482
40	·448	·450	·540
45	·534	·535	·594
50	·619	·613	·643
55	·684	·680	·688
60	·729	·736	·727
65	·770	·778	·761
70	·803	·808	·789
75	·823	·826	·811
80	·835	·836	·827
85	·839	·839	·838
90	·840	·840	·840

In the 2d column of this Table are contained the actual experimented Resistances, in ounces, to a plane of 32 square inches, or $\frac{2}{9}$ of a square foot, moved through the air with a velocity of exactly 12 feet per second, when the plane was set so as to make, with the direction of its path, the corresponding angles in the first column.

And from these I have deduced this formula, or theorem, viz, $\cdot 84 s^{1\cdot 842c}$, which brings out very nearly the same numbers, and is a general theorem for every angle, for the same plane of $\frac{2}{9}$ of a foot, and moved with the same velocity of 12 feet in a second of time; where s is the sine, and c the cosine of the angles of inclination in the first column.

If

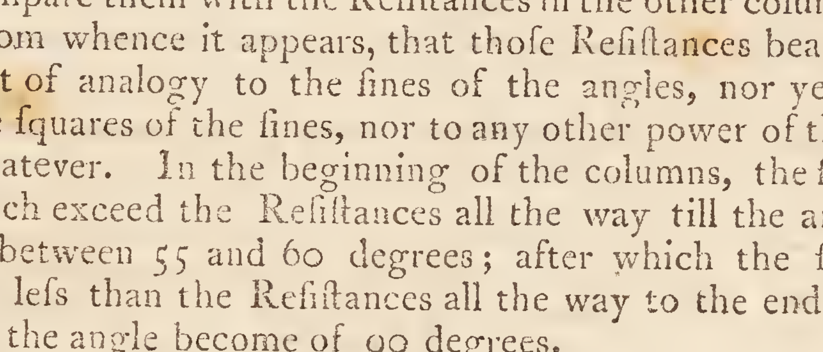
If a theorem be defired for any other velocity v, and any other plane whofe area is a, it will be this: $\frac{1}{38}av^2s^{1\cdot842c}$, or more nearly $\frac{1}{42}av^{2\cdot04}s^{1\cdot842c}$; which denotes the Refiftance nearly to any plane furface whofe area is a, moved through the air with the velocity v, in a direction making with that plane an angle, whofe fine is s, and cofine c.

If it be water or any other fluid, different from air, this formula will be varied in proportion to the denfity of it.

By this theorem were computed the numbers in the 3d column; which it is evident agree very nearly with the experiment Refiftances in the 2d column, excepting in two or three of the fmall numbers near the beginning, which are of the leaft confequence. In all other cafes, the theorem gives the true Refiftance very nearly. In the 4th or laft column are entered the fines of the angles of the firft column, to the radius ·84, in order to compare them with the Refiftances in the other columns. From whence it appears, that thofe Refiftances bear no fort of analogy to the fines of the angles, nor yet to the fquares of the fines, nor to any other power of them whatever. In the beginning of the columns, the fines much exceed the Refiftances all the way till the angle be between 55 and 60 degrees; after which the fines are lefs than the Refiftances all the way to the end, or till the angle become of 90 degrees.

Mr. James Bernoulli gave fome theorems for the Refiftances of different figures, in the Acta Erud. Lipf. for June 1693, pa. 252 &c. But as thefe are deduced from theory only, which we find to be fo different from experiment, they cannot be of much ufe. Meffieurs Euler, D'Alembert, Gravefande, and Simpfon, have alfo written pretty largely on the theory of Refiftances, befides what had been done by Newton.

Solid of Leaft Resistance. Sir Ifaac Newton, from his general theory of Refiftance, deduces the figure of a folid which fhall have the leaft Refiftance of the fame bafe, height and content.



The figure is this. Suppofe DNG to be a curve of fuch a nature, that if from any point N the ordinate NM be drawn perpendicular to the axis AB; and from a given point G there be drawn GR parallel to a tangent at N, and meeting the axis produced in R; then if MN be to GR, as GR^3 to $4BR \times BG^2$, a folid defcribed by the revolution of this figure about its axis AB, moving in a medium from A towards B, is lefs refifted than any other circular folid of the fame bafe, &c.

This theorem, which Newton gave without a demonftration, has been demonftrated by feveral mathematicians, as Facio, Bernoulli, Hofpital, &c. See Maclaurin's Flux. fect. 606 and 607; alfo Horfley's edit. of Newton, vol. 2, pag. 390. See alfo Act. Erud. 1699, pa. 514; and Mem. de l'Acad. &c; alfo Robins's View of Newton's method for comparing the Refiftance of Solids, 8vo, 1734; and Simpfon's Fluxions, art. 413; or my Principles of Bridges, prop. 11 and 12.

M. Bouguer has refolved this problem in a very general manner; not in fuppofing the folid to be formed by a revolution, of any figure whatever. The problem, as enunciated and refolved by M. Bouguer, is this: Any bafe being given, to find what kind of folid muft be formed upon it, fo that the impulfe upon it may be the leaft poffible. Properly however it ought to be the retardive force, or the impulfe divided by the weight or mafs of matter in the body, that ought to be the minimum.

RESOLUTION, in Phyfics, the reduction of a body into its original or natural ftate, by a diffolution or feparation of its aggregated parts. Thus, fnow and ice are faid to be refolved into water; water refolves in vapour by heat; and vapour is again refolved into water by cold; alfo any compound is refolved into its ingredients, &c.

Some of the modern philofophers, particularly Boyle, Mariotte, Boerhaave, &c, maintain, that the natural ftate of water is to be congealed, or in ice; in as much as a certain degree of heat, which is a foreign and violent agent, is required to make it fluid: fo that near the pole, where this foreign agent is wanting, it conftantly retains its fixed or icy ftate.

Resolution, or Solution, in Mathematics, is an orderly enumeration of feveral things to be done, to obtain what is required in a problem.

Wolfius makes a problem to confift of three parts: The *propofition* (or what is properly called the *problem*), the *Refolution*, and the *demonftration*.

As foon as a problem is demonftrated, it is converted into a theorem; of which the Refolution is the hypothefis; and the propofition the thefis.

For the procefs of a mathematical Refolution, fee the following article.

Resolution in *Algebra*, or *Algebraical* Resolution, is of two kinds; the one practifed in numerical problems, the other in geometrical ones.

In Refolving a Numerical Problem Algebraically, the method is this. Firft, the given quantities are diftinguifhed from thofe that are fought; and the former denoted by the initial letters of the alphabet, but the latter by the laft letters.—2. Then as many equations are formed as there are unknown quantities. If that cannot be done from the propofition or data, the problem is indeterminate; and certain arbitrary affumptions muft be made, to fupply the defect, and which can fatisfy the queftion. When the equations are not contained in the problem itfelf, they are to be found by particular theorems concerning equations, ratios, proportions, &c.—Since, in an equation, the known and unknown quantities are mixed together, they muft be feparated in fuch a manner, that the unknown one remain alone on one fide, and the known ones on the other. This reduction, or feparation, is made by addition, fubtraction, multiplication, divifion, extraction of

of roots, and raifing of powers; refolving every kind of combination of the quantities, by their counter or reverfe ones, and performing the fame operation on all the quantities or terms, on both fides of the equation, that the equality may ftill be preferved.

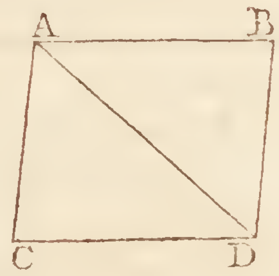
To Refolve a Geometrical Problem Algebraically.—The fame fort of operations are to be performed, as in the former article; befides feveral others, that depend upon the nature of the diagram, and geometrical properties. As 1ft, the thing required or propofed, muft be fuppofed done, the diagram being drawn or conftructed in all its parts, both known and unknown. 2. We muft then examine the geometrical relations which the lines of the figure have among themfelves, without regarding whether they are known or unknown, to find what equations arife from thofe relations, for finding the unknown quantities. 3. It is often neceffary to form fimilar triangles and rectangles, fometimes by producing of lines, or drawing parallels and perpendiculars, and forming equal angles, &c; till equations can be formed, from them, including both the known and unknown quantities.

If we do not thus arrive at proper equations, the thing is to be tried in fome other way. And fometimes the thing itfelf, that is required, is not to be fought directly, but fome other thing, bearing certain relations to it, by means of which it may be found.

The final equation being at laft arrived at, the geometrical conftruction is to be deduced from it, which is performed in various ways according to the different kinds of equations.

RESOLUTION *of Forces*, or *of Motion*, is the refolving or dividing of any one force or motion, into feveral others, in other directions, but which, taken together, fhall have the fame effect as the fingle one; and it is the reverfe of the compofition of forces or motions. See thefe articles.

Any fingle direct force AD, may be refolved into two oblique forces, whofe quantities and directions are AB, AC, having the fame effect, by defcribing any parallelogram ABDC, whofe diagonal is AD. And each of thefe may, in like manner, be refolved into two others; and fo on, as far as we pleafe. And all thefe new forces, or motions, fo found, when acting together, will produce exactly the fame effect as the fingle original one. See alfo COLLISION, PERCUSSION, MOTION, &c.



REST, in Phyfics, the continuance of a body in the fame place; or its continual application or contiguity to the fame parts of the ambient and contiguous bodies.—See SPACE.

Reft is either *abfolute* or *relative*, as place is.

Some define Reft to be the ftate of a thing without motion; and hence again Reft becomes either abfolute or relative, as motion is.

Newton defines true or abfolute Reft to be the continuance of a body in the fame part of abfolute and immoveable fpace; and relative Reft to be the continuance of a body in the fame part of relative fpace.

Thus, in a fhip under fail, relative Reft is the continuance of a body in the fame part of the fhip. But true or abfolute Reft is its continuance in the fame part of univerfal fpace in which the fhip itfelf is contained.

Hence, if the earth be really and abfolutely at Reft, the body relatively at Reft in the fhip will really and abfolutely move, and that with the fame velocity as the fhip itfelf. But if the earth do likewife move, there will then arife a real and abfolute motion of the body at Reft; partly from the real motion of the earth in abfolute fpace, and partly from the relative motion of the fhip on the fea. Laftly, if the body be likewife relatively moved in the fhip, its real motion will arife partly from the real motion of the earth in immoveable fpace, and partly from the relative motion of the fhip on the fea, and of the body in the fhip.

It is an axiom in philofophy, that matter is indifferent as to Reft or motion. Hence Newton lays it down, as a law of nature, that every body perfeveres in its ftate, either of Reft or uniform motion, except fo far as it is difturbed by external caufes.

The Cartefians affert, that firmnefs, hardnefs, or folidity of bodies, confifts in this, that their parts are at Reft with regard to each other; and this Reft they eftablifh as the great nexus, or principle of cohefion, by which the parts are connected together. On the other hand, they make fluidity to confift in a perpetual motion of the parts, &c. But the Newtonian philofophy furnifhes us with much better folutions.

Maupertuis afferts, that when bodies are in equilibrio, and any fmall motion is impreffed on them, the quantity of action refulting will be the leaft poffible. This he calls the law of *Reft*; and from this law he deduces the fundamental propofition of ftatics. See Berlin Mem. tom. 2, pa. 294. And from the fame principle too he deduces the laws of percuffion.

RESTITUTION, in Phyfics, the returning of elaftic bodies, forcibly bent, to their natural ftate; by fome called the *motion of Reftitution*.

RETARDATION, in Phyfics, the act of retarding, that is, of delaying the motion or progrefs of a body, or of diminifhing its velocity.

The Retardation of moving bodies arifes from two great caufes, the refiftance of the medium, and the force of gravity.

The RETARDATION *from the Refiftance* is often confounded with the refiftance itfelf; becaufe, with refpect to the fame moving body, they are in the fame proportion.

But with refpect to different bodies, the fame refiftance often generates different Retardations. For if bodies of equal bulk, but different denfities, be moved through the fame fluid with equal velocity, the fluid will act equally on each; fo that they will have equal refiftances, but different Retardations; and the Retardations will be to each other, as the velocities which might be generated by the fame forces in the bodies propofed; that is, they are inverfely as the quantities of matter in the bodies, or inverfely as the denfities.

Suppofe then bodies of equal denfity, but of unequal bulk, to move equally faft through the fame fluid; then their refiftances increafe according to their fuperficies, that is as the fquares of their diameters; but the quantities of matter are increafed according to their mafs or magnitude, that is as the cubes of their diameters: the refiftances are the quantities of motion; the

the Retardations are the celerities arifing from them; and dividing the quantities of motion by the quantities of matter, we fhall have the celerities; therefore the Retardations are directly as the fquares of the diameters, and inverfely as the cubes of the diameters, that is inverfely as the diameters themfelves.

If the bodies be of equal magnitude and denfity, and moved through different fluids, with equal celerity, their Retardations are as the denfities of the fluids. And when equal bodies are carried through the fame fluid with different velocities, the Retardations are as the fquares of the velocities.

So that, if s denote the fuperficies of a body, w its weight, d its diameter, v the velocity, and n the denfity of the fluid medium, and N that of the body; then, in fimilar bodies, the refiftance is as nsv^2 or as nd^2v^2, and the Retardation, or retarding force, as $\frac{nsv^2}{w}$, or as $\frac{nd^2v^2}{Nd^3} = \frac{nv^2}{Nd}$.

The RETARDATION *from Gravity* is peculiar to bodies projected upwards. A body thrown upwards is retarded after the fame manner as a falling body is accelerated; only in the one cafe the force of gravity confpires with the motion acquired, and in the other it acts contrary to it.

As the force of gravity is uniform, the Retardation from that caufe will be equal in equal times. Hence, as it is the fame force which generates motion in the falling body, and diminifhes it in the rifing one, a body rifes till it lofe all its motion; which it does in the fame time in which a body falling would have acquired a velocity equal to that with which the body was thrown up.

Alfo, a body thrown up, will rife to the fame height from which, in falling, it would acquire the fame velocity with which it was thrown up: therefore the heights which bodies can rife to, when thrown up with different velocities, are to each other as the fquares of the velocities.

Hence, the Retardations of motions may be compared together. For they are, firft, as the fquares of the velocities; 2dly, as the denfities of the fluids through which the bodies are moved; 3dly, inverfely as the diameters of thofe bodies; 4thly, inverfely as the denfities of the bodies themfelves; as expreffed by the theorem above, viz, $\frac{nv^2}{Nd}$.

The Laws of RETARDATION, are the very fame as thofe for acceleration; motion and velocity being deftroyed in the one cafe, in the very fame quantity and proportion as it is generated in the other.

RETICULA, or RETICULE, in Aftronomy, a contrivance for meafuring very nicely the quantity of eclipfes, &c.

This inftrument, introduced fome years fince by the Paris Acad. of Sciences, is a little frame, confifting of 13 fine filken threads, parallel to, and equidiftant from each other; placed in the focus of object-glaffes of telefcopes; that is, in the place where the image of the luminary is painted in its full extent. Confequently the diameter of the fun or moon is thus feen divided into 12 equal parts or digits: fo that, to find the quantity of the eclipfe, there is nothing to do but to number the parts that are dark, or that are luminous.

As a fquare Reticule is only proper for the diameter of the luminary, not for the circumference of it, it is fometimes made circular, by drawing 6 concentric equidiftant circles; which reprefents the phafes of the eclipfe perfectly.

But it is evident that the Reticule, whether fquare or circular, ought to be perfectly equal to the diameter or circumference of the fun or ftar, fuch as it appears in the focus of the glafs; otherwife the divifion cannot be juft. Now this is no eafy matter to effect, becaufe the apparent diameter of the fun and moon differs in each eclipfe; nay that of the moon differs from itfelf in the progrefs of the fame eclipfe.—Another imperfection in the Reticule is, that its magnitude is determined by that of the image in the focus; and of confequence it will only fit one certain magnitude.

But M. de la Hire has found a remedy for all thefe inconveniences, and contrived that the fame Reticule fhall ferve for all telefcopes, and all magnitudes of the luminary in the fame eclipfe. The principle upon which his invention is founded, is that two object-glaffes applied againft each other, having a common focus, and thefe forming an image of a certain magnitude, this image will increafe in proportion as the diftance between the two glaffes is increafed, as far as to a certain limit. If therefore a Reticule be taken of fuch a magnitude, as juft to comprehend the greateft diameter the fun or moon can ever have in the common focus of two object-glaffes applied to each other, there needs nothing but to remove them from each other, as the ftar comes to have a lefs diameter, to have the image ftill exactly comprehended in the fame Reticule.

Farther, as the filken threads are fubject to fwerve from the parallelifm, &c, by the different temperature of the air, another improvement is, to make the Reticule of a thin looking-glafs, by drawing lines or circles upon it with the fine point of a diamond. See MICROMETER.

RETIRED FLANK, in Fortification. See FLANK.

RETROCESSION *of Curves*, &c. See RETROGRADATION.

RETROCESSION *of the Equinox*. See PRECESSION.

RETROGRADATION, or RETROGRESSION, in Aftronomy, is an apparent motion of the planets, by which they feem to go backwards in the ecliptic, and to move contrary to the order or fucceffion of the figns.

When a planet moves in confequentia, or according to the order of the figns, as from Aries to Taurus, from Taurus to Gemini, &c, which is from weft to eaft, it is faid to be *direct*.—When it appears for fome days in the fame place, or point of the heavens, it is faid to be *ftationary*.—And when it goes in antecedentia, or backwards to the following figns, or contrary to the order of the figns, which is from eaft to weft, it is faid to be *retrograde*. All thefe different affections or circumftances, may happen in all the planets, except the fun and moon, which are feen to go direct only. But the times of the fuperior and inferior planets being retrograde are different; the former appearing fo about their oppofition, and the latter about their conjunction.

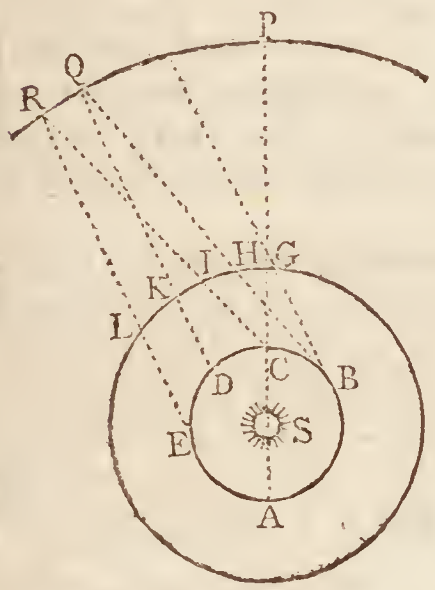
tion. The intervals of time alſo between two Retrogradations of the ſeveral planets, are very unequal;

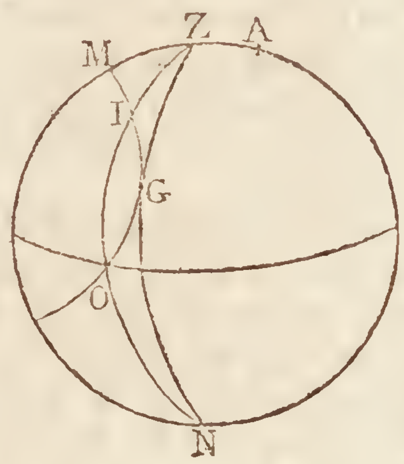
In Saturn	it is	1 year	13 days,
In Jupiter	- -	1 - -	43
In Mars	- -	2 - -	50
In Venus	- -	1 - -	220
In Mercury	-	0 - -	115

Again, Saturn continues retrograde 140 days, Jupiter 120, Mars 73, Venus 42, and Mercury 22; or nearly ſo; for the ſeveral Retrogradations of the ſame planet are not conſtantly equal.

Theſe various circumſtances however in the motions of the planets are not real, but only apparent; as the inequalities ariſe from the motion and poſition of the earth, from whence they are viewed; for when they are conſidered as ſeen from the ſun, their motions appear always uniform and regular. Theſe inequalities are thus explained:

Let S denote the ſun; and ABCD &c the path or orbit of the earth, moving from weſt to eaſt, and in that order; alſo GK &c the orbit of a ſuperior planet, as Saturn for inſtance, moving the ſame way, or in the direction GKLG, but with a much leſs celerity than





the earth's motion. Now when the earth is at the point A of its orbit, let Saturn be at G, in conjunction with the ſun, when it will be ſeen at P in the zodiac, or among the ſtars; and when the earth has moved from A to B, let Saturn have moved from G to H in its orbit, when it will be ſeen in the line BHQ, and will appear to have moved from P to Q in the zodiac; alſo when the earth has got to C, let Saturn be arrived at I, but found at R in the zodiac, where being ſeen in the line CIR, it appears ſtationary, or without motion in the zodiac at R. But after this, Saturn will appear for ſome time in Retrogradation, viz, moving backwards, or the contrary way: for when the earth has moved to D, Saturn will have got to K, and, being ſeen in the line DKQ, will appear to have moved retrograde in the zodiac from R to Q; about which place the planet, ceaſing to recede any farther, again becomes ſtationary, and afterward proceeds forward again; for while the earth moves from D to E, and Saturn from K to L, this latter, being now ſeen in the line ELR, appears to have moved forward in the zodiac from Q to R. And ſo on; the ſuperior planets always becoming retrograde a little before they are in oppoſition to the ſun, and continuing ſo till ſome time after the oppoſition: the retrograde motion being ſwifteſt when the planet is in the very oppoſition itſelf; and the direct motion ſwifteſt when in the conjunction. The arch RQ which the planet deſcribes while thus retrograde, is called the arch of Retrogradation. Theſe arches are unequal in all the planets, being greateſt in the moſt diſtant, and gradually leſs in the nearer ones.

In like manner may be ſhewn the circumſtances of the Retrogradations of the inferior planets; by which it will appear, they become ſtationary a little before their inferior conjunction, and go retrograde till a little time after it; moving the quickeſt retrograde juſt at that conjunction, and the quickeſt direct juſt at the ſuperior or further conjunction.

Retrogradation *of the Nodes of the Moon*, is a motion of the line of the nodes of her orbit, by which it continually ſhifts its ſituation from eaſt to weſt, contrary to the order of the ſigns, completing its retrograde circulation in the period of about 19 years: after which time, either of the nodes, having receded from any point of the ecliptic, returns to the ſame again.—Newton has demonſtrated, in his Principia, that the Retrogradation of the moon's nodes is cauſed by the action of the ſun, which continually drawing this planet from her orbit, deflects this orbit from a plane, and cauſes its interſection with the ecliptic continually to vary; and his determinations on this point have been confirmed by obſervation.

Retrogradation *of the Sun*, a motion by which in ſome ſituations, in the torrid zone, he ſeems to move retrograde or backwards.

When the ſun is in the torrid zone, and has his declination AM greater than the latitude of the place AZ, but either northern or ſouthern as that is (laſt fig. above), the ſun will appear to go retrograde, or backwards, both before and after noon. For draw the vertical circle ZGN to be a tangent to the ſun's diurnal circle MGO in G, and another ZON through the ſun's riſing, at O: then it is evident, that all the intermediate vertical circles cut the ſun's diurnal circle twice: firſt in the arc GO, and the ſecond time in the arc GI. So that, as the ſun aſcends through the arc GO, he continually arrives at farther and farther verticals. But as he continues his aſcent through the arc GI, he returns to his former verticals; and therefore is ſeen retrograde for ſome time before noon. And in like manner it may be ſhewn that he does the ſame thing for ſome time after noon. Hence, as the ſhadow always tends oppoſite to the ſun, the ſhadow will be retrograde twice every day in all places of the torrid zone, where the ſun's declination exceeds the latitude.

But the ſame thing can never happen without the tropics, in a natural way.

Retrogradation, or Retrogression, in the Higher Geometry, is the ſame with what is otherwiſe called *contrary flexion* or *flexure*. See Flexure, and Inflexion.

RETROGRADE, denotes backward, or contrary to the forward or natural direction. See Retrogradation.

RETROGRESSION, or Retrocession. The fame with Retrogradation.

RETURNING *Stroke*, in Electricity, is an expreffion ufed by lord Mahon (now earl Stanhope) to denote the effect produced by the return of the electric fire into a body from which, in certain circumftances, it has been expelled.

To underftand properly the meaning of thefe terms, it muft be premifed that, according to the noble author's experiments, an infulated fmooth body, immerged within the electrical atmofphere, but beyond the ftriking diftance of another body, charged pofitively, is at the fame time in a ftate of threefold electricity. The end next to the charged body acquires negative electricity; the farther end is pofitively electrified; while a certain part of the body, fomewhere between its two extremes, is in a natural, unelectrified, or neutral ftate; fo that the two contrary electricities balance each other. It may farther be added, that if the body be not infulated, but have a communication with the earth, the whole of it will be in a negative ftate. Suppofe then a brafs ball, which may be called A, to be conftantly placed at the ftriking diftance of a prime conductor; fo that the conductor, the inftant when it becomes fully charged, explodes into it. Let another large or fecond conductor be fufpended, in a perfectly infulated ftate, farther from the prime conductor than the ftriking diftance, but within its electrical atmofphere: let a perfon ftanding on an infulated ftool touch this fecond conductor very lightly with a finger of his right hand; while, with a finger of his left hand, he communicates with the earth, by touching very lightly a fecond brafs ball fixed at the top of a metallic ftand, on the floor, which may be called B. Now while the prime conductor is receiving its electricity, fparks pafs (at leaft if the diftance between the two conductors is not too great) from the fecond conductor to the right hand of the infulated perfon; while fimilar and fimultaneous fparks pafs out from the finger of his left hand into the fecond metallic ball B, communicating with the earth. At length however the prime conductor, having acquired its full charge, fuddenly ftrikes into the ball A, of the firft metallic ftand, placed for that purpofe at the ftriking diftance. The explofion being made, and the prime conductor fuddenly robbed of its elaftic atmofphere, its preffure or action on the fecond conductor, and on the infulated perfon, as fuddenly ceafes; and the latter inftantly feels a fmart Returning Stroke, though he has no direct or vifible communication (except by the floor) with either of the two bodies, and is placed at the diftance of 5 or 6 feet from both of them. This Returning Stroke is evidently occafioned by the fudden re-entrance of the electric fire naturally belonging to his body and to the fecond conductor, which had before been expelled from them by the action of the charged prime conductor upon them; and which returns to its former place in the inftant when that action or elaftic preffure ceafes. When the fecond conductor and the infulated perfon are placed in the denfeft part of the electrical atmofphere of the prime conductor, or juft beyond the ftriking diftance, the effects are ftill more confiderable; the Returning Stroke being extremely fevere and pungent, and appearing confiderably fharper than even the main ftroke itfelf, received directly from the prime conductor. Lord Mahon obferves, that perfons and animals may be deftroyed, and particular parts of buildings may be much damaged, by an electrical Returning Stroke, occafioned even by fome very diftant explofion from a thunder cloud; poffibly at the diftance of a mile or more. It is certainly not difficult to conceive that a charged extenfive thunder cloud muft be productive of effects fimilar to thofe produced by the prime conductor; but perhaps the effects are not fo great, nor the danger fo terrible, as it feems have been apprehended. If the quantity of electric fluid naturally contained, for example, in the body of a man, were immenfe or indefinite, then the eftimate between the effects producible by a cloud, and thofe caufed by a prime conductor, might be admitted; but furely no electrical cloud can expel from a body more than the natural quantity of electricity which this contains. On the fudden removal therefore of the preffure by which this natural quantity had been expelled, in confequence of the explofion of the cloud into the earth, no more (at the utmoft) than his whole natural ftock of electricity can re-enter his body, provided it be fo fituated, that the returning fire of other bodies muft neceffarily pafs through his body. But perhaps we have no reafon to fuppofe that this quantity is fo great, as that its fudden re-entrance into his body fhould deftroy or injure him.

Allowing therefore the exiftence of the Returning Stroke, as fufficiently afcertained, and well illuftrated, in a variety of circumftances, by the author's experiments, the magnitude and danger of it do not feem to be fo alarming as he apprehends. See Lord Mahon's Principles of Electricity, &c. 4to. 1779, pa. 76, 113, and 131. Alfo Monthly Review, vol. 62, pa. 436.

REVERSION *of Series*, in Algebra, is the finding the value of the root, or unknown quantity, whofe powers enter the terms of an infinite feries, by means of another infinite feries in which it is not contained. As, in the infinite feries $z = ax + bx^2 + cx^3 + dx^4$ &c; then if there be found $x = Az + Bz^2 + Cz^3$ &c, that feries is inverted, or its root x is found in an infinite feries of other terms.

This was one of Newton's improvements in analyfis, the firft fpecimen of which was given in his Analyfis per Æquationes Numero Terminorum Infinitas; and it is of great ufe in refolving many problems in various parts of the mathematics.

The moft ufual and general way of Reverfion, is to affume a feries, of a proper form, for the value of the required unknown quantity; then fubftitute the powers of this value, inftead of thofe of that quantity into the given feries; laftly compare the refulting terms with the faid given feries, and the values of the affumed coefficients will thus be obtained. So, to revert the feries $z = ax + bx^2 + cx^3$, &c, or to find the value of x in terms of z; affume it thus, $x = Az + Bz^2 + Cz^3$ &c;

&c; then by involving this feries, for the feveral powers of x, and multiplying the correfponding powers by a, b, c, &c, there refults

$$\begin{aligned} z = aAz &+ aBz^2 + aCz^3 + aDz^4, \text{ \&c.} \\ &+ bA^2z^2 + 2bABz^3 + 2bACz^4 \\ &\qquad\qquad\qquad\quad + bB^2z^4 \\ &\qquad\quad + cA^3z^3 + 3cA^2Bz^4 \\ &\qquad\qquad\qquad\quad + dA^4z^4 \end{aligned}$$

Then by comparing the correfponding terms of this laft feries, or making their coefficients equal, there are obtained thefe equations, viz,

$aA = 1$, and $aB + bA^2 = 0$, and $aC + 2bAB + cA^3 = 0$, &c, which give thefe values of the affumed coefficients, viz,

$$A = \frac{1}{a};\ B = -\frac{bA^2}{a} = -\frac{b}{a^3};$$

$$C = -\frac{2bAB + cA^3}{a} = \frac{2bb - a}{a^5}c;$$

$$D = -\frac{2bAC + bB^2 + 3cA^2B + dA^4}{a}$$

$$= -\frac{5abc - 5b^3 - a^2d}{a^7};\ \text{\&c.}$$

and confequently

$$x = \frac{1}{a}z - \frac{b}{a^3}z^2 + \frac{2bb - ac}{a^5}z^3 - \frac{5abc - 5b^3 - a^2d}{a^7}z^4$$

&c; which is therefore a general formula or theorem for every feries of the fame kind, as to the powers of the quantity x. Thus, for

Ex. Suppofe it were required to revert the feries $z = x - x^2 + x^3 - x^4$, &c.

Here $a = 1$, $b = -1$, $c = 1$, $d = -1$, &c; which values of thefe letters being fubftituted in the theorem, there refults $x = z + z^2 + z^3 + z^4$, &c, which is that feries reverted, or the value of x in it.

In the fame way it will be found that the theorem for reverting the feries

$z = ax + bx^3 + cx^5 + dx^7$ &c, is

$$x = \frac{1}{a}z - \frac{b}{a^4}z^3 + \frac{3bb - ac}{a^7}z^5 - \frac{a^2d + 12b^3 - 8abc}{a^{10}},$$

&c.

And if $z = ax^{m} + bx^{m+n} + cx^{m+2n} +$ &c, then is

$$x = y^{\frac{1}{m}} - \frac{b}{ma}y^{\frac{1+n}{m}} + \frac{(1 + 2n + m)\,bb - 2mac}{2mmaa}$$

$\times\, y^{\frac{1+2n}{m}}$ &c. where y is $= \frac{z}{a}$.

Various methods of Reverfion may be feen as given by De Moivre in the Philof. Tranf. number 240; or Maclaurin's Algebra pa. 263; or Stuart's Explanation of Newton's Analyfis, &c. pa. 455; or Coulfon's Comment on Newton's Flux. pa. 219; or Horfley's ed. of Newton's works vol. 1, pa. 291; or Simpfon's Flux. vol. 2, pa. 302: or moft authors on Algebra.

REVETEMENT, in Fortification, a ftrong wall built on the outfide of the rampart and parapet, to fupport the earth, and prevent its rolling into the ditch.

REVOLUTION, in Geometry, the motion of rotation of a line about a fixed point or centre, or of any figure about a fixed axis, or upon any line or furface. Thus, the Revolution of a given line about a fixed centre, generates a circle; and that of a right-angled triangle about one fide, as an axis, generates a cone; and that of a femicircle about its diameter, generates a fphere or globe, &c.

REVOLUTION, in Aftronomy, is the period of a ftar, planet, or comet, &c; or its courfe from any point of its orbit, till it return to the fame again.

The planets have a twofold Revolution. The one about their own axis, ufually called their *diurnal rotation*, which conftitutes their day. The other about the fun, called their *annual Revolution*, or *period*, conftituting their year.

REYNEAU (CHARLES-RENE), commonly called Father Reyneau, a noted French mathematician, was born at Briffac in the province of Anjou, in the year 1656. At 20 years of age he entered himfelf among the Oratorians, a kind of religious order, in which the members lived in community without making any vows, and applied themfelves chiefly to the education of youth. He was foon after fent, by his fuperiors, to teach philofophy at Pezenas, and then at Toulon. This requiring fome acquaintance with geometry, he contracted a great affection for this fcience, which he cultivated and improved to a great extent; in confequence he was called to Angers in 1683, to fill the mathematical chair; and the Academy of Angers elected him a member in 1694.

In this occupation Father Reyneau, not content with making himfelf mafter of every thing worth knowing, which the modern analyfis, fo fruitful in fublime fpeculations and ingenious difcoveries, had already produced, undertook to reduce into one body, for the ufe of his fcholars, the principal theories fcattered here and there in Newton, Defcartes, Leibnitz, Bernoulli, the Leipfic Acts, the Memoirs of the Paris Academy, and in other works; treafures which by being fo widely difperfed, proved much lefs ufeful than they otherwife might have been. The fruit of this undertaking, was his *Analyfe Demontrée*, or Analyfis Demonftrated, which he publifhed in 2 volumes 4to, 1708.

Father Reyneau called this ufeful work, Analyfis Demonftrated, becaufe he demonftrates in it feveral methods which had not been demonftrated by the authors of them, or at leaft not with fufficient perfpicuity and exactnefs; for it often happens that, in matters of this kind, a perfon is clear in a thing, without being able to demonftrate it. Some perfons too have been fo miftakingly fond of glory as to make a fecret of their demonftrations, in order to perplex thofe, whom it would become them much better to inftruct. This book of Reyneau's was fo well approved, that it foon became a maxim, at leaft in France, that to follow him was the beft, if not the only way, to make any extra-

 ordinary

ordinary progrefs in the mathematics. This was confidering him as the firft mafter, as the Euclid of the fublime geometry.

Reyneau, after thus giving leffons to thofe who underftood fomething of geometry, thought proper to draw up fome for fuch as were utterly unacquainted with that fcience. This was in fome meafure a condefcenfion in him, but his paffion to be ufeful made it eafy and agreeable. In 1714 he publifhed a volume in 4to on calculation, under the title of *Science du Calcul des Grandeurs*, of which the then Cenfor Royal, a moft intelligent and impartial judge, fays, in his approbation of it, that "though feveral books had already appeared upon the fame fubject, fuch a treatife as that before him was ftill wanting, as in it every thing was handled in a manner fufficiently extenfive, and at the fame time with all poffible exactnefs and perfpicuity." In fact, though moft branches of the mathematics had been well treated of before that period, there were yet no good elements, even of practical geometry. Thofe who knew no more than what precifely fuch a book ought to contain, knew too little to complete a good one; and thofe who knew more, thought themfelves probably above the tafk; whereas Reyneau poffeffed at once all the learning and modefty neceffary to undertake and execute fuch a work.

As foon as the Royal Academy of Sciences at Paris, in confequence of a regulation made in the year 1716, opened its doors to other learned men, under the title of *Free Affociates*, Father Reyneau was admitted of the number. The works however which we have already mentioned, befides a fmall piece upon *Logic*, are the only ones he ever publifhed, or probably ever compofed, except moft of the materials for a fecond volume of his *Science du Calcul*, which he left behind him in manufcript. The laft years of his life were attended with too much ficknefs to admit of any extraordinary application. He died in 1728, at 72 years of age, not more regretted on account of his great learning, than of his many virtues, which all confpired in an eminent degree to make that learning agreeable to thofe about him, and ufeful to the world. The firft men in France deemed it an honour and a happinefs to count him among their friends. Of this number were the chancellor of that kingdom, and Father Mallebranche, of whom Reyneau was a zealous and faithful difciple.

RHABDOLOGY, or Rabdology, in Arithmetic, a name given by Napier to a method of performing fome of the more difficult operations of numbers by means of certain fquare little rods. Upon thefe are infcribed the fimple numbers; then by fhifting them according to certain rules, thofe operations are performed by fimply adding or fubtracting of the numbers as they ftand upon the rods. See Napier's Rabdologia, printed in 1617. See alfo the article Napier's *Bones*.

RHEO-STATICS, is ufed by fome for the ftatics, or the fcience of the equilibrium of fluids.

RHETICUS (George Joachim), a noted German aftronomer and mathematician, who was the colleague of Reinhold in the univerfity of Wittemberg, being joint profeffors of mathematics there together. He was born at Feldkirk in Tyrol the 15th of February 1514. After imbibing the elements of the mathematics at Tiguri with Ofwald Mycone, he went to Wittemberg, where he diligently cultivated that fcience. Here he was made mafter of philofophy in 1535, and profeffor in 1537. He quitted this fituation however two years after, and went to Fruenburg to put him under the affiftance of the celebrated Copernicus, being induced to this ftep by his zeal for aftronomical purfuits, and the great fame which Copernicus had then acquired. Rheticus affifted this aftronomer for fome years, and conftantly exhorted him to perfect his work, *De Revolutionibus*, which he publifhed after the death of Copernicus, viz, in 1543, folio, at Norimberg, together with an illuftration of the fame in a narration, dedicated to Schoner. Here too, to render aftronomical calculations more accurate, he began his very elaborate canon of fines, tangents and fecants, to 15 places of figures, and to every 10 feconds of the quadrant, a defign which he did not live quite to complete. The canon of fines however to that radius, for every 10 feconds, and for every fingle fecond in the firft and laft degree of the quadrant, computed by him, was publifhed in folio at Francfort 1613 by Pitifcus, who himfelf added a few of the firft fines computed to 22 places of figures. But the larger work, or canon of fines, tangents and fecants, to every 10 feconds, was perfected and publifhed after his death, viz, in 1596, by his difciple Valentine Otho, mathematician to the Electoral Prince Palatine; a particular account and analyfis of which work may be feen in the Hiftorical Introduction to my Logarithms, pa. 9.

After the death of Copernicus, Rheticus returned to Wittemberg, viz, in 1541 or 1542, and was again admitted to his office of profeffor of mathematics. The fame year, by the recommendation of Melancthon, he went to Norimberg, where he found certain manufcripts of Werner and Regiomontanus. He afterwards taught mathematics at Leipfic. From Saxony he departed a fecond time, for what reafon is not known, and went to Poland; and from thence to Caffovia in Hungary, where he died December the 4th, 1576, near 63 years of age.

His *Narratio de Libris Revolutionum Copernici*, was firft publifhed at Gedunum in 4to, 1540; and afterwards added to the editions of Copernicus's work. He alfo compofed and publifhed *Ephemerides*, according to the doctrine of Copernicus, till the year 1551.

Rheticus alfo projected other works, and partly executed them, though they were never publifhed, of various kinds, aftronomical, aftrological, geographical, chemical, &c; as they are more particularly mentioned in his letter to Peter Ramus in the year 1568, which Adrian Romanus inferted in the preface to the firft part of his Idea of Mathematics.

RHOMB Solid, confifts of two equal and right cones joined together at their bafes.

RHOMBOID, or Rhomboides, in Geometry, a quadrilateral figure, whofe oppofite fides and angles are equal; but which is neither equilateral nor equiangular.

RHOMBUS, is an oblique equilateral parallelogram; or

or a quadrilateral figure, whofe fides are equal and parallel, but the four angles not all equal, two of the oppofite ones being obtufe, and the other two oppofite ones acute.

The two diagonals of a Rhombus interfect at right angles; but not of a rhomboides.

As to the area of the Rhombus or rhomboides, it is found, like that of all other parallelograms, by multiplying the length or bafe by the perpendicular breadth.

RHOMBUS-*Solid.* See RHOMB-*Solid.*

RHUMB, RUMB, or RUM, in Navigation, a vertical circle of any given place; or the interfection of a part of fuch a circle with the horizon. Rhumbs therefore coincide with points of the world, or of the horizon. And hence mariners diftinguifh the Rhumbs by the fame names as the points and winds. But we may obferve, that the Rhumbs are denominated from the points of the compafs in a different manner from the winds: thus, at fea, the north-eaft wind is that which blows from the north-eaft point of the horizon towards the fhip in which we are; but we are faid to fail upon the north-eaft Rhumb, when we go towards the north-eaft.

They ufually reckon 32 Rhumbs, which are reprefented by the 32 lines in the rofe or card of the compafs.

Aubin defines a Rhumb to be a line on the terreftrial globe, or fea-compafs, or fea-chart, reprefenting one of the 32 winds which ferve to conduct a veffel. So that the Rhumb a veffel purfues is conceived as its route, or courfe.

Rhumbs are divided and fubdivided like points of the compafs. Thus, the whole Rhumb anfwers to the cardinal point. The half Rhumb to a collateral point, or makes an angle of 45 degrees with the former. And the quarter Rhumb makes an angle of 22° 30′ with it. Alfo the half-quarter Rhumb makes an angle of 11° 15′ with the fame.

For a table of the Rhumbs, or points, and their diftances from the meridian, fee WIND.

RHUMB-LINE, *Loxodromia,* in Navigation, is a line prolonged from any point of the compafs in a nautical chart, except the four cardinal points: or it is the line which a fhip, keeping in the fame collateral point, or rhumb, defcribes throughout its whole courfe.

The chief property of the Rhumb-line, or loxodromia, and that from which fome authors define it, is, that it cuts all the meridians in the fame angle.

This angle is called the *angle of the Rhumb,* or the *loxodromic* angle. And the angle which the Rhumb-line makes with any parallel to the equator, is called the *complement of the Rhumb.*

An idea of the origin and properties of the Rhumb-line, the great foundation of Navigation, may be conceived thus: a veffel beginning its courfe, the wind by which it is driven makes a certain angle with the meridian of the place; and as we fhall fuppofe that the veffel runs exactly in the direction of the wind, it makes the fame angle with the meridian which the wind makes. Suppofing then the wind to continue the fame, as each point or inftant of the progrefs may be efteemed the beginning, the veffel always makes the fame angle with the meridian of the place where it is each moment, or in each point of its courfe which the wind makes.

Now a wind, for example, that is north-eaft, and which confequently makes an angle of 45 degrees with the meridian, is equally north-eaft wherever it blows, and makes the fame angle of 45 degrees with all the meridians it meets. And therefore a veffel, driven by the fame wind, always makes the fame angle with all the meridians it meets with on the furface of the earth.

If the veffel fail north or fouth, it defcribes the great circle of a meridian. If it runs eaft or weft, it cuts all the meridians at right angles, and defcribes either the circle of the equator, or elfe a circle parallel to it.

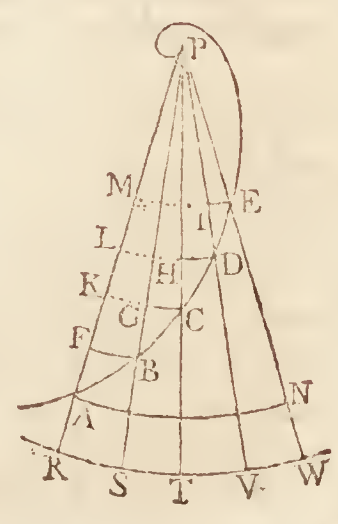
But if the veffel fails between the two, it does not then defcribe a circle; fince a circle, drawn obliquely to a meridian, would cut all the meridians at unequal angles, which the veffel cannot do. It defcribes therefore another curve, the effential property of which is, that it cuts all the meridians in the fame angle, and it is called the *loxodromy,* or *loxodromic curve,* or *Rhumb-line.*

This curve, on the globe, is a kind of fpiral, tending continually nearer and nearer to the pole, and making an infinite number of circumvolutions about it, but without ever arriving exactly at it. But the fpiral Rhumbs on the globe become proportional fpirals in the ftereographic projection on the plane of the equator.

The length of a part of this Rhumb-line, or fpiral, then, is the diftance run by the fhip while fhe keeps in the fame courfe. But as fuch a fpiral line would prove very perplexing in the calculation, it was neceffary to have the fhip's way in a right line; which right line however muft have the effential properties of the curve line, viz, to cut all the meridians at right angles. The method of effecting which, fee under the article CHART.

The arc of the Rhumb-line is not the fhorteft diftance between any two places through which it paffes; for the fhorteft diftance, on the furface of the globe, is an arc of the great circle paffing through thofe places; fo that it would be a fhorter courfe to fail on the arc of this great circle: but then the fhip cannot be kept in the great circle, becaufe the angle it makes with the meridians is continually varying, more or lefs.

Let P be the pole, RW the equator, ABCDEP a fpiral Rhumb, divided into an indefinite number of equal parts at the points B, C, D, &c; through which are drawn the meridians, PS, PT, PV, &c, and the parallels FB, KC, LD, &c, alfo draw the parallel AN. Then, as a fhip fails along the Rhumb-line towards the pole, or in the direction ABCD &c, from A to E, the diftance failed AE is



is made up of all the small equal parts of the Rhumb $AB + BC + CD + DE$; and

the sum of all the small differences of latitude $AF + BG + CH + DI$ make up the whole difference of latitude AM or EN; and

the sum of all the small parallels $FB + GC + HD + IE$ is what is called the departure in plane sailing; and ME is the meridional distance, or distance between the first and last meridians, measured on the last parallel; also RW is the difference of longitude, measured on the equator. So that these last three are all different, viz, the departure, the meridional distance, and the difference of longitude.

If the ship sail towards the equator, from E to A; the departure, difference of latitude, and difference of longitude, will be all three the same as before; but the meridional distance will now be AN, instead of ME; the one of these AN being greater than the departure $FB + GC + HD + IE$, and the other ME is less than the same; and indeed that departure is nearly a mean proportional between the two meridional distances ME, AN. Other properties are as below.

1. All the small elementary triangles ABF, BCG, CDH, &c, are mutually similar and equal in all their parts. For all the angles at A, B, C, D, &c are equal, being the angles which the Rhumb makes with the meridians, or the angles of the course; also all the angles F, G, H, I, are equal, being right angles; therefore the third angles are equal, and the triangles all similar. Also the hypotenuses AB, BC, CD, &c, are all equal by the hypothesis; and consequently the triangles are both similar and equal.

2. As radius : distance run AE
:: sine of course $\angle A$: departure $FB + GC$ &c,
:: cosin. of course $\angle A$: dif. of lat. AM.

For in any one ABF of the equal elementary triangles, which may be considered as small right-angled plane triangles, it is, as rad. or sin. $\angle F$: sin. course A :: AB : FB :: (by composition) the sum of all the distances $AB + BC + CD$ &c : the sum of all the departures $FB + GC + HD$ &c.

And, in like manner, as radius : cos. course A :: AB : AF :: $AB + BC$ &c : $AF + BG$ &c.

Hence, of these four things, the course, the difference of latitude, the departure, and the distance run, having any two given, the other two are found by the proportions above in this article.

By means of the departure, the length of the Rhumb, or distance run, may be connected with the longitude and latitude, by the following two theorems.

3. As radius : half the sum of the cosines of both the latitudes, of A and E :: dif. of long. RW : departure.

Because RS : FB :: radius : sine of PA or cos. RA,
and VW : IE :: radius : sine of PE or cos. EW.

4. As radius : cos. middle latitude :: dif. of longitude : departure.—Because cosine of middle latitude is nearly equal to half the sum of the cosines of the two extreme latitudes.

RICCIOLI (Joannes Baptista), a learned Italian astronomer, philosopher, and mathematician, was born in 1598, at Ferrara, a city in Italy, in the dominions of the Pope. At 16 years of age he was admitted into the society of the Jesuits. He was endowed with uncommon talents, which he cultivated with extraordinary application; so that the progress he made in every branch of literature and science was surprising. He was first appointed to teach rhetoric, poetry, philosophy, and scholastic divinity, in the Jesuits' colleges at Parma and Bologna; yet applied himself in the mean time to making observations in geography, chronology, and astronomy. This was his natural bent, and at length he obtained leave from his superiors to quit all other employment, that he might devote himself entirely to those sciences.

He projected a large work, to be divided into three parts, and to contain as it were a complete system of philosophical, mathematical, and astronomical knowledge. The first of these parts, which regards astronomy, came out at Bologna in 1651, 2 vols. folio, with this title, *J. B. Riccioli Almagestum Novum, Astronomiam veterem novamque complectens, observationibus aliorum et propriis, novisque theorematibus, problematibus ac tabulis promotam*. Riccioli imitated Ptolomy in this work, by collecting and digesting into proper order, with observations, every thing ancient and modern, which related to his subject; so that Gassendus very justly called his work, "Promptuarium et thesaurum ingentem Astronomiæ."

In the first volume of this work, he treats of the sphere of the world, of the sun and moon, with their eclipses; of the fixed stars, of the planets, of the comets and new stars, of the several mundane systems, and six sections of general problems serving to astronomy, &c. —In the second volume, he treats of trigonometry, or the doctrine of plane and spherical triangles; proposes to give a treatise of astronomical instruments, and the optical part of astronomy (which part was never published); treats of geography, hydrography, with an epitome of chronology.—The third, comprehends observations of the sun, moon, eclipses, fixed stars and planets, with precepts and tables of the primary and secondary motions, and other astronomical tables.

Riccioli printed also, two other works, in folio, at Bologna, viz,

2. *Astronomia Reformata*, 1665: the design of which was, that of considering the various hypotheses of several astronomers, and the difficulty thence arising of concluding any thing certain, by comparing together all the best observations, and examining what is most certain in them, thence to reform the principles of astronomy.

3. *Chronologia Reformata*, 1669.

Riccioli died in 1671, at 73 years of age.

RICOCHET Firing, in the Military Art, is a method of firing with small charges, and pieces elevated but in a small degree, as from 3 to 6 degrees. The word signifies duck-and-drake, or rebounding, because the ball or shot, thus discharged, goes bounding and rolling along, and killing or destroying every thing in its way, like the bounding of a flat stone along the surface of water when thrown almost horizontally.

RIDEAU,

RIDEAU, in Fortification, a ſmall elevation of earth, extending itſelf lengthways on a plain; ſerving to cover a camp, or give an advantage to a poſt.

Rideau is ſometimes alſo uſed for a trench, the earth of which is thrown up on its ſide, to ſerve as a parapet for covering the men.

RIFLE Guns, in the Military Art, are thoſe whoſe barrels, inſtead of being ſmooth on the inſide, are formed with a number of ſpiral channels, making each about a turn and a half in the length of the barrel. Theſe carry their balls both farther and truer than the common pieces. For the nature and qualities of them, ſee Robins's Tracts, vol. 1 pa. 328 &c.

RIGEL, in Aſtronomy. See Regel.

RIGHT, in Geometry, ſomething that lies evenly or equally, without inclining or bending one way or another. Thus, a Right-line is that whoſe ſmall parts all tend the ſame way. In this ſenſe, Right means the ſame as ſtraight, as oppoſed to curved or crooked.

Right-*Angle*, that which one line makes with another upon which it ſtands ſo as to incline neither to one ſide nor the other. And in this ſenſe the word Right ſtands oppoſed to oblique.

Right-*angled*, is ſaid of a figure when its ſides are at Right angles or perpendicular to each other.—This ſometimes holds in all the angles of the figure, as in ſquares and rectangles; ſometimes only in part, as in right-angled triangles.

Right *Cone*, or *Cylinder*, or priſm, or pyramid, one whoſe axis is at right-angles to the baſe.

Right-*lined Angle*, one formed by Right lines.

Right *Sine*, one that ſtands at Right-angles to the diameter; as oppoſed to verſed ſine.

Right *Sphere*, is that where the equator cuts the horizon at Right angles; or that which has the poles in the horizon, and the equinoctial in the zenith.

Such is the poſition of the ſphere with regard to thoſe who live at the equator, or under the equinoctial. The conſequences of which are; that they have no latitude, nor elevation of the pole; they ſee both poles of the world, and all the ſtars riſe, culminate and ſet; alſo the ſun always riſes and deſcends at Right angles, and makes their days and nights equal. In a Right ſphere, the horizon is a meridian; and if the ſphere be ſuppoſed to revolve, all the meridians ſucceſſively become horizons, one after another.

Right Aſcenſion, Deſcenſion, Parallax, &c. See the reſpective Articles.

Right *Circle*, in the Stereographic Projection of the Sphere, is a circle at Right angles to the plane of projection, or that is projected into a Right line.

Right *Sailing*, is that in which a voyage is performed on ſome one of the four cardinal points, eaſt, weſt, north, or ſouth.

If the ſhip ſail on a meridian, that is, north or ſouth, ſhe does not alter her longitude, but only changes the latitude, and that juſt as much as the number of degrees ſhe has run.

But if ſhe ſail on the equator, directly eaſt or weſt, ſhe varies not her latitude, but only changes the longitude, and that juſt as much as the number of degrees ſhe has run.

And if ſhe ſail directly eaſt or weſt upon any parallel, ſhe again does not change her latitude, but only the longitude; yet not the ſame as the number of degrees of a great circle ſhe hath ſailed, as on the equator, but more, according as the parallel is remoter from the equinoctial towards the pole. For the leſs any parallel is, the greater is the difference of longitude anſwering to the diſtance run.

RIGIDITY, a brittle hardneſs; or that kind of hardneſs which is ſuppoſed to ariſe from the mutual indentation of the component particles within one another. Rigidity is oppoſed to ductility, malleability, &c.

RING, in Aſtronomy and Navigation, an inſtrument uſed for taking the ſun's altitude &c. It is uſually of braſs, about 9 inches diameter, ſuſpended by a little ſwivel, at the diſtance of 45° from the point of which is a perforation, which is the centre of a quadrant of 90° divided in the inner concave ſurface.

To uſe it, let it be held up by the ſwivel, and turned round to the ſun, till his rays, falling through the hole, make a ſpot among the degrees, which marks the altitude required.

This inſtrument is preferred before the aſtrolabe, becauſe the diviſions are here larger than on that inſtrument.

Ring, of Saturn, is a thin, broad, opaque circular arch, encompaſſing the body of that planet, like the wooden horizon of an artificial globe, without touching it, and appearing double, when ſeen through a good teleſcope.

This Ring was firſt diſcovered by Huygens, who, after frequent obſervation of the planet, perceived two lucid points, like anſæ or handles, ariſing out from the body in a right line. Hence as in ſubſequent obſervations he always found the ſame appearance, he concluded that Saturn was encompaſſed with a permanent Ring; and accordingly produced his New Syſtem of Saturn, in 1659. However, Galileo firſt diſcovered that the figure of Saturn was not round.

Huygens makes the ſpace between the globe of Saturn and the Ring equal to the breadth of the Ring, or rather more, being about 22000 miles broad; and the greateſt diameter of the Ring, in proportion to that of the globe, as 9 to 4. But Mr. Pound, by an excellent micrometer applied to the Huygenian glaſs of 123 feet, determined this proportion, more exactly, to be as 7 to 3.

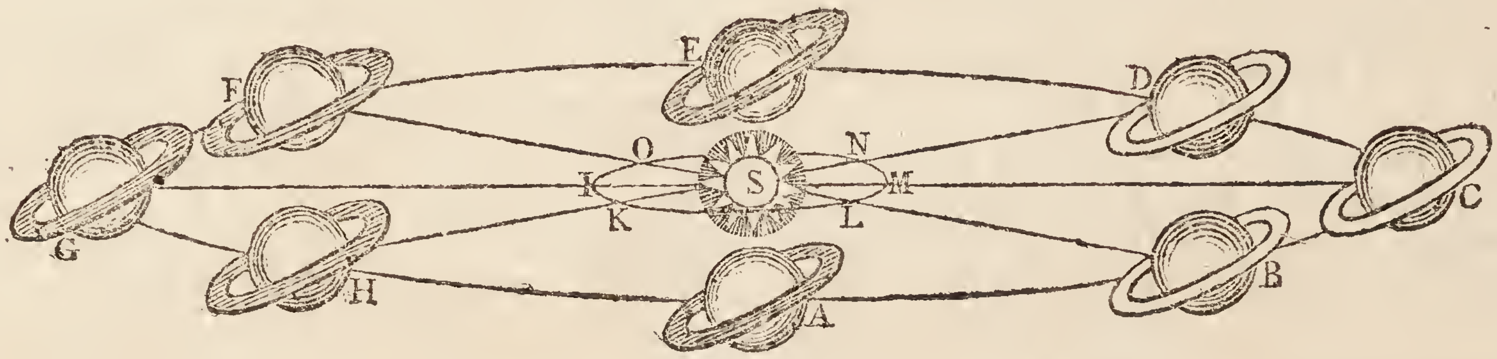
Obſervations have alſo determined, that the plane of the Ring is inclined to the plane of the ecliptic in an angle of 30 degrees; that the Ring probably turns, in the direction of its plane, round its axis, becauſe when it is almoſt edgewiſe to us, it appears rather thicker on one ſide of the planet than on the other; and the thickeſt edge has been ſeen on different ſides at different times: the ſun ſhines almoſt 15 of our years together on one ſide of Saturn's Ring without ſetting, and as long on the other in its turn; ſo that the Ring is viſible to the inhabitants of that planet for almoſt 15

of

of our years, and as long inviſible, by turns, if its axis has no inclination to its Ring; but if the axis of the planet be inclined to the Ring, ex. gr. about 30 degrees, the Ring will appear and diſappear once every natural day to all the inhabitants within 30 degrees of the equator, on both ſides, frequently eclipſing the ſun in a Saturnian day. Moreover, if Saturn's axis be ſo inclined to his Ring, it is perpendicular to his orbit; by which the inconvenience of different ſeaſons to that planet is avoided.

This Ring, ſeen from Saturn, appears like a large luminous arch in the heavens, as if it did not belong to the planet.

When we ſee the Ring moſt open, its ſhadow upon the planet is broadeſt; and from that time the ſhadow grows narrower, as the Ring appears to do to us; until, by Saturn's annual motion, the ſun comes to the plane of the Ring, or even with its edge; which, being then directed towards us, becomes inviſible, on account of its thinneſs.



The phenomena of Saturn's Ring are illuſtrated by a view of this figure. Let S be the ſun, ABCDEFGH Saturn's orbit, and IKLMNO the earth's orbit. Both Saturn and the earth move according to the order of the letters; and when Saturn is at A, his Ring is turned edgewiſe to the ſun S, and he is then ſeen from the earth as if he had loſt his Ring, let the earth be in any part of its orbit whatever, except between N and O; for whilſt it deſcribes that ſpace, Saturn is apparently ſo near the ſun as to be hid in his beams. As Saturn goes from A to C, his Ring appears more and more open to the earth; at C the Ring appears moſt open of all; and ſeems to grow narrower and narrower as Saturn goes from C to E; and when he comes to E, the Ring is again turned edgewiſe both to the ſun and earth; and as neither of its ſides is illuminated, it is inviſible to us, becauſe its edge is too thin to be perceptible; and Saturn appears again as if he had loſt his Ring. But as he goes from E to G, his Ring opens more and more to our view on the under ſide; and ſeems juſt as open at G as it was at C, and may be ſeen in the night time from the earth in any part of its orbit, except about M, when the ſun hides the planet from our view.

As Saturn goes from G to A, his Ring turns more and more edgewiſe to us, and, therefore, it ſeems to grow narrower and narrower; and at A it diſappears as before.

Hence, while Saturn goes from A to E, the ſun ſhines on the upper ſide of his Ring, and the under ſide is dark; and whilſt he goes from E to A, the ſun ſhines on the under ſide of his Ring, and the upper ſide is dark. The Ring diſappears twice in every annual revolution of Saturn, viz. when he is in the 19th degree of Piſces and of Virgo, and when Saturn is in the middle between theſe points, or in the 19th degree either of Gemini or of Sagittarius, his Ring appears moſt open to us; and then its longeſt diameter is to its ſhorteſt, as 9 to 4. Ferguſon's Aſtr. ſect. 204.

There are various hypotheſes concerning this Ring. Kepler, in his Epitom. Aſtron. Copern. and after him Dr. Halley, in his Enquiry into the Cauſes of the Variation of the Needle, Phil. Tranſ. No 195, ſuppoſe our earth may be compoſed of ſeveral cruſts or ſhells, one within another, and concentric to each other. If this be the caſe, it is poſſible the Ring of Saturn may be the fragment or remaining ruin of his formerly exterior ſhell, the reſt of which is broken or fallen down upon the body of the planet. And ſome have ſuppoſed that the Ring may be a congeries or ſeries of moons revolving about the planet.

Later obſervations have thrown much more light upon this curious phenomenon, eſpecially reſpecting its dimenſions, and rotation, and diviſion into two or more parts. De la Lande and De la Place ſay, that Caſſini ſaw the breadth of the Ring divided into two ſeparate parts that are equal, or nearly ſo. Mr. Short aſſured M. De la Lande, that he had ſeen many diviſions upon the Ring, with his 12 feet teleſcope. And Mr. Hadley, with an excellent $5\frac{1}{2}$ feet reflector, ſaw the Ring divided into two parts. Several excellent theories have been given in the French Memoirs, particularly by De la Place, contending for the diviſion of the Ring into many parts. But finally the obſervations of Dr. Herſchel, in ſeveral volumes of the Philoſ. Tranſ. ſeem to confirm the diviſion into two concentric parts only. The dimenſions of theſe two Rings, and the ſpace between them, he ſtates in the following proportion to each other.

	Miles.
Inner diam. of ſmaller Ring	146345
Outſide diam. of ditto	184393
Inner diam. of larger Ring	190248
Outſide diam. of ditto	204883
Breadth of the inner Ring	20000
Breadth of the outer Ring	7200
Breadth of the vacant ſpace	2839

Ring revolves in its own plane, in $10^h\ 32'\ 15''\cdot4$.

So that the outſide diameter of the larger Ring is almoſt 26 times the diameter of the earth.

Dr. Herſchel adds, Some theories and obſervations, of

of other perſons, "lead us to conſider the queſtion, whether the conſtruction of this Ring is of a nature ſo as permanently to remain in its preſent ſtate? or whether it be liable to continual and frequent changes, in ſuch a manner as in the courſe of not many years, to be ſeen ſubdivided into narrow ſlips, and then again as united into one or two circular planes only. Now, without entering into a diſcuſſion, the mind ſeems to revolt, even at firſt ſight, againſt an idea of the chaotic ſtate in which ſo large a maſs as the Ring of Saturn muſt needs be, if phenomena like theſe can be admitted. Nor ought we to indulge a ſuſpicion of this being a reality, unleſs repeated and well-confirmed obſervations had proved, beyond a doubt, that this Ring was actually in ſo fluctuating a condition." But from his own obſervations he concludes, "It does not appear to me that there is a ſufficient ground for admitting the Ring of Saturn to be of a very changeable nature, and I gueſs that its phenomena will hereafter be ſo fully explained, as to reconcile all obſervations. In the mean while, we muſt withhold a final judgment of its conſtruction, till we can have more obſervations. Its diviſion however into two very unequal parts, can admit of no doubt." See Philoſ. Tranſ. vol. 80 pa. 4, 481 &c, and the vol. for 1792, pa. 1 &c. alſo Hiſt. de l'Acad. des Scienc. de Paris, 1787, pa. 249 &c.

RINGS *of Colours*, in Optics, a phenomenon firſt obſerved in thin plates of various ſubſtances, by Boyle, and Hook, but afterwards more fully explained by Newton.

Mr. Boyle having exhibited a variety of colours in colourleſs liquors, by ſhaking them till they roſe in bubbles, as well as in bubbles of ſoap and water, and alſo in turpentine, procured glaſs blown ſo thin as to exhibit ſimilar colours; and he obſerves, that a feather of a proper ſhape and ſize, and alſo a black ribband, held at a proper diſtance between his eye and the ſun, ſhewed a variety of little rainbows, as he calls them, with very vivid colours. Boyle's Works by Shaw, vol. 2, p. 70. Dr. Hook, about nine years after the publication of Mr. Boyle's Treatiſe on Colours, exhibited the coloured bubbles of ſoap and water, and obſerved, that though at firſt it appeared white and clear, yet as the film of water became thinner, there appeared upon it all the colours of the rainbow. He alſo deſcribed the beautiful colours that appear in thin plates of Muſcovy glaſs; which appeared, through the microſcope, to be ranged in Rings ſurrounding the white ſpecks or flaws in them, and with the ſame order of colours as thoſe of the rainbow, and which were often repeated ten times. He alſo took two thin pieces of glaſs, ground plane and poliſhed, and putting them one upon another, preſſed them till there began to appear a red coloured ſpot in the middle; and preſſing them cloſer, he obſerved ſeveral Rings of colours encompaſſing the firſt place, till, at laſt, all the colours diſappeared out of the middle of the circles, and the central ſpot appeared white. The firſt colour that appeared was red, then yellow, then green, then blue, then purple; then again red, yellow, green, blue, and purple; and again in the ſame order; ſo that he ſometimes counted nine or ten of theſe circles, the red immediately next to the purple; and the laſt colour that appeared before the white was blue; ſo that it began with red, and ended with purple. Theſe Rings, he ſays, would change their places, by changing the poſition of the eye, ſo that, the glaſſes remaining the ſame, that part which was red in one poſition of the eye, was blue in a ſecond, green in the third, &c. Birch's Hiſt. of the Royal Society, vol. 3, pa. 54.

Newton, having demonſtrated that every different colour conſiſts of rays which have a different and ſpecific degree of refrangibility, and that natural bodies appear of this or that colour, according to their diſpoſition to reflect this or that ſpecies of rays (ſee COLOUR), purſued the hint ſuggeſted by the experiments of Dr. Hook, already recited, and caſually noticed by himſelf, with regard to thin tranſparent ſubſtances. Upon compreſſing two priſms hard together, in order to make their ſides touch one another, he obſerved, that in the place of contact they were perfectly tranſparent, which appeared like a dark ſpot, and when it was looked through, it ſeemed like a hole in that air, which was formed into a thin plate, by being impreſſed between the glaſſes. When this plate of air, by turning the priſms about their common axis, became ſo little inclined to the incident rays, that ſome of them began to be tranſmitted, there aroſe in it many ſlender arcs of colours, which increaſed, as the motion of the priſms was continued, and bended more and more about the tranſparent ſpot, till they were completed into circles, or Rings, ſurrounding it; and afterwards they became continually more and more contracted.

By another experiment, with two object glaſſes, he was enabled to obſerve diſtinctly the order and quality of the colours from the central ſpot, to a very conſiderable diſtance. Next to the pellucid central ſpot, made by the contact of the glaſſes, ſucceeded blue, white, yellow, and red. The next circuit immediately ſurrounding theſe, conſiſted of violet, blue, green, yellow, and red. The third circle of colours was purple, blue, green, yellow, and red. The fourth circle conſiſted of green and red. All the ſucceeding colours became more and more imperfect and dilute, till, after three or four revolutions, they ended in perfect whiteneſs.

When theſe Rings were examined in a darkened room, by the coloured light of a priſm caſt on a ſheet of white paper, they became more diſtinct, and viſible to a far greater number than in the open air. He ſometimes ſaw more than twenty of them, whereas in the open air he could not diſcern above eight or nine.

From other curious obſervations on theſe Rings, made by different kinds of light thrown upon them, he inferred, that the thickneſſes of the air between the glaſſes, where the Rings are ſucceſſively made, by the limits of the ſeven colours, red, orange, yellow, green, blue, indigo, and violet, in order, are one to another as the cube roots of the ſquares of the eight lengths of a chord, which ſound the notes in an octave, ſol, la, fa, ſol, la, mi, fa, ſol; that is, as the cube roots of the ſquares of the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$. Theſe Rings appeared of that priſmatic colour, with which they were illuminated, and by projecting the priſmatic colours immediately upon the glaſſes, he found that the light, which fell on the dark ſpaces between

the coloured Rings, was tranſmitted through the glaſſes without any change of colour. From this circumſtance he thought that the origin of theſe Rings is manifeſt; becauſe the air between the glaſſes is diſpoſed according to its various thickneſs, in ſome places to reflect, and in others to tranſmit the light of any particular colour, and in the ſame place to reflect that of one colour, where it tranſmits that of another.

In examining the phenomena of colours made by a denſer medium ſurrounded by a rarer, ſuch as thoſe which appear in plates of Muſcovy glaſs, bubbles of ſoap and water, &c, the colours were found to be much more vivid than the others, which were made with a rarer medium ſurrounded by a denſer.

From the preceding phenomena it is an obvious deduction, that the tranſparent parts of bodies, according to their ſeveral ſeries, reflect rays of one colour and tranſmit thoſe of another; on the ſame account that thin plates, or bubbles, reflect or tranſmit thoſe rays, and this Newton ſuppoſed to be the reaſon of all their colours. Hence alſo he has inferred, that the ſize of thoſe component parts of natural bodies that affect the light, may be conjectured by their colours. See Colour and Reflection.

Newton, purſuing his diſcoveries concerning the colours of thin ſubſtances, found that the ſame were alſo produced by plates of a conſiderable thickneſs, diviſible into leſſer thickneſſes. The Rings formed in both caſes have the ſame origin, with this difference, that thoſe of the thin plates are made by the alternate reflexions and tranſmiſſions of the rays at the ſecond ſurface of the plate, after one paſſage through it; but that, in the caſe of a glaſs ſpeculum, concave on one ſide, and convex on the other, and quickſilvered over on the convex ſide, the rays go through the plate and return before they are alternately reflected and tranſmitted. Newton's Optics, p. 169, &c. or Newton's Opera, Horſley's edit. vol. 4, p. 121, &c. p. 184, &c.

The abbé Mazeas, in his experiments on the Rings of colours that appear in thin plates, has diſcovered ſeveral important circumſtances attending them, which were overlooked by the ſagacious Newton, and which tend to invalidate his theory for explaining them. In rubbing the flat ſide of an object-glaſs againſt another piece of flat and ſmooth glaſs, he found that they adhered very firmly together after this friction, and that the ſame colours were exhibited between theſe plane glaſſes, which Newton had obſerved between the convex object glaſs of a teleſcope, and another that was plane; and that the colours were in proportion to their adheſion. When the ſurfaces of pieces of glaſs, that are tranſparent and well poliſhed, are equally preſſed, a reſiſtance will be perceived; and wherever this is felt, two or three very fine curve lines will be diſcovered, ſome of a pale red, and others of a faint green. If the friction be continued, the red and green lines increaſe in number at the place of contact; the colours being ſometimes mixed without any order, and ſometimes diſpoſed in a regular manner; in which caſe the coloured lines are generally concentric circles, or ovals, more or leſs elongated, as the ſurfaces are more or leſs united.

When the colours are formed, the glaſſes adhere with conſiderable force; but if the glaſſes be ſeparated ſuddenly, the colours will appear immediately upon their being put together, without the leaſt friction. Beginning with the ſlighteſt touch, and increaſing the preſſure by inſenſible degrees, there firſt appears an oval plate of a faint red, and in the centre of it a ſpot of light green, which enlarges by the preſſure, and becomes a green oval, with a red ſpot in the centre; and this enlarging in its turn, diſcovers a green ſpot in its centre. Thus the red and green ſucceed one another in turns, aſſuming different ſhades, and having other colours mixed with them. The greateſt difference between theſe colours exhibited between plane ſurfaces, and thoſe by curve ones, is, that, in the former caſe, preſſure alone will not produce them, except in the caſe above mentioned.

In rubbing together two priſms, with very ſmall refracting angles, which were joined ſo as to form a parallelopiped, the colours appeared with a ſurpriſing luſtre at the places of contact, and differently coloured ovals appeared.

In the centre there was a black ſpot, bordered by a deep purple; next to this appeared violet, blue, orange, red tinged with purple, light green, and faint purple.

The other Rings appeared to the naked eye to conſiſt of nothing but faint reds and greens. When theſe coloured glaſſes were ſuſpended over the flame of a candle, the colours diſappeared ſuddenly, though they ſtill adhered; but being ſuffered to cool, the colours returned to their former places, in the ſame order as before. At firſt the abbé Mazeas had no doubt but that theſe colours were owing to a thin plate of air between the glaſſes, to which Newton has aſcribed them; but the remarkable difference in the circumſtances attending thoſe produced by the flat plates and thoſe produced by the object glaſſes of Newton, convinced him that the air was not the cauſe of this appearance. The colours of the flat plates vaniſhed at the approach of flame, but thoſe of the object glaſſes did not. Nor was this difference owing to the plane glaſſes being leſs compreſſed than the convex ones; for though the former were compreſſed ever ſo much by a pair of forceps, it did not in the leaſt hinder the effect of the flame. Afterwards he put both the plane glaſſes and the convex ones into the receiver of an air-pump, ſuſpending the former by a thread, and keeping the latter compreſſed by two ſtrings; but he obſerved no change in the colours of either of them, in the moſt perfect vacuum that he could make. Suſpecting ſtill that the air adhered to the ſurface of the glaſſes, ſo as not to be ſeparated from them by the force of the pump, he had recourſe to other experiments, which rendered it ſtill more improbable that the air ſhould be the cauſe of theſe colours. Having laid the coloured plates, after warming them gradually, on burning coals; and thus, when they were nearly red, rubbing them together, he obſerved the ſame coloured circles and ovals as before. When he ceaſed to preſs upon them, the colours ſeemed to vaniſh; but they returned, as he renewed the friction. In order to determine whether the colours were owing to the thickneſs of ſome matter interpoſed between the glaſſes, he rubbed them together

ther with fuet and other foft fubftances between them; yet his endeavour to produce the colours had no effect. However by continuing the friction with fome degree of violence, he obferved, that a candle appeared through them encompaffed with two or three concentric greens, and with a lively red inclining to yellow, and a green like that of an emerald, and at length the Rings affumed the colours of blue, yellow, and violet. The abbé was confirmed in his opinion that there muft be fome error in Newton's hypothefis, by confidering that, according to his meafures, the colours of the plates varied with the difference of a millionth part of an inch; whereas he was fatiffied that there muft have been much greater differences in the diftance between his glaffes, when the colours remained unchanged. From other experiments he concluded, that the plate of water introduced between the glaffes was not the caufe of their colours, as Newton apprehended; and that the coloured Rings could not be owing to the compreffion of the glaffes. After all, he adds, that the theory of light, thus reflected from thin plates, is too delicate a fubject to be completely afcertained by a fmall number of obfervations. Berlin Mem. for 1752, or Memoires Prefentés, vol. 2, pa. 28—43. M. du Tour repeated the experiments of the abbé Mazeas, and added fome obfervations of his own. See Mem. Pref. vol. 4, pa. 288.

Muffchenbroeck is alfo of opinion, that the colours of thin plates do not depend upon the air; but as to the caufe of them, he acknowledges that he could not fatisfy himfelf about it. Introd. ad Phil. Nat. vol. 2, p. 738.

See on this fubject Prieftley's Hift. of Light, &c. per. 6, fect. 5, pa. 498, &c.

For an account of the Rings of colours produced by electrical explofions, fee *Colours* of *natural bodies*, CIRCULAR *fpots*, and FAIRY *circles*.

RISING, in Aftronomy, the appearance of the fun, or a ftar, or other luminary, above the horizon, which before was hid beneath it.

By reafon of the refraction of the atmofphere, the heavenly bodies always appear to rife before their time; that is, they are feen above the horizon, while they are really below it, by about 33′ of a degree.

There are three poetical kinds of Rifing of the ftars. See ACRONICAL, COSMICAL, and HELIACAL.

RIVER, in Geography, a ftream or current of frefh water, flowing in a bed or channel, from a fource or fpring, into the fea.

When the ftream is not large enough to bear boats, or fmall veffels, loaden, it is properly called by the diminutive, *rivulet* or *brook*; but when it is confiderable enough to carry larger veffels, it is called by the general name River.

Rivulets have their rife fometimes from great rains, or great quantities of thawed fnow, efpecially in mountainous places; but they more ufually arife from fprings.

Rivers themfelves all arife either from the confluence of feveral rivulets, or from lakes.

RIVER, in Phyfics, denotes a ftream of water running by its own gravity, from the more elevated parts of the earth towards the lower parts, in a natural bed or channel open above.

When the channel is artificial, or cut by art, it is called a canal; of which there are two kinds, viz, that whofe channel is every where open, without fluices, called an artificial River, and that whofe water is kept up and let off by means of fluices, which is properly a *canal*.

Modern philofophers endeavour to reduce the motion and flux of Rivers to precife laws; and with this view they have applied geometry and mechanics to this fubject; fo that the doctrine of Rivers is become a part of the new philofophy.

The authors who have moft diftinguifhed themfelves in this branch, are the Italians, the French, and the Dutch, but efpecially the firft, and among them more efpecially Gulielmini, and Ximenes.

Rivers, fays Gulielmini, ufually have their fources in mountains or elevated grounds; in the defcent from which it is moftly that they acquire the velocity, or acceleration, which maintains their future current. In proportion as they advance farther, this velocity diminifhes, on account of the continual friction of the water againft the bottom and fides of the channel; as well as from the various obftacles they meet with in their progrefs, and from their arriving at length in plains where the defcent is lefs, and confequently their inclination to the horizon greater. Thus the Reno, a River in Italy, which he fays gave occafion, in fome meafure, to his fpeculations, is found to have near its mouth a declivity of fcarce 52 feconds.

When the acquired velocity is quite fpent, through the many obftacles, fo that the current becomes horizontal, there will then nothing remain to propagate the motion, and continue the ftream, but the depth, or the perpendicular preffure of the water, which is always proportional to the depth. And, happily for us, this refource increafes, as the occafion for it increafes; for in proportion as the water lofes of the velocity acquired by the defcent, it rifes and increafes in its depth.

It appears from the laws of motion pertaining to bodies moved on inclined planes, that when water flows freely upon an inclined bed, it acquires a velocity, which is always as the fquare root of the quantity of defcent of the bed. But in an horizontal bed, opened by fluices or otherwife, at one or both ends, the water flows out by its gravity alone.

The upper parts of the water of a River, and thofe at a diftance from the banks, may continue to flow, from the fimple caufe or principle of declivity, how fmall foever it be; for not being detained by any obftacle, the minuteft difference of level will have its effect; but the lower parts, which roll along the bottom, will fcarce be fenfible of fo fmall a declivity; and will only have what motion they receive from the preffure of the fuperincumbent waters.

The greateft velocity of a River is about the middle of its depth and breadth, or that point which is the fartheft poffible from the furface of the water, and from the bottom and fides of the bed or channel. Whereas, on the contrary, the leaft velocity of the water is at the bottom and fides of the bed, becaufe there the refiftance arifing from friction is the greateft, which is communicated to the other parts of the fection of the

River inverſely as the diſtances from the bottom and ſides.

To find whether the water of a River, almoſt horizontal, flows by means of the velocity acquired in its deſcent, or by the preſſure of its depth; ſet up an obſtacle perpendicular to it; then if the water riſe and ſwell immediately againſt the obſtacle, it runs by virtue of its fall; but if it firſt ſtop a little while, in virtue of its preſſure.

Rivers, according to this author, almoſt always make their own beds. If the bottom have originally been a large declivity, the water, hence falling with a great force, will have ſwept away the moſt elevated parts of the ſoil, and carrying them lower down, will gradually render the bottom more nearly horizontal.

The water having made its bed horizontal, becomes ſo itſelf, and conſequently rakes with the leſs force againſt the bottom, till at length that force becomes only equal to the reſiſtance of the bottom, which is now arrived at a ſtate of permanency, at leaſt for a conſiderable time; and the longer according to the quality of the ſoil, clay and chalk reſiſting longer than ſand or mud.

On the other hand, the water is continually wearing away the brims of its channel, and this with the more force, as, by the direction of its ſtream, it impinges more directly againſt them. By this means it has a continual tendency to render them parallel to its own courſe. At the ſame time that it has thus rectified its edges, it has widened its own bed, and thence becoming leſs deep, it loſes part of its force and preſſure: this it continues to do till there is an equilibrium between the force of the water and the reſiſtance of its banks, and then they will remain without farther change. And it appears by experience that theſe equilibriums are all real; as we find that Rivers only dig and widen to a certain pitch.

The very reverſe of all theſe things does alſo on ſome occaſions happen. Rivers, whoſe waters are thick and muddy, raiſe their bed, by depoſiting part of the heterogeneous matters contained in them: they alſo contract their banks, by a continual oppoſition of the ſame matter, in bruſhing over them. This matter, being thrown aſide far from the ſtream of water, might even ſerve, by reaſon of the dullneſs of the motion, to form new banks.

If theſe various cauſes of reſiſtance to the motion of flowing waters did not exiſt, viz, the attraction and continual friction of the bottom and ſides, the inequalities in both, the windings and angles that occur in their courſe, and the diminution of their declivity the farther they recede from their ſprings, the velocity of their currents would be accelerated to 10, 15, or even 20 times more than it is at preſent in the ſame Rivers, by which they would become abſolutely unnavigable.

The union of two Rivers into one, makes the whole flow the ſwifter, becauſe, inſtead of the friction of four ſhores, they have only two to overcome, and one bottom inſtead of two; alſo the ſtream, being farther diſtant from the banks, goes on with the leſs interruption, beſides, that a greater quantity of water, moving with a greater velocity, digs deeper in the bed, and of courſe retrenches of its former width. Hence alſo it is, that Rivers, by being united, take up leſs ſpace on the ſurface of the earth, and are more advantageous to low grounds, which drain their ſuperfluous moiſture into them, and have alſo leſs occaſion for dykes to prevent their overflowing.

A very good and ſimple method of meaſuring the velocity of the current of a River, or canal, is the following. Take a cylindrical piece of dry, light wood, and of a length ſomething leſs than the depth of the water in the River; about one end of it let there be ſuſpended as many ſmall weights, as may keep the cylinder in a vertical or upright poſition, with its head juſt above water. To the centre of this end fix a ſmall ſtraight rod, preciſely in the direction of the cylinder's axis; to the end that, when the inſtrument is ſuſpended in the water, the deviations of the rod from a perpendicularity to the ſurface of it, may indicate which end of the cylinder goes foremoſt, by which may be diſcovered the different velocities of the water at different depths; for when the rod inclines forward, according to the direction of the current, it is a proof that the ſurface of the water has the greateſt velocity; but when it reclines backward, it ſhews that the ſwifteſt current is at the bottom; and when it remains perpendicular, it is a ſign that the velocities at the top and bottom are equal.

This inſtrument, being placed in the current of a River or canal, receives all the percuſſions of the water throughout the whole depth, and will have an equal velocity with that of the whole current from the ſurface to the bottom at the place where it is put in, and by that means may be found, both with exactneſs and eaſe, the mean velocity of that part of the River for any determinate diſtance and time.

But to obtain the mean velocity of the whole ſection of the River, the inſtrument muſt be put ſucceſſively both in the middle and towards the ſides, becauſe the velocities at thoſe places are often very different from each other. Having by this means found the ſeveral velocities, from the ſpaces run over in certain times, the arithmetical mean proportional of all theſe trials, which is found by dividing the common ſum of them all by the number of the trials, will be the mean velocity of the River or canal. And if this medium velocity be multiplied by the area of the tranſverſe ſection of the waters at any place, the product will be the quantity running through that place in a ſecond of time.

If it be required to find the velocity of the current only at the ſurface, or at the middle, or at the bottom, a ſphere of wood loaded, or a common bottle corked with a little water in it, of ſuch a weight as will remain ſuſpended in equilibrium with the water at the ſurface or depth which we want to meaſure, will be better for the purpoſe than the cylinder, becauſe it is only affected by the water of that ſole part of the current where it remains ſuſpended.

It follows from what has been ſaid in the former part of this article, that the deeper the waters are in their bed in proportion to its breadth, the more their motion is accelerated; ſo that their velocity increaſes in the inverſe ratio of the breadth of the bed, and alſo of

of the magnitude of the fection; whence, in order to augment the velocity of water in a River or canal, without augmenting the declivity of the bed, we muft increafe the depth of the channel, and diminifh its breadth. And thefe principles are agreeable to obfervation; as it is well known, that the velocity of flowing waters depends much more on the quantity and depth of the water, and on the compreffion of the upper parts on the lower, than on the declivity of the bed; and therefore the declivity of a River muft be made much greater in the beginning than toward the end of its courfe; where it fhould be almoft infenfible. If the depth or volume of water in a River or canal be confiderable, it will fuffice, in the part next the mouth, to allow one foot of declivity through 6000, or 8000, or even (according to Dechales, De Fontibus et Fluviis, prop. 49) 10000 feet in horizontal extent; at moft it need not be above 1 in 6 or 7 thoufand. From hence the quantity of declivity in equal fpaces muft flowly and gradually increafe as far as the current is to be made fit for navigation; but in fuch a manner, as that at this upper end there may not be above one foot of perpendicular declivity in 4000 feet of horizontal extent.

To conclude this article, M. de Buffon obferves, that people accuftomed to Rivers can eafily foretell when there is going to be a fudden increafe of water in the bed from floods produced by fudden falls of rain in the higher countries through which the Rivers pafs. This they perceive by a particular motion in the water, which they exprefs by faying, that the River's bottom moves, that is, the water at the bottom of the channel runs off fafter than ufual; and this increafe of motion at the bottom of a River always announces a fudden increafe of water coming down the ftream. Nor, fays he, is their opinion ill grounded; becaufe the motion and weight of the waters coming down, though not yet arrived, muft act upon the waters in the lower parts of the River, and communicate by impulfion part of their motion to them, within a certain diftance.

On the fubject of this article, fee an elaborate treatife on Rivers and canals, in the Philof. Tranf. vol. 69, pa. 555 &c, by Mr. Mann, who has availed himfelf of the obfervations of Gulielmini, and moft other writers.

RIXDOLLAR, a filver coin, ftruck in feveral ftates and free cities in Germany, as alfo in Flanders, Poland, Denmark, Sweden, &c.

There is but little difference between the Rixdollar and the dollar, another filver coin ftruck in Germany, each being nearly equal to the French crown of three livres, or the Spanifh piece of eight, or 4s. 6d. fterling.

ROBERVAL (Giles-Personne), an eminent French mathematician, was born in 1602, at Roberval, a parifh in the diocefe of Beauvais. He was firft profeffor of mathematics at the College of Maitre-Gervais, and afterwards at the College-royal. A fimilarity of tafte connected him with Gaffendi and Morin; the latter of whom he fucceeded in the mathematical chair at the Royal College, without quitting however that of Ramus.

Roberval made experiments on the Torricellian vacuum: he invented two new kinds of balance, one of which was proper for weighing air; and made many other curious experiments. He was one of the firft members of the ancient Academy of Sciences of 1666; but died in 1675, at 73 years of age. His principal works are,

I. A treatife on Mechanics.

II. A work entitled Ariftarchus Samos.

He had feveral memoirs inferted in the volumes of the Academy of Sciences of 1666, viz,

1. Experiments concerning the Preffure of the Air.
2. Obfervations on the Compofition of Motion, and on the Tangents of Curve Lines.
3. The Recognition of Equations.
4. The Geometrical Refolution of Plane and Cubic Equations.
5. Treatife on Indivifibles.
6. On the Trochoid, or Cycloid.
7. A Letter to Father Merfenne.
8. Two Letters from Torricelli.
9. A new kind of Balance.

ROBERVALLIAN *Lines*, a name given to certain lines, ufed for the transformation of figures: thus called from their inventor Roberval.

Thefe lines bound fpaces that are infinitely extended in length, which are neverthelefs equal to other fpaces that are terminated on all fides.

The abbot Gallois, in the Memoirs of the Royal Academy, anno 1693, obferves, that the method of transforming figures, explained at the latter end of Roberval's treatife of Indivifibles, was the fame with that afterwards publifhed by James Gregory, in his Geometria Univerfalis, and alfo by Barrow in his Lectiones Geometricæ; and that, by a letter of Torricelli, it appears, that Roberval was the inventor of this manner of transforming figures, by means of certain lines, which Torricelli therefore called Robervallian Lines.

He adds, that it is highly probable, that J. Gregory firft learned the method in the journey he made to Padua in 1668, the method itfelf having been known in Italy from the year 1646, though the book was not publifhed till the year 1692.

This account David Gregory has endeavoured to refute, in vindication of his uncle James. His anfwer is inferted in the Philof. Tranf. of 1694, and the abbot rejoined in the French Memoirs of the Academy of 1703.

ROBINS (Benjamin), an Englifh mathematician and philofopher of great genius and eminence, was born at Bath in Somerfetfhire, 1707. His parents were of low condition, and Quakers; and confequently neither able from their circumftances, nor willing from their religious profeffion, to have him much inftructed in that kind of learning which they are taught to defpife as human. Neverthelefs, he made an early and furprifing progrefs in various branches of fcience and literature, particularly in the mathematics; and his friends being defirous that he might continue his purfuits, and that his merit might not be buried in obfcurity, wifhed that he could be properly recommended to

to teach that ſcience in London. Accordingly, a ſpecimen of his abilities in this way was ſent up thither, and ſhewn to Dr. Pemberton, the author of the "View of Sir Iſaac Newton's Philoſophy;" who, thence conceiving a good opinion of the writer, for a farther trial of his ſkill ſent him ſome problems, which Robins reſolved very much to his ſatisfaction. He then came to London, where he confirmed the opinion which had been preconceived of his abilities and knowledge.

But though Robins was poſſeſſed of much more ſkill than is uſually required in a common teacher; yet being very young, it was thought proper that he ſhould employ ſome time in peruſing the beſt writers upon the ſublimer parts of the mathematics, before he ſhould undertake publicly the inſtruction of others. In this interval, beſides improving himſelf in the modern languages, he had opportunities of reading in particular the works of Archimedes, Apollonius, Fermat, Huygens, De Witt, Sluſius, Gregory, Barrow, Newton, Taylor, and Cotes. Theſe authors he readily underſtood without any aſſiſtance, of which he gave frequent proofs to his friends: one was, a demonſtration of the laſt propoſition of Newton's treatiſe on Quadratures, which was thought not undeſerving a place in the Philoſophical Tranſactions for 1727.

Not long after, an opportunity offered him of exhibiting to the public a ſpecimen alſo of his knowledge in Natural Philoſophy. The Royal Academy of Sciences at Paris had propoſed, among their prize queſtions in 1724 and 1726, to demonſtrate the laws of motion in bodies impinging on one another. John Bernoulli here condeſcended to be a candidate; and as his diſſertation loſt the reward, he appealed to the learned world by printing it in 1727. In this piece he endeavoured to eſtabliſh Leibnitz's opinion of the force of bodies in motion from the effects of their ſtriking againſt ſpringy materials; as Poleni had before attempted to evince the ſame thing from experiments of bodies falling on ſoft and yielding ſubſtances. But as the inſufficiency of Poleni's arguments had been demonſtrated in the Philoſophical Tranſactions, for 1722; ſo Robins publiſhed in the Preſent State of the Republic of Letters, for May 1728, a Confutation of Bernoulli's performance, which was allowed to be unanſwerable.

Robins now began to take ſcholars; and about this time he quitted the garb and profeſſion of a Quaker; for, having neither enthuſiaſm nor ſuperſtition in his nature, as became a mathematician, he ſoon ſhook off the prejudices of ſuch early habits. But though he profeſſed to teach the mathematics only, he would frequently aſſiſt particular friends in other matters; for he was a man of univerſal knowledge: and the confinement of this way of life not ſuiting his diſpoſition, which was active, he gradually declined it, and went into other courſes, that required more exerciſe. Hence he tried many laborious experiments in gunnery; believing that the reſiſtance of the air had a much greater effect on ſwift projectiles, than was generally ſuppoſed. And hence he was led to conſider thoſe mechanic arts that depend upon mathematical principles, in which he might employ his invention: as, the conſtructing of mills, the building of bridges, draining of fens, rendering of rivers navigable, and making of harbours. Among other arts of this kind, fortification very much engaged his attention; in which he met with opportunities of perfecting himſelf, by a view of the principal ſtrong places of Flanders, in ſome journeys he made abroad with perſons of diſtinction.

On his return home from one of theſe excurſions, he found the learned here amuſed with Dr. Berkeley's treatiſe, printed in 1734, entitled, "The Analyſt;" in which an examination was made into the grounds of the doctrine of Fluxions, and occaſion thence taken to explode that method. Robins was therefore adviſed to clear up this affair, by giving a full and diſtinct account of Newton's doctrines, in ſuch a manner, as to obviate all the objections, without naming them, which had been advanced by Berkeley; and accordingly he publiſhed, in 1735, *A Diſcourſe concerning the Nature and Certainty of Sir Iſaac Newton's Method of Fluxions, and of Prime and Ultimate Ratios.* This is a very clear, neat, and elegant performance: and yet ſome perſons, even among thoſe who had written againſt The Analyſt, taking exception at Robins's manner of defending Newton's doctrine, he afterwards wrote two or three additional diſcourſes.

In 1738, he defended Newton againſt an objection, contained in a note at the end of a Latin piece, called "Matho, ſive Coſmotheoria puerilis," written by Baxter, author of the "Inquiry into the Nature of the Human Soul:" and the year after he printed *Remarks on Euler's Treatiſe of Motion*, on *Smith's Syſtem of Optics.* and on *Jurin's Diſcourſe of Diſtinct and Indiſtinct Viſion*, annexed to Dr. Smith's work.

In the mean time Robins's performances were not confined to mathematical ſubjects: for, in 1739, there came out three pamphlets upon political affairs, which did him great honour. The firſt was entitled, *Obſervations on the preſent Convention with Spain:* the ſecond, *A Narrative of what paſſed in the Common Hall of the Citizens of London, aſſembled for the Election of a Lord Mayor:* the third, *An Addreſs to the Electors and other free Subjects of Great Britain, occaſioned by the late Succeſſion; in which is contained a Particular Account of all our Negociations with Spain, and their Treatment of us for above ten years paſt.* Theſe were all publiſhed without our author's name; and the firſt and laſt were ſo univerſally eſteemed, that they were generally reputed to have been the production of the great man himſelf, who was at the head of the oppoſition to Sir Robert Walpole. They proved of ſuch conſequence to Mr. Robins, as to occaſion his being employed in a very honourable poſt; for, the patriots at length gaining ground againſt Sir Robert, and a committee of the Houſe of Commons being appointed to examine into his paſt conduct, Robins was choſen their ſecretary. But after the committee had preſented two reports of their proceedings, a ſudden ſtop was put to their farther progreſs, by a compromiſe between the contending parties.

In 1742, being again at leiſure, he publiſhed a ſmall treatiſe, entitled, *New Principles of Gunnery;* containing the reſult of many experiments he had made, by which are diſcovered the force of gunpowder, and the difference in the reſiſting power of the air to ſwift and

ſlow

ſlow motions. To this treatiſe was prefixed a full and learned account of the progreſs which modern fortification had made from its firſt riſe; as alſo of the invention of gunpowder, and of what had already been performed in the theory of gunnery. It ſeems that the occaſion of this publication, was the diſappointment of a ſituation at the Royal Military Academy at Woolwich. On the new modelling and eſtabliſhing of that Academy, in 1741, our author and the late Mr. Muller were competitors for the place of profeſſor of fortification and gunnery. Mr. Muller held then ſome poſt in the Tower of London, under the Board of Ordnance, ſo that, notwithſtanding the great knowledge and abilities of our author, the intereſt which Mr. Muller had with the Board of Ordnance carried the election in his favour. Upon this diſappointment Mr. Robins, indignant at the affront, determined to ſhew them, and the world, by his military publications, what ſort of a man he was that they had rejected.

Upon a diſcourſe containing certain experiments being publiſhed in the Philoſophical Tranſactions, with a view to invalidate ſome of Robins's opinions, he thought proper, in an account he gave of his book in the ſame Tranſactions, to take notice of thoſe experiments: and in conſequence of this, ſeveral diſſertations of his on the reſiſtance of the air were read, and the experiments exhibited before the Royal Society, in 1746 and 1747; for which he was preſented with the annual gold medal by that Society.

In 1748 came out Anſon's Voyage round the World; which, though it bears Walter's name in the title-page, was in reality written by Robins. Of this voyage the public had for ſome time been in expectation of ſeeing an account, compoſed under that commander's own inſpection: for which purpoſe the reverend Richard Walter was employed, as having been chaplain on board the Centurion the greateſt part of the expedition. Walter had accordingly almoſt finiſhed his taſk, having brought it down to his own departure from Macao for England; when he propoſed to print his work by ſubſcription. It was thought proper however that an able judge ſhould firſt review and correct it, and Robins was appointed; when, upon examination, it was reſolved, that the whole ſhould be written entirely by Robins, and that what Walter had done, being moſtly taken verbatim from the journals, ſhould ſerve as materials only. Hence it was that the whole of the introduction, and many diſſertations in the body of the work, were compoſed by Robins, without receiving the leaſt hint from Walter's manuſcript; and what he had tranſcribed from it regarded chiefly the wind and weather, the currents, courſes, bearings, diſtances, offings, ſoundings, moorings, the qualities of the ground they anchored on, and ſuch particulars as uſually fill up a ſeaman's account. No production of this kind ever met with a more favourable reception, four large impreſſions having been ſold off within a year: it was alſo tranſlated into moſt of the European languages; and it ſtill ſupports its reputation, having been repeatedly reprinted in various ſizes. The fifth edition at London in 1749 was reviſed and corrected by Robins himſelf; and the 9th edition was printed there in 1761.

Thus becoming famous for his elegant talents in writing, he was requeſted to compoſe an apology for the unfortunate affair at Preſtonpans in Scotland. This was added as a preface to the Report of the Proceedings and Opinion of the Board of General Officers on their Examination into the Conduct of Lieutenant General Sir John Cope, &c, printed at London in 1749; and this preface was eſteemed a maſter-piece in its kind.

Robins had afterwards, by the favour of lord Anſon, opportunities of making farther experiments in Gunnery; which have been publiſhed ſince his death, in the edition of his works by his friend Dr. Wilſon. He alſo not a little contributed to the improvements made in the Royal Obſervatory at Greenwich, by procuring for it, through the intereſt of the ſame noble perſon, a ſecond mural quadrant, and other inſtruments; by which it became perhaps the completeſt of any obſervatory in the world.

His reputation being now arrived at its full height, he was offered the choice of two very conſiderable employments. The firſt was to go to Paris, as one of the commiſſaries for adjuſting the limits in Acadia; the other, to be engineer general to the Eaſt India Company, whoſe forts, being in a moſt ruinous condition, wanted an able perſon to put them into a proper ſtate of defence. He accepted the latter, as it was ſuitable to his genius, and as the Company's terms were both advantageous and honourable. He deſigned, if he had remained in England, to have written a ſecond part of the Voyage round the World; as appears by a letter from lord Anſon to him, dated Bath, Oct. 22, 1749, as follows.

"Dear Sir, when I laſt ſaw you in town, I forgot to aſk you, whether you intended to publiſh the ſecond volume of my Voyage before you leave us; which I confeſs I am very ſorry for. If you ſhould have laid aſide all thoughts of favouring the world with more of your works, it will be much diſappointed, and no one in it more than your very obliged humble ſervant,

"Anson."

Robins was alſo preparing an enlarged edition of his New Principles of Gunnery: but, having provided himſelf with a complete ſet of aſtronomical and other inſtruments, for making obſervations and experiments in the Indies, he departed hence at Chriſtmas in 1749; and after a voyage, in which the ſhip was near being caſt away, he arrived at India in July following. There he immediately ſet about his proper buſineſs with the greateſt diligence, and formed complete plans for Fort St. David and Madras: but he did not live to put them into execution. For the great difference of the climate from that of England being beyond his conſtitution to ſupport, he was attacked by a fever in September the ſame year; and though he recovered out of this, yet about eight months after he fell into a languiſhing condition, in which he continued till his death, which happened the 29th of July 1751, at only 44 years of age.

By his laſt will, Mr. Robins left the publiſhing of his Mathematical Works to his honoured and intimate friend Martin Folkes, Eſq. preſident of the Royal Society, and to Dr. James Wilſon; but the former of theſe gentlemen

gentlemen being incapacitated by a paralytic diforder, for fome time before his death, they were afterwards publifhed by the latter, in 2 volumes 8vo, 1761. To this collection, which contains his mathematical and philofophical pieces only, Dr. Wilfon has prefixed an account of Mr. Robins, from which this memoir is chiefly extracted. He added alfo a large appendix at the end of the fecond volume, containing a great many curious and critical matters in various interefting parts of the mathematics. As to Mr. Robins's own papers in thefe two volumes, they are as follow: viz, in vol. I,

1. New Principles of Gunnery. Firft printed in 1742.

2. An Account of that book. Read before the Royal Society, April the 14th and 21ft 1743.

3. Of the Refiftance of the Air. Read the 12th of June 1746.

4. Of the Refiftance of the Air; together with the Method of computing the Motions of Bodies projected in that Medium. Read June 19, 1746.

5. Account of Experiments relating to the Refiftance of the Air. Read the 4th of June 1747.

6. Of the Force of Gunpowder, with the Computation of the Velocities thereby communicated to military projectiles. Read the 25th of June 1747.

7. A Comparifon of the Experimental Ranges of Cannon and Mortars, with the Theory contained in the preceding papers. Read the 27th of June 1751.

8. Practical Maxims relating to the Effects and Management of Artillery, and the Flight of Shells and Shot.

9. A Propofal for increafing the Strength of the Britifh Navy. Read the 2d of April 1747.

10. A Letter to Martin Folkes, Efq. Prefident of the Royal Society. Read the 7th of January 1748.

11. A Letter to Lord Anfon. Read the 26th of October 1749.

12. On Pointing, or Directing of Cannon to ftrike diftant objects.

13. Obfervations on the Height to which Rockets afcend. Read the 4th of May 1749.

14. An Account of fome Experiments on Rockets, by Mr. Ellicott.

15. Of the Nature and Advantage of Rifled Barrel Pieces, by Mr. Robins. Read the 2d of July 1747.

In volume II are,

16. A Difcourfe concerning the Nature and Certainty of Sir Ifaac Newton's Methods of Fluxions, and of Prime and Ultimate Ratios.

17. An Account of the preceding Difcourfe.

18. A Review of fome of the principal Objections, that have been made to the Doctrine of Fluxions and Ultimate Proportions, with fome Remarks on the different Methods, that have been taken to obviate them.

19. A Differtation fhewing, that the Account of the Doctrines of Fluxions and of Prime and Ultimate Ratios, delivered in Mr. Robins's Difcourfe, is agreeable to the real Meaning of their great Inventor.

20. A Demonftration of the Eleventh Propofition of Sir Ifaac Newton's Treatife of Quadratures.

21. Remarks on Bernoulli's Difcourfe upon the Laws of the Communication of Motion.

22. An Examination of a Note concerning the Sun's Parallax, publifhed at the end of Baxter's Matho.

23. Remarks on Euler's Treatife of Motion; Dr. Smith's Syftem of Optics; and Dr. Jurin's Effay on Diftinct and Indiftinct Vifion.

24. Appendix by the Publifher.

It is but juftice to fay, that Mr. Robins was one of the moft accurate and elegant mathematical writers that our language can boaft of; and that he made more real improvements in Artillery, the flight and the refiftance of projectiles, than all the preceding writers on that fubject. His New Principles of Gunnery were tranflated into feveral other languages, and commented upon by feveral eminent writers. The celebrated Euler tranflated the work into the German language, accompanied with a large and critical commentary; and this work of Euler's was again tranflated into Englifh in 1714, by Mr. Hugh Brown, with Notes, in one volume 4to.

ROBINS, or Robyns (John), an Englifh mathematician, was born in Staffordfhire about the clofe of the 15th century, as he was entered a ftudent at Oxford in 1516, where he was educated for the church. But the bent of his genius lay to the fciences, and he foon made fuch a progrefs, fays Wood, in "the pleafant ftudies of mathematics and aftrology, that he became the ableft perfon in his time for thofe ftudies, not excepted his friend Record, whofe learning was more general. At length, taking the degree of bachelor of divinity in 1531, he was the year following made by king Henry the VIIIth (to whom he was chaplain) one of the canons of his college in Oxon, and in December 1543 canon of Windfor, and in fine chaplain to Queen Mary, who had him in great veneration for his learning. Among feveral things that he hath written relating to aftrology (or aftronomy) I find thefe following:

"*De Culminatione Fixarum Stellarum*, &c.
De Ortu & Occafu Stellarum Fixarum, &c.
Annotationes Aftrologicæ, &c. *lib.* 3.
Annotationes Edwardo VI.
Tractatus de Prognofticatione per Eclipfin.

"All which books, that are in MS, were fome time in the choice library of Mr. Thomas Allen of Glocefter Hall. After his death, coming into the hands of Sir Kenelm Digby, they were by him given to the Bodleian library, where they yet remain. It is alfo faid, that he the faid Robyns hath written a book intitled, *De Portentofis Cometis*, but fuch a thing I have not yet feen, nor do I know any thing elfe of the author, only that paying his laft debt to nature the 25th of Auguft 1558, he was buried in the chappel of St. George at Windfore."

ROCKET, in Pyrotechny, an artificial firework, ufually confifting of a cylindrical cafe of paper, filled with a compofition of certain combuftible ingredients; which being tied to a rod, mounts into the air to a confiderable height, and there burfts. Thefe are called *Sky Rockets*. Befide which, there are others called *Water Rockets*, from their acting in water.

The

The compoſition with which Rockets are filled, conſiſts of the three following ingredients, viz., ſaltpetre, charcoal, and ſulphur, all well ground; and in the ſmaller ſizes, gunpowder duſt is alſo added. But the proportions of all the ingredients vary with the weight of the Rocket, as in the following Table.

Compoſitions for Rockets of Various Sizes.

The General Compoſition for Rockets is,

Saltpetre	4 lb.
Sulphur	1 lb.
Charcoal	1 lb.

But for large Rockets,

Saltpetre	4 lb.
Sulphur	1 lb.
Mealpowder	1 lb.

For Rockets of a Middle Size,

Saltpetre	3 lb.
Sulphur	2 lb.
Mealpowder	1 lb.
Charcoal	1 lb.

When Rockets are intended to mount upwards, they have a long ſlender rod fixed to the lower end, to direct their motion.

Theory of the Flight of Rockets.—Mariotte takes the riſe of Rockets to be owing to the impulſe or reſiſtance of the air againſt the flame. Deſaguliers accounts for it thus.

Conceive the Rocket to have no vent at the choke, and to be ſet on fire in the conical bore; the conſequence would be, either that the Rocket would burſt in the weakeſt place, or that, if all parts were equally ſtrong, and able to ſuſtain the impulſe of the flame, the Rocket would burn out immoveable. Now, as the force of the flame is equable, ſuppoſe its action downwards, or that upwards, ſufficient to lift 40 pounds; as theſe forces are equal, but their directions contrary, they will deſtroy each other's action.

Imagine then the Rocket opened at the choke; by this means the action of the flame downwards is taken away, and there remains a force equal to 40 pounds acting upwards, to carry up the Rocket, and the ſtick or rod it is tied to. Accordingly we find that if the compoſition of the Rocket be very weak, ſo as not to give an impulſe greater than the weight of the Rocket and ſtick, it does not riſe at all; or if the compoſition be ſlow, ſo that a ſmall part of it only kindles at firſt, the Rocket will not riſe.

The ſtick ſerves to keep it perpendicular; for if the Rocket ſhould begin to tumble, moving round a point in the choke, as being the common centre of gravity of Rocket and ſtick, there would be ſo much friction againſt the air, by the ſtick between the centre and the point, and the point would beat againſt the air with ſo much velocity, that the reaction of the medium would reſtore it to its perpendicularity. When the compoſition is burnt out, and the impulſe upwards has ceaſed, the common centre of gravity is brought lower towards the middle of the ſtick; by which means the velocity of the point of the ſtick is decreaſed, and that of the point of the Rocket is increaſed; ſo that the whole will tumble down, with the Rocket end foremoſt.

All the while the Rocket burns, the common centre of gravity is ſhifting and getting downwards, and ſtill the faſter and the lower as the ſtick is lighter; ſo that it ſometimes begins to tumble before it is quite burnt out: but when the ſtick is too heavy, the common centre of gravity will not get ſo low, but that the Rocket will riſe ſtraight, though not ſo faſt.

From the experiments of Mr. Robins, and other gentlemen, it appears that the Rockets of 2, 3, or 4 inches diameter, riſe the higheſt; and they found them riſe to all heights in the air, from 400 to 1254 yards, which is about three quarters of a mile. See Robins's Tracts, vol. 2, pa. 317, and the Philoſ. Tranſ. vol. 46, pa. 578.

ROD, or *Pole*, is a long meaſure, of $16\frac{1}{2}$ feet, or $5\frac{1}{2}$ yards, or the 4th part of a Gunter's chain, for land-meaſuring.

ROEMER (Olaus), a noted Daniſh aſtronomer and mathematician, was born at Arhuſen in Jutland, 1644; and at 18 years of age was ſent to the univerſity of Copenhagen. He applied aſſiduouſly to the ſtudy of the mathematics and aſtronomy, and became ſo expert in thoſe ſciences, that when Picard was ſent by Lewis the XIVth in 1671, to make obſervations in the north, he was greatly ſurpriſed and pleaſed with him. He engaged him to return with him to France, and had him preſented to the king, who honoured him with the dauphin as a pupil in mathematics, and ſettled a penſion upon him. He was joined with Picard and Caſſini, in making aſtronomical obſervations; and in 1672 he was admitted a member of the academy of ſciences.

During the ten years he reſided at Paris, he gained great reputation by his diſcoveries; yet it is ſaid he complained afterwards, that his coadjutors ran away with the honour of many things which belonged to him. Here it was that Roemer, firſt of any one, found out the velocity with which light moves, by means of the eclipſes of Jupiter's ſatellites. He had obſerved for many years that, when Jupiter was at his greateſt diſtance from the earth, where he could be obſerved, the emerſions of his firſt ſatellite happened conſtantly 15 or 16 minutes later than the calculation gave them. Hence he concluded that the light reflected by Jupiter took up this time in running over the exceſs of diſtance, and conſequently that it took up 16 or 18 minutes in running over the diameter of the earth's orbit, and 8 or 9 in coming from the ſun to us, provided its velocity was nearly uniform. This diſcovery had at firſt many oppoſers; but it was afterwards confirmed by Dr. Bradley in the moſt ingenious and beautiful manner.

In 1681 Roemer was recalled back to his own country by Chriſtian the Vth, king of Denmark, who made him profeſſor of aſtronomy at Copenhagen. The king employed him alſo in reforming the coin and the architecture, in regulating the weights and meaſures, and in meaſuring and laying out the high roads throughout the kingdom; offices which he diſcharged with the greateſt credit and ſatisfaction. In conſequence he was honoured by the king with the appointment of chancellor of the exchequer and other dignities. Finally he became counſellor of ſtate and burgomaſter of Copen-

hagen, under Frederic the IVth, the succeffor of Christian. Roemer was preparing to publifh the refult of his obfervations, when he died the 19th of September 1710, at 66 years of age: but this lofs was fupplied by Horrebow, his difciple, then profeffor of aftronomy at Copenhagen, who publifhed, in 4to, 1753, various obfervations of Roemer, with his method of obferving, under the title of *Bafis-Aftronomiæ.*—He had alfo printed various aftronomical obfervations and pieces, in feveral volumes of the Memoirs of the Royal Academy of Sciences at Paris, of the inftitution of 1666, particularly vol. 1 and 10 of that collection.

ROHAULT (JAMES), a French philofopher, was the fon of a rich merchant at Amiens, where he was born in 1620. He cultivated the languages and belles lettres in his own country, and then was fent to Paris to ftudy philofophy. He feems to have been a great lover of truth, at leaft what he thought fo, and to have fought it with much impartiality. He read the ancient and modern philofophers; but Des Cartes was the author who moft engaged his notice. Accordingly he became a zealous follower of that great man, and drew up an abridgment and explanation of his philofophy with great clearnefs and method. In the preface to his *Phyfics*, for fo his work is called, he makes no fcruple to fay, that "the abilities and accomplifhments of this philofopher muft oblige the whole world to confefs, that France is at leaft as capable of producing and raifing men verfed in all arts and branches of knowledge, as ancient Greece." Clerfelier, well known for his tranflation of many pieces of Des Cartes, conceived fuch an affection for Rohault, on account of his attachment to this philofopher, that he gave him his daughter in marriage againft all the remonftrances of his family.

Rohault's Phyfics were written in French, but have been tranflated into Latin by Dr. Samuel Clarke, with notes, in which the Cartefian errors are corrected upon the Newtonian fyftem. The fourth and beft edition of *Rohault's Phyfica*, by Clarke, is that of 1718, in 8vo. He wrote alfo,

Elemens de Mathematiques,
Traité de Mechanique, and
Entretiens fur la Philofophie.

But thefe dialogues are founded and carried on upon the principles of the Cartefian philofophy, which has now little other merit, than that of having corrected the errors of the Ancients. Rohault died in 1675, and left behind him the character of an amiable, as well as a learned and philofophic man.

His pofthumous works were collected and printed in two neat little volumes, firft at Paris, and then at the Hague in 1690. The contents of them are, 1. The firft 6 books of Euclid. 2. Trigonometry. 3. Practical Geometry. 4. Fortification. 5. Mechanics. 6. Perfpective. 7. Spherical Trigonometry. 8. Arithmetic.

ROLLE (MICHEL), a French mathematician, was born at Ambert, a fmall town in Auvergne, the 21ft of April 1652. His firft ftudies and employments were under notaries and attorneys; occupations but little fuited to his genius. He went to Paris in 1675, with the only refource of fine penmanfhip, and fubfifted by giving leffons in writing. But as his inclination for the mathematics had drawn him to that city, he attended the mafters in this fcience, and foon became one himfelf. Ozanam propofed a queftion in arithmetic to him, to which Rolle gave fo clear and good a folution, that the minifter Colbert made him a handfome gratuity, which at laft grew into a fixed penfion. He then abandoned penmanfhip, and gave himfelf up entirely to algebra and other branches of the mathematics. His conduct in life gained him many friends; in which his fcientific merit, his peaceable and regular behaviour, with an exact and fcrupulous probity of manners, were his only folicitors.

Rolle was chofen a member of the Ancient Academy of Sciences in 1685, and named fecond geometrical-penfionary on its renewal in 1699; which he enjoyed till his death, which happened the 5th of July 1719, at 67 years of age.

The works publifhed by Rolle, were,

I. A Treatife of Algebra; in 4to, 1690.

II. A method of refolving Indeterminate Queftions in Algebra; in 1699. Befides a great many curious pieces inferted in the Memoirs of the Academy of Sciences, as follow:

1. A Rule for the Approximation of Irrational Cubes: an. 1666, vol. 10.

2. A Method of Refolving Equations of all Degrees which are expreffed in General Terms: an. 1666, vol. 10.

3. Remarks upon Geometric Lines: 1702 and 1703.

4. On the New Syftem of Infinity: 1703, pa. 312.

5. On the Inverfe Method of Tangents: 1705, pa. 25, 171, 222.

6. Method of finding the Foci of Geometric Lines of all kinds: 1706, pa. 284.

7. On Curves, both Geometrical and Mechanical, with their Radii of Curvature: 1707, pa. 370.

8. On the Conftruction of Equations: 1708, and 1709.

9. On the Extermination of the Unknown Quantities in the Geometrical Analyfis: 1709, pa. 419.

10. Rules and Remarks for the Conftruction of Equations: 1711, pa. 86.

11. On the Application of Diophantine Rules to Geometry: 1712.

12. On a Paradox in Geometric Effections: 1713, pa. 243.

13. On Geometric Conftructions: 1713, pa. 261, and 1714, pa. 5.

ROLLING, or *Rotation*, in Mechanics, a kind of circular motion, by which the moveable body turns round its own axis, or centre, and continually applies new parts of its furface to the body it moves upon. Such is that of a wheel, a fphere, a garden roller, or the like.

The motion of Rolling is oppofed to that of fliding; in which latter motion the fame furface is continually applied to the plane it moves along.

In a wheel, it is only the circumference that properly and fimply rolls; the reft of the wheel proceeds in a compound angular kind of motion, and partly rolls,

rolls, partly slides. The want of distinguishing between which two motions, occasioned the difficulty of that celebrated problem of Aristotle's Wheel.

The friction of a body in rolling, is much less than the friction in sliding. And hence arises the great use of wheels, rolls, &c, in machines; as much of the action as possible being laid upon it, to make the resistance the less.

ROMAN *Order*, in Architecture, is the same as the composite. It was invented by the Romans, in the time of Augustus; and it is made up of the Ionic and Corinthian orders, being more ornamental than either.

RONDEL, in Fortification, a round tower, sometimes erected at the foot of a bastion.

ROOD, a square measure, being a quantity of land just equal to the 4th part of an acre, or equal to 40 perches or square poles.

ROOF, in Architecture, the uppermost part of a building; being that which forms the covering of the whole. In this sense, the Roof comprises the timber work, together with its furniture, of slate, or tile, or lead, or whatever else serves for a covering: though the carpenters usually restrain Roof to the timber-work only.

The form of a Roof is various: viz, 1. *Pointed*, when the ridge, or angle formed by the two sides, is an acute angle.—2. *Square*, when the pitch or angle of the ridge is a right angle, called the true pitch.—3. *Flat* or pediment Roof, being only pediment pitch, or the angle very obtuse. There are also various other forms, as hip Roofs, valley Roofs, hopper Roofs, double ridges, platforms, round, &c.—In the true pitch, when the sides form a square or right angle, the girt over both sides of the Roof, is accounted equal to the breadth of the building and the half of the same.

ROOKE (Lawrence), an English astronomer and geometrician, was born at Deptford in Kent, 1623, and educated at Eton school. From hence he removed to King's College, Cambridge, in 1639. After taking the degree of master of arts in 1647, he retired into the country. But in the year 1650 he went to Oxford, and settled in Wadham College, that he might have the company of, and receive improvement from Dr. Wilkins, and Mr. Seth Ward the Astronomy Professor; and that he might also accompany Mr. Boyle in his chemical operations.

After the death of Mr. Foster, he was chosen Astronomy Professor in Gresham College, London, in the year 1652. He made some observations upon the comet at Oxford, which appeared in the month of December that year; which were printed by Mr. Seth Ward the year following. And, in 1655, Dr. Wallis publishing his treatise on Conic Sections, he dedicated that work to those two gentlemen.

In 1657, Mr. Rooke was permitted to exchange the astronomy professorship for that of geometry. This step might seem strange, as astronomy still continued to be his favourite study; but it was thought to have been from the convenience of the lodgings, which opened behind the reading hall, and therefore were proper for the reception of those gentlemen after the lectures, who in the year 1660 formed the Royal Society there.

Mr. Rooke having thus successively enjoyed those two places some years before the restoration in 1658, most of those gentlemen who had been accustomed to assemble with him at Oxford, coming to London, joined with other philosophical gentlemen, and usually met at Gresham College to hear Mr. Rooke's lectures, and afterwards withdrew into his apartment; till their meetings were interrupted by the quartering of soldiers in the college that year. And after the Royal Society came to be formed and settled into a regular body, Mr. Rooke was very zealous and serviceable in promoting that great and useful institution; though he did not live till it received its establishment by the Royal charter.

The Marquis of Dorchester, who was not only a patron of learning, but learned himself, used to entertain Mr. Rooke at his seat at Highgate after the restoration, and bring him every Wednesday in his coach to the Royal Society, which then met on that day of the week at Gresham College. But the last time Mr. Rooke was at Highgate, he walked from thence; and it being in the summer, he overheated himself, and taking cold after it, he was thrown into a fever, which cost him his life. He died at his apartments at Gresham College the 27th of June 1662, in the 40th year of his age.

One other very unfortunate circumstance attended his death, which was, that it happened the very night that he had for some years expected to finish his accurate observations on the satellites of Jupiter. When he found his illness prevented him from making that observation, Dr. Pope says, he sent to the Society his request, that some other person, properly qualified, might be appointed for that purpose; so intent was he to the last on making those curious and useful discoveries, in which he had been so long engaged.

Mr. Rooke made a nuncupatory will, leaving what he had to Dr. Ward, then lately made bishop of Exeter: whom he permitted to receive what was due upon bond, if the debtors offered payment willingly, otherwise he would not have the bonds put in suit: "for, said he, as I never was in law, nor had any contention with any man, in my life-time; neither would I be so after my death."

Few persons have left behind them a more agreeable character than Mr. Rooke, from every person that was acquainted with him, or with his qualifications; and in nothing more than for his veracity: for what he asserted positively, might be fully relied on: but if his opinion was asked concerning any thing that was dubious, his usual answer was, "I have no opinion." Mr. Hook has given this copious, though concise character of him: "I never was acquainted with any person who knew more, and spoke less, being indeed eminent for the knowledge and improvement of astronomy." Dr. Wren and Dr. Seth Ward describe him, as a man of profound judgment, a vast comprehension, prodigious memory, and solid experience. His skill in the mathematics was reverenced by all the lovers of those studies, and his perfection in many other sorts of learning deserves no less admiration; but above all, as another writer characterizes him, his extensive know-

 ledge

ledge had a right influence on the temper of his mind, which had all the humility, goodneſs, calmneſs, ſtrength, and ſincerity, of a ſound and unaffected philoſopher. Theſe accounts give us his picture only in miniature; but his ſucceſſor, Dr. Iſaac Barrow, has drawn it in full proportion, in his oration at Greſham College; which is too long to be inſerted in this place.

His writings were chiefly;

1. *Obſervations on the Comet* of Dec. 1652. This was printed by Dr. Seth Ward, in his Lectures on Comets, 4to, 1653.

2. *Directions for Seamen going to the Eaſt and Weſt Indies.* Publiſhed in the Philoſophical Tranſactions for Jan. 1665.

3. *A Method of Obſerving the Eclipſes of the Moon &c.* In the Philoſ. Tranſ. for Feb. 1666.

4. *A Diſcourſe concerning the Obſervations of the Eclipſes of the Satellites of Jupiter.* In the Hiſtory of the Royal Society, pa. 183.

5. *An Account of an Experiment made with Oil in a long Tube.* Read to the Royal Soc. April 23, 1662.—By this experiment it was found, that the oil ſunk when the ſun ſhone out, and roſe when he was clouded; the proportions of which are ſet down in the account.

ROOT, in Arithmetic and Algebra, denotes a quantity which being multiplied by itſelf produces ſome higher power; or a quantity conſidered as the baſis or foundation of a higher power, out of which this ariſes and grows, like as a plant from its Root.

In the involution of powers, from a given Root, the Root is alſo called the firſt power; when this is once multiplied by itſelf, it produces the ſquare or ſecond power; this multiplied by the Root again, makes the cube or 3d power; and ſo on. And hence the Roots alſo come to be denominated the ſquare-Root, or cube-Root, or 2d Root, or 3d Root, &c, according as the given power or quantity is conſidered as the ſquare, or cube, or 2d power, or 3d power, &c. Thus, 2 is the ſquare-Root or 2d Root of 4, and the cube-Root or 3d Root of 8, and the 4th Root of 16, &c.

Hence, the ſquare-Root is the mean proportional between 1 and the ſquare or given power; and the cube-Root is the firſt of two mean proportionals between 1 and the given cube; and ſo on.

To Extract the Root of a given number or power. This is the ſame thing as to find a number or quantity, which being multiplied the proper number of times, will produce the given number or power. So, to find the cube Root of 8, is finding the number 2, which multiplied twice by itſelf produces the given number 8.

For the uſual methods of extracting the Roots of Numbers, ſee the common treatiſes on Arithmetic.

A Root, of any power, that conſiſts of two parts, is called a binomial Root; as 12 or 10 + 2. If it conſiſt of three parts, it is a trinomial Root; as 126 or 100 + 20 + 6. And ſo on.

The extraction of the Roots of algebraic quantities, is alſo performed after the ſame manner as that of numbers; as may be ſeen in any treatiſe on algebra. See alſo the article EXTRACTION of Roots.

A general method for all Roots, is alſo by Newton's binomial theorem. See BINOMIAL *Theorem.*

Finite approximating rules for the extraction of Roots have alſo been given by ſeveral authors, as Raphſon, De Lagney, Halley, &c. See the articles APPROXIMATION and EXTRACTION. See alſo Newton's Univerſal Arith. the Appendix; Philoſ. Tranſ. numb. 210, or Abridg. vol. 1, pa. 81; Maclaurin's Alg. pa. 242; Simpſon's Alg. pa. 155; or his Eſſays, pa. 82, or his Diſſertations, pa. 102, or his Select Exerc. pa. 215: where various general theorems for approximating to the Roots of pure powers are given. See alſo EQUATION and REDUCTION of Equations, APPROXIMATION, and CONVERGING.

But the moſt commodious and general rule of any, for ſuch approximations, I believe, is that which has been invented by myſelf, and explained in my Tracts, vol. 1, pa. 49: which theorem is this;

$\frac{\overline{n+1}.N + \overline{n-1}.a^n}{\overline{n-1}.N + \overline{n+1}.a^n} a = \sqrt[n]{N}$. That is, having to extract the nth Root of the given number N; take a^n the neareſt rational power to that given quantity N, whether greater or leſs, its Root of the ſame kind being a; then the required Root, or $\sqrt[n]{N}$, will be as is expreſſed in this formula above; or the ſame expreſſed in a proportion will be thus:

$$\overline{n-1}.N + \overline{n+1}.a^n : \overline{n+1}.N + \overline{n-1}.a^n :: a : \sqrt[n]{N}$$

the Root ſought very nearly. Which rule includes all the particular rational formulas of De Lagney, and Halley, which were ſeparately inveſtigated by them; and yet this general formula is perfectly ſimple and eaſy to apply, and more eaſily kept in mind than any one of the ſaid particular formulas.

Ex. Suppoſe it be required to double the cube, or to extract the cube Root of the number 2.

Here N = 2, $n = 3$, the neareſt Root $a = 1$, alſo $a^3 = 1$; hence, for the cube Root the formula becomes $\frac{4N + 2a^3}{2N + 4a^3} a$ or $\frac{2N + a^3}{N + 2a^3} a = \sqrt[3]{N}$.

But $N + 2a^3 = 4$, and $2N + a^3 = 5$; therefore as $4 : 5 :: 1 : \frac{5}{4} = 1{\cdot}25$ = the Root nearly by a firſt approximation.

Again, for a ſecond approximation, take $a = \frac{5}{4}$, and conſequently $a^3 = \frac{125}{64}$;

hence $2N + a^3 = 4 + \frac{125}{64} = \frac{381}{64}$,

and $N + 2a^3 = 2 + \frac{250}{64} = \frac{378}{64}$;

therefore as $378 : 381$, or as $126 : 127 :: \frac{5}{4} : \frac{635}{504} = 1{\cdot}259921$ &c, for the required cube Root of 2, which is true even in the laſt place of decimals.

ROOT *of an Equation*, denotes the value of the unknown quantity in an equation; which is ſuch a quantity,

quantity, as being substituted instead of that unknown letter, into the equation, shall make all the terms to vanish, or both sides equal to each other. Thus, of the equation $3x + 5 = 14$, the Root or value of x is 3, because substituting 3 for x, makes it become $9 + 5 = 14$. And the Root of the equation $2x^2 = 32$ is 4, because $2 \times 4^2 = 32$. Also the Root of the equation $x^2 = a^2 + c^2$ is $x = \sqrt{a^2 + c^2}$.

For the Nature of Roots, and for extracting the several Roots of equations, see EQUATION.

Every equation has as many Roots, or values of the unknown quantity, as are the dimensions or highest power in it. As a simple equation one Root, a quadratic two, a cubic three, and so on.

Roots are positive or negative, real or imaginary, rational or radical, &c. See EQUATION.

Cubic ROOT. This is threefold, even for a simple cubic. So the cube Root of a^3, is either

$$a, \text{ or } \frac{-1 + \sqrt{-3}}{2}a, \text{ or } \frac{-1 - \sqrt{-3}}{2}a.$$

And even the cube Root of 1 itself is either

$$1, \text{ or } \frac{-1 + \sqrt{-3}}{2}, \text{ or } \frac{-1 - \sqrt{-3}}{2}.$$

Real and Imaginary ROOTS. The odd Roots, as the 3d, 5th, 7th, &c Roots, of all real quantities, whether positive or negative, are real, and are respectively positive or negative. So the cube Root of a^3 is a, and of $-a^3$ is $-a$.

But the even Roots, as the 2d, 4th, 6th, &c, are only real when the quantity is positive; being imaginary or impossible when the quantity is negative. So the square Root of a^2 is a, which is real; but the square Root of $-a^2$, that is, $\sqrt{-a^2}$, is imaginary or impossible; because there is no quantity, neither $+a$ nor $-a$, which by squaring will make the given negative square $-a^2$.

TABLE *of* ROOTS, &c.

THE following Table of Roots, Squares, and Cubes, is very useful in many calculations, and will serve to find square-Roots and cube Roots, as well as square and cubic powers. The Table consists of three columns: in the first column are the series of common numbers, or Roots, 1, 2, 3, 4, 5, 6, &c; in the second column are the squares, and in the third column the cubes of the same. For example, to find the square or the cube of the number or Root 49. Finding this number 49 in the first column; upon the same line with it, stands its square 2401 in the second column, and its cube 117649 in the third column.

Again, to find the square Root of the number 700. Near the beginning of the Table, it appears that the next less and greater tabular squares are 676 and 729, whose Roots are 26 and 27, and therefore the square Root of 700 is between 26 and 27. But a little further on, viz, among the hundreds, it appears that the required Root lies between 26·4 and 26·5, the tabular squares of these being 696·96 and 702·25, cutting off the proper part of the figures for decimals. Take the difference between the less square 696·96 and the given number 700, which gives 3·04, and divide the half of it, viz 1·52, by the less given tabular Root, viz 26·4, and the quotient 575 gives as many more figures of the Root, to be joined to the first three, and thus making the Root equal to 26·4575, which is true in all its places.

Also to find the cube Root of the number 7000; near the beginning of the Table, among the tens, it appears that the cube Root of this number is between 19 and 20; but farther on, among the hundreds, it appears that it lies between 19·1 and 19·2, allowing for the proper number of integers. But if more figures are required; from the given number 7000 take the next less tabular one, or the cube of 19·1, viz 6967871, and there remains 32·129, the 3d part of which, or 10·730, divide by the square of 19·1, viz 364·81, found on the same line, and the quotient 293 is the next three figures of the Root, and therefore the whole cubic Root is 19·1293, which is true in all its figures.—The Table follows.

TABLE

TABLE *of Square and Cubic* ROOTS.

Root.	Square	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.
1	1	1	64	4096	262144	127	16129	2048383	190	36100	6859000
2	4	8	65	4225	274625	128	16384	2097152	191	36481	6967871
3	9	27	66	4356	287496	129	16641	2146689	192	36864	7077888
4	16	64	67	4489	300763	130	16900	2197000	193	37429	7189057
5	25	125	68	4624	314432	131	17161	2248091	194	37636	7301384
6	36	216	69	4761	328509	132	17424	2299968	195	38025	7414875
7	49	343	70	4900	343000	133	17689	2352637	196	38416	7529536
8	64	512	71	5041	357911	134	17956	2406104	197	38809	7645373
9	81	729	72	5184	373248	135	18225	2460375	198	39204	7762392
10	100	1000	73	5329	389017	136	18496	2515456	199	39601	7880599
11	121	1331	74	5476	405224	137	18769	2571353	200	40000	8000000
12	144	1728	75	5625	421875	138	19044	2628072	201	40401	8120601
13	169	2197	76	5776	438976	139	19321	2685619	202	40804	8242408
14	196	2744	77	5929	456533	140	19600	2744000	203	41209	8365427
15	225	3375	78	6084	474552	141	19881	2803221	204	41616	8489664
16	256	4096	79	6241	493039	142	20164	2863288	205	42025	8615125
17	289	4913	80	6400	512000	143	20449	2924207	206	42436	8741816
18	324	5832	81	6561	531441	144	20736	2985984	207	42849	8869743
19	361	6859	82	6724	551368	145	21025	3048625	208	43264	8998912
20	400	8000	83	6889	571787	146	21316	3112136	209	43681	9123329
21	441	9261	84	7056	592704	147	21609	3176523	210	44100	9261000
22	484	10648	85	7225	614125	148	21904	3241792	211	44521	9393931
23	529	12167	86	7396	636056	149	22201	3307949	212	44944	9528128
24	576	13824	87	7569	658503	150	22500	3375000	213	45369	9663597
25	625	15625	88	7744	681472	151	22801	3442951	214	45796	9800344
26	676	17576	89	7921	704969	152	23104	3511808	215	46225	9938375
27	729	19683	90	8100	729000	153	23409	3581577	216	46656	10077696
28	784	21952	91	8281	753571	154	23716	3652264	217	47089	10218313
29	841	24389	92	8464	778688	155	24025	3723875	218	47524	10360282
30	900	27000	93	8649	804357	156	24336	3796416	219	47961	10503459
31	961	29791	94	8836	830584	157	24649	3869893	220	48400	10648000
32	1024	32768	95	9025	857375	158	24964	3944312	221	48841	10793861
33	1089	35937	96	9216	884736	159	25281	4019679	222	49284	10941048
34	1156	39304	97	9409	912673	160	25600	4096000	223	49729	11089567
35	1225	42875	98	9604	941192	161	25921	4173281	224	50176	11239424
36	1296	46656	99	9801	970299	162	26244	4251528	225	50625	11390625
37	1369	50653	100	10000	1000000	163	26569	4330747	226	51076	11543176
38	1444	54872	101	10201	1030301	164	26896	4410944	227	51529	11697083
39	1521	59319	102	10404	1061208	165	27225	4492125	228	51984	11852352
40	1600	64000	103	10609	1092727	166	27556	4574296	229	52441	12008989
41	1681	68921	104	10816	1124864	167	27889	4657463	230	52900	12167000
42	1764	74088	105	11025	1157625	168	28224	4741632	231	53361	12326391
43	1849	79507	106	11236	1191016	169	28561	4826809	232	53824	12487168
44	1936	85184	107	11449	1225043	170	28900	4913000	233	54289	12649337
45	2025	91125	108	11664	1259712	171	29241	5000211	234	54756	12812904
46	2116	97336	109	11881	1295029	172	29584	5088448	235	55225	12977875
47	2209	103823	110	12100	1331000	173	29929	5177717	236	55696	13144256
48	2304	110592	111	12321	1367631	174	30276	5268024	237	56169	13312053
49	2401	117649	112	12544	1404928	175	30625	5359375	238	56644	13481272
50	2500	125000	113	12769	1442897	176	30976	5451776	239	57121	13651919
51	2601	132651	114	12996	1481544	177	31329	5545233	240	57600	13824000
52	2704	140608	115	13225	1520875	178	31684	5639752	241	58081	13997521
53	2809	148877	116	13456	1560896	179	32041	5735339	242	58564	14172488
54	2916	157464	117	13689	1601613	180	32400	5832000	243	59049	14348907
55	3025	166375	118	13924	1643032	181	32761	5929741	244	59536	14526784
56	3136	175616	119	14161	1685159	182	33124	6028568	245	60025	14706125
57	3249	185193	120	14400	1728000	183	33489	6128487	246	60516	14886936
58	3364	195112	121	14641	1771561	184	33856	6229504	247	61009	15069223
59	3481	205379	122	14884	1815848	185	34225	6331625	248	61504	15252992
60	3600	216000	123	15129	1860867	186	34596	6434856	249	62001	15438249
61	3721	226981	124	15376	1906624	187	34969	6539203	250	62500	15625000
62	3844	238328	125	15625	1953125	188	35344	6644672	251	63001	15813251
63	3969	250047	126	15876	2000376	189	35721	6751269	252	63504	16003008

Table of Square and Cubic ROOTS.

Root.	Square.	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.
253	64009	16194277	316	99856	31554496	379	143641	54439939	442	195364	86350888
254	64516	16387064	317	100489	31855013	380	144400	54872000	443	196249	86938307
255	65025	16581375	318	101124	32157432	381	145161	55306341	444	197136	87528384
256	65536	16777216	319	101761	32461759	382	145924	55742968	445	198025	88121125
257	66049	16974593	320	102400	32768000	383	146689	56181887	446	198916	88716536
258	66564	17173512	321	103041	33076161	384	147456	56623104	447	199809	89314623
259	67081	17373979	322	103684	33386248	385	148225	57066625	448	200704	89915392
260	67600	17576000	323	104329	33698267	386	148996	57512456	449	201601	90518849
261	68121	17779581	324	104976	34012224	387	149769	57960603	450	202500	91125000
262	68644	17984728	325	105625	34328125	388	150544	58411072	451	203401	91733851
263	69169	18191447	326	106276	34645976	389	151321	58863869	452	204304	92345408
264	69696	18399744	327	106929	34965783	390	152100	59319000	453	205209	92959677
265	70225	18609625	328	107584	35287552	391	152881	59776471	454	206116	93576664
266	70756	18821096	329	108241	35611289	392	153664	60236288	455	207025	94196375
267	71289	19034163	330	108900	35937000	393	154449	60698457	456	207936	94818816
268	71824	19248832	331	109561	36264691	394	155236	61162984	457	208849	95443993
269	72361	19465109	332	110224	36594368	395	156025	61629875	458	209764	96071912
270	72900	19683000	333	110889	36926037	396	156816	62099136	459	210681	96702579
271	73441	19902511	334	111556	37259704	397	157609	62570773	460	211600	97336000
272	73984	20123648	335	112225	37595375	398	158404	63044792	461	212521	97972181
273	74529	20346417	336	112896	37933056	399	159201	63521199	462	213444	98611128
274	75076	20570824	337	113569	38272753	400	160000	64000000	463	214369	99252847
275	75625	20796875	338	114244	38614472	401	160801	64481201	464	215296	99897344
276	76176	21024576	339	114921	38958219	402	161604	64964808	465	216225	100544625
277	76729	21253933	340	115600	39304000	403	162409	65450827	466	217156	101194696
278	77284	21484952	341	116281	39651821	404	163216	65939264	467	218089	101847563
279	77841	21717639	342	116964	40001688	405	164025	66430125	468	219024	102503232
280	78400	21952000	343	117649	40353607	406	164836	66923416	469	219961	103161709
281	78961	22188041	344	118336	40707584	407	165649	67419143	470	220900	103823000
282	79524	22425768	345	119025	41063625	408	166464	67917312	471	221841	104487111
283	80089	22665187	346	119716	41421736	409	167281	68417929	472	222784	105154048
284	80656	22906304	347	120409	41781923	410	168100	68921000	473	223729	105823817
285	81225	23149125	348	121104	42144192	411	168921	69426531	474	224676	106496424
286	81796	23393656	349	121801	42508549	412	169744	69934528	475	225625	107171875
287	82369	23639903	350	122500	42875000	413	170569	70444997	476	226576	107850176
288	82944	23887872	351	123201	43243551	414	171396	70957944	477	227529	108531333
289	83521	24137569	352	123904	43614208	415	172225	71473375	478	228484	109215352
290	84100	24389000	353	124609	43986977	416	173056	71991296	479	229441	109902239
291	84681	24642171	354	125316	44361864	417	173889	72511713	480	230400	110592000
292	85264	24897088	355	126025	44738875	418	174724	73034632	481	231361	111284641
293	85849	25153757	356	126736	45118016	419	175561	73560059	482	232324	111980168
294	86436	25412184	357	127449	45499293	420	176400	74088000	483	233289	112678587
295	87025	25672375	358	128164	45882712	421	177241	74618461	484	234256	113379904
296	87616	25934336	359	128881	46268279	422	178084	75151448	485	235225	114084125
297	88209	26198073	360	129600	46656000	423	178929	75686967	486	236196	114791256
298	88804	26463592	361	130321	47045881	424	179776	76225024	487	237169	115501303
299	89401	26730899	362	131044	47437928	425	180625	76765625	488	238144	116214272
300	90000	27000000	363	131769	47832147	426	181476	77308776	489	239121	116930169
301	90601	27270901	364	132496	48228544	427	182329	77854483	490	240100	117649000
302	91204	27543608	365	133225	48627125	428	183184	78402752	491	241081	118370771
303	91809	27818127	366	133956	49027896	429	184041	78953589	492	242064	119095488
304	92416	28094464	367	134689	49430863	430	184900	79507000	493	243049	119823157
305	93025	28372625	368	135424	49836032	431	185761	80062991	494	244036	120553784
306	93636	28652616	369	136161	50243409	432	186624	80621568	495	245025	121287375
307	94249	28934443	370	136900	50653000	433	187489	81182737	496	246016	122023936
308	94864	29218112	371	137641	51064811	434	188356	81746504	497	247009	122763473
309	95481	29503629	372	138384	51478848	435	189225	82312875	498	248004	123505992
310	96100	29791000	373	139129	51895117	436	190096	82881856	499	249001	124251499
311	96721	30080231	374	139876	52313624	437	190969	83453453	500	250000	125000000
312	97344	30371328	375	140625	52734375	438	191844	84027672	501	251001	125751501
313	97969	30664297	376	141376	53157376	439	192721	84604519	502	252004	126506008
314	98596	30959144	377	142129	53582633	440	193600	85184000	503	253009	127263527
315	99225	31255875	378	142884	54010152	441	194481	85766121	504	254016	128024064

Table of Square and Cube Roots.

Root.	Square.	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.	Root.	Square.	Cube.
505	255025	128787625	568	322624	183250432	631	398161	251239591	694	481636	334255384
506	256036	129554216	569	323761	184220009	632	399424	252435968	695	483025	335702375
507	257049	130323843	570	324900	185193000	633	400689	253636137	696	484416	337153536
508	258064	131096512	571	326041	186169411	634	401956	254840104	697	485809	338608873
509	259081	131872229	572	327184	187149248	635	403225	256047875	698	487204	340068392
510	260100	132651000	573	328329	188132517	636	404496	257259456	699	488601	341532099
511	261121	133432831	574	329476	189119224	637	405769	258474853	700	490000	343000000
512	262144	134217728	575	330625	190109375	638	407044	259694072	701	491401	344472101
513	263169	135005697	576	331776	191102976	639	408321	260917119	702	492804	345948008
514	264196	135796744	577	332929	192100033	640	409600	262144000	703	494209	347428927
515	265225	136590875	578	334084	193100552	641	410881	263374721	704	495616	348913664
516	266256	137388096	579	335241	194104539	642	412164	264609288	705	497025	350402625
517	267289	138188413	580	336400	195112000	643	413449	265847707	706	498436	351895816
518	268324	138991832	581	337561	196122941	644	414736	267089984	707	499849	353393243
519	269361	139798359	582	338724	197137368	645	416025	268336125	708	501264	354894912
520	270400	140608000	583	339889	198155287	646	417316	269586136	709	502681	356400829
521	271441	141420761	584	341056	199176704	647	418609	270840023	710	504100	357911000
522	272484	142236648	585	342225	200201625	648	419904	272097792	711	505521	359425431
523	273529	143055667	586	343396	201230056	649	421201	273359449	712	506944	360944128
524	274576	143877824	587	344569	202262003	650	422500	274625000	713	508369	362467097
525	275625	144703125	588	345744	203297472	651	423801	275894451	714	509796	363994344
526	276676	145531576	589	346921	204336469	652	425104	277167808	715	511225	365525875
527	277729	146363183	590	348100	205379000	653	426409	278445077	716	512656	367061696
528	278784	147197952	591	349281	206425071	654	427716	279726264	717	514089	368601813
529	279841	148035889	592	350464	207474688	655	429025	281011375	718	515524	370146232
530	280900	148877000	593	351649	208527857	656	430336	282300416	719	516961	371694959
531	281961	149721291	594	352836	209584584	657	431649	283593393	720	518400	373248000
532	283024	150568768	595	354025	210644875	658	432964	284890312	721	519841	374805361
533	284089	151419437	596	355216	211708736	659	434281	286191179	722	521284	376367048
534	285156	152273304	597	356409	212776173	660	435600	287496000	723	522729	377933067
535	286225	153130375	598	357604	213847192	661	436921	288804781	724	524176	379503424
536	287296	153990656	599	358801	214921799	662	438244	290117528	725	525625	381078125
537	288369	154854153	600	360000	216000000	663	439569	291434247	726	527076	382657176
538	289444	155720872	601	361201	217081801	664	440896	292754944	727	528529	384240583
539	290521	156590819	602	362404	218167208	665	442225	294079625	728	529984	385828352
540	291600	157464000	603	363609	219256227	666	443556	295408296	729	531441	387420489
541	292681	158340421	604	364816	220348864	667	444889	296740963	730	532900	389017000
542	293764	159220088	605	366025	221445125	668	446224	298077632	731	534361	390617891
543	294849	160103007	606	367236	222545016	669	447561	299418309	732	535824	392223168
544	295936	160989184	607	368449	223648543	670	448900	300763000	733	537289	393832837
545	297025	161878625	608	369664	224755712	671	450241	302111711	734	538756	395446904
546	298116	162771336	609	370881	225866529	672	451584	303464448	735	540225	397065375
547	299209	163667323	610	372100	226981000	673	452929	304821217	736	541696	398688256
548	300304	164566592	611	373321	228099131	674	454276	306182024	737	543169	400315553
549	301401	165469149	612	374544	229220928	675	455625	307546875	738	544644	401947272
550	302500	166375000	613	375769	230346397	676	456976	308915776	739	546121	403583419
551	303601	167284151	614	376996	231475544	677	458329	310288733	740	547600	405224000
552	304704	168196608	615	378225	232608375	678	459684	311665752	741	549081	406869021
553	305809	169112377	616	379456	233744896	679	461041	313046839	742	550564	408518488
554	306916	170031464	617	380689	234885113	680	462400	314432000	743	552049	410172407
555	308025	170953875	618	381924	236029032	681	463761	315821241	744	553536	411830784
556	309136	171879616	619	383161	237176659	682	465124	317214568	745	555025	413493625
557	310249	172808693	620	384400	238328000	683	466489	318611987	746	556516	415160936
558	311364	173741112	621	385641	239483061	684	467856	320013504	747	558009	416832723
559	312481	174676879	622	386884	240641848	685	469225	321419125	748	559504	418508992
560	313600	175616000	623	388129	241804367	686	470596	322828856	749	561001	420189749
561	314721	176558481	624	389376	242970624	687	471969	324242703	750	562500	421875000
562	315844	177504328	625	390625	244140625	688	473344	325660672	751	564001	423564751
563	316969	178453547	626	391876	245314376	689	474721	327082769	752	565504	425259008
564	318096	179406144	627	393129	246491883	690	476100	328509000	753	567009	426957777
565	319225	180362125	628	394384	247673152	691	477481	329939371	754	568516	428661064
566	320356	181321496	629	395641	248858189	692	478864	331373888	755	570025	430368875
567	321489	182284263	630	396900	250047000	693	480249	332812557	756	571536	432081216

Table of Square and Cubic Roots.

Root	Square	Cube	Root	Square	Cube	Root	Square	Cube	Root	Square	Cube
757	573049	433798093	820	672400	551368000	883	779689	688465387	946	894916	846590536
758	574564	435519512	821	674041	553387661	884	781456	690807104	947	896809	849278123
759	576081	437245479	822	675684	555412248	885	783225	693154125	948	898704	851971392
760	577600	438976000	823	677329	557441767	886	784996	695506456	949	900601	854670349
761	579121	440711081	824	678976	559476224	887	786769	697864103	950	902500	857375000
762	580644	442450728	825	680625	561515625	888	788544	700227072	951	904401	860085351
763	582169	444194947	826	682276	563559976	889	790321	702595369	952	906304	862801408
764	583696	445943744	827	683929	565609283	890	792100	704969000	953	908209	865523177
765	585225	447697125	828	685584	567663552	891	793881	707347971	954	910116	868250664
766	586756	449455096	829	687241	569722789	892	795664	709732288	955	912025	870983875
767	588289	451217663	830	688900	571787000	893	797449	712121957	956	913936	873722816
768	589824	452984832	831	690561	573856191	894	799236	714516984	957	915849	876467493
769	591361	454756609	832	692224	575930368	895	801025	716917375	958	917764	879217912
770	592900	456533000	833	693889	578009537	896	802816	719323136	959	919681	881974079
771	594441	458314011	834	695556	580093704	897	804609	721734273	960	921600	884736000
772	595984	460099648	835	697225	582182875	898	806404	724150792	961	923521	887503681
773	597529	461889917	836	698896	584277056	899	808201	726572699	962	925444	890277128
774	599076	463684824	837	700569	586376253	900	810000	729000000	963	927369	893056347
775	600625	465484375	838	702244	588480472	901	811801	731432701	964	929296	895841344
776	602176	467288576	839	703921	590589719	902	813604	733870808	965	931225	898632125
777	603729	469097433	840	705600	592704000	903	815409	736314327	966	933156	901428696
778	605284	470910952	841	707281	594823321	904	817216	738763264	967	935089	904231063
779	606841	472729139	842	708964	596947688	905	819025	741217625	968	937024	907039232
780	608400	474552000	843	710649	599077107	906	820836	743677416	969	938961	909853209
781	609961	476379541	844	712336	601211584	907	822649	746142643	970	940900	912673000
782	611524	478211768	845	714025	603351125	908	824464	748613312	971	942841	915498611
783	613089	480048687	846	715716	605495736	909	826281	751089429	972	944784	918330048
784	614656	481890304	847	717409	607645423	910	828100	753571000	973	946729	921167317
785	616225	483736625	848	719104	609800192	911	829921	756058031	974	948676	924010424
786	617796	485587656	849	720801	611960049	912	831744	758550528	975	950625	926859375
787	619369	487443403	850	722500	614125000	913	833569	761048497	976	952576	929714176
788	620944	489303872	851	724201	616295051	914	835396	763551944	977	954529	932574833
789	622521	491169069	852	725904	618470208	915	837225	766060875	978	956484	935441352
790	624100	493039000	853	727609	620650477	916	839056	768575296	979	958441	938313739
791	625681	494913671	854	729316	622835864	917	840889	771095213	980	960400	941192001
792	627264	496793088	855	731025	625026375	918	842724	773620632	981	962361	944076141
793	628849	498677257	856	732736	627222016	919	844561	776151559	982	964324	946966168
794	630436	500566184	857	734449	629422793	920	846400	778688000	983	966289	949862087
795	632025	502459875	858	736164	631628712	921	848241	781229961	984	968256	952763904
796	633616	504358336	859	737881	633839779	922	850084	783777448	985	970225	955671625
797	635209	506261573	860	739600	636056000	923	851929	786330467	986	972196	958585256
798	636804	508169592	861	741321	638277381	924	853776	788889024	987	974169	961504803
799	638401	510082399	862	743044	640503928	925	855625	791453125	988	976144	964430272
800	640000	512000000	863	744769	642735647	926	857476	794022776	989	978121	967361669
801	641601	513922401	864	746496	644972544	927	859329	796597983	990	980100	970299000
802	643204	515849608	865	748225	647214625	928	861184	799178752	991	982081	973242271
803	644809	517781627	866	749956	649461896	929	863041	801765089	992	984064	976191488
804	646416	519718464	867	751689	651714363	930	864900	804357000	993	986049	979146657
805	648025	521660125	868	753424	653972032	931	866761	806954491	994	988036	982107784
806	649636	523606616	869	755161	656234909	932	868624	809557568	995	990025	985074875
807	651249	525557943	870	756900	658503000	933	870489	812166237	996	992016	988047936
808	652864	527514112	871	758641	660776311	934	872356	814780504	997	994009	991026973
809	654481	529475129	872	760384	663054848	935	874225	817400375	998	996004	994011992
810	656100	531441000	873	762129	665338617	936	876096	820025856	999	998001	997002999
811	657721	533411731	874	763876	667627624	937	877969	822656953	1000	1000000	1000000000
812	659344	535387328	875	765625	669921875	938	879844	825293672	1001	1002001	1003003001
813	660969	537366797	876	767376	672221376	939	881721	827936019	1002	1004004	1006012008
814	662596	539353144	877	769129	674526133	940	883600	830584000	1003	1006009	1009027027
815	664225	541343375	878	770884	676836152	941	885481	833237621	1004	1008016	1012048064
816	665856	543338496	879	772641	679151439	942	887364	835896888	1005	1010025	1015075125
817	667489	545338513	880	774400	681472000	943	889249	838561807	1006	1012036	1018108216
818	669124	547343432	881	776161	683797841	944	891136	841232384	1007	1014049	1021147343
819	670761	549353259	882	777924	686128968	945	893025	843908625	1008	1016064	1024192512

The following is another Table of the Square Roots of the first 1000 Numbers to 10 places of decimal figures beside the integers, which needs no farther explanation, as Numbers stand always in the first column, and their Square Roots in the next.

Table of Square ROOTS *to ten Decimal Places.*

No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.
1	1·0000000000	64	8·0000000000	127	11·2694276696	190	13·7840487521
2	1·4142135624	65	8·0622577483	128	11·3137084990	191	13·8202749611
3	1·7320508076	66	8·1240384046	129	11·3578166916	192	13·8564064606
4	2·0000000000	67	8·1853527719	130	11·4017542510	193	13·8924439894
5	2·2360679775	68	8·2462112512	131	11·4455231423	194	13·9283882772
6	2·4494897428	69	8·3066238629	132	11·4891252931	195	13·9642400438
7	2·6457513111	70	8·3666002653	133	11·5325625947	196	14·0000000000
8	2·8284271247	71	8·4261497732	134	11·5758369028	197	14·0356688441
9	3·0000000000	72	8·4852813742	135	11·6189500386	198	14·0712472795
10	3·1622776602	73	8·5440037453	136	11·6619037897	199	14·1067359797
11	3·3166247904	74	8·6023252670	137	11·7046999111	200	14·1421356237
12	3·4641016151	75	8·6602540378	138	11·7473443808	201	14·1774468788
13	3·6055512755	76	8·7177978871	139	11·7898261226	202	14·2126704036
14	3·7416573868	77	8·7749643874	140	11·8321595662	203	14·2478068488
15	3·8729833462	78	8·8317608663	141	11·8743420870	204	14·2828568571
16	4·0000000000	79	8·8881944173	142	11·9163752878	205	14·3178210633
17	4·1231056256	80	8·9442719100	143	11·9582607431	206	14·3527000944
18	4·2426406871	81	9·0000000000	144	12·0000000000	207	14·3874945699
19	4·3588989435	82	9·0553851381	145	12·0415945788	208	14·4222051019
20	4·4721359550	83	9·1104335791	146	12·0830459736	209	14·4568322948
21	4·5825756950	84	9·1651513899	147	12·1243556530	210	14·4913767462
22	4·6904157598	85	9·2195444573	148	12·1655250606	211	14·5258390463
23	4·7958315233	86	9·2736184955	149	12·2065556153	212	14·5602197786
24	4·8989794856	87	9·3273790531	150	12·2474487139	213	14·5945195193
25	5·0000000000	88	9·3808315196	151	12·2882057274	214	14·6287388383
26	5·0990195136	89	9·4339811321	152	12·3288280059	215	14·6628782986
27	5·1961524227	90	9·4868329805	153	12·3693168769	216	14·6969384567
28	5·2915026221	91	9.5393920142	154	12·4096736460	217	14·7309198627
29	5·3851648071	92	9·5916630466	155	12·4498995980	218	14·7648230602
30	5·4772255751	93	9.6436507610	156	12·4899959968	219	14·7986485869
31	5·5677643628	94	9·6953597148	157	12·5299640861	220	14·8323969742
32	5·6568542495	95	9 7467943448	158	12·5698050900	221	14·8660687473
33	5·7445626465	96	9·7979589711	159	12·6095202129	222	14·8996644258
34	5·8309518948	97	9·8488578018	160	12·6491106407	223	14·9331845231
35	5·9160797831	98	9·8994949366	161	12·6885775404	224	14·9666295471
36	6·0000000000	99	9·9498743711	162	12·7279220614	225	15·0000000000
37	6·0827625303	100	10·0000000000	163	12·7671453348	226	15·0332963784
38	6·1644140030	101	10·0498756211	164	12·8062484749	227	15·0665191733
39	6·2449979984	102	10·0995049384	165	12·8452325787	228	15·0996688705
40	6·3245553203	103	10·1488915651	166	12·8840987267	229	15·1327459504
41	6·4031242374	104	10·1980390272	167	12·9228479833	230	15·1657508881
42	6·4807406984	105	10·2469507660	168	12·9614813968	231	15·1986841536
43	6·5574385243	106	10·2956301410	169	13.0000000000	232	15·2315462117
44	6·6332495807	107	10·3440804328	170	13·0384048104	233	15·2643375225
45	6·7082039325	108	10.3923048454	171	13·0766968306	234	15·2970585408
46	6·7823299831	109	10.4403065089	172	13·1148770486	235	15·3297097168
47	6·8556546004	110	10·4880884817	173	13·1529464380	236	15·3622914957
48	6 9282032303	111	10·5356537529	174	13·1909059583	237	15·3948043183
49	7·0000000000	112	10·5830052443	175	13.2287565553	238	15·4272486209
50	7·0710678119	113	10·6301458127	176	13.2664991614	239	15·4596248337
51	7·1414284285	114	10·6770782520	177	13·3041346957	240	15·4919333848
52	7·2111025509	115	10·7238052948	178	13·3416640641	241	15·5241746963
53	7·2801098893	116	10·7703296143	179	13·3790881603	242	15·5563491861
54	7·3484692283	117	10·8166538264	180	13.4164078650	243	15·5884572681
55	7·4161984871	118	10·8627804912	181	13.4536240471	244	15·6204993518
56	7·4833147735	119	10·9087121146	182	13·4907375632	245	15·6524758425
57	7·5498344353	120	10·9544511501	183	13·5277492585	246	15·6843871414
58	7·6157731059	121	11·0000000000	184	13·5646599663	247	15·7162336455
59	7.6811457479	122	11·0453610172	185	13.6014705087	248	15·7480157480
60	7·7459666924	123	11·0905365064	186	13.6381816970	249	15·7797338381
61	7·8102496759	124	11·1355287257	187	13·6747943312	250	15·8113883008
62	7·8740078740	125	11·1803398875	188	13·7113092008	251	15·8429795178

Table of Square Roots.

No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.
253	15·9059737206	316	17·7763888346	379	19·4679223339	442	21·0237960416
254	15·9373774505	317	17·8044938148	380	19·4935886896	443	21·0475651798
255	15·9687194227	318	17·8325545001	381	19·5192212959	444	21·0713075057
256	16·0000000000	319	17·8605710995	382	19·5448202857	445	21·0950231097
257	16·0312195419	320	17·8885438200	383	19·5703857908	446	21·1187120819
258	16·0623784042	321	17·9164728672	384	19·5959179423	447	21·1423745119
259	16·0934769394	322	17·9443584449	385	19·6214168703	448	21·1660104885
260	16·1245154966	323	17·9722007556	386	19·6468827044	449	21·1896201004
261	16·1554944214	324	18·0000000000	387	19·6723155729	450	21·2132034356
262	16·1864140562	325	18·0277563773	388	19·6977156036	451	21·2367605816
263	16·2172747402	326	18·0554700853	389	19·7230829231	452	21.2602916255
264	16·2480768092	327	18·0831413200	390	19·7484176581	453	21·2837966538
265	16·2788205961	328	18·1107702763	391	19·7737199333	454	21·3072757527
266	16·3095064303	329	18·1383571472	392	19·7989898732	455	21·3307290077
267	16·3401346384	330	18·1659021246	393	19·8242276016	456	21·3541565041
268	16·3707055437	331	18·1934053987	394	19·8494332413	457	21·3775583264
269	16·4012194669	332	18·2208671583	395	19·8746069144	458	21·4009345590
270	16·4316767252	333	18·2482875909	396	19·8997487421	459	21·4242852856
271	16·4620776332	334	18·2756668825	397	19·9248588452	460	21·4476105895
272	16·4924225025	335	18·3030052177	398	19·9499373433	461	21·4709105536
273	16·5227116419	336	18·3303027798	399	19·9749843554	462	21·4941852579
274	16·5529453560	337	18·3575597507	400	20·0000000000	463	21·5174347914
275	16·5831239518	338	18·3847763109	401	20·0249843945	464	21·5406592285
276	16·6132477258	339	18·4119526395	402	20·0499376558	465	21·5638586528
277	16·6433169771	340	18·4390889146	403	20·0748598999	466	21·5870331449
278	16·6733320005	341	18·4661853126	404	20·0997512422	467	21·6101827850
279	16·7032930885	342	18·4932420089	405	20·1246117975	468	21·6333076528
280	16·7332005307	343	18·5202591775	406	20·1494416796	469	21·6564078277
281	16·7630546142	344	18·5472369910	407	20·1742410018	470	21·6794833887
282	16·7928556237	345	18·5741756210	408	20·1990098767	471	21·7025344142
283	16·8226038413	346	18·6010752377	409	20·2237484162	472	21·7255609824
284	16·8522995464	347	18·6279360102	410	20·2484567313	473	21·7485631709
285	16·8819430161	348	18·6547581062	411	20·2731349327	474	21·7715410571
286	16·9115345253	349	18·6815416923	412	20·2977831302	475	21·7944947177
287	16.9410743461	350	18·7082869339	413	20·3224014329	476	21·8174242293
288	16·9705627485	351	18·7349939952	414	20·3469899494	477	21·8403296678
289	17·0000000000	352	18·7616630393	415	20·3715487875	478	21·8632111091
290	17·0293863659	353	18·7882942281	416	20·3960780544	479	21·8860686282
291	17·0587221092	354	18·8148877222	417	20·4205778567	480	21·9089023002
292	17.0880074906	355	18·8414436814	418	20·4450483003	481	21·9317121995
293	17·1172427686	356	18·8679622641	419	20·4694894905	482	21·9544984024
294	17·1464281995	357	18·8944436277	420	20·4939015319	483	21·9772609758
295	17·1755640373	358	18·9208879284	421	20·5182845287	484	22·0000000000
296	17·2046505341	359	18·9472953215	422	20·5426385842	485	22·0227155455
297	17·2336879396	360	18·9736659610	423	20·5669638012	486	22·0454076850
298	17·2626765016	361	19·0000000000	424	20·5912602820	487	22·0680764907
299	17·2916164658	362	19·0262975904	425	20·6155281281	488	22.0907220344
300	17·3205080757	363	19·0525588833	426	20·6397674406	489	22·1133443875
301	17·3493515729	364	19·0787840283	427	20·6639783198	490	22·1359436212
302	17.3781471969	365	19·1049731745	428	20·6881608656	491	22·1585198062
303	17·4068951855	366	19·1311264697	429	20·7123151772	492	22·1810730128
304	17·4355957742	367	19·1572440607	430	20·7364413533	493	22·2036033112
305	17·4642491966	368	19·1833260933	431	20·7605394920	494	22·2261107709
306	17·4928556845	369	19·2093727123	432	20·7846096908	495	22·2485954613
307	17·5214154679	370	19·2353840617	433	20·8086520467	496	22·2710574513
308	17·5499287748	371	19·2613602843	434	20·8326666560	497	22·2934968096
309	17·5783958312	372	19·2873015220	435	20·8566536146	498	22·3159136044
310	17·6068168617	373	19·3132079158	436	20·8806130178	499	22·3383079039
311	17·6351920885	374	19·3390537514	437	20·9045449604	500	22.3606797750
312	17·6635217327	375	19·3649167310	438	20·9284495365	501	22·3830292856
313	17·6918060130	376	19·3907194297	439	20·9523268398	502	22·4053565024
314	17·7200451467	377	19·4164878389	440	20·9761769634	503	22·4276614920
315	17·7482393493	378	19·4422220952	441	21·0000000000	504	22·4499441206

Table of Square Roots.

No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.
505	22·4722050542	568	23·8327505756	631	25·1197133742	694	26·3438797446
506	22·4944437584	569	23·8537208838	632	25·1396101800	695	26·3628526529
507	22·5166604984	570	23.8746727726	633	25·1594912508	696	26·3818119165
508	22·5388553392	571	23·8956062907	634	25·1793566201	697	26·4007575649
509	22·5610283454	572	23·9165214862	635	25·1992063367	698	26·4196896272
510	22·5831795813	573	23·9374184072	636	25·2190404258	699	26·4386081328
511	22·6053091109	574	23·9582971014	637	25·2388589282	700	26·4575131106
512	22·6274169980	575	23·9791576166	638	25·2586618806	701	26·4764045897
513	22·6495033058	576	24·0000000000	639	25·2784493195	702	26·4952825990
514	22·6715680975	577	24·0208242989	640	25·2982212813	703	26·5141471671
515	22·6936114358	578	24·0416305603	641	25·3179778023	704	26·5329983228
516	22·7156333832	579	24·0624188310	642	25·3377189186	705	26·5518360947
517	22·7376340018	580	24·0831683962	643	25·3574446662	706	26·4706605112
518	22·7596133535	581	24.1039415864	644	25·3771550809	707	26·5894716006
519	22·7815714998	582	24·1246761636	645	25·3968501984	708	26·6082693913
520	22·8035085020	583	24·1453929353	646	25·4165300543	709	26·6270539114
521	22·8254244210	584	24·1660919472	647	25·4361946840	710	26·6458251889
522	22·8473193176	585	24·1867732449	648	25·4558441227	711	26·6645832519
523	22·8691932521	586	24·2074368736	649	25·4754784057	712	26·6833281283
524	22·8910462845	587	24·2280828792	650	25·4950975680	713	26·7020598456
525	22·9128784748	588	24·2487113060	651	25·5147016443	714	26·7207784318
526	22·9346898824	589	24·2693221990	652	25·5342906696	715	26·7394839142
527	22·9564805665	590	24·2899156030	653	25·5538646784	716	26·7581763205
528	22·9782505862	591	24·3104915623	654	25·5734237051	717	26·7768556780
529	23·0000000000	592	24·3310501212	655	25·5929677841	718	26·7955220139
530	23·0217288664	593	24·3515913238	656	25·6124969497	719	26·8141753556
531	23·0434372436	594	24·3721152139	657	25·6320112360	720	26·8328157300
532	23·0651251893	595	24·3926218353	658	25·6515106768	721	26·8514431642
533	23·0867927612	596	24·4131112315	659	25·6709953060	722	26·8700576851
534	23·1084400166	597	24·4335834457	660	25·6904651573	723	26·8886593195
535	23·1300670124	598	24·4540385213	661	25·7099202644	724	26·9072480941
536	23·1516738056	599	24·4744765010	662	25·7293606605	725	26·9258240357
537	23.1732604525	600	24·4948974278	663	25·7487863792	726	26·9443871706
538	23·1948270095	601	24·5153013443	664	25·7681974535	727	26·9629375254
539	23·2163735325	602	24·5356882928	665	25·7875939165	728	26·9814751265
540	23·2379000772	603	24·5560583156	666	25·8069758011	729	27·0000000000
541	23·2594066992	604	24·5764114549	667	25·8263431403	730	27·0185121722
542	23·2808934536	605	24·5967477525	668	25·8456959666	731	27·0370116692
543	23·3023603955	606	24·6170672502	669	25·8650343128	732	27·0554985169
544	23·3238075794	607	24·6373699895	670	25·8843582111	733	27·0739727414
545	23·3452350599	608	24·6576560119	671	25·9036676940	734	27·0924343683
546	23·3666428911	609	24·6779253585	672	25·9229627936	735	27·1108834235
547	23·3880311271	610	24·6981780705	673	25·9422435421	736	27·1293199325
548	23·4093998214	611	24·7184141886	674	25·9615099715	737	27·1477439210
549	23·4307490277	612	24·7386337537	675	25·9807621135	738	27·1661554144
550	23·4520787991	613	24·7588368063	676	26·0000000000	739	27·1845544381
551	23·4733891886	614	24·7790233867	677	26·0192236625	740	27·2029410175
552	23·4946802489	615	24·7991935353	678	26·0384331326	741	27·2213151776
553	23·5159520326	616	24·8193472920	679	26·0576284416	742	27·2396769438
554	23·5372045919	617	24·8394846967	680	26·0768096208	743	27·2580263409
555	23·5584379788	618	24·8596057893	681	26·0959767014	744	27·2763633940
556	23·5796522451	619	24·8797106092	682	26·1151297144	745	27·2946881279
557	23·6008474424	620	24·8997991960	683	26·1342686907	746	27·3130005675
558	23·6220236220	621	24·9198715888	684	26·1533936612	747	27·3313007374
559	23·6431808351	622	24·9399278267	685	26·1725046566	748	27·3495886624
560	23·6643191324	623	24·9599679487	686	26·1916017074	749	27·3678643668
561	23·6854385647	624	24·9799919936	687	26·2106848442	750	27·3861278753
562	23·7065391823	625	25·0000000000	688	26·2297540972	751	27·4043792121
563	23·7276210354	626	25·0199920064	689	26·2488094968	752	27·4226184016
564	23·7486841741	627	25·0399680511	690	26·2678510731	753	27·4408454680
565	23·7697286480	628	25·0599281723	691	26·2868788562	754	27·4590604355
566	23·7907545067	629	25·0798724080	692	26·3058928759	755	27·4772633281
567	23·8117617996	630	25·0998007960	693	26·3248931622	756	27·4954541697

Table of Square Roots.

No.	Square Root.	No.	Square Root.	No.	Square Root.	No.	Square Root.
757	27·5136329844	818	28·6006992922	879	29·6479324743	940	30·6594194335
758	27·5317997959	819	28·6181760425	880	29·6647939484	941	30·6757233004
759	27·5499546279	820	28·6356421266	881	29·6816441593	942	30·6920185064
760	27·5680975042	821	28·6530975638	882	29·6984848098	943	30·7083050656
761	27·5862284483	822	28·6705423737	883	29·7153159162	944	30·7245829915
762	27·6043474837	823	28·6879765756	884	29·7321374946	945	30·7408522979
763	27·6224546339	824	28·7054001888	885	29·7489495613	946	30·7571129985
764	27·6405499222	825	28·7228132327	886	29·7657521323	947	30·7733651069
765	27·6586333719	826	28·7402157264	887	29·7825452237	948	30·7896086367
766	27·6767050062	827	28·7576076891	888	29·7993288515	949	30·8058436015
767	27·6947648483	828	28·7749891399	889	29·8161030318	950	30·8220700148
768	27·7128129211	829	28·7923600978	890	29·8328677804	951	30·8382878902
769	27·7308492477	830	28·8097205818	891	29·8496231132	952	30·8544972417
770	27·7488738510	831	28·8270706108	892	29·8663690461	953	30·8706980809
771	27·7668867538	832	28·8444102037	893	29·8831055950	954	30·8868904230
772	27·7848879789	833	28·8617393793	894	29·8998327755	955	30·9030742807
773	27·8028775489	834	28·8790581564	895	29·9165506033	956	30·9192496675
774	27·8208554865	835	28·8963665536	896	29·9332590942	957	30·9354165965
775	27·8388218142	836	28·9136645896	897	29·9499582637	958	30·9515750811
776	27·8567765544	837	28·9309522830	898	29·9666481275	959	30·9677251344
777	27·8747197295	838	28·9482296523	899	29·9833287011	960	30·9838667697
778	27·8926513620	839	28·9654967159	900	30·0000000000	961	31·0000000000
779	27·9105714739	840	28·9827534924	901	30·0166620396	962	31·0161248385
780	27·9284800875	841	29·0000000000	902	30·0333148354	963	31·0322412984
781	27·9463772250	842	29·0172362571	903	30·0499584026	964	31·0483493925
782	27·9642629082	843	29·0344622819	904	30·0665927567	965	31·0644491340
783	27·9821371593	844	29·0516780927	905	30·0832179130	966	31·0805405358
784	28·0000000000	845	29·0688837075	906	30·0998338866	967	31·0966236109
785	28·0178514522	846	29·0860791445	907	30·1164406928	968	31·1126983722
786	28·0356915378	847	29·1032644217	908	30·1330383466	969	31·1287648325
787	28·0535202782	848	29·1204395571	909	30·1496268634	970	31·1448230048
788	28·0713376881	849	29·1376045687	910	30·1662062580	971	31·1608729018
789	28·0891438104	850	29·1547594742	911	30·1827765456	972	31·1769145362
790	28·1069386451	851	29·1719042916	912	30·1993377411	973	31·1929479210
791	28·1247222209	852	29·1890390387	913	30·2158898595	974	31·2089730687
792	28·1424945589	853	29·2061637330	914	30·2324329157	975	31·2249899920
793	28·1602556807	854	29·2232783924	915	30·2489669245	976	31·2409987036
794	28·1780056072	855	29·2403830344	916	30·2654919008	977	31·2569992162
795	28·1957443597	856	29·2574776767	917	30·2820078595	978	31·2729915422
796	28·2134719593	857	29·2745623366	918	30·2985148151	979	31·2889756943
797	28·2311884270	858	29·2916370318	919	30·3150127824	980	31·3049516850
798	28·2488937837	859	29·3087017795	920	30·3315017762	981	31·3209195267
799	28·2665880502	860	29·3257565972	921	30·3479818110	982	31·3368792320
800	28·2842712475	861	29·3428015022	922	30·3644529014	983	31·3528308132
801	28·3019433962	862	29·3598365118	923	30·3809150619	984	31·3687742827
802	28·3196045170	863	29·3768616431	924	30·3973683071	985	31·3847096530
803	28·3372546306	864	29·3938769134	925	30·4138126515	986	31·4006369362
804	28·3548937575	865	29·4108823397	926	30·4302481094	987	31·4165561448
805	28·3725219182	866	29·4278779391	927	30·4466746953	988	31·4324672910
806	28·3901391332	867	29·4448637287	928	30·4630924235	989	31·4483703870
807	28·4077454227	868	29·4618397253	929	30·4795013083	990	31·4642654451
808	28·4253408071	869	29·4788059460	930	30·4959013640	991	31·4801524774
809	28·4429253067	870	29·4957624075	931	30·5122926048	992	31·4960314960
810	28·4604989415	871	29·5127091267	932	30·5286750449	993	31·5119025132
811	28·4780617318	872	29·5296461205	933	30·5450486986	994	31·5277655409
812	28·4956136976	873	29·5465734054	934	30·5614135799	995	31·5436205912
813	28·5131548588	874	29·5634909982	935	30·5777697028	996	31·5594676761
814	28·5306852354	875	29·5803989155	936	30·5941170816	997	31·5753068077
815	28·5482048472	876	29·5972971739	937	30·6104557300	998	31·5911379979
816	28·5657137142	877	29·6141857899	938	30·6267856622	999	31·6069612586
817	28·5832118559	878	29·6310647801	939	30·6431068921	1000	31·6227766017

ROTA, in Mechanics. See WHEEL.

ROTA *Ariſtotelica*, or *Ariſtotle's Wheel*, denotes a celebrated problem in mechanics, concerning the motion or rotation of a wheel about its axis; ſo called becauſe firſt noticed by Ariſtotle.

The difficulty is this. While a circle makes a revolution on its centre, advancing at the ſame time in a right line along a plane, it deſcribes, on that plane, a right line which is equal to its circumference. Now if this circle, which may be called the deferent, carry with it another ſmaller circle, concentric with it, like the nave of a coach wheel; then this little circle, or nave, will deſcribe a line in the time of the revolution, which ſhall be equal to that of the large wheel or circumference itſelf; becauſe its centre advances in a right line as faſt as that of the wheel does, being in reality the ſame with it.

The ſolution given by Ariſtotle, is no more than a good explication of the difficulty.

Galileo, who next attempted it, has recourſe to an infinite number of infinitely little vacuities in the right line deſcribed by the two circles; and imagines that the little circle never applies its circumference to thoſe vacuities; but in reality only applies it to a line equal to its own circumference; though it appears to have applied it to a much larger. But all this is nothing to the purpoſe.

Tacquet will have it, that the little circle, making its rotation more ſlowly than the great one, does on that account deſcribe a line longer than its own circumference; yet without applying any point of its circumference to more than one point of its baſe. But this is no more ſatisfactory than the former.

After the fruitleſs attempts of ſo many great men, M. Dortous de Meyran, a French gentleman, had the good fortune to hit upon a ſolution, which he ſent to the Academy of Sciences; where being examined by Meſſ. de Louville and Soulmon, appointed for that purpoſe, they made their report that it was ſatisfactory. The ſolution is to this effect:

The wheel of a coach is only acted on, or drawn in a right line; its rotation or circular motion ariſes purely from the reſiſtance of the ground upon which it is applied. Now this reſiſtance is equal to the force which draws the wheel in the right line, inaſmuch as it defeats that direction; of conſequence the cauſes of the two motions, the one right and the other circular, are equal. And hence the wheel deſcribes a right line on the ground equal to its circumference.

As for the nave of the wheel, the caſe is otherwiſe. It is drawn in a right line by the ſame force as the wheel; but it only turns round becauſe the wheel does ſo, and can only turn in the ſame time with it. Hence it follows, that its circular velocity is leſs than that of the wheel, in the ratio of the two circumferences; and therefore its circular motion is leſs than the rectilinear one. Since then it neceſſarily deſcribes a right line equal to that of the wheel, it can only do it partly by ſliding, and partly by revolving, the ſliding part being more or leſs as the nave itſelf is ſmaller or larger. See CYCLOID.

ROTATION, *Rolling*, in Mechanics. See ROLLING.

ROTATION, in Geometry, the circumvolution of a ſurface round an immoveable line, called the *axis of Rotation*. By ſuch Rotation of planes, the figures of certain regular ſolids are formed or generated. Such as, a cylinder by the Rotation of a rectangle, a cone by the Rotation of a triangle, a ſphere or globe by the Rotation of a ſemicircle, &c.

The method of cubing ſolids that are generated by ſuch Rotation, is laid down by Mr. Demoivre, in his ſpecimen of the uſe of the doctrine of fluxions, Philoſ. Tranſ. numb. 216; and indeed by moſt of the writers on Fluxions. In every ſuch ſolid, all the ſections perpendicular to the axis are circles, and therefore the fluxion of the ſolid, at any ſection, is equal to that circle multiplied by the fluxion of the axis. So that, if x denote an abſciſs of that axis, and y an ordinate to it in the revolving plane, which will alſo be the radius of that circle; then, n being put for 3·1416, the area of the circle is ny^2, and conſequently the fluxion of the ſolid is $ny^2\dot{x}$; the fluent of which will be the content of the ſolid.

Such ſolid may alſo be expreſſed in terms of the generating plane and its centre of gravity; for the ſolid is always equal to the product ariſing from the generating plane multiplied by the path of its centre of gravity, or by the line deſcribed by that centre in the Rotation of the plane. And this theorem is general, by whatever kind of motion the plane is moved, in deſcribing a ſolid.

ROTATION, *Revolution*, in Aſtronomy. See REVOLUTION.

Diurnal ROTATION. See DIURNAL, and EARTH.

ROTONDO, or ROTUNDO, in Architecture, a popular term for any building that is round both within and withoutſide, whether it be a church, hall, a ſaloon, a veſtibule, or the like.

ROUND, ROUNDNESS, ROTUNDITY, the property of a circle and ſphere or globe &c.

ROWNING (JOHN), an ingenious Engliſh mathematician and philoſopher, was fellow of Magdalen College, Cambridge, and afterwards Rector of Anderby in Lincolnſhire, in the gift of that ſociety. He was a conſtant attendant at the meetings of the Spalding Society, and was a man of a great philoſophical habit and turn of mind, though of a cheerful and companionable diſpoſition. He had a good genius for mechanical contrivances in particular. In 1738 he printed at Cambridge, in 8vo, *A Compendious Syſtem of Natural Philoſophy*, in 2 vols 8vo; a very ingenious work, which has gone through ſeveral editions. He had alſo two pieces inſerted in the Philoſophical Tranſactions, viz, 1. *A Deſcription of a Barometer wherein the Scale of Variation may be increaſed at pleaſure*; vol. 38, pa. 39. And 2. *Direction for making a Machine for finding the Roots of Equations univerſally, with the Manner of uſing it*; vol. 60, pa. 240.—Mr. Rowning died at his lodgings in Carey-ſtreet near Lincoln's Inn Fields, the latter end of November 1771, at 72 years of age.

Though a very ingenious and pleaſant man, he had but an unpromiſing and forbidding appearance: he was tall, ſtooping in the ſhoulders, and of a ſallow down-looking countenance.

ROYAL *Oak*, *Robur Carolinum*, in Aſtronomy, one of the new ſouthern conſtellations, the ſtars of which,

which, according to Sharp's catalogue, annexed to the Britannic, are 12.

ROYAL *Society of England*, is an academy or body of perfons, fuppofed to be eminent for their learning, inftituted by king Charles the IId, for promoting natural knowledge.

This once illuftrious body originated from an affembly of ingenious men, refiding in London, who, being inquifitive into natural knowledge, and the new and experimental philofophy, agreed, about the year 1645, to meet weekly on a certain day, to difcourfe upon fuch fubjects. Thefe meetings, it is faid, were fuggefted by Mr. Theodore Haak, a native of the Palatinate in Germany; and they were held fometimes at Dr. Goddard's lodgings in Wood-ftreet, fometimes at a convenient place in Cheapfide, and fometimes in or near Grefham College. This affembly feems to be that mentioned under the title of the *Invifible, or Philofophical College*, by Mr. Boyle, in fome letters written in 1646 and 1647. About the years 1648 and 1649, the company which formed thefe meetings, began to be divided, fome of the gentlemen removing to Oxford, as Dr. Wallis, and Dr. Goddard, where, in conjunction with other gentlemen, they held meetings alfo, and brought the ftudy of natural and experimental philofophy into fafhion there; meeting firft in Dr. Petty's lodgings, afterwards at Dr. Wilkins's apartments in Wadham College, and, upon his removal, in the lodgings of Mr. Robert Boyle; while thofe gentlemen who remained in London continued their meetings as before. The greater part of the Oxford Society coming to London about the year 1659, they met once or twice a week in Term-time at Grefham College, till they were difperfed by the public diftractions of that year, and the place of their meeting was made a quarter for foldiers. Upon the reftoration, in 1660, their meetings were revived, and attended by many gentlemen, eminent for their character and learning.

They were at length noticed by the government, and the king granted them a charter, firft the 15th of July 1662, then a more ample one the 22d of April 1663, and thirdly the 8th of April 1669; by which they were erected into a corporation, *confifting of a prefident, council, and fellows, for promoting natural knowledge*, and endued with various privileges and authorities.

Their manner of electing members is by ballotting; and two-thirds of the members prefent are neceffary to carry the election in favour of the candidate. The council confifts of 21 members, including the prefident, vice-prefident, treafurer, and two fecretaries; ten of which go out annually, and ten new members are elected inftead of them, all chofen on St. Andrew's day. They had formerly alfo two curators, whofe bufinefs it was to perform experiments before the fociety.

Each member, at his admiffion, fubfcribes an engagement, that he will endeavour to promote the good of the fociety; from which he may be freed at any time, by fignifying to the prefident that he defires to withdraw.

The charges are five guineas paid to the treafurer at admiffion; and one fhilling per week, or 52s. per year, as long as the perfon continues a member; or, in lieu of the annual fubfcription, a compofition of 25 guineas in one payment.

The ordinary meetings of the fociety, are once a week, from November till the end of Trinity term the next fummer. At firft, the meeting was from 3 o'clock till 6 afternoon. Afterwards, their meeting was from 6 till 7 in the evening, to allow more time for dinner, which continued for a long feries of years, till the hour of meeting was removed, by the prefent prefident, to between 8 and 9 at night, that gentlemen of fafhion, as was alleged, might have the opportunity of coming to attend the meetings after dinner.

Their defign is to "make faithful records of all the "works of nature or art, which come within their "reach; fo that the prefent, as well as after ages, "may be enabled to put a mark on errors which have "been ftrengthened by long prefcription; to reftore "truths that have been long neglected; to pufh thofe "already known to more various ufes; to make the "way more paffable to what remains unrevealed, "&c."

To this purpofe they have made a great number of experiments and obfervations on moft of the works of nature; as eclipfes, comets, planets, meteors, mines, plants, earthquakes, inundations, fprings, damps, fires, tides, currents, the magnet, &c: their motto being *Nullius in Verba*. They have regiftered experiments, hiftories, relations, obfervations, &c, and reduced them into one common ftock. They have, from time to time, publifhed fome of the moft ufeful of thefe, under the title of Philofophical Tranfactions, &c. ufually one volume each year, which were, till lately, very refpectable, both for the extent or magnitude of them, and for the excellent quality of their contents. The reft, that are not printed, they lay up in their regifters.

They have a good library of books, which has been formed, and continually augmenting, by numerous donations. They had alfo a mufeum of curiofities in nature, kept in one of the rooms of their own houfe in Crane Court Fleet-ftreet, where they held their meetings, with the greateft reputation, for many years, keeping regifters of the weather, and making other experiments; for all which purpofes thofe apartments were well adapted. But, difpofing of thefe apartments, in order to remove into thofe allotted them in Somerfet Place, where having neither room nor convenience for fuch purpofes, the mufeum was obliged to be difpofed of, and their ufeful meteorological regifters difcontinued for many years.

Sir Godfrey Copley, bart. left 5 guineas to be given annually to the perfon who fhould write the beft paper in the year, under the head of experimental philofophy: this reward, which is now changed to a gold medal, is the higheft honour the fociety can beftow; and it is conferred on St. Andrew's day: but the communications of late years have been thought of fo little importance, that the prize medal remains fometimes for years undifpofed of.

Indeed this once very refpectable fociety, now confifting of a great proportion of honorary members, who do not ufually communicate papers; and many fcientific members being difcouraged from making their

ufual

uſual communications, by what is deemed the preſent arbitrary government of the ſociety; the annual volumes have in conſequence become of much leſs importance, both in reſpect of their bulk and the quality of their contents.

ROYAL *Society of Scotland.* See SOCIETY.

RUDOLPHINE *Tables*, a ſet of aſtronomical tables that were publiſhed by the celebrated Kepler, and ſo called from the emperor Rudolph or Rudolphus.

RULE, *The Carpenters*, a folding ruler generally uſed by carpenters and other artificers; and is otherwiſe called the ſliding Rule.

This inſtrument conſiſts of two equal pieces of box-wood, each one foot in length, connected together by a folding joint. One ſide or face, of the Rule, is divided into inches, and half-quarters, or eighths. On the ſame face alſo are ſeveral plane ſcales, divided into 12th parts by diagonal lines; which are uſed in planning dimenſions that are taken in feet and inches. The edge of the Rule is commonly divided decimally, or into 10ths; viz, each foot into 10 equal parts, and each of theſe into 10 parts again, or 100dth parts of the foot: ſo that by means of this laſt ſcale, dimenſions are taken in feet and tenths and hundreds, and multiplied together as common decimal numbers, which is the beſt way.

On the one part of the other face are four lines, marked A, B, C, D, the two middle ones B and C being on a ſlider, which runs in a groove made in the ſtock. The ſame numbers ſerve for both theſe two middle lines, the one line being above the numbers, and the other below them.

Theſe four lines are logarithmic ones, and the three A, B, C, which are all equal to one another, are double lines, as they proceed twice over from 1 to 10. The loweſt line D is a ſingle one, proceeding from 4 to 40. It is alſo called the girt line, from its uſe in caſting up the contents of trees and timber: and upon it are marked WG at 17·15, and AG at 18·95, the wine and ale gauge points, to make this inſtrument ſerve the purpoſe of a gauging rule.

Upon the other part of this face is a table of the value of a load, or 50 cubic feet, of timber, at all prices, from 6 pence to 2s. a foot.

When 1 at the beginning of any line is accounted only 1, then the 1 in the middle is 10, and the 10 at the end 100; and when the 1 at the beginning is accounted 10, then 1 in the middle is 100, and the 10 at the end 1000; and ſo on. All the ſmaller diviſions being alſo altered proportionally.

By means of this Rule all the uſual operations of arithmetic may be eaſily and quickly performed, as multiplication, diviſion, involution, evolution, finding mean proportionals, 3d and 4th proportionals, or the Rule-of-three, &c. For all which, ſee my Menſuration, part 5, ſect. 3, 2d edition.

RULES *of Philoſophizing.* See PHILOSOPHIZING.

RULE, in Arithmetic, denotes a certain mode of operation with figures to find ſums or numbers unknown, and to facilitate computations.

Each Rule in arithmetic has its particular name, according to the uſe for which it is intended. The firſt four, which ſerve as a foundation of the whole art, are called *addition*, *ſubtraction*, *multiplication*, and *diviſion*.

From theſe ariſe numerous other Rules, which are indeed only applications of theſe to particular purpoſes and occaſions; as the Rule-of-three, or Golden Rule, or Rule of Proportion; alſo the Rules of Fellowſhip, Intereſt, Exchanges, Poſition, Progreſſions, &c, &c. For which, ſee each article ſeverally.

RULE-*of-Three*, or *Rule of Proportion*, commonly called the *Golden Rule* from its great uſe, is a Rule that teaches how to find a 4th proportional number to three others that are given.

As, if 3 degrees of the equator contain 208 miles, how many are contained in 360 degrees, or the whole circumference of the earth?

The Rule is this: State, or ſet the three given terms down in the form of the firſt three terms of a proportion, ſtating them proportionally, thus:

$$\begin{array}{rrrrrrr} & \text{deg.} & & \text{mil.} & & \text{deg.} & \text{miles.} \\ \text{as} & 3 & : & 208 & :: & 360 & : 24960 \\ & & & 360 & & & \\ \hline & & & 12480 & & & \\ & & & 624 & & & \\ \hline & & 3\,) & 74880 & & & \\ \hline & & & 24960 & & & \end{array}$$

Then multiply the 2d and 3d terms together, and divide the product by the 1ſt term, ſo ſhall the quotient be the 4th term in proportion, or the anſwer to the queſtion, which in this example is 24960 or nearly 25 thouſand miles, for the circumference of the earth.

This rule is often conſidered as of two kinds, viz. *Direct*, and *Inverſe*.

Rule-of-Three Direct, is that in which more requires more, or leſs requires leſs. As in this; if 3 men mow 21 yards of graſs in a certain time, how much will 6 men mow in the ſame time? Here more requires more, that is, 6 men, which are more than 3 men, will alſo perform more work, in the ſame time. Or if it were thus; if 6 men mow 42 yards, how much will 3 men mow in the ſame time? here then leſs requires leſs, or 3 men will perform proportionally leſs work, in the ſame time. In both theſe caſes then, the Rule, or the proportion, is direct; and the ſtating muſt be

thus, as 3 : 21 : : 6 : 42,
or thus, as 6 : 42 : : 3 : 21.

Rule-of-Three Inverſe, is when more requires leſs, or leſs requires more. As in this; if 3 men mow a certain quantity of graſs in 14 hours, in how many hours will 6 men mow the like quantity? Here it is evident that 6 men, being more than 3, will perform the ſame work in leſs time, or fewer hours; hence then more requires leſs, and the Rule or queſtion is inverſe, and muſt be ſtated by making the number of men change places, thus, as 6 : 14 : : 3 : 7 hours, the time in which 6 men will perform the work; ſtill multiplying the 2d and 3d terms together, and dividing by the 1ſt.

For various abbreviations, and other particulars relating

lating to these Rules, see any of the common books of arithmetic.

RULE-*of Five*, or *Compound Rule-of-Three*, is where two Rules-of-three are required to be wrought, or to be combined together, to find out the number sought.

This Rule may be performed, either by working the two statings or proportions separately, making the result or 4th term of the 1st operation to be the 2d term of the last proportion; or else by reducing the two statings into one, by multiplying the two first terms together, and the two third terms together, and using the products as the 1st and 3d terms of the compound stating. As, if the question be this: If 100l. in 2 years yield 9l. interest, how much will 500l. yield in 6 years. Here, the two statings are,

$$\left.\begin{matrix}100\\2\end{matrix}\right\} : 9 :: \left\{\begin{matrix}500\\6\end{matrix}\right.$$

Then, to work the two statings separately,

as 100 : 9 :: 500 : 45l.
and 2 : 45 :: 6 : 135l.

so that 135l. is the interest or answer sought. But to work by one stating, it will be thus,

$$\begin{array}{l}100 \qquad\quad 500\\ \underline{\quad 2} \qquad\quad \underline{\quad 6}\\ 200 : 9 :: \underline{3000} : 135\text{l. the answer.}\end{array}$$

2·00) 270.00 (135l.

See the books of arithmetic for more particulars.

Central RULE. See CENTRAL *Rule*.

Parallel RULER. See PARALLEL *Ruler*.

RUMB, or RUM. See RHUMB.

RUMB-*Line*, or *Loxodromic*. See RHUMB-*Line*.

RUSTIC, in Architecture, denotes a manner of building in imitation of simple or rude nature, rather than according to the rules of art.

RUSTIC *Quoins*. See QUOIN.

RUSTIC *Work* is where the stones in the face &c of a building, instead of being smooth, are hatched or picked with the point of an instrument.

Regular RUSTICS, are those in which the stones are chamfered off at the edges, and form angular or square recesses of about an inch deep at their jointings, or beds, and ends.

RUSTIC *Order*, is an order decorated with rustic quoins, or rustic work, &c.

RUTHERFORD (THOMAS, D. D.), an ingenious English philosopher, was the son of the Rev. Thomas Rutherford, rector of Papworth Everard in the county of Cambridge, who had made large collections for the history of that county.

Our author was born the 13th of October 1712. He studied at Cambridge, and became fellow of St. John's college, and regius professor of divinity, in that university; afterwards rector of Shenfield in Essex, and of Barley in Hertfordshire, and archdeacon of Essex. He died the 5th of October 1771, at 59 years of age.

Dr. Rutherford, besides a number of theological writings, published, at Cambridge,

1. *Ordo Institutionum Physicarum*, 1743, in 4to.

2. *A System of Natural Philosophy*, in 2 vols, 4to, 1748. A work which has been much esteemed.

3. He communicated also to the Gentleman's Society at Spalding, a curious correction of Plutarch's description of the instrument used to renew the Vestal fire, as relating to the triangle with which the instrument was formed. It was nothing else, it seems, but a concave speculum, whose principal focus, which collected the rays, is not in the centre of concavity, but at the distance of half a diameter from its surface. But some of the Ancients thought otherwise, as appears from prop. 31 of Euclid's Catoptrics.

The writer of his epitaph says, "He was eminent no less for his piety and integrity, than his extensive learning; and filled every public station in which he was placed with general approbation. In private life, his behaviour was truly amiable. He was esteemed, beloved, and honoured by his family and friends; and his death was sincerely lamented by all who had ever heard of his well deserved character."

S.

S, IN books of Navigation, &c, denotes fouth. So alfo S. E. is fouth-eaft; S. W. fouth-weft; and S. S. E. fouth-fouth-eaft, &c. See COMPASS.

SAGITTA, in Aftronomy, the *Arrow* or *Dart*, a conftellation of the northern hemifphere near the eagle, and one of the 48 old afterifms. The Greeks fay that this conftellation owes its origin to one of the arrows of Hercules, with which he killed the eagle or vulture that gnawed the liver of Prometheus.

The ftars in this conftellation, in the catalogues of Ptolomy, Tycho, and Hevelius, are only 5, but in Flamfteed's they are extended to 18.

SAGITTA, in Geometry, is a term ufed by fome writers for the abfcifs of a curve.

SAGITTA, in Trigonometry &c, is the fame as the verfed fine of an arch; being fo called becaufe it is like a dart or arrow, ftanding on the chord of the arch.

SAGITTARIUS, SAGITTARY, the *Archer*, one of the figns of the zodiac, being the 9th in order, and marked with the character ♐ of a dart or arrow. This conftellation is drawn in the figure of a Centaur, or an animal half man and half horfe, in the act of fhooting an arrow from a bow. This figure the Greeks feign to be Crotus, the fon of Eupheme, the nurfe of the mufes. Among more ancient nations the figure was probably meant for a hunter, to denote the hunting feafon, when the fun enters this fign.

The ftars in this conftellation are, in Ptolomy's catalogue 31, in Tycho's 14, in Hevelius's 22, and in the Britannic catalogue 69.

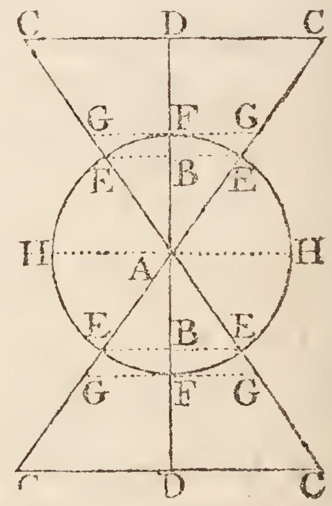
SAILING, in a general fenfe, denotes the movement by which a veffel is wafted along the furface of the water, by the action of the wind upon her fails.

Sailing is alfo ufed for the art or act of navigating; or of determining all the cafes of a fhip's motion, by means of fea-charts &c. Thefe charts are conftructed either on the fuppofition that the earth is a large extended flat furface, whence we obtain thofe that are called plane charts; or on the fuppofition that the earth is a fphere, whence are derived globular charts. Accordingly, Sailing may be diftinguifhed into two general kinds, viz, *plane Sailing*, and *globular Sailing*. Sometimes indeed a third fort is added, viz, *fpheroidical Sailing*, which proceeds upon the fuppofition of the fpheroidical figure of the earth.

Plane SAILING is that which is performed by means of a plane chart; in which cafe the meridians are confidered as parallel lines, the parallels of latitude are at right angles to the meridians, the lengths of the degrees on the meridians, equator, and parallels of latitude, are every where equal.

In Plane Sailing, the principal terms and circumftances made ufe of, are, courfe, diftance, departure, difference of latitude, rhumb, &c; for as to longitude, that has no place in plane Sailing, but belongs properly to globular or fpherical failing. For the explanation of all which terms, fee the refpective articles.

If a fhip fails either due north or fouth, fhe fails on a meridian, her diftance and difference of latitude are the fame, and fhe makes no departure; but where the fhip fails either due eaft or weft, fhe runs on a parallel of latitude, making no difference of latitude, and her departure and diftance are the fame. It may farther be obferved, that the departure and difference of latitude always make the legs of a right-angled triangle, whofe hypotenufe is the diftance the fhip has failed; and the angles are the courfe, its complement, and the right angle; therefore among thefe four things, courfe, diftance, difference of latitude, and departure, any two of them being given, the reft may be found by plane trigonometry.



Thus, in the annexed figure, fuppofe the circle FHFH to reprefent the horizon of the place A, from whence a fhip fails; AC the rhumb fhe fails upon, and C the place arrived at: then HH reprefents the parallel of latitude fhe failed from, and CC the parallel of the latitude arrived in: fo that

AD becomes the difference of latitude,
DC the departure,
AC the diftance failed,
∠DAC is the courfe, and
∠DCA the comp. of the courfe.

And all thefe particulars will be alike reprefented, whether the fhip fails in the NE, or NW, or SE, or SW quarter of the horizon.

From the fame figure, in which
AE or AF or AH reprefents the rad. of the tables,
EB the fine of the courfe,
AB the cofine of the courfe,

we may eafily deduce all the proportions or canons, as they are ufually called by mariners, that can arife in Plane Sailing; becaufe the triangles ADC and ABE and AFG are evidently fimilar. Thefe proportions are exhibited in the following Table, which confifts of 6 cafes, according to the varieties of the two parts that can be given.

Cafe

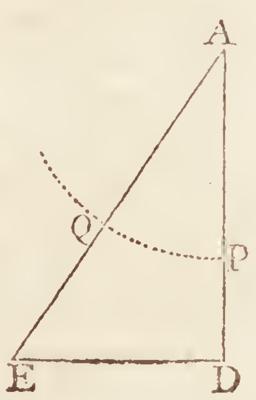
Case.	Given.	Required	Solutions.
1	∠A and AC, i. e. courſe and diſtance.	AD and DC, i. e. difference of latitude and departure.	AE : AB : : AC : AD, i. e. rad. : ſ. courſe : : diſt. : dif. lat. AE : EB : : AC : DC, i. e. rad. : coſ. courſe : : diſt. : depart.
2	∠A and AD, i. e. courſe and difference of latitude.	AC and DC, i. e. diſtance and departure.	AB : AE : : AD : AC, i. e. coſ. cour. : rad. : : dif. lat. : diſt. AB : BE : : AD : DC, i. e. coſ. cour. : ſ. cour. : : dif. lat. : dep.
3	∠A and DC, i. e. courſe and departure.	AC and AD, i. e. diſtance and difference of latitude.	BE : AE : : DC : AC, i. e. ſ. cour. : rad. : : depart. : diſt. BE : AB : : DC : AD, i. e. ſ. cour. : coſ. cour. : : dep. : dif. lat.
4	AC and AD, i. e. diſtance and difference of latitude.	∠A and DC, i. e. courſe and departure.	AC : AD : : AE : AB, i. e. diſt. : dif. lat. : : rad. : coſ. courſe. AE : EB : : AC : DC, i. e. rad. : ſ. courſe : : diſt. : depart.
5	AC and DC, i. e. diſtance and departure.	∠A and AD, i. e. courſe and difference of latitude.	AC : DC : : AE : EB, i. e. diſt. : dep. : : rad. : ſ. courſe. AE : AB : : AC : AD, i. e. rad. : coſ. cour. : : diſt. : dif. lat.
6	AD and DC, i. e. difference of latitude and departure.	∠A and AC, i. e. courſe and diſtance.	AD : DC : : AF : FG, i. e. dif. lat. : dep. : : rad. : tang. courſe. BE : AE : : DC : AC, i. e. ſ. cour. : rad. : : dep. : diſt.

For the ready working of any ſingle courſe, there is a table, called a *Traverſe Table*, uſually annexed to treatiſes of navigation; which is ſo contrived, that by finding the given courſe in it, and a diſtance not exceeding 100 or 120 miles, the uſual extent of the table; then the difference of latitude and the departure are had by inſpection. And the ſame table will ſerve for greater diſtances, by doubling, or trebling, or quadrupling, &c, or taking proportional parts. See TRAVERSE *Table*.

An ex. to the firſt caſe may ſuffice to ſhew the method. Thus, A ſhip from the latitude 47° 30′ N, has ſailed SW by S 98 miles; required the departure made, and the latitude arrived in.

1. *By the Traverſe Table.* In the column of the courſe, viz 3 points, againſt the diſtance 98, ſtands the number 54·45 miles for the departure, and 81·5 miles for the diff. of lat.; which is 1° 21′½; and this being taken from the given lat. 47° 30′, leaves 46° 8′½ for the lat. come to.

2. *By Conſtruction.* Draw the meridian AD; and drawing an arc, with the chord of 60, make PQ or angle A equal to 3 points; through Q draw the diſtance AQE = 98 miles, and through E the departure ED perp. to AD. Then, by meaſuring, the diff. of lat. AD meaſures about 81½ miles, and the departure DE about 54½ miles.



3. *By Computation.*

Firſt, as radius - - - - - -	10·00000
to ſin. courſe 33° 45′ - - -	9·74474
ſo diſt. 98 - - - - - - - -	1·99123
to depart. 54·45 - - - -	1·73597
Again, as radius - - - - -	10·00000
to coſ. courſe - - - - - -	9·91985
ſo diſt. 98 - - - - - - - -	1·99123
to diff. of lat. 81·48 - - -	1·91108

4. *By Gunter's Scale.* The extent from radius, or 8 points, to 3 points, on the line of ſine rhumbs, applied to the line of numbers, will reach from 98 to 54½ the departure. And the extent from 8 points to 5 points, of the rhumbs, reaches from 98 to 81½ on the line of numbers, for the difference of latitude.

And in like manner for other caſes.

Traverſe SAILING, or *Compound Courſes*, is the uniting of ſeveral caſes of plane ſailing together into one; as when a ſhip ſails in a zigzag manner, certain diſtances upon ſeveral different courſes, to find the whole difference of latitude and departure made good on all of them. This is done by working all the caſes ſeparately, by means of the traverſe table, and conſtructing the figure as in this example.

 Ex. As

Ex. A ſhip ſailing from a place in latitude 24° 32′ N, has run five different courſes and diſtances, as ſet down in the 1ſt and 2d columns of the following traverſe table; required her preſent latitude, with the departure, and the direct courſe and diſtance, between the place ſailed from, and the place come to.

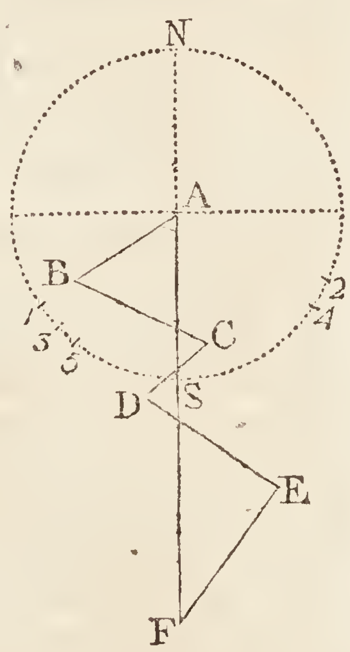
Traverſe Table.					
Courſes.	Diſt.	N	S	E	W
SW b S	45		25·0		37·4
ESE	50		19·1	46·2	
SW	30		21·2		21·2
SE b E	60		33·3	49·9	
SW b S ¼ W	63		50·6		37·5
			149·2	96·1	96·1

Here, by finding, in the general traverſe table, the difference of latitude and departure anſwering to each courſe and diſtance, they are ſet down on the ſame lines with each courſe, and in their proper columns of northing, ſouthing, eaſting, or weſting, according to the quarter of the compaſs the ſhip ſails in, at each courſe. As here, there is no northing, the differences of latitude are all ſouthward, alſo two departures are eaſtward, and three are weſtward. Then, adding up the numbers in each column, the ſum of the eaſtings appears to be exactly equal to the ſum of the weſtings, conſequently the ſhip is arrived in the ſame meridian, without making any departure; and the ſouthings, or difference of latitude being 149·2 miles or minutes,

that is - - - - - - - - 2° 29′,
which taken from - - 24 32, the latitude dep. from,

leaves - - - - - - - - - 22 3 N, the latitude come to.

To Conſtruct this Traverſe. With the chord of 60 degrees deſcribe the circle N 135 S &c, and quarter it by the two perpendicular diameters; then from S ſet upon it the ſeveral courſes, to the points marked 1, 2, 3, 4, 5, through which points draw lines from the centre A, or conceive them to be drawn; laſtly, upon the firſt line lay off the firſt diſtance 45 from A to B, alſo draw BC = 50 and parallel to A 2, and CD = 30 parallel to A 3, and DE = 60 parallel to A 4, and EF = 63, parallel to A 5; then it is found that the point F falls exactly upon the meridian NAF produced, thereby ſhewing that there is no departure; and by meaſuring AF, it gives 149 miles for the difference of latitude.

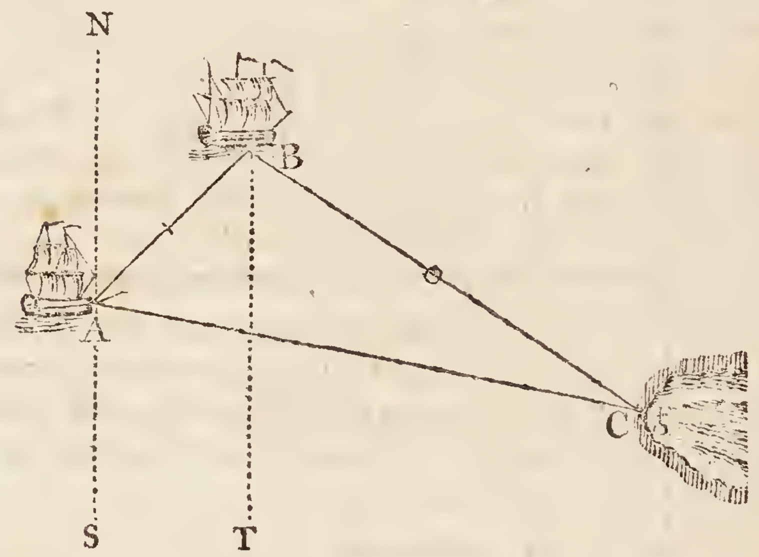


Oblique SAILING, is the reſolution of certain caſes and problems in Sailing by oblique triangles, or in which oblique triangles are concerned.

In this kind of Sailing, it may be obſerved, that *to ſet an object*, means to obſerve what rhumb or point of the nautical compaſs is directed to it. And the *bearing* of an object is the rhumb on which it is ſeen; alſo the bearing of one place from another, is reckoned by the name of the rhumb paſſing through thoſe two places.

In every figure relating to any caſe of plane Sailing, the bearing of a line, not running from the centre of the circle or horizon, is found by drawing a line parallel to it, from the centre, and towards the ſame quarter.

Ex. A ſhip ſailing at ſea, obſerved a point of land to bear E by S; and then after ſailing NE 12 miles, its bearing was found to be SE by E. Required the place of that point, and its diſtance from the ſhip at the laſt obſervation.



Conſtruction. Draw the meridian line NAS, and, aſſuming A for the firſt place of the ſhip, draw AC the E by S rhumb, and AB the NE one, upon which lay off 12 miles from A to B; then draw the meridian BT parallel to NS, from which ſet off the SE by E point BC, and the point C will be the place of the land required; then the diſtance BC meaſures 26 miles.

By Computation. Here are given the ſide AB, and the two angles A and B, viz. the $\angle A = 5$ points or 56° 15′, and the $\angle B = 9$ points or 101° 15′; conſequently the $\angle C = 2$ points or 22° 30′. Then, by plane trigonometry,

As ſin. ∠C	22° 30′	- - - -	9·58284
To ſin. ∠B	56 15	- - - -	9·91985
So is AB	12 miles	- - - -	1·07918
To BC	26·073 miles	- - - -	1·41619

SAILING *to Windward*, is working the ſhip towards that quarter of the compaſs from whence the wind blows.

For rightly underſtanding this part of navigation, it will be neceſſary to explain the terms that occur in it, though moſt of them may be ſeen in their proper places in this work.

When the wind is directly, or partly, againſt a ſhip's direct courſe for the place ſhe is bound to, ſhe reaches her port by a kind of zigzag or z like courſe; which is made by ſailing with the wind firſt on one ſide of the ſhip, and then on the other ſide.

In a ſhip, when you look towards the head,
Starboard denotes the right hand ſide.

Larboard

Larboard the left hand ſide.

Forwards, or *afore*, is towards the head.

Aft, or *abaft*, is towards the ſtern.

The *beam* ſignifies athwart or acroſs the middle of the ſhip.

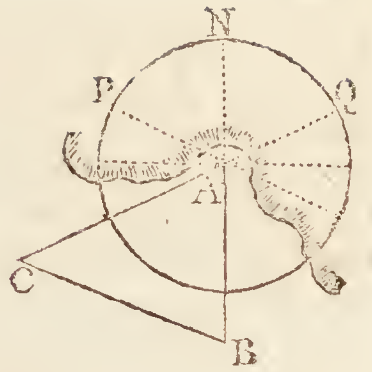
When a ſhip ſails the ſame way that the wind blows, ſhe is ſaid to ſail or run before the wind; and the wind is ſaid to be *right aft*, or *right aſtern;* and her courſe is then 16 points, or the fartheſt poſſible, from the wind, that is from the point the wind blows from.—When the ſhip ſails with the wind blowing directly acroſs her, ſhe is ſaid to have the *wind on the beam;* and her courſe is 8 points from the wind.—When the wind blows obliquely acroſs the ſhip, the wind is ſaid to be *abaft the beam* when it purſues her, or blows more on the hinder part, but *before the beam* when it meets or oppoſes her courſe, her courſe being more than 8 points from the wind in the former caſe, but leſs than 8 points in the latter caſe.—When a ſhip endeavours to ſail towards that point of the compaſs from which the wind blows, ſhe is ſaid to *ſail on a wind*, or to *ply to windward*.—And a veſſel ſailing as near as ſhe can to the point from which the wind blows, ſhe is ſaid to be *cloſe hauled*. Moſt ſhips will lie within about 6 points of the wind; but ſloops, and ſome other veſſels, will lie much nearer. To know how near the wind a ſhip will lie; obſerve the courſe ſhe goes on each tack, when ſhe is cloſe hauled; then half the number of points between the two courſes, will ſhew how near the wind the ſhip will lie.

The *windward*, or *weather ſide*, is that ſide of the ſhip on which the wind blows; and the other ſide is called the *leeward*, or *lee ſide*.—*Tacks* and *ſheets* are large ropes faſtened to the lower corners of the fore and main ſails; by which either of theſe corners is hauled fore or aft.—When a ſhip ſails on a wind, the windward tacks are always hauled forwards, and the leeward ſheets aft.—The *ſtarboard tacks are aboard*, when the ſtarboard ſide is to windward, and the larboard ſide to leeward. And the *larboard tacks are aboard*, when the larboard ſide is to windward, and the ſtarboard to leeward.

The moſt common caſes in turning to windward may be conſtructed by the following precepts. Having drawn a circle with the chord of 60°, for the compaſs, or the horizon of the place, quarter it by drawing the meridian and parallel of latitude perpendicular to each other, and both through the centre; mark the place of the wind in the circumference; draw the rhumb paſſing through the place bound to, and lay on it, from the centre, the diſtance of that place. On each ſide of the wind lay off, in the circumference, the points or degrees ſhewing how near the wind the ſhip can lie; and draw theſe rhumbs.—Now the firſt courſe will be on one of theſe rhumbs, according to the tack the ſhip leads with. Draw a line through the place bound to, parallel to the other rhumb, and meeting the firſt; and this will ſhew the courſe and diſtance on the other tack.

Ex. The wind being at north, and a ſhip bound to a port 25 miles directly to windward; beginning with the ſtarboard tacks, what muſt be the courſe and diſtance on each of two tacks to reach the port?

Conſtruction. Having drawn the circle &c, as above deſcribed, where A is the port, AP and AQ the two rhumbs, each within 6 points of AN; in NA produced take AB = 25 miles, then B is the place of the ſhip; draw BC parallel to AP, and meeting QA produced in C; ſo ſhall BC and CA be the diſtances on the two tacks; the former being WNW, and the latter ENE.



Computation.

Here $\angle B = NAP = 6$ points,
and $\angle A = NAQ = 6$ points,
theref. $\angle C = 4$ points.

So that all the angles are given, and the ſide AB, to find the other two ſides AC and BC, which are equal to each other, becauſe their oppoſite angles A and B are equal. Hence

as ſin. C : AB : : ſin. A : BC,

i. e. s. 45° : 25 : : s. 67° 30′ : $32\frac{2}{3}$ = BC or AC, the diſtance to be run on each tack.

Sailing *in Currents*, is the method of determining the true courſe and diſtance of a ſhip when her own motion is affected and combined with that of a current.

A *current* or *tide* is a progreſſive motion of the water, cauſing all floating bodies to move that way towards which the ſtream is directed.—The *ſetting* of a tide, or current, is that point of the compaſs towards which the waters run; and the *drift* of the current is the rate at which it runs per hour.

The drift and ſetting of the moſt remarkable tides and currents, are pretty well known; but for unknown currents, the uſual way to find the drift and ſetting, is thus: Let three or four men take a boat a little way from the ſhip; and by a rope, faſtened to the boat's ſtem, let down a heavy iron pot, or loaded kettle, into the ſea, to the depth of 80 or 100 fathoms, when it can be done: by which means the boat will ride almoſt as ſteady as at anchor. Then heave the log, and the number of knots run out in half a minute will give the current's rate, or the miles which it runs per hour; and the bearing of the log ſhews the ſetting of the current.

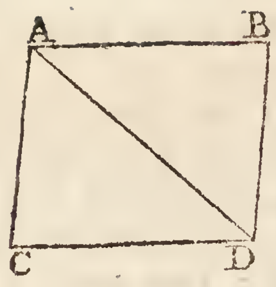
A body moving in a current, may be conſidered in three caſes: viz,

1. Moving with the current, or the ſame way it ſets.
2. Moving againſt it, or the contrary way it ſets.
3. Moving obliquely to the current's motion.

In the 1ſt caſe, or when a ſhip ſails with a current, its velocity will be equal to the ſum of its proper motion, and the current's drift. But in the 2d caſe, or when a ſhip ſails againſt a current, its velocity will be equal to the difference of her own motion and the drift of the current: ſo that if the current drives ſtronger than the wind, the ſhip will drive aſtern, or loſe way. In the 3d caſe, when the current ſets oblique to the courſe of the ſhip, her real courſe, or that made good, will be ſomewhere between that in which the ſhip endeavours to go, and the track in which the current tries to drive her; and indeed it will always be along the diagonal of a parallelogram, of which one ſide repreſents the ſhip's courſe ſet, and the other adjoining ſide is the current's drift.

Thus,

Thus, if ABDC be a parallelogram. Now if the wind alone would drive the ſhip from A to B in the ſame time as the current alone would drive her from A to C: then, as the wind neither helps nor hinders the ſhip from coming towards the line CD, the current will bring her there in the ſame time as if the wind did not act. And as the current neither helps nor hinders the ſhip from coming towards the line BD, the wind will bring her there in the ſame time as if the current did not act. Therefore the ſhip muſt, at the end of that time, be found in both thoſe lines, that is, in their meeting D. Conſequently the ſhip muſt have paſſed from A to D in the diagonal AD.



Hence, drawing the rhumbs for the proper courſe of the ſhip and of the current, and ſetting the diſtances off upon them, according to the quantity run by each in the given time; then forming a parallelogram of theſe two, and drawing its diagonal, this will be the real courſe and diſtance made good by the ſhip.

Ex. 1. A ſhip ſails E. 5 miles an hour, in a tide ſetting the ſame way 4 miles an hour: required the ſhip's courſe, and the diſtance made good.

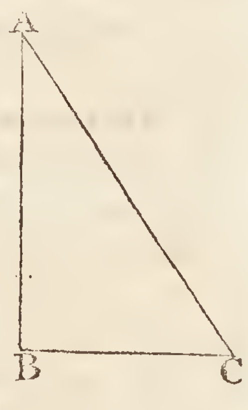
The ſhip's motion is	5 m. E.
The current's motion is	4 m. E.
Theref. the ſhip's run is	9 m. E.

Ex. 2. A ſhip ſails SSW. with a briſk gale, at the rate of 9 miles an hour, in a current ſetting NNE. 2 miles an hour: required the ſhip's courſe, and the diſtance made good.

The ſhip's motion is	SSW. 9 m.
The current's motion is	NNE. 2 m.
Theref. ſhip's true run is	SSW. 7 m.

Ex. 3. A ſhip running ſouth at the rate of 5 miles an hour, in 10 hours croſſes a current, which all that time was ſetting eaſt at the rate of 3 miles an hour: required the ſhip's true courſe and diſtance ſailed.

Here the ſhip is firſt ſuppoſed to be at A, her imaginary courſe is along the line AB, which is drawn ſouth, and equal to 50 miles, the run in 10 hours; then draw BC eaſt, and equal to 30 miles, the run of the current in 10 hours. Then the ſhip is found at C, and her true path is in the line AC = 58·31 her diſtance, and her courſe is the angle at A = 30° 58′ from the ſouth towards the eaſt.



Globular Sailing is the eſtimating the ſhip's motion and run upon principles derived from the globular figure of the earth, viz, her courſe, diſtance, and difference of latitude and longitude.

The principles of this method are explained under the articles Rhumb-*line*, *Mercator's* Chart, and Meridional *Parts*; which ſee.

Globular Sailing, in the extenſive ſenſe here applied to the term, comprehends *Parallel Sailing*, *Middle-latitude Sailing*, and *Mercator's Sailing*; to which may be added *Circular Sailing*, or *Great-circle Sailing*. Of each of which it may be proper here to give a brief account.

Parallel Sailing is the art of finding what diſtance a ſhip ſhould run due eaſt or weſt, in ſailing from the meridian of one place to that of another place, in any parallel of latitude.

The computations in parallel ſailing depend on the following rule:

As radius,
To coſine of the lat. of any parallel;
So are the miles of long. between any two meridians,
To the diſt. of theſe meridians in that parallel.

Alſo, for any two latitudes,
As the coſine of one latitude,
Is to the coſine of another latitude;
So is a given meridional diſt. in the 1ſt parallel,
To the like meridional diſt. in the 2d parallel.

Hence, counting 60 nautical miles to each degree of longitude, or on the equator; then, by the firſt rule the number of miles in each degree on the other parallels, will come out as in the following table.

Table of Meridional Diſtances.

Lat.	Miles.	Lat.	Miles.	Lat.	Miles.
1	59·99	31	51·43	61	29·09
2	59·96	32	50·88	62	28·17
3	59·92	33	50·32	63	27·24
4	59·85	34	49·74	64	26·30
5	59·77	35	49·15	65	25·36
6	59·67	36	48·54	66	24·41
7	59·56	37	47·92	67	23·44
8	59·42	38	47·28	68	22·48
9	59·26	39	46·63	69	21·50
10	59·09	40	45·96	70	20·52
11	58·89	41	45·28	71	19·53
12	58·69	42	44·59	72	18·54
13	58·46	43	43·88	73	17·54
14	58·22	44	43·16	74	16·54
15	57·95	45	42·43	75	15·53
16	57·67	46	41·68	76	14·51
17	57·38	47	40·92	77	13·50
18	57·06	48	40·15	78	12·48
19	56·73	49	39·36	79	11·45
20	56·38	50	38·57	80	10·42
21	56·01	51	37·76	81	9·38
22	55·63	52	36·94	82	8·35
23	55·23	53	36·11	83	7·32
24	54·81	54	35·27	84	6·28
25	54·38	55	34·41	85	5·23
26	53·93	56	33·55	86	4·18
27	53·46	57	32·68	87	3·14
28	52·97	58	31·79	88	2·09
29	52·47	59	30·90	89	1·05
30	51·96	60	30·00	90	0·00

See

See another table of this kind, allowing $69\frac{1}{13}$ English miles to one degree, under the article DEGREE.

To find the meridional distance to any number of minutes between any of the whole degrees in the table, as for instance in the parallel of 48° 26′; take out the tabular distances for the two whole degrees between which the parallel or the odd minutes lie, as for 48° and 49°; subtract the one from the other, and take the proportional part of the remainder for the odd minutes, by multiplying it by those minutes, and dividing by 60; and lastly, subtract this proportional part from the greater tabular number. Thus,

Lat. 48° - -	40·15	
Lat. 49. - -	39·36	
As 60′ : 26′ : :	0·79 rem. : 0.34	
	26	
	474	
	158	
60)	20·54	
	0·34	pro. part
Taken from -	40·15	for lat. 48°
Leaves merid. dist.	39·81	for lat. 48° 26′

And, in like manner, by the counter operation, to find what latitude answers to a given meridional distance. As, for ex. in what latitude 46·08 miles answer to a degree of longitude.

From 46·63 for 39°	from 46·63 for 39°
Take 45·96 for 40°	take 46·08 given number.
Then as 0·67 : 60′ : :	0·55 : 49′
	60
	67) 3300
	49′ pro. part.

Therefore the latitude sought is 39° 49′.

Ex. 3. Given the latitude and meridional distance; to find the corresponding difference of longitude. As, if a ship, in latitude 53° 36′, and longitude 10° 18 east, sail due west 236 miles; required her present longitude.

Here, by the first rule,

As cos. lat.	53° 36′ comp.	0·22664
To radius -	90 00 - -	10·00000
So merid. dist.	236 m. -	2·37291
To diff. long. 397·7 - -		2·59955

Its 60th gives	6°	38′ W. diff. long.
Taken from	10	18 E. long. from
Leaves - -	3	40 E. long. come to.

By the table; the length of a degree on the parallel of 53° 36′ is 35·6.

Then as 35·6 : 60 : : 236 : 397·7, the diff. of long. the same as before.

Middle-latitude SAILING, is a method of resolving the cases of globular Sailing by means of the middle latitude between the latitude departed from, and that come to. This method is not quite accurate, being only an approximation to the truth, and it makes use of the principles of plane Sailing and parallel Sailing conjointly.

The method is founded on the supposition that the departure is reckoned as a meridional distance in that latitude which is a middle parallel between the latitude sailed from, and the latitude come to. And the method is not quite accurate, because the arithmetical mean, or half sum of the cosines of two distant latitudes, is not exactly the cosine of the middle latitude, or half the sum of those latitudes; nor is the departure between two places, on an oblique rhumb, equal to the meridional distance in the middle latitude; as is presumed in this method. Yet when the parallels are near the equator, or near to each other, in any latitude, the error is not considerable.

This method seems to have been invented on account of the easy manner in which the several cases may be resolved by the traverse table, and when a table of meridional parts is wanting. The computations depend on the following rules:

1. Take half the sum, or the arithmetical mean, of the two given latitudes, for the middle latitude. Then,

2. As cosine of middle latitude,
 Is to the radius;
 So is the departure,
 To the diff. of longitude. And,

3. As cosine of middle latitude,
 Is to the tangent of the course;
 So is the difference of latitude,
 To the difference of longitude.

Mercator's SAILING, is the art of resolving the several cases of globular Sailing, by plane trigonometry, with the assistance of a table of meridional parts, or of logarithmic tangents. And the computations are performed by the following rules:

1. As meridional diff. lat.
 To diff. of longitude;
 So is the radius,
 To tangent of the course.

2. As the proper diff lat.
 To the departure;
 So is merid. diff. lat.
 To diff. of longitude.

3. As diff. log. tang. half colatitudes,
 To tang. of 51° 38′ 09″;
 So is a given diff. longitude,
 To tangent of the course.

The manner of working with the meridional parts and logarithmic tangents, will appear from the two following cases.

1. Given the latitudes of two places; to find their meridional difference of latitude.

By the Merid. Parts. When the places are both on the

the ſame ſide of the equator, take the difference of the meridional parts anſwering to each latitude; but when the places are on oppoſite ſides of the equator, take the ſum of the ſame parts, for the meridional difference of latitude ſought.

By the Log. Tangents. In the former caſe, take the difference of the long. tangents of the half colatitudes; but in the latter caſe, take the ſum of the ſame; then the ſaid difference or ſum divided by 12·63, will give the meridional difference of latitude ſought.

2. Given the latitude of one place, and the meridional difference of latitude between that and another place; to find the latitude of this latter place.

By the Merid. Parts. When the places have like names, take the ſum of the merid. parts of the given lat. and the given diff.; but take the difference between the ſame when they have unlike names; then the reſult, being found in the table of meridional parts, will give the latitude ſought.

By the Log. Tangents. Multiply the given meridional diff. of lat. by 12·63; then in the former caſe ſubtract the product from the log. tangent of the given half colatitude, but in the latter caſe add them; then ſeek the degrees and minutes anſwering to the reſult among the log. tangents, and theſe degrees, &c. doubled will be the colatitude ſought.

Circular SAILING, or *Great-circle* SAILING, is the art of finding what places a ſhip muſt go through, and what courſes to ſteer, that her track may be in the arc of a great circle on the globe, or nearly ſo, paſſing through the place ſailed from and the place bound to.

This method of Sailing has been propoſed, becauſe the ſhorteſt diſtance between two places on the ſphere, is an arc of a great circle intercepted between them, and not the ſpiral rhumb paſſing through them, unleſs when that rhumb coincides with a great circle, which can only be on a meridian, or on the equator.

As the ſolutions of the caſes in Mercator's Sailing are performed by plane triangles, in this method of Sailing they are reſolved by means of ſpherical triangles. A great variety of caſes might be here propoſed, but thoſe that are the moſt uſeful, and moſt commonly occur, pertain to the following problem.

Problem I. Given the latitudes and longitudes of two places on the earth; to find their neareſt diſtance on the ſurface, together with the angles of poſition from either place to the other.

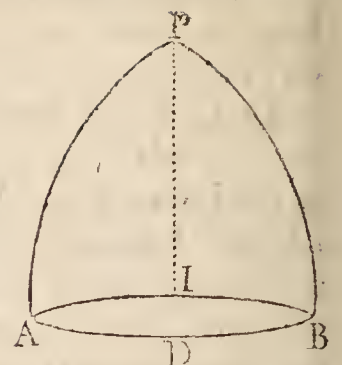
This problem comprehends 6 caſes.

Caſe 1. When the two places lie under the ſame meridian; then their difference of latitude will give their diſtance, and the poſition of one from the other will be directly north and ſouth.

Caſe 2. When the two places lie under the equator; their diſtance is equal to their difference of longitude, and the angle of poſition is a right angle, or the courſe from one to the other is due eaſt or weſt.

Caſe 3. When both places are in the ſame parallel of latitude. Ex. gr. The places both in 37° north, but the longitude of the one 25° weſt, and of the other 76° 23′ weſt.

Let P denote the north pole, and A and B the two places on the ſame parallel BDA, alſo BIA their diſtance aſunder, or the arc of a great circle paſſing through them. Then is the angle A or B that of poſition, and the angle BPA = 51° 23′ the difference of longitude, and the ſide PA or PB = 53° the colatitude.

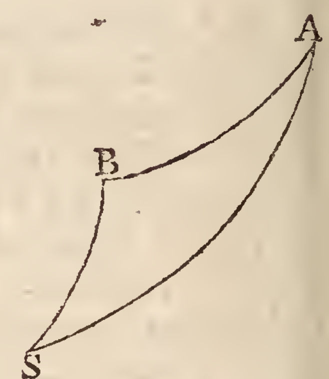


Draw PI perp. to AB, or biſecting the angle at P. Then in the triangle API, right-angled at I, are given the hypotenuſe AP = 53°, and the angle API = 25° 41′ 30″; to find the angle of poſition A or B = 73° 51′; and the half diſtance AI = 20° 15½′; this doubled gives 40° 31′ for the whole diſtance AB, or 2431 nautical miles, which is 31 miles leſs than the diſtance along ADB, or by parallel Sailing.

Caſe 4. When one place has latitude, and the other has none, or is under the equator. For example, ſuppoſe the Iſland of St. Thomas, lat. 0°, and long. 1° 0′ eaſt, and Port St. Julian, in lat. 48° 51′ ſouth, and long. 65° 10′ weſt.

Port St. Julian, lat.	48° 51′ S.	long.	65° 10′ W.
Iſle St. Thomas	0 00		1 00 E
Julian's colat.	41 09	Diff. long.	66 10

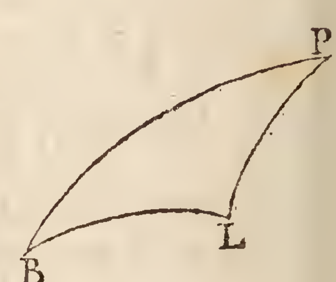
Hence, if S denote the ſouth pole, A the Iſle St. Thomas at the equator, and B St. Julian; then in the triangle are given SA a quadrant or 90°, BS = 41° 9′ the colat. of St. Julian, and the ∠S = 66° 10′ the dif. of longitude; to find AB = 74° 35′ = 4475 miles, which is leſs by 57 miles than the diſtance found by Mercator's Sailing; alſo the angle of poſition at A = 51°22′, and the angle of poſition B = 108° 24′.



Caſe 5. When the two given places are both on the ſame ſide of the equator; for example the Lizard, and the iſland of Bermudas.

The Lizard, lat.	49° 57′N.	long.	5° 21′ W.
Bermudas,	32 35 N.		63 32 W.
			58 11

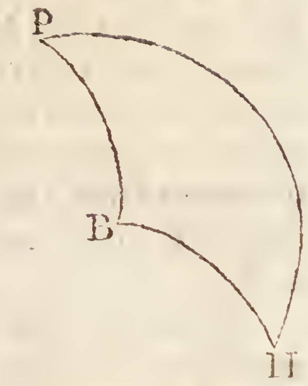
Here, if P be the north pole, L the Lizard, and B Bermudas; there are given,
PL = 40° 03′ colat. of the Lizard,
PB = 57 25 colat. of Bermudas,
∠P = 58 11 diff. of longitude;
to find BL = 45° 44 = 2744 miles the diſtance, and
∠ of poſition B = 49° 27′, alſo
∠ of poſition L = 90° 31′.



Caſe 6. When the given places lie on different ſides of the equator; as ſuppoſe St. Helena and Bermudas. Here

PB

PB = 57° 25′ polar diſt. Bermudas,
PH = 105 55 polar diſt. St. Helena,
∠P = 57 43 diff. long.
To find BH = 73° 26′ = 4406 miles, the diſtance, alſo the angle of poſition H = 48° 0′, and the angle of poſition B = 121° 59′.



From the ſolutions of the foregoing caſes it appears, that to ſail on the arc of a great circle, the ſhip muſt continually alter her courſe ; but as this is a difficulty too great to be admitted into the practice of navigation, it has been thought ſufficiently exact to effect this buſineſs by a kind of approximation, that is, by a method which nearly approaches to the ſailing on a great circle : namely, upon this principle, that in ſmall arcs, the difference between the arc and its chord or tangent is ſo ſmall, that they may be taken for one another in any nautical operations : and accordingly it is ſuppoſed that the great circles on the earth are made up of ſhort right lines, each of which is a ſegment of a rhumb line. On this ſuppoſition the ſolution of the following problem is deduced.

Problem II. Having given the latitudes and longitudes of the places ſailed from and bound to ; to find the ſucceſſive latitudes on the arc of a great circle in thoſe places where the alteration in longitude ſhall be a given quantity ; together with the courſes and diſtances between thoſe places.

1. Find the angle of poſition at each place, and their diſtance, by one of the preceding caſes.

2. Find the greateſt latitude the great circle runs through, i. e. find the perpendicular from the pole to that circle ; and alſo find the ſeveral angles at the pole, made by the given alterations of longitude between this perpendicular and the ſucceſſive meridians come to.

3. With this perpendicular and the polar angles ſeverally, find as many correſponding latitudes, by ſaying, as radius : tang. greateſt lat. : : coſ. 1ſt polar angle : tang. 1ſt lat. : : coſ. 2d polar angle : tang. of 2d lat. &c.

4. Having now the ſeveral latitudes paſſed through, and the difference of longitude between each, then by Mercator's Sailing find the courſes and diſtances between thoſe latitudes. And theſe are the ſeveral courſes and diſtances the ſhip muſt run, to keep nearly on the arc of a great circle.

The ſmaller the alterations in longitude are taken, the nearer will this method approach to the truth ; but it is ſufficient to compute to every 5 degrees of difference of longitude ; as the length of an arc of 5 degrees differs from its chord, or tangent, only by 0·002.

The track of a ſhip, when thus directed nearly in the arc of a great circle, may be delineated on the Mercator's chart, by marking on it, by help of the latitudes and longitudes, the ſucceſſive places where the ſhip is to alter her courſe ; then thoſe places or points, being joined by right lines, will ſhew the path along which the ſhip is to ſail, under the propoſed circumſtances.

On the ſubject of theſe articles, ſee Robertſon's Elements of Navigation, vol. 2.

Spheroidical SAILING, is computing the caſes of navigation on the ſuppoſition or principles of the ſpheroidical figure of the earth. See Robertſon's Navigation, vol. 2, b. 8. ſect. 8.

SAILING, *in a more confined ſenſe*, is the art of conducting a ſhip from place to place, by the working or handling of her ſails and rudder.

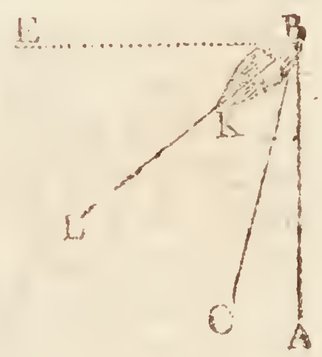
To bring Sailing to certain rules, M. Renau computes the force of the water, againſt the ſhip's rudder, ſtem, and ſide ; and the force of the wind againſt her ſails. In order to this, he firſt conſiders all fluid bodies, as the air, water, &c, as compoſed of little particles, which when they act upon, or move againſt any ſurface, do all move parallel to one another, or ſtrike againſt the ſurface after the ſame manner. Secondly, that the motion of any body, with regard to the ſurface it ſtrikes, muſt be either perpendicular, parallel, or oblique.

From theſe principles he computes, that the force of the air or water, ſtriking perpendicularly upon a ſail or rudder, is to the force of the ſame ſtriking obliquely, in the duplicate ratio of radius to the ſine of the angle of incidence : and conſequently that all oblique forces of the wind againſt the ſails, or of the water againſt the rudder, will be to one another in the duplicate ratio of the ſines of the angles of incidence.

Such are the concluſions from theory ; but it is very different in real practice, or experiments, as appears from the tables of experiments inſerted at the article RESISTANCE.

Farther, when the different degrees of velocity are conſidered, it is alſo found that the forces are as the ſquares of the velocities of the moving air or water nearly ; that is, a wind that blows twice as ſwift, as another, will have 4 times the force upon the ſail ; and when 3 times as ſwift, 9 times the force, &c. And it being alſo indifferent, whether we conſider the motion of a ſolid in a fluid at reſt, or of the fluid againſt the ſolid at reſt ; the reciprocal impreſſions being always the ſame ; if a ſolid be moved with different velocities in the ſame fluid matter, as water, the different reſiſtances which it will receive from that water, will be in the ſame proportion as the ſquares of the velocities of the moving body.

He then applies theſe principles to the motions of a ſhip, both forwards and ſideways, through the water, when the wind, with certain velocities, ſtrikes the ſails in various poſitions. After this, the author proceeds to demonſtrate, that the beſt poſition or ſituation of a ſhip, ſo as ſhe may make the leaſt lee-way, or ſide motion, but go to windward as much as poſſible, is this : that, let the ſail have what ſituation it will, the ſhip be always in a line biſecting the complement of the wind's angle of incidence upon the ſail. That is, ſuppoſing the ſail in the poſition BC, and the wind blowing from A to B, and conſequently the angle of the wind's incidence on the ſail is ABC, the complement of which is CBE : then muſt the ſhip be put in the poſition BK, or move in the line BL, biſecting the ∠ CBE.



He ſhews farther, that the angle which the ſail ought to make with the wind, i. e. the angle ABC, ought to be but 24 degrees; that being the moſt advantageous ſituation to go to windward the moſt poſſible.

To this might be added many curious particulars from Borelli de Vi Percuſſionis, concerning the different directions given to a veſſel by the rudder, when ſailing with a wind, or floating without ſails in a current: in the former caſe, the head of the ſhip always coming to the rudder, and in the latter always flying off from it; as alſo from Euler, Bouguer, and Juan, who have all written learnedly on this ſubject.

SALIANT, in Fortification, is ſaid of an angle that projects its point outwards; in oppoſition to a re-entering angle, which has its point turned inwards. Inſtances of both kinds of theſe we have in tenailles and ſtar-works.

SALON, or Saloon, in Architecture, a grand, lofty, ſpacious ſort of hall, vaulted at top, and uſually comprehending two ſtories, with two ranges of windows. It is ſometimes built ſquare, ſometimes round or oval, ſometimes octagonal, as at Marly, and ſometimes in other forms.

SAP, or Sapp, in Building, as to ſap a wall, &c, is to dig out the ground from beneath it, ſo as to bring it down all at once for want of ſupport.

Sap, in the Military Art, denotes a work carried on under cover of gabions and faſcines on the flank, and mantlets or ſtuffed gabions on the front, to gain the deſcent of a ditch, or the like.

It is performed by digging a deep trench, deſcending by ſteps from top to bottom, under a corridor, carrying it as far as the bottom of the ditch, when that is dry; or as far as the ſurface of the water, when wet.

SAROS, in Chronology, a period of 223 lunar months. The etymology of the word is ſaid to be Chaldean, ſignifying reſtitution, or return of eclipſes; that is, conjunctions of the ſun and moon in nearly the ſame place of the ecliptic. The Saros was a cycle like to that of Meto.

SARRASIN, or Sarrazin, in Fortification, a kind of port-cullis, otherwiſe called a herſe, which is hung with ropes over the gate of a town or fortreſs, to be let fall in caſe of a ſurpriſe.

SATELLITES, in Aſtronomy, are certain ſecondary planets, moving round the other planets, as the moon does round the earth. They are ſo called becauſe always found attending them, from riſing to ſetting, and making the tour about the ſun together with them.

The words moon and Satellite are ſometimes uſed indifferently: thus we ſay, either Jupiter's moons, or Jupiter's Satellites; but uſually we diſtinguiſh, reſtraining the term moon to the earth's attendant, and applying the term Satellite to the little moons more recently diſcovered about Jupiter, Saturn, and the Georgian planet, by the aſſiſtance of the teleſcope, which is neceſſary to render them viſible.

The Satellites move round their primary planets, as their centres, by the ſame laws as thoſe primary ones do round their centre the ſun; viz, in ſuch manner that, in the Satellites of the ſame planet, the ſquares of the periodic times are proportional to the cubes of their diſtances from the primary planet. For the phyſical cauſe of their motions, ſee Gravity. See alſo Planets.

We know not of any Satellites beſide thoſe above mentioned; what other diſcoveries may be made by farther improvements in teleſcopes, time only can bring to light.

Satellites *of Jupiter.* There are are four little moons, or ſecondary planets now known performing their evolutions about Jupiter, as that planet does about the Sun.

Simon Marius, mathematician of the elector of Brandenburg, about the end of November 1609, obſerved three little ſtars moving round Jupiter's body, and proceeding along with him; and in January 1610, he found a 4th. In January 1610 Galileo alſo obſerved the ſame in Italy, and in the ſame year publiſhed his obſervations. Theſe Satellites were alſo obſerved in the ſame month of January 1710, by Thomas Harriot, the celebrated author of a work upon algebra, and who made conſtant obſervations of theſe Satellites, from that time till the 26th of February 1612; as appears by his curious aſtronomical papers, lately diſcovered by Dr. Zach, at the ſeat of the earl of Egremont, at Petworth in Suſſex.

One Antony Maria Schyrlæus di Reita, a capuchin of Cologne, imagined that, beſides the four known Satellites of Jupiter, he had diſcovered five more, on December 29, 1642. But the obſervation being communicated to Gaſſendus, who had obſerved Jupiter on the ſame day, he ſoon perceived that the monk had miſtaken five fixed ſtars, in the effuſion of the water of Aquarius, marked in Tycho's catalogue 24, 25, 26, 27, 28, for Satellites of Jupiter.

When Jupiter comes into a line between any of his Satellites and the ſun, the Satellite diſappears, being then *eclipſed*, or involved in his ſhadow.—When the Satellite goes behind the body of Jupiter, with reſpect to an obſerver on the earth, it is then ſaid to be *occulted*, being hid from our ſight by his body, whether in his ſhadow or not.—And when the Satellite comes into a poſition between Jupiter and the Sun, it caſts a ſhadow upon the face of that planet, which we ſee as an obſcure round ſpot.—And laſtly, when the Satellite comes into a line between Jupiter and us, it is ſaid to *tranſit* the diſc of the planet, upon which it appears as a round black ſpot.

The periods or revolutions of Jupiter's Satellites, are found out from their conjunctions with that planet; after the ſame manner, as thoſe of the primary planets are diſcovered from their oppoſitions to the ſun. And their diſtances from the body of Jupiter, are meaſured by a micrometer, and eſtimated in ſemidiameters of that planet, and thence in miles.

By the lateſt and moſt exact obſervations, the periodical times and diſtances of theſe Satellites, and the angles under which their orbits are ſeen from the earth, at its mean diſtance from Jupiter, are as below:

Satel-

Satellites of Jupiter.				
Satel-lites.	Periodic Times.	Distances in		Angles of Orbit.
		Semidiameters.	Miles.	
1	1d 18h 27′ 34″	$5\frac{2}{3}$	266,000	3′ 55″
2	3 13 13 42	$9\frac{1}{59}$	423,000	6 14
3	7 3 42 36	$14\frac{5}{13}$	676,000	9 58
4	16 16 32 0	$25\frac{3}{10}$	1,189,000	17 30

The eclipses of the Satellites, especially of those of Jupiter, are of very great use in astronomy. First, in determining pretty exactly the distance of Jupiter from the earth. A second advantage still more considerable, which is drawn from these eclipses, is the proof which they give of the progressive motion of light. It is demonstrated by these eclipses, that light does not come to us in an instant, as the Cartesians pretended; although its motion is extremely rapid. For if the motion of light were infinite, or came to us in an instant, it is evident that we should see the commencement of an eclipse of a Satellite at the same moment, at whatever distance we might be from it; but, on the contrary, if light move progressively, then it is as evident, that the farther we are from a planet, the later we shall be in seeing the moment of its eclipse, because the light will take up a longer time in arriving at us; and so it is found in fact to happen, the eclipses of these Satellites appearing always later and later than the true computed times, as the earth removes farther and farther from the planet. When Jupiter and the earth are at their nearest distance, being in conjunction both on the same side of the sun, then the eclipses are seen to happen the soonest; and when the sun is directly between Jupiter and the earth, they are at their greatest distance asunder, the distance being more than before by the whole diameter of the earth's annual orbit, or by double the earth's distance from the sun, then the eclipses are seen to happen the latest of any, and later than before by about a quarter of an hour. Hence therefore it follows, that light takes up a quarter of an hour in travelling across the orbit of the earth, or near 8 minutes in passing from the sun to the earth; which gives us about 12 millions of miles per minute, or 200,000 miles per second, for the velocity of light. A discovery that was first made by M. Roemer.

The third and greatest advantage derived from the eclipses of the Satellites, is the knowledge of the longitudes of places on the earth. Suppose two observers of an eclipse, the one, for example, at London, the other at the Canaries; it is certain that the eclipse will appear at the same moment to both observers; but as they are situated under different meridians, they count different hours, being perhaps 9 o'clock to the one, when it is only 8 to the other; by which observations of the true time of the eclipse, on communication, they find the difference of their longitudes to be one hour in time, which answers to 15 degrees of longitude.

Satellites *of Saturn*, are 7 little secondary planets revolving about him.

One of them, which till lately was reckoned the 4th in order from Saturn, was discovered by Huygens, the 25th of March 1655, by means of a telescope 12 feet long; and the 1st, 2d, 3d, and 5th, at different times, by Cassini; viz, the 5th in October 1671, by a telescope of 17 feet; the 3d in December 1672, by a telescope of Campani's, 35 feet long; and the first and second in March 1684, by help of Campani's glasses, of 100 and 136 feet. Finally, the 6th and 7th Satellites have lately been discovered by Dr. Herschel, with his 40 feet reflecting telescope, viz, the 6th on the 19th of August 1787, and the 7th on the 17th of September 1788. These two he has called the 6th and 7th Satellites, though they are nearer to the planet Saturn than any of the former five, that the names or numbers of these might not be mistaken or confounded, with regard to former observations of them.

Moreover, the great distance between the 4th and 5th Satellite, gave occasion to Huygens to suspect that there might be some intermediate one, or else that the 5th might have some other Satellite moving round it, as its centre. Dr. Halley, in the Philos. Trans. (numb. 145, or Abr. vol. 1. pa. 371) gives a correction of the theory of the motions of the 4th or Huygenian Satellite. Its true period he makes 11d 22h 41′ 6″.

The periodical revolutions, and distances of these Satellites from the body of Saturn, expressed in semidiameters of that planet, and in miles, are as follow.

Satellites of Saturn.				
Satel-lites.	Periods.	Distances in		Diam. of Orbit.
		Semidiameters.	Miles.	
1	1d 21h 18′ 27″	$4\frac{3}{8}$	170,000	1′ 27″
2	2 17 41 22	$5\frac{1}{2}$	217,000	1 52
3	4 12 25 12	8	303,000	2 36
4	15 22 41 13	18	704,000	6 18
5	79 7 48 0	54	2,050,000	17 4
6	1 8 53 9	$3\frac{5}{9}$	135,000	1 14
7	0 22 40 46	$2\frac{5}{6}$	107,000	0 57

The four first describe ellipses like to those of the ring, and are in the same plane. Their inclination to the ecliptic is from 30 to 31 degrees. The 5th describes an orbit inclined from 17 to 18 degrees with the orbit of Saturn; his plane lying between the ecliptic and those of the other Satellites, &c. Dr. Herschel observes that the 5th Satellite turns once round its axis exactly in the time in which it revolves about the planet Saturn; in which respect it resembles our moon, which does the same thing. And he makes the angle of its distance from Saturn, at his mean distance, 17′ 2″. Philos. Trans. 1792, pa. 22. See a long account of observations of these Satellites, with tables of their mean motions, by Dr. Herschel, Philos. Trans. 1790, pa. 427 &c.

Satellites *of the Georgian Planet*, or *Herschel*, are two little moons that revolve about him, like those of

Jupiter and Saturn. These Satellites were discovered by Dr. Herschel, in the month of January 1787, who gave an account of them in the Philos. Trans. of that year, pa. 125 &c; and a still farther account of them in the vol. for 1788, pa. 364 &c; from which it appears that their synodical periods, and angular distances from their primary, are as follow:

Satellite.	Periods.	Dist.
1	8d 17h 1′ 19″	0′ 33″
2	13 11 5 1½	0 44⅖

The orbits of these Satellites are nearly perpendicular to the ecliptic; and in magnitude they are probably not less than those of Jupiter.

SATELLITE *of Venus*. Cassini thought he saw one, and Mr. Short and other astronomers have suspected the same thing. (Hist. de l'Acad. 1741, Philos. Trans. numb. 459). But the many fruitless searches that have been since made to discover it, leave room to suspect that it has been only an optical illusion, formed by the glasses of telescopes; as appears to be the opinion of F. Hell, at the end of his Ephemeris for 1766, and Boscovich, in his 5th Optical Dissertation.

Neither has it been discovered that either of the other planets Mars and Mercury have any Satellites revolving about them.

SATURDAY, the 7th or last day of the week, so called, as some have supposed, from the idol Seater, worshipped on this day by the ancient Saxons, and thought to be the same as the Saturn of the Latins. In astronomy, every day of the week is denoted by some one of the planets, and this day is marked with the planet ♄ Saturn. Saturday answers to the Jewish sabbath.

SATURN, one of the primary planets, being the 6th in order of distance from the sun, and the outermost of all, except the Georgian planet, or Herschel, lately discovered; and is marked with the character ♄, denoting an old man supporting himself with a staff, representing the ancient god Saturn.

Saturn shines with but a feeble light, partly on account of his great distance, and partly from its dull red colour. This planet is perhaps one of the most engaging objects that astronomy offers to our view; it is surrounded with a double ring, one without the other, and beyond these by 7 Satellites, all in the plane of the rings; the rings and planets being all dark and dense bodies, like Saturn himself, these bodies casting their shadows mutually one upon another; though the reflected light of the rings is usually brighter than that of the planet itself.

Saturn has also certain obscure zones, or belts, appearing at times across his disc, like those of Jupiter, which are changeable, and are probably obscurations in his atmosphere. Dr. Herschel, Philos. Trans. 1790, shews that Saturn has a dense atmosphere; that he revolves about an axis, which is perpendicular to the plane of the rings; that his figure is, like the other planets, the oblate spheroid, being flatted at the poles, the polar diameter being to the equatorial one as 10 to 11; that his ring has a motion of rotation in its own plane, its axis of motion being the same as that of Saturn himself, and its periodical time equal to $10^h\ 32'\ 15''\cdot4$. See also RING, and SATELLITE.

Concerning the discovery of the ring and figure of Saturn; we find that Galileo first perceived that his figure is not round: but Huygens shewed, in his Systema Saturniana 1659, that this was owing to the positions of his ring; for his spheroidical form could only be seen by Herschel's telescope; though indeed Cassini, in an observation made June 19, 1692, saw the oval figure of Saturn's shadow upon his ring.

Mr. Bugge determines (Philos. Trans. 1787, pa. 42) the heliocentric longitude of Saturn's descending node to be $9^s\ 21°\ 5'\ 8''\frac{1}{2}$; and that the planet was in that node August 21, 1784, at $18^h\ 20'\ 10''$, time at Copenhagen.

The annual period of Saturn about the sun, is 10759 days 7 hours, or almost 30 years; and his diameter is about 67000 miles, or near 8½ times the diameter of the earth; also his distance is about 9½ times that of the earth. Hence some have concluded that his light and heat are entirely unfit for rational inhabitants. But that their light is not so weak as we imagine, is evident from their brightness in the night time. Besides, allowing the sun's light to be 45000 times as strong, with respect to us, as the light of the moon when full, the sun will afford 500 times as much light to Saturn as the full moon does to us, and 1600 times as much to Jupiter. So that these two planets, even without any moon, would be much more enlightened than we at first imagine; and by having so many, they may be very comfortable places of residence. Their heat, so far as it depends on the force of the sun's rays, is certainly much less than ours; to which no doubt the bodies of their inhabitants are as well adapted as ours are to the seasons we enjoy. And if it be considered that Jupiter never has any winter, even at his poles, which probably is also the case with Saturn, the cold cannot be so intense on these two planets as is generally imagined. To this may be added, that there may be something in the nature of their mould warmer than in that of our earth; and we find that all our heat does not depend on the rays of the sun; for if it did, we should always have the same months equally hot or cold at their annual return, which is very far from being the case.

See the articles PLANET, PERIOD, RING, SATELLITE.

SAUCISSE, in Artillery, a long train of powder inclosed in a roll or pipe of pitched cloth, and sometimes of leather, about 2 inches in diameter; serving to set fire to mines or caissons. It is usually placed in a wooden pipe, called an auget, to prevent its growing damp.

SAUCISSON, in Fortification, a kind of faggot, made of thick branches of trees, or of the trunks of shrubs, bound together; for the purpose of covering the men, and to serve as epaulements; and also to repair breaches, stop passages, make traverses over a wet ditch, &c.

The Saucisson differs from the fascine, which is only made of small branches; and by its being bound at both ends, and in the middle.

SAVILLE (Sir HENRY), a very learned Englishman,

man, the second son of Henry Saville, Esq. was born at Bradley, near Halifax, in Yorkshire, November the 30th, 1549. He was entered of Merton-college, Oxford, in 1561, where he took the degrees in arts, and was chosen fellow. When he proceeded master of arts in 1570, he read for that degree on the Almagest of Ptolomy, which procured him the reputation of a man eminently skilled in mathematics and the Greek language; in the former of which he voluntarily read a public lecture in the university for some time.

In 1578 he travelled into France and other countries; where, diligently improving himself in all useful learning, in languages, and the knowledge of the world, he became a most accomplished gentleman. At his return, he was made tutor in the Greek tongue to queen Elizabeth, who had a great esteem and liking for him.

In 1585 he was made warden of Merton-college, which he governed six-and-thirty years with great honour, and improved it by all the means in his power.—In 1596 he was chosen provost of Eton-college; which he filled with many learned men.—James the First, upon his accession to the crown of England, expressed a great regard for him, and would have preferred him either in church or state; but Saville declined it, and only accepted the ceremony of knighthood from the king at Windsor in 1604. His only son Henry dying about that time, he thenceforth devoted his fortune to the promoting of learning. Among other things, in 1619, he founded, in the university of Oxford, two lectures, or professorships, one in geometry, the other in astronomy; which he endowed with a salary of 160l. a year each, besides a legacy of 600l. to purchase more lands for the same use. He also furnished a library with mathematical books near the mathematical school, for the use of his professors; and gave 100l. to the mathematical chest of his own appointing: adding afterwards a legacy of 40l. a year to the same chest, to the university, and to his professors jointly. He likewise gave 120l. towards the new-building of the schools, beside several rare manuscripts and printed books to the Bodleian library; and a good quantity of Greek types to the printing-press at Oxford.

After a life thus spent in the encouragement and promotion of science and literature in general, he died at Eton-college the 19th of February 1622, in the 73d year of his age, and was buried in the chapel there. On this occasion, the university of Oxford paid him the greatest honours, by having a public speech and verses made in his praise, which were published soon after in 4to, under the title of *Ultima Linea Savilii.*

As to the character of Saville, the highest encomiums are bestowed on him by all the learned of his time: by Casaubon, Mercerus, Meibomius, Joseph Scaliger, and especially the learned bishop Montague; who, in his *Diatribæ* upon Selden's History of Tythes, styles him, "that magazine of learning, whose memory shall be honourable amongst not only the learned, but the righteous for ever."

Several noble instances of his munificence to the republic of letters have already been mentioned: in the account of his publications many more, and even greater, will appear. These are,

1. *Four Books of the Histories of Cornelius Tacitus, and the Life of Agricola;* with Notes upon them, in folio, dedicated to Queen Elizabeth, 1581.

2. *A View of certain Military Matters*, or Commentaries concerning Roman Warfare, 1598.

3. *Rerum Anglicarum Scriptores post Bedam*, &c. 1596. This is a collection of the best writers of our English history; to which he added chronological tables at the end, from Julius Cæsar to William the Conqueror.

4. *The Works of St. Chrysostom*, in Greek, in 8 vols. folio, 1613. This is a very fine edition, and composed with great cost and labour. In the preface he says, "that having himself visited, about 12 years before, all the public and private libraries in Britain, and copied out thence whatever he thought useful to this design, he then sent some learned men into France, Germany, Italy, and the East, to transcribe such parts as he had not already, and to collate the others with the best manuscripts." At the same time, he makes his acknowledgments to several eminent men for their assistance; as Thuanus, Velserus, Schottus, Casaubon, Ducæus, Gruter, Hoeschelius, &c. In the 8th volume are inserted Sir Henry Saville's own notes, with those of other learned men. The whole charge of this edition, including the several sums paid to learned men, at home and abroad, employed in finding out, transcribing, and collating the best manuscripts, is said to have amounted to no less than 8000l. Several editions of this work were afterwards published at Paris.

5. In 1618 he published a Latin work, written by Thomas Bradwardin, abp. of Canterbury, against Pelagius, intitled, *De Causa Dei contra Pelagium, et de virtute causarum;* to which he prefixed the life of Bradwardin.

6. In 1621 he published a collection of his own Mathematical Lectures on Euclid's Elements; in 4to.

7. *Oratio coram Elizabetha Regina Oxoniæ habita*, anno 1592. Printed at Oxford in 1658, in 4to.

8. He translated into Latin king James's *Apology for the Oath of Allegiance.* He also left several manuscripts behind him, written by order of king James; all which are in the Bodleian library. He wrote notes likewise upon the margin of many books in his library, particularly Eusebius's *Ecclesiastical History;* which were afterwards used by Valesius, in his edition of that work in 1659.—Four of his letters to Camden are published by Smith, among *Camden's Letters*, 1691, 4to.

Sir Henry Saville had a younger brother, *Thomas* Saville, who was admitted probationer fellow of Merton college, Oxford, in 1580. He afterwards travelled abroad into several countries. Upon his return he was chosen fellow of Eton-college; but he died at London in 1593. Thomas Saville was also a man of great learning, and an intimate friend of Camden; among whose letters, just mentioned, there are 15 of Mr. Saville's to him.

SAUNDERSON (Dr. Nicholas), an illustrious professor of mathematics in the university of Cambridge, and a fellow of the Royal Society, was born at Thurlston in Yorkshire in 1682. When he was but twelve months old, he lost not only his eye-sight, but his very eye-balls, by the small-pox; so that he could retain no more ideas of vision than if he had been born blind. At an early age, however, being of very pro-

miſing parts, he was ſent to the free-ſchool at Penniſton, and there laid the foundation of that knowledge of the Greek and Latin languages, which he afterwards improved ſo far, by his own application to the claſſic authors, as to hear the works of Euclid, Archimedes, and Diophantus read in their original Greek.

Having acquired a grammatical education, his father, who was in the exciſe, inſtructed him in the common rules of arithmetic. And here it was that his excellent mathematical genius firſt appeared: for he very ſoon became able to work the common queſtions, to make very long calculations by the ſtrength of his memory, and to form new rules to himſelf for the better reſolving of ſuch queſtions as are often propoſed to learners as trials of ſkill.

At the age of 18, our author was introduced to the acquaintance of Richard Weſt, of Underbank, Eſq. a lover of mathematics, who, obſerving Mr. Saunderſon's uncommon capacity, took the pains to inſtruct him in the principles of algebra and geometry, and gave him every encouragement in his power to the proſecution of theſe ſtudies. Soon after this he became acquainted alſo with Dr. Nettleton, who took the ſame pains with him. And it was to theſe two gentlemen that Mr. Saunderſon owed his firſt inſtitution in the mathematical ſciences: they furniſhed him with books, and often read and expounded them to him. But he ſoon ſurpaſſed his maſters, and became fitter to teach, than to learn any thing from them.

His father, otherwiſe burdened with a numerous family, finding a difficulty in ſupporting him, his friends began to think of providing both for his education and maintenance. His own inclination led him ſtrongly to Cambridge, and it was at length determined he ſhould try his fortune there, not as a ſcholar, but as a maſter: or, if this deſign ſhould not ſucceed, they promiſed themſelves ſucceſs in opening a ſchool for him at London. Accordingly he went to Cambridge in 1707, being then 25 years of age, and his fame in a ſhort time filled the univerſity. Newton's Principia, Optics, and Univerſal Arithmetic, were the foundations of his lectures, and afforded him a noble field for the diſplaying of his genius; and great numbers came to hear a blind man give lectures on optics, diſcourſe on the nature of light and colours, explain the theory of viſion, the effect of glaſſes, the phenomenon of the rainbow, and other objects of ſight.

As he inſtructed youth in the principles of the Newtonian philoſophy, he ſoon became acquainted with its incomparable author, though he had ſeveral years before left the univerſity; and frequently converſed with him on the moſt difficult parts of his works: he alſo held a friendly communication with the other eminent mathematicians of the age, as Halley, Cotes, Demoivre, &c.

Mr. Whiſton was all this time in the mathematical profeſſor's chair, and read lectures in the manner propoſed by Mr. Saunderſon on his ſettling at Cambridge; ſo that an attempt of this kind looked like an encroachment on the privilege of his office; but, as a good-natured man, and an encourager of learning, he readily conſented to the application of friends made in behalf of ſo uncommon a perſon.

Upon the removal of Mr. Whiſton from his profeſſorſhip, Mr. Saunderſon's merit was thought ſo much ſuperior to that of any other competitor, that an extraordinary ſtep was taken in his favour, to qualify him with a degree, which the ſtatute requires: in conſequence he was choſen in 1711, Mr. Whiſton's ſucceſſor in the Lucaſian profeſſorſhip of mathematics, Sir Iſaac Newton intereſting himſelf greatly in his favour. His firſt performance, after he was ſeated in the chair, was an inaugural ſpeech made in very elegant latin, and a ſtyle truly Ciceronian; for he was well verſed in the writings of Tully, who was his favourite in proſe, as Virgil and Horace were in verſe. From this time he applied himſelf cloſely to the reading of lectures, and gave up his whole time to his pupils. He continued to reſide among the gentlemen of Chriſt-college till the year 1723, when he took a houſe in Cambridge, and ſoon after married a daughter of Mr. Dickens, rector of Boxworth in Cambridgeſhire, by whom he had a ſon and a daughter.

In the year 1728, when king George viſited the univerſity, he expreſſed a deſire of ſeeing ſo remarkable a perſon; and accordingly our profeſſor attended the king in the ſenate, and by his favour was there created doctor of laws.

Dr. Saunderſon was naturally of a ſtrong healthy conſtitution; but being too ſedentary, and conſtantly confining himſelf to the houſe, he became a valetudinarian: and in the ſpring of the year 1739 he complained of a numbneſs in his limbs, which ended in a mortification in his foot, of which he died the 19th of April that year, in the 57th year of his age.

There was ſcarcely any part of the mathematics on which Dr. Saunderſon had not compoſed ſomething for the uſe of his pupils. But he diſcovered no intention of publiſhing any thing till, by the perſuaſion of his friends, he prepared his Elements of Algebra for the preſs, which after his death were publiſhed by ſubſcription in 2 vols 4to, 1740.

He left many other writings, though none perhaps prepared for the preſs. Among theſe were ſome valuable comments on Newton's Principia, which not only explain the more difficult parts, but often improve upon the doctrines. Theſe are publiſhed in Latin at the end of his poſthumous Treatiſe on Fluxions, a valuable work, publiſhed in 8vo, 1756.—His manuſcript lectures too, on moſt parts of natural philoſophy, which I have ſeen, might make a conſiderable volume, and prove an acceptable preſent to the public if printed.

Dr. Saunderſon, as to his character, was a man of much wit and vivacity in converſation, and eſteemed an excellent companion. He was endued with a great regard to truth; and was ſuch an enemy to diſguiſe, that he thought it his duty to ſpeak his thoughts at all times with unreſtrained freedom. Hence his ſentiments on men and opinions, his friendſhip or diſregard, were expreſſed without reſerve; a ſincerity which raiſed him many enemies.

A blind man, moving in the ſphere of a mathematician, ſeems a phenomenon difficult to be accounted for, and has excited the admiration of every age in which it has appeared. Tully mentions it as a thing ſcarce credible in his own maſter in philoſophy, Diodotus; that he exerciſed himſelf in it with more aſſi-

duity after he became blind; and, what he thought next to impoffible to be done without fight, that he profeffed geometry, defcribing his diagrams fo exactly to his fcholars, that they could draw every line in its proper direction. St. Jerome relates a ftill more remarkable inftance in Didymus of Alexandria, who, though blind from his infancy, and therefore ignorant of the very letters, not only learned logic, but geometry alfo to very great perfection, which feems moft of all to require fight. But, if we confider that the ideas of extended quantity, which are the chief objects of mathematics, may as well be acquired by the fenfe of feeling as that of fight, that a fixed and fteady attention is the principal qualification for this ftudy, and that the blind are by neceffity more abftracted than others (for which reafon it is faid that Democritus put out his eyes, that he might think more intenfely), we fhall perhaps find reafon to fuppofe that there is no branch of fcience fo much adapted to their circumftances.

At firft, Dr. Saunderfon acquired moft of his ideas by the fenfe of feeling; and this, as is commonly the cafe with the blind, he enjoyed in great perfection. Yet he could not, as fome are faid to have done, diftinguifh colours by that fenfe; for, after having made repeated trials, he ufed to fay, it was pretending to impoffibilities. But he could with great nicety and exactnefs obferve the fmalleft degree of roughnefs or defect of polifh in a furface. Thus, in a fet of Roman medals, he diftinguifhed the genuine from the falfe, though they had been counterfeited with fuch exactnefs as to deceive a connoiffeur who had judged by the eye. By the fenfe of feeling alfo, he diftinguifhed the leaft variation; and he has been feen in a garden, when obfervations have been making on the fun, to take notice of every cloud that interrupted the obfervation almoft as juftly as they who could fee it. He could alfo tell when any thing was held near his face, or when he paffed by a tree at no great diftance, merely by the different impulfe of the air on his face.

His ear was alfo equally exact. He could readily diftinguifh the 5th part of a note. By the quicknefs of this fenfe he could judge of the fize of a room, and of his diftance from the wall. And if ever he walked over a pavement, in courts or piazzas which reflected a found, and was afterwards conducted thither again, he could tell in what part of the walk he ftood, merely by the note it founded.

Dr. Saunderfon had a peculiar method of performing arithmetical calculations, by an ingenious machine and method which has been called his Palpable Arithmetic, and is particularly defcribed in a piece prefixed to the firft volume of his Algebra. That he was able to make long and intricate calculations, both arithmetical and algebraical, is a thing as certain as it is wonderful. He had contrived for his own ufe, a commodious notation for any large numbers, which he could exprefs on his abacus, or calculating table, and with which he could readily perform any arithmetical operations, by the fenfe of feeling only, for which reafon it was called his Palpable Arithmetic.

His calculating table was a fmooth thin board, a little more than a foot fquare, raifed upon a fmall frame fo as to lie hollow; which board was divided into a great number of little fquares, by lines interfecting one another perpendicularly, and parallel to the fides of the table, and the parallel ones only one-tenth of an inch from each other; fo that every fquare inch of the table was thus divided into 100 little fquares. At every point of interfection the board was perforated by fmall holes, capable of receiving a pin; for it was by the help of pins, ftuck up to the head through thefe holes, that he expreffed his numbers. He ufed two forts of pins, a larger and a fmaller fort; at leaft their heads were different, and might eafily be diftinguifhed by feeling. Of thefe pins he had a large quantity in two boxes, with their points cut off, which always ftood ready before him when he calculated. The writer of that account defcribes particularly the whole procefs of ufing the machine, and concludes, "He could place and difplace his pins with incredible nimblenefs and facility, much to the pleafure and furprize of all the beholders. He could even break off in the middle of a calculation, and refume it when he pleafed, and could prefently know the condition of it, by only drawing his fingers gently over the table."

SAURIN (Joseph), an ingenious French mathematician, was born in 1659, at Courtaifon, in the principality of Orange. His father, minifter at Grenoble, was a man of a very ftudious difpofition, and was the firft preceptor or inftructor to our author; who made a rapid progrefs in his ftudies, and at a very early age was admitted a minifter at Eure in Dauphiny. But preaching an offenfive fermon, he was obliged to quit France in 1683. On this occafion he retired to Geneva; from whence he went into the State of Berne, and was appointed to a living at Yverdun. He was no fooner eftablifhed in this his poft, than certain theologians raifed a ftorm againft him. Saurin, difgufted with the controverfy, and ftill more with the Swifs, where his talents were buried, paffed into Holland, and from thence into France, where he put himfelf under the protection of the celebrated Boffu, to whom he made his abjuration in 1690, as it is fufpected, that he might find protection, and have an opportunity of cultivating the fciences at Paris. And he was not difappointed: he met with many flattering encouragements; was even much noticed by the king, had a penfion from the court, and was admitted of the Academy of Sciences in 1707, in the quality of geometrician. This fcience was now his chief ftudy and delight; with many writings upon which he enriched the volumes of the Journal des Savans, and the Memoirs of the Academy of Sciences. Thefe were the only works of this kind that he publifhed: he was author of feveral other pieces of a controverfial nature, againft the celebrated Rouffeau, and other antagonifts, over whom with the affiftance of government he was enabled to triumph. The latter part of his life was fpent in more peace, and in cultivating the mathematical fciences; and he died the 29th of December 1737, of a lethargic fever, at 78 years of age.

The character of Saurin was lively and impetuous, endued with a confiderable degree of that noble independence and loftinefs of manner, which is apt to be miftaken for haughtinefs or infolence; in confequence of which, his memory was attacked after his death, as his reputation had been during his life; and it was even faid

faid he had been guilty of crimes, by his own confeffion, that ought to have been punifhed with death.

Saurin's mathematical and philofophical papers, printed in the Memoirs of the Academy of Sciences, which are pretty numerous, are to be found in the volumes for the years following; viz, 1709, 1710, 1713, 1716, 1718, 1720, 1722, 1723, 1725, 1727.

SAUVEUR (Joseph), an eminent French mathematician, was born at La Fleche the 24th of March 1653. He was abfolutely dumb till he was feven years of age; and then the organs of fpeech did not difengage fo effectually, but that he was ever after obliged to fpeak very flowly and with difficulty. He very early difcovered a great turn for mechanics, and was always inventing and conftructing fomething or other in that way.

He was fent to the college of the Jefuits to learn polite literature, but made very little progrefs in poetry and eloquence. Virgil and Cicero had no charms for him; but he read with greedinefs books of arithmetic and geometry. However, he was prevailed on to go to Paris in 1670, and, being intended for the church, there he applied himfelf for a time to the ftudy of philofophy and theology; but ftill fucceeded no better. In fhort, mathematics was the only ftudy he had any paffion or relifh for, and this he cultivated with extraordinary fuccefs; for during his courfe of philofophy, he learned the firft fix books of Euclid in the fpace of a month, without the help of a mafter.

As he had an impediment in his voice, though otherwife endued with extraordinary abilities, he was advifed by M. Boffuet, to give up all defigns upon the church, and to apply himfelf to the ftudy of phyfic: but this being utterly againft the inclination of his uncle, from whom he drew his principal refources, Sauveur determined to devote himfelf to his favourite ftudy, and to perfect himfelf in it, fo as to teach it for his fupport; and in effect he foon became the fafhionable preceptor in mathematics, fo that at 23 years of age he had prince Eugene for his fcholar.—He had not yet read the geometry of Des Cartes; but a foreigner of the firft quality defiring to be taught it, he made himfelf mafter of it in an inconceivably fmall fpace of time.—Baffet being a fafhionable game at that time, the marquis of Dangeau afked him for fome calculations relating to it, which gave fuch fatisfaction, that Sauveur had the honour to explain them to the king and queen.

In 1681 he was fent with M. Mariotte to Chantilli, to make fome experiments upon the waters there, which he did with much applaufe. The frequent vifits he made to this place infpired him with the defign of writing a treatife on fortification; and, in order to join practice with theory, he went to the fiege of Mons in 1691, where he continued all the while in the trenches. With the fame view alfo he vifited all the towns of Flanders; and on his return he became the mathematician in ordinary at the court, with a penfion for life.—In 1680 he had been chofen to teach mathematics to the pages of the Dauphinefs. In 1686 he was appointed mathematical profeffor in the Royal College. And in 1696 admitted a member of the Academy of Sciences, where he was in high efteem with the members of that fociety.—He became alfo particularly acquainted with the prince of Condé, from whom he received many marks of favour and affection. Finally, M. Vauban having been made marfhal of France, in 1703, he propofed Sauveur to the king as his fucceffor in the office of examiner of the engineers; to which the king agreed, and honoured him with a penfion, which our author enjoyed till his death, which happened the 9th of July 1716, in the 64th year of his age

Sauveur, in his character, was of a kind obliging difpofition, of a fweet, uniform, and unaffected temper; and although his fame was pretty generally fpread abroad, it did not alter his humble deportment, and the fimplicity of his manners. He ufed to fay, that what one man could accomplifh in mathematics, another might do alfo, if he chofe it.

He was twice married. The firft time he took a very fingular precaution; for he would not meet the lady till he had been with a notary to have the conditions, he intended to infift on, reduced into a written form; for fear the fight of her fhould not leave him enough mafter of himfelf. This was acting very wifely, and like a true mathematician; who always proceeds by rule and line, and makes his calculations when his head is cool.—He had children by both his wives; and by the latter a fon who, like himfelf, was dumb for the firft feven years of his life.

An extraordinary part of Sauveur's character is, that though he had neither a mufical voice nor ear, yet he ftudied no fcience more than mufic, of which he compofed an entire new fyftem. And though he was obliged to borrow other people's voice and ears, yet he amply repaid them with fuch demonftrations as were unknown to former muficians. He alfo introduced a new diction in mufic, more appropriate and extenfive. He invented a new doctrine of founds. And he was the firft that difcovered, by theory and experiment, the velocity of mufical ftrings, and the fpaces they defcribe in their vibrations, under all circumftances of tenfion and dimenfions. It was he alfo who firft invented for this purpofe the monochord and the echometer. In fhort, he purfued his refearches even to the mufic of the ancient Greeks and Romans, to the Arabs, and to the very Turks and Perfians themfelves; fo jealous was he, left any thing fhould efcape him in the fcience of founds.

Sauveur's writings, which confift of pieces rather than of fet works, are all inferted in the volumes of the Memoirs of the Academy of Sciences, from the year 1700 to the year 1716, on various geometrical, mathematical, philofophical, and mufical fubjects.

SCALE, a mathematical inftrument, confifting of certain lines drawn on wood, metal, or other matter, divided into various parts, either equal or unequal. It is of great ufe in laying down diftances in proportion, or in meafuring diftances already laid down.

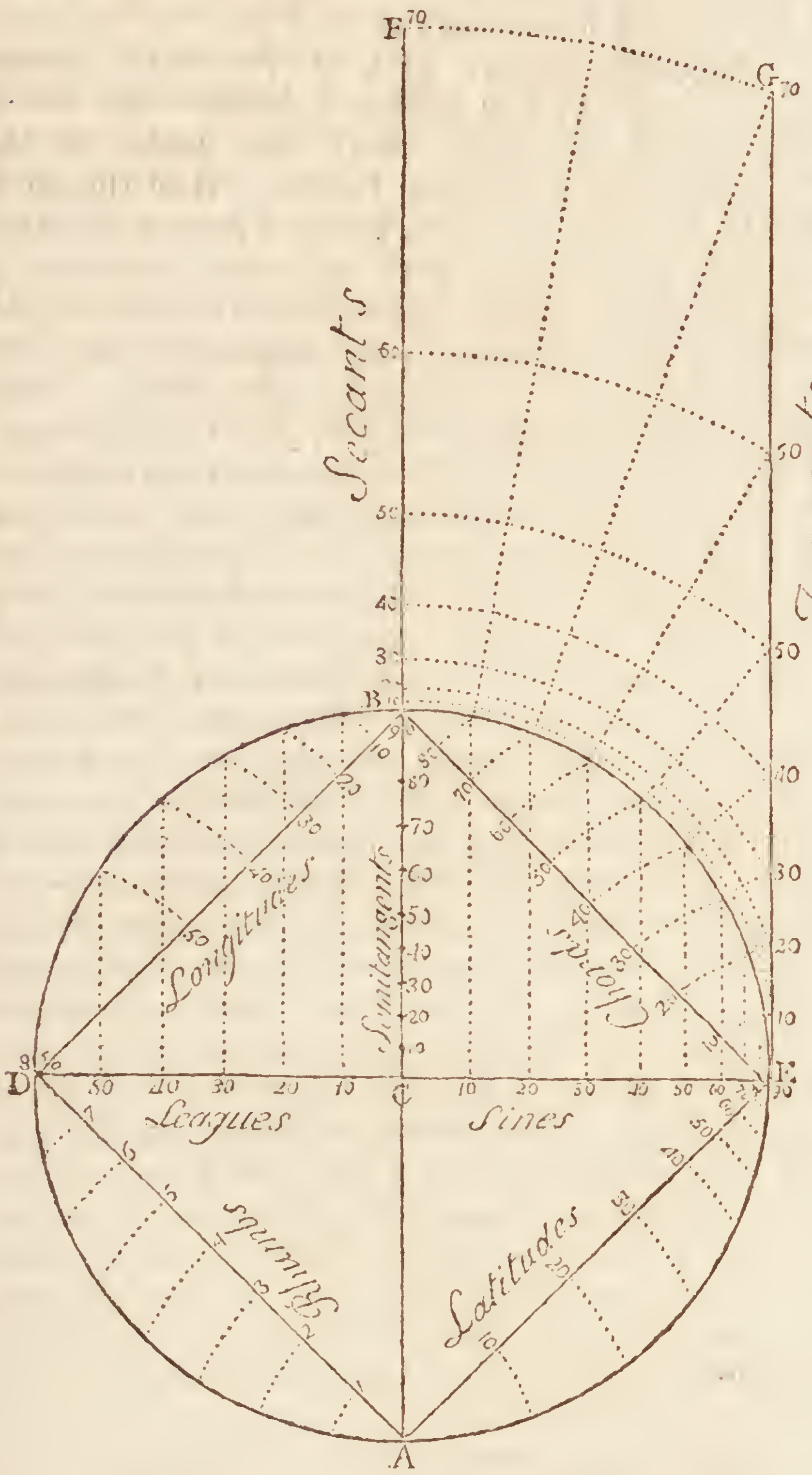
There are Scales of various kinds, accommodated to the feveral ufes: the principal are the *plane Scale*, the *diagonal Scale*, *Gunter's Scale*, and the *plotting Scale*.

Plane or Plain Scale, a mathematical inftrument of very extenfive ufe and application; which is commonly made of 2 feet in length; and the lines ufually drawn upon it are the following, viz,

1. Lines

1	Lines of Equal parts, and marked	E. P.
2	- - Chords - - -	Cho.
3	- - Rhumbs - - -	Ru.
4	- - Sines - - -	Sin.
5	- - Tangents - -	Tan.
6	- - Secants - -	Sec.
7	- - Semitangents - -	S. T.
8	- - Longitude - -	Long.
9	- - Latitude - -	Lat.

1. The lines of equal parts are of two kinds, viz, ſimply divided, and diagonally divided. The firſt of theſe are formed by drawing three lines parallel to one another, and dividing them into any equal parts by ſhort lines drawn acroſs them, and in like manner ſubdividing the firſt diviſion or part into 10 other equal ſmall parts; by which numbers or dimenſions of two figures may be taken off. Upon ſome rulers, ſeveral of theſe ſcales of equal parts are ranged parallel to each other, with figures ſet to them to ſhew into how many equal parts they divide the inch; as 20, 25, 30, 35, 40, 45, &c. The 2d or diagonal diviſions are formed by drawing eleven long parallel and equidiſtant lines, which are divided into equal parts, and croſſed



by other ſhort lines, as the former; then the firſt of the equal parts have the two outermoſt of the eleven parallels divided into 10 equal parts, and the points of diviſion being connected by lines drawn diagonally, the whole ſcale is thus divided into dimenſions or numbers of three places of figures.

The other lines upon the ſcales are ſuch as are commonly uſed in trigonometry, navigation, aſtronomy, dialling, projection of the ſphere, &c, &c; and their conſtructions are moſtly taken from the diviſions of a circle, as follow:

Deſcribe a circle with any convenient radius, and quarter it by drawing the diameters AB and DE at right angles to each other; continue the diameter AB out towards F, and draw the tangent line EG parallel to it; alſo draw the chords AD, DB, BE, EA. Then,

2. For the line of chords, divide a quadrant BE into 90 equal parts; on E as a centre, with the compaſſes transfer theſe diviſions to the chord line EB, which mark with the correſponding numbers, and it will become a line of chords, to be transferred to the ruler.

3. For the line of rhumbs, divide the quadrant AD into 8 equal parts; then with the centre A transfer the diviſions to the chord AD, for the line of rhumbs.

4. For the line of ſines, through each of the diviſions of the arc BE, draw right lines parallel to the radius BC, which will divide the radius CE into the ſines, or verſed ſines, numbering it from C to E for the ſines, and from E to C for the verſed ſines.

5. For the line of tangents, lay a ruler on C, and the ſeveral diviſions of the arc BE, and it will interſect the line EG, which will become a line of tangents, and numbered from E to G with 10, 20, 30, 40, &c.

6. For the line of ſecants, transfer the diſtances between the centre C and the diviſions on the line of tangents to the line BF, from the centre C, and theſe will give the diviſions of the line of ſecants, which muſt be numbered from B towards F, with 10, 20, 30, &c.

7. For the line of ſemitangents, lay a ruler on D and the ſeveral diviſions of the arc EB, which will interſect the radius CB in the diviſions of the ſemitangents, which are to be marked with the correſponding figures of the arc EB.

The chief uſes of the ſines, tangents, ſecants, and ſemitangents, are to find the poles and centres of the ſeveral circles repreſented in the projections of the ſphere.

8. For the line of longitude, divide the radius CD into 60 equal parts; through each of theſe, parallels to the radius BC will interſect the arc BD in as many points: from D as a centre the diviſions of the arc BD being transferred to the chord BD, will give the diviſions of the line of longitude.

If this line be laid upon the ſcale cloſe to the line of chords, both inverted, ſo that 60° in the ſcale of longitude be againſt 0° in the chords, &c; and any degree of latitude be counted on the chords, there will ſtand oppoſite to it, in the line of longitude, the miles contained in one degree of longitude, in that latitude; the meaſure of 1 degree under the equator being 60 geographical miles.

9. For the line of latitude, lay a ruler on B, and the feveral divifions on the fines on CE, and it will interfect the arc AE in as many points; on A as a centre transfer the interfections of the arc AE to the chord AE, for the line of latitude.

See alfo Robertfon's Defcription and ufe of Mathematical Inftruments.

Decimal, or *Gunter's*, or *Plotting*, or *Proportional*, or *Reducing* SCALE. See the feveral articles.

SCALE, in Architecture and Geography, a line divided into equal parts, placed at the bottom of a map or draught, to ferve as a common meafure to all the parts of the building, or all the diftances and places of the map.

In maps of large tracts, as kingdoms and provinces, &c, the Scale ufually confifts of miles; whence it is denominated a Scale of miles.—In more particular maps, as thofe of manors, &c, the Scale is ufually of chains &c.—The Scales ufed in draughts of buildings moftly confift of modules, feet, inches, palms, fathoms, or the like.

To find the diftance between two towns &c, in a map, the interval is taken in the compaffes, and fet off in the fcale; and the number of divifions it includes gives the diftance. The fame method ferves to find the height of a ftory, or other part in a defign.

Front SCALE, in Perfpective, is a right line in the draught, parallel to the horizontal line; divided into equal parts, reprefenting feet, inches, &c.

Flying SCALE, is a right line in the draught, tending to the point of view, and divided into unequal parts, reprefenting feet, inches, &c.

Differential SCALE, is ufed for the fcale of relation fubtracted from unity. See SERIES.

SCALE *of Relation*, in Algebra, an expreffion denoting the relation of the terms of recurring feries to each other. See SERIES.

Hour SCALE. See HOUR.

SCALE, in Mufic, is a denomination given to the arrangement of the fix fyllables, invented by Guido Aratino, *ut re mi fa fol la;* called alfo gammut. It is called Scale, or ladder, becaufe it reprefents a kind of ladder, by means of which the voice rifes to acute, or finks to grave; each of the fix fyllables being as it were one ftep of the ladder.

SCALE is alfo ufed for a feries of founds rifing or falling towards acutenefs or gravity, from any given pitch of tune, to the greateft diftance that is fit or practicable, through fuch intermediate degrees as to make the fucceffion moft agreeable and perfect, and in which we have all the harmonical intervals moft commodioufly divided.

The fcale is otherwife called an *univerfal fyftem*, as including all the particular fyftems belonging to mufic. See SYSTEM.

There were three different Scales in ufe among the Ancients, which had their denominations from the three feveral forts of mufic, viz, the *diatonic*, *chromatic*, and *inharmonic*. Which fee.

SCALENE, or SCALENOUS *triangle*, is a triangle whofe fides and angles are all unequal.—A cylinder or cone, whofe axis is oblique or inclined to its bafe, is alfo faid to be fcalenous: though more frequently it is called oblique.

SCALIGER (JOSEPH JUSTUS), a celebrated French chronologer and critic, was the fon of Julius Cæfar Scaliger, and born at Agen in France, in 1540. He ftudied in the college of Bourdeaux; after which his father took him under his own care, and employed him in tranfcribing his poems; by which means he obtained fuch a tafte for poetry, that before he was 17 years old, he wrote a tragedy upon the fubject of Oedipus, in which he introduced all the poetical ornaments of ftyle and fentiment.

His father dying in 1558, he went to Paris the year following, with a defign to apply himfelf to the Greek tongue. For this purpofe he for two months attended the lectures of Turnebus; but finding that in the ufual courfe he fhould be a long time in gaining his point, he fhut himfelf up in his clofet, and by conftant application for two years gained a perfect knowledge of the Greek language. After which he applied himfelf to the Hebrew, which he learned by himfelf with great facility. And in like manner he ran through many other languages, till he could fpeak it is faid no lefs than 13 ancient and modern ones. He made no lefs progrefs in the fciences; and his writings procured him the reputation of one of the greateft men of that or any other age. He embraced the reformed religion at 22 years of age. In 1563, he attached himfelf to Lewis Cafteignier de la Roch Pazay, whom he attended in feveral journies. And, in 1593, the curators of the univerfity of Leyden invited him to an honorary profefforfhip in that univerfity, where he lived 16 years, and where he died of a dropfy in 1609, at 69 years of age.

Scaliger was a man of great temperance; was never married; and was fo clofe a ftudent, that he often fpent whole days in his ftudy without eating: and though his circumftances were always very narrow, he conftantly refufed the prefents that were offered him.

He was author of many ingenious works on various fubjects. His elaborate work, *De Emendatione Temporum;* his exquifite animadverfions on Eufebius; with his *Canon Ifagogicus Chronologiæ;* and his accurate comment upon Manilius's *Aftronomicon*, fufficiently evince his knowledge in the aftronomy, and other branches of learning, among the Ancients, and who, according to the opinion of the celebrated Vieta, was far fuperior to any of that age. And he had no lefs a character given him by the learned Cafaubon.—He wrote *Cyclometrica et Diatriba de Equinoctiorum Anticipatione.* He wrote alfo notes upon Seneca, Varro, and Aufonius's Poems. But that which above all things renders the name of Scaliger memorable to pofterity, is the invention of the Julian period, which confifts of 7980 years, being the continued product of the three cycles, of the fun 28, the moon 19, and Roman indiction 15. This period had its beginning fixed to the 764th year before the creation, and is not yet completed, and comprehends all other cycles, periods, and epochas, with the times of all memorable actions and hiftories.—The collections intitled *Scaligeriana*, were collected from his converfations by one of his friends; and being ranged in alphabetical order, were publifhed by Ifaac Voffius.

SCANTLING, a meafure, fize, or ftandard, by which the dimenfions &c of things are to be determined.

The

The term is particularly applied to the dimenſions of any piece of timber, with regard to its breadth and thickneſs.

SCAPEMENT, in Clock-work, a general term for the manner of communicating the impulſe of the wheels to the pendulum. The ordinary Scapements conſiſt of the ſwing-wheel and pallets only; but modern improvements have added other levers or detents, chiefly for the purpoſes of diminiſhing friction, or for detaching the pendulum from the preſſure of the wheels during part of the time of its vibration. Notwithſtanding the very great importance of the Scapement to the performance of clocks, no material improvement was made in it from the firſt application of the pendulum to clocks to the days of Mr. George Graham; nothing more was attempted before his time, than to apply the impulſe of the ſwing-wheel in ſuch manner as was attended with the leaſt friction, and would give the greateſt motion to the pendulum. Dr. Halley diſcovered, by ſome experiments made at the Royal Obſervatory at Greenwich, that by adding more weight to the pendulum, it was made to vibrate larger arcs, and the clock went faſter; by diminiſhing the weight of the pendulum, the vibrations became ſhorter, and the clock went ſlower; the reſult of theſe experiments being diametrically oppoſite to what ought to be expected from the theory of the pendulum, probably firſt rouſed the attention of Mr. Graham, and led him to ſuch farther trials as convinced him, that this ſeeming paradox was occaſioned by the retrograde motion, which was given to the ſwing-wheel by every conſtruction of Scapement that was at that time in uſe; and his great ſagacity ſoon produced a remedy for this defect, by conſtructing a Scapement which prevented all recoil of the wheels, and reſtored to the clock pendulum, wholly in theory, and nearly in practice, all its natural properties in its detached ſimple ſtate; this Scapement was named by its celebrated inventor the *dead beat*, and its great ſuperiority was ſo univerſally acknowledged, that it was ſoon introduced into general uſe, and ſtill continues in univerſal eſteem. The importance of the Scapement to the accurate going of clocks, was by this improvement rendered ſo unqueſtionable, that artiſts of the firſt rate all over Europe, were forward in producing each his particular conſtruction, as may be ſeen in the works of Thiout l'ainé, M. J. A. Lepante, M. le Roy, M. Ferdinand Bertoud, and Mr. Cummings' Elements of Clock and Watchwork, in which we have a minute deſcription of ſeveral new and ingenious conſtructions of Scapements, with an inveſtigation of the principles on which their claim to merit is founded; and a comparative view of the advantages or defects of the ſeveral conſtructions. Beſides the Scapements deſcribed in the above works, many curious conſtructions have been produced by eminent artiſts, who have not publiſhed any account of them, nor of the motives which have induced each to prefer his favourite conſtruction: Mr. Harriſon, Mr. Hindley of York, Mr. Ellicot, Mr. Mudge, Mr. Arnold, Mr. Whitehurſt, and many other ingenious artiſts of this country, have made Scapements of new and peculiar conſtructions, of which we are unable, for the above reaſon, to give any farther account than that thoſe of Mr. Harriſon and Mr. Hindley had ſcarce any friction, with a certain mode and quantity of recoil; thoſe of all the other gentlemen, we believe, have been on the principle of the dead beat, with ſuch other improvements as they ſeverally judged moſt conducive to a good performance.

Count Bruhl has juſt publiſhed (in 1794) a ſmall pamphlet, "On the Inveſtigation of Aſtronomical Circles," to which he has annexed, "a Deſcription of the Scapement in Mr. Mudge's firſt Timekeeper, drawn up in Auguſt 1771." Before entering upon the Deſcription, the Count premiſes a few obſervations, in one of which he recognizes a hint concerning the nature of Mr. Mudge's Scapement, thrown out by this artiſt in a ſmall tract printed by him in the year 1763, which is this: "The force derived from the main-ſpring ſhould be made as equal as poſſible, by making the mainſpring wind up another ſmaller ſpring at a leſs diſtance from the balance, at ſhort intervals of time. *I think it would not be impracticable to make it wind up at every vibration, a ſmall ſpring ſimilar to the pendulum ſpring, that ſhould immediately act on the balance, by which the whole force acting on the balance would be reduced to the greateſt ſimplicity, with this advantage, that the force would increaſe in proportion to the arch.*" From this hint, Count Bruhl is ſurpriſed that no other artiſts have taken up Mr. Mudge's invention. He then gives the Deſcription of that invention as follows: "Mr. Mudge's Timekeeper has five wheels, with numbers high enough to admit pinions of twelve, and yet to go eight days. The Scapement conſiſts of a wheel almoſt like that of a common crown wheel, and acts on pallets, each of which has a ſeparate axis lying in the ſame line. To each pallet a ſpring is fixed in the ſhape of a pendulum ſpring; theſe ſprings are wound up alternately by the action of the laſt wheel upon the pallets, which is performed in the following manner: —Whenever one of the pallets (for inſtance the upper one) is ſet in motion by a tooth of the wheel ſliding upon it, and then reſting againſt a hook, or, rather a bearing at its end, the balance is entirely detached from it, being then employed in carrying the other pallet the contrary way. When the balance returns from that vibration (partly by the force of the pendulum ſpring, and partly by that of one of the two ſmall ſprings which it had bent by the motion of that pallet which it had carried along with itſelf) it lays hold of the upper pallet and carries it round in the ſame manner as it did before the lower one, and, of courſe, in the ſame direction which the upper pallet had received from the power of the mainſpring at the time that it was quite unconnected with the balance. The communication of motion from the balance to the pallets, and vice verſa, is effected by two pins fixed to a crank, which in following the balance, hit each its proper pallet alternately. By what has been ſaid, it is evident that whatever inequality there may be in the power derived from the mainſpring (provided the latter be ſufficient to wind up thoſe little pallet ſprings) it can never interfere with the regularity of the balance's motion, but at the inſtant of unlocking the pallets, which is ſo inſtantaneous an operation, and the reſiſtance ſo exceedingly ſmall, that it cannot poſſibly amount to any ſenſible error. The removal of this great obſtacle was certainly never ſo effectually done by any other contrivance

vance, and deserves the highest commendation, as a probable means to perfect a portable machine that will measure time correctly. But this is not the only, nor indeed the principal advantage which this timekeeper will possess over any other; for, as it is impossible to reduce friction to so small a quantity as not to affect the motion of a balance, the consequence of which is, that it describes sometimes greater and sometimes smaller arcs, it became necessary to think of some method by which the balance might be brought to describe those different arcs in the same time. If a balance could be made to vibrate without friction or resistance from the medium in which it moves, the mere expanding and contracting of the pendulum spring, would probably produce the so much wished-for effect, as its force is supposed to be proportional to the arcs described; but as there is no machine void of friction, and as from that cause, the velocity of every balance decreases more rapidly than the spaces gone through decrease, this inequality could only be removed by a force acting on the balance, which assuming different ratios in its different stages, could counterbalance that inequality. This very material and important remedy, Mr. Mudge has effected by the construction of his Scapement; for his pallet springs having a force capable of being increased almost at pleasure, at the commencement of every vibration, the proportion in their different degrees of tension may be altered till it answers the intended purpose. This shews how effectually Mr. Mudge's Scapement removes the two greatest difficulties that have hitherto baffled the attempts of every other artist, namely, the inequalities of the power derived from the main spring, and the irregularities arising from friction, and the variable resistance of the medium in which the balance moves. Although at the time I am writing this account of his invention, the machine is not yet finished; I am not the less confident that whenever it is, it will be found to be one of the most useful of any which has as yet appeared."

SCARP, in Fortification, the interior slope of the ditch of a place; that is, the slope of that side of a ditch which is next to the place, or on the outside of the rampart at its foot, facing the champaign or open country. The slope on the outer side of the ditch is called the *counterscarp*.

SCENOGRAPHY, in Perspective, the perspective representation of a body on a plane; or a description and view of it in all its parts and dimensions, such as it appears to the eye in any oblique view.

This differs essentially from the ichnography and the orthography. The ichnography of a building, &c, represents the plan or ground work of the building, or section parallel to it; and the orthography the elevation, or front, or one side, also in its natural dimensions; but the Scenography exhibits the whole of the building that appears to the eye, front, sides, height, and all, not in their real dimensions or extent, but raised on the geometrical plan in perspective.

In architecture and fortification, Scenography is the manner of delineating the several parts of a building or fortress, as they are represented in perspective.

To exhibit the SCENOGRAPHY *of any body*. 1. Lay down the basis, ground-plot, or plan, of the body, in the perspective ichnography, that is, draw the perspective appearance of the plan or basement, by the proper rules of perspective. 2. Upon the several points of the said perspective plan, raise the perspective heights, and connect the tops of them by the proper slope or oblique lines. So will the Scenography of the body be completed, when a proper shade is added. See PERSPECTIVE.

SCHEINER (CHRISTOPHER), a considerable German mathematician and astronomer, was born at Mundeilheim in Schwaben in 1575. He entered into the society of the Jesuits at 20 years of age; and afterwards taught the Hebrew tongue and the mathematics at Ingolstadt, Friburg, Brisac, and Rome. At length he became confessor to the archduke Charles, and rector of the college of the Jesuits at Neisse in Silesia, where he died in 1650, at 75 years of age.

Scheiner was chiefly remarkable for being one of the first who observed the spots in the sun with the telescope, though not the very first; for his observations of those spots were first made, at Ingolstadt, in the latter part of the year 1611, whereas Galileo and Harriot both observed them in the latter part of the year before, or 1610. Scheiner continued his observations on the solar phenomena for many years afterwards at Rome, with great assiduity and accuracy, constantly making drawings of them on paper, describing their places, figures, magnitude, revolutions and periods, so that Riccioli delivered it as his opinion that there was little reason to hope for any better observations of those spots. Des Cartes and Hevelius also say, that in their judgment, nothing can be expected of that kind more satisfactory. These observations were published in one volume folio, 1630, under the title of *Rosa Ursina*, &c; almost every page of which is adorned with an image of the sun with the spots. He wrote also several smaller pieces relating to mathematics and philosophy, the principal of which are,

2. *Oculus, sive Fundamentum Opticum*, &c; which was reprinted at London, in 1652, in 4to.

3. *Sol Ecllpticus, Disquisitiones Mathematicæ*.

4. *De Controversiis et Novitatibus Astronomicis*.

SCHEME, a draught or representation of any geometrical or astronomical figure, or problem, by lines sensible to the eye; or of the celestial bodies in their proper places for any moment; otherwise called a diagram.

SCHEME *Arches*. See ARCH.

SCHOLIUM, a note, remark, or annotation, occasionally made on some passage, proposition, or the like.

The term is much used in geometry, and other parts of the mathematics; where, after demonstrating a proposition, it is used to point out how it might be done some other way; or to give some advice or precaution, in order to prevent mistakes; or to add some particular use or application of it.

Wolfius has given abundance of curious and useful arts and methods, and a good part of the modern philosophy, with the description of mathematical instruments, &c; all by way of Scholia to the respective propositions in his Elementa Matheseos.

SCHONER (JOHN), a noted German philosopher and mathematician, was born at Carolostadt in the year 1477, and died in 1547, at 70 years of age.—His early propensity to those sciences may be deemed a just prognostication of the great progress which

which he afterwards made in them. So that from his uncommon acquirements, he was chofen mathematical profeffor at Nuremburg when he was but a young man. He wrote a great many works, and was particularly famous for his aftronomical tables, which he publifhed after the manner of thofe of Regiomontanus, and to which he gave the title of *Refolutæ*, on account of their clearnefs. But notwithftanding his great knowledge, he was, after the fafhion of the times, much addicted to judicial aftrology, which he took great pains to improve. The lift of his writings is chiefly as follows:

1. Three Books of Judicial Aftrology.
2. The Aftronomical Tables named *Refolutæ*.
3. *De Ufu Globi Stelliferi; De Compofitione Globi Celeftis; De Ufu Globi Terreftris, et de Compofitione ejufdem.*
4. *Æquatorium Aftronomicum.*
5. *Libellus de Diftantiis Locorum per Inftrumentum et Numeros Inveftigandis.*
6. *De Compofitione Torqueti.*
7. *In Conftructionem et Ufum Rectanguli five Radii Aftronomici Annotationes.*
8. *Horarii Cylindri Canones.*
9. *Planifphærium, feu Meteorifcopium.*
10. *Organum Uranicum.*
11. *Inftrumentum Impedimentorum Lunæ.*

All printed at Nuremburg, in folio, 1551.

Of thefe, the large treatife of dialling rendered him more known in the learned world than all his other works befides; in which he difcovers a furprifing genius and fund of learning of that kind.

SCHOOL, a place where the languages, humanities, or arts and fciences, &c, are taught.

School is alfo ufed for a whole faculty, univerfity, or fect; as Plato's fchool, the fchool of Epicurus, the fchool of Paris, &c.—The fchool of Tiberias was famous among the ancient Jews; and it is to this we owe the Maffora, and Mafforetes.

School *Philofophy*, &c. the fame with *fcholaftic.*

SCIAGRAPHY, or Sciography, the profile or vertical fection of a building; ufed to fhew the infide of it.

Sciagraphy, in Aftronomy &c, is a term ufed by fome authors for the art of finding the hour of the day or night, by the fhadow of the fun, moon, ftars, &c. See Dial.

SCIENCE, a clear and certain knowledge of any thing, founded on demonftration, or on felf evident principles.—In this fenfe, *doubting* is oppofed to fcience; and *opinion* is the middle between the two.

Science is more particularly ufed for a formed fyftem of any branch of knowledge, comprehending the doctrine, reafon, and theory of the thing, without any immediate application of it to any ufes or offices of life. And in this fenfe, the word is ufed in oppofition to *art*.

Science may be divided into thefe three forts: Firft, the knowledge of things, their conftitutions, properties, and operations, whether material or immaterial. And this, in a little more enlarged fenfe of the word, may be called phyfics, or natural philofophy. Secondly, the fkill of rightly applying our own powers and actions for the attainment of good and ufeful things, as *Ethics*. Thirdly, the doctrine of figns; as words, logic, &c.

SCIENTIFIC, or Scientifical, fomething relating to the pure and fublimer fciences; or that abounds in fcience, or knowledge.

A work, or method, &c, is faid to be fcientifical, when it is founded on the pure reafon of things, or conducted wholly on the principles of them. In which fenfe the word ftands oppofed to narrative, arbitrary, opinionative, pofitive, tentative, &c.

SCIOPTIC, or Scioptric *Ball*, a fphere or globe of wood, with a circular hole or perforation, where a lens is placed. It is fo fitted that, like the eye of an animal, it may be turned round every way, to be ufed in making experiments of the darkened room.

SCIOPTRICS. See Camera Obscura.

SCIOTHERICUM *Telefcopium*, is an horizontal dial, adapted with a telefcope for obferving the true time both by day and night, to regulate and adjuft pendulum clocks, watches, and other time-keepers. It was invented by Mr. Molyneux, who publifhed a book with this title, which contains an accurate defcription of this inftrument, with all its ufes and applications.

SCLEROTICA, one of the common membranes of the eye, on its hinder part. It is a large, thick, firm, hard, opaque membrane, extended from the external circumference of the cornea to the optic nerve, and forms much the greater part of the external globe of the eye. The Sclerotica and the cornea compofe the cafe in which all the internal coats of the eye and its humours are contained.

SCONCES, fmall forts, built for the defence of fome pafs, river, or other place. Some Sconces are made regular, of four, five, or fix baftions; others are of fmaller dimenfions, fit for paffes, or rivers; and others for the field.

SCORE, in Mufic, denotes partition, or the original draught of the whole compofition, in which the feveral parts, viz the treble, fecond treble, bafs, &c. are diftinctly fcored, and marked.

SCORPIO, the *Scorpion*, the 8th fign of the zodiac, denoted by the character ♏, being a rude defign of the animal of that name.

The Greeks, who would be fuppofed the inventors of aftronomy, and who have, with that intent, fathered fome ftory or other of their own upon every one of the conftellations, give a very fingular account of the origin of this one. They tell us that this is the creature which killed Orion. The ftory goes, that the famous hunter of that name boafted to Diana and Latona, that he would deftroy every animal that was upon the earth; the earth, they fay, enraged at this, fent forth the poifonous reptile the Scorpion, which infignificant creature ftung him, that he died. Jupiter, they fay, raifed the Scorpion to the heavens, giving him this place among the conftellations; and that afterwards Diana requefted of him to do the fame honour to Orion, which he at laft confented to, but placed him in fuch a fituation, that when the Scorpion rifes, he fets.

But the Egyptians, or whatever early nation it was that framed the zodiac, probably placed this poifonous reptile in that part of the heavens to denote that when the fun arrived at it, fevers and fickneffes, the maladies of autumn, would begin to rage. This they reprefented by an animal whofe fting was of power to occafion fome

of

of them; and it was thus they formed all the conftellations.

The ancients allotted one of the twelve principal among their deities to be the guardian for each of the 12 figns of the zodiac. The Scorpion, as their hiftory of it made it a fierce and fatal animal that had killed the great Orion, fell naturally to the protection of the god of war; Mars is therefore its tutelary deity; and to this fingle circumftance is owing all that jargon of the aftrologers, who tell us that there is a great analogy between the planet Mars and the conftellation Scorpio. To this alfo is owing the doctrine of the alchymifts, that iron, which they call Mars, is alfo under the dominion of the fame conftellation, and that the tranfmutation of that metal into gold can only be performed when the fun is in this fign.

The ftars in Scorpio, in Ptolomy's catalogue, are 24; in that of Tycho 10, in that of Hevelius 20, but in that of Flamfteed and Sharp 44.

Scorpion is alfo the name of an ancient military engine, ufed chiefly in the defence of walls, &c.

Marcellinus defcribes the Scorpion, as confifting of two beams bound together by ropes. From the middle of the two, rofe a third beam, fo difpofed, as to be pulled up and let down at pleafure; and on the top of this were faftened iron hooks, where a fling was hung, either of iron or hemp; and under the third beam lay a piece of hair-cloth full of chaff, tied with cords. It had its name Scorpio, becaufe when the long beam or tiller was erected, it had a fharp top in manner of a fting.

To ufe the engine, a round ftone was put into the fling, and four perfons on each fide, loofening the beams bound by the ropes, drew back the erect beam to the hook; then the engineer, ftanding on an eminence, gave a ftroke with a hammer on the chord to which the beam was faftened with its hook, which fet it at liberty; fo that hitting againft the foft hair-cloth, it ftruck out the ftone with a great force.

SCOTIA, in Architecture, a femicircular cavity or channel between the tores, in the bafes of columns; and fometimes under the larmier or drip, in the cornice of the Doric order. The workmen often call it the Cafement, and it is alfo otherwife called the Trochilus.

SCREW, or Scrue, one of the fix mechanical powers; chiefly ufed in preffing or fqueezing bodies clofe, though fometimes alfo in raifing weights.

The Screw is a fpiral thread or groove cut round a cylinder, and everywhere making the fame angle with the length of it. So that, if the furface of the cylinder, with this fpiral thread upon it, were unfolded and ftretched into a plane, the fpiral thread would form a ftraight inclined plane, whofe length would be to its height, as the circumference of the cylinder is to the diftance between two threads of the Screw; as is evident by confidering, that in making one round, the fpiral rifes along the cylinder the diftance between the two threads.

Hence the threads of a Screw may be traced upon the fmooth furface of a cylinder thus: Cut a fheet of paper into the form of a right-angled triangle, having its bafe to its height in the above proportion, viz, as the circumference of the cylinder of the Screw is to the intended diftance between two threads; then wrap this paper triangle about the cylinder, and the hypothenufe of it will trace out the line of the fpiral thread.

When the fpiral thread is upon the outfide of a cylinder, the Screw is faid to be a *male* one. But if the thread be cut along the inner furface of a hollow cylinder, or a round perforation, it is faid to be *female*. And this latter is alfo fometimes called the *box* or *nut*.

When motion is to be given to fomething, the male and female Screw are neceffarily conjoined; that is, whenever the fcrew is to be ufed as a fimple engine, or mechanical power. But when joined with an axis in peritrochio, there is no occafion for a female; but in that cafe it becomes part of a compound engine.

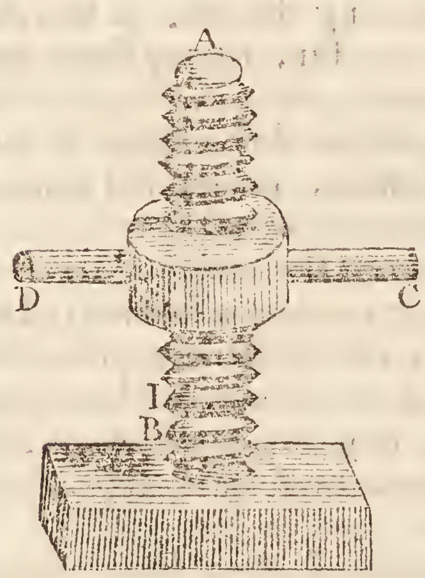
The Screw cannot properly be called a fimple machine, becaufe it is never ufed without the application of a lever, or winch, to affift in turning it.

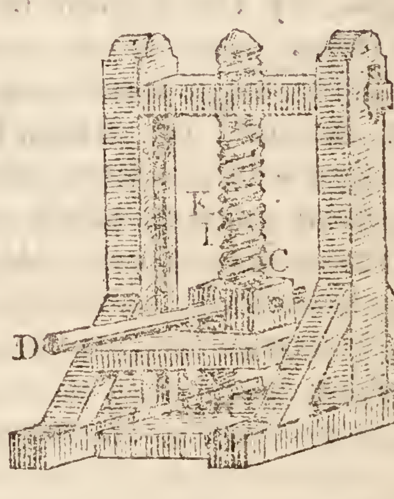
Of the Force and Power of the Screw.

1. The force of a power applied to turn a Screw round, is to the force with which it preffes upwards or downwards, fetting afide the friction, as the diftance between two threads is to the circumference where the power is applied.

For, the Screw being only an inclined plane, or half wedge, whofe height is the diftance between two threads, and its bafe the faid circumference; and the force in the horizontal direction being to that in the vertical one as the lines perpendicular to them, viz, as the height of the plane, or diftance of the two threads, is to the bafe of the plane, or circumference at the place where the power is applied; therefore the power is to the preffure, as the diftance of two threads, is to that circumference.

2. Hence, when the Screw is put in motion; then the power is to the weight which would keep it in equilibrio, as the velocity of the latter is to that of the former. And hence their two momenta are equal, which are produced by multiplying each weight or power by its own velocity. Two different forms of Screw preffes, are as below.





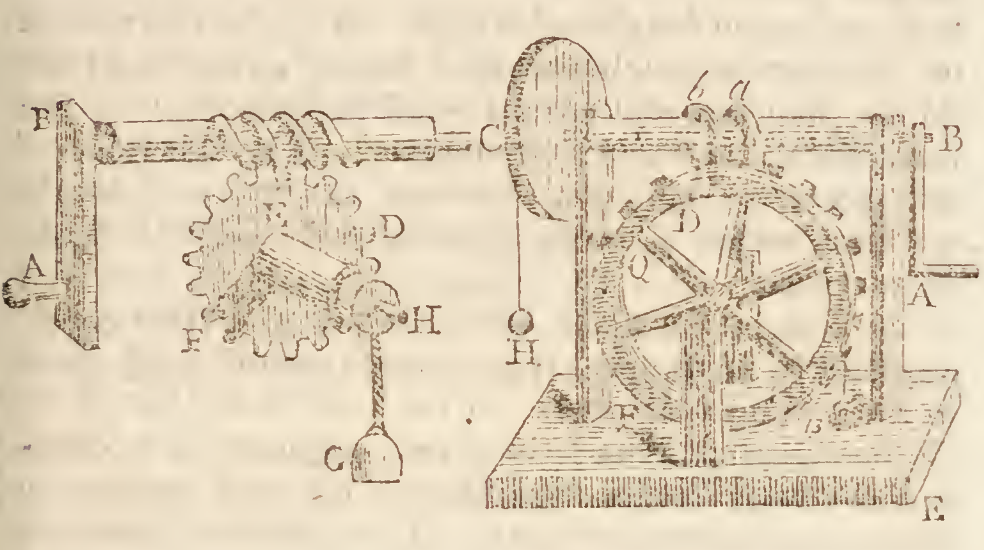
3. Hence we can eafily compute the force of any machine turned by a Screw. Let the annexed figure reprefent a prefs driven by a Screw, whofe threads are each a quarter of an inch afunder; and let the Screw be turned by a handle of 4 feet long from C to D; then if the natural force of a man, by which he can lift, or pull,

pull, or draw, be 150 pounds; and it be required to determine with what force the Screw will prefs on the board, when the man turns the handle at C and D with his whole force. The diameter CD of the power being 4 feet, or 48 inches, its circumference is $48 \times 3{\cdot}1416$ or $150\frac{4}{5}$ nearly; and the diftance of the threads being $\frac{1}{4}$ of an inch; therefore the power is to the preffure, as $\frac{1}{4}$ to $150\frac{4}{5}$ or as 1 to $603\frac{1}{5}$: but the power is equal to 150lb; therefore as $1 : 603\frac{1}{5} :: 150 : 90{,}480$; and confequently the preffure at the bottom of the Screw, is equal to a weight of 90,480 pounds, independent of friction.

But the power has to overcome, not only the weight, or other refiftance, but alfo the friction of the Screw, which in this machine is very great, in fome cafes equal to the weight itfelf, fince it is fometimes fufficient to fuftain the weight, when the power is taken off.

Mr. Hunter has defcribed a new method of applying the Screw with advantage in particular cafes, in the Philof. Tranf. vol. 71, pa. 58 &c.

The Endlefs SCREW, or *Perpetual* SCREW, is one which works in, and turns, a dented wheel DF, without a concave or female Screw; being fo called becaufe it may be turned for ever, without coming to an end. From the following fchemes it is evident, that while the Screw turns once round, the wheel only advances the diftance of one tooth.



1. If the power applied to the lever, or handle, of an endlefs Screw AB, be to the weight, in a ratio compounded of the periphery of the axis of the wheel EH, to the periphery defcribed by the power in turning the handle, and of the revolutions of the wheel DF to the revolutions of the Screw CB, the power will balance the weight. Hence,

2. As the motion of the wheel is very flow, a fmall power may raife a very great weight, by means of an endlefs Screw. And therefore the chief ufe of fuch a Screw is, either where a great weight is to be raifed through a little fpace; or where only a flow gentle motion is wanted. For which reafon it is very ufeful in clocks and watches.

3. Having given the number of teeth, the diftance of the power from the centre of the Screw B, the radius of the axis HE, and the power; to find the weight it will raife. Multiply the diftance of the power from the centre of the Screw AB, by the number of the teeth, and the product will be the fpace paffed through by the power, while the weight paffes through a fpace equal to the periphery of the axis: then fay, as the radius of the axis is to the fpace of the power juft found, fo is the power to a 4th proportional, which will be the weight the power is able to fuftain. Thus, if $AB = 3$, the radius of the axis $HE = 1$, the power 150 pounds, and the number of teeth of the wheel DF 48; then the weight will be found $= 21{,}600 = 3 \times 150 \times 48$. Whence it appears that the endlefs Screw exceeds all others in increafing the force of a power.

4. A machine for fhewing the power of the Screw may be contrived in the following manner. Let the wheel C (laft fig.) have a Screw *a b* on its axis, working in the teeth of the wheel D, which fuppofe to be 48 in number. It is plain that for every revolution of the wheel C, and Screw *ab*, by the winch A, the wheel D will be moved one tooth by the Screw; and therefore in 48 revolutions of the winch, the wheel D will be turned once round. Then if the circumference of a circle, defcribed by the handle of the winch, be equal to the circumference of a groove *e* round the wheel D, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Confequently when a line G goes round the groove *e*, and has a weight of 48lb hung to it below the pedeftal EF, a power equal to one pound at the handle will balance and fupport the weight.

Archimedes's SCREW, is a fpiral pump, being a machine for raifing water, firft invented by Archimedes.

Its ftructure and ufe will be underftood by the following defcription of it.

ABCD (Pl. xxiii, fig. 6) is a wheel, which is turned round, according to the order of thofe letters, by the fall of water EF, which need not be more than 3 feet. The axis G of the wheel is raifed fo as to make an angle of about 44° with the horizon; and on the top of that axle is a wheel H, which turns fuch another wheel I of the fame number of teeth; the axle K of this laft wheel being parallel to the axle G of the two former wheels. The axle G is cut into a double threaded Screw, as in the annexed figure (fig. 7), exactly refembling the Screw on the axis of the fly of a common jack, which muft be what is called a right-handed Screw, if the firft wheel turns in the direction ABCD; but a left-handed Screw, if the ftream turns the wheel the contrary way; and the Screw on the axle G muft be cut in a contrary way to that on the axle K, becaufe thefe axles turn in contrary directions. Thefe Screws muft be covered clofe over with boards, like thefe of a cylindrical cafk; and then they will be fpiral tubes. Or they may be made of tubes or pipes of lead, and wrapt round the axles in fhallow grooves cut in it, like the figure 8. The lower end of the axle G turns conftantly in the ftream that turns the wheel, and the lower ends of the fpiral tubes are open into the water. So that, as the wheel and axle are turned round, the water rifes in the fpiral tubes, and runs out at L through the holes M, N, as they come about below the axle. Thefe holes, of which there may be any number, as 4 or 6, are in a broad clofe ring on the top of the axle, into which ring the water is delivered from the upper open ends of the Screw tubes, and falls into the open box N. The lower end of the axle K turns on a gudgeon in the water in N; and the fpiral tubes in that axle take up the water from N, and deliver it into another fuch box under the top of K; on which there may be fuch another wheel

wheel as I, to turn a third axle by ſuch a wheel upon it. And in this manner may water be raiſed to any propoſed height, when there is a ſtream ſufficient for that purpoſe to act on the broad float boards of the firſt wheel. Archimedes's Screw, or a ſtill ſimpler form of it, is alſo repreſented in fig. 9.

SCROLLS, or Scrowls, or *Volutes*, a term in Architecture. See Volutes.

SCRUE. See Screw.

SCRUPLE, the leaſt of the weights uſed by the ancients. Among the Romans it was the 24th part of an ounce, or the third part of a drachm.

Scruple is ſtill a ſmall weight among us, equal to 20 grains, or the 3d part of a drachm. Among goldſmiths the ſcruple is 24 grains.

Scruple, in Chronology, a ſmall portion of time much uſed by the Chaldeans, Jews, Arabs, and other eaſtern people, in computations of time. It is the 1080th part of an hour, and by the Hebrews called *helakin*.

Scruples, in Aſtronomy. As

Scruples *Eclipſed*, denote that part of the moon's diameter which enters the ſhadow, expreſſed in the ſame meaſure in which the apparent diameter of the moon is expreſſed. See Digit.

Scruples *of Half Duration*, an arch of the moon's orbit, which the moon's centre deſcribes from the beginning of an eclipſe to its middle.

Scruples *of Immerſion*, or *Incidence*, an arch of the moon's orbit, which her centre deſcribes from the beginning of the eclipſe, to the time when the centre falls into the ſhadow. See Immersion.

Scruples *of Emerſion*, an arch of the moon's orbit, which her centre deſcribes in the time from the firſt emerſion of the moon's limb, to the end of the eclipſe.

SCYTALA, in Mechanics, a term which ſome writers uſe for a kind of radius, or ſpoke, ſtanding out from the axis of a machine, as a lever or handle, to turn it round, and work it by.

SEA, in Geography, is frequently uſed for that vaſt tract of water encompaſſing the whole earth, more properly called ocean. But

Sea is more properly uſed for a particular part or diviſion of the ocean, denominated from the countries it waſhes, or from other circumſtances. Thus we ſay, the Iriſh ſea, the Mediterranean ſea, the Baltic ſea, the Red ſea, &c.

Sea among ſailors is variouſly applied, to a ſingle wave, or to the agitation produced by a multitude of waves in a tempeſt, or to their particular progreſs and direction. Thus they ſay, a heavy ſea broke over our quarter, or we ſhipped a heavy ſea; there is a great ſea in the offing; the ſea ſets to the ſouthward. Hence a ſhip is ſaid to head the ſea, when her courſe is oppoſed to the ſetting or direction of the ſurges. A *Long Sea* implies a ſteady and uniform motion of long and extenſive waves. On the contrary, a *Short Sea* is when they run irregularly, broken, and interrupted, ſo as frequently to burſt over a veſſel's ſide or quarter.

Properties and Affections of the Sea.

1. *General Motion of the Sea.* M. Daſſie of Paris, in a work long ſince publiſhed, has been at great pains to prove that the Sea has a general motion, independent of winds and tides, and of more conſequence in navigation than is uſually ſuppoſed. He affirms that this motion is from eaſt to weſt, inclining toward the north when the ſun is on the north ſide of the equinoctial, but toward the ſouth when he is on the ſouth ſide of it. Philoſ. Tranſ. No. 135.

2. *Baſon or Bottom of the* Sea, or *Fundus Maris*, a term uſed to expreſs the bed or bottom of the ſea in general. Mr. Boyle has publiſhed a treatiſe on this ſubject, in which he has given an account of its irregularities and various depths ſounded on the obſervations communicated to him by mariners.

Count Marſigli has, ſince his time, given a much fuller account of this part of the globe. The materials which compoſe the bottom of the Sea, may reaſonably be ſuppoſed, in ſome degree, to influence the taſte of its waters; and this author has made many experiments to prove that foſſil coal, and other bituminous ſubſtances, which are found in plenty at the bottom of the Sea, may communicate in great part its bitterneſs to it.

It is a general rule among ſailors, and is found to hold true in many inſtances, that the more the ſhores of any place are ſteep and high, forming perpendicular cliffs, the deeper the Sea is below; and that on the contrary, level ſhores denote ſhallow Seas. Thus the deepeſt part of the Mediterranean is generally allowed to be under the height of Malta. And the obſervation of the ſtrata of earth and other foſſils, on and near the ſhores, may ſerve to form a good judgment as to the materials to be found in its bottom. For the veins of ſalt and of bitumen doubtleſs run on the ſame, and in the ſame order, as we ſee them at land; and the ſtrata of rocks that ſerve to ſupport the earth of hills and elevated places on ſhore, ſerve alſo, in the ſame continued chain, to ſupport the immenſe quantity of water in the baſon of the Sea.

The coral fiſheries have given occaſion to obſerve that there are many, and thoſe very large caverns or hollows in the bottom of the Sea, eſpecially where it is rocky; and that the like caverns are ſometimes found in the perpendicular rocks which form the ſteep ſides of thoſe fiſheries. Theſe caverns are often of great depth, as well as extent, and have ſometimes wide mouths, and ſometimes only narrow entrances into large and ſpacious hollows.

The bottom of the Sea is covered with a variety of matters, ſuch as could not be imagined by any but thoſe who have examined into it, eſpecially in deep water, where the ſurface only is diſturbed by tides and ſtorms, the lower part, and conſequently its bed at the bottom, remaining for ages perhaps undiſturbed. The ſoundings, when the plummet firſt touches the ground on approaching the ſhores, give ſome idea of this. The bottom of the plummet is hollowed, and in that hollow there is placed a lump of tallow; which being the part that firſt touches the ground, the ſoft nature of the fat receives into it ſome part of thoſe ſubſtances which it meets with at the bottom: this matter, thus brought up, is ſometimes pure ſand, ſometimes a kind of ſand made of the fragment of ſhells, beaten to a ſort of powder, ſometimes it is made of a like powder of the ſeveral ſorts of corals, and ſometimes it is compoſed of

of fragments of rocks; but beside these appearances, which are natural enough, and are what might well be expected, it brings up substances which are of the most beautiful colours. Marsigli Hist. Phys. de la Mer.

Dr. Donati, in an Italian work, containing an essay towards a natural history of the Adriatic Sea, printed at Venice in 1750, has related many curious observations on this subject, and which confirm the observations of Marsigli: having carefully examined the soil and productions of the various countries that surround the Adriatic Sea, and compared them with those which he took up from the bottom of the Sea, he found that there is very little difference between the former and the latter. At the bottom of the water there are mountains, plains, vallies, and caverns, similar to those upon land. The soil consists of different strata placed one upon another, and mostly parallel and correspondent to those of the rocks, islands, and neighbouring continents. They contain stones of different sorts, minerals, metals, various putrefied bodies, pumice stones, and lavas formed by volcanos.

One of the objects which most excited his attention, was a crust, which he discovered under the water, composed of crustaceous and testaceous bodies, and beds of polypes of different kinds, confusedly blended with earth, sand, and gravel; the different marine bodies which form this crust, are found at the depth of a foot or more, entirely petrified and reduced into marble; these he supposes are naturally placed under the Sea when it covers them, and not by means of volcanos and earthquakes, as some have conjectured. On this account he imagines that the bottom of the Sea is constantly rising higher and higher, with which other obvious causes of increase concur; and from this rising of the bottom of the Sea, that of its level or surface naturally results; in proof of which this writer recites a great number of facts. Philos. Trans. vol. 49, pa. 585.

3. *Luminousness of the* Sea. This is a phenomenon that has been noticed by many nautical and philosophical writers. Mr. Boyle ascribes it to some cosmical law or custom of the terrestrial globe, or at least of the planetary vortex.

Father Bourzes, in his voyage to the Indies, in 1704, took particular notice of this phenomenon, and very minutely describes it, without assigning the true cause.

The Abbé Nollet was long of opinion, that the light of the Sea proceeded from electricity; and others have had recourse to the same principle, and shewn that the luminous points in the surface of the Sea are produced merely by friction.

There are however two other hypotheses, which have more generally divided between them the solution of this phenomenon; the one of these ascribes it to the shining of luminous insects or animalcules, and the other to the light proceeding from the putrefaction of animal substances. The Abbé Nollet, who at first considered this luminousness as an electrical phenomenon, having had an opportunity of observing the circumstances of it, when he was at Venice in 1749, relinquished his former opinion, and concluded that it was occasioned either by the luminous aspect, or by some liquor or effluvia of an insect which he particularly describes, though he does not altogether exclude other causes, and especially the spawn or fry of fish.

The same hypothesis had also occurred to M. Vianelli; and both he and Grizellini, a physician in Venice, have given drawings of the insects from which they imagined this light to proceed.

A similar conjecture is proposed by a correspondent of Dr. Franklin, in a letter read at the Royal Society in 1756; the writer of which apprehends, that this appearance may be caused by a great number of little animals, floating on the surface of the Sea. And Mr. Forster, in his account of a voyage round the world with captain Cook, in the years 1772, 3, 4, and 5, describes this phenomenon as a kind of blaze of the Sea; and, having attentively examined some of the shining water, expresses his conviction that the appearance was occasioned by innumerable minute animals of a round shape, moving through the water in all directions, which show separately as so many luminous sparks when taken up on the hand: he imagines that these small gelatinous luminous specks may be the young fry of certain species of some medusæ, or blubber. And M. Dagelat and M. Rigaud observed several times, and in different parts of the ocean, such luminous appearances by vast masses of different animalcules; and a few days after the Sea was covered, near the coasts, with whole banks of small fish in innumerable multitudes, which they supposed had proceeded from the shining animalcules.

But M. le Roi, after giving much attention to this phenomenon, concludes that it is not occasioned by any shining insects, especially as, after carefully examining with a microscope some of the luminous points, he found them to have no appearance of an animal; and he also found that the mixture of a little spirit of wine with water just drawn from the Sea, would give the appearance of a great number of little sparks, which would continue visible longer than those in the ocean: the same effect was produced by all the acids, and various other liquors. M. le Roi is far from asserting that there are no luminous insects in the Sea; for he allows that several gentlemen have found them; but he is satisfied that the Sea is luminous chiefly on some other account, though he does not so much as offer a conjecture with respect to the true cause.

Other authors, equally dissatisfied with the hypothesis of luminous insects, for explaining the phenomenon which is the subject of this article, have ascribed it to some substance of the phosphoric kind, arising from putrefaction. The observations of F. Bourzes, above referred to, render it very probable, that the luminousness of the Sea arises from slimy and other putrescent matter, with which it abounds, though he does not mention the tendency to putrefaction, as a circumstance of any consequence to the appearance. But the experiments of Mr. Canton, which have the advantage of being easily made, seem to leave no room to doubt that the luminousness of the Sea is chiefly owing to putrefaction. And his experiments confirm an observation of Sir John Pringle's, that the quantity of salt contained in Sea-water hastens putrefaction; but since that precise quantity of salt which promotes putrefaction

faction the most, is less than that which is found in Sea-water, it is probable, Mr. Canton observes, that if the Sea were less salt, it would be more luminous. See Philos. Trans. vol. 59, pa. 446, and Franklin's Exper. and Observ. pa. 274.

Of the Depth of the Sea, its Surface, &c.

What proportion the superficies of the Sea bears to that of the land, is not accurately known, though it is said to be somewhat more than two to one. This proportion of the surface of the Sea to the land, has been found by experiment thus: taking the printed paper map or covering of a terrestrial globe, with a pair of scissors clip out the parts that are land, and those that are water; then weighing these parcels separately in a pair of fine scales, the land is found to be near $\frac{1}{3}$, and the water rather more than $\frac{2}{3}$ of the whole.

With regard to the profundity or depth of the Sea, Varenius affirms, that it is in some places unfathomable, and in others very various, being in certain places from $\frac{1}{20}$th of a mile to $4\frac{1}{2}$ miles in depth, in other places deeper, but much less in bays than in oceans. In general, the depths of the Sea bear a great analogy to the height of mountains on the land, so far as is hitherto discovered.

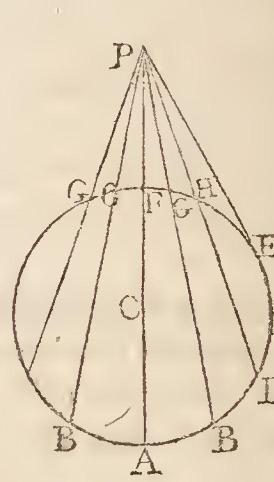
There are two special reasons why the Sea does not increase by means of rivers, &c, running every where into it. The first is, because waters return from the Sea by subterranean cavities and aqueducts, through various parts of the earth. Secondly, because the quantity of vapours raised from the Sea, and falling in rain upon the land, only cause a circulation of the water, but no increase of it. It has been found by experiment and calculation, that in a summer's day, there may be raised in vapours from the surface of the Mediterranean Sea, 528 millions of tuns of water; and yet this Sea receiveth not, from all its nine great rivers, above 183 millions of tuns per day, which is but about a third part of what is exhausted in vapours; and this defect in the supply by the rivers, may serve to account for the continual influx of a current by the mouth or straits at Gibraltar. Indeed it is rather probable, that the waters of the Sea suffer a continual slow decrease as to their quantity, by sinking always deeper into the earth, by filtering through the fissures in the strata and component parts.

SEASONS, certain portions or quarters of the year, distinguished by the signs which the sun then enters. Upon them depend the different temperatures of the air, different works in tillage, &c.

The year is divided into four Seasons, spring, summer, autumn, winter, which take their beginnings when the sun enters the first point of the signs Aries, Cancer, Libra, Capricorn.

The Seasons are very well illustrated by fig. 1, plate viii; where the candle at I represents the sun in the centre, about which the earth moves in the ecliptic ABCD, which cuts the equinoctial *abcd* in the two equinoxes E and G. When the earth is in these two points, it is evident that the sun equally illuminates both the poles, and makes the days and nights equal all over the earth. But while the earth moves from G by C to ♑, the upper or north pole becomes more and more enlightened, the days become longer, and the nights shorter; so that when the earth is at ♑, or the sun at ♋, our days are at the longest, as at midsummer. While the earth moves from ♑ by D to E, our days continually decrease, by the north pole gradually declining from the sun, till at E or autumn they become equal to the nights, or 12 hours long. Again, while the earth moves from E by A to F, the north pole becomes always more and more involved in darkness, and the days grow always shorter, till at F or ♋, when it is midwinter to the inhabitants of the northern hemisphere. Lastly, while the earth moves from ♋ by B to G, the north parts come more and more out of darkness, and the days grow continually longer, till at G the two poles are equally enlightened, and the days equal to the nights again. And so on continually year after year.

SECANT, in Geometry, a line that cuts another, whether right or curved; Thus the line PA or PB, &c, is a Secant of the circle ABD, because cutting it in the point F, or G, &c. Properties of such Secants to the circle are as follow:



1. Of several Secants PA, PB, PD, &c, drawn from the same point P, that which passes through the centre C is the greatest; and from thence they decrease more and more as they recede farther from the centre; viz. PB less than PA, and PD less than PB, and so on, till they arrive at the tangent at E, which is the limit of all the Secants.

2. Of these Secants, the external parts PF, PG, PH, &c, are in the reverse order, increasing continually from F to E, the greater Secant having the less external part, and in such sort, that any Secant and its external part are in reciprocal proportion, or the whole is reciprocally as its external part, and consequently that the rectangle of every Secant and its external part is equal to a constant quantity, viz, the square of the tangent. That is,

$$PA : \frac{1}{PF} :: PB : \frac{1}{PG} :: PD : \frac{1}{PH} \text{ \&c,}$$

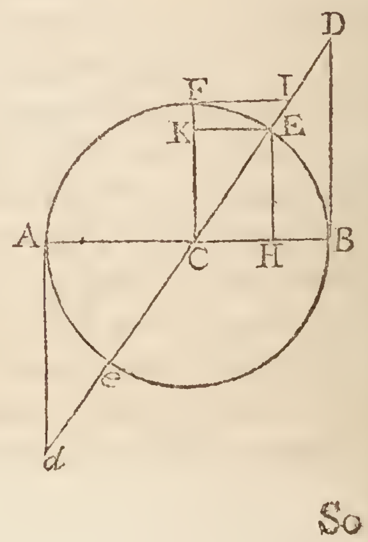
$$\text{or } PA \times PF = PB \times PG = PD \times PH = PE^2.$$

3. The tangent PE is a mean proportional between any Secant and its external part; as between PA and PF, or PB and PG, or PD and PH, &c.

4. The angle DPB, formed by two Secants, is measured by half the difference of its intercepted arcs DB and GH.

SECANT, in Trigonometry, denotes a right line drawn from the centre of a circle, and, cutting the circumference, proceeds till it meets with a tangent to the same circle.

Thus, the line CD, drawn from the centre C, till it meets the tangent BD, is called a Secant; and particularly the Secant of the arc BE, to which BD is a tangent. In like manner, by producing DC to meet the tangent A*d* in *d*, then C*d*, equal to CD, is the Secant of the arch AE which is the supplement of the arch BE.



So

So that an arch and its ſupplement have their Secants equal, only the latter one is negative to the former, being drawn the contrary way. And thus the Secants in the 2d and 3d quadrant are negative, while thoſe in the 1ſt and 4th quadrants are poſitive.

The Secant CI of the arc EF, which is the complement of the former arch BE, is called the *coſecant* of BE, or the Secant of its complement. The coſecants in the 1ſt and 2d quadrants are affirmative, but in the 3d and 4th negative.

The Secant of an arc is reciprocally as the coſine, and the coſecant reciprocally as the ſine; or the rectangle of the Secant and coſine, and the rectangle of the coſecant and ſine, are each equal to the ſquare of the radius.

For CD : CE : : CB : CH, or $ſ : r : : r : c$,
and CI : CE : : CF : CK, or $\sigma : r : : r : s$;

and conſequently $r^2 = cſ = s\sigma$; where r denotes the radius, s the ſine, c the coſine, $ſ$ the Secant, and σ the coſecant.

An arc a, to the radius r, being given, the Secant $ſ$, and coſecant σ, and their logarithms, or the logarithmic Secant and coſecant, may be expreſſed in infinite ſeries, as follows, viz,

$$ſ = r + \frac{a^2}{2r} + \frac{5a^4}{24r^3} + \frac{61a^6}{720r^5} + \frac{277a^8}{8064r^7} \text{ \&c.}$$

$$\sigma = \frac{r^2}{a} + \frac{a}{6} + \frac{7a^3}{360r^2} + \frac{31a^5}{15120r^4} + \frac{127a^7}{604800r^6} \text{ \&c.}$$

$$log.\ ſ = m \times \left(\frac{a^2}{2} + \frac{a^4}{12} + \frac{a^6}{45} + \frac{17a^8}{2520} \text{ \&c.}\right)$$

$$log.\sigma = -log.a + m \times \left(\frac{a^2}{6} + \frac{a^4}{180} + \frac{a^6}{2835} + \frac{a^8}{37800} \text{\&c.}\right)$$

where m is the modulus of the ſyſtem of logarithms.

SECANTS, *Figure of*. See FIGURE *of Secants*.

SECANTS, *Line of*. See SECTOR, and SCALE.

SECOND, in Geometry, or Aſtronomy, &c, the 60th part of a prime or minute: either in the diviſion of circles, or in the meaſure of time. A degree, or an hour, are each divided into 60 minutes, marked thus ′; a minute is ſubdivided into 60 Seconds, marked thus ″; a Second into 60 thirds, marked thus ‴; &c.

We ſometimes ſay a *Second minute*, a *third minute*, &c, but more uſually only *Second*, *third*, &c.

The Seconds pendulum, or pendulum that vibrates Seconds, in the latitude of London, is $39\frac{1}{8}$ inches long.

SECONDARY *Circles of the Ecliptic*, are circles of longitude of the ſtars; or circles which, paſſing through the poles of the ecliptic, are at right angles to the ecliptic.

By means of theſe Secondary circles, all points in the heavens are referred to the ecliptic; that is, any ſtar, planet, or other phenomenon, is underſtood to be in that point of the ecliptic, which is cut by the Secondary circle that paſſes through ſuch ſtar, &c.

If two ſtars be thus referred to the ſame point of the ecliptic, they are ſaid to be in conjunction; if in oppoſite points, they are in oppoſition; if they are referred to two points at a quadrant's diſtance, they are ſaid to be in a quartile aſpect, if the points differ a 6th part of the ecliptic, they are in ſextile aſpect, &c.

In general, all circles that interſect one of the ſix greater circles of the ſphere at right angles, may be called Secondary circles. As the azimuth or vertical circles in reſpect of the horizon, &c; the meridian in reſpect of the equator, &c.

SECONDARY *Planets*, or *Satellites*, are thoſe moving round other planets as the centres of their motion, and along with them round the ſun.

SECTION, in Geometry, denotes a ſide or ſurface appearing of a body, or figure, cut by another; or the place where lines, planes, &c, cut each other.

The common Section of two planes is always a right line; being the line ſuppoſed to be drawn by one plane in its cutting or entering the other. If a ſphere be cut in any manner by a plane, the figure of the Section will be a circle; alſo the common interſection of the ſurfaces of two ſpheres, is the circumference of a circle; and the two common Sections of the ſurfaces of a right cone and a ſphere, are the circumferences of circles if the axis of the cone paſs through the centre of the ſphere, otherwiſe not; moreover, of the two common Sections of a ſphere and a cone, whether right or oblique, if the one be a circle the other will be a circle alſo, otherwiſe not. See my Tracts, tract 7, prop. 7, 8, 9.

The Sections of a cone by a plane, are five; viz, a triangle, circle, ellipſe, hyperbola, and parabola. See each of theſe terms, as alſo CONIC SECTION.

Sections of Buildings and Bodies, &c, are either vertical, or horizontal, &c. The

Vertical SECTION, or ſimply the SECTION, of a building, denotes its profile, or a delineation of its heights and depths raiſed on the plan; as if the fabric had been cut aſunder by a vertical plane, to diſcover the inſide. And

Horizontal SECTION is the ichnography or ground plan, or a Section parallel to the horizon.

SECTOR, of a Circle, is a portion of the circle comprehended between two radii and their included arc. Thus, the mixt triangle ABC, contained between the two radii AC and BC, and the arc AB, is a Sector of the circle.

The Sector of a circle, as ABC, is equal to a triangle, whoſe baſe is the arc AB, and its altitude the radius AC or BC. And therefore the radius being drawn into the arc, half the product gives the area.

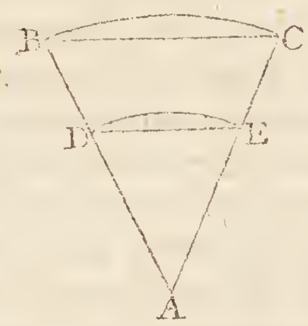
Similar SECTORS, are thoſe which have equal angles included between their radii. Theſe are to each other as the ſquares of their bounding arcs, or as their whole circles.

SECTOR alſo denotes a mathematical inſtrument, which is of great uſe in geometry, trigonometry, ſurveying, &c, in meaſuring and laying down and finding proportional quantities of the ſame kind: as between lines and lines, ſurfaces and ſurfaces, &c: whence the French call it the *compaſs of proportion*.

 The

The great advantage of the Sector above the common scales, &c, is, that it is contrived so as to suit all radii, and all scales. By the lines of chords, sines, &c, on the Sector, we have lines of chords, sines, &c, to any radius between the length and breadth of the Sector when open.

The Sector is founded on the 4th proposition of the 6th book of Euclid; where it is demonstrated, that similar triangles have their like sides proportional. An idea of the theory of its construction may be conceived thus. Let the lines AB, AC represent the legs of the Sector; and AD, AE, two equal sections from the centre: then if the points BC and DE be connected, the lines BC and DE will be parallel; therefore the triangles ABC, ADE will be similar, and consequently the sides AB, BC, AD, DE proportional, that is, as AB : BC : : AD : DE; so that if AD be the half, 3d, or 4th part of AB, then DE will be a half, 3d, or 4th part of BC: and the same holds of all the rest. Hence, if DE be the chord, sine or tangent, of any arc, or of any number of degrees, to the radius AD, then BC will be the same to the radius AB.



The Sector, it is supposed, was the invention of Guido Baldo or Ubaldo, about the year 1568. The first printed account of it was in 1584, by Gaspar Mordente at Antwerp, who indeed says that his brother Fabricius Mordente invented it, in the year 1554. It was next treated of by Daniel Speckle, at Strasburgh, in 1589; after that by Dr. Thomas Hood, at London, in 1598: and afterwards by many other writers on practical geometry, in all the nations of Europe.

Description of the SECTOR. This instrument consists of two rules or legs, the longer the better, made of box, or ivory, or brass, &c, representing the radii, moveable round an axis or joint, the middle of which represents the centre; from whence several scales are drawn on the faces. See the fig. 1, plate xxvi.

The scales usually set upon Sectors, may be distinguished into single and double. The single scales are such as are set upon plane scales: the double scales are those which proceed from the centre; each of these being laid twice on the same face of the instrument, viz, once on each leg. From these scales, dimensions or distances are to be taken, when the legs of the instrument are set in an angular position.

The scales set upon the best Sectors are

Single	1	A line of Inches, each divided into 8 and 10 parts,	
	2	Decimals, containing 100 parts.	
	3	Chords	marked Cho.
	4	Sines	Sin.
	5	Tangents	Tang.
	6	Rhumbs	Rhum.
	7	Latitude	Lat.
	8	Hours	Hou.
	9	Longitude	Lon.
	10	Inclin. Merid.	In. mer.
	11	the logarithms of Numbers	Num.
	12	Sines	Sin.
	13	Versed Sines	V. Sin.
	14	Tangents	Tan.

Double	1	a line of Lines, or equal parts	marked Lin.
	2	Chords	Cho.
	3	Sines	Sin.
	4	Tangents to 45°	Tan.
	5	Secants	Sec.
	6	Tangents to above 45°	Tan.
	7	Polygons	Pol.

The manner in which these scales are disposed on the Sector, is best seen in the figure.

The scales of lines, chords, sines, tangents, rhumbs, latitudes, hours, longitude, incl. merid. may be used, with the instrument either shut or open, each of these scales being contained on one of the legs only. The scales of inches, decimals, log. numbers, log. sines, log. versed sines, and log. tangents, are to be used with the Sector quite open, with the two rulers or legs stretched out in the same direction, part of each scale lying on both legs.

The double scales of lines, chords, sines, and lower tangents, or tangents under 45°, are all of the same radius or length; they begin at the centre of the instrument, and are terminated near the other extremity of each leg; viz, the lines at the division 10, the chords at 60, the sines at 90, and the tangents at 45; the remainder of the tangents, or those above 45°, are on other scales beginning at $\frac{1}{4}$ of the length of the former, counted from the centre, where they are marked with 45, and run to about 76 degrees.

The secants also begin at the same distance from the centre, where they are marked with 10, and are from thence continued to as many degrees as the length of the Sector will allow, which is about 75°.

The angles made by the double scales of lines, of chords, of sines, and of tangents to 45 degrees, are always equal. And the angles made by the scales of upper tangents, and of secants, are also equal.

The scales of polygons are set near the inner edge of the legs; and where these scales begin, they are marked with 4, and from thence are figured backwards, or towards the centre, to 12.

From this disposition of the double scales, it is plain, that those angles that are equal to each other while the legs of the Sector were close, will still continue to be equal, although the Sector be opened to any distance.

The scale of inches is laid close to the edge of the Sector, and sometimes on the edge; it contains as many inches as the instrument will receive when opened; each inch being usually divided into 8, and also into 10 equal parts. The decimal scale lies next to this: it is of the length of the Sector when opened, and is divided into 10 equal parts, or primary divisions, and each of these into 10 other equal parts; so that the whole is divided into 100 equal parts: and by this decimal scale, all the other scales, that are taken from tables, may be laid down. The scales of chords, rhumbs, sines, tangents, hours, &c, are such as are described under Plane Scale.

The scale of logarithmic or artificial numbers, called Gunter's scale, or Gunter's line, is a scale expressing the logarithms of common numbers, taken in their natural order.

The construction of the double scale will be evident by inspecting the instrument. As to the scale of polygons,

gons, it usually comprehends the sides of the polygons from 6 to 12 sides inclusive: the divisions are laid down by taking the lengths of the chords of the angles at the centre of each polygon, and laying them down from the centre of the instrument. When the polygons of 4 and 5 sides are also introduced, this line is constructed from a scale of chords, where the length of 90° is equal to that of 60° of the double scale of chords on the Sector.

In describing the use of the Sector, the terms *lateral distance* and *transverse distance* often occur. By the former is meant the distance taken with the compasses on one of the scales only, beginning at the centre of the Sector; and by the latter, the distance taken between any two corresponding divisions of the scales of the same name, the legs of the Sector being in an angular position.

Uses of the SECTOR.

Of the Line of Lines. This is useful, to divide a given line into any number of equal parts, or in any proportion, or to make scales of equal parts, or to find 3d and 4th proportionals, or mean proportionals, or to increase or decrease a given line in any proportion. Ex. 1. To divide a given line into any number of equal parts, as suppose 9: make the length of the given line a transverse distance to 9 and 9, the number of parts proposed; then will the transverse distance of 1 and 1 be one of the equal parts, or the 9th part of the whole; and the transverse distance of 2 and 2 will be 2 of the equal parts, or $\frac{2}{9}$ of the whole line; and so on. 2. Again, to divide a given line into any number of parts that shall be in any assigned proportion, as suppose three parts, in the proportion of 2, 3, and 4. Make the given line a transverse distance to 9, the sum of the proposed numbers 2, 3, 4; then the transverse distances of these numbers severally will be the parts required.

Of the Scale of Chords. 1. To open the Sector to any angle, as suppose 50 degrees: Take the distance from the joint to 50 on the chords, the number of degrees proposed; then open the Sector till the transverse distance from 60 to 60, on each leg, be equal to the said lateral distance of 50; so shall the scale of chords make the proposed angle of 50 degrees.—By the converse of this operation, may be known the angle the Sector is opened to; viz, taking the transverse distance of 60, and applying it laterally from the joint.

2. To protract or lay down an angle of any given number of degrees. At any opening of the Sector, take the transverse distance of 60°, with which extent describe an arc; then take the transverse distance of the number of degrees proposed, and apply it to that arc; and through the extremities of this distance on the arc draw two lines from the centre, and they will form the angle as proposed. When the angle exceeds 60°, lay it off at twice or thrice.—By the converse operation any angle may be measured; viz, With any radius describe an arc from the angular point; set that radius transversely from 60 to 60; then take the distance of the intercepted arc and apply it transversely to the chords, which will shew the degrees in the given angle.

Of the Line of Polygons. 1. In a given circle to inscribe a regular polygon, for example an octagon. Open the legs of the Sector till the transverse distance from 6 to 6 be equal to the radius of the circle; then will the transverse distance of 8 and 8 be the side of the inscribed octagon. 2. Upon a line given to describe a regular polygon. Make the given line a transverse dis. to 5 and 5; and at that opening of the Sector take the transverse distance of 6 and 6; with which as a radius, from the extremities of the given line describe arcs to intersect each other, which intersection will be the centre of a circle in which the proposed polygon may be inscribed; then from that centre describe the said circle through the extremities of the given line, and apply this line continually round the circumference, for the several angular points of the polygon.—3. On a given right line as a base, to describe an isosceles triangle, having the angles at the base double the angle at the vertex. Open the Sector till the length of the given line fall transversely on 10 and 10 on each leg; then take the transverse distance to 6 and 6, and it will be the length of each of the equal sides of the triangle.

Of the Sines, Tangents, and Secants. By the several lines disposed on the sector, we have scales of several radii. So that, 1. Having a length or radius given, not exceeding the length of the Sector when opened, we can find the chord, sine, &c, to the same: for ex. suppose the chord, sine, or tangent of 20 degrees to a radius of 3 inches be required. Make 3 inches the opening or transverse distance to 60 and 60 on the chords; then will the same extent reach from 45 to 45 on the tangents, and from 90 to 90 on the sines; so that to whatever radius the line of chords is set, to the same are all the others set also. In this disposition therefore, if the transverse distance between 20 and 20 on the chords be taken with the compasses, it will give the chord of 20 degrees; and if the transverse of 20 and 20 be in like manner taken on the sines, it will be the sine of 20 degrees; and lastly, if the transverse distance of 20 and 20 be taken on the tangents, it will be the tangent of 20 degrees, to the same radius.—2. If the chord or tangent of 70 degrees were required. For the chord, the transverse distance of half the arc, viz 35, must be taken, as before; which distance taken twice gives the chord of 70 degrees. To find the tangent of 70 degrees, to the same radius, the scale of upper tangents must be used, the under one only reaching to 45: making therefore 3 inches the transverse distance to 45 and 45 at the beginning of that scale, the extent between 70 and 70 degrees on the same, will be the tangent of 70 degrees to 3 inches radius.—3. To find the secant of an arc; make the given radius the transverse distance between 0 and 0 on the secants; then will the transverse distance of 20 and 20, or 70 and 70, give the secant of 20 or 70 degrees.—4. If the radius, and any line representing a sine, tangent, or secant, be given, the degrees corresponding to that line may be found by setting the Sector to the given radius, according as a sine, tangent, or secant is concerned; then taking the given line between the compasses, and applying the two feet transversely to the proper scale, and sliding the feet along till they both rest on like divisions on both legs; then the divisions will shew the degrees and parts corresponding to the given line.

Use

Use of the Sector in Trigonometry, or in working any other proportions.

By means of the double ſcales, which are the parts more peculiar to the Sector, all proportions are worked by the property of ſimilar triangles, making the ſides proportional to the baſes, that is, on the Sector, the lateral diſtances proportional to the tranſverſe ones; thus, taking the diſtance of the firſt term, and applying it to the 2d, then the diſtance of the 3d term, properly applied, will give the 4th term: obſerving that the ſides of triangles are taken off the line of numbers laterally, and the angles are taken tranſverſely, off the ſines or tangents or ſecants, according to the nature of the proportion. For example, in a plane triangle ABC, given two ſides and an angle oppoſite to one of them, to find the reſt; viz, given AB = 56, AC = 64, and ∠B = 46° 30′, to find BC and the angles A and C. In this caſe, the ſides are proportional to the ſines of their oppoſite angles; hence theſe proportions,

as AC (64) : ſin. ∠B (46° 30′) : : AB (56) : ſin. ∠C, and as ſin. B : AC : : ſin. A : BC.

Therefore, to work theſe proportions by the Sector, take the lateral diſtance of 64 = AC from the lines, and open the Sector to make this a tranſverſe diſtance of 46° 30′ = ∠B, on the ſines; then take the lateral diſtance of 56 = AB on the lines, and apply it tranſverſely on the ſines, which will give 39° 24′ = ∠C. Hence, the ſum of the angles B and C, which is 85° 54′, taken from 180°, leaves 94° 6′ = ∠A. Then, to work the 2d proportion, the Sector being ſet at the ſame opening as before, take the tranſverſe diſtance of 94° 6′ = ∠A, on the ſines, or, which is the ſame thing, the tranſverſe diſtance of its ſupplement 85° 54′; then this applied laterally to the lines, gives 88 = the ſide BC ſought.

For the complete hiſtory of the Sector, with its more ample and particular conſtruction and uſes, ſee Robertſon's *Treatiſe of ſuch Mathematical Inſtruments, as are uſually put into a Portable Caſe*, the Introduction.

SECTOR *of a Sphere*, is the ſolid generated by the revolution of the Sector of a circle about one of its radii; the other radius deſcribing the ſurface of a cone, and the circular arc a circular portion of the ſurface of the ſphere of the ſame radius. So that the ſpherical Sector conſiſts of a right cone, and of a ſegment of the ſphere having the ſame common baſe with the cone. And hence the ſolid content of it will be found by multiplying the baſe or ſpherical ſurface by the radius of the ſphere, and taking a 3d part of the product.

SECTOR *of an ellipſe*, or *of an hyperbola*, &c, is a part reſembling the circular Sector, being contained by three lines, two of which are radii, or lines drawn from the centre of the figure to the curve, and the intercepted arc or part of that curve.

Aſtronomical SECTOR, an inſtrument invented by Mr. George Graham, for finding the difference in right aſcenſion and declination between two objects, whoſe diſtance is too great to be obſerved through a fixed

teleſcope, by means of a micrometer. This inſtrument (fig. 2, pl. 26,) conſiſts of a braſs plate, called the Sector, formed like a T, having the ſhank CD, as a radius, about 2½ feet long, and 2 inches broad at the end D, and an inch and a half at C; and the croſs-piece AB, as an arch, about 6 inches long, and one and a half broad; upon which, with a radius of 30 inches, is deſcribed an arch of 10 degrees, each degree being divided in as many parts as are convenient. Round a ſmall cylinder C, containing the centre of this arch, and fixed in the ſhank, moves a plate of braſs, to which is fixed a teleſcope CE, having its line of collimation parallel to the plane of the Sector, and paſſing over the centre C of the arch AB, and the index of a Vernier's dividing plate, whoſe length, being equal to 16 quarters of a degree, is divided into 15 equal parts, fixed to the eye end of the teleſcope, and made to ſlide along the arch; which motion is performed by a long ſcrew, G, at the back of the arch, communicating with the Vernier through a ſlit cut in the braſs, parallel to the divided arch. Round the centre F of a circular braſs plate *abc*, of 5 inches diameter, moves a braſs croſs KLMN, having the oppoſite ends O and P of one bar turned up perpendicularly about 3 inches, to ſerve as ſupporters to the Sector, and ſcrewed to the back of its radius; ſo that the plane of the Sector is parallel to the plane of the circular plate, and can revolve round the centre of that plate in this parallel poſition. A ſquare iron axis HIF, 18 inches long, is ſcrewed flat to the back of the circular plate along one of its diameters, ſo that the axis is parallel to the plane of the Sector. The whole inſtrument is ſupported on a proper pedeſtal, ſo that the ſaid axis ſhall be parallel to the earth's axis, and proper contrivances are annexed to fix it in any poſition. The inſtrument, thus ſupported, can revolve round its axis HI, parallel to the earth's axis, with a motion like that of the ſtars, the plane of the Sector being always parallel to the plane of ſome hour circle, and conſequently every point of the teleſcope deſcribing a parallel of declination; and if the Sector be turned round the joint F of the circular plate, its graduated arch may be brought parallel to an hour-circle; and conſequently any two ſtars, whoſe difference of declination does not exceed the degrees in that arch, will paſs over it.

To obſerve their paſſage, direct the teleſcope to the preceding ſtar, and fix the plane of the Sector a little to the weſtward of it; move the teleſcope by the ſcrew G, and obſerve at the tranſit of each over the croſs wires the time ſhewn by the clock, and alſo the diviſion upon the arch AB, ſhewn by the index; then is the difference of the arches the difference of the declination; and that of the times ſhews the difference of the right aſcenſion of thoſe ſtars. For a more particular deſcription of this inſtrument, ſee Smith's Optics, book iii, chap. 9.

SECULAR *Year*, the ſame with Jubilee.

SECUNDANS, an infinite ſeries of numbers, beginning from nothing, and proceeding according to the ſquares of numbers in arithmetical progreſſion, as 0, 1, 4, 9, 16, 25, 36, 49, 64, &c.

SEEING, the act of perceiving objects by the organ of ſight; or the ſenſe we have of external objects by means of the eye.

For

For the apparatus, or difpofition of the parts neceffary to Seeing, fee EYE. And for the manner in which Seeing is performed, and the laws of it, fee VISION.

Our beft anatomifts differ greatly as to the caufe why we do not fee double with the two eyes? Galen, and others after him, afcribe it to a coalition, or decuffation, of the optic nerve, behind the os fphenoides. But whether they decuffate or coalefce, or only barely touch one another, is not well agreed upon.

The Bartholines and Vefalius fay exprefsly, they are united by a perfect confufion of their fubftance; Dr. Gibfon allows them to be united by the clofeft conjunction, but not by a confufion of their fibres.

Alhazen, an Arabian philofopher of the 12th century, accounts for fingle vifion by two eyes, by fuppofing that when two correfponding parts of the retina are affected, the mind perceives but one image.

Des Cartes and others account for the effect another way; viz, by fuppofing that the fibrillæ conftituting the medullary part of thofe nerves, being fpread in the retina of each eye, have each of them correfponding parts in the brain, fo that when any of thofe fibrillæ are ftruck by any part of an image, the correfponding parts of the brain are affected by it. Somewhat like which is the opinion of Dr. Briggs, who takes the optic nerves of each eye to confift of homologous fibres, having their rife in the thalamus nervorum opticorum, and being thence continued to both the retinæ, which are compofed of them; and farther, that thofe fibrillæ have the fame parallelifm, tenfion, &c, in both eyes; confequently when an image is painted on the fame correfponding fympathizing parts of each retina, the fame effects are produced, the fame notice carried to the thalamus, and fo imparted to the foul. Hence it is, that double vifion enfues upon an interruption of the parallelifm of the eyes; as when one eye is depreffed by the finger, or their fymphony is interrupted by difeafe: but Dr. Briggs maintains, that it is but in few fubjects there is any decuffation; and in none any conjunction more than mere contact; though his notion is by no means confonant to facts, fnd it is attended with many improbable circumftances.

It was the opinion of Sir Ifaac Newton, and of many others, that objects appear fingle, becaufe the two optic nerves unite before they reach the brain. But Dr. Porterfield fhews, from the obfervation of feveral anatomifts, that the optic nerves do not mix or confound their fubftance, being only united by a clofe cohefion; and objects have appeared fingle, where the optic nerves were found to be disjoined. To account for this phenomenon, this ingenious writer fuppofes, that, by an original law in our natures, we imagine an object to be fituated fomewhere in a right line drawn from the picture of it upon the retina, through the centre of the pupil; confequently the fame object appearing to both eyes to be in the fame place, we cannot diftinguifh it into two. In anfwer to an objection to this hypothefis, from objects appearing double when one eye is diftorted, he fays, the mind miftakes the pofition of the eye, imagining, that it had moved in a manner correfponding to the other, in which cafe the conclufion would have been juft: in this he feems to have recourfe to the power of habit, though he difclaims that hypothefis. This principle however has been thought fufficient to account for this appearance.

Originally, every object making two pictures, one in each eye, is imagined to be double; but, by degrees, we find that when two correfponding parts of the retina are impreffed, the object is but one; but if thofe correfponding parts be changed by the diftortion of one of the eyes, the object muft again appear double as at the firft. This feems to be verified by Mr. Chefelden, who informs us, that a gentleman, who, from a blow on his head, had one eye diftorted, found every object to appear double, but by degrees the moft familiar ones came to appear fingle again, and in time all objects did fo without amendment of the diftortion. A fimilar cafe is mentioned by Dr. Smith.

On the other hand, Dr. Reid is of opinion, that the correfpondence of the centres of two eyes, on which fingle vifion depends, does not arife from cuftom, but from fome natural conftitution of the eye, and of the mind.

M. du Tour adopts an opinion, long before fuggefted by Gaffendi, that the mind attends to no more than the image made in one eye at a time; in fupport of which, he produces feveral curious experiments; but as M. Buffon obferves, it is a fufficient anfwer to this hypothefis, that we fee more diftinctly with two eyes than with one; and that when a round object is near us, we plainly fee more of the furface in one cafe than in the other.

With refpect to fingle vifion with two eyes, Dr. Hartley obferves, that it deferves particular attention, that the optic nerves of man, and fuch other animals as look the fame way with both eyes, unite in the fella turrica in a ganglion, or little brain, as it may be called, peculiar to themfelves, and that the affociations between fynchronous impreffions on the two retinas, muft be made fooner and cemented ftronger on this account; alfo that they ought to have a much greater power over one another's image, than in any other part of the body. And thus an impreffion made on the right eye alone by a fingle object, propagates itfelf into the left, and there raifes up an image almoft equal in vividnefs to itfelf; and, confequently, when we fee with one eye only, we may however have pictures in both eyes.

It is a common obfervation, fays Dr. Smith, that objects feen with both eyes appear more vivid and ftronger than they do to a fingle eye, efpecially when both of them are equally good. Porterfield on the Eye, vol. ii, pa. 285, 315. Smith's Optics, Remarks pa. 31. Reid's Inquiry, pa. 267. Mem. Préfentes, pa. 514. Acad. Par. 1747. Mem. Pr. 334. Hartley on Man, vol. i, pa. 207. Prieftley's Hift. of Light and Colours, pa. 663, &c.

Whence it is that we fee objects erect, when it is certain, that the images thereof are painted invertedly on the retina, is another difficulty in the theory of Seeing. Des Cartes accounts for it hence, that the notice which the foul takes of the object, does not depend on any image, nor any action coming from the object, but merely on the fituation of the minute parts of the brain, whence the nerves arife. Ex. gr. the fituation of a capillament brain, which occafions the foul

ſoul to ſee all thoſe places lying in a right line with it.

But Mr. Molyneux gives another account of this matter. The eye, he obſerves, is only the organ, or inſtrument; it is the ſoul that ſees. To enquire then, how the ſoul perceives the object erect by an inverted image, is to enquire into the ſoul's faculties. Again, imagine that the eye receives an impulſe on its lower part, by a ray from the upper part of an object; muſt not the viſive faculty be hereby directed to conſider this ſtroke as coming from the top, rather than the bottom of the object, and conſequently be determined to conclude it the repreſentation of the top?

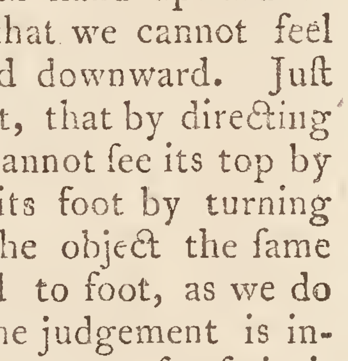
Upon theſe principles, we are to conſider, that inverted is only a relative term, and that there is a very great difference between the real object, and the means or image by which we perceive it. When all the parts of a diſtant proſpect are painted upon the retina (ſuppoſing that to be the ſeat of viſion), they are all right with reſpect to one another, as well as the parts of the proſpect itſelf; and we can only judge of an object being inverted, when it is turned reverſe to its natural poſition with reſpect to other objects which we ſee and compare it with.

The eye or viſive faculty (ſays Molyneux) takes no notice of the internal ſurface of its own parts, but uſes them as an inſtrument only, contrived by nature for the exerciſe of ſuch a faculty. If we lay hold of an upright ſtick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and very well know that we cannot feel the upper end by moving our hand downward. Juſt ſo, we find by experience and habit, that by directing our eyes towards a tall object, we cannot ſee its top by turning our eyes downward, nor its foot by turning our eyes upward; but muſt trace the object the ſame way by the eye to ſee it from head to foot, as we do by the hand to feel it; and as the judgement is informed by the motion of the hand in one caſe, ſo it is alſo by the motion of the eye in the other.

Molyneux's Dioptr. pa. 105, &c. Muſſchenbroek's Int. ad Phil. Nat. vol. ii, pa. 762. Ferguſon's Lectures, pa. 132. See SIGHT, VISIBLE, &c.

SEGMENT, in Geometry, is a part cut off the top of a figure by a line or plane; and the part remaining at the bottom, after the Segment is cut off, is called a *fruſtum*, or a *zone*. So, a

SEGMENT *of a Circle*, is a part of the circle cut off by a chord, or a portion comprehended by an arch and its chord; and may be either greater or leſs than a ſemicircle. Thus, the portion ABCA is a Segment leſs than a ſemicircle; and ADCA a Segment greater.



The angle formed by lines drawn from the extremities of a chord to meet in any point of the arc, is called an angle *in* the Segment. So the angle ABC is an angle *in* the Segment ABCA; and the angle ADC, an angle *in* the Segment ADCA.

Alſo the angle B is ſaid to be the angle *upon* the Segment ADC, and D the angle *on* the Segment ABC.

The angle which the chord AC makes with a tangent EF, is called the angle *of* a Segment; and it is equal to the angle in the alternate or ſupplemental Segment, or equal to the ſupplement of the angle in the ſame Segment. So the angle ACE is the angle *of* the Segment ABC, and is equal to the angle ADC, or to the ſupplement of the angle B; alſo the angle ACF is the angle *of* the Segment ADC, and is equal to the angle B, or to the ſupplement of the angle D.

The area of a Segment ABC, is evidently equal to the difference between the ſector OABC of the ſame arc, and the triangle OAC on the ſame chord; the triangle being ſubtracted from the ſector, to give the Segment, when leſs than a ſemicircle; but to be added when greater. See more rules for the Segment in my Menſuration, pa. 122 &c, 2d edition.

Similar SEGMENTS, are thoſe that have their chords directly proportional to their radii or diameters, or that have ſimilar arcs, or ſuch as contain the ſame number of degrees.

SEGMENT *of a Sphere*, is a part cut off by a plane.

The baſe of a Segment is always a circle. And the convex ſurfaces of different Segments, are to each other as their altitudes, or verſed ſines. And as the whole convex ſurface of the ſphere is equal to 4 of its great circles, or 4 circles of the ſame diameter; ſo the ſurface of any Segment, is equal to 4 circles on a diameter equal to the chord of half the arc of the Segment. So that if d denote the diameter of the ſphere, or the chord of half the circumference, and c the chord of half the arc of any other Segment, alſo a the altitude or verſed ſine of the ſame; then,

$3{\cdot}1416d^2$ is the ſurface of the whole ſphere, and
$3{\cdot}1416c^2$, or $3{\cdot}1416ad$, the ſurface of the Segment.

For the ſolid content of a Segment, there are two rules uſually given; viz, 1. To 3 times the ſquare of the radius of its baſe, add the ſquare of its height; multiply the ſum by the height, and the product by ·5236. Or, 2dly, From 3 times the diameter of the ſphere, ſubtract twice the height of the fruſtum; multiply the remainder by the ſquare of the height, and the product by ·5236. That is, in ſymbols, the ſolid content is either

$= {\cdot}5236a \times \overline{3r^2 + a^2}$, or $= {\cdot}5236a^2 \times \overline{3d - 2a}$;
where a is the altitude of the Segment, r the radius of its baſe, and d the diameter of the whole ſphere.

Line of SEGMENTS, are two particular lines, ſo called, on Gunter's ſector. They lie between the lines of ſines and ſuperficies, and are numbered with 5, 6, 7, 8, 9, 10. They repreſent the diameter of a circle, ſo divided into 100 parts, as that a right line drawn through thoſe parts, and perpendicular to the diameter, ſhall cut the circle into two Segments, the greater of which ſhall have the ſame proportion to the whole circle, as the parts cut off have to 100.

SELENOGRAPHY, the deſcription and repreſentation of the moon, with all the parts and appearances of her diſc or face; like as geography does thoſe of the earth.

Since the invention of the telescope, Selenography is very much improved. We have now distinct names for most of the regions, seas, lakes, mountains, &c, visible in the moon's body. Hevelius, a celebrated astronomer of Dantzic, and who published the first Selenography, named the several places of the moon from those of the earth. But Riccioli afterwards called them after the names of the most celebrated astronomers and philosophers. Thus, what the one calls *mons Porphyrites*, the other calls *Aristarchus*; what the one calls *Ætna*, *Sinai*, *Athos*, *Apenninus*, &c, the other calls, *Copernicus*, *Posidonius*, *Tycho*, *Gassendus*, &c.

M. Cassini has published a work called *Instructions Seleniques*, and has published the best map of the moon.

SELEUCIDÆ, in Chronology, the era of the Seleucidæ, or the Syro-Macedonian era, which is a computation of time, commencing from the establishment of the Seleucidæ, a race of Greek kings, who reigned as successors of Alexander the Great, in Syria, as the Ptolemies did in Egypt. According to the best accounts, the first year of this era falls in the year 311 before Christ, which was 12 years after the death of Alexander.

SELL, in Building, is of two kinds, viz, *Ground-Sell*, which denotes the lowest piece of timber in a wooden building, and that upon which the whole superstructure is raised And Sell of a window, or of a door, which is the bottom piece in the frame of them, upon which they rest.

SEMICIRCLE, in Geometry, is half a circle, or a figure comprehended between the diameter of a circle, and half the circumference.

SEMICIRCLE is also an instrument in Surveying, sometimes called the *graphometer*.

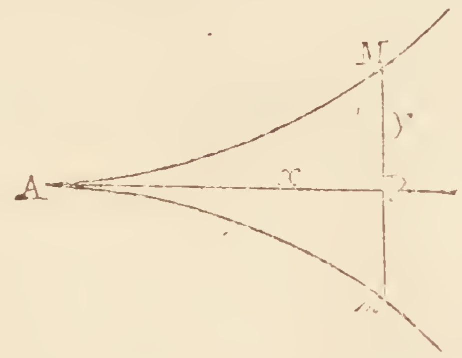
It consists of a semicircular limb or arch, as FIG (fig. 3, pl. 26) divided into 180 degrees, and sometimes subdivided diagonally or otherwise into minutes. This limb is subtended by a diameter FG, having two sights erected at its extremities. In the centre of the Semicircle, or the middle of the diameter, is fixed a box and needle; and on the same centre is fitted an alidade, or moveable index, carrying two other sights, as H, I: the whole being mounted on a staff, with a ball and socket &c.

Hence it appears, that the Semicircle is nothing but half a theodolite; with this only difference, that whereas the limb of the theodolite, being an entire circle, takes in all the 360° successively; while in the Semicircle the degrees only going from 1 to 180, it is usual to have the remaining 180°, or those from 180° to 360°, graduated in another line on the limb within the former.

To take an Angle with a Semicircle.—Place the instrument in such manner, as that the radius CG may hang over one leg of the angle to be measured, with the centre C over the vertex of the same The first is done by looking through the sights F and G, at the extremities of the diameter, to a mark fixed up in one extremity of the leg; and the latter is had by letting fall a plummet from the centre of the instrument. This done, turn the moveable index HI on its centre towards the other leg of the angle, till through the sights fixed in it, you see a mark in the extremity of the leg. Then the degree which the index cuts on the limb, is the quantity or measure of the angle.

Other uses are the same as in the theodolite.

SEMICUBICAL PARABOLA, a curve of the 2d order, of such a nature that the cubes of the ordinates are proportional to the squares of the abscisses. Its equation is $ay^2 = x^3$. This curve, AMm, is one of



Newton's five diverging parabolas, being his 70th species; having a cusp at its vertex at A. It is otherwise named the Neilian parabola, from the name of the author who first treated of it.

The area of the space APM, is $= \frac{4}{5}xy = \frac{4}{5}AP \times PM$, or $\frac{4}{5}$ of the circumscribing rectangle.

The content of the solid generated by the revolution of the space APM about the axis AP, is $^1pxy^2 = \cdot 7854 AP \times PM^2$, or $\frac{1}{4}$ of the circumscribing cylinder. And a circle equal to the surface of that solid may be found from the quadrature of an hyperbolic space.

Also the length of any arc AM of the curve may be easily obtained from the quadrature of a space contained under part of the curve of the common parabola, two semiordinates to the axis, and the part of the axis contained between them.

This curve may be described by a continued motion, viz, by fastening the angle of a square in the vertex of a common parabola; and then carrying the intersection of one side of this square and a long ruler (which ruler always moves perpendicularly to the axis of the parabola) along the curve of that parabola. For the intersection of the ruler, and the other side of the square will describe a Semicubical parabola. Maclaurin performs this without a common parabola, in his Geometria Organica.

SEMIDIAMETER, or *Radius*, of a circle or sphere, is a line drawn from the centre to the circumference. And in any curve that has diameters and a centre, it is the radius, or half diameter, or a line drawn from the centre to some point in the curve.

The distances, diameters, &c, of the heavenly bodies, are usually estimated by astronomers in Semidiameters of the earth; the number of which terrestrial Semidiameters, contained in that of each of those planets, is as below.

The Earth	- -	1 Semidiam.
The Sun	- -	111¼
The Moon	- -	0·27
Mercury	- -	0·38
Venus	- -	1·15
Mars	- -	0·65
Jupiter	- -	11·81
Saturn	- -	9·77
Herschel	- -	4·32

SEMIDIAPENTE, in Music, a defective or imperfect fifth, called usually by the Italians, *falsa quinta*, and by us a false fifth.

SEMIDIAPASON, in Mufic, a defective or imperfect octave; or an octave diminifhed by a leffer femitone, or 4 commas.

SEMIDIATESSARON, in Mufic, a defective fourth, called alfo a falfe fourth.

SEMIDITONE, in Mufic, is the leffer third, having its terms as 6 to 5.

SEMIORDINATES, in Geometry, the halves of the ordinates or applicates, being the lines applied between the abfcifs and the curve.

SEMIPARABOLA, &c, in Geometry, the half of the whole parabola, &c.

SEMIQUADRATE, or SEMIQUARTILE, is an afpect of the planets, when diftant from each other one fign and a half, or 45 degrees.

SEMIQUAVER, in Mufic, the half of a quaver.

SEMIQUINTILE, is an afpect of the planets when diftant from each other the half of a 5th of the circle, or by 36 degrees.

SEMISEXTILE, an afpect of two planets, when they are diftant from each other 30 degrees, or the half of a fextile, which is 2 figns or 60°. The Semifextile is marked *s. s.*

SEMITONE, in Mufic, a half tone or half note, one of the degrees or intervals of concords.

There are three degrees, or lefs intervals, by which a found can move upwards and downwards, fucceffively from one extreme of any concord to the other, and yet produce true melody. Thefe degrees are the greater tone, the lefs tone, and the femitone. The ratios defining thefe intervals are thefe, viz, the greater tone 8 to 9, the lefs tone 9 to 10, and the Semitone 15 to 16. Its compafs is 5 commas, and it has its name from being nearly half a whole, though it is really fomewhat more.

There are feveral fpecies of Semitones; but thofe that ufually occur in practice are of two kinds, diftinguifhed by the addition of greater and lefs. The firft is expreffed by the ratio of 16 to 15, or $\frac{16}{15}$; and the fecond by 25 to 24, or $\frac{25}{24}$. The octave contains 10 Semitones major, and 2 diefes, nearly, or 17 Semitones minor, nearly; for the meafure of the octave being expreffed by the logarithm - 1,00000, the Semitone major will be meafured by 0,09311, and the Semitone minor by - - 0,05889. Thefe two differ by a whole enharmonic diefis; which is an interval practicable by the voice. It was much in ufe among the Ancients, and is not unknown among modern practitioners. Euler Tent. Nov. Theor. Muf. pa. 107. See INTERVAL.

Thefe Semitones are called *fictitious notes;* and, with refpect to the natural ones, they are expreffed by characters called *flats* and *fharps*. The ufe of them is to remedy the defects of inftruments, which, having their founds fixed, cannot always be made to anfwer to the diatonic fcale. By means of thefe, we have a new kind of fcale, called the

SEMITONIC *Scale*, or the *Scale of Semitones*, which is a fcale or fyftem of mufic, confifting of 12 degrees, or 13 notes, in the octave, being an improvement on the natural or diatonic fcale, by inferting between each two notes of it, another note, which divides the interval or tone into two unequal parts, called Semitones.

The ufe of this fcale is for inftruments that have fixed founds, as the organ, harpfichord, &c, which are exceedingly defective on the foot of the natural or diatonic fcale. For the degrees of the fcale being unequal, from every note to its octave there is a different order of degrees; fo that from any note we cannot find every interval in a feries of fixed founds; which yet is neceffary, that all the notes of a piece of mufic, carried through feveral keys, may be found in their juft tune, or that the fame fong may be begun indifferently at any note, as may be neceffary for accommodating fome inftrument to others, or to the voice, when they are to accompany each other in unifon.

The diatonic fcale, beginning at the loweft note, being firft fettled on an inftrument, and the notes of it diftinguifhed by their names *a, b, c, d, e, f, g*; the inferted notes, or Semitones, are called fictitious notes, and take the name or letter below with a ✻, as *c* ✻ called *c* fharp; fignifying that it is a femitone higher than the found of *c* in the natural feries; or this mark ♭, called a flat, with the name of the note above fignifying it to be a Semitone lower.

Now $\frac{15}{16}$ and $\frac{128}{135}$ being the two Semitones the greater tone is divided into, and $\frac{15}{16}$ and $\frac{24}{25}$, the Semitones the lefs tone is divided into, the whole octave will ftand as in the following fcheme, where the ratios of each term to the next are written fraction-wife between them below.

Scale of Semitones.

c.	*c*✻.	*d.*	*d*✻.	*e.*	*f.*	*f*✻.	*g.*	*g*✻.	*a*♭.	*b.*	*cc.*
$\frac{15}{16}$	$\frac{128}{135}$	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{128}{135}$	$\frac{15}{16}$	$\frac{15}{16}$	$\frac{24}{25}$	$\frac{15}{16}$	$\frac{128}{135}$	$\frac{15}{16}$

for the names of the intervals in this fcale, it may be confidered, that as the notes added to the natural fcale are not defigned to alter the fpecies of melody, but leave it ftill diatonic, and only correct certain defects arifing from fomething foreign to the office of the fcale of mufic, viz, the fixing and limiting the founds; we fee the reafon why the names of the natural fcale are continued, only making a diftinction of each into a greater and lefs. Thus an interval of one Semitone, is called a lefs fecond; of two Semitones, a greater fecond; of three Semitones, a lefs third; of four, a greater third, &c.

A fecond kind of Semitonic fcale we have from another divifion of the octave into Semitones, which is performed by taking an harmonical mean between the extremes of the greater and lefs tone of the natural fcale, which divides it into two Semitones nearly equal. Thus, the greater tone 8 to 9 is divided into two Semitones, which are 16 to 17, and 17 to 18; where 16, 17, 18, is an arithmetical divifion, the numbers reprefenting the lengths of the chords; but if they reprefent the vibration, the lengths of the chords are reciprocal; viz as 1, $\frac{16}{17}$, $\frac{8}{9}$; which puts the greater Semitone

mitone $\frac{16}{17}$ next the lower part of the tone, and the leſſer $\frac{17}{18}$ next the upper, which is the property of the harmonical diviſion. And after the ſame manner the leſs tone 9 to 10 is divided into two Semitones, 18 to 19, and 19 to 20; and the whole octave ſtands thus:

c.	*c*※.	*d.*	*d*※.	*e.*	*f.*	*f*※.	*g.*	*g*※.	*a.*	♭.	*b.*	*c.*
$\frac{16}{17}$	$\frac{17}{18}$	$\frac{18}{19}$	$\frac{19}{20}$	$\frac{15}{16}$	$\frac{16}{17}$	$\frac{17}{18}$	$\frac{18}{19}$	$\frac{19}{20}$	$\frac{16}{17}$	$\frac{17}{18}$	$\frac{15}{16}$.	

This ſcale, Mr. Salmon tells us, in the Philoſophical Tranſactions, he made an experiment of before the Royal Society, on chords, exactly in theſe proportions, which yielded a perfect concert with other inſtruments, touched by the beſt hands. Mr. Malcolm adds, that, having calculated the ratios of them, for his own ſatiſfaction, he found more of them falſe than in the preceding ſcale, but then their errors were conſiderably leſs, which made amends. Malcolm's Muſic, chap. 10. § 2.

SENSIBLE *Horizon*, or *Point*, or *Quality*, &c. See the ſubſtantives.

SEPTUAGESIMA, in the Calendar, is the 9th Sunday before Eaſter, ſo called, as ſome have ſuppoſed, becauſe it is near 70 days, though in reality it is only 63 days, before it.

SERIES, in Algebra, denotes a rank or progreſſion of quantities or terms, which uſually proceed according to ſome certain law.

As the Series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ &c,

or the Series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ &c.

where the former is a geometrical Series, proceeding by the conſtant diviſion by 2, or the denominators multiplied by 2; and the latter is an harmonical Series, being the reciprocals of the arithmetical Series 1, 2, 3, 4, &c, or the denominators being continually increaſed by 1.

The doctrine and uſe of Series, one of the greateſt improvements of the preſent age, we owe to Nicholas Mercator; though it ſeems he took the firſt hint of it from Dr. Wallis's Arithmetic of Infinites; but the genius of Newton firſt gave it a body and a form.

It is chiefly uſeful in the quadrature of curves; where, as we often meet with quantities which cannot be expreſſed by any preciſe definite numbers, ſuch as is the ratio of the diameter of a circle to the circumference, we are glad to expreſs them by a Series, which, infinitely continued, is the value of the quantity ſought, and which is called an Infinite Series.

The Nature, Origin, &c, of Series.

Infinite Series commonly ariſe, either from a continued diviſion, as was practiſed by Mercator, or the extraction of roots, as firſt performed by Newton, who alſo explained other general ways for the expanding of quantities into infinite Series, as by the binomial theorem. Thus, to divide 1 by 3, or to expand the fraction $\frac{1}{3}$ into an infinite Series; by diviſion in decimals in the ordinary way, the ſeries is 0·3333 &c, or $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000}$ &c, where the law of continuation is manifeſt. Or, if the ſame fraction $\frac{1}{3}$ be ſet in this form $\frac{1}{2+1}$, and diviſion be performed in the algebraic manner, the quotient will be

$$\frac{1}{3} = \frac{1}{2+1} = \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} \text{ \&c.}$$

Or, if it be expreſſed in this form $\frac{1}{3} = \frac{1}{4-1}$, by a like diviſion there will ariſe the Series,

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \text{ \&c} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} \text{ \&c.}$$

And, thus, by dividing 1 by 5 — 2, or 6 — 3, or 7 — 4, &c, the Series anſwering to the fraction $\frac{1}{3}$, may be found in an endleſs variety of infinite Series; and the finite quantity $\frac{1}{3}$ is called the value or radix of the Series, or alſo its ſum, being the number or ſum to which the Series would amount, or the limit to which it would tend or approximate, by ſumming up its terms, or by collecting them together one after another.

In like manner, by dividing 1 by the algebraic ſum $a + c$, or by $a - c$, the quotient will be in theſe two caſes, as below, viz,

$$\frac{1}{a+c} = \frac{1}{a} - \frac{c}{a^2} + \frac{c^2}{a^3} - \frac{c^3}{a^4} \text{ \&c,}$$

$$\frac{1}{a-c} = \frac{1}{a} + \frac{c}{a^2} + \frac{c^2}{a^3} + \frac{c^3}{a^4} \text{ \&c.}$$

where the terms of each Series are the ſame, and they differ only in this, that the ſigns are alternately poſitive and negative in the former, but all poſitive in the latter.

And hence, by expounding a and c by any numbers whatever, we obtain an endleſs variety of infinite Series, whoſe ſums or values are known. So, by taking a or c equal to 1 or 2 or 3 or 4, &c, we obtain theſe Series, and their values;

$$\frac{1}{1+1} = \frac{1}{2} = 1 - 1 + 1 - 1 + 1 - 1 \text{ \&c,}$$

$$\frac{1}{3-1} = \frac{1}{2} = \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \frac{1}{3^4} \text{ \&c,}$$

$$\frac{1}{2+1} = \frac{1}{3} = \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^3} - \frac{1}{2^4} \text{ \&c,}$$

$$\frac{1}{1+2} = \frac{1}{3} = 1 - 2 + 2^2 - 2^3 \text{ \&c,}$$

$$\frac{1}{3+1} = \frac{1}{4} = \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} \text{ \&c.}$$

And hence it appears, that the ſame quantity or radix may be expreſſed by a great variety of infinite Series, or that many different Series may have the ſame radix or ſum.

Another way in which an infinite Series ariſes, is by the extraction of roots. Thus, by extracting the ſquare root of the number 3 in the common way, we obtain its value in a ſeries as follows, viz, $\sqrt{3} = 1{\cdot}73205 \text{ \&c} = 1 + \frac{7}{10} + \frac{3}{100} + \frac{2}{1000} + \frac{5}{100000}$ &c;

in which way of reſolution the law of the progreſſion of

of the Series is not vifible, as it is when found by divifion. And the fquare root of the algebraic quantity $a^2 + c^2$ gives

$$\sqrt{a^2 + c^2} = a + \frac{c^2}{2a} - \frac{c^4}{8a^3} + \frac{c^6}{16a^5} \text{ \&c.}$$

And a 3d way is by Newton's binomial theorem, which is a univerfal method, that ferves for all forts of quantities, whether fractional or radical ones: and by this means the fame root of the laft given quantity becomes $\sqrt{a^2 + c^2} =$

$$= a + \frac{c^2}{2a} - \frac{1.c^4}{2.4a^3} + \frac{1.3c^6}{2.4.6a^5} + \frac{1.3.5c^8}{2.4.6.8a^7} \text{ \&c.}$$

where the law of continuation is vifible.

See EXTRACTION *of Roots*, and BINOMIAL *Theorem*.

From the fpecimens above given, it appears that the figns of the terms may be either all plus, or alternately plus and minus. Though they may be varied in many other ways. It alfo appears that the terms may be either continually fmaller and fmaller, or larger and larger, or elfe all equal. In the firft cafe therefore the Series is faid to be a *decreafing* one, in the 2d cafe an *increafing* one, and in the 3d cafe an *equal* one. Alfo the firft Series is called a *converging* one, becaufe that by collecting its terms fucceffively, taking in always one term more, the fucceffive fums approximate or converge to the value or fum of the whole infinite Series. So, in the Series

$$\frac{1}{3-1} = \frac{1}{2} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}, \text{ \&c,}$$

the firft term $\frac{1}{3}$ is too little, or below $\frac{1}{2}$ which is the value or fum of the whole infinite Series propofed; the fum of the firft two terms $\frac{1}{3} + \frac{1}{9}$ is $\frac{4}{9} = \cdot 4444$ &c, is alfo too little, but nearer to $\frac{1}{2}$ or $\cdot 5$ than the former; and the fum of three terms $\frac{1}{3} + \frac{1}{9} + \frac{1}{27}$ is $\frac{13}{27} = \cdot 481481$ &c, is nearer than the laft, but ftill too little; and the fum of four terms

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}, \text{ is } \frac{40}{81} = \cdot 493827 \text{ \&c.}$$

which is again nearer than the former, but ftill too little; which is always the cafe when the terms are all pofitive. But when the converging Series has its terms alternately pofitive and negative, then the fucceffive fums are alternately too great and too little, though ftill approaching nearer and nearer to the final fum or value. Thus in the Series

$$\frac{1}{3+1} = \frac{1}{4} = 0\cdot 25 = \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} \text{ \&c,}$$

the 1ft term $\frac{1}{3} = \cdot 333$ &c, is too great,

two terms $\frac{1}{3} - \frac{1}{9} = \cdot 222$ &c, are too little,

three terms $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} = \cdot 259259$ &c, are too great,

four terms $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} = \cdot 246913$ &c, are too great, and fo on, alternately too great and too fmall, but every fucceeding fum ftill nearer than the former, or converging.

In the fecond cafe, or when the terms grow larger and larger, the Series is called a *diverging* one, becaufe that by collecting the terms continually, the fucceffive fums diverge, or go always farther and farther from the true value or radix of the Series; being all too great when the terms are all pofitive, but alternately too great and too little when they are alternately pofitive and negative. Thus, in the Series

$$\frac{1}{1+2} = \frac{1}{3} = 1 - 2 + 4 - 8 \text{ \&c.}$$

the firft term $+ 1$ is too great,
two terms $1 - 2 = - 1$ are too little,
three terms $1 - 2 + 4 = + 3$ are too great,
four terms $1 - 2 + 4 - 8 = - 5$ are too little,

and fo on continually, after the 2d term, diverging more and more from the true value or radix $\frac{1}{3}$, but alternately too great and too little, or pofitive and negative. But the alternate fums would be always more and more too great if the terms were all pofitive, and always too little if negative.

But in the third cafe, or when the terms are all equal, the Series of equals, with alternate figns, is called a *neutral* one, becaufe the fucceffive fums, found by a continual collection of the terms, are always at the fame diftance from the true value or radix, but alternately pofitive and negative, or too great and too little. Thus, in the Series

$$\frac{1}{1+1} = \frac{1}{2} = 1 - 1 + 1 - 1 + 1 - 1 \text{ \&c,}$$

the firft term 1 is too great,
two terms $1 - 1 = 0$ are too little,
three terms $1 - 1 + 1 = 1$ too great,
four terms $1 - 1 + 1 - 1 = 0$ too little,

and fo on continually, the fucceffive fums being alternately 1 and 0, which are equally different from the true value or radix $\frac{1}{2}$, the one as much above it, as the other below it.

A Series may be terminated and rendered finite, and accurately equal to the fum or value, by affuming the fupplement, after any particular term, and combining it with the foregoing terms. So, in the Series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16}$ &c, which is equal to $\frac{1}{3}$, and found by dividing 1 by $2 + 1$, after the firft term, $\frac{1}{2}$, of the quotient, the remainder is $- \frac{1}{2}$, which divided by $2 + 1$, or 3, gives $- \frac{1}{6}$ for the fupplement, which combined

combined with the first term $\frac{1}{2}$, gives $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ the true sum of the Series. Again, after the first two terms $\frac{1}{2} - \frac{1}{4}$, the remainder is $+ \frac{1}{4}$, which divided by the same divisor 3, gives $\frac{1}{12}$ for the supplement, and this combined with those two terms $\frac{1}{2} - \frac{1}{4}$, makes $\frac{1}{2} - \frac{1}{4} + \frac{1}{12} = \frac{1}{4} + \frac{1}{2} = \frac{4}{12}$ or $\frac{1}{3}$ the same sum or value as before. And in general, by dividing 1 by $a + c$, there is obtained

$$\frac{1}{a+c} = \frac{1}{a} - \frac{c}{a^2} + \frac{c^2}{a^3} \cdots \pm \frac{c^n}{a^{n+1}} \mp \frac{c^{n+1}}{a^{n+1}(a+c)};$$

where, stopping the division at any term as $\frac{c^n}{a^{n+1}}$, the remainder after this term is $\frac{c^{n+1}}{a^{n+1}}$, which being divided by the same divisor $a + c$, gives $\frac{c^{n+1}}{a^{n+1}(a+c)}$ for the supplement as above.

The Law of Continuation.—A Series being proposed, one of the chief questions concerning it, is to find the law of its continuation. Indeed, no universal rule can be given for this; but it often happens that the terms of the Series, taken two and two, or three and three, or in greater numbers, have an obvious and simple relation, by which the Series may be determined and produced indefinitely. Thus, if 1 be divided by $1 - x$, the quotient will be a geometrical progression, viz, $1 + x + x^2 + x^3$ &c, where the succeeding terms are produced by the continual multiplication by x. In like manner, in other cases of division, other progressions are produced.

But in most cases the relation of the terms of a Series is not constant, as it is in those that arise by division. Yet their relation often varies according to a certain law, which is sometimes obvious on inspection, and sometimes it is found by dividing the successive terms one by another, &c. Thus, in the Series

$1 + \frac{2}{3}x + \frac{8}{15}x^2 + \frac{16}{35}x^3 + \frac{128}{315}x^4$ &c, by dividing the 2d term by the 1st, the 3d by the 2d, the 4th by the 3d, and so on, the quotients will be

$$\frac{2}{3}x, \ \frac{4}{5}x, \ \frac{6}{7}x, \ \frac{8}{9}x, \ \&c;$$

and therefore the terms may be continued indefinitely by the successive multiplication by these fractions. Also in the following Series

$1 + \frac{1}{6}x + \frac{3}{40}x^2 + \frac{5}{128}x^3 + \frac{35}{1152}x^4$ &c, by dividing the adjacent terms successively by each other, the Series of quotients is

$$\frac{1}{6}x, \ \frac{9}{20}x, \ \frac{25}{42}x, \ \frac{49}{72}x, \ \&c, \text{ or}$$

$$\frac{1 \cdot 1}{2 \cdot 3}x, \ \frac{3 \cdot 3}{4 \cdot 5}x, \ \frac{5 \cdot 5}{6 \cdot 7}x, \ \frac{7 \cdot 7}{8 \cdot 9}x, \ \&c;$$

and therefore the terms of the Series may be continued by the multiplication of these fractions.

Another method of expressing the law of a Series, is one that defines the Series itself, by its *general term*, shewing the relation of the terms generally by their distances from the beginning, or by differential equations. To do this, Mr. Stirling conceives the terms of the Series to be placed as so many ordinates on a right line given by position, taking unity as the common interval between these ordinates. The terms of the Series he denotes by the initial letters of the alphabet, A, B, C, D, &c; A being the first, B the 2d, C the 3d, &c: and he denotes any term in general by the letter T, and the rest following it in order by T', T'', T''', T'''', &c; also the distance of the term T from any given term, or from any given intermediate point between two terms, he denotes by the indeterminate quantity z: so that the distances of the terms T', T'', T''', &c, from the said term or point, will be $z + 1$, $z + 2$, $z + 3$, &c; because the increment of the abscis is the common interval of the ordinates, or terms of the Series, applied to the abscis.

These things being premised, let this Series be proposed, viz,

$$1, \ \frac{1}{2}x, \ \frac{3}{8}x^2, \ \frac{5}{16}x^3, \ \frac{35}{128}x^4, \ \frac{63}{256}x^5, \ \&c;$$

in which it is found, by dividing the terms by each other, that the relations of the terms are,

$$B = \frac{1}{2}Ax, \ C = \frac{3}{4}Bx, \ D = \frac{5}{6}Cx, \ E = \frac{7}{8}Dx, \ \&c:$$

then the relation in general will be defined by the equation $T' = \frac{2z+1}{2z+2}Tx$ or $\frac{z+\frac{1}{2}}{z+1}Tx$, where z denotes the distance of T from the first term of the Series. For by substituting 0, 1, 2, 3, 4, &c, successively instead of z, the same relations will arise as in the proposed Series above. In like manner, if z be the distance of T from the 2d term of the Series, the equation will be $T' = \frac{2z+3}{2z+4}Tx$ or $\frac{z+\frac{3}{2}}{z+2}Tx$, as will appear by substituting the numbers -1, 0, 1, 2, 3, &c, successively for z. Or, if z denote the place or number of the term T in the Series, its successive values will be 1, 2, 3, 4, &c, and the equation or general term will be $T' = \frac{2z-1}{2z}Tx$.

It appears therefore, that innumerable differential equations may define one and the same Series, according to the different points from whence the origin of the abscis z is taken. And, on the contrary, the same equation defines innumerable different Series, by taking different successive values of z. For in the equation $T' = \frac{2z-1}{2z}Tx$, which defines the foregoing Series when

when 1, 2, 3, 4, &c are the fucceffive values of the abfciffes; if $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, &c, be fucceffively fubftituted for z, the relations of the terms arifing will be,

$B = \frac{2}{3} Ax$, $C = \frac{4}{5} Bx$, $D = \frac{6}{7} Cx$, &c, from whence will arife the Series

A, $\frac{2}{3} Ax$, $\frac{8}{15} Ax^2$, $\frac{16}{35} Ax^3$, $\frac{128}{315} Ax^4$, &c,

which is different from the former.

And thus the equation will always determine the Series from the given values of the abfcifs and of the firft term, when the equation includes but two terms of the Series, as in the laft example, where the firft term being given, all the reft will be given.

But when the equation includes three terms, then two muft be given; and three muft be given, when it includes four; and fo on. So, if there be propofed the

Series x, $\frac{1}{6}x^3$, $\frac{3}{40}x^5$, $\frac{5}{128}x^7$, $\frac{35}{1152}x^9$, &c,

where the relations of the terms are,

$B = \frac{1 \cdot 1}{2 \cdot 3} Ax^2$, $C = \frac{3 \cdot 3}{4 \cdot 5} Bx^2$, $D = \frac{5 \cdot 5}{6 \cdot 7} Cx^2$, &c.

the equation defining this Series will be

$$T' = \frac{\overline{2z-1} \cdot \overline{2z-1}}{2z \cdot \overline{2z+1}} Tx^2 = \frac{4zz - 4z + 1}{4zz + 2z} Tx^2,$$

where the fucceffive values of z are 1, 2, 3, 4, &c. See Stirling's Methodus Differentialis, in the introduction.

This may fuffice to give a notion of thefe differential equations, defining the nature of Series. But as to the application of thefe equations in interpolations, and finding the fums of Series, it would require a treatife to explain it. We muft therefore refer to that excellent one juft quoted, as alfo to De Moivre's Mifcellanea Analytica; and feveral curious papers by Euler in the Acta Petropolitana.

A Series often converges fo flowly, as to be of no ufe in practice. Thus, if it were required to find the fum of the Series

$$\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8} + \frac{1}{9 \cdot 10} \text{ \&c,}$$

which Lord Brouncker found for the quadrature of the hyperbola, true to 9 figures, by the mere addition of the terms of the Series; Mr. Stirling computes that it would be neceffary to add a thoufand millions of terms for that purpofe; for which the life of man would be too fhort. But by that gentleman's method, the fum of the Series may be found by a very moderate computation. See Method. Differ. pa. 26.

Series are of various kinds or defcriptions. So,

An *Afcending* SERIES, is one in which the powers of the indeterminate quantity increafe; as

$$1 + ax + bx^2 + cx^3 \text{ \&c.}$$ And a

Defcending SERIES, is one in which the powers decreafe, or elfe increafe in the denominators, which is the fame thing; as

$$1 + ax^{-1} + bx^{-2} + cx^{-3} \text{ \&c, or } 1 + \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} \text{ \&c.}$$

A Circular SERIES, which denotes a Series whofe fum depends on the quadrature of the circle. Such is the Series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c: See Demoivre Mifcel. Analyt. pa. 111, or my Menfur. pa. 119. Or the fum of the Series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$ &c, continued ad infinitum, according to Euler's difcovery.

Continued Fraction or *Series*, is a fraction of this kind, to infinity,

$$\cfrac{a}{b + \cfrac{c}{d + \cfrac{e}{f + \cfrac{g}{h \text{ \&c.}}}}}$$

The firft Series of this kind was given by Lord Brouncker, firft prefident of the Royal Society, for the quadrature of the circle, as related by Dr. Wallis, in his Algebra, pa. 317. His feries is

$$1 + \cfrac{1}{2 + \cfrac{9}{2 + \cfrac{25}{2 + \cfrac{49}{2 + \cfrac{81}{2 + \text{\&c,}}}}}}$$

which denotes the ratio of the fquare of the diameter of a circle to its area. Mr. Euler has treated on this kind of Series, in the Peterfburgh Commentaries, vol. 11, and in his Analyf. Infinit. vol. 1, pa. 295, where he fhews various ufes of it, and how to transform ordinary fractions and common Series into continued fractions. A common fraction is transformed into a continued one, after the manner of feeking the greateft common meafure of the numerator and denominator, by dividing the greater by the lefs, and the laft divifor always by the laft remainder. Thus to change $\frac{1461}{59}$ to a continued fraction.

```
59) 1461 (24
    118
    ---
    281
    236
    ---
    45) 59 (1
        45
        --
        14) 45 (3
            42
            --
            3) 14 (4
               12
               --
               2) 3 (1
                  2
                  -
                  1) 2 (2
                     2
                     -
                     0
```

Theref. $\frac{1461}{59} = 24 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{1 + \frac{1}{2}.}}}}$

Alfo $\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2} \text{ \&c.}}}}$

Converging

Converging SERIES, is a Series whose terms continually decrease, or the successive sums of whose terms approximate or converge always nearer to the ultimate sum of the whole Series. And, on the contrary, a

Diverging SERIES, is one whose terms continually increase, or that has the successive sums of its terms diverging, or going off always the farther, from the sum or value of the Series.

Determinate SERIES, is a Series whose terms proceed by the powers of a determinate quantity; as $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} +$ &c. If that determinate quantity be unity, the Series is said to be determined by unity. De Moivre, Miscel. Analyt. pa. 111. And an

Indeterminate SERIES is one whose terms proceed by the powers of an indeterminate quantity x; as $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$ &c; or sometimes also with indeterminate exponents, or indeterminate coefficients.

The *Form of a* SERIES, is used for that affection of an indeterminate Series, such as $ax^n + bx^{n+r} + cx^{n+2r} + dx^{n+3r}$ &c, which arises from the different values of the indices of x. Thus,

If $n = 1$, and $r = 1$, the Series will take the form
$$ax + bx^2 + cx^3 + dx^4 \text{ \&c.}$$
If $n = 1$, and $r = 2$, the form will be
$$ax + bx^3 + cx^5 + dx^7 \text{ \&c.}$$
If $n = \frac{1}{2}$, and $r = 1$, the form is
$$ax^{\frac{1}{2}} + bx^{\frac{3}{2}} + cx^{\frac{5}{2}} + dx^{\frac{7}{2}} \text{ \&c.}$$ And
If $n = 0$, and $r = -1$, the form will be
$$a + bx^{-1} + cx^{-2} + dx^{-3} \text{ \&c.}$$

When the value of a quantity cannot be found exactly, it is of use in algebra, as well as in common arithmetic, to seek an approximate value of that quantity, which may be useful in practice. Thus, in arithmetic, as the true value of the square root of 2 cannot be assigned, a decimal fraction is found to a sufficient degree of exactness in any particular case; which decimal fraction is in reality, no more than an infinite series of fractions converging or approximating to the true value of the root sought. For the expression $\sqrt{2} = 1{\cdot}414213$ &c, is equivalent to this $\sqrt{2} = 1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000}$ &c; or supposing $x = 10$, to this
$$\sqrt{2} = 1 + \frac{4}{x} + \frac{1}{x^2} + \frac{4}{x^3} + \frac{2}{x^4} \text{ \&c.}$$
$$\text{or} = 1 + 4x^{-1} + x^{-2} + 4x^{-3} + 2x^{-4} \text{ \&c,}$$
which last Series is a particular case of the more general indeterminate Series $ax^n + bx^{n+r} + cx^{n+2r}$ &c, viz, when $n = 0$, $r = -1$, and the coefficients $a = 1$, $b = 4$, $c = 1$, $d = 4$, &c.

But the application of the notion of approximations in numbers, to species, or to algebra, is not so obvious. Newton, with his usual sagacity, took the hint, and prosecuted it; by which were discovered general methods in the doctrine of infinite Series, which had before been treated only in a particular manner, though with great acuteness, by Wallis and a few others. See Newton's Method of Fluxions and Infinite Series, with Colson's Comment; as also the Analysis per Æquationes Numero Terminorum Infinitas, published by Jones in 1711, and since translated and explained by Stewart, together with Newton's Tract on Quadratures, in 1745. To these may be added Maclaurin's Algebra, part 2, chap. 10, pa. 244; and Cramer's Analyse des Lignes Courbes Algebraiques, chap. 7, pa. 148; and many other authors.

Among the various methods for determining the value of a quantity by a converging Series, that seems preferable to the rest, which consists in assuming an indeterminate Series as equal to the quantity whose value is sought, and afterwards determining the values of the terms of this assumed Series. For instance, suppose a logarithm were given, to find the natural number answering to it. Suppose the logarithm to be z, and the corresponding number sought $1 + x$: then by the nature of logarithms and fluxions, $\dot{z} = \frac{\dot{x}}{1 + x}$, or $\dot{z} + x\dot{z} = \dot{x}$. Now assume a Series for the value of the unknown quantity x, and substitute it and its fluxion instead of x and $\dot{x}$ in the last equation, then determine the assumed coefficients by comparing or equating the like terms of the equation. Thus,

assume $x = az + bz^2 + cz^3 + dz^4$ &c,
then $\dot{x} = a\dot{z} + 2bz\dot{z} + 3cz^2\dot{z} + 4dz^3\dot{z}$ &c;
and $\dot{x} = (\dot{z} + x\dot{z}) = \dot{z} + az\dot{z} + bz^2\dot{z} + cz^3\dot{z}$ &c;

hence, comparing the like terms of these two values of $\dot{x}$, there arises $a = 1$, $b = \frac{1}{2}$, $c = \frac{1}{6}$, $d = \frac{1}{24}$, &c; which values being substituted for a, b, c, &c, in the assumed Series $ax + bx^2 + cx^3$ &c, it gives
$$x = z + \frac{1}{2}z^2 + \frac{1}{6}z^3 + \frac{1}{24}z^4 + \frac{1}{120}z^5, \text{ \&c, or}$$
$$x = z + \frac{1}{1.2}z^2 + \frac{1}{1.2.3}z^3 + \frac{1}{1.2.3.4}z^4 + \frac{1}{1.2.3.4.5}z^5$$
&c; and consequently the number sought will be
$$1 + x = 1 + z + \frac{1}{1.2}z^2 + \frac{1}{1.2.3}z^3 \text{ \&c.}$$

But the indeterminate Series $az + bz^2 + cz^3$ &c, was here assumed arbitrarily, with regard to its exponents 1, 2, 3, &c, and will not succeed in all cases, because some quantities require other forms for the exponents. For instance, if from an arc given, it were required to find the tangent. Let $x =$ the tangent, and z the arc, the radius being $= 1$. Then, from the nature of the circle we shall have $\frac{\dot{x}}{1 + x^2} = \dot{z}$, or $\dot{x} = \dot{z} + x^2\dot{z}$. Now if, to find the value of x, we suppose $x = az + bz^2 + cz^3$ &c, and proceed as before, we shall find all the alternate coefficients b, d, f, &c, or those of the even powers of z, to be each $= 0$; and therefore the Series assumed is not of a proper form. But

But suppofing $x = az + bz^3 + cz^5 + dz^7$, &c, then we find $a = 1$, $b = \frac{1}{3}$, $c = \frac{2}{15}$, $d = \frac{17}{315}$, &c, and consequently $x = z + \frac{1}{3}z^3 + \frac{2}{15}z^5 + \frac{17}{315}z^7$ &c.

And other quantities require other forms of Series.

Now to find a proper indeterminate Series in all cafes, tentatively, would often be very laborious, and even impracticable. Mathematicians have therefore endeavoured to find out a general rule for this purpofe; though till lately the method has been but imperfectly underftood and delivered. Moft authors indeed have explained the manner of finding the coefficients a, b, c, d, &c, of the indeterminate Series $ax^n + bx^{n+r} + cx^{n+2r}$ &c, which is eafy enough; but the values of n and r, in which the chief difficulty lies, have been affigned by many in a manner as if they were felf-evident, or at leaft difcoverable by an eafy trial or two, as in the laft example.

As to the number n, Newton himfelf has fhewn the method of determining it, by his rule for finding the firft term of a converging Series, by the application of his parallelogram and ruler. For the particulars of this method, fee the authors above cited; fee alfo PARALLELOGRAM.

Taylor, in his Methodus Incrementorum, inveftigates the number r; but Stirling obferves that his rule fometimes fails. Lineæ Tert. Ordin. Newton. pa. 28. Mr. Stirling gives a correction of Taylor's rule, but fays he cannot affirm it to be univerfal, having only found it by chance. And again

Gravefande obferves, that though he thinks Stirling's rule never leads into an error, yet that it is not perfect. See Gravefande, De Determin. Form. Seriei Infinit. printed at the end of his Mathefeos Univerfalis Elementa. This learned profeffor has endeavoured to rectify the rule. But Cramer has fhewn that it is ftill defective in feveral refpects; and he himfelf, to avoid the inconveniences to which the methods of former authors are fubject, has had recourfe to the firft principles of the method of infinite Series, and has entered into a more exact and inftructive detail of the whole method, than is to be met with elfewhere; for which reafon, and many others, his treatife deferves to be particularly recommended to beginners.

But it is to be obferved, that in determining the value of a quantity by a converging Series, it is not always neceffary to have recourfe to an indeterminate Series: for it is often better to find it by divifion, or by extraction of roots. See Newton's Meth. of Flux. and Inf. Series, above cited. Thus, if it were required to find the arc of a circle from its given tangent, that is, to find the value of z in the given fluxional equation, $\dot{z} = \frac{\dot{x}}{1 + xx}$, by an infinite Series: dividing $\dot{x}$ by $1 + xx$, the quotient will be the Series $\dot{x} - x^2\dot{x} + x^4\dot{x} - x^6\dot{x}$ &c $= \dot{z}$; and taking the fluents of the terms, there refults $z = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7$ &c, which is the Series often ufed for the quadrature of the circle. If $x = 1$, or the tangent of 45°, then will $z = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ &c = the length of an arc of 45°, or $\frac{1}{8}$ of the circumference, to the radius 1, or $\frac{1}{4}$ of the circumference to the diameter 1. Confequently, if 1 be the diameter, then $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ &c will be the area of the circle, becaufe $\frac{1}{4}$ of the circumference multiplied by the diameter, gives the area of the circle. And this Series was firft given by Leibnitz and James Gregory.

See the form of the Series for the binomial theorem, determined, both as to the coefficients and exponents, in my Tracts, vol. 1, pa. 79.

Harmonical SERIES, the reciprocal of arithmeticals. See HARMONICAL.

Hyperbolic SERIES, is ufed for a Series whofe fum depends upon the quadrature of the hyperbola. Such is the Series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ &c. De Moivre's Mifcel. Analyt. pa. 111.

Interpolation of SERIES, the inferting of fome terms between others, &c. See INTERPOLATION.

Interfcendent SERIES. See INTERSCENDENT.

Mixt SERIES, one whofe fum depends partly on the quadrature of the circle, and partly on that of the hyperbola. De Moivre, Mifcel. Analyt. pa. 111.

Recurring SERIES, is ufed for a Series which is fo conftituted, that having taken at pleafure any number of its terms, each following term fhall be related to the fame number of preceding terms according to a conftant law of relation. Thus, in the following Series,

$$\begin{array}{cccccc} a & b & c & d & e & f \\ 1 + & 2x + & 3x^2 + & 10x^3 + & 34x^4 + & 97x^5 \text{ \&c,} \end{array}$$

in which the terms being refpectively reprefented by the letters a, b, c, &c, fet over them, we fhall have

$$\begin{aligned} d &= 3cx - 2bx^2 + 5ax^3, \\ e &= 3dx - 2cx^2 + 5bx^3, \\ f &= 3ex - 2dx^2 + 5cx^3, \\ &\text{\&c, \&c,} \end{aligned}$$

where it is evident that the law of relation between d and e, is the fame as between e and f, each being formed in the fame manner from the three terms which precede it in the Series.

The quantities $3x - 2x^2 + 5x^3$, taken together and connected by their proper figns, form what De Moivre calls the *index*, or the *fcale of relation*; though fometimes the bare coefficients $3 - 2 + 5$ are called the fcale of relation. And the fcale of relation fubtracted from unity, is called the *differential fcale*. On the fubject of Recurring Series, fee De Moivre's Mifcel. Analyt. pa. 27 and 72, and his Doctrine of Chances, 3d edit. pa. 220; alfo Euler's Analyf. Infinit. tom. 1, pa. 175.

Having given a recurring Series, with its fcale of relation, the fum of the whole infinite Series will alfo be given. For inftance, fuppofe a Series

$a +$

$a + bx + cx^2 + dx^3$ &c, where the relation between the coefficient of any term and the coefficients of any two preceding terms may be expreſſed by $f - g$; that is, $e = fd - gc$, and $d = fc - gb$, &c; then will the ſum of the Series, infinitely continued, be $\frac{a + (b - fa)\, x}{1 - fx + gx^2}$.

Thus, for example, aſſume 2 and 5 for the coefficients of the firſt two terms of a recurring Series; and ſuppoſe f and g to be reſpectively 2 and 1; then the recurring Series will be

$$2 + 5x + 8x^2 + 11x^3 + 14x^4 + 18x^5 \ \&c,$$

and its ſum $= \frac{2 + 5x - 4x}{1 - 2x + xx} = \frac{2 + x}{(1 - x)^2}$. For the proof of which divide $2 + x$ by $(1 - x)^2$, and there ariſes the ſaid Series $2 + 5x + 8x^2 + 11x^3$ &c. And ſimilar rules might be derived for more complex caſes.

De Moivre's general rule is this: 1. Take as many terms of the Series as there are parts in the ſcale of relation. 2. Subtract the ſcale of relation from unity, and the Remainder is the differential ſcale. 3. Multiply the terms taken in the Series by the differential ſcale, beginning at unity, and ſo proceeding orderly, remembering to leave out what would naturally be extended beyond the laſt of the terms taken. Then will the product be the numerator, and the differential ſcale will be the denominator of the fraction expreſſing the ſum required.

But it muſt here be obſerved, that when the ſum of a recurring Series extended to infinity, is found by De Moivre's rule, it ought to be ſuppoſed that the Series converges indefinitely, that is, that the terms may become leſs than any aſſigned quantity. For if the Series diverge, that is, if its terms continually increaſe, the rule does not give the true ſum. For the ſum in ſuch caſe is infinite, or greater than any given quantity, whereas the ſum exhibited by the rule, will often be finite. The rule therefore in this caſe only gives a fraction expreſſing the radix of the Series, by the expanſion of which the Series is produced. Thus $\frac{1}{(1 - x)^2}$ by expanſion becomes the recurring Series $1 + 2x + 3x^2$ &c, whoſe ſcale of relation is $2 - 1$, and its ſum by the rule will be $\frac{a + bx - fax}{1 - fx + gxx} = \frac{1 + 2x - 2x}{1 - 2x + xx} = \frac{1}{(1 - x)^2}$, the quantity from which the Series aroſe. But this quantity cannot in all caſes be deemed equal to the infinite Series $1 + 2x + 3x^2$ &c: for ſtop where you will, there will always want a ſupplement to make the product of the quotient by the diviſor equal to the dividend. Indeed when the Series converges infinitely, the ſupplement, diminiſhing continually, becomes leſs than any aſſigned quantity, or equal to nothing; but in a diverging Series, this ſupplement becomes infinitely great, and the Series deviates indefinitely from the truth. See Colſon's Comment on Newton's Method of Fluxions and Infinite Series, pa. 152; Stirling's Method. Differ. pa. 36; Bernoulli de Serieb. Infin. pa. 249; and Cramer's Analyſe des Lignes Courbes, pa. 174.

A recurring Series being given, the ſum of any finite number of the terms of that Series may be found. This is prob. 3, pa. 73, De Moivre's Miſcel. Analyt. and prob. 5, pa. 223 of his Doctrine of Chances. The ſolution is effected, by taking the difference between the ſums of two infinite Series, differing by the terms anſwering to the given number; viz, from the ſum of the whole infinite Series, commencing from the beginning, ſubtract the ſum of another infinite number of terms of the ſame Series, commencing after ſo many of the firſt terms whoſe ſum is required; and the difference will evidently be the ſum of that number of terms of the Series. For example, to find the ſum of n terms of the infinite geometrical Series $a + ax + ax^2 + ax^3$ &c. Here are two infinite Series; the one beginning with a, and the other with ax^n, which is the next term after the firſt n terms of the original Series. By the rule, the ſum of the firſt infinite progreſſion will b $\frac{a}{1 - x}$, and the ſum of the ſecond $\frac{ax^n}{1 - x}$; the difference of which is $\frac{a - ax^n}{1 - x}$, which is therefore the ſum of the firſt n terms of the Series. This quantity $\frac{a - ax^n}{1 - x}$ is equal to $\frac{ax^n - a}{x - 1}$ which laſt expreſſion, putting $ax^{n-1} = l$, will be equivalent to this, $\frac{lx - a}{x - 1}$, which is the common rule for finding the ſum of any geometric progreſſion, having given the firſt term a, the laſt term l, and the ratio x. See Miſcel. Analyt. pa. 167, 168.

In a recurring Series, any term may be obtained whoſe place is aſſigned. For after having taken ſo many terms of the Series as there are terms in the ſcale of relation, the Series may be protracted till it reach the place aſſigned. But when that place is very diſtant from the beginning of the Series, the continuing the terms is very laborious; and therefore other methods have been contrived. See Miſcel. Analyt. pa. 33; and Doctrine of Chances, pa. 224.

Theſe queſtions have been reſolved in many caſes, beſides thoſe of recurring Series. But as there is no univerſal method for the quadrature of curves, neither is there one for the ſummation of Series; indeed there is a great analogy between theſe things, and ſimilar difficulties ariſing in both. See the authors above cited.

The inveſtigation of Daniel Bernoulli's method for finding the roots of algebraic equations, which is inſerted in the Peterſburgh Acts, tom. 3, pa. 92, depends upon the doctrine of recurring Series. See Euler's Analyſis Infinitorum, tom. 1, pa. 276.

Reverſion of SERIES. See REVERSION *of Series.*

Summable SERIES, is one whoſe ſum can be accurately found. Such is the Series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c, the ſum of which is ſaid to be unity, or, to ſpeak more accurately, the limit of its ſum is unity or 1.

An indefinite number of ſummable infinite Series may

may be assigned: such are, for instance, all infinite recurring converging Series, and many others, for which, consult De Moivre, Bernoulli, Stirling, Euler, and Maclaurin; viz, Miscel. Analyt. pa. 110; De Serieb. Infinit. passim; Method. Different. pa. 34; Acta Petrop. passim; Fluxions, art. 350.

The obtaining the sums of infinite Serieses of fractions has been one of the principal objects of the modern method of computation; and these sums may often be found, and often not. Thus the sums of the two following Series of geometrical progressionals are easily found to be 1 and $\frac{1}{2}$,

viz, $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ &c,

and $\frac{1}{2} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$ &c.

But the Serieses of fractions that occur in the solution of problems, can seldom be reduced to geometric progressions; nor can any general rule, in cases so infinitely various, be given. The art here, as in most other cases, is only to be acquired by examples, and by a careful observation of the arts used by great authors in the investigation of such Series of fractions as they have considered. And the general methods of infinite Series, which have been carried so far by De Moivre, Stirling, Euler, &c, are often found necessary to determine the sum of a very simple Series of fractions. See the quotations above.

The sum of a Series of fractions, though decreasing continually, is not always finite. This is the case of the Series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ &c, which is the harmonic Series, consisting of the reciprocals of arithmeticals, the sum of which exceeds any given number whatever; and this is shewn from the analogy between this progression and the space comprehended by the common hyperbola and its asymptote; though the same may be shewn also from the nature of progressions. See James Bernoulli, de Seriebus Infin. But, what is curious, the square of it is finite, for if the same terms of the harmonic Series, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$ &c, be squared, forming the Series $\frac{1}{1} + \frac{1}{4} + \frac{1}{9}$ &c, being the reciprocals of the squares of the natural Series of numbers; the sum of this Series of fractions will not only be limited, but it is remarkable that this sum will be precisely equal to the 6th part of the number which expresses the ratio of the square of the circumference of a circle to the square of its diameter. That is, if c denote 3·14159 &c, the ratio of the circumference to the diameter, then is $\frac{1}{6}c^2 = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$ &c. This property was first discovered by Euler; and his investigation may be seen in the Acta Petrop. vol. 7. And Maclaurin has since observed, that this may easily be deduced from his Fluxions, art. 822. Philos. Trans. numb. 469.

It would require a whole treatise to enumerate the various kinds of Series of fractions which may or may not be summed. Sometimes the sum cannot be assigned, either because it is infinite, as in the harmonic Series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ &c, or although its sum be finite (as in the Series $\frac{1}{1} + \frac{1}{4} + \frac{1}{9}$ &c), yet its sum cannot be assigned in finite terms, or by the quadrature of the circle or hyperbola, which was the case of this Series before Euler's discovery; but yet the sum of any given number of the terms of the Series may be expeditiously found, and the whole sum may be assigned by approximation, independent of the circle. See Stirling's Method. Different. and De Moivre's Miscel. Analyt.

Besides the Serieses of fractions, the sums of which converge to a certain quantity, there sometimes occur others, which converge by a continued multiplication. Of this kind is the Series found by Wallis, for the quadrature of the circle, which he expresses thus,

$$\square = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 9 \times 9 \times \&c}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times 8 \times 10 \times \&c},$$

where the character $\square$ denotes the ratio of the square of the diameter to the area of the circle. Hence the denominator of this fraction, is to its numerator, both infinitely continued, as the circle is to the square of the diameter. It may farther be observed that this Series is equivalent to

$\frac{9}{8} \times \frac{25}{24} \times \frac{49}{48} \times$ &c, or to $\frac{3^2}{3^2-1} \times \frac{5^2}{5^2-1} \times \frac{7^2}{7^2-1} \times$ &c, that is, the product of the squares of all the odd numbers 3, 5, 7, 9, &c, is to the product of the same squares severally diminished by unity, as the square of the diameter is to the area of the circle. See Arithmet. Infinit. prop. 191, Oper. vol. 1, pa. 469. Id. Oper. vol. 2, pa. 819. And these products of fractions, and the like quantities arising from the continued multiplication of certain factors, have been particularly considered by Euler, in his Analysis Infinit. vol. 1, chap. 15, pa. 221.

For an easy and general method of summing all alternate Series, such as $a - b + c - d$ &c, see my Tracts, vol. 1, pa. 11; and in the same vol. may be seen many other curious tracts on infinite Series.

Summation of Infinite SERIES, is the finding the value of them, or the radix from which they may be raised. For which, consult all the authors upon this science.

To find an infinite Series by extracting of roots; and to find an infinite Series by a presupposed Series; see QUADRATURE *of the Circle*.

To extract the roots of an infinite Series, see EXTRACTION *of Roots*.

To raise an infinite Series to any power, see INVOLUTION, and POWER.

Transcendental SERIES. See TRANSCENDENTAL.

There are many other important writings upon the subject of Infinite Series, besides those above quoted. A very good elementary tract on this science is that of James Bernoulli, intituled, *Tractatus de Seriebus Infinitis*,

lis, and annexed to his *Ars Conjectandi*, published in 4to, 1713.

SERPENS, in Astronomy, a constellation in the northern hemisphere, being one of the 48 old constellations mentioned by all the Ancients, and is called more particularly *Serpens Ophiuchi*, being grasped in the hands of the constellation Ophiuchus. The Greeks, in their fables, have ascribed it sometimes to one of Triptolemus's dragons, killed by Carnabos; and sometimes to the serpent of the river Segaris, destroyed by Hercules. This is by some supposed to be the same as the author of the book of Job calls the *Crooked Serpent*; but this expression more probably meant the constellation Draco, near the north pole.

The stars in the constellation Serpens, in Ptolomy's catalogue are 18, in Tycho's 13, in Hevelius's 22, and in the Britannic catalogue 64.

SERPENTARIUS, a constellation of the northern hemisphere, being one of the 48 old constellations mentioned by all the Ancients. It is called also Ophiuchus, and anciently Æsculapius. It is in the figure of a man grasping the serpent.

The Greeks had different fables about this, and other constellations, because they were ignorant of the true meaning of them. Some of them say, it represents Carnabos, who killed one of the dragons of Triptolemus. Others say, it was Hercules, killing the serpent at the river Segaris. And others again say, it represents the celebrated physician Æsculapius, to denote his skill in medicine to cure the bite of the serpent.

The stars in the constellation Serpentarius, in Ptolomy's catalogue are 29, in Tycho's 15, in Hevelius's 40, and in the Britannic catalogue they are 74.

SERPENTINE *Line*, the same with spiral.

SESQUI, an expression of a certain ratio, viz, the second ratio of inequality, called also *superparticular* ratio; being that in which the greater term contains the less once, and some certain part over; as 3 to 2, where the first term contains the second once, and unity over, which is a quota part of 2. Now if this part remaining be just half the less term, the ratio is called *sesquialtera*; if the remaining part be a 3d part of the less term, as 4 to 3, the ratio is called *sesquitertia*, or *sesquiterza*; if a 4th part, as 5 to 4, the ratio is called *sesquiquarta*; and so on continually, still adding to Sesqui the ordinal number of the smaller term.

In English we sometimes say, *sesquialteral*, or *sesquialterate*, *sesquithird*, *sesquifourth*, &c.

As to the kinds of triples expressed by the particle *sesqui*, they are these:

SESQUIALTERATE, *the greater perfect*, which is a triple, where the breve is three measures, or semibreves.

Sesquialterate, *greater imperfect*, which is where the breve, when pointed, contains three measures, and without any point, two.

Sesquialterate, *less imperfect*, a triple, where the semibreve with a point contains three measures, and two without.

Sesquialterate, in Arithmetic and Geometry, is a ratio between two numbers, or lines, &c, where the greater is equal to once and a half of the less. Thus 6 and 9 are in a Sesquialterate ratio, as also 20 and 30.

SESQUIDITONE, in Music, a concord resulting from the sounds of two strings whose vibrations, in equal times, are to each other in the ratio of 5 to 6.

SESQUIDUPLICATE *Ratio*, is that in which the greater term contains the less, twice and a half; as the ratio of 15 to 6, or 50 to 20.

SESQUIQUADRATE, an aspect or position of the planets, when they are distant by 4 signs and a half, or 135 degrees.

SESQUIQUINTILE, is an aspect of the planets when they are distant $\frac{1}{5}$ of the circle and a half, or 108 degrees.

SESQUITERTIONAL *Proportion*, is that in which the greater term contains the less once and one third; as 4 to 3, or 12 to 9.

SETTING, in Astronomy, the withdrawing of a star or planet, or its sinking below the horizon.

Astronomers and poets count three different kinds of Setting of the stars, viz, Achronical, Cosmical, and Heliacal. See these terms respectively.

Setting, in Navigation, Surveying, &c, denotes the observing the bearing or situation of any distant object by the compass, &c, to discover the angle it makes with the nearest meridian, or with some other line. See Bearing.

Thus, to *set the land*, or *the sun*, by the compass, is to observe how the land bears on any point of the compass, or on what point of the compass the sun is. Also, when two ships come in sight of each other, to mark on what point the chace bears, is termed *Setting the chace by the compass*.

Setting also denotes the direction of the wind, current, or sea, particularly of the two latter.

SEVEN Stars, a common denomination given to the cluster of stars in the neck of the sign Taurus, the bull, properly called the pleiades. They are so called from their number Seven which appear to the naked eye, though some eyes can discover only 6 of them; but by the help of telescopes there appears to be a great multitude of them.

SEVENTH, *Septima*, an interval in Music, called by the Greeks *heptachordon*.

SEXAGENARY, something relating to the number 60.

Sexagenary *Arithmetic*. See Sexagesimal.

Sexagenary *Tables*, are tables of proportional parts, shewing the product of two Sexagenaries that are to be multiplied, or the quotient of two that are to be divided.

SEXAGESIMA, the eighth Sunday before Easter; being so called because near 60 days before it.

SEXAGESIMAL or Sexagenary *Arithmetic*, a method of computation proceeding by 60ths. Such is that used in the division of a degree into 60 minutes, of the minute into 60 seconds, of the second into 60 thirds, &c.

SEXAGESIMALS, or Sexagesimal *Fractions*, are fractions whose denominators proceed in a sexagecuple ratio; that is, a prime, or the first minute $= \frac{1}{60}$, a second $= \frac{1}{3600}$, a third $= \frac{1}{216000}$.

Anciently there were no other than Sexagesimals used in astronomical operations, for which reason they are sometimes called *astronomical fractions*, and they are still retained in many cases, as in the divisions of time and of a circle; but decimal arithmetic is now much

 used

used in the calculations. Sexagesimals were probably first used for the divisions of a circle, 360, or 6 times 60 making up the whole circumference, on account that 360 days made up the year of the Ancients, in which time the sun was supposed to complete his course in the circle of the ecliptic.

In these fractions, the denominator being always 60, or a multiple of it, it is usually omitted, and the numerator only written down: thus, $3^\circ\ 45'\ 24''\ 40'''$ &c, is to be read, 3 degrees, 45 minutes, 24 seconds, 40 thirds, &c.

SEXANGLE, in Geometry, a figure having 6 angles, and consequently 6 sides also.

SEXTANS, a sixth part of certain things.

The Romans divided their *as*, which was a pound of brass, into 12 ounces, called *uncia*, from *unum*; and the quantity of 2 ounces was called *sextans*, as being the 6th part of the pound.

SEXTANS was also a measure, which contained 2 ounces of liquor, or 2 cyathi.

SEXTANS, the Sextant, in Astronomy, a new constellation, placed across the equator, but on the south side of the ecliptic, and by Hevelius made up of some unformed stars, or such as were not included in any of the 48 old constellations. In Hevelius's catalogue it contains 11 stars, but in the Britannic catalogue 41.

SEXTANT, denotes the 6th part of a circle, or an arch containing 60 degrees.

SEXTANT is more particularly used for an astronomical instrument. It is made like a quadrant, excepting that its limb only contains 60 degrees. Its use and application are the same with those of the QUADRANT; which see.

SEXTARIUS, an ancient Roman measure, containing 2 cotylæ, or 2 heminæ.

SEXTILE, the aspect or position of two planets, when they are distant the 6th part of the circle, viz, 2 signs or 60 degrees; and it is marked thus ✱.

SEXTUPLE, denotes 6 fold in general. But in music it denotes a mixed sort of triple time, which is beaten in double time.

SHADOW, *Shade*, in Optics, a certain space deprived of light, or where the light is weakened by the interposition of some opaque body before the luminary.

The doctrine of Shadows makes a considerable article in optics, astronomy, and geography; and is the general foundation of dialling.

As nothing is seen but by light, a mere shadow is invisible; and therefore when we say we see a shadow, we mean, partly that we see bodies placed in the Shadow, and illuminated by light reflected from collateral bodies, and partly that we see the confines of the light.

When the opaque body, that projects the Shadow, is perpendicular to the horizon, and the plane it is projected on is horizontal, the Shadow is called a *right* one: such as the Shadows of men, trees, buildings, mountains, &c. But when the body is placed parallel to the horizon, it is called a *versed Shadow*; as the arms of a man when stretched out, &c.

Laws of the Projection of Shadows.

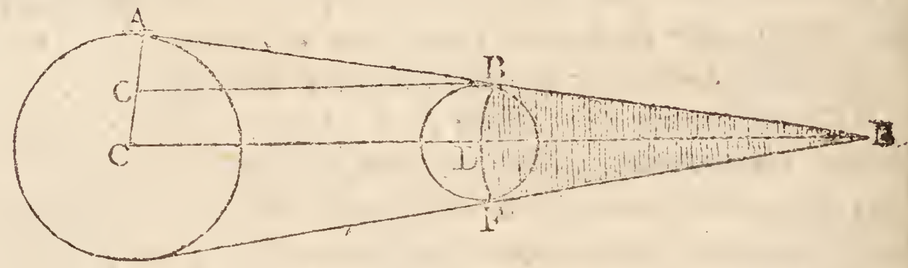
1. Every opaque body projects a Shadow in the same direction with the rays of light; that is, towards the part opposite to the light. Hence, as either the luminary or the body changes place, the Shadow likewise changes its place.

2. Every opaque body projects as many Shadows as there are luminaries to enlighten it.

3. As the light of the luminary is more intense, the shadow is the deeper. Hence, the intensity of the Shadow is measured by the degrees of light that space is deprived of. In reality, the Shadow itself is not deeper; but it appears so, because the surrounding bodies are more intensely illuminated.

4. When the luminous body and opaque one are equal, the Shadow is always of the same breadth with the opaque body. But when the luminous body is the larger, the Shadow grows always less and less, the farther from the body. And when the luminous body is the smaller of the two, the Shadow increases always the wider, the farther from the body. Hence, the Shadow of an opaque globe is, in the first case a cylinder, in the second case it is a cone verging to a point, and in the third case a truncated cone that enlarges still the more the farther from the body. Also, in all these cases, a transverse Section of the Shadow, by a plane, is a circle, respectively, in the three cases, equal, less, or greater than a great circle of the globe.

5. To find the length of the Shadow, or the axis of the shady cone, projected by a sphere, when it is illuminated by a larger one; the diameters and distance of the two spheres being known. Let C and D be the



centres of the two spheres, CA the semidiameter of the larger, and DB that of the smaller, both perpendicular to the side of the conical Shadow BEF, whose axis is DE, continued to C; and draw BG parallel to the same axis. Then, the two triangles AGB and BDE being similar, it will be AG : GB or CD : : BD : DE, that is, as the difference of the semidiameters is to the distance of the centres, so is the semidiameter of the opaque sphere to the axis of the Shadow, or the distance of its vertex from the said opaque sphere.

Ex. gr. If BD = 1 be the semidiameter of the earth, and AC = 101 the mean semidiameter of the sun, also their distance CD or GB = 24000; then as 100 : 24000 : : 1 : 240 = DE, which is the mean height of the earth's Shadow, in semidiameters of the base.

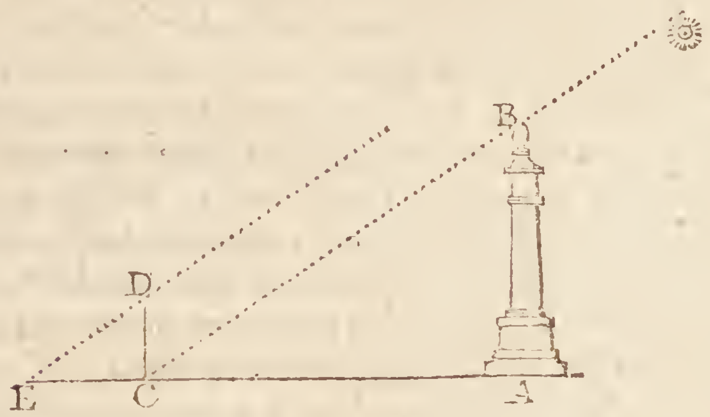
6. To find the length of the shadow AC projected by an opaque body AB; having given the altitude of the luminary, for ex. of the sun, above the horizon, viz, the angle C, and the height of the object AB. Here the proportion is, as tang. ∠ C : radius : : AB : AC.

Or, if the length of the Shadow AC be given, to find the height AB, it will be,

as radius : tang. ∠ C : : AC : AB.

Or,

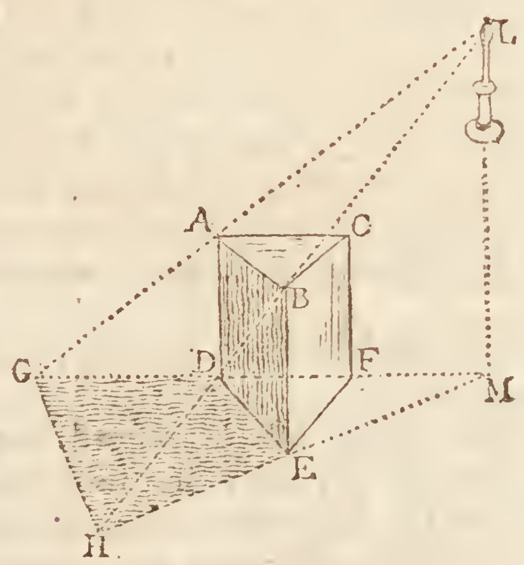
Or, if the length of the Shadow AC, and of the object AB, be given, to find the sun's altitude above the horizon, or the angle at C. It will be, as AC : AB : : radius : tang. ∠ C fought.



7. To meafure the height of any object, ex. gr. a tower AB, by means of its fhadow projected on an horizontal plane.—At the extremity of the fhadow, at C, erect a ftick or pole CD, and meafure the length of its fhadow CE ; alfo meafure the length of the Shadow AC of the tower. Then, by fimilar triangles, it will be, as EC : CD : : CA : AB. So if EC = 10 feet, CD = 6 feet, and CA = 95 feet; then as 10 : 6 : : 95 : 57 feet = AB, the height of the tower fought.

SHADOW, in Geography. The inhabitants of the earth are divided, with refpect to their fhadows, into ASCII, AMPHISCII, HETEROSCII, and PERISCII. See thefe terms in their places.

SHADOW, in Perfpective, is of great ufe in this art. —Having given the appearance of an opaque body, and a luminous one, whofe rays diverge, as a candle, or lamp, &c ; to find the juft appearance of the Shadow, according to the laws of perfpective. The method is this : From the luminous body, which is here confidered as a point, let fall a perpendicular to the perfpective plane or table ; and from the feveral angles, or raifed points of the body, let fall perpendiculars to the fame plane ; then connect the points on which thefe latter perpendiculars fall, by right lines, with the point on which the firft falls ; continuing thefe lines beyond the fide oppofite to the luminary, till they meet with as many other lines drawn from the centre of the luminary through the faid angles or raifed points ; fo fhall the points of interfection of thefe lines be the extremes or bounds of the Shadow.



For Example, to project the appearance of the Shadow of a prifm ABCDEF, fcenographically delineated. Here M is the place of the perpendicular of the light L, and D, E, F thofe of the raifed points A, B, C, of the prifm ; therefore, draw MEH, MDG, &c, and LBH, LAG, &c, which will give DEGH &c for the appearance of the Shadow.

As for thofe Shadows that are intercepted by other objects, it may be obferved, that when the Shadow of a line falls upon any object, it muft neceffarily take the form of that object. If it fall upon another plane, it will be a right line ; if upon a globe, it will be circular ; and if upon a cylinder or cone, it will be circular, or oval, &c. If the body intercepting it be a plane, whatever be the fituation of it, the fhadow falling upon it might be found by producing that plane till it intercepted the perpendicular let fall upon it from the luminous body ; for then a line drawn from that point would determine the Shadow, juft as if no other plane had been concerned. But the appearance of all thefe Shadows may be drawn with lefs trouble, by firft drawing it through thefe intercepted objects, as if they had not been in the way, and then making the Shadow to afcend perpendicularly up every perpendicular plane, and obliquely on thofe that are fituated obliquely, in the manner defcribed by Dr. Prieftley, in his Perfpective, pa. 73 &c.

Here we may obferve in general, that fince the Shadows of all objects which are caft upon the ground, will vanifh into the horizontal line ; fo, for the fame reafon, the vanifhing points of all Shadows, which are caft upon any inclined or other plane, will be fomewhere in the vanifhing line of that plane.

When objects are not fuppofed to be viewed by the light of the fun, or of a candle, &c, but only in the light of a cloudy day, or in a room into which the fun does not fhine, there is no fenfible Shadow of the upper part of the object, and the lower part only makes the neighbouring parts of the ground, on which it ftands, a little darker than the reft. This imperfect obfcure kind of Shadow is eafily made, being nothing more than a fhade on the ground, oppofite to the fide on which the light comes ; and it may be continued to a greater or lefs diftance, according to the fuppofed brightnefs of the light by which it is made. It is in this manner (in order to fave trouble, and fometimes to prevent confufion) that the Shadows in moft drawings are made. On this fubject, fee Prieftley's Perfpect. above quoted ; alfo Kirby's Perfp. book 2, ch. 4.

SHAFT *of a Column*, in Building, is the body of it ; thus called from its ftraightnefs : but by architects more commonly the Fuft.

SHAFT is alfo ufed for the fpire of a church fteeple ; and for the fhank or tunnel of a chimney.

SHARP (ABRAHAM), an eminent mathematician, mechanift, and aftronomer, was defcended from an ancient family at Little-Horton, near Bradford, in the Weft Riding of Yorkfhire, where he was born about the year 1651. At a proper age he was put apprentice to a merchant at Manchefter ; but his genius led him fo ftrongly to the ftudy of mathematics, both theoretical and practical, that he foon became uneafy in that fituation of life. By the mutual confent therefore of his mafter and himfelf, though not altogether with that of his father, he quitted the bufinefs of a merchant. Upon this he removed to Liverpool, where he

give

gave himself up wholly to the study of mathematics, astronomy, &c; and where, for a subsistance, he opened a school, and taught writing and accounts, &c.

He had not been long at Liverpool when he accidentally fell in company with a merchant or tradesman visiting that town from London, in whose house it seems the astronomer Mr. Flamsteed then lodged. With the view therefore of becoming acquainted with this eminent man, Mr. Sharp engaged himself with the merchant as a book-keeper. In consequence he soon contracted an intimate acquaintance and friendship with Mr. Flamsteed, by whose interest and recommendation he obtained a more profitable employment in the dock-yard at Chatham; where he continued till his friend and patron, knowing his great merit in astronomy and mechanics, called him to his assistance, in contriving, adapting, and fitting up the astronomical apparatus in the Royal Observatory at Greenwich, which had been lately built, namely about the year 1676; Mr. Flamsteed being then 30 years of age, and Mr. Sharp 25.

In this situation he continued to assist Mr. Flamsteed in making observations (with the mural arch, of 80 inches radius, and 140 degrees on the limb, contrived and graduated by Mr. Sharp) on the meridional zenith distances of the fixed stars, sun, moon, and planets, with the times of their transits over the meridian; also the diameters of the sun and moon, and their eclipses, with those of Jupiter's satellites, the variation of the compass, &c. He assisted him also in making a catalogue of near 3000 fixed stars, as to their longitudes and magnitudes, their right ascensions and polar distances, with the variations of the same while they change their longitude by one degree.

But from the fatigue of continually observing the stars at night, in a cold thin air, joined to a weakly constitution, he was reduced to a bad state of health; for the recovery of which he desired leave to retire to his house at Horton; where, as soon as he found himself on the recovery, he began to fit up an observatory of his own; having first made an elegant and curious engine for turning all kinds of work in wood or brass, with a maundril for turning irregular figures, as ovals, roses, wreathed pillars, &c. Beside these, he made himself most of the tools used by joiners, clockmakers, opticians, mathematical instrument-makers, &c. The limbs or arcs of his large equatorial instrument, sextant, quadrant, &c, he graduated with the nicest accuracy, by diagonal divisions into degrees and minutes. The telescopes he made use of were all of his own making, and the lenses ground, figured, and adjusted with his own hands.

It was at this time that he assisted Mr. Flamsteed in calculating most of the tables in the second volume of his *Historia Cœlestis*, as appears by their letters, to be seen in the hands of Mr. Sharp's friends at Horton. Likewise the curious drawings of the charts of all the constellations visible in our hemisphere, with the still more excellent drawings of the planispheres both of the northern and southern constellations. And though these drawings of the constellations were sent to be engraved at Amsterdam by a masterly hand, yet the originals far exceeded the engravings in point of beauty and elegance: these were published by Mr. Flamsteed, and both copies may be seen at Horton.

5

The mathematician meets with something extraordinary in Sharp's elaborate treatise of *Geometry Improved* (in 4to 1717, signed A. S. Philomath.), 1st, by a large and accurate table of segments of circles, its construction and various uses in the solution of several difficult problems, with compendious tables for finding a true proportional part; and their use in these or any other tables exemplified in making logarithms, or their natural numbers, to 60 places of figures; there being a table of them for all primes to 1100, true to 61 figures. 2d, His concise treatise of Polyedra, or solid bodies of many bases, both the regular ones and others: to which are added twelve new ones, with various methods of forming them, and their exact dimensions in surds, or species, and in numbers: illustrated with a variety of copper-plates, neatly engraved by his own hands. Also the models of these polyedra he cut out in box-wood with amazing neatness and accuracy. Indeed few or none of the mathematical instrument-makers could exceed him in exactly graduating or neatly engraving any mathematical or astronomical instrument, as may be seen in the equatorial instrument above mentioned, or in his sextant, quadrants and dials of various sorts; also in a curious armillary sphere, which, beside the common properties, has moveable circles &c, for exhibiting and resolving all spherical triangles; also his double sector, with many other instruments, all contrived, graduated and finished, in a most elegant manner, by himself. In short, he possessed at once a remarkably clear head for contriving, and an extraordinary hand for executing, any thing, not only in mechanics, but likewise in drawing, writing, and making the most exact and beautiful schemes or figures in all his calculations and geometrical constructions.

The quadrature of the circle was undertaken by him for his own private amusement in the year 1699, deduced from two different series, by which the truth of it was proved to 72 places of figures; as may be seen in the introduction to Sherwin's tables of logarithms; that is, if the diameter of a circle be 1, the circumference will be found equal to 3·14159265358979323846264338327950288419716939937510582097494459230781640 5, &c. In the same book of Sherwin's may also be seen his ingenious improvements on the making of logarithms, and the constructing of the natural sines, tangents, and secants.

He also calculated the natural and logarithmic sines, tangents, and secants, to every second in the first minute of the quadrant: the laborious investigation of which may probably be seen in the archives of the Royal Society, as they were presented to Mr. Patrick Murdoch for that purpose; exhibiting his very neat and accurate manner of writing and arranging his figures, not to be equalled perhaps by the best penman now living.

The late ingenious Mr. Smeaton says (Philos. Trans. an. 1786, pa. 5, &c):

" In the year 1689, Mr. Flamsteed completed his mural arc at Greenwich; and, in the Prolegomena to his Historia Cœlestis, he makes an ample acknowledgment of the particular assistance, care, and industry of Mr. Abraham Sharp; whom, in the month of August 1688, he brought into the observatory, as his amanuensis; and being as Mr. Flamsteed tells us, not only

only a very ſkilful mathematician, but exceedingly expert in mechanical operations, he was principally employed in the conſtruction of the mural arc; which in the compaſs of 14 months he finiſhed, ſo greatly to the ſatisfaction of Mr. Flamſteed, that he ſpeaks of him in the higheſt terms of praiſe.

"This celebrated inſtrument, of which he alſo gives the figure at the end of the Prolegomena, was of the radius of 6 feet $7\frac{1}{2}$ inches; and, in like manner as the ſextant, was furniſhed both with ſcrew and diagonal diviſions, all performed by the accurate hand of Mr. Sharp. But yet, whoever compares the different parts of the table for converſion of the revolutions and parts of the ſcrew belonging to the mural arc into degrees, minutes, and ſeconds, with each other, at the ſame diſtance from the zenith on different ſides; and with their halves, quarters, &c, will find as notable a diſagreement of the ſcrew-work from the hand diviſions, as had appeared before in the work of Mr. Tompion: and hence we may conclude, that the method of Dr. Hook, being executed by two ſuch maſterly hands as Tompion and Sharp, and found defective, is in reality not to be depended upon in nice matters.

"From the account of Mr. Flamſteed it appears alſo, that Mr. Sharp obtained the zenith point of the inſtrument, or line of collimation, by obſervation of the zenith ſtars, with the face of the inſtrument on the eaſt and on the weſt ſide of the wall: and that having made the index ſtronger (to prevent flexure) than that of the ſextant, and thereby heavier, he contrived, by means of pulleys and balancing weights, to relieve the hand that was to move it from a great part of its gravity. Mr. Sharp continued in ſtrict correſpondence with Mr. Flamſteed as long as he lived, as appeared by letters of Mr. Flamſteed's found after Mr. Sharp's death; many of which I have ſeen.

"I have been the more particular relating to Mr. Sharp, in the buſineſs of conſtructing this mural arc; not only becauſe we may ſuppoſe it the firſt good and valid inſtrument of the kind, but becauſe I look upon Mr. Sharp to have been the firſt perſon that cut accurate and delicate diviſions upon aſtronomical inſtruments; of which, independent of Mr. Flamſteed's teſtimony, there ſtill remain conſiderable proofs: for, after leaving Mr. Flamſteed, and quitting the department above-mentioned, he retired into Yorkſhire, to the village of Little Horton, near Bradford, where he ended his days about the year 1743 (ſhould be, in 1742); and where I have ſeen not only a large and very fine collection of mechanical tools, the principal ones being made with his own hands, but alſo a great variety of ſcales and inſtruments made with them, both in wood and braſs, the diviſions of which were ſo exquiſite, as would not diſcredit the firſt artiſts of the preſent times: and I believe there is now remaining a quadrant, of 4 or 5 feet radius, framed of wood, but the limb covered with a braſs plate; the ſubdiviſions being done by diagonals, the lines of which are as finely cut as thoſe upon the quadrants at Greenwich. The delicacy of Mr. Sharp's hand will indeed permanently appear from the copper-plates in a quarto book, publiſhed in the year 1718, intituled *Geometry Improved* by A. Sharp, Philomath." (or rather 1717, by A. S. Philomath.) "whereof not only the geometrical lines upon the plates, but the whole of the engraving of letters and figures, were done by himſelf, as I was told by a perſon in the mathematical line, who very frequently attended Mr. Sharp in the latter part of his life. I therefore look upon Mr. Sharp as the firſt perſon that brought the affair of hand diviſion to any degree of perfection."

Mr. Sharp kept up a correſpondence by letters with moſt of the eminent mathematicians and aſtronomers of his time, as Mr. Flamſteed, Sir Iſaac Newton, Dr. Halley, Dr. Wallis, Mr. Hodgſon, Mr. Sherwin, &c, the anſwers to which letters are all written upon the backs, or empty ſpaces, of the letters he received, in a ſhort-hand of his own contrivance. From a great variety of letters (of which a large cheſt full remain with his friends) from theſe and many other celebrated mathematicians, it is evident that Mr. Sharp ſpared neither pains nor time to promote real ſcience. Indeed, being one of the moſt accurate and indefatigable computers that ever exiſted, he was for many years the common reſource for Mr. Flamſteed, Sir Jonas Moore, Dr. Halley, and others, in all ſorts of troubleſome and delicate calculations.

Mr. Sharp continued all his life a bachelor, and ſpent his time as recluſe as a hermit. He was of a middle ſtature, but very thin, being of a weakly conſtitution; he was remarkably feeble the laſt three or four years before he died, which was on the 18th of July 1742, in the 91ſt year of his age.

In his retirement at Little Horton, he employed four or five rooms or apartments in his houſe for different purpoſes, into which none of his family could poſſibly enter at any time without his permiſſion. He was ſeldom viſited by any perſons, except two gentlemen of Bradford, the one a mathematician, and the other an ingenious apothecary: theſe were admitted, when he choſe to be ſeen by them, by the ſignal of rubbing a ſtone againſt a certain part of the outſide wall of the houſe. He duly attended the diſſenting chapel at Bradford, of which he was a member, every Sunday; at which time he took care to be provided with plenty of halfpence, which he very charitably ſuffered to be taken ſingly out of his hand, held behind him during his walk to the chapel, by a number of poor people who followed him, without his ever looking back, or aſking a ſingle queſtion.

Mr. Sharp was very irregular as to his meals, and remarkably ſparing in his diet, which he frequently took in the following manner. A little ſquare hole, ſomething like a window, made a communication between the room where he was uſually employed in calculations, and another chamber or room in the houſe where a ſervant could enter; and before this hole he had contrived a ſliding board: the ſervant always placed his victuals in this hole, without ſpeaking or making any the leaſt noiſe; and when he had a little leiſure he viſited his cupboard to ſee what it afforded to ſatisfy his hunger or thirſt. But it often happened, that the breakfaſt, dinner, and ſupper have remained untouched by him, when the ſervant has gone to remove what was left—ſo deeply engaged had he been in calculations.—Cavities might eaſily be perceived in an old Engliſh oak table where he ſat to write, by the frequent rubbing and wearing of his elbows.—*Gutta cavat lapidem*, &c.

By

By Mr. Sharp's epitaph it appears that he was related to archbishop Sharp. And Mr. Sharp the eminent surgeon, who, it seems has lately retired from business, is the nephew of our author. Another nephew was the father of Mr. Ramsden, the present celebrated instrument maker, who says that his grand uncle Abraham, our author, was some time in his younger days an exciseman; which occupation he quitted on coming to a patrimonial estate of about 200l. a year.

Sharp, in Music, a kind of artificial note, or character, thus formed ✻: this being prefixed to any note, shews that it is to be sung or played a semitone or half note higher than the natural note is. When a Sharp is placed at the beginning of a stave or movement, it shews that all notes that are found on the same line, or space, throughout, are to be raised half a tone above their natural pitch, unless a natural intervene. When a Sharp occurs accidentally, it only affects as many notes as follow it on the same line or space, without a natural, in the compass of a bar.

SHEAVE, in Mechanics, a solid cylindrical wheel, fixed in a channel, and moveable about an axis, as being used to raise or increase the mechanical powers applied to remove any body.

SHEERS, aboard a ship, an engine used to hoist or displace the lower masts of a ship.

SHEKEL, or Shekle, an ancient Hebrew coin and weight, equal to 4 Attic drachmas, or 4 Roman denarii, or 2s. $9\frac{1}{2}$d. sterling. According to father Mersenne, the Hebrew Shekel weighs 268 grains, and is composed of 20 oboli, each obolus weighing 16 grains of wheat.

SHILLING, an English silver coin, equal to 12 pence, or the 20th part of a pound sterling.

This was a Saxon coin, being the 48th part of their pound weight. Its value at first was 5 pence; but it was reduced to 4 pence about a century before the conquest. After the conquest, the French solidus of 12 pence, which was in use among the Normans, was called by the English name of Shilling; and the Saxon Shilling of 4 pence took a Norman name, and was called the *groat*, or *great* coin, because it was the largest English coin then known in England. From this time, the Shilling underwent many alterations.

Many other nations have also their Shillings. The English Shilling is worth about 23 French sols; those of Holland and Germany about half as much, or $11\frac{1}{2}$ sols; those of Flanders about 9. The Dutch Shillings are also called *sols de gros*, because equal to 12 gross. The Danes have copper Shillings, worth about one fourth of a farthing sterling.

In the time of Edward the 1st, the pound troy was the same as the pound sterling of silver, consisting of 20 Shillings; so that the Shilling weighed the 20th part of a pound, or more than half an ounce troy. But some are of opinion, there were no coins of this denomination, till Henry the 7th, in the year 1504, first coined silver pieces of 12 pence value, which we call Shillings. Since the reign of Elizabeth, a Shilling weighs the 62nd part of a pound troy, or 3 dwts. $20\frac{8}{31}$ grs. the pound weight of silver making 62 Shillings. And hence the ounce of silver is worth 5s. 2d. or $5\frac{1}{6}$ Shillings.

SHIVERS, in a ship, the seamen's term for those little round wheels, in which the rope of a pully or block runs. They turn with the rope, and have pieces of brass in their centres, into which the pin of the block goes, and on which they turn.

SHORT-SIGHTEDNESS, *myopia*, a defect in the conformation of the eye, when the crystalline &c being too convex, the rays that enter the eye are refracted too much, and made to converge too fast, so as to unite before they reach the retina, by which means vision is rendered dim and confused.

It is commonly thought that Short-sightedness wears off in old age, on account of the eye becoming flatter; but Dr. Smith questions whether this be matter of fact, or only hypothesis. It is remarkable that Short-sighted persons commonly write a small hand, and love a small print, because they can see more of it at one view. That it is customary with them not to look at the person they converse with, because they cannot well see the motion of his eyes and features, and are therefore attentive to his words only. That they see more distinctly, and somewhat farther off, by a strong light, than by a weak one; because a strong light causes a contraction of the pupil, and consequently of the pencils, both here and at the retina, which lessens their mixture, and consequently the apparent confusion; and therefore, to see more distinctly, they almost close their eye-lids, for which reason they were anciently called *myopes*. Smith's Optics, vol. 2, Rem. p. 10.

Dr. Jurin observes, that persons who are much and long accustomed to view objects at small distances, as students in general, watchmakers, engravers, painters in miniature, &c, see better at small distances, and worse at great distances, than other people. And he gives the reasons, from the mechanical effect of habit in the eye. Essay on Dist. and Indist. Vision.

The ordinary remedy for Short-sightedness is a concave lens, held before the eye; for this causing the rays to diverge, or at least diminishing much of their convergency, it makes a compensation for the too great convexity of the crystalline. Dr. Hook suggests another remedy; which is to employ a convex glass, in a position between the object and the eye, by means of which, the object may be made to appear at any distance from the eye, and so the eye be made to contemplate the picture in the same manner as if the object itself were in its place. But here unfortunately the image will appear inverted: for this however he has some whimsical expedients; viz, in reading to turn the book upside down, and to learn to write upside down. As to distant objects, the Doctor asserts, from his own experience, that with a little practice in contemplating inverted objects, one gets as good an idea of them as if seen in their natural posture.

SHOT, in the Military Art, includes all sorts of balls or bullets for fire arms, from the cannon to the pistol. As to those for mortars, they are usually called shells.

Shot are mostly of a round form, though there are other shapes. Those for cannon are of iron; but those for muskets and pistols are of lead.

Cannon shot and shells are usually set up in piles, or heaps, tapering from the base towards the top; the base being either a triangle, a square, or a rectangle; from

from which the number in the pile is easily computed. See PILE.

The weight and dimensions of balls may be found, the one from the other, whether they are of iron or of lead. Thus,

The weight of an iron ball of 4 inches diameter, is 9lb, and because the weight is as the cube of the diameter, therefore as $4^3 : 9 :: d^3 : \frac{9}{64} d^3 = w$, the weight of the iron ball whose diameter is d; that is, $\frac{9}{64}$ of the cube of its diameter. And, conversely, if the weight be given, to find the diameter, it will be $\sqrt[3]{\frac{64}{9} w} = d$; that is, take $\frac{64}{9}$ or $7\frac{1}{9}$ of the weight, and the cube root of that will be the diameter of the iron ball.

For leaden balls; one of $4\frac{1}{4}$ inches diameter weighs 17 pounds; therefore as the cube of $4\frac{1}{4}$ is to 17, or nearly as $9 : 2 :: d^3 : \frac{2}{9} d^3 = w$, the weight of the leaden ball whose diameter is d, that is, $\frac{2}{9}$ of the cube of the diameter. On the contrary, if the weight be given, to find the diameter, it will be $\sqrt[3]{\frac{9}{2} w} = d$; that is, $\frac{9}{2}$ or $4\frac{1}{2}$ of the weight, and the cube root of the product. See my Conic Sections and Select Exercises, pa. 141.

SHOULDER *of a Bastion*, in Fortification, is the angle where the face and the flank meet.

SHOULDERING, in Fortification. See *Epaulement*.

SHWAN-*pan*, a Chinese instrument, composed of a number of wires, with beads upon them, which they move backwards and forwards, and which serves to assist them in their computations. See ABACUS.

SIDE, *latus*, in Geometry. The side of a figure is a line making part of the periphery of any superficial figure, viz, a part between two successive angles.

In triangles, the sides are also called *legs*. In a right-angled triangle, the two sides that include the right angle, are called *catheti*, or sometimes the *base* and *perpendicular*; and the third side, the *hypothenuse*.

SIDE *of a Polygonal Number*, is the number of terms in the arithmetical progression that are summed up to form the number.

SIDE *of a Power*, is what is usually called the root or radix.

SIDES of *Horn-works*, *Crown-works*, *Double-tenailles*, &c, are the ramparts and parapets which inclose them on the right and left, from the gorge to the head.

SIDEREAL, or SIDERIAL, something relating to the stars. As Sidereal year, day, &c, being those marked out by the stars.

SIDEREAL *Year*. See YEAR.

SIDEREAL *Day*, is the time in which any star appears to revolve from the meridian to the meridian again; which is 23 hours 56′ 4″ 6‴ of mean solar time; there being 366 Sidereal days in a year, or in the time of 365 diurnal revolutions of the sun; that is, exactly, if the equinoctial points were at rest in the heavens. But the equinoctial points go backward, with respect to the stars, at the rate of 50″ of a degree in a Julian year; which causeth the stars to have an apparent progressive motion eastward 50″ in that time. And as the sun's mean motion in the ecliptic is only 11 signs 29° 45′ 40″ 15‴ in 365 days, it follows, that at the end of that time he will be 14′ 19″ 45‴ short of that point of the ecliptic from which he set out at the beginning; and the stars will be advanced 50″ of a degree with respect to that point.

Consequently, if the sun's centre be on the meridian with any star on any given day of the year, that star will be 14′ 19″ 45‴ + 50″ or 15′ 9″ 45‴ east of the sun's centre, on the 365th day afterward, when the sun's centre is on the meridian; and therefore that star will not come to the meridian on that day till the sun's centre has passed it by 1′ 0″ 38‴ 57⁗ of mean solar time; for the sun takes so much time to go through an arc of 15′ 9″ 45‴; and then, in 365da 0^{h} 1′ 0″ 38‴ 57⁗ the star will have just completed its 366th revolution to the meridian.

In the following table, of Sidereal revolutions, the first column contains the number of revolutions of the stars; the others next it shew the times in which these revolutions are made, as shewn by a well regulated clock; and those on the right hand shew the daily accelerations of the stars, that is, how much any star gains upon the time shewn by such a clock, in the corresponding revolutions.

Revol. of the stars.	Times in which the revolutions are made.						Accelerations of the stars.				
	da.	ho.	m.	sec.	th.	fo.	ho.	m.	sec.	th.	fo.
1	0	23	56	4	6	0	0	3	55	54	0
2	1	23	52	8	12	1	0	7	51	47	59
3	2	23	48	12	18	1	0	11	47	41	59
4	3	23	44	16	24	2	0	15	43	35	58
5	4	23	40	20	30	2	0	19	39	29	58
6	5	23	36	24	36	3	0	23	35	23	57
7	6	23	32	28	42	3	0	27	31	17	57
8	7	23	28	32	48	4	0	31	27	11	56
9	8	23	24	36	54	4	0	35	23	5	56
10	9	23	20	41	0	5	0	39	18	59	55
11	10	23	16	45	6	5	0	43	14	53	55
12	11	23	12	49	12	6	0	47	10	47	54
13	12	23	8	53	18	6	0	51	6	41	54
14	13	23	4	57	24	7	0	55	2	35	53
15	14	23	1	1	30	7	0	58	58	29	53
16	15	22	57	5	36	8	1	2	54	23	52
17	16	22	53	9	42	8	1	6	50	17	52
18	17	22	49	13	48	9	1	10	46	11	51
19	18	22	45	17	54	9	1	14	42	5	51
20	19	22	41	22	0	10	1	18	37	59	50
21	20	22	37	26	6	10	1	22	33	53	50
22	21	22	33	30	12	11	1	26	29	47	49
23	22	22	29	34	18	11	1	30	25	41	49
24	23	22	25	38	24	12	1	34	21	35	48
25	24	22	21	42	30	12	1	38	17	29	48
26	25	22	17	46	36	13	1	42	13	23	47
27	26	22	13	50	42	13	1	46	9	17	47
28	27	22	9	54	48	14	1	50	5	11	46
29	28	22	5	58	54	14	1	54	1	5	46
30	29	22	2	3	0	15	1	57	56	59	45
40	39	21	22	44	0	19	2	37	15	59	41
50	49	20	43	25	0	24	3	16	34	59	36
100	99	17	26	50	0	48	6	33	9	59	12
200	199	10	53	40	1	37	13	6	19	58	23
300	299	4	20	30	2	25	19	39	29	57	35
360	359	0	24	36	2	54	23	35	23	57	6
365	364	0	4	56	32	56	23	55	3	27	4
366	365	0	1	0	38	57	23	58	59	21	3

This table will not differ the 27993600000oth part of a second of time from the truth in a whole year. It was calculated by Mr. Ferguson; and it is the only table of the kind in which the recession of the equinoctial points has been taken into the calculation.

SIDUS *Georgium*, a new primary planet, discovered by Dr. Herschel at Bath, in the night of March 13, 1781. It is sometimes also called the *Georgian Planet*, and the *New Planet*, from its having been newly or lately discovered, also *Herschel's Planet*, from the name of its discoverer, and the *Planet Herschel*, or simply *Herschel*, by which name it is distinguished by the astronomers of almost all foreign nations. The planet is denoted by this character ♅, a Roman H as the initial of the name, the horizontal bar being crossed by a perpendicular line, forming a kind of cross, the emblem of Christianity, meaning thereby perhaps that its discovery was made by a Christian, or since the birth of Christ, as all the other planets were discovered long before that period.

This planet is the remotest of all those that are yet known, though not the largest, being in point of magnitude less than Saturn and Jupiter. Its light, says Dr. Herschel, is of a blueish-white colour, and its brilliancy between that of Venus and the moon. With a telescope that magnifies about 300 times, it appears to have a very well defined visible disk; but with instruments of a small power, it can hardly be distinguished from a fixed star of between the 6th and 7th magnitude. In a very fine clear night, when the moon is absent, a good eye will perceive it without a telescope.

From the observations and calculations of Dr. Herschel and other astronomers, the elements and dimensions &c of this planet, have been collected as below.

Place of the node - - - - -	2^s	$11°$	$49'$	$30''$	
Place of the aphelion in 1795	11	23	33	55	
Inclination of the orbit - -	·	·	43	35	
Time of the perihelion passage	Sep. 7, 1799.				
Eccentricity of the orbit - -	·8203				
Half the greater axis - - - -	19·0818	of Earth's dist.			
Revolution - - - - - - - - -	$83\frac{1}{3}$	sidereal years			
Diameter of the planet - - -	34217	miles			
Propor. of diam. to the earth's	4·3177	to 1			
Its bulk to the earth's - - -	80·4926	to 1			
Its density as - - - - - - - -	·2204	to 1			
Its quantity of matter - - -	17·7406	to 1			

And heavy bodies fall on its surface 18 feet 8 inches in one second of time.

SIGN, in Algebra, a symbol or CHARACTER.

SIGNS, *like*, *positive*, *negative*, *radical*, &c. See the adjectives.

SIGN, in Astronomy, a 12th part of the ecliptic, or zodiac; or a portion containing 30 degrees of the same.

The ancients divided the zodiac into 12 segments, called Signs; commencing at the point where the ecliptic and equinoctial intersect, and so counting forward from west to east, according to the course of the sun; these Signs they named from the 12 constellations which possessed those segments in the time of Hipparchus. But the constellations have since so changed their places, by the precession of the equinox, that Aries is now found in the sign called Taurus, and Taurus in that of Gemini, &c.

The names, and characters, of the 12 Signs, and their order, are as follow: Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aquarius ♒, Pisces ♓; each of which, with the stars in them, see under its proper article, ARIES, TAURUS, &c.

The Signs are distinguished, with regard to the season of the year when the sun is in them, into vernal, æstival, autumnal, and brumal.

Vernal or *Spring* SIGNS, are Aries, Taurus, Gemini.

Æstival or *Summer* SIGNS, are Cancer, Leo, Virgo.

Autumnal SIGNS, are Libra, Scorpio, Sagittary.

Brumal or *Winter* SIGNS, are Capricorn, Aquarius, Pisces.

The vernal and summer Signs are also called *northern* Signs, because they are on the north side of the equinoctial; and the autumnal and winter Signs are called *southern* ones, because they are on the south side of the same.

The Signs are also distinguished into *ascending* and *descending*, according as they are ascending toward the north, or descending toward the south. Thus, the

Ascending SIGNS, are the winter and spring signs, or those six from the winter solstice to the summer solstice, viz, the Signs Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini. And the

Descending SIGNS are the summer and autumn Signs, or the Signs Cancer, Leo, Virgo, Libra, Scorpio, Sagittary.

SIGNS, *Fixed*, *Masculine*, &c; see the adjectives.

SILLON, in Fortification, an elevation of earth, made in the middle of the moat, to fortify it, when too broad. It is more usually called the Envelope.

SIMILAR, in Arithmetic and Geometry, the same with like. Similar things have the same disposition or conformation of parts, and differ in nothing but as to their quantity or magnitude; as two squares, or two circles, &c.

In Mathematics, Similar parts, as A, *a*, have the same ratio to their wholes B, *b*; and if the wholes have the same ratio to the parts, the parts are Similar.

SIMILAR *angles*, are also equal angles.

SIMILAR *arcs*, of circles, are such as are like parts of their whole peripheries. And, in general, similar arcs of any like curves, are the like parts of the wholes.

SIMILAR *bodies*, in Natural Philosophy, are such as have their particles of the same kind and nature one with another.

SIMILAR *Curves*. Two segments of two curves are said to be Similar when, any right-lined figure being inscribed within one of them, we can inscribe always a Similar rectilineal figure in the other.

SIMILAR *Conic Sections*, are such as are of the same kind, and have their principal axes and parameters proportional. So, two ellipses are figures of the same kind, but they are not Similar unless the axes of the one have the same ratio as the axes of the other. And the same of two hyperbolas, or two parabolas. And generally, those curves are Similar, that are of the same kind, and have their corresponding dimensions in the same ratio.—All circles are Similar figures.

SIMILAR *Diameters of Conic Sections*, are such as make equal angles with their ordinates.

SIMILAR *Figures*, or plane figures, are such as have all their angles equal respectively, each to each, and their

their sides about the equal angles proportional. And the same of Similar polygons.—Similar plane figures have their areas or contents, in the duplicate ratio of their like sides, or as the squares of those sides.

SIMILAR *Plane Numbers*, are such as may be ranged into the form of Similar rectangles; that is, into rectangles whose sides are proportional. Such are 12 and 48; for the sides of 12 are 6 and 2, and the sides of 48 are 12 and 4, which are in the same proportion, viz, 6 : 2 : : 12 : 4.

SIMILAR *Polygons*, are polygons of the same number of angles, and the angles in the one equal severally to the angles in the other, also the sides about those angles proportional.

SIMILAR *Rectangles*, are those that have their sides about the like angles proportional.—All squares are Similar.

SIMILAR *Segments of circles*, are such as contain equal angles.

SIMILAR *Solids*, are such as are contained under the same number of Similar planes, alike situated.—Similar solids are to each other as the cubes of their like linear dimensions.

SIMILAR *Solid Numbers*, are those whose little cubes may be so ranged, as to form Similar parallelopipedons.

SIMILAR *Triangles*, are such as are equiangular ones, or have all their three angles respectively equal in each triangle For it is sufficient for triangles to be similar, that they be equiangular, because that being equiangular, they necessarily have their sides proportional, which is a condition of Similarity in all figures. As to other figures, having more sides than three, they may be equiangular, without having their sides proportional, and therefore without being similar.—Similar triangles are as the squares of their like sides.

SIMILITUDE, in Arithmetic and Geometry, denotes the relation of things that are similar to each other.

Euclid and, after him, most other authors, demonstrate every thing in geometry from the principle of congruity. Wolfius, instead of it, substitutes that of Similitude; which, he says, was communicated to him by Leibnitz, and which he finds of very considerable use in geometry, as serving to demonstrate many things directly, which are only demonstrable from the principle of congruity in a very tedious manner.

SIMPLE, something not mixed, or not compounded; in which sense it stands opposed to compound.

The elements are Simple bodies, from the composition of which there result all sorts of mixed bodies.

SIMPLE *Equation*, *Fraction*, and *Surd*. See the substantives.

SIMPLE *Quantities*, in Algebra, are those that consist of one term only; as a, or $-ab$, or $3abc$. In opposition to compound quantities, which consist of two or more terms; as $a + b$, or $a + 2b - 3ac$.

SIMPLE *Flank*, and *Tenaille*, in Fortification. See the substantives.

SIMPLE *Machine*, *Motion*, *Pendulum*, and *Wheel*, in Mechanics. See the substantives.

The simplest machines are always the most esteemed. And in geometry, the most simple demonstrations are the best.

SIMPLE *Problem*, in Mathematics. See LINEAR *Problem*.

SIMPLE *Vision*, in Optics. See VISION.

SIMPSON (THOMAS), F.R.S. a very eminent mathematician, and professor of Mathematics in the Royal Military Academy at Woolwich, was born at Market Bosworth, in the county of Leicester, the 20th of August 1710. His father was a stuff-weaver in that town; and though in tolerable circumstances, yet, intending to bring up his son Thomas to his own business, he took so little care of his education, that he was only taught to read English. But nature had furnished him with talents and a genius for far other pursuits; which led him afterwards to the highest rank in the mathematical and philosophical sciences.

Young Simpson very soon gave indications of his turn for study in general, by eagerly reading all books he could meet with, teaching himself to write, and embracing every opportunity he could find of deriving knowledge from other persons. His father observing him thus to neglect his business, by spending his time in reading what he thought useless books, and following other such like pursuits, used all his endeavours to check such proceedings, and to induce him to follow his profession with steadiness and better effect. But after many struggles for this purpose, the differences thus produced between them at length rose to such a height, that our author quitted his father's house entirely.

Upon this occasion he repaired to Nuneaton, a town at a small distance from Bosworth, where he went to lodge at the house of a taylor's widow, of the name of Swinfield, who had been left with two children, a daughter and a son, by her husband, of whom the son, who was the younger, being but about two years older than Simpson, had become his intimate friend and companion. And here he continued some time, working at his trade, and improving his knowledge by reading such books as he could procure.

Among several other circumstances which, long before this, gave occasion to shew our author's early thirst for knowledge, as well as proving a fresh incitement to acquire it, was that of a large solar eclipse, which took place on the 11th day of May, 1724. This phenomenon, so awful to many who are ignorant of the cause of it, struck the mind of young Simpson with a strong curiosity to discover the reason of it, and to be able to predict the like surprising events. It was however several years before he could obtain his desire, which at length was gratified by the following accident. After he had been some time at Mrs. Swinfield's, at Nuneaton, a travelling pedlar came that way, and took a lodging at the same house, according to his usual custom. This man, to his profession of an itinerant merchant, had joined the more profitable one of a fortune-teller, which he performed by means of judicial astrology. Every one knows with what regard persons of such a cast are treated by the inhabitants of country villages; it cannot be surprising therefore that an untutored lad of nineteen should look upon this man as a prodigy, and, regarding him in this light, should endeavour to ingratiate himself into his favour; in which he succeeded so well, that the sage was no less taken with the quick natural parts and genius of his new acquaintance. The pedlar, intending a journey to Bristol fair, left in the hands of young Simpson an old edition of Cocker's Arithmetic, to which was subjoined a short Appendix on Algebra, and a book upon Genitures, by Partridge the almanac maker. These books he had perused to so good purpose, during the absence of his friend

friend, as to excite his amazement upon his return; in consequence of which he set himself about erecting a genethliacal figure, in order to a presage of Thomas's future fortune.

This position of the heavens having been maturely considered *secundum artem*, the wizard, with great confidence, pronounced, that, "within two years time Simpson would turn out a greater man than himself!"

In fact, our author profited so well by the encouragement and assistance of the pedlar, afforded him from time to time when he occasionally came to Nuneaton, that, by the advice of his friend, he at length made an open profession of casting nativities himself; from which, together with teaching an evening school, he derived a pretty pittance, so that he greatly neglected his weaving, to which indeed he had never manifested any great attachment, and soon became the oracle of Nuneaton, Bosworth, and the environs. Scarce a courtship advanced to a match, or a bargain to a sale, without previously consulting the infallible Simpson about the consequences. But as to helping people to stolen goods, he always declared that above his skill; and over life and death he declared he had no power: all those called *lawful questions* he readily resolved, provided the persons were certain as to the horary *data* of the horoscope: and, he has often declared, with such success, that if from very cogent reasons he had not been thoroughly convinced of the vain foundation and fallaciousness of his art, he never should have dropt it, as he afterwards found himself in conscience bound to do.

About this time he married the widow Swinfield, in whose house he lodged, though she was then almost old enough to be his grandmother, being upwards of fifty years of age. After this the family lived comfortably enough together for some short time, Simpson occasionally working at his business of a weaver in the day-time, and teaching an evening school or telling fortunes at night; the family being also farther assisted by the labours of young Swinfield, who had been brought up in the profession of his father.

But this tranquillity was soon interrupted, and our author driven at once from his home and the profession of astrology, by the following accident. A young woman in the neighbourhood had long wished to hear or know something of her lover, who had been gone to sea; but Simpson had put her off from time to time, till the girl grew at last so importunate, that he could deny her no longer. He asked her if she would be afraid if he should raise the devil, thinking to deter her; but she declared she feared neither ghost nor devil: so he was obliged to comply. The scene of action pitched upon was a barn, and young Swinfield was to act the devil or ghost; who being concealed under some straw in a corner of the barn, was, at a signal given, to rise slowly out from among the straw, with his face marked so that the girl might not know him. Every thing being in order, the girl came at the time appointed; when Simpson, after cautioning her not to be afraid, began muttering some mystical words, and chalking round about them, till, on the signal given, up rises the taylor slow and solemn, to the great terror of the poor girl, who, before she had seen half his shoulders, fell into violent fits, crying out it was the very image of her lover; and the effect upon her was so dreadful, that it was thought either death or madness must be the consequence. So that poor Simpson was obliged immediately to abandon at once both his home and the profession of a conjuror.

Upon this occasion it would seem he fled to Derby, where he remained some two or three years, viz, from 1733 till 1735 or 1736; instructing pupils in an evening school, and working at his trade by day.

It would seem that Simpson had an early turn for versifying, both from the circumstance of a song written here in favour of the Cavendish family, on occasion of the parliamentary election at that place, in the year 1733; and from his first two mathematical questions that were published in the Ladies Diary, which were both in a set of verses, not ill written for the occasion. These were printed in the Diary for 1736, and therefore must at latest have been written in the year 1735. These two questions, being at that time pretty difficult ones, shew the great progress he had even then made in the mathematics; and from an expression in the first of them, viz, where he mentions his residence as being in latitude 52°, it appears he was not then come up to London, though he must have done so very soon after.

Together with his astrology, he had soon furnished himself with arithmetic, algebra, and geometry sufficient to be qualified for looking into the Ladies Diary (of which he had afterwards for several years the direction), by which he came to understand that there was a still higher branch of the mathematical knowledge than any he had yet been acquainted with; and this was the method of *Fluxions*. But our young analyst was quite at a loss to discover any English author who had written on the subject, except Mr. Hayes; and his work being a folio, and then pretty scarce, exceeded his ability of purchasing: however an acquaintance lent him Mr. Stone's *Fluxions*, which is a translation of the *Marquis de l'Hospital's Analyse des Infiniment Petits*: by this one book, and his own penetrating talents, he was, as we shall see presently, enabled in a very few years to compose a much more accurate treatise on this subject than any that had before appeared in our language.

After he had quitted astrology and its emoluments, he was driven to hardships for the subsistence of his family, while at Derby, notwithstanding his other industrious endeavours in his own trade by day, and teaching pupils at evenings. This determined him to repair to London, which he did in 1735 or 1736.

On his first coming to London, Mr. Simpson wrought for some time at his business in Spitalfields, and taught mathematics at evenings, or any spare hours. His industry turned to so good account, that he returned down into the country, and brought up his wife and three children, she having produced her first child to him in his absence. The number of his scholars increasing, and his abilities becoming in some measure known to the public, he was encouraged to make proposals for publishing by subscription, A new Treatise of Fluxions: wherein the Direct and Inverse Methods are demonstrated after a new, clear, and concise Manner, with their Application to Physics and Astronomy: also the Doctrine of Infinite Series and Reverting Series universally, are amply explained, Fluxionary and Exponential Equations solved: together with a variety of new and curious Problems.

When

When Mr. Simpfon firft propofed his intentions of publifhing fuch a work, he did not know of any Englifh book, founded on the true principles of Fluxions, that contained any thing material, efpecially the practical part; and though there had been fome very curious things done by feveral learned and ingenious gentlemen, the principles were neverthelefs left obfcure and defective, and all that had been done by any of them in *infinite feries*, very inconfiderable.

The book was publifhed in 4to, in the year 1737, although the author had been frequently interrupted from furnifhing the prefs fo faft as he could have wifhed, through his unavoidable attention to his pupils for his immediate fupport. The principles of fluxions treated of in this work, are demonftrated in a method accurately true and genuine, not effentially different from that of their great inventor, being entirely expounded by finite quantities.

In 1740, Mr. Simpfon publifhed a Treatife on The Nature and Laws of Chance, in 4to. To which are annexed, Full and clear Inveftigations of two important Problems added in the 2d Edition of Mr. De Moivre's Book on Chances, as alfo two New Methods for the Summation of Series.

Our author's next publication was a 4to volume of Effays on feveral curious and interefting Subjects in Speculative and Mixed Mathhmatics; printed in the fame year 1740: dedicated to Francis Blake, Efq. fince Fellow of the Royal Society, and our author's good friend and patron.—Soon after the publication of this book, he was chofen a member of the Royal Academy at Stockholm.

Our author's next work was, The Doctrine of Annuities and Reverfions, deduced from general and evident Principles: with ufeful Tables, fhewing the Values of Single and Joint Lives, &c. in 8vo, 1742. This was followed in 1743, by an Appendix containing fome Remarks on a late book on the fame Subject (by Mr. Abr. De Moivre, F. R. S.) with Anfwers to fome perfonal and malignant Reprefentations in the Preface thereof. To this anfwer Mr. De Moivre never thought fit to reply. A new edition of this work has lately been publifhed, augmented with the tract upon the fame fubject that was printed in our author's Select Exercifes.

In 1743 alfo was publifhed his Mathematical Differtations on a variety of Phyfical and Analytical Subjects, in 4to; containing, among other particulars,

A Demonftration of the true Figure which the Earth, or any Planet, muft acquire from its rotation about an Axis. A general Inveftigation of the Attraction at the Surfaces of Bodies nearly fpherical. A Determination of the Meridional Parts, and the Lengths of the feveral Degrees of the Meridian, according to the true Figure of the Earth. An Inveftigation of the Height of the Tides in the Ocean. A new Theory of Aftronomical Refractions, with exact Tables deduced from the fame. A new and very exact Method for approximating the Roots of Equations in Numbers; which quintuples the number of Places at each Operation. Several new Methods for the Summation of Series. Some new and very ufeful Improvements in the Inverfe Method of Fluxions. The work being dedicated to Martin Folkes, Efq. Prefident of the Royal Society.

His next book was A Treatife of Algebra, wherein the fundamental Principles are demonftrated, and applied to the Solution of a variety of Problems. To which he added, The Conftruction of a great Number of Geometrical Problems, with the Method of refolving them numerically.

This work, which was defigned for the ufe of young beginners, was infcribed to William Jones, Efq. F. R. S. and printed in 8vo, 1745. And a new edition appeared in 1755, with additions and improvements; among which was a new and general method of refolving all Biquadratic Equations, that are complete, or having all their terms. This edition was dedicated to James Earl of Morton, F. R. S. Mr. Jones being then dead. The work has gone through feveral other editions fince that time: the 6th, or laft, was in 1790.

His next work was, "Elements of Geometry, with their Application to the Menfuration of Superficies and Solids, to the Determination of Maxima and Minima, and to the Conftruction of a great Variety of geometrical Problems:" firft publifhed in 1747, in 8vo. And a fecond edition of the fame came out in 1760, with great alterations and additions, being in a manner a new work, defigned for young beginners, particularly for the gentlemen educated at the Royal Military Academy at Woolwich, and dedicated to Charles Frederick, Efq. Surveyor General of the Ordnance. And other editions have appeared fince.

Mr. Simpfon met with fome trouble and vexation in confequence of the firft edition of his Geometry. Firft, from fome reflections made upon it, as to the accuracy of certain parts of it, by Dr. Robert Simfon, the learned profeffor of mathematicks in the univerfity of Glafgow, in the notes fubjoined to his edition of Euclid's Elements. This brought an anfwer to thofe remarks from Mr. Simpfon, in the notes added to the 2d edition as above; to fome parts of which Dr. Simfon again replied in his notes on the next edition of the faid Elements of Euclid.

The fecond was by an illiberal charge of having ftolen his Elements from Mr. Muller, the profeffor of fortification and artillery at the fame academy at Woolwich, where our author was profeffor of geometry and mathematics. This charge was made at the end of the preface to Mr. Muller's Elements of Mathematics, in two volumes, printed in 1748; which was fully refuted by Mr. Simpfon in the preface to the 2d edition of his Geometry.

In 1748 came out Mr. Simpfon's Trigonometry, Plane and Spherical, with the Conftruction and Application of Logarithms, 8vo. This little book contains feveral things new and ufeful.

In 1750 came out, in two volumes, 8vo, The Doctrine and Application of Fluxions, containing, befides what is common on the Subject, a Number of new Improvements in the Theory, and the Solution of a Variety of new and very interefting Problems in different Branches of the Mathematics.—In the preface the author offers this to the world as a new book, rather than a fecond edition of that which was publifhed in 1737, in which he acknowledges, that, befides errors of the prefs, there are feveral obfcurities and defects, for want of experience, and the many difadvantages he then laboured under, in his firft fally.

The

The idea and explanation here given of the first principles of Fluxions, are not essentially different from what they are in his former treatise, though expressed in other terms. The consideration of *time* introduced into the general definition, will, he says, perhaps be disliked by those who would have fluxions to be *mere velocities:* but the advantage of considering them otherwise, viz, not as the velocities themselves, but as magnitudes they would uniformly generate in a given time, appears to obviate any objection on that head. By taking fluxions as mere velocities, the imagination is confined as it were to a point, and without proper care insensibly involved in metaphysical difficulties. But according to this other mode of explaining the matter, less caution in the learner is necessary, and the higher orders of fluxions are rendered much more easy and intelligible. Besides, though Sir Isaac Newton defines fluxions to be the velocities of motions, yet he has recourse to the increments or moments generated in equal particles of time, in order to determine those velocities; which he afterwards teaches to expound by finite magnitudes of other kinds. This work was dedicated to George earl of Macclesfield.

In 1752 appeared, in 8vo, the *Select Exercises for young Proficients in the Mathematics.* This neat volume contains, A great Variety of algebraical Problems, with their Solutions. A select Number of Geometrical Problems, with their Solutions, both algebraical and geometrical. The Theory of Gunnery, independent of the Conic Sections. A new and very comprehensive Method for finding the Roots of Equations in Numbers. A short Account of the first Principles of Fluxions. Also the Valuation of Annuities for single and joint Lives, with a Set of new Tables, far more extensive than any extant. This last part was designed as a supplement to his Doctrine of Annuities and Reversions; but being thought too small to be published alone, it was inserted here at the end of the Select Exercises; from whence however it has been removed in the last editions, and referred to its proper place, the end of the Annuities, as before mentioned. The examples that are given to each problem in this last piece, are according to the London bills of mortality; but the solutions are general, and may be applied with equal facility and advantage to any other table of observations. The volume is dedicated to John Bacon, Esq. F. R. S.

Mr. Simpson's Miscellaneous Tracts, printed in 4to, 1757, were his last legacy to the public: a most valuable bequest, whether we consider the dignity and importance of the subjects, or his sublime and accurate manner of treating them.

The first of these papers is concerned in determining the Precession of the Equinox, and the different Motions of the Earth's Axis, arising from the Attraction of the Sun and Moon. It was drawn up about the year 1752, in consequence of another on the same subject, by M. de Sylvabelle, a French gentleman. Though this gentleman had gone through one part of the subject with success and perspicuity, and his conclusions were perfectly conformable to Dr. Bradley's observations; it nevertheless appeared to Mr. Simpson, that he had greatly failed in a very material part, and that indeed the only very difficult one; that is, in the determination of the momentary alteration of the position of the earth's axis, caused by the forces of the sun and moon; of which forces, the quantities, but not the effects, are truly investigated. The second paper contains the Investigation of a very exact Method or Rule for finding the Place of a Planet in its Orbit, from a Correction of Bishop Ward's circular Hypothesis, by Means of certain Equations applied to the Motion about the upper Focus of the Ellipse. By this Method the Result, even in the Orbit of Mercury, may be found within a Second of the Truth, and that without repeating the Operation. The third shews the Manner of transferring the Motion of a Comet from a parabolic Orbit, to an elliptic one; being of great Use, when the observed Places of a (new) Comet are found to differ sensibly from those computed on the Hypothesis of a parabolic Orbit. The fourth is an Attempt to shew, from mathematical Principles, the Advantage arising from taking the Mean of a Number of Observations, in practical Astronomy; wherein the Odds that the Result in this Way, is more exact than from one single Observation, is evinced, and the Utility of the Method in Practice clearly made appear. The fifth contains the Determination of certain Fluents, and the Resolution of some very useful Equations, in the higher Orders of Fluxions, by Means of the Measures of Angles and Ratios, and the right and versed Sines of circular Arcs. The 6th treats of the Resolution of algebraical Equations, by the Method of Surd-divisors; in which the Grounds of that Method, as laid down by Sir Isaac Newton, are investigated and explained. The 7th exhibits the Investigation of a general Rule for the Resolution of Isoperimetrical Problems of all Orders, with some Examples of the Use and Application of the said Rule. The 8th, or last part, comprehends the Resolution of some general and very important Problems in Mechanics and Physical Astronomy; in which, among other Things, the principal Parts of the 3d and 9th Sections of the first Book of Newton's Principia are demonstrated in a new and concise Manner. But what may perhaps best recommend this excellent tract, is the application of the general equations, thus derived, to the determination of the Lunar Orbit.

According to what Mr. Simpson had intimated at the conclusion of his Doctrine of Fluxions, the greatest part of this arduous undertaking was drawn up in the year 1750. About that time M. Clairaut, a very eminent mathematician of the French Academy, had started an objection against Newton's general law of gravitation. This was a motive to induce Mr. Simpson (among some others) to endeavour to discover whether the motion of the moon's apogee, on which that objection had its whole weight and foundation, could not be truly accounted for, without supposing a change in the received law of gravitation, from the inverse ratio of the squares of the distances. The success answered his hopes, and induced him to look farther into other parts of the theory of the moon's motion, than he had at first intended: but before he had completed his design, M. Clairaut arrived in England, and made Mr. Simpson a visit; from whom he learnt, that he had a little before printed a piece on that subject, a copy of which Mr. Simpson afterwards received as a present, and found in it the same things demonstrated, to which he himself had directed his enquiry, besides several others.

The

The facility of the method Mr. Simpson fell upon, and the extensiveness of it, will in some measure appear from this, that it not only determines the motion of the apogee, in the same manner, and with the same ease, as the other equations, but utterly excludes all that dangerous kind of terms that had embarrassed the greatest mathematicians, and would, after a great number of revolutions, entirely change the figure of the moon's orbit. From whence this important consequence is derived, that the moon's mean motion, and the greatest quantities of the several equations, will remain unchanged, unless disturbed by the intervention of some foreign or accidental cause. These tracts are inscribed to the Earl of Macclesfield, President of the Royal Society.

Besides the foregoing, which are the whole of the regular books or treatises that were published by Mr. Simpson, he wrote and composed several other papers and fugitive pieces, as follow:

Several papers of his were read at the meetings of the Royal Society, and printed in their Transactions: but as most, if not all of them, were afterwards inserted, with alterations or additions, in his printed volumes, it is needless to take any farther notice of them here.

He proposed, and resolved many questions in the Ladies Diaries, &c; sometimes under his own name, as in the years 1735 and 1736; and sometimes under feigned or fictitious names; such as, it is thought, Hurlothrumbo, Kubernetes, Patrick O'Cavenah, Marmaduke Hodgson, Anthony Shallow, Esq, and probably several others; see the Diaries for the years 1735, 1736, 42, 43, 53, 54, 55, 56, 57, 58, 59, and 60. Mr. Simpson was also the editor or compiler of the Diaries from the year 1754 till the year 1760, both inclusive, during which time he raised that work to the highest degree of respect. He was succeeded in the Editorship by Mr. Edw. Rollinson. See my Diarian Miscellany, vol. 3.

It has also been commonly supposed that he was the real editor of, or had a principal share in, two other periodical works of a miscellaneous mathematical nature; viz, the Mathematician, and Turner's Mathematical Exercises, two volumes, in 8vo, which came out in periodical numbers, in the years 1750 and 1751, &c. The latter of these seems especially to have been set on foot to afford a proper place for exposing the errors and absurdities of Mr. Robert Heath, the then conductor of the Ladies Diary and the Palladium; and which controversy between them ended in the disgrace of Mr. Heath, and expulsion from his office of editor to the Ladies Diary, and the substitution of Mr. Simpson in his stead, in the year 1753.

In the year 1760, when the plans proposed for erecting a new bridge at Blackfriars were in agitation, Mr. Simpson, among other gentlemen, was consulted upon the best form for the arches, by the New-bridge Committee. Upon this occasion he gave a preference to the semicircular form; and, besides his report to the Committee, some letters also appeared, by himself and others, on the same subject, in the public newspapers, particularly in the Daily Advertiser, and in Lloyd's Evening Post. The same were also collected in the Gentleman's Magazine for that year, page 143 and 144.

It is probable that this reference to him, gave occasion to the turning his thoughts more seriously to this subject, so as to form the design of composing a regular treatise upon it: for his family have often informed me, that he laboured hard upon this work for some time before his death, and was very anxious to have completed it, frequently remarking to them, that this work, when published, would procure him more credit than any of his former publications. But he lived not to put the finishing hand to it. Whatever he wrote upon this subject, probably fell, together with all his other remaining papers, into the hands of major Henry Watson, of the engineers, in the service of the India Company, being in all a large chest full of papers. This gentleman had been a pupil of Mr. Simpson's, and had lodged in his house. After Mr. Simpson's death, Mr. Watson prevailed upon the widow to let him have the papers, promising either to give her a sum of money for them, or else to print and publish them for her benefit. But neither of these was ever done; this gentleman always declaring, when urged on this point by myself and others, that no use could be made of any of the papers, owing to the very imperfect state in which he said they were left. And yet he persisted in his refusal to give them up again.

From Mr. Simpson's writings, I now return to himself. Through the interest and solicitations of the before-mentioned William Jones, Esq, he was, in 1743, appointed professor of mathematics, then vacant by the death of Mr. Derham, in the Royal Academy at Woolwich; his warrant bearing date August 25th. And in 1745 he was admitted a fellow of the Royal Society, having been proposed as a candidate by Martin Folkes, Esq. President, William Jones, Esq. Mr. George Graham, and Mr. John Machin, Secretary; all very eminent mathematicians. The president and council, in consideration of his very moderate circumstances, were pleased to excuse his admission fees, and likewise his giving bond for the settled future payments.

At the academy he exerted his faculties to the utmost, in instructing the pupils who were the immediate objects of his duty, as well as others, whom the superior officers of the ordnance permitted to be boarded and lodged in his house. In his manner of teaching, he had a peculiar and happy address; a certain dignity and perspicuity, tempered with such a degree of mildness, as engaged both the attention, esteem and friendship of his scholars; of which the good of the service, as well as of the community, was a necessary consequence.

It must be acknowledged however, that his mildness and easiness of temper, united with a more inactive state of mind, in the latter years of his life, rendered his services less useful; and the same very easy disposition, with an innocent, unsuspecting simplicity, and playfulness of mind, rendered him often the dupe of the little tricks of his pupils. Having discovered that he was fond of listening to little amusing stories, they took care to furnish themselves with a stock; so that, having neglected to learn their lessons perfect, they would get round him in a crowd, and, instead of demonstrating a proposition, would amuse him with some comical story, at which he would laugh and shake very heartily, especially if it were tinctured with somewhat of the ludicrous

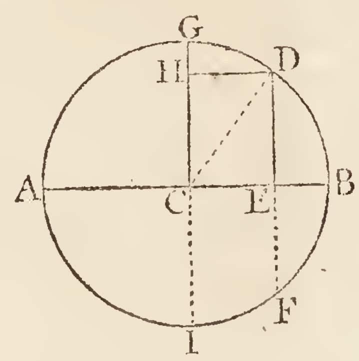
ludicrous or ſmutty; by which device they would contrive imperceptibly to wear out the hours allotted for inſtruction, and ſo avoid the trouble of learning and repeating their leſſon. They tell alſo of various tricks that were practiſed upon him in conſequence of the loſs of his memory in a great degree, in the latter ſtage of his life.

It has been ſaid that Mr. Simpſon frequented low company, with whom he uſed to guzzle porter and gin: but it muſt be obſerved that the miſconduct of his family put it out of his power to keep the company of gentlemen, as well as to procure better liquor.

In the latter ſtage of his exiſtence, when his life was in danger, exerciſe and a proper regimen were preſcribed him, but to little purpoſe; for he ſunk gradually into ſuch a lowneſs of ſpirits, as often in a manner deprived him of his mental faculties, and at laſt rendered him incapable of performing his duty, or even of reading the letters of his friends; and ſo trifling an accident as the dropping of a tea-cup would flurry him as much as if a houſe had tumbled down.

The phyſicians adviſed his native air for his recovery; and in February, 1761, he ſet out, with much reluctance (believing he ſhould never return) for Boſworth, along with ſome relations. The journey fatigued him to ſuch a degree, that upon his arrival he betook himſelf to his chamber, where he grew continually worſe and worſe, to the day of his death, which happened the 14th of May, in the fifty-firſt year of his age.

SINE, or *Right* SINE, of an arc, in Trigonometry, a right line drawn from one extremity of the arc, perpendicular to the radius drawn to the other extremity of it: Or, it is half the chord of double the arc. Thus the line DE is the ſine of the arc BD; either becauſe it is drawn from one end D of that arc, perpendicular to CB the radius drawn to the other end B of the arc; or alſo becauſe it is half the chord DF of double the arc DBF. For the ſame reaſon alſo DE is the Sine of the arc AD, which is the ſupplement of BD to a ſemicircle or 180 degrees; that is, every Sine is common to two arcs, which are ſupplements to each other, or whoſe ſum make up a ſemicircle, or 180 degrees.



Hence the Sines increaſe always from nothing at B till they become the radius CG, which is the greateſt, being the Sine of the quadrant BG. From hence they decreaſe all the way along the ſecond quadrant from G to A, till they quite vaniſh at the point A, thereby ſhewing that the Sine of the ſemicircle BGA, or 180 degrees, is nothing. After this they are negative all the way along the next ſemicircle, or 3d and 4th quadrants AFB, being drawn on the oppoſite ſide, or downwards from the diameter AB.

Whole SINE, or *Sinus Totus*, is the Sine of the quadrant BG, or of 90 degrees; that is, the Whole Sine is the ſame with the radius CG.

SINE-*Complment*, or *Coſine*, is the ſine of an arc DG, which is the complement of another arc BD, to a quadrant. That is, the line DH is the Coſine of the arc BD; becauſe it is the ſine of DG which is the complement of BD. And for the ſame reaſon DE is the Coſine of DG. Hence the ſine and Coſine and radius, of any arc, form a right-angled triangle CDE or CDH, of which the radius CD is the hypotenuſe; and therefore the ſquare of the radius is equal to the ſum of the ſquares of the ſine and Coſine of any arc, that is, $CD^2 = CE^2 + ED^2$ or $= CH^2 + DH^2$.

It is evident that the Coſine of 0 or nothing, is the whole radius CB. From B, where this Coſine is greateſt, the Coſine decreaſes as the arc increaſes from B along the quadrant BDG, till it become 0 for the complete quadrant BG. After this, the Coſines, decreaſing, become negative more and more all the way to the complete ſemicircle at A. Then the Coſines increaſe again all the way from A through I to B; at I the negation is deſtroyed, and the Coſine is equal to 0 or nothing; from I to B it is poſitive, and at B it is again become equal to the radius. So that, in general, the Coſines in the 1ſt and 4th quadrants are poſitive, but in the 2d and 3d negative.

Verſed-SINE, is the part of the diameter between the ſine and the arc. So BE is the Verſed Sine of the arc BD, and AE the Verſed Sine of AD, alſo GH the Verſed Sine of DG, &c. All Verſed Sines are affirmative. The ſum of the Verſed Sine and coſine, of any arc or angle, is equal to the radius, that is, $BE + EC = AC$.—The ſine, coſine, and Verſed Sine, of an arc, are alſo the ſame of an angle, or the number of degrees &c, which it meaſures.

The Sines &c, of every degree and minute in a quadrant, are calculated to the radius 1, and ranged in tables for uſe. But becauſe operations with theſe natural Sines require much labour in multiplying and dividing by them, the logarithms of them are taken, and ranged in tables alſo; and theſe logarithmic Sines are commonly uſed in practice, inſtead of the natural ones, as they require only additions and ſubtractions, inſtead of the multiplications and diviſions. For the method of conſtructing the ſcales of Sines &c, ſee the article SCALE.

The Sines were introduced into trigonometry by the Arabians. And for the etymology of the word *Sine* ſee Introduction to my Logarithms, pa. 17 &c. And the various ways of calculating tables of the Sines, may be ſeen in the ſame place, pa. 13 &c.

Theorems for the Sines, Coſines, &c, one from another. From the definitions of them, and the common property of right-angled triangles, with that of the circle, viz, that $DE^2 = CD^2 - CE^2 = AE \times EB$, are eaſily deduced theſe following values of the Sines, &c, viz, putting

$s =$ the ſine DE,
$c =$ the coſine CE,
$v =$ verſed ſine BE,
$\mathrm{v} =$ ſuppl. verſed ſine AE,
$r =$ radius AC or CB,
$a =$ arc BD; then

$$s = \sqrt{r^2 - c^2} = \sqrt{\mathrm{v}v} = \sqrt{2rv - vv} = \sqrt{2r\mathrm{v} - \mathrm{vv}}$$
$$c = \sqrt{r^2 - s^2} = r - v = \mathrm{v} - r = \tfrac{1}{2}\mathrm{v} - \tfrac{1}{2}v.$$
$$v = r - c = 2r - \mathrm{v} = r - \sqrt{r^2 - s^2} = \mathrm{v} - 2c.$$
$$\mathrm{v} = r + c = 2r - v = r + \sqrt{r^2 - s^2} = v + 2c.$$

The

The tangent $= \frac{rs}{\sqrt{r^2 - s^2}}$. And Cotang. $= \frac{r\sqrt{r^2 - s^2}}{s}$.

The Secant $= \frac{rr}{\sqrt{r^2 - s^2}}$. And Cofec. $= \frac{rr}{s}$.

$$s = a - \frac{a^3}{2.3\,r^2} + \frac{a^5}{2.3.4.5\,r^4} - \frac{a^7}{2.3.4.5.6.7\,r^6} \text{ \&c.}$$

$$a = s + \frac{s^3}{2.3\,r^2} + \frac{1.3\,s^5}{2.4.5\,r^4} + \frac{1.3.5\,s^7}{2.4.6.7\,r^6} \text{ \&c.}$$

$$\text{Log. } s = \text{log. } a - \text{M}\left(\frac{a^2}{6} + \frac{a^4}{180} + \frac{a^6}{2835} + \frac{a^8}{37800} \text{ \&c}\right)$$

or $\text{Log. } s = -\frac{1}{2}\text{M}\left(c^2 + \frac{1}{2}c^4 + \frac{1}{3}c^6 + \frac{1}{4}c^8 \text{ \&c}\right)$

or $\text{Log. } s = -2\text{M}\left(z + \frac{1}{3}z^3 + \frac{1}{5}z^5 + \frac{1}{7}z^7 \text{ \&c.}\right)$

when $z = \frac{1 - s}{1 + s}$, radius 1, and $\text{M} = {\cdot}43429448$ &c.

If A be any other arc, S its fine, and C its cofine. Then

Sin. $\overline{\text{A} + a} = \frac{\text{S}c + s\text{C}}{r}$. Cof. $\overline{\text{A} + a} = \frac{\text{C}c - \text{S}s}{r}$.

Sin. $\overline{\text{A} - a} = \frac{\text{S}c - s\text{C}}{r}$. Cof. $\overline{\text{A} - a} = \frac{\text{C}c + \text{S}s}{r}$.

Sin. A $\times$ cof. $a = \frac{1}{2}$ fin. $\overline{\text{A} - a} + \frac{1}{2}$ fin. $\overline{\text{A} + a}$.

Sin. A $\times$ fin. $a = \frac{1}{2}$ cof. $\overline{\text{A} - a} - \frac{1}{2}$ cof. $\overline{\text{A} + a}$.

Cof. A $\times$ cof. $a =$ cof. $\frac{\text{A} - a}{2} +$ cof. $\frac{\text{A} + a}{2}$.

If $h = 2{\cdot}718281828$ &c, the number whofe hyp. log. is 1; then

$$\text{Sin. } a = s = \frac{h^{a\sqrt{-1}} - h^{-a\sqrt{-1}}}{2\sqrt{-1}}.$$

$$\text{Cof. } a = c = \frac{h^{a\sqrt{-1}} + h^{-a\sqrt{-1}}}{2}.$$

See many other curious expreffions of this kind in Bougainville's Calcul Integral, and in Bertrand's Mathematics.

From fome of the foregoing theorems the Sines of a great variety of angles, or number of degrees, may be computed. Ex. gr. as below.

Angles.	*Sines.*
90°	r
75	$\frac{1}{2}r\sqrt{2 + \sqrt{3}} = r \times \frac{\sqrt{6} + \sqrt{2}}{4}$
72	$\frac{1}{2}r\sqrt{\frac{5 + \sqrt{5}}{2}}$
$67\frac{1}{2}$	$\frac{1}{2}r\sqrt{2 + \sqrt{2}}$
60	$\frac{1}{2}r\sqrt{3}$
54	$\frac{1}{2}r\sqrt{\frac{3 + \sqrt{5}}{2}} = r \times \frac{\sqrt{5} + 1}{4}$
45	$\frac{1}{2}r\sqrt{2}$
36	$\frac{1}{2}r\sqrt{\frac{5 - \sqrt{5}}{2}}$
30	$\frac{1}{2}r$
$22\frac{1}{2}$	$\frac{1}{2}r\sqrt{2 - \sqrt{2}}$
18	$\frac{1}{2}r\sqrt{\frac{3 - \sqrt{5}}{2}} = r \times \frac{\sqrt{5} - 1}{4}$
15	$\frac{1}{2}r\sqrt{2 - \sqrt{3}} = r \times \frac{\sqrt{6} - \sqrt{2}}{4}$

Radius being 1. Then for multiple arcs:

the Sin. $\overline{n + 1}.a = 2c \times$ fin. $na -$ fin. $\overline{n - 1}.a$,

and Cof. $\overline{n + 1}.a = 2c \times$ cof. $na -$ cof. $\overline{n - 1}.a$;

That is, multiplying any Sine or cofine by $2c$, and the next preceding Sine or cofine fubtracted from it, it gives the next following Sine or cofine. Hence

fin. $0a = 0$.	cof. $0a = 1$ or radius.
fin. $a = s$.	cof. $a = c$.
fin. $2a = 2sc$.	cof. $2a = c^2 - s^2$.
fin. $3a = 3sc^2 - s^3$.	cof. $3a = c^3 - 3cs^2$.
fin. $4a = 4sc^3 - 4s^3c$.	cof. $4a = c^4 - 6c^2s^2 + s^4$.
fin. $5a = 5sc^4 - 10s^3c^2 + s^5$.	cof. $5a = c^5 - 10c^3s^2 + 5cs^4$.
&c.	&c.

And, in general,

$$\text{Sin. } na = nsc^{n-1} - \frac{n.\overline{n-1}.\overline{n-2}}{1.2.3}s^3c^{n-3} + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}.\overline{n-4}}{1.2.3.4.5}s^5c^{n-5} \text{ \&c.}$$

$$\text{or Sin. } na = ns - \frac{n.\overline{n^2-1^2}}{2.3}s^3 + \frac{n.\overline{n^2-1^2}.\overline{n^2-3^2}}{2.3.4.5}s^5 \text{ \&c.}$$

$$\text{Cof. } na = c^n - \frac{n.\overline{n-1}}{2}s^2c^{n-2} + \frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{2.3.4}s^4c^{n-4} \text{ \&c.}$$

$$\text{or Cof. } na = 1 - \frac{n^2}{2}s^2 + \frac{n^2.\overline{n^2-2^2}}{2.3.4}s^4 - \frac{n^2.\overline{n^2-2^2}.\overline{n^2-4^2}}{2.3.4.5.6}s^6 \text{ \&c.}$$

Sin. $\frac{1}{2}a = r\sqrt{\frac{r - c}{2r}}$. And cof. $\frac{1}{2}a = r\sqrt{\frac{r + c}{2r}}$. Radius being r.

Of the Tables of Sines, &c.

In eftimating the quantity of the Sines &c, we affume radius for unity; and then compute the quantity of the Sines, tangents, and fecants, in fractions of it. From Ptolomy's Almageft we learn, that the ancients divided the radius into 60 parts, which they called degrees, and thence determined the chords in minutes,

nutes, seconds, and thirds; that is, in sexagesimal fractions of the radius, which they likewise used in the resolution of triangles. As to the Sines, tangents and secants, they are modern inventions; the Sines being introduced by the Moors or Saracens, and the tangents and secants afterwards by the Europeans. See Introd. to my Logs. pa. 1 to 19.

Regiomontanus, at first, with the ancients, divided the radius into 60 degrees; and determined the Sines of the several degrees in decimal fractions of it. But he afterwards found it would be more convenient to assume 1 for radius, or 1 with any number of cyphers, and take the Sines in decimal parts of it; and thus he introduced the present method in trigonometry. In this way, different authors have divided the radius into more or fewer decimal parts; but in the common tables of Sines and tangents, the radius is conceived as divided into 10000000 parts; by which all the Sines are estimated.

An idea of some of the modes of constructing the tables of Sines, may be conceived from what here follows: First, by common geometry the sides of some of the regular polygons inscribed in the circle are computed, from the given radius, which will be the chords of certain portions of the circumference, denoted by the number of the sides; viz, the side of the triangle the chord of the 3d part, or 120 degrees; the side of the pentagon the chord of the 5th part, or 72 degrees; the side of the hexagon the chord of the 6th part, or 60 degrees; the side of the octagon the chord of the 8th part, or 45 degrees; and so on. By this means there are obtained the chords of several of such arcs; and the halves of these chords will be the Sines of the halves of the same arcs. Then the theorem $c = \sqrt{1 - s^2}$ will give the cosines of the same half arcs. Next, by bisecting these arcs continually, there will be found the Sines and cosines of a continued series as far as we please by these two theorems,

$$\text{Sin. } \tfrac{1}{2} a = \sqrt{\frac{1-c}{2}}; \text{ and cos. } \tfrac{1}{2} a = \sqrt{\frac{1+c}{2}}.$$

Then, by the theorems for the sums and differences of arcs, from the foregoing series, will be derived the Sines and cosines of various other arcs, till we arrive at length at the arc of 1′, or 1″, &c, whose Sine and cosine thus become known.

Or, rather, the sine of 1 minute will be much more easily found from the series

$$s = a - \frac{a^3}{6} + \frac{a^5}{120} \text{ \&c},$$

because the arc is equal to its Sine in small arcs; whence $s = a$ only in such small arcs. But the length of the arc of 180° or 10800′ is known to be 3·14159265, &c; therefore, by proportion, as 10800′ : 1′ :: 3·14159265 : 0·0002908882 = a the arc or s the sine of 1′, which number is true to the last place of decimals. Then, for the cosine of 1′, it is $c = \sqrt{1 - s^2}$ = 0·9999999577 the cosine of the same 1′.

Hence we shall readily obtain the Sines and cosines of all the multiples of 1′, as of 2′, 3′ 4′ 5′, &c, by the application of these two theorems,

$$\text{Sin. } \overline{n+1}. a = 2c \times \text{sin. } na - \text{sin. } \overline{n-1}. a,$$
$$\text{Cos. } \overline{n+1}. a = 2c \times \text{cos. } na - \text{cos. } \overline{n-1}. a;$$

for supposing a = the arc of 1, then c = 0·9999999577, and taking n successively equal to 1, 2, 3, 4, &c, the theorems for the Sines and cosines give severally the Sines and cosines of 1′, 2′, 3′, 4′, &c; viz, the Sines thus:

sin. 1′ = s - - - - - - - - - - = ·0002908882
sin. 2′ = $2c$ × sin. 1′ — sin. 0′ = ·0005817764
sin. 3′ = $2c$ × sin. 2′ — sin. 1′ = ·0008726645
sin. 4′ = $2c$ × sin. 3′ — sin. 2′ = ·0011635526
sin. 5′ = $2c$ × sin. 4′ — sin. 3′ = ·0014544406
&c.

And the Cosines thus,

cos. 1′ = c - - - - - - - - - - - = ·9999999577
cos. 2′ = $2c$ × cos. 1′ — cos. 0′ = ·9999998308
cos. 3′ = $2c$ × cos. 2′ — cos. 1′ = ·9999996192
cos. 4′ = $2c$ × cos. 3′ — cos. 2′ = ·9999993231
cos. 5′ = $2c$ × cos. 4′ — cos. 3′ = ·9999989423
&c.

In this manner then all the Sines and cosines are made, by only one constant multiplication and a subtraction, up to 30 degrees, forming thus the Sines of the first and last 30 degrees of the quadrant, or from 0 to 30° and from 60° to 90°; or, which will be much the same thing, the Sines only may be thus computed all the way up to 60°.

Then the Sines of the remaining 30°, from 60 to 90, will be found by one addition only for each of them, by means of this theorem, viz,

$$\text{Sin. } \overline{60 + a} = \text{sin. } \overline{60 - a} + \text{sin. } a;$$

that is, to the sine of any arc below 60°, add the Sine of its defect below 60, and the sum will be the Sine of another arc which is just as much above 60.

The Sines of all arcs being thus found, they give also very easily the versed sines, the tangents, and the secants. The versed sines are only the arithmetical complements to 1, that is, each cosine taken from the radius 1.

The tangents are found by these three theorems:

1. As cosine to sine, so is radius to tangent.

2. Radius is a mean proportional between the tangent and cotangent.

3. Half the difference between the tangent and cotangent, is equal to the tangent of the difference between the arc and its complement. Or, the sum arising from the addition of double the tangent of an arc with the tangent of half its complement, is equal to the tangent of the sum of that arc and the said half complement.

By the 1st and 2d of these theorems, the tangents are to be found for one half of the quadrant: then the other half of them will be found by one single addition, or subtraction, for each, by the 3d theorem.

This done, the secants will be all found by addition or subtraction only, by these two theorems: 1st. The secant of an arc, is equal to the sum of its tangent and the tangent of half its complement. 2nd. The secant of

of an arc, is equal to the difference between the tangent of that arc and the tangent of the arc added to half its complement.

Artificial SINES, are the logarithmic Sines, or the logarithms of the Sines.

Curve or *Figure of the* SINES. See FIGURE *of the Sines*, &c. To what is there faid of the figure of the Sines, may be here added as follows, from a property juft given above, viz, if x denote the abfcifs of this curve, or the correfponding circular arc, and y its ordinate, or the Sine of that arc; then the equation of the curve will be this,

$$y = \text{fin. } x = \frac{h^{x\sqrt{-1}} - h^{-x\sqrt{-1}}}{2\sqrt{-1}};$$

where $h = 2\cdot718281828$, &c, the number whofe hyp. log. is 1.

Line of SINES, is a line on the fector, or Gunter's fcale, &c, divided according to the Sines, or expreffing the Sines. See thofe articles.

SINE *of Incidence*, or *of Reflection*, or *of Refraction*, is ufed for the Sine of the *angle* of incidence, &c.

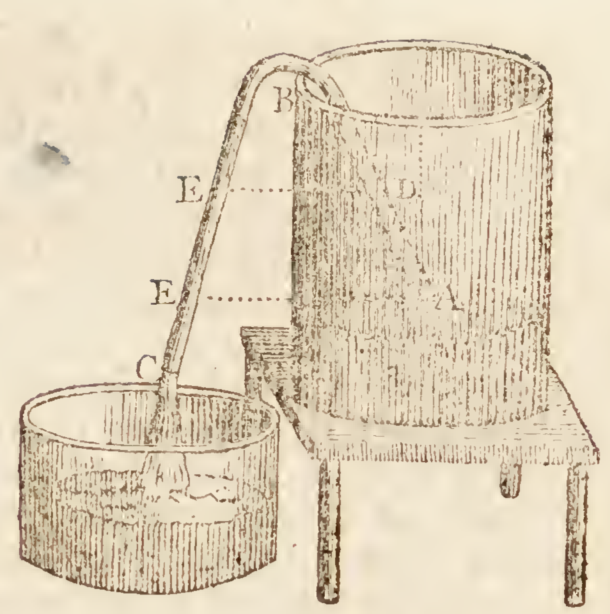
SINICAL *Quadrant*, is a quadrant, made of wood or metal, with lines drawn from each fide interfecting one another, with an index, divided by fines, alfo with 90 degrees on the limb, and two fights at the edge. Its ufe is to take the altitude of the fun. Inftead of the fines, it is fometimes divided all into equal parts; and then it is ufed by feamen to refolve, by infpection, any problem of plane failing.

SIPHON, or SYPHON, in Hydraulics, a crooked pipe or tube ufed in the raifing of fluids, emptying of veffels, and in various hydroftatical experiments. It is otherwife called a crane.

Wolfius defcribes two veffels under the name of Siphons; the one cylindrical in the middle and conical at the two extremes; the other globular in the middle, with two narrow tubes fitted to it axis-wife; both ferving to take up a quantity of liquid, and to retain it when up.

But the moft ufual Syphon is that which is here reprefented; where ABC is any crooked tube, having two legs of unequal lengths; but fuch however that, in any pofition, the perpendicular altitude BD, of B above A, when AB is filled with any fluid, the weight of that fluid may not be more than about 15lb. upon every fquare inch of the bafe, or equal to the preffure of the atmofphere, becaufe the preffure of the atmofphere will raife or fufpend the fluid fo high, when the tube is exhaufted of air. This height is about 30 inches when the fluid is quickfilver, and about 34 feet when it is water; and fo on for other fluids, according to the rarity of them.

To ufe the Siphon, in drawing off any fluid; immerfe the fhorter end A into the fluid, then fuck or draw the air out by the other or lower end C, and the fluid will prefently follow, and run out by the Siphon, from the veffel at A to the veffel at C; till fuch time as the furface of the fluid fink as low as the orifice at A,



when the decanting will ceafe, and the Siphon will empty itfelf of the fluid, the whole of that which is in it running out at C. The principle upon which the Siphon acts, is this: when the tube is exhaufted of air, the preffure of the atmofphere upon the furface of the fluid at D, forces it into the tube by the orifice at A, as in the barometer tube, and down the leg BC, if B is not above the furface at D more than 34 feet for water, or 30 inches for quickfilver, &c. Here, if the external leg of the Siphon terminate at E, on a horizontal level with the immerfed end at A, or rather on a level with the water at D, the perpendicular preffures of the fluid in each leg, and of the external air, againft each orifice, being alike in both, the fluid will be at reft in the Siphon, completely filling it, but without running or preponderating either way. But if the external end be the lower, terminating at C, then the fluid in this end being the heavier, or having more preffure, will preponderate and run out by the orifice at C; this would leave a vacuum at B but for the continual preffure of the atmofphere at D, which forces the fluid up by A to B, and fo producing a continued motion of it through the tube, and a difcharge or ftream at C.

Inftead of fucking out the air at C, another method is, firft to fill the tube completely with the fluid, in an inverted pofition with the angle B downward; and, ftopping the two orifices with the fingers, revert the tube again, and immerge the end A in the fluid; then take off the fingers, and immediately the ftream commences from the end C.

Either of the two foregoing methods can be conveniently practifed when the Siphon is fmall, and eafily managed by the hand; as in decanting off liquors from cafks, &c. But when the Siphon is very large, and many feet in height, as in exhaufting water from a valley or pit, the following method is then recommended: Stop the orifice C, and, by means of an opening made in the top at B, fill the tube completely with water; then ftop the opening at B with a plug, and open that at C; upon which the water will prefently flow out at C, and

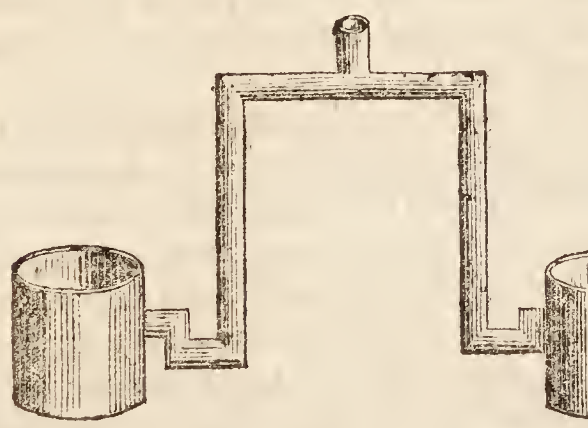
fo continue till that at A is exhaufted. And this method of conveying water over a hill, from one valley to another, is defcribed by Hero, the chief author of any confequence upon this fubject among the Ancients. But in this experiment it muft be noted, that the effect will not be produced when the hill at B is more than 33 or 34 feet above the furface of the water at A.

In an experiment of this kind, it is even faid the water in the legs, unlefs it be purged of its air, will not reft at a height of quite 30 feet above the water in the veffels; becaufe air will extricate itfelf out of the water, and getting above the water in the legs, prefs it downward, fo that its height will be lefs to balance the preffure of the atmofphere. But with very fine, or capillary tubes, the experiment will fucceed to a height fomewhat greater; becaufe the attraction of the matter of the very fine tube will attract the fluid, and fupport it at fome certain height, independent of the preffure of the atmofphere. For which reafon alfo it is, that the experiment fucceeds for fmall heights in the exhaufted receiver; as has been tried both with water and mercury, by Defaguliers and many other philofophers. Exper. Philof. vol. 2, pa. 168.

The figure of the veffel may be varied at pleafure, provided the orifice C be but below the level of the furface of the water to be drawn up, but ftill the farther it is below it, the quicker will the fluid run off. And if, in the courfe of the efflux, the orifice A be drawn out of the fluid; all the liquor in the Siphon will iffue out at the lower orifice C; that in the leg BC dragging, as it were, that in the fhorter leg AB after it.

But if a filled Siphon be fo difpofed, as that both orifices, A and C, be in the fame horizontal line; the fluid will remain pendant in each leg, how unequal foever the length of the legs may be. So that fluids in Siphons feem, as it were, to form one continued body; the heavier part defcending like a chain, and drawing the lighter after it.

The *Wirtemberg* Siphon, is a very extraordinary



machine, performing feveral things which the common Siphon will not reach. This Siphon was projected by Jordan Pelletier, and executed at the expence of prince Frederic Charles, adminiftrator of Wirtemberg, by his mathematician Shahackard, who made each branch 20 feet long, and fet them 18 feet apart; and the defcription of it was publifhed by Reifelius, the duke's phyfician. This gave occafion to Papin to invent another, which performed the fame things, and is defcribed in the Philof. Tranf. vol. 14, or Abr. vol. 1. Reifelius, in another paper in the fame volume, ingenuoufly owns that this is the fame with the Wirtemberg Siphon.

In this engine, though the legs be on the fame level, yet the water rifes up the one, and defcends through the other: The water rifes even through the aperture if the lefs leg be only half immerged in water: The Siphon has its effect after continuing dry a long time: Either of the apertures being opened, the other remaining fhut for a whole day, and then opened, the water flows out as ufual: Laftly, the water rifes and falls indifferently through either leg.

Muffchenbroek, in accounting for the operation of this Siphon, obferves that no difcharge could be made by it, unlefs the water applied to either leg caufe the one to be fhorter, and the other longer by its own weight. Introd. ad Phil. Nat. tom. 2, pa. 853, ed. 4to. 1762.

SIRIUS, the *Dog-ftar*; a very bright ftar of the firft magnitude, in the mouth of the conftellation *Canis Major*, or the Great Dog.

This is the brighteft of all the ftars in our firmanent, and therefore probably, fays Dr. Mafkelyne, the aftronomer royal, the neareft to us of them all, in a paper recommending the difcovery of its parallax, Philof. Tranf. vol. 2, pa. 889. Some however fuppofe Arcturus to be the neareft.

The Arabs call it *Afchere*, *Elfcheere*, *Scera*; the Greeks, *Sirius*; and the Latins, *Canicula*, or *Canis candens*. See CANICULA.

This is one of the earlieft named ftars in the whole heavens. Hefiod and Homer mention only four or five conftellations, or ftars, and this is one of them. Sirius and Orion, the Hyades, Pleiades, and Arcturus are almoft the whole of the old poetical aftronomy. The three laft the Greeks formed of their own obfervation, as appears by the names; the two others were Egyptian. Sirius was fo called from the Nile, one of the names of that river being Siris; and the Egyptians, feeing that river begin to fwell at the time of a particular rifing of this ftar, paid divine honours to the ftar, and called it by a name derived from that of the river, expreffing the ftar of the Nile.

SITUS, in Algebra and Geometry, denotes the fituation of lines, furfaces, &c. Wolfius delivers fome things in geometry, which are not deduced from the common analyfis, particularly matters depending on the *Situs* of lines and figures. Leibnitz has even founded a particular kind of analyfis upon it, called *Calculus Situs*.

SKY, the blue expanfe of the air or atmofphere.

The azure colour of the fky is attributed, by Newton, to vapours beginning to condenfe, having attained confiftence enough to reflect the moft reflexible rays, viz, the violet ones; but not enough to reflect any of the lefs reflexible ones.

De la Hire attributes it to our viewing a black object, viz the dark fpace beyond the regions of the atmofphere, through a white or lucid one, viz the air illuminated by the fun; a mixture of black and white always appearing blue. But this hypothefis is not originally his; being as old as Leonardo da Vinci.

SLIDING, in Mechanics, is when the fame point of a body, moving along a furface, defcribes a line on that

that furface. Such is the motion of a parallelopipedon moved along a plane.

From Sliding arifes friction.

Sliding *Rule*, a mathematical inftrument ferving to perform computations in gauging, meafuring, &c, without the ufe of compaffes; merely by the fliding of the parts of the inftrument one by another, the lines and divifions of which give the anfwer or amount by infpection.

This inftrument is varioufly contrived and applied by different authors, particularly Gunter, Partridge, Hunt, Everard, and Coggefhall; but the moft ufual and ufeful ones are thofe of the two latter.

Everard's Sliding *Rule* is chiefly ufed in cafk gauging. It is commonly made of box, 12 inches long, 1 inch broad, and $\frac{6}{10}$ of an inch thick. It confifts of three parts; viz, the ftock juft mentioned, and two thin flips, of the fame length, fliding in fmall grooves in two oppofite fides of the ftock: confequently, when both thefe pieces are drawn out to their full extent, the inftrument is 3 feet long.

On the firft broad face of the inftrument are four logarithmic lines of numbers; for the properties &c, of which, fee Gunter's *Line*. The firft, marked A, confifting of two radii numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1; and then 2, 3, 4, 5, &c, to 10. On this line are four brafs centre-pins, two in each radius; one in each of them being marked MB, for malt-bufhel, is fet at 2150·42 the number of cubic inches in a malt-bufhel; the other two are marked with A, for ale-gallon, at 282, the number of cubic inches in an ale gallon. The 2d and 3d lines of numbers are on the fliding pieces, and are exactly the fame with the firft; but they are diftinguifhed by the letter B. In the firft radius is a dot, marked *Si*, at ·707, the fide of a fquare infcribed in a circle whofe diameter is 1. Another dot, marked *Se.* ftands at ·886, the fide of a fquare equal to the area of the fame circle. A third dot, marked W, is at 231, the cubic inches in a wine gallon. And a fourth, marked C, at 3·14, the circumference of the fame circle whofe diameter is 1. The fourth line of numbers, marked MD, to fignify malt-depth, is a broken line of two radii, numbered 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, &c; the number 1 being fet directly againft MB on the firft radius.

On the fecond broad face, marked *cd*, are feveral lines: as 1ft, a line marked D, and numbered 1, 2, 3, &c, to 10. On this line are four centre pins: the firft, marked WG, for wine-gauge, is at 17·15, the gauge-point for wine gallons, being the diameter of a cylinder whofe height is one inch, and content 231 cubic inches, or a wine gallon: the fecond centre-pin, marked AG, for ale-gauge, is at 18·95, the like diameter for an ale gallon: the 3d, marked MS, for malt fquare, is at 46·3, the fquare root of 2150·42, or the fide of a fquare whofe content is equal to the number of inches in a folid bufhel: and the fourth, marked MR, for malt-round, is at 52·32, the diameter of a cylinder, or bufhel, the area of whofe bafe is the fame 2150·42, the inches in a bufhel. 2dly, Two lines of numbers on the fliding piece, on the other fide, marked C. On thefe are two dots; the one, marked *c*, at ·0795, the area of a circle whofe circumference is 1; and the other, marked *d*, at ·785, the area of the circle whofe diameter is 1. 3dly, Two lines of fegments, each numbered 1, 2, 3, to 100; the firft for finding the ullage of a cafk, taken as the middle fruftum of a fpheroid, lying with its axis parallel to the horizon; and the other for finding the ullage of a cafk ftanding.

Again, on one of the narrow fides, noted *c*, are, 1ft, a line of inches, numbered 1, 2, 3, &c to 12, each fubdivided into 10 equal parts. 2dly, A line by which, with that of inches, we find a mean diameter for a cafk, in the figure of the middle fruftum of a fpheroid: it is marked *Spheroid*, and numbered 1, 2, 3, &c to 7. 3dly, A line for finding the mean diameter of a cafk, in the form of the middle fruftum of a parabolic fpindle, which gaugers call the fecond variety of cafks; it is therefore marked *Second Variety*, and is numbered 1, 2, 3, &c.

4thly, A line by which is found the mean diameter of a cafk of the third variety, confifting of the fruftums of two parabolic conoids, abutting on a common bafe; it is therefore marked *Third Variety*, and is numbered 1, 2, 3, &c.

On the other narrow face, marked *f*, are 1ft, a line of a foot divided into 100 equal parts, marked FM. 2dly, A line of inches, like that before mentioned, marked IM. 3dly, A line for finding the mean diameter of the fourth variety of cafks, which is formed of the fruftums of two cones, abutting on a common bafe. It is numbered 1, 2, 3, &c; and marked FC, for fruftum of a cone.

On the backfide of the two fliding pieces is a line of inches, from 12 to 36, for the whole extent of the 3 feet, when the pieces are put endwife, and againft that, the correfpondent gallons, and 100th parts, that any fmall tub, or the like open veffel, will contain at 1 inch deep.

For the various ufes of this inftrument, fee the authors mentioned above, and moft other writers on Gauging.

Coggefhall's Sliding *Rule* is chiefly ufed in meafuring the fuperficies and folidity of timber, mafonry, brick-work, &c.

This confifts of two rulers, each a foot long, which are united together in various ways. Sometimes they are made to flide by one another, like glaziers' rules: fometimes a groove is made in the fideof a common two-foot joint rule, and a thin fliding piece in one fide, and Coggefhall's lines added on that fide; thus forming the common or Carpenter's rule: and fometimes one of the two rulers is made to flide in a groove made in the fide of the other.

On the Sliding fide of the rule are four lines of numbers, three of which are double, that is, are lines to two radii, and the fourth is a fingle broken line of numbers. The firft three, marked A, B, C, are figured 1, 2, 3, &c to 9; then 1, 2, 3, &c to 10; the conftruction and ufe of them being the fame as thofe on Everard's Sliding rule. The fingle line, called the *girt line*, and marked D, whofe radius is equal to the two radii of any of the other lines, is broken for the eafier meafuring of timber, and figured 4, 5, 6, 7, 8, 9, 10, 20, 30, &c. From 4 to 5 it is divided into 10 parts, and each 10th fubdivided into 2; and fo on from 5 to 10, &c.

On the backfide of the rule are, 1ft, a line of inch meafure, from 1 to 12; each inch being divided and fubdivided. 2dly, A line of foot meafure, confifting of

of one foot divided into 100 equal parts, and figured 10, 20, 30, &c.

The backside of the sliding-piece is divided into inches, halves, &c, and figured from 12 to 24; so that when the slide is out, there may be a measure of 2 feet.

In the Carpenter's rule, the inch measure is on one side, continued all the way from 1 to 24, when the rule is unfolded, and subdivided into 8ths or half-quarters: on this side are also some diagonal scales of equal parts. And upon the edge, the whole length of 2 feet is divided into 200 equal parts, or 100ths of a foot.

SLING, a string instrument, serving for the casting of stones &c with the greater violence.

Pliny, lib. 76, chap. 5, attributes the invention of the Sling to the Phœnicians; but Vegetius ascribes it to the inhabitants of the Balearic islands, who were celebrated in antiquity for the dextrous management of it. Florns and Strabo say, those people bore three kinds of Slings; some longer, others shorter, which they used according as their enemies were more remote or nearer hand. Diodorus adds, that the first served them for a head-band, the 2d for a girdle, and that the third they constantly carried with them in the hand. But it must be impossible to tell who were the first inventors of the Sling, as the instrument is so simple, and has been in general use by almost all nations. The instrument is much spoken of in the wars and history of the Israelites. David was so expert a slinger, that he ventured to go out, with one in his hand, against the giant and champion Goliath, and at a distance struck him on the forehead with the stone. And there were a number of left-handed men of one of the tribes of Israel, who it is said could Sling a stone at an hair's breadth.

The motion of a stone discharged from a Sling arises from its centrifugal force, when whirled round in a circle. The velocity with which it is discharged, is the same as that which it had in the circle, and is much greater than what can be given to it by the hand alone. And the direction in which it is discharged, is that of the tangent to the circle at the point of discharge. Whence its motion and effect may be computed as a projectile.

SLUSE, or Slusius (*René Francis Walter*) of Vise, a small town in the county of Liege, where he enjoyed honours and preferment. He then became abbé of Amas, canon, councellor and chancellor of Liege, and made his name famous for his knowledge in theology, physics, and mathematics. The Royal Society of London elected him one of their members, and inserted several of his compositions in their Transactions. This very ingenious and learned man died at Liege in 1683, at 63 years of age.

Of Slusius's works there have been published, some learned letters, and a work intitled, *Mesolabium et Problemata solida*; beside the following pieces in the Philosophical Transactions, viz,

1. Short and Easy Method of drawing Tangents to all Geometrical Curves; vol. 7, pa. 5143.
2. Demonstration of the same; vol. 8, pa. 6059, 6119.
3. On the Optic Angle of Alhazen; vol. 8, pa. 6139.

SMEATON (John), F. R. S. and a very celebrated civil engineer, was born the 28th of May 1724, at Austhorpe, near Leeds, in a house built by his grandfather, where the family have resided ever since, and where our author died the 28th of October 1792, in the 68th year of his age.

Mr. Smeaton seems to have been born an engineer. The originality of his genius and the strength of his understanding appeared at a very early age. His playthings were not those of children, but the tools men work with; and he had always more amusement in observing artificers work, and asking them questions, than in any thing else. Having watched some mill-wrights at work, he was one day, soon after, seen (to the distress of his family) on the top of his father's barn, fixing up something like a windmill. Another time, attending some men who were fixing a pump at a neighbouring village, and observing them cut off a piece of bored pipe, he contrived to procure it, of which he made a working pump that actually raised water. These anecdotes refer to circumstances that happened when he was hardly out of petticoats, and probably before he had reached the 6th year of his age. About his 14th or 15th year, he had made for himself an engine to turn rose-work; and he made several presents to his friends of boxes in ivory and wood, turned by him in that way.

His friend and partner in the Deptford Waterworks, Mr. John Holmes, an eminent clock and watch maker in the Strand, says, he visited Mr. Smeaton and spent a month with him at his father's house, in the year 1742, when consequently our author was about 18 years of age. Mr. Holmes could not but view young Smeaton's works with astonishment: he forged his own iron and steel, and melted his own metals; he had tools of every sort, for working in wood, ivory, and metals: he had made a lathe, by which he had cut a perpetual screw in brass, a thing very little known at that day.

Thus had Mr. Smeaton, by the strength of his genius, and indefatigable industry, acquired, at 18 years of age, an extensive set of tools, and the art of working in most of the mechanical trades, without the assistance of any master, and which he continued to do a part of every day when at the place where his tools were: and few men could work better.

Mr. Smeaton's father was an attorney, and was desirous of bringing him up to the same profession. He therefore came up to London in 1742, and for some time attended the courts in Westminster Hall. But finding that the profession of the law did not suit *the bent of his genius*, as his usual expression was, he wrote a strong memorial to his father on the subject, whose good sense from that moment left Mr. Smeaton to pursue the bent of his genius in his own way.

Mr. Smeaton after this continued to reside in London, and about 1750 he commenced philosophical instrument maker, which he continued for some time, and became acquainted with most of the ingenious men of that time; and this same year he made his first communication to the Royal Society, being an account of Dr. Knight's improvements of the mariner's compass. Continuing his very useful labours, and making experiments, he communicated to that learned body, the two following years, a number of other ingenious improvements,

ments, as will be enumerated in the lift of his writings, at the end of this account of him.

In 1751 he began a courfe of experiments, to try a machine of his invention, for meafuring a fhip's way at fea; and alfo made two voyages in company with Dr. Knight to try it, as well as a compafs of his own invention.

In 1753 he was elected a member of the Royal Society; and in 1759 he was honoured with their gold medal, for his paper concerning the natural powers of water and wind to turn mills, and other machines depending on a circular motion. This paper, he fays, was the refult of experiments made on working models in the years 1752 and 1753, but not communicated to the Society till 1759, having in the interval found opportunities of putting the refult of thefe experiments into real practice, in a variety of cafes, and for various purpofes, fo as to affure the Society he had found them to anfwer.

In 1754 his great thirft after experimental knowledge led him to undertake a voyage to Holland and the Low Countries, where he made himfelf acquainted with moft of the curious works of art fo frequent in thofe places.

In December 1755, the Edyftone lighthoufe was burnt down, and the proprietors, being defirous of rebuilding it in the moft fubftantial manner, enquired of the earl of Macclesfield, then prefident of the Royal Society, who he thought might be the fitteft perfon to rebuild it. when he immediately recommended our author. Mr. Smeaton accordingly undertook the work, which he completed with ftone in the fummer of 1759. Of this work he gives an ample defcription in a folio volume, with plates, publifhed in 1791. A work which contains, in a great meafure, the hiftory of four years of his life, in which the originality of his genius is fully difplayed, as well as his activity, induftry, and perfeverance.

Though Mr. Smeaton completed the building of the Edyftone lighthoufe in 1759, yet it feems he did not foon get into full bufinefs as a civil engineer; for in 1764, while in Yorkfhire, he offered himfelf a candidate for one of the receivers of the Derwentwater eftate; in which he fucceeded, though two other perfons, ftrongly recommended and powerfully fupported, were candidates for the employment. In this appointment he was very happy, by the affiftance and abilities of his partner Mr. Walton the younger, of Farnacres near Newcaftle, one of the prefent receivers, who, taking upon himfelf the management and the accounts, left Mr. Smeaton leifure and opportunity to exert his abilities on public works, as well as to make many improvements in the mills, and in the eftates of Greenwich hofpital.

By the year 1775, he had fo much bufinefs, as a civil engineer, that he was defirous of refigning the appointment for that hofpital, and would have done it then, had not his friends prevailed upon him to continue in the office about two years longer.

Mr. Smeaton having thus got into full bufinefs as a civil engineer, it would be an endlefs tafk to enumerate all the variety of concerns he was engaged in. A very few of them however may be juft mentioned in this place. —He made the river Calder navigable: a work that required great fkill and judgment; owing to the very impetuous floods in that river.—He planned and attended the execution of the great canal in Scotland, for conveying the trade of the country, either to the Atlantic or German ocean; and having brought it to a conclufion, he declined a handfome yearly falary, that he might not be prevented from attending to the multiplicity of his other bufinefs.

On opening the great arch at London bridge, the excavation around and under the fterlings was fo confiderable, that it was thought the bridge was in great danger of falling; the apprehenfions of the people on this head being fo great, that few would pafs over or under it. He was then in Yorkfhire, where he was fent for by exprefs, and he arrived in town with the greateft expedition. He applied himfelf immediately to examine it, and to found about the fterlings as minutely as he could. The committee being called together, adopted his advice, which was, to repurchafe the ftones that had been taken from the middle pier, then lying in Moorfields, and to throw them into the river to guard the fterlings, a practice he had before adopted on other occafions. Nothing fhews the apprehenfions of the bridge falling, more than the alacrity with which his advice was purfued: the ftones were repurchafed that day; horfes, carts, and barges were got ready, and the work inftantly begun though it was Sunday morning. Thus Mr. Smeaton, in all human probability, faved London bridge from falling, and fecured it till more effectual methods could be taken.

In 1771, he became, jointly with his friend Mr. Holmes above mentioned, proprietor of the works for fupplying Deptford and Greenwich with water; which by their united endeavours they brought to be of general ufe to thofe they were made for, and moderately beneficial to themfelves.

About the year 1785, Mr. Smeaton's health began to decline; in confequence he then took the refolution to endeavour to avoid any new undertakings in bufinefs as much as he could, that he might thereby alfo have the more leifure to publifh fome account of his inventions and works. Of this plan however he got no more executed than the account of the Edyftone lighthoufe, and fome preparations for his intended treatife on mills; for he could not refift the folicitations of his friends in various works; and Mr. Aubert, whom he greatly loved and refpected, being chofen chairman of Ramfgate harbour, prevailed upon him to accept the office of engineer to that harbour; and to their joint efforts the public are chiefly indebted for the improvements that have been made there within thefe few years; which fully appears in a report that Mr. Smeaton gave in to the board of truftees in 1791, which they immediately publifhed.

It had for many years been the practice of Mr. Smeaton to fpend part of the year in town, and the remainder in the country, at his houfe at Aufthorpe; on one of thefe excurfions in the country, while walking in his garden, on the 16th of September 1792, he was ftruck with the palfy, which put an end to his ufeful life the 28th of October following, to the great regret of a numerous fet of friends and acquaintances.

The great variety of mills conftructed by Mr. Smeaton, fo much to the fatisfaction and advantage of the owners, will fhew the great ufe he made of his experiments

ments in 1752 and 1753. Indeed he fcarcely trufted to theory in any cafe where he could have an opportunity to inveftigate it by experiment; and for this purpofe he built a fteam-engine at Aufthorpe, that he might make experiments exprefsly to afcertain the power of Newcomen's fteam-engine, which he improved and brought to a much greater degree of certainty, both in its conftruction and powers, than it was before.

During many years of his life, Mr Smeaton was a conftant attendant on parliament, his opinion being continually called for. And here his natural ftrength of judgment and perfpicuity of expreffion had their full difplay. It was his conftant practice, when applied to, to plan or fupport any meafure, to make himfelf fully acquainted with it, and be convinced of its merits, before he would be concerned in it. By this caution, joined to the clearnefs of his defcription, and the integrity of his heart, he feldom failed having the bill he fupported carried into an act of parliament. No perfon was heard with more attention, nor had any one ever more confidence placed in his teftimony. In the courts of law he had feveral compliments paid to him from the bench, by the late lord Mansfield and others, on account of the new light he threw upon difficult fubjects.

As a civil engineer, he was perhaps unrivalled, certainly not excelled by any one, either of the prefent or former times. His building the Edyftone lighthoufe, were there no other monument of his fame, would eftablifh his character. The Edyftone rocks have obtained their name from the great variety of contrary *fets* of the tide or current in their vicinity. They are fituated nearly S. S. W. from the middle of Plymouth Sound. Their diftance from the port of Plymouth is about 14 miles. They are almoft in the line which joins the Start and the Lizard points; and as they lie nearly in the direction of veffels coafting up and down the channel, they were unavoidably, before the eftablifhment of a light-houfe there, very dangerous, and often fatal to fhips. Their fituation with regard to the Bay of Bifcay and the Atlantic is fuch, that they lie open to the fwells of the bay and ocean, from all the fouth-weftern points of the compafs; fo that all the heavy feas from the fouth-weft quarter come uncontroled upon the Edyftone rocks, and break upon them with the utmoft fury. Sometimes, when the fea is to all appearance fmooth and even, and its furface unruffled by the flighteft breeze, the *ground fwell* meeting the flope of the rocks, the fea beats upon them in a frightful manner, fo as not only to obftruct any work being done on the rock, or even landing upon it, when, figuratively fpeaking, you might go to fea in a walnut-fhell. That circumftances fraught with danger furrounding it fhould lead mariners to wifh for a light-houfe, is not wonderful; but the danger attending the erection leads us to wonder that any one could be found hardy enough to undertake it. Such a man was firft found in the perfon of Mr. H. Winftanley, who, in the year 1696, was furnifhed by the Trinity-houfe with the neceffary powers. In 1700 it was finifhed; but in the great ftorm of November 1703, it was deftroyed, and the projector perifhed in the ruins. In 1709 another, upon a different conftruction, was erected by a Mr. Rudyerd, which, in 1755, was unfortunately confumed by fire. The next building was under the direction of Mr. Smeaton, who, having confidered the errors of the former conftructions, has judicioufly guarded againft them, and erected a building, the demolition of which feems little to be dreaded, unlefs the rock on which it is erected fhould perifh with it.——Of his works, in conftructing bridges, harbours, mills, engines, &c, &c, it were endlefs to fpeak. Of his inventions and improvements of philofophical inftruments, as of the air-pump, the pyrometer, hygrometer, &c, &c, fome idea may be formed from the lift of his writings inferted below.

In his perfon, Mr. Smeaton was of a middle ftature, but broad and ftrong made, and poffeffed of an excellent conftitution. He had a great fimplicity and plainnefs in his manners: he had a warmth of expreffion that might appear, to thofe who did not know him well, to border on harfhnefs; but fuch as were more clofely acquainted with him, knew it arofe from the intenfe application of his mind, which was always in the purfuit of truth, or engaged in the inveftigation of difficult fubjects. He would fometimes break out haftily, when any thing was faid that was contrary to his ideas of the fubject; and he would not give up any thing he argued for, till his mind was convinced by found reafoning.

In all the focial duties of life, Mr. Smeaton was exemplary; he was a moft affectionate hufband, a good father, a warm, zealous and fincere friend, always ready to affift thofe he refpected, and often before it was pointed out to him in what way he could ferve them. He was a lover and an encourager of merit wherever he found it; and many perfons now living are in a great meafure indebted for their prefent fituation to his affiftance and advice. As a companion, he was always entertaining and inftructive, and none could fpend their time in his company without improvement.

As to the lift of his writings; befide the large work abovementioned, being the Hiftory of Edyftone Lighthoufe, and numbers of reports and memorials, many of which were printed, his communications to the Royal Society, and inferted in their Tranfactions, are as follow:

1. An Account of Dr. Knight's Improvements of the Mariner's Compafs; an. 1750, pa. 513.
2. Some improvements in the Air-pump; an. 1752, pa. 413.
3. An Engine for raifing Water by Fire; being an improvement on Savary's conftruction, to render it capable of working itfelf: invented by M. de Moura, of Portugal. Ib. pa. 436.
4. Defcription of a new Tackle, or Combination of Pulleys. Ib. 494.
5. Experiments upon a machine for meafuring the Way of a Ship at Sea. An. 1754, pa. 532.
6. Defcription of a new Pyrometer. Ib. pa. 598.
7. Effects of Lightning on the Steeple and Church of Leftwithial in Cornwall. An. 1757, pa. 198.
8. Remarks on the different Temperature of the Air at Edyftone Light-houfe, and at Plymouth. An. 1758, pa. 488.
9. Experimental enquiry concerning the natural powers of Water and Wind to turn mills and other machines depending on a circular motion. An. 1759, pa. 100.

10. On

10. On the Menſtrual Parallax ariſing from the mutual gravitation of the earth and moon, its influence on the obſervation of the ſun and planets, with a method of obſerving it. An. 1768, pa. 156.

11. Deſcription of a new method of Obſerving the heavenly bodies out of the meridian. An. 1768, pa. 170.

12. Obſervations on a Solar Eclipſe. An. 1769, pa. 286.

13. Deſcription of a new Hygrometer. An. 1771, pa. 198.

14. An Experimental Examination of the quantity and proportion of Mechanical Power, neceſſary to be employed in giving different degrees of velocity to heavy bodies from a ſtate of reſt. An. 1776, pa. 450.

SMOKE, or *Smoak*, a humid matter exhaled in form of vapour by the action of heat, either external or internal; or Smoke conſiſts of palpable particles, elevated by means of the rarefying heat, or by the force of the aſcending current of air, from certain bodies expoſed to heat; which particles vary much in their properties, according to the ſubſtances from which they are produced.

Sir Iſaac Newton obſerves, that Smoke aſcends in the chimney by the impulſe of the air it floats in: for that air, being rarefied by the heat of the fire underneath, has its ſpecific gravity diminiſhed; and thus, being diſpoſed to aſcend itſelf, it carries up the Smoke along with it. The tail of a comet, the ſame author ſuppoſes, aſcends from the nucleus after the ſame manner.

Smoke of fat unctuous woods, as fir, beech, &c, makes what is called lamp-black.

There are various inventions for preventing and curing ſmoky chimneys: as the æolipiles of Vitruvius, the ventiducts of Cardan, the windmills of Bernard, the capitals of Serlio, the little drums of Paduanus, and ſeveral artifices of De Lorme. See alſo the philoſophical works of Dr. Franklin. Pans, reſembling ſugar pans, placed over the tops of chimneys, are uſeful to make them draw better; and the fire-grates called regiſter-ſtoves, are always a ſure remedy.

In the Philoſophical Tranſactions is the deſcription of an engine, invented by M. Daleſme, which conſumes the Smoke of all ſorts of wood ſo effectually, that the eye cannot diſcover it in the room, nor the noſe diſtinguiſh the ſmell of it, though the fire be made in the middle of the room. It conſiſts of ſeveral iron hoops, 4 or 5 inches in diameter, which ſhut into one another, and is placed on a trevet.

The late invention called Argand's lamp, alſo conſumes the Smoke, and gives a very ſtrong light. Its principle is a thin broad cotton wick, rolled into the form of a hollow cylinder; the air paſſes up the hollow of it, and the Smoke is almoſt all conſumed.

Smoke *Jack*, is a jack for turning a ſpit, turned by the Smoke of the kitchen fire, by means of thin iron ſails ſet obliquely on an axis in the flue of the chimney. See Jack.

SNELL (Rodolph), a reſpectable Dutch philoſopher, was born at Oudenwater in 1546. He was ſome time profeſſor of Hebrew and mathematics at Leyden, where he died in 1613, at 67 years of age. He was author of ſeveral works on geometry, and on all parts of the philoſophy of his time; but I have not obtained a particular liſt of them.

Snell (*Willebrord*), ſon of Rodolph above mentioned, an excellent mathematician, was born at Leyden in 1591, where he ſucceeded his father in the mathematical chair in 1613, and where he died in 1626, at only 35 years of age.

Willebrord Snell was author of ſeveral ingenious works and diſcoveries. Thus, it was he who firſt diſcovered the true law of the refraction of the rays of light; a diſcovery which he made before it was announced by Des Cartes, as Huygens aſſures us. Though the work which Snell prepared upon this ſubject, and upon optics in general, was never publiſhed, yet the diſcovery was very well known to belong to him, by ſeveral authors about his time, who had ſeen it in his manuſcripts.—He undertook alſo to meaſure the earth. This he effected by meaſuring a ſpace between Alcmaer and Bergen-op-zoom, the difference of latitude between theſe places being 1° 11′ 30″. He alſo meaſured another diſtance between the parallels of Alcmaer and Leyden; and from the mean of both theſe meaſurements, he made a degree to conſiſt of 55021 French toiſes or fathoms. Theſe meaſures were afterwards repeated and corrected by Muſſchenbroek, who found the degree to contain 57033 toiſes.—He was author of a great many learned mathematical works, the principal of which are,

1. *Apollonius Batavus*; being the reſtoration of ſome loſt pieces of Apollonius, concerning Determinate Section, with the Section of a Ratio and Space: in 4to, 1608, publiſhed in his 17th year.

2. *Eratoſthenes Batavus*; in 4to, 1617. Being the work in which he gives an account of his operations in meaſuring the earth.

3. A tranſlation out of the Dutch language, into Latin, of Ludolph van Collen's book *De Circulo & Adſcriptis*, &c; in 4to, 1619.

4. *Cyclometricus, De Circuli Dimenſione* &c; 4to, 1621. In this work, the author gives ſeveral ingenious approximations to the meaſure of the circle, both arithmetical and geometrical.

5. *Tiphis Batavus*; being a treatiſe on Navigation and Naval Affairs; in 4to, 1624.

6. A poſthumous treatiſe, being four books *Doctrinæ Triangulorum Canonicæ*; in 8vo, 1627. In which are contained the canon of ſecants; and in which the conſtruction of ſines, tangents, and ſecants, with the dimenſion or calculation of triangles, both plane and ſpherical, are briefly and clearly treated.

7. Heſſian and Bohemian Obſervations; with his own notes.

8. *Libra Aſtronomica & Philoſophica*; in which he undertakes the examination of the principles of Galileo concerning comets.

9. Concerning the Comet which appeared in 1618, &c.

SNOW, a well known meteor, formed by the freezing of the vapours in the atmoſphere. It differs from hail and hoar-froſt in being as it were cryſtallized, which they are not. This appears on examination of a flake of Snow by a magnifying glaſs; when the whole of it appears to be compoſed of fine ſhining ſpicula diverging like rays from a centre. As the flakes deſcend

through the atmosphere, they are continually joined by more of these radiated spicula, and thus increase in bulk like the drops of rain or hailstones; so that it seems as if the whole body of Snow were an infinite mass of icicles irregularly figured.

The lightness of Snow, although it is firm ice, is owing to the excess of its surface, in comparison to the matter contained under it; as even gold itself may be extended in surface, till it will float upon the least breath of air.

According to Beccaria, clouds of Snow differ in nothing from clouds of rain, but in the circumstance of cold that freezes them. Both the regular diffusion of the Snow, and the regularity of the structure of its parts, shew that clouds of Snow are acted upon by some uniform cause like electricity; and he endeavours to shew how electricity is capable of forming these figures. He was confirmed in his conjectures by observing, that his apparatus for shewing the electricity of the atmosphere, never failed to be electrified by Snow as well as by rain. Professor Wintrop sometimes found his apparatus electrified by Snow when driven about by the wind, though it had not been affected by it when the Snow itself was falling. A more intense electricity, according to Beccaria, unites the particles of hail more closely than the more moderate electricity does those of Snow, in the same manner as we see that the drops of rain which fall from the thunder-clouds, are larger than those which fall from others, though the former descend through a less space.

In the northern countries, the ground is covered with snow for several months; which proves exceedingly favourable for vegetation, by preserving the plants from those intense frosts which are common in such countries, and which would certainly destroy them. Bartholin ascribes great virtues to Snow-water, but experience does not seem to warrant his assertions. Snow-water, or ice-water, is always deprived of its fixed air: and those nations who live among the Alps, and use it for their constant drink, are subject to affections of the throat, which it is thought are occasioned by it.

From some late experiments on the quantity of water yielded by Snow, it appears that the latter gives only about one-tenth of its bulk in water.

SOCIETY, an assemblage or union of several learned persons, for their mutual assistance, improvement, or information, and for the promotion of philosophical or other knowledge. There are various philosophical Societies instituted in different parts of the world. See ROYAL *Society*.

American Philosophical SOCIETY, was established at Philadelphia in the year 1769, for promoting useful knowledge, under the direction of a patron, a president, three vice-presidents, a treasurer, four secretaries, and three curators. The first volume of their Transactions comprehends a period of two years, viz, from Jan. 1, 1769, to Jan. 1, 1771. Their labours seem to have been interrupted during the troubles in America, which commenced soon after; but since their termination, some more volumes have been published, containing a number of very ingenious and useful memoirs.

American Academy of Arts and Sciences, was established by a law of the Commonwealth of Massachusetts in North America, in the year 1780.

Boston Academy of Arts and Sciences. This is a Society similar to the former, which has lately been established at Boston in New England, under the title of the Academy of Arts and Sciences &c.

Berlin SOCIETY. The Society of Natural Historians at Berlin, was founded by Dr. Martini. There is also a Philosophical Society in the same place.

Brussels SOCIETY. The Imperial and Royal Academy of Sciences and Belles Lettres of Brussels was founded in 1773. Several volumes of their Transactions have now been published.

Dublin SOCIETY. This is an Experimental Society, for promoting natural knowledge, which was instituted in 1777: the members meet once a week, and distribute three honorary gold medals annually for the most approved discovery, invention, or essay, on any mathematical or philosophical subject. The Society is under the direction of a president, two vice-presidents, and a secretary.

Edinburgh Philosophical SOCIETY, succeeded the Medical Society, and was formed upon the plan of including all the different branches of natural knowledge and the antiquities of Scotland. The meetings of this Society, interrupted in 1745, were revived in 1752; and in 1754 the first volume of their collection was published, under the title of Essays or Observations Physical and Literary, which has been succeeded by other volumes. This Society has been lately incorporated by royal charter, under the name of the Royal Society of Scotland, instituted for the advancement of learning and useful knowledge. The members are divided into two classes, physical and literary; and those who are near enough to Edinburgh to attend the meetings, pay a guinea on admission, and the same sum annually. The first meeting was held on the first Monday of August 1783; when there were chosen, a president, two vice-presidents, a secretary, treasurer, and a council of 12 persons. Three of the volumes of their Transactions have been published, which are very respectable both for their magnitude and contents.

In *France* there have been several institutions of this kind for the improvement of science, besides those recounted under the word ACADEMY: As, the Royal Academy at Soissons, founded in 1674; at Villefranche, Beaujolois, in 1679; at Nismes, in 1682; at Angers, in 1685; the Royal Society at Montpelier, in 1706, which is so intimately connected with the Royal Academy of Sciences of Paris, as to form with it, in some respects, one body; the literary productions of this Society are published in the memoirs of the academy: the Royal Academy of Sciences and Belles Lettres at Lyons, in 1700; at Bourdeaux, in 1703; at Marseilles, in 1726; at Rochelle, in 1734; at Dijon, in 1740; at Pau in Bern, in 1721; at Beziers, in 1723; at Montauban, in 1744; at Rouen, in 1744; at Amiens, in 1750; at Toulouse, in 1750; at Besançon, in 1752; at Metz, in 1760; at Arras, in 1773; and at Chalons sur Maine, in 1775. For other institutions of a similar nature, and their literary productions, see the articles ACADEMY, JOURNAL, and TRANSACTIONS.

Manchester Literary and Philosophical SOCIETY, is of considerable reputation, and has been lately established there, under the direction of two presidents, four vice-presidents, and two secretaries. The number of

of members is limited to 50; besides these there are several honorary members, all of whom are elected by ballot; and the officers are chosen annually in April. Several valuable essays have been already read at the meetings of this Society.

Newcastle-upon-Tyne Literary and Philosophical Society. This Society was instituted the 7th of February 1793, under the direction of a president, four vice-presidents, two secretaries, a treasurer, which together with four of the ordinary members form a committee, all annually elected at a general meeting. The subjects proposed for the consideration and improvement of this Society, comprehend the mathematics, natural philosophy and history, chemistry, polite literature, antiquities, civil history, biography, questions of general law and policy, commerce, and the arts. From such ample scope in the objects of the Society, with the known respectability, zeal, and talents of the members, the greatest improvements and discoveries may be expected to be made in those important branches of useful knowledge.

SOCRATES, the chief of the ancient philosophers, was born at Alopece, a small village of Attica, in the 4th year of the 77th olympiad, or about 467 years before Christ. Sophroniscus, his father, being a statuary or carver of images in stone, our author followed the same profession for some time, for a subsistence. But being naturally averse to this profession, he only followed it when necessity compelled him; and upon getting a little before-hand, would for a while lay it aside. These intermissions of his trade were bestowed upon philosophy, to which he was naturally addicted; and this being observed by Crito, a rich philosopher of Athens, Socrates was at length taken from his shop, and put into a condition of philosophising at his ease and leisure.

He had various instructors in the sciences, as Anaxagoras, Archylaus, Damon, Prodicus, to whom may be added the two learned women Diotyma and Aspasia, of the last of whom he learned rhetoric: of Euenus he learned poetry; of Ichomachus, husbandry; and of Theodorus, geometry.

At length he began himself to teach; and was so eloquent, that he could lead the mind to approve or disapprove whatever he pleased; but never used this talent for any other purpose than to conduct his fellow citizens into the path of virtue. The academy of the Lycæum, and a pleasant meadow without the city on the side of the river Ilyssus, were places where he chiefly delivered his instructions, though it seems he was never out of his way in that respect, as he made use of all times and places for that purpose.

He is represented by Xenophon as excellent in all kinds of learning, and particularly instances arithmetic, geometry, and astrology or astronomy: Plato mentions natural philosophy; Idomeneus, rhetoric; Laertius, medicine. Cicero affirms, that by the testimony of all the learned, and the judgment of all Greece, he was, as well in wisdom, acuteness, politeness, and subtlety, as in eloquence, variety, and richness, in whatever he applied himself to, without exception, the prince of all.

It has been observed by many, that Socrates little affected travel; his life being wholly spent at home, excepting when he went out upon military services. In the Peloponnesian war he was thrice personally engaged: upon which occasions it is said he outwent all the soldiers in hardiness: and if at any time, saith Alcibiades, as it often happens in war, the provisions failed, there were none who could bear the want of meat and drink like Socrates; yet, on the other hand, in times of feasting, he alone seemed to enjoy them; and though of himself he would not drink, yet being invited, he far outdrank every one, though he was never seen intoxicated.

To this great philosopher Greece was principally indebted for her glory and splendor. He formed the manners of the most celebrated persons of Greece, as Alcibiades, Xenophon, Plato, &c. But his great services and the excellent qualities of his mind could not secure him from envy, persecution, and calumny. The thirty tyrants forbad his instructing youth; and as he derided the plurality of the Pagan deities, he was accused of impiety. The day of trial being come, Socrates made his own defence, without procuring an advocate, as the custom was, to plead for him. He did not defend himself with the tone and language of a suppliant or guilty person, but, as if he were master of the judges themselves, with freedom, firmness, and some degree of contumacy. Many of his friends also spoke in his behalf; and lastly, Plato went up into the chair, and began a speech in these words: "Though I, Athenians, am the youngest of those that come up into this place"—but they stopped him, crying out, "of those that go down," which he was thereupon constrained to do; and then proceeding to vote, they condemned Socrates to death, which was effected by means of poison, when he was 70 years of age. Plato gives an affecting account of his imprisonment and death, and concludes, "This was the end of the best, the wisest, and the justest of men." And that account of it by Plato, Tully professes, he could never read without tears.

As to the person of Socrates, he is represented as very homely; he was bald, had a dark complexion, a flat nose, eyes sticking out, and a severe downcast look. But the defects of his person were amply compensated by the virtues and accomplishments of his mind. Socrates was indeed a man of all virtues; and so remarkably frugal, that how little soever he had, it was always enough. When he was amidst a great variety of rich and expensive objects, he would often say to himself, "How many things are there which I do not want!"

Socrates had two wives, one of which was the noted Xantippe; whom Aulus Gellius describes as an accursed froward woman, always chiding and scolding, by day and by night, and whom it was said he made choice of as a trial and exercise of his temper. Several instances are recorded of her impatience and his forbearance. One day, before some of his friends, she fell into the usual extravagances of her passion; when he, without answering a word, went abroad with them: but on his going out of the door, she ran up into the chamber, and threw down water upon his head; upon which, turning to his friends, "Did not I tell you

(says he), that after so much thunder we should have rain?" Another time she pulled his cloak from his shoulders in the open forum; and some of his friends advising him to beat her, "Yes (says he), that while we two fight, you may all stand by, and cry, Well done, Socrates; to him, Xantippe."

They who affirm that Socrates wrote nothing, mean only in respect to his philosophy; for it is attested and allowed, that he assisted Euripides in composing tragedies, and was the author of some pieces of poetry. Dialogues also and epistles are ascribed to him: but his philosophical disputations were committed to writing only by his scholars; and that chiefly by Plato and Xenophon. The latter set the example to the rest in doing it first, and also with the greatest punctuality; as Plato did it with the most liberty, intermixing so much of his own, that it is hardly possible to know what part belongs to each. Hence Socrates, hearing him recite his Lysis, cried out, "How many things doth this young man feign of me!" Accordingly, the greatest part of his philosophy is to be found in the writings of Plato. To Socrates is ascribed the first introduction of moral philosophy. Man having a twofold relation to things divine and human, his doctrines were with regard to the former metaphysical, to the latter moral. His metaphysical opinions were chiefly, that, There are three principles of all things, God, matter, and idea. God is the universal intellect; matter the subject of generation and corruption; idea, an incorporeal substance, the intellect of God; God the intellect of the world. God is one, perfect in himself, giving the being and well-being of every creature.—That God, not chance, made the world and all creatures, is demonstrable from the reasonable disposition of their parts, as well for use as defence; from their care to preserve themselves, and continue their species.—That he particularly regards man in his body, appears from his noble upright form, and from the gift of speech; in his soul, from the excellency of it above others.—That God takes care of all creatures, is demonstrable from the benefit he gives them of light, water, fire, and fruits of the earth in due season. That he hath a particular regard of man, from the destination of all plants and creatures for his service; from their subjection to man, though they may exceed him ever so much in strength; from the variety of man's sense, accommodated to the variety of objects, for necessity, use, and pleasure; from reason, by which he discourseth through reminiscenee from sensible objects; from speech, by which he communicates all he knows, gives laws, and governs states. Finally, that God, though invisible himself, at once sees all, hears all, is every where, and orders all.

As to the other great object of metaphysical research, the soul, Socrates taught, that it is pre-existent to the body, endued with the knowledge of eternal ideas, which in its union to the body it loseth, as stupefied, until awakened by discourse from sensible objects; on which account, all its learning is only reminiscence, a recovery of its first knowledge. That the body, being compounded, is dissolved by death; but that the soul, being simple, passeth into another life, incapable of corruption. That the souls of men are divine. That the souls of the good after death are in a happy state, united to God in a blessed inaccessible place; that the bad in convenient places suffer condign punishment.

All the Grecian sects of philosophers refer their origin to the discipline of Socrates; particularly the Platonics, Peripatetics, Academics, Cyrenaics, Stoics, &c.

SOL, in Astrology, &c, signifies the sun.

SOLAR, something relating to the sun. Thus, we say Solar fire in contradistinction to culinary fire.

SOLAR *Civil Month*. See MONTH.

SOLAR *Cycle*. See CYCLE.

SOLAR *Comet*. See DISCUS.

SOLAR *Eclipse*, is a privation of the light of the sun, by the interposition of the opake body of the moon. See ECLIPSE.

SOLAR *Month*, *Rising*, *Spots*. See the substantives.

SOLAR *System*, the order and disposition of the several heavenly bodies, which revolve round the sun as the centre of their motion; viz, the planets, primary and secondary, and the comets. See SYSTEM.

SOLAR *Year*. See YEAR.

SOLID, in Physics, a body whose minute parts are connected together, so as not to give way, or slip from each other, on the smallest impression. The word is used in this sense, in contradistinction to fluid.

SOLID, in Geometry, is a magnitude extended in every possible direction, quite around. Though it is commonly said to be endued with three dimensions only, length, breadth, and depth or thickness.

Hence, as all bodies have these three dimensions, and nothing but bodies, Solid and body are often used indiscriminately.

The extremes of Solids are surfaces. That is, Solids are terminated either by one surface, as a globe, or by several, either plane or curved. And from the circumstances of these, Solids are distinguished into regular and irregular.

Regular SOLIDS, are those that are terminated by regular and equal planes. These are the tetraedron, hexaedron, or cube, octaedron, dodecaedron, and icosaedron; nor can there possibly be more than these five regular Solids or bodies, unless perhaps the sphere or globe be considered as one of an infinite number of sides. See these articles severally, also the article *Regular* BODY.

Irregular SOLIDS, are all such as do not come under the definition of regular ones: such as cylinder, cone, prism, pyramid, &c.

Similar Solids are to one another in the triplicate ratio of their like sides, or as the cubes of the same. And all sorts of prisms, as also pyramids, are to one another in the compound ratio of their bases and altitudes.

SOLID *Angle*, is that formed by three or more plane angles meeting in a point; like an angle of a die, or the point of a diamond well cut.

The sum of all the plane angles forming a Solid angle, is always less than 360°; otherwise they would constitute the plane of a circle, and not a Solid.

Atmosphere of SOLIDS. See ATMOSPHERE.

SOLID *Bastion*. See BASTION.

Cubature of SOLIDS. See CUBATURE and SOLIDITY.

Measure

Measure of a Solid. See Measure.

Solid *Foot.* See Foot.

Solid *Numbers*, are those which arise from the multiplication of a plane number, by any other number whatever. Thus, 18 is a Solid number, produced from the plane number 6 and 3, or from 9 and 2.

Solid *Place.* See Locus.

Solid *Problem*, is one which cannot be constructed geometrically; but by the intersection of a circle and a conic section, or by the intersection of two conic sections. Thus, to describe an isosceles triangle on a given base, so that either angle at the base shall be triple of that at the vertex, is a Solid problem, resolved by the intersection of a parabola and circle, and it serves to inscribe a regular heptagon in a given circle.

In like manner, to describe an isosceles triangle having its angles at the base each equal to 4 times that at the vertex, is a Solid problem, effected by the intersection of an hyperbola and a parabola, and serves to inscribe a regular nonagon in a given circle.

And such a problem as this has four solutions, and no more; because two conic sections can intersect but in 4 points.

How all such problems are constructed, is shewn by Dr. Halley, in the Philos. Trans. num. 188.

Solid *of Least Resistance.* See Resistance.

Surfaces of Solids. See Area and Superficies.

Solid *Theorem.* See Theorem.

SOLIDITY, in Physics, a property of matter or body, by which it excludes every other body from that place which is possessed by itself.

Solidity in this sense is a property common to all bodies, whether solid or fluid. It is usually called *impenetrability*; but Solidity expresses it better, as carrying with it somewhat more of positive than the other, which is a negative idea.

The idea of Solidity, Mr. Locke observes, arises from the resistance we find one body makes to the entrance of another into its own place. Solidity, he adds, seems the most extensive property of body, as being that by which we conceive it to fill space; it is distinguished from mere space, by this latter not being capable of resistance or motion.

It is distinguished from hardness, which is only a firm cohesion of the solid parts.

The difficulty of changing situation gives no more Solidity to the hardest body than to the softest; nor is the hardest diamond properly a jot more solid than water. By this we distinguish the idea of the extension of body, from that of the extension of space: that of body is the continuity or cohesion of solid, separable, moveable parts; that of space the continuity of unsolid, inseparable, immoveable parts.

The Cartesians however will, by all means, deduce Solidity, or as they call it impenetrability, from the nature of extension; they contend, that the idea of the former is contained in that of the latter; and hence they argue against a vacuum. Thus, say they, one cubic foot of extension cannot be added to another without having two cubic feet of extension; for each has in itself all that is required to constitute that magnitude. And hence they conclude, that every part of space is solid, or impenetrable, as of its own nature it excludes all others. But the conclusion is false, and the instance they give follows from this, that the parts of space are immoveable, not from their being impenetrable or solid. See Matter.

Solidity is also used for hardness, or firmness; as opposed to fluidity; viz, when body is considered either as fluid or solid, or hard or firm.

Solidity, in Geometry, denotes the quantity of space contained in a solid body, or occupied by it; called also the *solid content*, or the *cubical content*; for all solids are measured by cubes, whose sides are inches, or feet, or yards, &c; and hence the Solidity of a body is said to be so many cubic inches, feet, yards, &c, as will fill its capacity or space, or another of an equal magnitude.

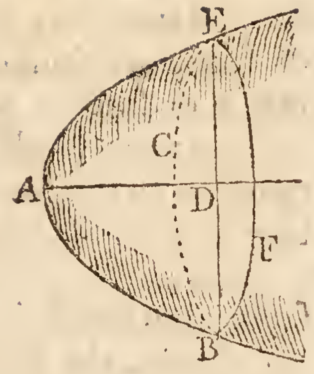
The Solidity of a cube, parallelopipedon, cylinder, or any other prismatic body, i. e. one whose parallel sections are all equal and similar throughout, is found by multiplying the base by the height or perpendicular altitude. And of any cone or other pyramid, the Solidity is equal to one-third part of the same prism, because any pyramid is equal to the 3d part of its circumscribing prism. Also, because a sphere or globe may be considered as made up of an infinite number of pyramids, whose bases form the surface of the globe, and their vertices all meet in the centre, or having their common altitude equal to the radius of the globe; therefore the solid content of it is equal to one-third part of the product of its radius and surface. For the Solidity of other figures, see each figure separately.

The foregoing rules are such as are derived from common geometry. But there are in nature numberless other forms, which require the aid of other methods and principles, as follows.

Of the Solidity *of Bodies formed by a Plane revolving about any Axis, either within or without the Body.*—Concerning such bodies, there is a remarkable property or relation between their Solidity and the path or line described by the centre of gravity of the revolving plane; viz, the Solidity of the body generated, whether by a whole revolution, or only a part of one, is always equal to the product arising from the generating plane drawn into the path or line described by its centre of gravity, during its motion in describing the body. And this rule holds true for figures generated by all sorts of motion whatever, whether rotatory, or direct or parallel, or irregularly zigzag, &c, provided the generating plane vary not, but continue the same throughout. And the same law holds true also for all surfaces any how generated by the motion of a right line. This is called the Centrobaric method. See my Mensuration, sect. 3, part 4, pa. 501, 2d edit.

Of the Solidity *of Bodies by the Method of Fluxions.*—This method applies very advantageously in all cases also in which a body is conceived to be generated by the revolution of a plane figure about an axis, or, which is much the same thing, by the parallel motion of a circle, gradually expanding and contracting itself, according to the nature of the generating plane. And this method is particularly useful for the solids generated by any curvilineal plane figures. Thus, let the plane AED revolve about the axis AD; then it will generate the solid ABFEC. But as every ordinate DE, perpendicular

pendicular to the axis AD, deſcribes a circle BCEF in the revolution, therefore the ſame ſolid may be conceived as generated by a circle BCEF, gradually expanding itſelf larger and larger, and moving perpendicularly along the axis AD. Conſequently the area of that circle being drawn into the fluxion of the axis, will produce the fluxion of the ſolid; and therefore the fluent, when taken, will give the Solidity of that body. That is, $\dot{AD} \times$ circle BCF, (whoſe radius is DE, or diameter BE) is the fluxion of the Solidity.



Hence then, putting $AD = x$, $DE = y$, $c = 3{\cdot}1416$; becauſe cy^2 is equal to the area of the circle BCF; therefore $cy^2\dot{x}$ is the fluxion of the ſolid. Conſequently if the value of either y^2 or x be found in terms of each other, from the given equation expreſſing the nature of the curve, and that value be ſubſtituted for it in the fluxional expreſſion $cy^2\dot{x}$, the fluent of the reſulting quantity, being taken, will be the required Solidity of the body.

For Ex. Suppoſe the figure of a parabolic conoid, generated by the rotation of the common parabola ADE about its axis AD. In this caſe, the equation of the curve of the parabola is $px = y^2$, where p denotes the parameter of the axis. Subſtituting therefore px inſtead of y^2, in the fluxion $cy^2\dot{x}$, it becomes $cpx\dot{x}$; and the fluent of this is $\frac{1}{2}cpx^2 = \frac{1}{2}cxy^2$ for the Solidity; that is, half the product of the baſe of the ſolid drawn into its altitude; for cy^2 is the area of the circular baſe BCF, and x is the altitude. And ſo on for other ſuch figures. See the content of each ſolid under its proper article.

For the Solidity *of Irregular Solids*, or ſuch as cannot be conſidered as generated by ſome regular motion or deſcription; they muſt either be conſidered as cut or divided into ſeveral parts of known forms, as priſms, or pyramids, or wedges, &c, and the contents of theſe parts found ſeparately. Or, in the caſe of the ſmaller bodies, of forms ſo irregular as not to be eaſily divided in that way, put them into ſome hollow regular veſſel, as a hollow cylinder or parallelopipedon, &c; then pour in water or ſand ſo as it may fill the veſſel juſt up to the top of the incloſed irregular body, noting the height it riſes to; then take out the body, and note the height the fluid again ſtands at; the difference of theſe two heights is to be conſidered as the altitude of a priſm of the ſame baſe and form as the hollow veſſel; and conſequently the product of that altitude and baſe will be the accurate Solidity of the immerged body, be it ever ſo irregular.

SOLSTICE, in Aſtronomy, is the time when the ſun is in one of the ſolſtitial points, that is, when he is at the greateſt diſtance from the equator, which is now nearly 23° 28′ on either ſide of it. It is ſo called, becauſe the ſun then ſeems to ſtand ſtill, and not to change his place, as to declination, either way.

There are two Solſtices, in each year, when the ſun is at the greateſt diſtance on the north and ſouth ſides of the ecliptic; viz, the *eſtival* or *ſummer ſolſtice*, and the *hyemal* or *winter ſolſtice*.

The *Summer Solſtice* is when the ſun is in the tropic of Cancer; which is about the 21ſt of June, when he makes the longeſt day. And

The *Winter Solſtice* is when he enters the firſt degree of Capricorn; which is about the 22d day of December, when he makes the ſhorteſt day.

This is to be underſtood, as in our northern hemiſphere; for in the ſouthern, the ſun's entrance into Capricorn makes their ſummer Solſtice, and that into Cancer the winter one. So that it is more preciſe and determinate, to ſay the northern and ſouthern Solſtice.

SOLSTITIAL *Points*, are thoſe points of the ecliptic the ſun is in at the times of the two Solſtices, being the firſt points of Cancer and Capricorn, which are diametrically oppoſite to each other.

Solstitial *Colure*, is that which paſſes through the Solſtitial points.

SOLUTION, in Mathematics, is the anſwering or reſolving of a queſtion or problem that is propoſed. See Resolution, and Reduction *of Equations*.

Solution, in Phyſics, is the reduction of a ſolid or firm body, into a fluid ſtate, by means of ſome menſtruum.—Solution is often confounded with what is called diſſolution, though there is a difference.

SOSIGENES, was an Egyptian mathematician, whoſe principal ſtudies were chronology and the mathematics in general, and who flouriſhed in the time of Julius Cæſar. He is repreſented as well verſed in the mathematics and aſtronomy of the Ancients; particularly of thoſe celebrated mathematicians, Thales, Archimedes, Hipparchus, Calippus, and many others, who had undertaken to determine the quantity of the ſolar year; which they had aſcertained much nearer the truth than one can well imagine they ſhould, with inſtruments ſo very imperfect; as may appear by reference to Ptolomy's Almageſt.

It ſeems Soſigenes made great improvements, and gave proofs of his being able to demonſtrate the certainty of his diſcoveries; by which means he became popular, and obtained repute with thoſe who had a genius to underſtand and reliſh ſuch enquiries. Hence he was ſent for by Julius Cæſar, who being convinced of his capacity, employed him in reforming the calendar; and it was he who formed the Julian year which begins 45 years before the birth of Chriſt. His other works are loſt ſince that period.

SOUND, in Geography, denotes a ſtrait or inlet of the ſea, between two capes or head-lands.

The Sound is uſed, by way of eminence, for that celebrated ſtrait which connects the German ſea to the Baltic. It is ſituated between the iſland of Zealand and the coaſt of Schonen. It is about 16 leagues in length, and in general about 5 in breadth, except near the caſtle of Cronenberg, where it is but one; ſo that there is no paſſage for veſſels but under the cannon of the fortreſs.

Sound, in Phyſics, a perception of the mind, communicated by means of the ear; being an effect of the colliſion of bodies, and their conſequent tremulous motion, communicated to the ambient fluid; and ſo propagated through it to the organs of hearing.

To illuſtrate the cauſe of Sound, it is to be obſerved, 1ſt, That a motion is neceſſary in the ſonorous body for the production of ſound. 2dly, That this motion exiſts firſt in the ſmall and inſenſible parts of the ſonorous bodies

bodies, and is excited in them by their mutual collision against each other, which produces the tremulous motion so observable in bodies that have a clear sound, as bells, musical chords, &c. 3dly, That this motion is communicated to, or produces a like motion in the air, or such parts of it as are fit to receive and propagate it. Lastly, That this motion must be communicated to those parts that are the proper and immediate instruments of hearing.

Now that motion of a sonorous body, which is the immediate cause of Sound, may be owing to two different causes; either the percussion between it and other hard bodies, as in drums, bells, chords, &c; or the beating and dashing of the sonorous body and the air immediately against each other, as in flutes, trumpets, &c.

But in both these cases, the motion, which is the consequence of the mutual action, as well as the immediate cause of the sonorous motion which the air conveys to the ear, is supposed to be an invisible, tremulous or undulating motion, in the small and insensible parts of the body. Perrault adds, that the visible motion of the grosser parts contributes no otherwise to Sound, than as it causes the invisible motion of the smaller parts, which he calls particles, to distinguish them from the sensible ones, which he calls parts, and from the smallest of all, which are called corpuscles.

The sonorous body having made its impression on the contiguous air, that impression is propagated from one particle to another, according to the laws of pneumatics.

A few particles, for instance, driven from the surface of the body, push or press their adjacent particles into a less space; and the medium, as it is thus rarefied in one place, becomes condensed in the other; but the air thus compressed in the second place, is, by its elasticity, returned back again, both to its former place and its former state; and the air contiguous to that is compressed; and the like obtains when the air less compressed, expanding itself, a new compression is generated. Therefore from each agitation of the air there arises a motion in it, analogous to the motion of a wave on the surface of the water; which is called a *wave* or *undulation* of air.

In each wave, the particles go and return back again, through very short equal spaces; the motion of each particle being analogous to the motion of a vibrating pendulum while it performs two oscillations; and most of the laws of the pendulum, with very little alteration, being applicable to the former.

Sounds are as various as are the means that concur in producing them. The chief varieties result from the figure, constitution, quantity, &c, of the sonorous body; the manner of percussion, with the velocity &c, of the consequent vibrations; the state and constitution of the medium; the disposition, distance, &c, of the organ; the obstacles between the organ and the sonorous object and the adjacent bodies. The most notable distinction of Sounds, arising from the various degrees and combinations of the conditions above mentioned, are into *loud* and *low* (or strong and weak); into *grave* and *acute* (or sharp and flat, or high and low); and into *long* and *short*. The management of which is the office of music.

Euler is of opinion, that no Sound making fewer vibrations than 30 in a second, or more than 7520, is distinguishable by the human ear. According to this doctrine, the limit of our hearing, as to acute and grave, is an interval of 8 octaves. Tentam. Nov. Theor. Mus. cap. 1, sect. 13.

The velocity of Sound is the same with that of the aerial waves, and does not vary much, whether it go with the wind or against it. By the wind indeed a certain quantity of air is carried from one place to another; and the Sound is accelerated while its waves move through that part of the air, if their direction be the same as that of the wind. But as Sound moves vastly swifter than the wind, the acceleration it will hereby receive is but inconsiderable; and the chief effect we can perceive from the wind is, that it increases and diminishes the space of the waves, so that by help of it the Sound may be heard to a greater distance than otherwise it would.

That the air is the usual medium of Sound, appears from various experiments in rarefied and condensed air. In an unexhausted receiver, a small bell may be heard to some distance; but when exhausted, it can scarce be heard at the smallest distance. When the air is condensed, the Sound is louder in proportion to the condensation, or quantity of air crowded in; of which there are many instances in Hauksbee's experiments, in Dr. Priestley's, and others.

Besides, sounding bodies communicate tremors to distant bodies; for example, the vibrating motion of a musical string puts others in motion, whose tension and quantity of matter dispose their vibrations to keep time with the pulses of air, propagated from the string that was struck. Galileo explains this phenomenon by observing, that a heavy pendulum may be put in motion by the least breath of the mouth, provided the blasts be often repeated, and keep time exactly with the vibrations of the pendulum; and also by the like art in raising a large bell.

It is not air alone that is capable of the impressions of Sound, but water also; as is manifest by striking a bell under water, the Sound of which may plainly enough be heard, only not so loud, and also a fourth deeper, according to good judges in musical notes. And Mersenne says, a Sound made under water is of the same tone or note, as if made in air, and heard under the water.

The velocity of Sound, or the space through which it is propagated in a given time, has been very differently estimated by authors who have written concerning this subject. Roberval states it at the rate of 560 feet in a second; Gassendus at 1473; Mersenne at 1474; Duhamel, in the History of the Academy of Sciences at Paris, at 1338; Newton at 968; Derham, in whose measure Flamsteed and Halley acquiesce, at 1142.

The reason of this variety is ascribed by Derham, partly to some of those gentlemen using strings and plummets instead of regular pendulums; and partly to the too small distance between the sonorous body and the place of observation; and partly to no regard being had to the winds.

But by the accounts since published by M. Cassini de Thury, in the Memoirs of the Royal Acad. of Scien-

 ces

-ces at Paris, 1738, where cannon were fired at various as well as great distances, under many varieties of weather, wind, and other circumstances, and where the measures of the different places had been settled with the utmost exactness, it was found that Sound was propagated, on a medium, at the rate of 1038 French feet in a second of time. But the French foot is in proportion to the English as 15 to 16; and consequently 1038 French feet are equal to 1107 English feet. Therefore the difference of the measures of Derham and Cassini is 35 English feet, or 33 French feet, in a second. The medium velocity of Sound therefore is nearly at the rate of a mile, or 5280 feet, in $4\frac{3}{4}$ seconds, or a league in 14 seconds, or 13 miles in a minute. But sea miles are to land miles nearly as 7 to 6; and therefore Sound moves over a sea mile in $5\frac{1}{3}$ seconds nearly, or a sea league in 16 seconds.

Farther, it is a common observation, that persons in good health have about 75 pulsations, or beats of the artery at the wrist, in a minute; consequently in 75 pulsations, Sound flies about 13 land miles, or $11\frac{1}{7}$ sea miles, which is about 1 land mile in 6 pulses, or one sea mile in 7 pulses, or a league in 20 pulses.

And hence the distance of objects may be found, by knowing the time employed by Sound in moving from those objects to an observer. For Ex. On seeing the flash of a gun at sea, if 54 beats of the pulse at the wrist were counted before the report was heard; the distance of the gun will easily be found by dividing 54 by 20, which gives 2·7 leagues, or about 8 miles.

Upon the nature, production, and propagation of Sound, see the article PHONICS and ECHO; also the Memoirs of the Acad. and the Philos. Trans. in many places; Newton, Principia; Kircher, Mesurgia Universalis; Mersenne; Borelli, Del Suono; Priestley, Exper. and Observ. vol. 5; Hales, Sonorum Doctrina rationalis et experimentalis; 4to 1778. See also an ingenious treatise published 1790, by Mr. Geo. Saunders, on Theatres; in which he relates many experiments made by himself, on the nature and propagation of Sound. In this work, he shews the great effect of water, and some other bodies, in conducting of Sound, probably by rendering the air more dense near them. Some of his conclusions and observations are as follow:

Earth may be supposed to have a twofold property with respect to Sound. Being very porous, it absorbs Sound, which is counteracted by its property of conducting Sound, and occasions it to pass on a plane, in an equal proportion to its progress in air, unencumbered by any body.

If a Sound be sufficiently intense to impress the earth in its tremulous quality, it will be carried to a considerable distance, as when the earth is struck with any thing hard, as by the motion of a carriage, horses feet, &c.

Plaster is proportionally better than loose earth for conducting Sound, as it is more compact.

Clothes of every kind, particularly woollen cloths, are very prejudicial to Sound: their absorption of Sound, may be compared to that of water, which they greedily imbibe.

A number of people seated before others, as in the pit or gallery of a theatre, do considerably prevent the voice reaching those behind; and hence it is, that we hear so much better in the front of the galleries, or of any situation, than behind others, though we may be nearer to the speaker. Our seats, rising so little above each other, occasion this defect, which would be remedied, could we have the seats to rise their whole height above each other, as in the ancient theatres.

Paint has generally been thought unfavourable to Sound, from its being so to musical instruments, whose effects it quite destroys.

Musical instruments mostly depend on the vibrative or tremulous property of the material, which a body of colour hardened in oil must very much alter; but we should distinguish that this regards the formation of Sound, which may not altogether be the case in the progress of it.

Water has been little noticed, with respect to its conducting Sound; but it will be found to be of the greatest consequence. I had often perceived in newly-finished houses, that while they were yet damp, they produced echoes; but that the echoing abated as they dried.

Exp. When I made the following experiment there was a gentle wind; consequently the water was proportionally agitated. I chose a quiet part of the river Thames, near Chelsea Hospital, and with two boats tried the distance the voice would reach. On the water we could distinctly hear a person read at the distance of 140 feet, on land at that of 76. It should be observed, that on land no noise intervened; but on the river some noise was occasioned by the flowing of the water against the boats; so that the difference on land and on water must be much more.

Watermen observe, that when the water is still, and the weather quite calm, if no noise intervene, a whisper may be heard across the river; and that with the current it will be carried to a much greater distance, and vice versa against the current.

Mariners well know the difference of Sound on sea and land.

When a canal of water was laid under the pit floor of the theatre of Argentino, at Rome, a surprising difference was observed; the voice has since been heard at the end very distinctly, where it was before scarce distinguishable. It is observable that, in this part, the canal is covered with a brick arch, over which there is a quantity of earth, and the timber floor over all.

The villa Simonetta near Milan, so remarkable for its echoes, is entirely over arcades of water.

Another villa near Rouen, remarkable for its echo, is built over subterraneous cavities of water.

A reservoir of water domed over, near Stanmore, has a strong echo.

I do not remember ever being under the arches of a stone bridge that did not echo; which is not always the case with similar structures on land.

A house in Lambeth Marsh, inhabited by Mr. Tuttle, is very damp during winter, when it yields an echo which abates as the house becomes dry in summer.

Kircher observes, that echoes repeat more by night than during the day: he makes the difference to be double.

Dr. Plott says, the echo in Woodstock park repeated 17 times by day, and 20 by night. And Addison's experi-

experiment at the Villa Simonetta was in a fog, when it produced 56 repetitions.

After all these instances, I think little doubt can remain of the influence water has on Sound; and I conclude that it conducts Sound more than any other body whatever.

After water, stone may be reckoned the best conductor of Sound. To what cause it may be attributed, I leave to future enquiries: I have confined myself to speak of facts only as they appear.

Stone is sonorous, but gives a harsh disagreeable tone, unfavourable to music.

Brick, in respect to Sound, has nearly the same properties as stone. Part of the garden wall of the late W. Pitt, Esq. of Kingston in Dorsetshire, conveys a whisper to the distance of near 200 feet.

Wood is sonorous, conductive, and vibrative; of all materials it produces a tone the most agreeable and melodious; and it is therefore the fittest for musical instruments, and for lining of rooms and theatres.

The common notion that whispering at one end of a long piece of timber would be heard at the other end, I found by experiment to be erroneous. A stick of timber 65 feet long being slightly struck at one end, a sound was heard at the other, and the tremor very perceptible: which is easily accounted for, when we consider the number or length of the fibres that compose it, each of which may be compared to a string of catgut.

For the Reflection, Refraction, &c, of SOUND; see ECHO, and PHONICS.

Articulate SOUND. See ARTICULATE.

SOUND, in Music, denotes a quality in the several agitations of the air, so as to make music or harmony.

Sound is the object of music; which is nothing but the art of applying Sounds, under such circumstances of tone and time, as to raise agreeable sensations. The principal affection of Sound, by which it becomes fitted to have this end, is that by which it is distinguished into acute and grave. This difference depends on the nature of the sonorous body; the particular figure and quantity of it; and even in some cases, on the part of the body where it is struck: and it is this that constitutes what are called *different tones*.

The cause of this difference appears to be no other than the different velocities of the vibrations of the sounding body. Indeed the tone of a Sound is found, by numerous experiments, to depend on the nature of those vibrations, whose differences we can conceive no otherwise than as having different velocities: and since it is proved that the small vibrations of the same chord are all performed in equal times, and that the tone of a Sound, which continues for some time after the stroke, is the same from first to last, it follows, that the tone is necessarily connected with a certain quantity of time in making each vibration, or each wave; or that a certain number of vibrations or waves, made in a given time, constitute a certain and determinate tone. From this principle are all the phænomena of tune deduced.

If the vibrations be isochronous, or performed in the same time, the Sound is called musical, and is said to continue at the same pitch; and it is also accounted acuter, sharper, or higher than any other Sound, whose vibrations are slower, and therefore graver, flatter, or lower, than any other whose vibrations are quicker. See UNISON.

From the same principle arise what are called *concords*, &c; which result from the frequent unions and coincidences of the vibrations of two sonorous bodies, and consequently of the pulses or the waves of the air occasioned by them.

On the contrary, the result of less frequent coincidences of those vibrations, is what is called *discord*.

Another considerable distinction of musical Sounds, is that by which they are called *long* and *short*, owing to the continuation of the impulse of the efficient cause on the sonorous body for a longer or shorter time, as in the notes of a violin &c, which are made longer or shorter by strokes of different length or quickness. This continuity is properly a succession of several Sounds, or the effect of several distinct strokes, or repeated impulses, on the sonorous body, so quick, that we judge it one continued Sound, especially where it is continued in the same degree of strength; and hence arises the doctrine of *measure* and *time*.

Musical Sounds are also divided into *simple* and *compound*; and that in two different ways. In the first, a Sound is said to be compound, when a number of successive vibrations of the sonorous body, and the air, come so fast upon the ear, that we judge them the same continued Sound; like as in the phenomenon of the circle of fire, caused by putting the fired end of a stick in a quick circular motion; where supposing the end of the stick in any point of the circle, the idea we receive of it there continues till the impression is renewed by a sudden return.

A *Simple* SOUND then, with regard to this composition, should be the effect of a single vibration, or of as many vibrations as are necessary to raise in us the idea of Sound.

In the second sense of composition, a simple Sound is the product of one voice, or one instrument, &c.

A *Compound* SOUND consists of the Sounds of several distinct voices or instruments all united in the same individual time, and measure of duration, that is, all striking the ear together, whatever their other differences may be. But in this sense again, there is a twofold composition; a natural and an artificial one.

The natural composition is that proceeding from the manifold reflections of the first Sound from adjacent bodies, where the reflections are not so sudden as to occasion echoes, but are all in the same tune with the first note.

The artificial composition, which alone comes under the musician's province, is that mixture of several Sounds, which being made by art, the ingredient Sounds are separable, and distinguishable from one another. In this sense the distinct Sounds of several voices or instruments, or several notes of the same instrument, are called simple Sounds, in contradistinction to the compound ones, in which, to answer the end of

of mufic, the fimples muft have fuch an agreement in all relations, chiefly as to acutenefs and gravity, as that the ear may receive the mixture with pleafure.

Another diftinction of Sounds, with regard to mufic, is that by which they are faid to be *fmooth* or *even*, and *rough* or *harfh*, alfo *clear* and *hoarfe*: the caufe of which difference depends on the difpofition and ftate of the fonorous body, or the circumftances of the place; but the ideas of the differences muft be fought from obfervation.

Smooth and *Rough* Sounds depend chiefly on the founding body; of which we have a notable inftance in ftrings that are uneven, and not of the fame dimenfion and conftitution throughout.

As to *clear* and *hoarfe* Sounds, they depend on circumftances that are accidental to the fonorous body. Thus, a voice or inftrument will be hollow and hoarfe if founded within an empty hogfhead, that yet is clear and bright out of it: the effect is owing to the mixture of different Sounds, raifed by reflections, which corrupt and change the fpecies of the primitive Sound.

For Sounds to be fit to obtain the end of mufic, they ought to be fmooth and clear, efpecially the firft; fince, without this, they cannot have one certain and difcernible tone, capable of being compared to others, in a certain relation of acutenefs, which the ear may judge of. So that, with Malcolm, we call that an harmonic or mufical Sound which, being clear and even, is agreeable to the ear, and gives a certain and difcernible tune (hence called tunable Sound), which is the fubject of the whole theory of harmony.

Wood has a particular vibrating quality, owing to its elafticity; and all mufical inftruments made of this matter, are of a thicknefs proportioned to the fuperficies of the wood, and the tone they are to produce.

Metals are fonorous and vibrative, producing a harfh tone, very ferviceable to fome parts of mufic. Moft wind inftruments are made of metal, which is acted upon in its elaftic and tremulous quality, being capable of being reduced very thin for that purpofe. Inftruments of this kind are fuch as horns, trumpets, &c. Some inftruments however depend more on the form than the material; as flutes, for inftance, which, if their lengths and bore be the fame, have very little difference in their Sounds, whatever the matter of them may be. See Harmonical.

SOUND-Board, the principal part of an organ, and that which makes the whole machine play. This Sound-board, or fummer, is a refervoir into which the wind, drawn in by the bellows, is conducted by a portvent, and thence diftributed into the pipes placed over the holes of its upper part. This wind enters them by valves, which open by preffing upon the ftops or keys, after drawing the regifters, which prevent the air from going into any of the other pipes befide thofe it is required in.

Sound-*board* denotes alfo a thin broad board placed over the head of a public fpeaker, to enlarge and extend or ftrengthen his voice.

Sound-boards, in theatres, are found by experience to be of no fervice; their diftance from the fpeaker being too great, to be impreffed with fufficient force. But Sound-boards immediately over a pulpit have often a good effect, when the cafe is made of a juft thicknefs, and according to certain principles.

Sound-*Poft*, is a poft placed withinfide of a violin, &c, as a prop between the back and the belly of the inftrument, and nearly under the bridge.

SOUNDING, in Navigation, the act of trying the depth of the water, and the quality of the bottom, by a line and plummet, or other artifice.

At fea, there are two plummets ufed for this purpofe, both fhaped like the fruftum of a cone or pyramid. One of thefe is called the hand-lead, weighing about 8 or 9lb; and the other the deep-fea-lead, weighing from 25 to 30lb. The former is ufed in fhallow waters, and the latter at a great diftance from the fhore. The line of the hand-lead, is about 25 fathoms in length, and marked at every 2 or 3 fathoms, in this manner, viz, at 2 and 3 fathoms from the lead there are marks of black leather; at 5 fathoms a white rag, at 7 a red rag, at 10 and at 13 black leather, at 15 a white rag, and at 17 a red one.

Sounding with the hand-lead, which the feamen call heaving the lead, is generally performed by a man who ftands in the main-chains to windward. Having the line all ready to run out, without interruption, he holds it nearly at the diftance of a fathom from the plummet, and having fwung the latter backwards and forwards three or four times, in order to acquire the greater velocity, he fwings it round his head, and thence as far forward as is neceffary; fo that, by the lead's finking whilft the fhip advances, the line may be almoft perpendicular when it reaches the bottom. The perfon founding then proclaims the depth of the water in a kind of fong refembling the cries of hawkers in a city; thus, if the mark of 5 be clofe to the furface of the water, he calls, 'by the mark 5,' and as there is no mark at 4, 6, 8, &c, he eftimates thofe numbers, and calls, 'by the dip four, &c.' If he judges it to be a quarter or a half more than any particular number, he calls, 'and a quarter 5,' 'and a half 4' &c. If he conceives the depth to be three quarters more than a particular number, he calls it a quarter lefs than the next: thus, at 4 fathom $\frac{3}{4}$, he calls, 'a quarter lefs 5,' and fo on.

The deep-fea-lead line is marked with 2 knots at 20 fathom, 3 at 30, 4 at 40, &c to the end. It is alfo marked with a fingle knot at the middle of each interval, as at 25, 35, 45 fathoms, &c. To ufe this lead more effectually at fea, or in deep water on the fea-coaft, it is ufual previoufly to bring-to the fhip, in order to retard her courfe: the lead is then thrown as far as poffible from the fhip on the line of her drift, fo that, as it finks, the fhip drives more perpendicularly over it. The pilot feeling the lead ftrike the bottom, readily difcovers the depth of the water by the mark on the line neareft its furface. The bottom of the lead, which is a little hollowed there for the purpofe, being alfo well rubbed over with tallow, retains the diftinguifhing marks of the bottom, as fhells, ooze, gravel, &c, which naturally adhere to it.

The depth of the water, and the nature of the ground, which are called the Soundings, are carefully marked in the log-book, as well to determine the diftance of the

the place from the ſhore, as to correct the obſervations of former pilots. Falconer.

For a machine to meaſure unfathomable depths of the ſea, ſee ALTITUDE.

SOUNDING *the pump*, at ſea, is done by letting fall a ſmall line, with ſome weight at the end, down into the pump, to know what depth of water there is in it.

SOUTH, one of the four cardinal points of the wind, or compaſs, being that which is directly oppoſite to the north.

SOUTH *Direct Dials*. See PRIME *Verticals*.

SOUTHERN *Hemiſphere*, *Signs*, *&c*, thoſe in the ſouth ſide of the equator.

SOUTHING, in Navigation, the difference of latitude made by a ſhip in ſailing to the ſouthward.

SPACE, denotes room, place, diſtance, capacity, extenſion, duration, &c.

When Space is conſidered barely in length between any two bodies, it gives the ſame idea as that of diſtance. When it is conſidered in length, breadth, and thickneſs, it is properly called capacity. And when conſidered between the extremities of matter, which fills the capacity of Space with ſomething ſolid, tangible, and moveable, it is then called extenſion.

So that extenſion is an idea belonging to body only; but Space may be conſidered without it. Therefore Space, in the general ſignification, is the ſame thing with diſtance conſidered every way, whether there be any matter in it or not.

Space is uſually divided into *abſolute* and *relative*.

Abſolute SPACE is that which is conſidered in its own nature, without regard to any thing external, which always remains the ſame, and is infinite and immoveable.

Relative SPACE is that moveable dimenſion, or meaſure of the former, which our ſenſes define by its poſitions to bodies within it; and this the vulgar uſe for immoveable Space.

Relative Space, in magnitude and figure, is always the ſame with abſolute; but it is not neceſſary it ſhould be ſo numerically. Thus, when a ſhip is perfectly at reſt, then the places of all things within her are the ſame both abſolutely and relatively, and nothing changes its place: but, on the contrary, when the ſhip is under ſail, or in motion, ſhe continually paſſes through new parts of abſolute Space; though all things on board, conſidered relatively, in reſpect to the ſhip, may yet be in the ſame places, or have the ſame ſituation and poſition, in regard to one another.

The Carteſians, who make extenſion the eſſence of matter, aſſert, that the Space any body takes up, is the ſame thing with the body itſelf; and that there is no ſuch thing in the univerſe as mere Space, void of all matter; thus making Space or extenſion a ſubſtance. See this diſproved under VACUUM.

Among thoſe too who admit a vacuum, and conſequently an eſſential difference between Space and matter, there are ſome who aſſert that Space is a ſubſtance. Among theſe we find Graveſande, Introd. ad Philoſ. ſect. 19.

Others again put Space into the ſame claſs of beings as time and number; thus making it to be no more than a notion of the mind. So that according to theſe authors, abſolute Space, of which the Newtonians ſpeak, is a mere chimera. See the writings of the late biſhop Berkley.

Space and time, according to Dr. Clarke, are attributes of the Deity; and the impoſſibility of annihilating theſe, even in idea, is the ſame with that of the neceſſary exiſtence of the Deity.

SPACE, in Geometry, denotes the area of any figure; or that which fills the interval or diſtance between the lines that terminate or bound it. Thus,

The Parabolic Space is that included in the whole parabola. The conchoidal Space, or the ciſſoidal Space, is what is included within the cavity of the conchoid or ciſſoid. And the aſymptotic Space, is what is included between an hyperbolic curve and its aſymptote. By the new methods now introduced, of applying algebra to geometry, it is demonſtrated that the conchoidal and ciſſoidal Spaces, though infinitely extended in length, are yet only finite magnitudes or Spaces.

SPACE, in Mechanics, is the line a moveable body, conſidered as a point, is conceived to deſcribe by its motion.

SPANDREL, with Builders, is the ſpace included between the curve of an arch and the ſtraight or right lines which incloſe it; as the ſpace *a*, or *b*.

SPEAKING *Trumpet*. See *Speaking* TRUMPET.

SPECIES, in Algebra, are the letters, ſymbols, marks, or characters, which repreſent the quantities in any operation or equation.

This ſhort and advantageous way of notation was chiefly introduced by Vieta, about the year 1590; and by means of which he made many diſcoveries in algebra, not before taken notice of.

The reaſon why Vieta gave this name of Species to the letters of the alphabet uſed in algebra, and hence called Arithmetica Specioſa, ſeems to have been in imitation of the Civilians, who call caſes in law that are put abſtractedly, between John a Nokes and Tom a Stiles, between A and B; ſuppoſing thoſe letters to ſtand for any perſons indefinitely. Such caſes they call Species: whence, as the letters of the alphabet will alſo as well repreſent quantities, as perſons, and that alſo indefinitely, one quantity as well as another, they are properly enough called Species; that is general ſymbols, marks, or characters. From whence the literal algebra hath ſince been often called Specious Arithmetic, or Algebra in Species.

SPECIES, in Optics, the image painted on the retina by the rays of light reflected from the ſeveral points of the ſurface of an object, received in by the pupil, and collected in their paſſage through the cryſtalline, &c.

Philoſophers have been in great doubt, whether the Species of objects, which give the ſoul an occaſion of ſeeing, are an effuſion of the ſubſtance of the body; or a mere impreſſion which they make on all ambient bodies, and which theſe all reflect, when in a proper diſpoſition and diſtance; or laſtly, whether they are not ſome other more ſubtile body, as light, which receives all theſe impreſſions from bodies, and is continually ſent and returning from one to another, with the different impreſſions and figures it has taken. But the moderns have decided this point by their invention of

artificial eyes, inwhich the Species of objects are received on a paper, in the fame manner as they are received in the natural eye.

SPECIFIC, in Philofophy, that which is proper and peculiar to any thing; or that characterifes it, and diftinguifhes it from every other thing. Thus, the attracting of iron is Specific to the loadftone, or is a Specific property of it.

A juft definition fhould contain the Specific notion of the thing defined, or that which fpecifies and diftinguifhes it from every thing elfe.

SPECIFIC *Gravity*, in Hydroftatics, is the relative proportion of the weight of bodies of the fame bulk. See *Specific* GRAVITY.

SPECIFIC *Gravity of living men*. Mr. John Robertfon, late librarian to the Royal Society, in order to determine the Specific gravity of men, prepared a ciftern 78 inches long, 30 inches wide, 30 inches deep; and having procured 10 men for his purpofe, the height of each was taken and his weight; and afterwards they plunged fucceffively into the ciftern. A ruler or fcale, graduated to inches and decimal parts, was fixed to one end of the ciftern, and the height of the water fhown by it was noted before each man went in, and to what height it rofe when he immerfed himfelf under its furface. The following table contains the feveral refults of his experiments:

No. of men.	Height. Ft. In.	Weight. lbs.	Water raifed. Inches.	Solidity. Feet.	Wt. of water. lbs.	Specific gravity. (Wat. 1)
1	6 2	161	1·90	2·573	160·8	1·001
2	5 10$\frac{3}{8}$	147	1·91	2·586	161·6	0·901
3	5 9$\frac{1}{2}$	156	1·85	2·505	156·6	0·991
4	5 6$\frac{3}{4}$	140	2·04	2·763	172·6	0·801
5	5 5$\frac{7}{8}$	158	2·08	2·817	176·0	0·900
6	5 5$\frac{1}{2}$	158	2·17	2·939	183·7	0·849
7	5 4$\frac{3}{8}$	140	2·01	2·722	170·1	0·823
8	5 4$\frac{1}{8}$	121	1·79	2·424	151·5	0·800
9	5 3$\frac{1}{4}$	146	1·73	2·343	146·4	0·997
10	5 3$\frac{1}{8}$	132	1·85	2·505	156·6	0·843
medium of all.	5 6$\frac{2}{3}$	146	1·933	2·618	163·6	0·891

One of the reafons, Mr. Robertfon fays, that induced him to make thefe experiments, was a defire of knowing what quantity of timber would be fufficient to keep a man afloat in water, thinking that moft men were fpecifically heavier than river or common frefh water; but the contrary appears from the trials above recited; for, except the firft, every man was lighter than an equal bulk of frefh water, and much more fo than that of fea-water. So that, if perfons who fall into water had prefence of mind enough to avoid the fright ufual on fuch occafions, many might be preferved from drowning; and a piece of wood not larger than an oar, would buoy a man partly above water as long as he had ftrength or fpirits to keep his hold. Philof. Tranf. vol. 50, art. 5.

From the laft line of the table appears the medium of all the circumftances of height, weight, &c; particularly the mean Specific Gravity, 0·891, which is about $\frac{1}{9}$ lefs than common water.

SPECTACLES, an optical machine, confifting of two lenfes fet in a frame, and applied on the nofe, to affift in defects of the organ of fight.

Old people, and all prefbytæ, ufe Spectacles of convex lenfes, to make amends for the flatnefs of the eye, which does not make the rays converge enough to have them meet in the retina.

Short-fighted people, or myopes, ufe concave lenfes, to prevent the rays from converging fo faft, on account of the greater roundnefs of the eye, or fmallnefs of the fphere, which is fuch as to make them meet before they reach the retina.

F. Cherubin, a capuchin, defcribes a kind of Spectacle telefcopes, for viewing remote objects with both eyes; and hence called *binoculi*. Though F. Rheita had mentioned the fame before him, in his Oculus Enoch et Eliæ. See BINOCLE. The fame author invented a kind of Spectacles, with three or four glaffes, which performed very well.

The invention of Spectacles has been much difputed. They were certainly not known to the ancients. Francifco Redi, in a learned treatife on Spectacles, contends that they were firft invented between the years 1280 and 1311, probably about 1290; and adds, that Alexander de Spina, a monk of the order of Predicants of St. Catharine, at Pifa, firft communicated the fecret, which was of his own invention, upon learning that another perfon had it as well as himfelf.

The author tells us, that in an old manufcript ftill preferved in his library, compofed in 1299, Spectacles are mentioned as a thing invented about that time: and that a celebrated Jacobin, one Jourdon de Rivalto, in a treatife compofed in 1305, fays exprefsly, that it was not yet 20 years fince the invention of Spectacles. He likewife quotes Bernard Gordon in his Lilium Medicinæ, written the fame year, where he fpeaks of a collyrium, good to enable an old man to read without Spectacles.

Muffchenbroek obferves, (Introd. vol. 2, pa. 786) that it is infcribed on the tomb of Salvinus Armatus, a nobleman of Florence, who died in 1317, that he was the inventor of Spectacles.

Du-Cange, however, carries the invention of Spectacles farther back; affuring us, that there is a Greek poem in manufcript in the French king's library, which fhews that Spectacles were in ufe in the year 1150; however the dictionary of the Academy Della Crufca, under the word *occhiale*, inclines to Redi's fide; and quotes a paffage from Jourdon's fermons, which fays that Spectacles had not been 20 years in ufe; and Salvati has obferved that thofe fermons were compofed between the years 1330 and 1336.

It is probable that the firft hint of the conftruction and ufe of Spectacles, was derived from the writings either of Alhazen, who lived in the 12th century, or of our own countryman Roger Bacon, who was born in 1214, and died in 1292, or 1294. The following remarkable paffage occurs in Bacon's Opus Majus by Jebb, p. 352. Si vero homo afpiciat literas et alias res minutas per medium cryftalli, vel vitri, vel alterius perfpicui fuppofiti literis, et fit portio minor fpheræ, cujus convexitas fit verfus oculum et oculus fit in aëre, longe

longe melius videbit literas, et apparebunt ei majores.—Et ideo hoc inſtrumentum eſt utile ſenibus et habentibus oculos debiles: nam literam quantumcunque parvam poſſunt videre in ſufficienti magnitudine. Hence, and from other paſſages in his writings, much to the ſame purpoſe, Molyneux, Plott, and others, have attributed to him the invention of reading-glaſſes. Dr. Smith indeed, obſerving that there are ſome miſtakes in his reaſoning on this ſubject, has diſputed his claim. See Molyneux's Dioptr. p. 256. Smith's Optics, Rem. 86—89.

SPECULATIVE *Geometry*, *Mathematics*, *Muſic*, and *Philoſophy*. See the SUBSTANTIVES.

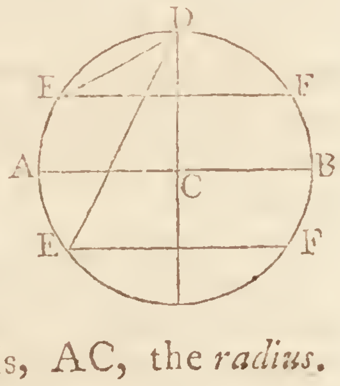
SPECULUM, or *Mirror*, in Optics, any poliſhed body, impervious to the rays of light: ſuch as poliſhed metals, and glaſſes lined with quickſilver, or any other opake matter, popularly called Looking-glaſſes; or even the ſurface of mercury or of water, &c.

For the ſeveral kinds and forms of Specula, plane, concave, and convex, with their theory and phenomena, ſee MIRROR. And for their laws and effects, ſee REFLECTION and BURNING *Glaſs*.

As for the Specula of reflecting teleſcopes, it may here be obſerved, that the perfection of the metal of which they ſhould be made, conſiſts in its hardneſs, whiteneſs, and compactneſs; for upon theſe properties the reflective powers and durability of the Specula depend. There are various compoſitions recommended for theſe Specula, in Smith's Optics, book 3, ch. 2, ſect. 787; alſo by Mr. Mudge in the Philoſ. Tranſ. vol. 67; and in various other places, as by Mr. Edwards, in the Naut. Alm. for 1787, whoſe metal is the whiteſt and beſt of any that I have ſeen.—For the method of grinding, ſee GRINDING.

Mr. Hearne's method of cleaning a tarniſhed Speculum was this: Get a little of the ſtrongeſt ſoap ley from the ſoap-makers, and having laid the Speculum on a table with its face upwards, put on as much of the ley as it will hold, and let it remain about an hour: then rub it ſoftly with a ſilk or muſlin, till the ley is all gone; then put on ſome ſpirit of wine, and rub it dry with another part of the ſilk or muſlin. If the Speculum will not perform well after this, it muſt be new poliſhed. A few faint ſpots of tarniſh may be rubbed off with ſpirit of wine only, without the ley. Smith's Optics, Rem. p. 107.

SPHERE, in Geometry, a ſolid body contained under one ſingle uniform ſurface, every point of which is equally diſtant from a certain point in the middle called its centre.



The Sphere may be ſuppoſed to be generated by the revolution of a ſemicircle ABD about its diameter AB, which is alſo called the *axis* of the Sphere, and the extreme points of the axis, A and B, the *poles* of the Sphere; alſo the middle of the axis C is the *centre*, and half the axis, AC, the *radius*.

Properties of the SPHERE, are as follow.

1. A Sphere may be conſidered as made up of an infinite number of pyramids, whoſe common altitude is equal to the radius of the Sphere, and all their baſes form the ſurface of the Sphere. And therefore the ſolid content of the Sphere is equal to that of a pyramid whoſe altitude is the radius, and its baſe is equal to the ſurface of the Sphere, that is, the ſolid content is equal to $\frac{1}{3}$ of the product of its radius and ſurface.

2. A Sphere is equal to $\frac{2}{3}$ of its circumſcribing cylinder, or of the cylinder of the ſame height and diameter, and therefore equal to the cube of the diameter multiplied by ·5236, or $\frac{2}{3}$ of ·7854; or equal to double a cone of the ſame baſe and height. Hence alſo different Spheres are to one another as the cubes of their diameters. And their ſurfaces as the ſquares of the ſame diameters.

3. The ſurface or ſuperficies of any Sphere, is equal to 4 times the area of its great circle, or of a circle of the ſame diameter as the Sphere. Or

4. The ſurface of the whole Sphere is equal to the area of a circle whoſe radius is equal to the diameter of the Sphere. And, in like manner, the curve ſurface of any ſegment EDF, whether greater or leſs than a hemiſphere, is equal to a circle whoſe radius is the chord line DE, drawn from the vertex D of the ſegment to the circumference of its baſe, or the chord of half its arc.

5. The curve ſurface of any ſegment or zone of a Sphere, is alſo equal to the curve ſurface of a cylinder of the ſame height with that portion, and of the ſame diameter with the Sphere. Alſo the ſurface of the whole Sphere, or of an hemiſphere, is equal to the curve ſurface of its circumſcribing cylinder. And the curve ſurfaces of their correſponding parts are equal, that are contained between any two places parallel to the baſe. And conſequently the ſurface of any ſegment or zone of a Sphere, is as its height or altitude.

Moſt of theſe properties are contained in Archimedes's treatiſe on the Sphere and cylinder. And many other rules for the ſurfaces and ſolidities of Spheres, their ſegments, zones, fruſtums, &c, may be ſeen in my Menſuration, part 3, ſect. 1, prob. 10, &c.

Hence, if d denote the diameter or axis of a Sphere, s its curve ſurface, c its ſolid content, and $a = ·7854$ the area of a circle whoſe diam. is 1; then we ſhall, from the foregoing properties, have theſe following general values or equations, viz,

$$s = 4ad^2 = \frac{6c}{d} = 6\sqrt[3]{\tfrac{2}{3}ac^2}.$$

$$c = \tfrac{1}{6}ds = \tfrac{2}{3}ad^3 = \tfrac{1}{12}\sqrt{\frac{s^3}{a}}.$$

$$d = \frac{6c}{s} = \sqrt{\frac{s}{4a}} = \sqrt[3]{\frac{3c}{2a}}.$$

Doctrine of the SPHERE. See SPHERICS.

Projection of the SPHERE. See PROJECTION.

SPHERE *of Activity*, of any body, is that determinate ſpace or extent all around it, to which, and no farther, the effluvia or the virtue of that body reaches, and in which it operates according to the nature of the body. See ACTIVITY.

SPHERE, in Aſtronomy, that concave orb or expanſe which inveſts our globe, and in which the heavenly

venly bodies, the ſun, moon, ſtars, planets, and comets, appear to be fixed at an equal diſtance from the eye. This is alſo called the Sphere of the world; and it is the ſubject of ſpherical aſtronomy.

This Sphere, as it includes the fixed ſtars, from whence it is ſometimes called the *Sphere of the fixed ſtars*, is immenſely great. So much ſo, that the diameter of the earth's orbit is vaſtly ſmall in reſpect of it; and conſequently the centre of the Sphere is not ſenſibly changed by any alteration of the ſpectator's place in the ſeveral parts of the orbit: but ſtill in all points of the earth's ſurface, and at all times, the inhabitants have the ſame appearance of the Sphere; that is, the fixed ſtars ſeem to poſſeſs the ſame points in the ſurface of the Sphere. For, our way of judging of the places &c of the ſtars, is to conceive right lines drawn from the eye, or from the centre of the earth, through the centres of the ſtars, and thence continued till they cut the Sphere; and the points where theſe lines ſo meet the Sphere, are the apparent places of thoſe ſtars.

The better to determine the places of the heavenly bodies in the Sphere, ſeveral circles are conceived to be drawn in the ſurface of it, which are called circles of the Sphere.

Sphere, in Geography, &c, denotes a certain diſpoſition of the circles on the ſurface of the earth, with regard to one another, which varies in the different parts of it.

The circles originally conceived on the ſurface of the Sphere of the world, are almoſt all transferred, by analogy, to the ſurface of the earth, where they are conceived to be drawn directly underneath thoſe of the Sphere, or in the ſame poſitions with them; ſo that, if the planes of thoſe of the earth were continued to the Sphere of the ſtars, they would coincide with the reſpective circles on it. Thus, we have an horizon, meridian, equator, &c, on the earth. And as the equinoctial, or equator, in the heavens, divides the Sphere into two equal parts, the one north and the other ſouth, ſo does the equator on the ſurface of the earth divide its globe in the ſame manner. And as the meridians in the heavens paſs through the poles of the equinoctial, ſo do thoſe on the earth, &c. With regard then to the poſition of ſome of theſe circles in reſpect of others, we have a *right*, an *oblique*, and a *parallel* Sphere.

A Right or Direct Sphere, (fig. 4, plate 26), is that which has the poles of the world PS in its horizon, and the equator EQ in the zenith and nadir. The inhabitants of this Sphere live exactly at the equator of the earth, or under the line. They have therefore no latitude, nor no elevation of the pole. They can ſee both poles of the world; all the ſtars do riſe, culminate, and ſet to them; and the ſun always riſes at right-angles to their horizon, making their days and nights always of equal length, becauſe the horizon biſects the circle of the diurnal revolution.

An Oblique Sphere, (fig. 5, plate 26), is that in which the equator EQ, as alſo the axis PS, cuts the horizon HO obliquely. In this Sphere, one pole P is above the horizon, and the other below it; and therefore the inhabitants of it ſee always the former pole, but never the latter; the ſun and ſtars &c all riſe and ſet obliquely; and the days and nights are always varying, and growing alternately longer and ſhorter.

A Parallel Sphere, (fig. 6, plate 26), is that which has the equator in or parallel to the horizon, as well as all the ſun's parallels of declination. Hence, the poles are in the zenith and nadir; the ſun and ſtars move always quite around parallel to the horizon, the inhabitants, if any, being juſt at the two poles, having 6 months continual day, and 6 months night, in each year; and the greateſt height to which the ſun riſes to them, is 23° 28′, or equal to his greateſt declination.

Armillary or Artificial Sphere, is an aſtronomical inſtrument, repreſenting the ſeveral circles of the Sphere in their natural order; ſerving to give an idea of the office and poſition of each of them, and to reſolve various problems relating to them.

It is thus called, as conſiſting of a number of rings of braſs, or other matter, called by the Latins *armillæ*, from their reſembling of bracelets or rings for the arm.

By this, it is diſtinguiſhed from the globe, which, though it has all the circles of the Sphere on its ſurface; yet is not cut into armillæ or rings, to repreſent the circles ſimply and alone; but exhibits alſo the intermediate ſpaces between the circles.

Armillary Spheres are of different kinds, with regard to the poſition of the earth in them; whence they become diſtinguiſhed into Ptolomaic and Copernican Spheres: in the firſt of which, the earth is in the centre, and in the latter near the circumference, according to the poſition which that planet obtains in thoſe ſyſtems.

The Ptolomaic Sphere, is that commonly in uſe, and is repreſented in fig. 6, plate 2, vol. 1, with the names of the ſeveral circles, lines, &c of the Sphere inſcribed upon it. In the middle, upon the axis of the Sphere, is a ball T, repreſenting the earth, on the ſurface of which are the circles &c of the earth. The Sphere is made to revolve about the ſaid axis, which remains at reſt; by which means the ſun's diurnal and annual courſes about the earth are repreſented according to the Ptolomaic hypotheſis: and even by means of this, all problems relating to the phenomena of the ſun and earth are reſolved, as upon the celeſtial globe, and after the ſame manner; which ſee deſcribed under Globe.

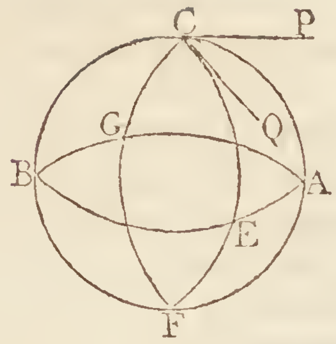
Copernican Sphere, fig. 7, plate 26, is very different from the Ptolomaic, both in its conſtitution and uſe; and is more intricate in both. Indeed the inſtrument is in the hands of ſo few people, and its uſe ſo inconſiderable, except what we have in the other more common inſtruments, particularly the globe and the Ptolomaic Sphere, that any farther account of it is unneceſſary.

Dr. Long had an Armillary Sphere of glaſs, of a very large ſize, which is deſcribed and repreſented in his Aſtronomy. And Mr. Ferguſon conſtructed a ſimilar one of braſs, which is exhibited in his Lectures, p. 194 &c.

SPHERICAL, ſomething relating to the ſphere. As,

Spherical *Angle*, is the angle formed on the ſurface of a Sphere or globe by the circumferences of two

two great circles. This angle, formed by the circumferences, is equal to that formed by the planes of the same circles, or equal to the inclination of those two planes; or equal to the angle made by their tangents at the angular point. Thus, the inclination of the two planes CAF, CEF, forms the Spherical Angle ACE, equal to the tangential angle PCQ.



The measure of a Spherical Angle, ACE, is an arc of a great circle AE, described from the vertex C, as from a pole, and intercepted between the legs CA and CE.

Hence, 1st, Since the inclination of the plane CEF to the plane CAF, is every where the same, the angles in the opposite intersections, C and F, are equal.—2d, Hence the measure of a Spherical Angle ACE, is an arc described at the interval of a quadrant CA or CE, from the vertex C between the legs CA, CE.—3d, If a circle of the sphere CEFG cut another AEBG, the adjacent angles AEC and BEC are together equal to two right angles; and the vertical angles AEC, BEF are equal to one another. Also all the angles formed at the same point, on the same side of a circle, are equal to two right angles, and all those quite around any point equal to four right angles.

SPHERICAL *Triangle*, is a triangle formed upon the surface of a sphere, by the intersecting arcs of three great circles; as the triangle ACE.

Spherical Triangles are either *right-angled*, *oblique*, *equilateral*, *isosceles*, or *scalene*, in the same manner as plane triangles. They are also said to be *quadrantal*, when they have one side a quadrant. Two sides or two angles are said to be of the *same affection*, when they are at the same time either both greater, or both less than a quadrant or a right angle or 90°; and of *different affections*, when one is greater and the other less than 90 degrees.

Properties of SPHERICAL *Triangles.*

1. Spherical Triangles have many properties in common with plane ones: Such as, That, in a triangle, equal sides subtend equal angles, and equal angles are subtended by equal sides: That the greater angles are subtended by the greater sides, and the less angles by the less sides.

2. In every Spherical Triangle, each side is less than a semicircle: any two sides taken together are greater than the third side: and all the three sides taken together are less than the whole circumference of a circle.

3. In every Spherical Triangle, any angle is less than 2 right angles; and the sum of all the three angles taken together, is greater than 2, but less than 6, right angles.

4. In an oblique Spherical Triangle, if the angles at the base be of the same affection, the perpendicular from the other angle falls within the triangle; but if they be of different affections, the perpendicular falls without the triangle.

Dr. Maskelyne's remarks on the properties of Spherical Triangles, are as follow: (See the Introd. to my Logs. pa. 160, 2d edition.)

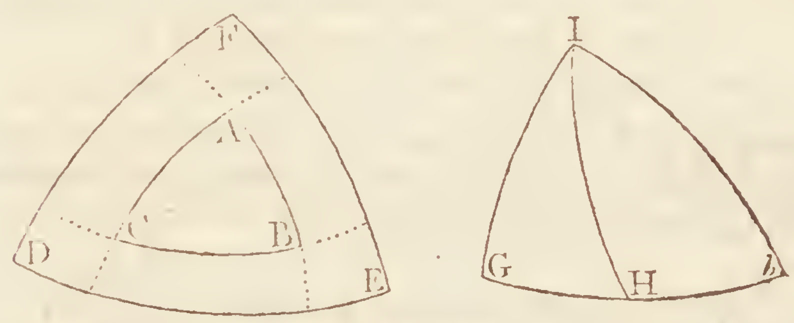
5. "A Spherical Triangle is equilateral, isoscelar, or scalene, according as it has its three angles all equal, or two of them equal, or all three unequal; and vice versa.

6. The greatest side is always opposite the greatest angle, and the smallest side opposite the smallest angle.

7. Any two sides taken together are greater than the third.

8. If the three angles are all acute, or all right, or all obtuse; the three sides will be, accordingly, all less than 90°, or equal to 90°, or greater than 90°; and vice versa.

9. If from the three angles A, B, C, of a triangle ABC, as poles, there be described, upon the surface of the sphere, three arches of a great circle DE, DF, FE, forming by their intersections a new Spherical Triangle DEF; each side of the new triangle will be the supplement of the angle at its pole; and each angle of the same triangle, will be the supplement of the side opposite to it in the triangle ABC.



10. In any triangle GHI or G*h*I, right angled in G, 1st, The angles at the hypotenuse are always of the same kind as their opposite sides; 2dly, The hypotenuse is less or greater than a quadrant, according as the sides including the right angle, are of the same or different kinds; that is to say, according as these same sides are either both acute, or both obtuse, or as one is acute and the other obtuse. And, vice versa, 1st, The sides including the right angle, are always of the same kind as their opposite angles; 2dly, The sides including the right angle will be of the same or different kinds, according as the hypotenuse is less or more than 90°; but one at least of them will be of 90°, if the hypotenuse is so."

Of the Area of a SPHERICAL *Triangle.* The mensuration of Spherical Triangles and polygons was first found out by Albert Girard, about the year 1600, and is given at large in his *Invention Nouvelle en l'Algebre*, pa. 50, &c; 4to, Amst. 1629. In any Spherical Triangle, the area, or surface inclosed by its three sides upon the surface of the globe, will be found by this proportion:

As 8 right angles or 720°,
Is to the whole surface of the sphere;
Or, as 2 right angles or 180°,
To one great circle of the sphere;
So is the excess of the 3 angles above 2 right angles,
To the area of the Spherical Triangle.

Hence, if a denote $\cdot 7854$,
$d =$ diam. of the globe, and
$s =$ sum of the 3 angles of the triangle;
then

then $add \times \frac{s-180}{180}$ = area of the Spherical Triangle.

Hence alſo, if r denote the radius of the ſphere, and c its circumference;
then the area of the triangle will thus be variouſly expreſſed; viz, Area =

$$ad^2 \times \frac{s-180}{180} = cd \times \frac{s-180}{720} = cr \times \frac{s-180}{360};$$

or barely $= r \times \overline{s-180^\circ}$, in ſquare degrees, when the radius r is eſtimated in degrees; for then the circumference c is $= 360^\circ$.

Farther, becauſe the radius r, of any circle, when eſtimated in degrees, is, $= \frac{180}{3 \cdot 14159 \text{ \&c.}} = 57 \cdot 2957795$, the laſt rule $r \times \overline{s-180}$, for expreſſing the area A of the Spherical Triangle, in ſquare degrees, will be barely

$$A = 57 \cdot 2957795\, s - 10313 \cdot 24 =$$
$$= 57\tfrac{50}{169}s - 10313\tfrac{1}{4} \text{ very nearly.}$$

Hence may be found the ſums of the three angles in any Spherical Triangle, having its area A known; for the laſt equation gives the ſum

$$s = \frac{A}{r} + 180 = \frac{A}{57 \cdot 29 \text{ \&c.}} + 180 = \frac{169A}{9683} + 180.$$

So that, for a Triangle on the ſurface of the earth, whoſe three ſides are known; if it be but ſmall, as of a few miles extent, its area may be found from the known lengths of its ſides, conſidering it as a plane Triangle, which gives the value of the quantity A; and then the laſt rule above will give the value of s, the ſum of the three angles; which will ſerve to prove whether thoſe angles are nearly exact, that have been taken with a very nice inſtrument, as in large and extenſive meaſurements on the ſurface of the earth.

Reſolution of SPHERICAL *Triangles.* See TRIANGLE, and TRIGONOMETRY.

SPHERICAL *Polygon*, is a figure of more than three ſides, formed on the ſurface of a globe by the interſecting arcs of great circles.

The area of any Spherical Polygon will be found by the following proportion; viz,

As 8 right-angles or 720°,
To the whole ſurface of the ſphere;
Or, as 2 right angles or 180°,
To a great circle of the ſphere;
So is the exceſs of all the angles above the product of 180 and 2 leſs than the number of angles,
To the area of the ſpherical polygon.

That is, putting n = the number of angles,
s = ſum of all the angles,
d = diam. of the ſphere,
a = ·78539 &c;

Then $A = ad^2 \times \frac{s-(n-2)180}{180}$ = the area of the Spherical Polygon.

Hence other rules might be found, ſimilar to thoſe for the area of the Spherical Triangle.

Hence alſo, the ſum s of all the angles of any Spherical Polygon, is always leſs than $180n$, but greater than $180(n-2)$, that is leſs than n times 2 right angles, but greater than $n-2$ times 2 right angles.

SPHERICAL *Aſtronomy*, that part of aſtronomy which conſiders the univerſe ſuch as it appears to the eye. See ASTRONOMY.

Under Spherical Aſtronomy, then, come all the phenomena and appearances of the heavens and heavenly bodies, ſuch as we perceive them, without any enquiry into the reaſon, the theory, or truth of them. By which it is diſtinguiſhed from theorical aſtronomy, which conſiders the real ſtructure of the univerſe, and the cauſes of thoſe phenomena.

In the Spherical Aſtronomy, the world is conceived to be a concave Spherical ſurface, in whoſe centre is the earth, or rather the eye, about which the viſible frame revolves, with ſtars and planets fixed in the circumference of it. And on this ſuppoſition all the other phenomena are determined.

The theorical aſtronomy teaches us, from the laws of optics, &c, to correct this Scheme and reduce the whole to a juſter ſyſtem.

SPHERICAL *Compaſſes.* See COMPASSES.

SPHERICAL *Geometry*, the doctrine of the ſphere; particularly of the circles deſcribed on its ſurface, with the method of projecting the ſame on a plane; and meaſuring their arches and angles when projected.

SPHERICAL *Numbers.* See CIRCULAR *Numbers.*

SPHERICAL *Trigonometry.* See *Spherical* TRIGONOMETRY.

SPHERICITY, the quality of a ſphere; or that by which a thing becomes ſpherical or round.

SPHERICS, the Doctrine of the ſphere, particularly of the ſeveral circles deſcribed on its ſurface; with the method of projecting the ſame on a plane. See PROJECTION *of the Sphere.*

A circle of the ſphere is that which is made by a plane cutting it. If the plane paſs through the centre, it is a *great* circle: if not, it is a *little* circle.

The *pole* of a circle, is a point on the ſurface of the ſphere equidiſtant from every point of the circumference of the circle. Hence every circle has two poles, which are diametrically oppoſite to each other; and all circles that are parallel to each other have the ſame poles.

Properties of the Circles of the Sphere.

1. If a ſphere be cut in any manner by a plane, the ſection will be a circle. And a great circle when the ſection paſſes through the centre, otherwiſe it is a *little* circle. Hence, all great circles are equal to each other: and the line of ſection of two great circles of the ſphere, is a diameter of the ſphere: and therefore two great circles interſect each other in points diametrically oppoſite; and make equal angles at thoſe points; and divide each other into two equal parts; alſo any great circle divides the whole ſphere into two equal parts.

2. If a great circle be perpendicular to any other circle, it paſſes through its poles. And if a great circle paſs

pass through the pole of any other circle, it cuts it at right angles, and into two equal parts.

3. The distance between the poles of two circles, is equal to the angle of their inclination.

4. Two great circles passing through the poles of another great circle, cut all the parallels to this latter into similar arcs. Hence, an angle made by two great circles of the sphere, is equal to the angle of inclination of the planes of these great circles. And hence also the lengths of those parallels are to one another as the sines of their distances from their common pole, or as the cosines of their distances from their parallel great circle. Consequently, as radius is to the cosine of the latitude of any point on the globe, so is the length of a degree at the equator, to the length of a degree in that latitude.

5. If a great circle pass through the poles of another; this latter also passes through the poles of the former; and the two cut each other perpendicularly.

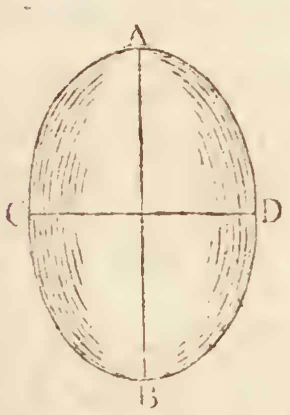
6. If two or more great circles intersect each other in the poles of another great circle; this latter will pass through the poles of all the former.

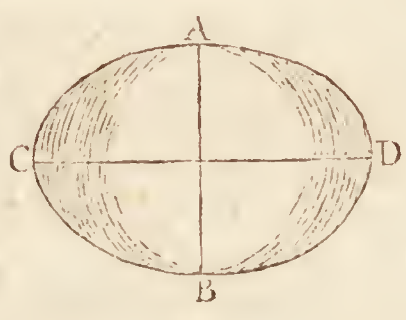
7. All circles of the sphere that are equally distant from the centre, are equal; and the farther they are distant from the centre, the less they are.

8. The shortest distance on the surface of a sphere, between any two points on that surface, is the arc of a great circle passing through those points. And the smaller the circle is that passes through the same points, the longer is the arc of distance between them. Hence the proper measure, or distance, of two places on the surface of the globe, is an arc of a great circle intercepted between the same. See Theodosius and other writers on Spherics.

SPHEROID, a solid body approaching to the figure of a sphere, though not exactly round, but having one of its diameters longer than the other.

This solid is usually considered as generated by the rotation of an oval plane figure about one of its axes. If that be the longer or transverse axis, the solid so generated is called an *oblong* Spheroid, and sometimes *prolate*, which resembles an egg, or a lemon; but if the oval revolve about its shorter axis, the solid will be an *oblate* Spheroid, which resembles an orange, and in this shape also is the figure of the earth, and the other planets.





The axis about which the oval revolves, is called the *fixed* axis, as AB; and the other CD is the *revolving* axis: whichever of them happens to be the longer.

When the revolving oval is a perfect ellipse, the solid generated by the revolution is properly called an *ellipsoid*, as distinguished from the Spheroid, which is generated from the revolution of any oval whatever, whether it be an ellipse or not. But generally speaking, in common acceptation, the term Spheroid is used for an ellipsoid; and therefore, in what follows, they are considered as one and the same thing.

Any section of a Spheroid, by a plane, is an ellipse (except the sections perpendicular to the fixed axe, which are circles); and all parallel sections are similar ellipses, or having their transverse and conjugate axes in the same constant ratio; and the sections parallel to the fixed axe are similar to the ellipse from which the solid was generated. See my Mensuration pa. 267 &c, 2d edit.

For the Surface of a Spheroid, whether it be oblong or oblate. Let f denote the fixed axe,

r the revolving axe,

$$a = \cdot 7854, \text{ and } q = \frac{ff \backsim rr}{ff};$$

then will the surface s be expressed by the following series, using the upper signs for the oblong spheroid, and the under signs for the oblate one; viz,

$$s = 4arf \times \left(1 \mp \frac{1}{2.3}q - \frac{1}{2.4.5}q^2 \mp \frac{3}{2.4.6.7}q^3 \text{ \&c}\right.;$$

where the signs of the terms, after the first, are all negative for the oblong Spheroid, but alternately positive and negative for the oblate one.

Hence, because the factor $4arf$ is equal to 4 times the area of the generating ellipse, it appears that the surface of the oblong Spheroid is less than 4 times the generating ellipse, but the surface of the oblate Spheroid is greater than 4 times the same: while the surface of the sphere falls in between the two, being just equal to 4 times its generating circle.

Huygens, in his Horolog. Oscillat. prop. 9, has given two elegant constructions for describing a circle equal to the superficies of an oblong and an oblate Spheroid, which he says he found out towards the latter end of the year 1657. As he gave no demonstrations of these, I have demonstrated them, and also rendered them more general, by extending and adapting them to the surface of any segment or zone of the Spheroid. See my Mensuration, pa. 308 &c, 2d ed. where also are several other rules and constructions for the surfaces of Spheroids, besides those of their segments, and frustums.

Of the Solidity of a Spheroid. Every Spheroid, whether oblong or oblate, is, like a sphere, exactly equal to two-thirds of its circumscribing cylinder. So that, if f denote the fixed axe, r the revolving axe, and $a = \cdot 7854$; then $\frac{2}{3}afr^2$ denotes the solid content of either Spheroid. Or, which comes to the same thing, if t denote the transverse, and c the conjugate axe of the generating ellipse;

then $\frac{2}{3}ac^2t$ is the content of the oblong Spheroid,

and $\frac{2}{3}act^2$ is the content of the oblate Spheroid.

Consequently, the proportion of the former solid to the latter, is as c to t, or as the less axis to the greater.

Farther, if about the two axes of an ellipse be generated

nerated two spheres and two spheroids, the four solids will be continued proportionals, and the common ratio will be that of the two axes of the ellipse; that is, as the greater sphere, or the sphere upon the greater axe, is to the oblate Spheroid, so is the oblate Spheroid to the oblong Spheroid, and so is the oblong Spheroid to the less sphere, and so is the transverse axis to the conjugate. See my Mensuration, pa. 327 &c, 2d ed. where may be seen many other rules for the solid contents of Spheroids, and their various parts. See also Archimedes on Spheroids and Conoids.

Dr. Halley has demonstrated, that in a sphere, Mercator's nautical meridian line is a scale of logarithmic tangents of the half complements of the latitudes. But as it has been found that the shape of the earth is spheroidal, this figure will make some alteration in the numbers resulting from Dr. Halley's theorem. Maclaurin has therefore given a rule, by which the meridional parts to any Spheroid may be found with the same exactness as in a sphere. There is also an ingenious tract by Mr. Murdoch on the same subject. See Philos. Trans. No. 219. Mr. Cotes has also demonstrated the same proposition, Harm. Mens. pa. 20, 21. See MERIDIONAL *Parts*.

Universal SPHEROID, a name given to the solid generated by the rotation of an ellipse about some other diameter, which is neither the transverse nor conjugate axis. This produces a figure resembling a heart. See my Mensuration, pa. 352, 2d ed.

SPINDLE, in Geometry, a solid body generated by the revolution of some curve line about its base or double ordinate AB; in opposition to a conoid, which is generated by the rotation of the curve about its axis or abscis, perpendicular to its ordinate.

The Spindle is denominated circular, elliptic, hyperbolic, or parabolic, &c, according to the figure of its generating curve. See my Mensur. in several places.

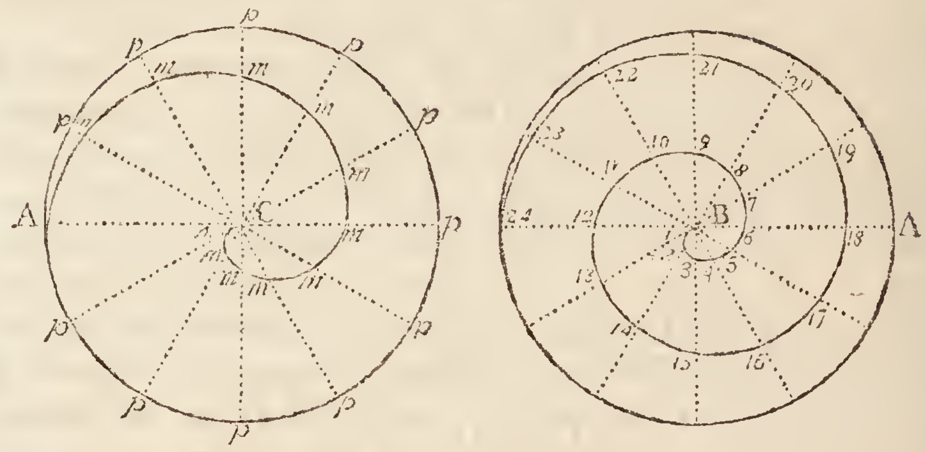
SPINDLE, in Mechanics, sometimes denotes the axis of a wheel, or roller, &c; and its ends are the pivots. See also *Double* CONE.

SPIRAL, in Geometry, a curve line of the circular kind, which, in its progress, recedes always more and more from a point within, called its centre; as in winding from the vertex of a cone down to its base.

The first treatise on a Spiral is by Archimedes, who thus describes it: Divide the circumference of a circle A*pp* &c into any number of equal parts, by a continual bisection at the points *pp* &c. Divide also the radius AC into the same number of equal parts, and make C*m*, C*m*, C*m*, &c, equal to 1, 2, 3, &c of these equal parts; then a line drawn, with a steady hand, drawn through all the points *m*, *m*, *m*, &c, will trace out the Spiral.

This is more particularly called the *first* Spiral, when it has made one complete revolution to the point A; and the space included between the Spiral and the radius CA, is the *Spiral space*.

The first Spiral may be continued to a *second*, by describing another circle with double the radius of the first; and the second may be continued to a *third*, by a third circle; and so on.



Hence it follows, that the parts of the circumference A*p* are as the parts of the radii C*m*; or A*p* is to the whole circumference, as C*m* is to the whole radius. Consequently, if c denote the circumference, r the radius, x = C*m*, and y = A*p*; then there arises this proportion $r : c :: x : y$, which gives $ry = cx$ for the equation of this Spiral; and which therefore it has in common with the quadratrix of Dinostrates, and that of Tschirnhausen: so that $r^n y^m = c^n x^m$ will serve for infinite Spirals and quadratrices. See QUADRATRIX.

The Spiral may also be conceived to be thus generated, by a continued uniform motion. If a right line, as AB (*last fig. above*) having one end moveable about a fixed point at B, be uniformly turned round, so as the other end A may describe the circumference of a circle; and at the same time a point be conceived to move uniformly forward from B towards A, in the right line or radius AB, so that the point may describe that line, while the line generates the circle; then will the point, with its two motions, describe the curve B, 1, 2, 3, 4, 5, &c, of the same Spiral as before.

Again, if the point B be conceived to move twice as slow as the line AB, so that it shall get but half way along BA, when that line shall have formed the circle; and if then you imagine a new revolution to be made of the line carrying the point, so that they shall end their motion at last together, there will be formed a *double* Spiral line, as in the last figure. From the manner of this description may easily be drawn these corollaries:

1. That the lines B12, B11, B10, &c, making equal angles with the first and second Spiral (as also B12, B10, B8), &c, are in arithmetical progression.

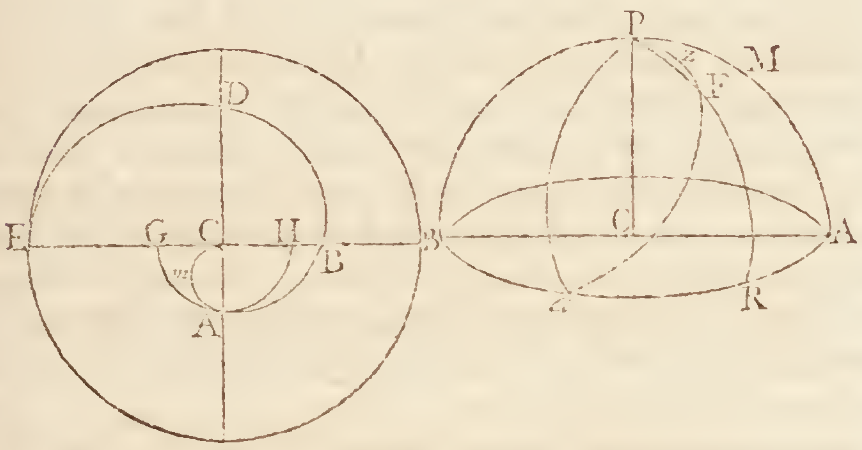
2. The lines B7, B10, &c, drawn any how to the first Spiral, are to one another as the arcs of the circle intercepted between BA and those lines; because whatever parts of the circumference the point A describes, as suppose 7, the point B will also have run over 7 parts of the line AB.

3. Any lines drawn from B to the second Spiral, as B18, B22, &c, are to each other as the aforesaid arcs, together with the whole circumference added on both sides: for at the same time that the point A runs over 12, or the whole circumference, or perhaps 7 parts more, shall the point B have run over 12, and 7 parts of the line AB, which is now supposed to be divided into 24 equal parts.

4. The

4. The first Spiral line is equal to half the circumference of the first circle; for the radii of the sectors, and consequently of the arcs, are in a simple arithmetic progression, while the circumference of the circle contains as many arcs equal to the greatest; therefore the circumference is in proportion to all those Spiral arcs, as 2 to 1.

5. The first Spiral space is equal to $\frac{1}{3}$ of the first or circumscribing circle. That is, the area CABDE of the Spiral, is equal to $\frac{1}{3}$ part of the circle described with the radius CE. In like manner, the whole Spiral area, generated by the ray drawn from the point C to the curve, when it makes two revolutions, is $\frac{7}{3}$ of the circle described with the radius 2CE.



And, generally, the whole area generated by the ray from the beginning of the motion, till after any number n of revolutions, is equal to $\frac{1}{3}$ of the circle whose radius is $n \times$ CE, that is equal to the 3d part of the space which is the same multiple of the circle described with the greatest ray, as the number of revolutions is of unity.

In like manner also, any sector or portion of the area of the Spiral, terminated by the curve CmA and the right line CA, is equal to $\frac{1}{3}$ of the circular sector CAG terminated by the right lines CA and CG, this latter being the situation of the revolving ray when the point that describes the curve sets out from C. See Maclaurin's Flux. Introd. pa. 30, 31. Se also Quadrature of the Spiral of Archimedes.

Spiral, *Logistic*, or *Logarithmic*. See Logistic and Quadrature.

Spiral *of Pappus*, a Spiral formed on the surface of a sphere, by a motion similar to that by which the Spiral of Archimedes is described on a plane. This Spiral is so called from its inventor Pappus. Collect. Mathem. lib. 4 prop. 30. Thus, if C be the centre of the sphere, ARBA a great circle, P its pole; and while the quadrant PMA revolves about the pole P with an uniform motion, if a point proceeding from P move with a given velocity along the quadrant, it will trace upon the spherical surface the Spiral PZFa.

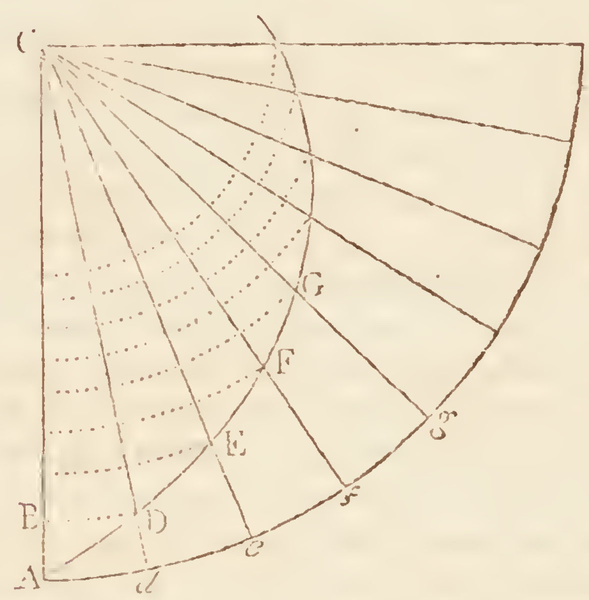
Now if we suppose the quadrant PMA to make a complete revolution in the same time that the point, which traces the Spiral on the surface of the sphere, describes the quadrant, which is the case considered by Pappus; then the portion of the spherical surface terminated by the whole Spiral, and the circle ARBA, and the quadrant PMA, will be equal to the square of the diameter AB. In any other case, the area PMAa FZP is to the square of that diameter AB, as the arc Aa is to the whole circumference ARBA. And this area is always to the spherical triangle PAa, as a square is to its circumscribing circle, or as the diameter of a circle is to half its circumference, or as 2 is to 3·14159 &c. See Maclaurin's Fluxions, Introd. pa. 31—33.

The portion of the spherical surface, terminated by the quadrant PMA, with the arches AR, FR, and the spiral PZF, admits of a perfect quadrature, when the ratio of the arch Aa to the whole circumference can be assigned. See Maclaurin, ibid. pa. 33.

Parabolic Spiral. See Helicoid.

Proportional Spiral, is generated by supposing the radius to revolve uniformly, and a point from the circumference to move towards the centre with a motion decreasing in geometrical progression. See Logistic.

From the nature of a decreasing geometrical progression, it is easy to conceive that the radius CA may be continually divided; and although each successive division becomes shorter than the next preceding one, yet there must be an infinite number of divisions or terms before the last of them become of no finite magnitude. Whence it follows, that this Spiral winds continually round the centre, without ever falling into it in any finite number of revolutions.



It is also evident that any Proportional Spiral cuts the intercepted radii at equal angles: for if the divisions Ad, de, ef, fg, &c, of the circumference be very small, the several radii will be so close to one another, that the intercepted parts AD, DE, EF, FG, &c, of the Spiral may be taken as right lines; and the triangles CAD, CDE, CEF, &c, will be similar, having equal angles at the point C, and the sides about those angles proportional; therefore the angles at A, D, E, F, &c, are equal, that is, the spiral cuts the radii at equal angles. Robertson's Elem. of Navig. book 2, pa. 87.

Proportional Spirals are such Spiral lines as the rhumb lines on the terraqueous globe; which, because they make equal angles with every meridian, must also make equal angles with the meridians in the stereographic projection on the plane of the equator, and therefore will be, as Dr. Halley observes, Proportional Spirals about the polar point. From whence he demonstrates, that the meridian line is a scale of log. tangents of

of the half complements of the latitudes. See Rhumb, Loxodromy, and Meridional *Parts*.

Spiral *Pump*. See *Archimedes*'s Screw.

Spiral, in Architecture and Sculpture, denotes a curve that afcends, winding about a cone, or fpire, fo that all the points of it continually approach the axis.

By this it is diftinguifhed from the Helix, which winds in the fame manner about a cylinder.

SPORADES, in Aftronomy, a name by which the ancients diftinguifhed fuch ftars as were not included in any conftellation. Thefe the moderns more ufually call *unformed*, or *extraconftellary* ftars.

Many of the Sporades of the ancients have been fince formed into new conftellations: thus, of thofe between Urfa Major and Leo, Hevelius has formed a conftellation named Leo Minor; and of thofe between Urfa Minor and Auriga, he alfo formed the Lynx; and of thofe under the tail of Urfa Minor, another called Canis Venaticus; &c.

SPOTS, in Aftronomy, are dark places obferved on the difks or faces of the fun, moon, and planets.

The Spots on the fun are feldom if ever vifible, except through a telefcope. I have indeed met with perfons whofe eyes were fo good that they have declared they could diftinguifh the folar Spots; and it is mentioned in Jofephus à Cofta's Natural and Moral Hiftory of the Weft Indies, book 1, ch. 2, before the ufe of telefcopes, that in Peru there are Spots to be feen in the fun, which are not to be feen in Europe. See a memoir by Dr. Zach, in the Aftronomical Ephemeris of the Acad. of Berlin for 1788, relating to the difcoveries and unpublifhed papers of Thomas Harriot the celebrated algebraift. In that memoir it is fhewn, for the firft time, that Harriot was alfo an excellent aftronomer, both theoretical and practical; that he made innumerable obfervations with telefcopes from the year 1610, and, amongft them, 199 obfervations of the folar Spots, with their drawings, calculations, and the determinations of the fun's revolution round his axis. Thefe Spots were alfo difcovered near about the fame time by Galileo and Scheiner. See Joh. Fabricius Phryfius De Maculis in Sole obfervatis & apparente eorum cum fole converfione narratio, 1611; alfo Galileo's Iftoria e Demonftrazioni intorne alle Machie Solare e loro accidenti, 1613.

Some diftinguifh the Spots into Maculæ, or dark Spots; and Faculæ, or bright Spots; but there feems but little foundation for any fuch divifion. They are very changeable as to number, form, &c; and are fometimes in a multitude, and fometimes none at all. Some imagine they may become fo numerous, as to hide the whole face of the Sun, or at leaft the greater part of it; and to this they afcribe what Plutarch mentions, viz, that in the firft year of the reign of Auguftus, the fun's light was fo faint and obfcure, that one might look fteadily at it with the naked eye. To which Kepler adds, that in 1547, the Sun appeared reddifh, as when viewed through a thick mift; and hence he conjectures that the Spots in the fun are a kind of dark fmoke, or clouds, floating on his furface.

Some again will have them ftars, or planets, paffing over the body of the fun: but others, with more probability, think they are opake bodies, in manner of crufts, formed like the fcums on the furface of liquors.

Dr. Derham, from a variety of particulars, which he has recited, concerning the folar Spots, and their congruity to what we obferve in our own globe, infers, that they are caufed by the eruption of fome new volcano in the fun, which pouring out at firft a prodigious quantity of fmoke and other opake matter, caufeth the Spots: and as that fuliginous matter decays and fpends itfelf, and the volcano at laft becomes more torrid and flaming, fo the Spots decay and become umbræ, and at laft faculæ: which faculæ he fuppofes to be no other than more flaming lighter parts than any other parts of the fun. Philof. Tranf. vol. 23, p. 1504, or Abr. vol. 4, p. 235.

Dr. Franklin (in his Exper. and Obferv. p. 266.) fuggefts a conjecture, that the parts of the Sun's fulphur feparated by fire, rife into the atmofphere, and there being freed from the immediate action of the fire, they collect into cloudy maffes, and gradually becoming too heavy to be longer fupported, they defcend to the fun, and are burnt over again. Hence, he fays, the Spots appearing on his face, which are obferved to diminifh daily in fize, their confuming edges being of particular brightnefs.

For another folution of thefe phenomena, fee Maculæ. Various other accounts and hypothefes of thefe Spots may be feen in many of the other volumes of the Philof. Tranf. In one of thefe, viz, vol. 57, pa. 398, Dr. Horfley attempts to determine the height of the fun's atmofphere from the height of the folar Spots above his furface.

By means of the obfervations of thefe Spots, has been determined the period of the fun's rotation about his axis, viz, by obferving their periodical return.

The lunar Spots are fixed: and aftronomers reckon about 48 of them on the moon's face; to each of which they have given names. The 21ft, called *Tycho*, is one of the moft confiderable.

Circular Spots, in Electricity. See Circular Spots and Colours.

Lucid Spots, *in the heavens*, are feveral little whitifh Spots, that appear magnified, and more luminous when feen through telefcopes; and yet without any ftars in them. One of thefe is in Andromeda's girdle, and was firft obferved in 1612, by Simon Marius: it has fome whitifh rays near its middle, is liable to feveral changes, and is fometimes invifible. Another is near the ecliptic, between the head and bow of Sagittarius; it is fmall, but very luminous. A third is in the back of the Centaur, which is too far fouth to be feen in Britain. A fourth, of a fmaller fize, is before Antinous's right foot, having a ftar in it, which makes it appear more bright. A fifth is in the conftellation Hercules, between the ftars ζ and η, which is vifible to the naked eye, though it is but fmall, when the fky is clear and the moon abfent. It is probable that with more powerful telefcopes thefe lucid Spots will be found to be congeries of very minute fixed ftars.

Planetary Spots, are thofe of the planets. Aftronomers find that the planets are not without their fpots. Jupiter, Mars, and Venus, when viewed through a telefcope, fhew feveral very remarkable ones: and it is by

by the motion of these Spots, that the rotation of the planets about their axes is concluded, in the same manner as that of the sun is deduced from the apparent motion of his maculæ.

SPOUT, or *Water* Spout, an extraordinary meteor, or appearance, consisting of a moving column or pillar of water; called by the Latins *typho*, and *sypho*; and by the French *trompe*, from its shape, which resembles a speaking trumpet, the widest end uppermost.

Its first appearance is in form of a deep cloud, the upper part of which is white, and the lower black. From the lower part of this cloud there hangs, or rather falls down, what is properly called the Spout, in manner of a conical tube, largest at top. Under this tube is always a great boiling and flying up of the water of the sea, as in a jet d'eau. For some yards above the surface of the sea, the water stands as a column, or pillar; from the extremity of which it spreads, and goes off, as in a kind of smoke. Frequently the cone descends so low as to the middle of this column, and continues for some time contiguous to it; though sometimes it only points to it at some distance, either in a perpendicular, or in an oblique line.

Frequently it can scarce be distinguished, whether the cone or the column appear the first, both appearing all of a sudden against each other. But sometimes the water boils up from the sea to a great height, without any appearance of a Spout pointing to it, either perpendicularly or obliquely. Indeed, generally, the boiling or flying up of the water has the priority, this always preceding its being formed into a column. For the most part the cone does not appear hollow till towards the end, when the sea water is violently thrown up along its middle, as smoke up a chimney: soon after this, the Spout or canal breaks and disappears; the boiling up of the water, and even the pillar, continuing to the last, and for some time afterwards; sometimes till the Spout form itself again, and appear anew, which it will do several times in a quarter of an hour. See a description of several Water-Spouts by Mr. Gordon, and by Dr. Stuart, in Phil. Transf. Abr. vol. iv, pa. 103 &c.

M. de la Pryme, from a near observation of two or three Spouts in Yorkshire, described in the Philosophical Transactions, num. 281, or Abr. vol. iv, pa. 106, concludes, that the Water Spout is nothing but a gyration of clouds by contrary winds meeting in a point, or centre; and there, where the greatest condensation and gravitation is, falling down into a pipe, or great tube, somewhat like Archimedes's spiral screw; and, in its working and whirling motion, absorbing and raising the water, in the same manner as the spiral screw does; and thus destroying ships &c.

Thus, June the 21st, he observed the clouds mightily agitated above, and driven together; upon which they became very black, and were hurried round; whence proceeded a most audible whirling noise like that usually heard in a mill. Soon after there issued a long tube, or Spout, from the centre of the congregated clouds, in which he observed a spiral motion, like that of a screw, by which the water was raised up.

Again, August 15, 1687, the wind blowing at the same time out of the several quarters, created a great vortex and whirling among the clouds, the centre of which every now and then dropt down, in shape of a long thin black pipe, in which he could distinctly behold a motion like that of a screw, continually drawing upwards, and screwing up, as it were, wherever it touched.

In its progress it moved slowly over a grove of trees, which bent under it like wands, in a circular motion. Proceeding, it tore off the thatch from a barn, bent a huge oak tree, broke one of its greatest limbs, and threw it to a great distance. He adds, that whereas it is commonly said, the water works and rises in a column, before the tube comes to touch it, this is doubtless a mistake, owing to the fineness and transparency of the tubes, which do most certainly touch the surface of the sea, before any considerable motion can be raised in it; but which do not become opake and visible, till after they have imbibed a considerable quantity of water.

The dissolution of Water-Spouts he ascribes to the great quantity of water they have glutted: which, by its weight, impeding their motion, upon which their force, and even existence depends, they break, and let go their contents; which use to prove fatal to whatever is found underneath.

A notable instance of this may be seen in the Philosophical Transactions (num. 363, or Abr. vol. iv. pa 108) related by Dr. Richardson. A Spout, in 1718, breaking on Emmotmoor, nigh Coln, in Lancashire, the country was immediately overflowed; a brook, in a few minutes, rose six feet perpendicularly high; and the ground upon which the Spout fell, which was 66 feet over, was torn up to the very rock, which was no less than 7 feet deep; and a deep gulf was made for above half a mile, the earth being raised in vast heaps on each side. See a description and figure of a Water-Spout, with an attempt to account for it in Franklin's Exp. and Obs. pa. 226, &c.

Signor Beccaria has taken pains to show that Water-Spouts have an electrical origin. To make this more evident, he first describes the circumstances attending their appearance, which are the following.

They generally appear in calm weather. The sea seems to boil, and to send up a smoke under them, rising in a hill towards the Spout. At the same time, persons who have been near them have heard a rumbling noise. The form of a Water-Spout is that of a speaking trumpet, the wider end being in the clouds, and the narrower end towards the sea.

The size is various, even in the same Spout. The colour is sometimes inclining to white, and sometimes to black. Their position is sometimes perpendicular to the sea, sometimes oblique; and sometimes the Spout itself is in the form of a curve. Their continuance is very various, some disappearing as soon as formed, and some continuing a considerable time. One that he had heard of continued a whole hour. But they often vanish, and presently appear again in the same place. The very same things that Water Spouts are at sea, are some kinds of whirlwinds and hurricanes by land. They have been known to tear up trees, to throw down buildings, and make caverns in the earth; and in all these cases, to scatter earth, bricks, stones, timber, &c,

to

to a great distance in every direction. Great quantities of water have been left, or raised by them, so as to make a kind of deluge; and they have always been attended by a prodigious rumbling noise.

That these phenomena depend upon electricity cannot but appear very probable from the nature of several of them; but the conjecture is made more probable from the following additional circumstances. They generally appear in months peculiarly subject to thunder-storms, and are commonly preceded, accompanied, or followed by lightning, rain, or hail, the previous state of the air being similar. Whitish or yellowish flashes of light have sometimes been seen moving with prodigious swiftness about them. And lastly, the manner in which they terminate exactly resembles what might be expected from the prolongation of one of the uniform protuberances of electrified clouds, mentioned before, towards the sea; the water and the cloud mutually attracting one another: for they suddenly contract themselves, and disperse almost at once; the cloud rising, and the water of the sea under it falling to its level. But the most remarkable circumstance, and the most favourable to the supposition of their depending on electricity, is, that they have been dispersed by presenting to them sharp pointed knives or swords. This, at least, is the constant practice of mariners, in many parts of the world, where these Water-Spouts abound, and he was assured by several of them, that the method has often been undoubtedly effectual.

The analogy between the phenomena of Water Spouts and electricity, he says, may be made visible, by hanging a drop of water to a wire communicating with the prime conductor, and placing a vessel of water under it. In these circumstances, the drop assumes all the various appearances of a Water Spout, both in its rise, form, and manner of disappearing. Nothing is wanting but the smoke, which may require a great force of electricity to become visible.

Mr. Wilcke also considers the Water-Spout as a kind of great electrical cone, raised between the cloud strongly electrified, and the sea or the earth, and he relates a very remarkable appearance which occurred to himself, and which strongly confirms his supposition. On the 20th of July 1758, at three o'clock in the afternoon, he observed a great quantity of dust rising from the ground, and covering a field, and part of the town in which he then was. There was no wind, and the dust moved gently towards the east, where appeared a great black cloud, which, when it was near its zenith, electrified his apparatus positively, and to as great a degree as ever he had observed it to be done by natural electricity. This cloud passed his zenith, and went gradually towards the west, the dust then following it, and continuing to rise higher and higher till it composed a thick pillar, in the form of a sugar-loaf, and at length seemed to be in contact with the cloud. At some distance from this, there came, in the same path, another great cloud, together with a long stream of smaller clouds, moving faster than the preceding. These clouds electrified his apparatus negatively, and when they came near the positive cloud, a flash of lightning was seen to dart through the cloud of dust, the positive cloud, the large negative cloud, and, as far as the eye could distinguish, the whole train of smaller negative clouds which followed it. Upon this, the negative clouds spread very much, and dissolved in rain, and the air was presently clear of all the dust. The whole appearance lasted not above half an hour. See Priestley's Electr. vol. 1, pa. 438, &c.

This theory of Water-Spouts has been farther confirmed by the account which Mr. Forster gives of one of them, in his Voyage Round the World, vol. 1, pa. 191, &c. On the coast of New Zealand he had an opportunity of seeing several, one of which he has particularly described. The water, he says, in a space of fifty or sixty fathoms, moved towards the centre, and there rising into vapour, by the force of the whirling motion, ascended in a spiral form towards the clouds. Directly over the whirlpool, or agitated spot in the sea, a cloud gradually tapered into a long slender tube, which seemed to descend to meet the rising spiral, and soon united with it into a straight column of a cylindrical form. The water was whirled upwards with the greatest violence in a spiral, and appeared to leave a hollow space in the centre; so that the water seemed to form a hollow tube, instead of a solid column; and that this was the case, was rendered still more probable by the colour, which was exactly like that of a hollow glass tube. After some time, this last column was incurvated, and broke like the others; and the appearance of a flash of lightning which attended its disjunction, as well as the hail stones which fell at the time, seemed plainly to indicate, that Water-Spouts either owe their formation to the electric matter, or, at least, that they have some connection with it.

In Pliny's time, the seamen used to pour vinegar into the sea, to assuage and lay the Spout when it approached them: our modern seamen think to keep it off, by making a noise with filing and scratching violently on the deck; or by discharging great guns to disperse it.

See the figure of a Water-Spout, fig. 1, plate 27.

SPRING, in Natural History, a fountain or source of water, rising out of the ground.

The most general and probable opinion among philosophers, on the formation of Springs, is, that they are owing to rain. The rain-water penetrates the earth till such time as it meets a clayey soil, or stratum; which proving a bottom sufficiently solid to sustain and stop its descent, it glides along it that way to which the earth declines, till, meeting with a place or aperture on the surface, through which it may escape, it forms a Spring, and perhaps the head of a stream or brook.

Now, that the rain is sufficient for this effect, appears from hence, that upon calculating the quantity of rain and snow which falls yearly on the tract of ground that is to furnish, for instance, the water of the Seine, it is found that this river does not take up above one-sixth part of it.

Springs commonly rise at the bottom of mountains; the reason is, that mountains collect the most waters, and give them the greatest descent the same way. And if we sometimes see Springs on high grounds, and even on the tops of mountains, they must come from other remoter places, considerably higher, along beds of clay, or clayey ground, as in their natural channels. So that if there happen to be a valley between a mountain on whose top is a Spring, and the mountain which is to furnish

furniſh it with water, the Spring muſt be conſidered as water conducted from a reſervoir of a certain height, through a ſubterraneous channel, to make a jet of an almoſt equal height.

As to the manner in which this water is collected, ſo as to form reſervoirs for the different kinds of Springs, it ſeems to be this: the tops of mountains uſually abound with cavities and ſubterraneous caverns, formed by nature to ſerve as reſervoirs; and their pointed ſummits, which ſeem to pierce the clouds, ſtop thoſe vapours which float in the atmoſphere; which being thus condenſed, they precipitate in water, and by their gravity and fluidity eaſily penetrate through beds of ſand and the lighter earth, till they become ſtopped in their deſcent by the denſer ſtrata, ſuch as beds of clay, ſtone, &c, where they form a baſon or cavern, and working a paſſage horizontally, or a little declining, they iſſue out at the ſides of the mountains. Many of theſe Springs diſcharge water, which running down between the ridges of hills, unite their ſtreams, and form rivulets or brooks, and many of theſe uniting again on the plain, become a river.

The perpetuity of divers Springs, always yielding the ſame quantity of water, equally when the leaſt rain or vapour is afforded as when they are the greateſt, furniſh, in the opinion of ſome, conſiderable objections to the univerſality or ſufficiency of the theory above. Dr. Derham mentions a Spring in his own pariſh of Upminſter, which he could never perceive by his eye was diminiſhed in the greateſt droughts, even when all the ponds in the country, as well as an adjoining brook, had been dry for ſeveral months together; nor ever to be increaſed in the moſt rainy ſeaſons, excepting perhaps for a few hours, or at moſt for a day, from ſudden and violent rains. Had this Spring, he thought, derived its origin from rain or vapours, there would be found an increaſe and decreaſe of its water correſponding to thoſe of its cauſes; as we actually find in ſuch temporary Springs, as have undoubtedly their riſe from rain and vapour.

Some naturaliſts therefore have recourſe to the ſea, and derive the origin of Springs immediately from thence. But how the ſea-water ſhould be raiſed up to the ſurface of the earth, and even to the tops of the mountains, is a difficulty, in the ſolution of which they cannot agree. Some fancy a kind of hollow ſubterranean rocks to receive the watery vapours raiſed from channels communicating with the ſea, by means of an internal fire, and to act the part of alembics, in freeing them from their ſaline particles, as well as condenſing and converting them into water. This kind of ſubterranean laboratory, ſerving for the diſtillation of ſea-water, was the invention of Des Cartes: ſee his Princip. part 4, § 64. Others, as De la Hire &c (Mem. de l'Acad. 1703) ſet aſide the alembics, and think it enough that there be large ſubterranean reſervoirs of water at the height of the ſea, from whence the warmth of the bottom of the earth, &c, may raiſe vapours; which pervade not only the intervals and fiſſures of the ſtrata, but the bodies of the ſtrata themſelves, and at length arrive near the ſurface; where, being condenſed by the cold, they glide along on the firſt bed of clay they meet with, till they iſſue forth by ſome aperture in the ground. De la Hire adds, that the ſalts of ſtones and minerals may contribute to the detaining and fixing the vapours, and converting them into water. Farther, it is urged by ſome, that there is a ſtill more natural and eaſy way of exhibiting the riſe of the ſea-water up into mountains &c, viz, by putting a little heap of ſand, or aſhes, or the like, into a baſon of water; in which caſe the ſand &c will repreſent the dry land, or an iſland; and the baſon of water, the ſea about it. Here, ſay they, the water in the baſon will riſe to the top of the heap, or nearly ſo, in the ſame manner, and from the ſame principle, as the waters of the ſea, lakes, &c, riſe in the hills. The principle of aſcent in both is accordingly ſuppoſed to be the ſame with that of the aſcent of liquids in capillary tubes, or between contiguous planes, or in a tube filled with aſhes; all which are now generally accounted for by the doctrine of attraction.

Againſt this laſt theory, Perrault and others have urged ſeveral unanſwerable objections. It ſuppoſes a variety of ſubterranean paſſages and caverns, communicating with the ſea, and a complicated apparatus of alembics, with heat and cold, &c, of the exiſtence of all which we have no ſort of proof. Beſides, the water that is ſuppoſed to aſcend from the depths of the ſea, or from ſubterranean canals proceeding from it, through the porous parts of the earth, as it riſes in capillary tubes, aſcends to no great height, and in much too ſmall a quantity to furniſh ſprings with water, as Perrault has ſufficiently ſhewn. And though the ſand and earth through which the water aſcends may acquire ſome ſaline particles from it, they are nevertheleſs incapable of rendering it ſo freſh as the water of our fountains is generally found to be. Not to add, that in proceſs of time the ſaline particles of which the water is deprived, either by ſubterranean diſtillation or filtration, muſt clog and obſtruct thoſe canals and alembics, by which it is ſuppoſed to be conveyed to our Springs, and the ſea muſt likewiſe gradually loſe a conſiderable quantity of its ſalt.

Different ſorts of Springs. Springs are either ſuch as run continually, called perennial; or ſuch as run only for a time, and at certain ſeaſons of the year, and therefore called *temporary* Springs. Others again are called *intermitting* Springs, becauſe they flow and then ſtop, and flow and ſtop again; and *reciprocating* Springs, whoſe waters riſe and fall, or flow and ebb, by regular intervals.

In order to account for theſe differences in Springs, let ABCDE (fig. 2, pl. 27) repreſent the declivity of a hill, along which the rain deſcends; paſſing through the fiſſures or channels BF, CG, DH, and LK, into the cavity or reſervoir FGHKMI; from this cavity let there be a narrow drain or duct KE, which diſcharges the water at E. As the capacity of the reſervoir is ſuppoſed to be large in proportion to that of the drain, it will furniſh a conſtant ſupply of water to the ſpring at E. But if the reſervoir FGHKMI be ſmall, and the drain large, the water contained in the former, unleſs it is ſupplied by rain, will be wholly diſcharged by the latter, and the Spring will become dry: and ſo it will continue, even though it rains, till the water has had time to penetrate through the earth, or to paſs through the channels into the reſervoir; and the time neceſſary for furniſhing a new ſupply to the drain KE will depend on the ſize of the fiſſures, the nature

ture of the soil, and the depth of the cavity with which it communicates. Hence it may happen, that the Spring at E may remain dry for a considerable time, and even while it rains; but when the water has found its way into the cavity of the hill, the Spring will begin to run. Springs of this kind, it is evident, may be dry in wet weather, especially if the duct KE be not exactly level with the bottom of the cavity in the hill, and discharge water in dry weather; and the intermissions of the Spring may continue several days. But if we suppose XOP to represent another cavity, supplied with water by the channel NO, as well as by fissures and clefts in the rock, and by the draining of the adjacent earth; and another channel STV, communicating with the bottom of it at S, ascending to T, and terminating on the surface at V, in the form of a siphon; this disposition of the internal cavities of the earth, which we may reasonably suppose that nature has formed in a variety of places, will serve to explain the principle of reciprocating Springs; for it is plain, that the cavity XOP must be supplied with water to the height QPT, before it can pass over the bend of the channel at T, and then it will flow through the longer leg of the siphon TV, and be discharged at the end V, which is lower than S. Now if the channel STV be considerably larger than NO, by which the water is principally conveyed into the reservoir XOP, the reservoir will be emptied of its water by the siphon; and when the water descends below its orifice S, the air will drive the remaining water out of the channel STV, and the Spring will cease to flow. But in time the water in the reservoir will again rise to the height QPT, and be discharged at V as before. It is easy to conceive, that the diameters of the channels NO and STV may be so proportioned to one another, as to afford an intermission and renewal of the Spring V at regular intervals. Thus, if NO communicates with a well supplied by the tide, during the time of flow, the quantity of water conveyed by it into the cavity XOP may be sufficient to fill it up to QPT; and STV may be of such a size as to empty it, during the time of ebb. It is easy to apply this reasoning to more complicated cases, where several reservoirs and siphons communicating with each other, may supply Springs with circumstances of greater variety. See Musschenbroek's Introd. ad Phil. Nat. tom. ii. pa. 1010. Desagu. Exp. Phil. vol. ii, pa. 173, &c.

We shall here observe, that Desaguliers calls those *reciprocating* Springs which flow constantly, but with a stream subject to increase and decrease; and thus he distinguishes them from *intermitting* Springs, which flow or stop alternately.

It is said that in the diocese of Paderborn, in Westphalia, there is a Spring which disappears after twenty-four hours, and always returns at the end of six hours with a great noise, and with so much force, as to turn three mills, not far from its source. It is called the Bolderborn, or boisterous Spring. Phil. Transf. num. 7, pa. 127.

There are many Springs of an extraordinary nature in our own country, which it is needless to recite, as they are explicable by the general principles already illustrated.

Spring, *Ver*, in Astronomy and Cosmography, denotes one of the seasons of the year; commencing, in the northern parts of the earth, on the day the sun enters the first degree of Aries, which is about the 21st day of March, and ending when the sun enters Cancer, at the summer solstice, about the 21st of June; Spring ending when the summer begins.

Or, more strictly and generally, for any part of the earth, or on either side of the equator, the Spring season begins when the meridian altitude of the sun, being on the increase, is at a medium between the greatest and least; and ends when the meridian altitude is at the greatest. Or the Spring is the season, or time, from the moment of the sun's crossing the equator till he rise to the greatest height above it.

Elater Spring, in Physics, denotes a natural faculty, or endeavour, of certain bodies, to return to their first state, after having been violently put out of the same by compressing, or bending them, or the like.

This faculty is usually called by philosophers, *elastic force*, or *elasticity*.

Spring, in Mechanics, is used to signify a body of any shape, perfectly elastic.

Elasticity of a Spring. See Elasticity.

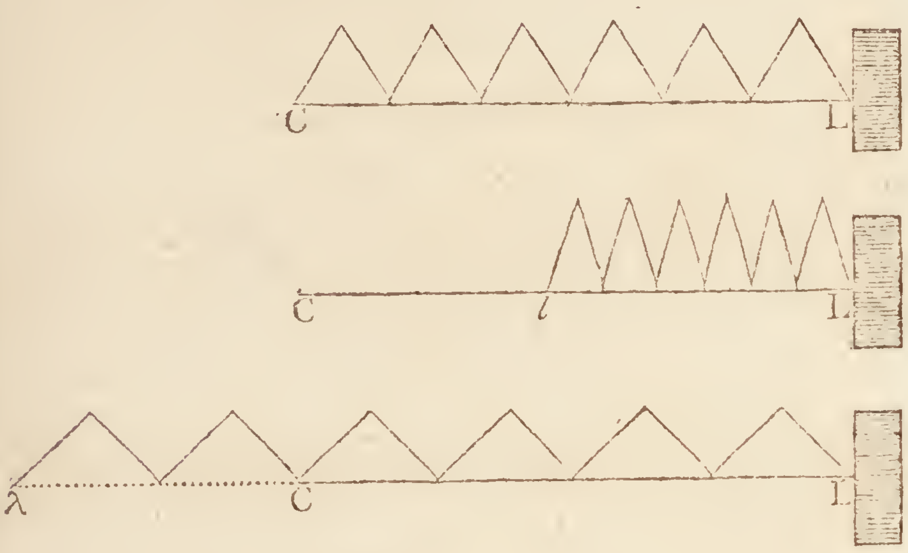
Length of a Spring, may, from its etymology, signify the length of any elastic body; but it is particularly used by Dr. Jurin to signify the greatest length to which a Spring can be forced inwards, or drawn outwards, without prejudice to its elasticity. He observes, this would be the whole length, were the Spring considered as a mathematical line; but in a material Spring, it is the difference between the whole length, when the Spring is in its natural situation, or the situation it will rest in when not disturbed by any external force, and the length or space it takes up when wholly compressed and closed, or when drawn out.

Strength or Force of a Spring, is used for the force or weight which, when the Spring is wholly compressed or closed, will just prevent it from unbending itself. Also the Force of a Spring partly bent or closed, is the force or weight which is just sufficient to keep the Spring in that state, by preventing it from unbending itself any farther.

The theory of Springs is founded on this principle, *ut intensio, sic vis*; that is, the intensity is as the compressing force; or if a Spring be any way forced or put out of its natural situation, its resistance is proportional to the space by which it is removed from that situation. This principle has been verified by the experiments of Dr. Hook, and since him by those of others, particularly by the accurate hand of Mr. George Graham. Lectures De Potentia Restitutiva, 1678.

For elucidating this principle, on which the whole theory of Springs depends, suppose a Spring CL, resting at L against any immoveable support, but otherwise lying in its natural situation, and at full liberty. Then if this Spring be pressed inwards by any force p, or from C towards L, through the space of one inch, and can be there detained by that force p, the resistance of the Spring, and the force p, exactly counterbalancing each other; then will the double force $2p$ bend the Spring through the space of 2 inches, and the triple force $3p$ through 3 inches, and the quadruple force $4p$ through 4 inches, and so on. The space CL through which the Spring is bent, or by which its end C is removed from its natural situation, being always

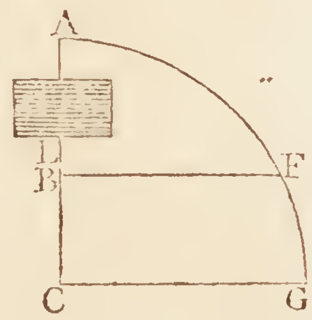
ways proportional to the force which will bend it ſo far, and will juſt detain it when ſo bent. On the other hand, if the end C be drawn outwards to any place λ, and be there detained from returning back by any force p, the ſpace Cλ, through which it is ſo drawn outwards, will be alſo proportional to the force p, which is juſt able to retain it in that ſituation.



It may here be obſerved, that the Spring of the air, or its elaſtic force, is a power of a different nature, and governed by different laws, from that of a palpable rigid Spring. For ſuppoſing the line LC to repreſent a cylindrical volume of air, which by compreſſion is reduced to Ll, or by dilatation is extended to Lλ, its elaſtic force will be reciprocally as Ll or Lλ; whereas the force or reſiſtance of a Spring is directly as Cl or Cλ.

This principle being premiſed, Dr. Jurin lays down a general theorem concerning the action of a body ſtriking on one end of a Spring, while the other end is ſuppoſed to reſt againſt an immoveable ſupport.

Thus, if a Spring of the ſtrength P, and the length CL, lying at full liberty upon an horizontal plane, reſt with one end L againſt an immoveable ſupport; and a body of the weight M, moving with the velocity V, in the direction of the axis of the Spring, ſtrike directly on the other end C, and ſo force the Spring inwards, or bend it through any ſpace CB; and if a mean proportional CG be taken between $\frac{M}{P} \times CL$ and $2a$, where a denotes the height to which a body would aſcend in vacuo with the velocity V; and farther, if upon the radius R = CG be deſcribed the quadrant of a circle GFA: then,



1. When the Spring is bent through the right ſine CB of any arc GF, the velocity v of the body M is to the original velocity V, as the coſine BF is to the radius CG; that is $v : V :: BF : CG$, or $v = \frac{BF}{R} \times V$.

2. The time t of bending the Spring through the ſame ſine CB, is to T, the time of a heavy body's aſcending in vacuo with the velocity V, as the correſponding arc is to $2a$; that is $t : T :: GF : 2a$, or $t = \frac{GF}{2a} \times T$.

The doctor gives a demonſtration of this theorem, and deduces a great many curious corollaries from it. Theſe he divides into three claſſes. The firſt contains ſuch corollaries as are of more particular uſe when the Spring is wholly cloſed before the motion of the body ceaſes: the ſecond comprehends thoſe relating to the caſe, when the motion of the body ceaſes before the Spring is wholly cloſed: and the third when the motion of the body ceaſes at the inſtant that the Spring is wholly cloſed.

We ſhall here mention ſome of the laſt claſs, as being the moſt ſimple; having firſt premiſed, that P = the ſtrength of the Spring, L = its length, V = the initial velocity of the body cloſing the Spring, M = its maſs, t = time ſpent by the body in cloſing the Spring, A = height from which a heavy body will fall in vacuo in a ſecond of time, a = the height to which a body would aſcend in vacuo with the velocity V, C = the velocity gained by the fall, m = the circumference of a circle, whoſe diameter is 1. Then, the motion of the ſtriking body ceaſing when the Spring is wholly cloſed, it will be,

1. $V = C\sqrt{\frac{PL}{2MA}}$.

2. $Vt = \frac{mCL}{4A} \times 1''$.

3. $MV = C\sqrt{\frac{PLM}{2A}}$ the firſt momentum.

4. If a quantity of motion MV bend a Spring through its whole length, and be deſtroyed by it; no other quantity of motion equal to the former, as $nM \times \frac{V}{n}$, will cloſe the ſame Spring, and be wholly deſtroyed by it.

5. But a quantity of motion, greater or leſs than MV, in any given ratio, may cloſe the ſame Spring, and be wholly deſtroyed in cloſing it; and the time ſpent in cloſing the Spring will be reſpectively greater or leſs, in the ſame given ratio.

6. The initial vis viva, or MV^2 is $= \frac{C^2PL}{2A}$; and $2aM = PL$; alſo the initial vis viva is as the rectangle under the length and ſtrength of the Spring, that is, MV^2 is as PL.

7. If the vis viva MV^2 bend a Spring through its whole length, and be deſtroyed in cloſing it; any other vis viva, equal to the former, as $n^2M \times \frac{V^2}{n^2}$, will cloſe the ſame Spring, and be deſtroyed by it.

8. But the time of cloſing the Spring by the vis viva $n^2M \times \frac{V^2}{n^2}$, will be to the time of cloſing it by the vis viva MV^2, as n to 1.

9. If the vis viva MV^2 be wholly conſumed in cloſing a Spring, of the length L, and ſtrength P; then the vis

vis viva n^2MV^2 will be fufficient to clofe, 1ft, Either a Spring of the length L and ftrength n^2P. 2d, Or a Spring of the length nL and ftrength nP. 3d, Or of the length n^2L and ftrength P. 4th, Or, if n be a whole number, the number n^2 of Springs, each of the length L and ftrength P.—It may be added, that it appears from hence, that the number of fimilar and equal Springs a given body in motion can wholly clofe, is always proportional to the fquares of the velocity of that body. And it is from this principle that the chief argument, to prove that the force of a body in motion is as the fquare of its velocity, is deduced. See FORCE.

The theorem given above, and its corollaries, will equally hold good, if the Spring be fuppofed to have been at firft bent through a certain fpace, and by unbending itfelf to prefs upon a body at reft, and thus to drive that body before it, during the time of its expanfion: only V, inftead of being the initial velocity with which the body ftruck the Spring, will now be the final velocity with which the body parts from the Spring when totally expanded.

It may alfo be obferved, that the theorem, &c, will equally hold good, if the Spring, inftead of being preffed inward, be drawn outward by the action of the body. The like may be faid, if the Spring be fuppofed to have been already drawn outward to a certain length, and in reftoring itfelf draw the body after it. And laftly, the theorem extends to a Spring of any form whatever, provided L be the greateft length it can be extended to from its natural fituation, and P the force which will confine it to that length. See Philof. Tranf. num. 472, fect. 10, or vol. 43, art. 10.

SPRING is more particularly ufed, in the Mechanic Arts, for a piece of tempered fteel, put into various machines to give them motion, by the endeavour it makes to unbend itfelf.

In watches, it is a fine piece of well-beaten fteel, coiled up in a cylindrical cafe, or frame; which by ftretching itfelf forth, gives motion to the wheels, &c.

SPRING *Arbor*, in a Watch, is that part in the middle of the Spring-box, about which the Spring is wound or turned, and to which it is hooked at one end.

SPRING *Box*, in a Watch, is the cylindrical cafe, or frame, containing within it the Spring of the watch.

SPRING-*Compaffes*. See COMPASSES.

SPRING *of the Air*, or its elaftic force. See AIR, and ELASTICITY.

SPRING-*Tides*, are the higher tides, about the times of the new and full moon. See TIDE.

SPRINGY, or *Elaftic Body*. See ELASTIC *Body*.

SQUARE, in Geometry, a quadrilateral figure, whofe angles are right, and fides equal. Or it is an equilateral rectangle. Or an equilateral rectangular parallelogram.

A Square, and indeed any other parallelogram, is bifected by its diagonal. And the fide of a Square is incommenfurable to its diagonal, being in the ratio of 1 to $\sqrt{2}$.

To find the Area of a SQUARE. Multiply the fide by itfelf, and the product is the area. So, if the fide be 10, the area is 100; and if the fide be 12, the area is 144.

SQUARE *Foot*, is a Square each fide of which is equal to a foot, or 12 inches; and the area, or Square foot is equal to 144 fquare inches.

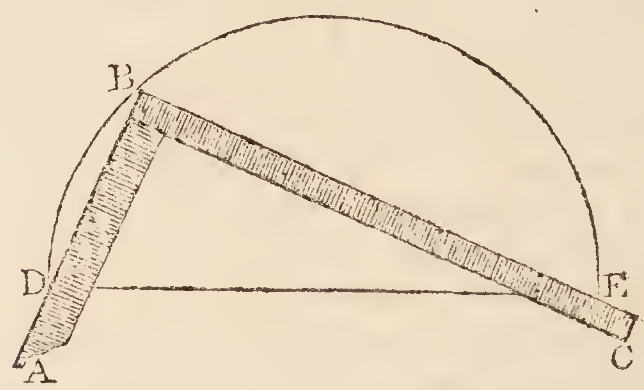
Geometrical SQUARE, a compartment often added on the face of a quadrant, called alfo *Line of* SHADOWS, and QUADRANT.

Gunner's SQUARE. See QUADRANT.

Magic SQUARE. See MAGIC *Square*.

SQUARE *Meafures*, the Squares of the lineal meafures; as in the following Table of Square Meafures:

Squa. Inches.	Sq. Feet.	Sq. Yards.	Sq. Poles.	S. Chs.	Acres.	S. Miles.
144	1					
1296	9	1				
39204	$272\frac{1}{4}$	$30\frac{1}{4}$	1			
627264	4356	484	16	1		
6272640	43560	4840	160	10	1	
4014489600	27878400	3097600	102400	6400	640	1

Normal SQUARE, is an inftrument, made of wood or metal, ferving to defcribe and meafure right angles; 

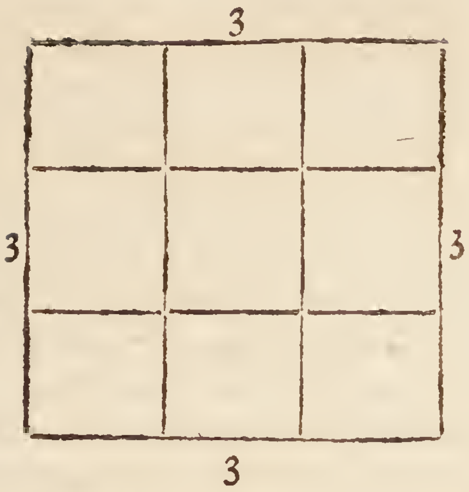
fuch is ABC. It confifts of two rulers or branches faftened together perpendicularly. When the two legs are moveable on a joint, it is called a bevel.

To examine whether the Square is exact or not. Defcribe a femicircle DBE, with any radius at pleafure; in the circumference of which apply the angle of the Square to any point as B, and the edge of one leg to one end of the diameter as D, then if the other leg pafs juft by the other extremity at E, the Square is true; otherwife not.

SQUARE *Number*, is the product arifing from a number multiplied by itfelf. Thus, 4 is the Square of 2, and 16 the Square of 4.

The feries of Square integers, is 1, 4, 9, 16, 25, 36, &c;
which are the Squares of - - 1, 2, 3, 4, 5, 6, &c.
Or the Square fractions - - $\frac{1}{4}$, $\frac{4}{9}$, $\frac{9}{16}$, $\frac{16}{25}$, $\frac{25}{36}$, $\frac{36}{49}$, &c,
which are the Squares of - - $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, &c.

A Square number is fo called, either becaufe it denotes the area of a Square, whofe fide is expreffed by the root of the Square number; as in the annexed Square, which



which consists of 9 little squares, the side being equal to 3; or else, which is much the same thing, because the points in the number may be ranged in the form of a Square, by making the root, or factor, the side of the Square.

Some properties of Squares are as follow: 1. Of the

Natural series of Squares,	1^2, 2^2, 3^2, 4^2, &c,
which are equal to	1, 4, 9, 16, &c;

The mean proportional mn between any two of these Squares m^2 and n^2, is equal to the less square *plus* its root multiplied by the difference of the roots; or also equal to the greater square *minus* its root multiplied by the said difference of the roots. That is,

$$mn = m^2 + dm = n^2 - dn;$$

where $d = n - m$ is the difference of their roots.

2. An arithmetical mean between any two Squares m^2 and n^2, exceeds their geometrical mean, by half the Square of the difference of their roots.

That is $\frac{1}{2}m^2 + \frac{1}{2}n^2 = mn + \frac{1}{2}d^2$.

3. Of three equidistant Squares in the Series, the geometrical mean between the extremes, is less than the middle Square by the Square of their common distance in the Series, or of the common difference of their roots.

That is, $mp = n^2 - d^2$;

where m, n, p, are in arithmetical progression, the common difference being d.

4. The difference between the two adjacent Squares m^2, and n^2, is $n^2 - m^2 = 2m + 1$;

in like manner, $p^2 - n^2 = 2n + 1$, the difference between the next two adjacent Squares n^2 and p^2; and so on, for the next following Squares. Hence the difference of these differences, or the second difference of the Squares, is $2n - 2m = 2 \times \overline{n - m} = 2$ only, because $n - m = 1$; that is, the second differences of the Squares are each the same constant number 2: therefore the first differences will be found by the continual addition of the number 2; and then the Squares themselves will be found by the continual addition of the first differences; and thus the whole series of Squares is constructed by addition only, as here below:

2d Diff.		2	2	2	2	2	2	&c.
1st Diff.	1	3	5	7	9	11	13	&c.
Squares.	1	4	9	16	25	36	49	&c.

And this method of constructing the table of Square numbers I find first noticed by Peletarius, in his Algebra.

5. Another curious property, also noted by the same author, is, that the sum of any number of the cubes of the natural series 1, 2, 3, 4, &c, taken from the beginning, always makes a Square number; and that the series of Squares, so formed, have for their roots the numbers 1, 3, 6, 10, 15, 21, &c, the diffs. of which are 1, 2, 3, 4, 5, 6, &c,

viz, $1^3 = 1^2$,

$1^3 + 2^3 = 3^2$,

$1^3 + 2^3 + 3^3 = 6^2$,

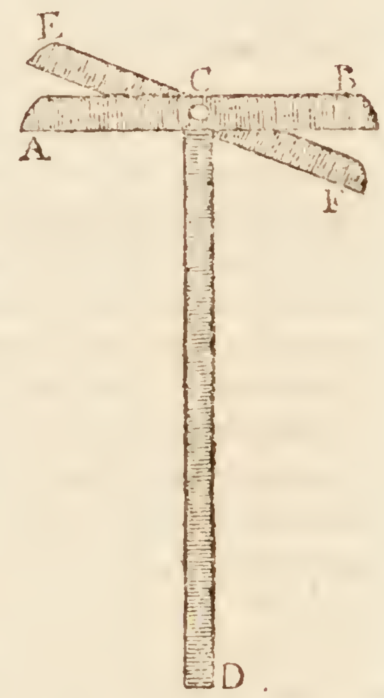
$1^3 + 2^3 + 3^3 + 4^3 = 10^2$; and in general

$1^3 + 2^3 + 3^3 + n^3 = (1 + 2 + 3 + n)^2 = \frac{1}{4}n.\overline{n+1}$;

where n is the number of the terms or cubes.

Square *Root*, a number considered as the root of a second power or Square number: or a number which multiplied by itself, produces the given number. See Extraction *of Roots*, and also the article Root, where tables of Squares and roots are inserted.

T. Square, or *Tee* Square, an instrument used in drawing, so called from its resemblance to the capital letter T.



This instrument consists of two straight rulers AB and CD, fixed at right angles to each other. To which is sometimes added a third EF, moveable about the pin C, to set it to make any angle with CD.—It is very useful for drawing parallel and perpendicular lines, on the face of a smooth drawing-board.

SQUARED *-square*, Squared-*cube*, &c. See Power.

SQUARING. See Quadrature.

Squaring *the Circle*, is the making or finding a Square whose area shall be equal to the area of a given circle.

The best mathematicians have not yet been able to resolve this problem accurately, and perhaps never will. But they can easily come to any proposed degree of approximation whatever; for instance, so near as not to err so much in the area, as a grain of sand would cover, in a circle whose diameter is equal to that of the orbit of Saturn. The following proportion is near enough the truth for any real use, viz, as 1 is to ·88622692, so is the diameter of any circle, to the side of the square of an equal area. Therefore, if the diameter of the circle be called d, and the side of the equal square s;

$$\text{then is } s = \cdot88622692d = \tfrac{39}{44}d \text{ nearly},$$

$$\text{and } d = \frac{s}{\cdot88622692} = \tfrac{44}{39}s \text{ nearly}.$$

See Circle, Diameter, and Quadrature.

 STADIUM,

STADIUM, an ancient Greek long meaſure, containing 125 geometrical paces, or 625 Roman feet; correſponding to our furlong.

Eight Stadia make a geometrical or Roman mile; and 20, according to Dacier, a French league: but according to others, 800 Stadia make $41\frac{2}{3}$ leagues.

Guilletiere obſerves, that the Stadium was only 600 Athenian feet, which amount to 625 Roman, or 566 French, or 604 Engliſh feet: ſo that the Stadium ſhould have been only 113 geometrical paces. It muſt be obſerved however, that the Stadium was different at different times and places.

STAFF, *Almucantar's*, *Augural*, *Back*, *Croſs*, *Fore*, *Offset*, *&c.* See theſe ſeveral articles.

STAR, STELLA, in Aſtronomy, a general name for all the heavenly bodies.

The Stars are diſtinguiſhed, from the phenomena, &c, into *fixed* and *erratic* or *wandering*.

Erratic or *Wandering* STARS, are thoſe which are continually changing their places and diſtances, with regard to each other. Theſe are what are properly called *planets*. Though to the ſame claſs may likewiſe be referred comets or blazing Stars.

Fixed STARS, called alſo barely *Stars*, by way of eminence, are thoſe which have uſually been obſerved to keep the ſame diſtance, with regard to each other.

The chief circumſtances obſervable in the fixed Stars, are their *diſtance*, *magnitude*, *number*, *nature*, and *motion*. Of each of which in their order.

Diſtance of the Fixed STARS. The fixed Stars are ſo extremely remote from us, that we have no diſtances in the planetary ſyſtem to compare to them. Their immenſe diſtance appears from hence, that they have no ſenſible parallax; that is, that the diameter of the earth's annual orbit, which is nearly 190 millions of miles, bears no ſenſible proportion to their diſtance.

Mr. Huygens (Coſmotheor. lib. 4) attempts to determine the diſtance of the Stars, by making the aperture of a teleſcope ſo ſmall, as that the ſun through it appears no larger than Sirius; which he found to be only as 1 to 27664 of his diameter, when ſeen with the naked eye. So that, were the ſun's diſtance 27664 times as much as it is, it would then be ſeen of the ſame diameter with Sirius. And hence, ſuppoſing Sirius to be a ſun of the ſame magnitude with our ſun, the diſtance of Sirius will be found to be 27664 times the diſtance of the ſun, or 345 million times the earth's diameter.

Dr. David Gregory inveſtigated the diſtance of Sirius, by ſuppoſing it of the ſame magnitude with the ſun, and of the ſame apparent diameter with Jupiter in oppoſition: as may be ſeen at large in his Aſtronomy, lib. 3, prop. 47.

Caſſini (Mem. Acad. 1717), by comparing Jupiter and Sirius, when viewed through the ſame teleſcope, inferred, that the diameter of that planet was 10 times as great as that of the Star; and the diameter of Jupiter being 50″, he concluded that the diameter of Sirius was about 5″; ſuppoſing then that the real magnitude of Sirius is equal to that of the ſun, and the diſtance of the ſun from us 12000 diameters of the earth, and the apparent diameter of Sirius being to that of the ſun as 1 to 384, the diſtance of Sirius becomes equal to 4608000 diameters of the earth.

Theſe methods of Huygens, Gregory, and Caſſini, are conjectural and precarious; both becauſe the ſun and Sirius are ſuppoſed of equal magnitude, and alſo becauſe it is ſuppoſed the diameter of Sirius is determined with ſufficient exactneſs.

Mr. Michell has propoſed an enquiry into the probable parallax and magnitude of the fixed Stars, from the quantity of light which they afford us, and the peculiar circumſtances of their ſituation. With this view he ſuppoſes, that they are, on a medium, equal in magnitude and natural brightneſs to the ſun; and then proceeds to inquire, what would be the parallax of the ſun, if he were to be removed ſo far from us, as to make the quantity of the light, which we ſhould then receive from him, no more than equal to that of the fixed Stars. Accordingly, he aſſumes Saturn in oppoſition, as equal, or nearly equal in light to the brighteſt fixed Star. As the mean diſtance of Saturn from the ſun is equal to about 2082 of the ſun's ſemidiameters, the denſity of the ſun's light at Saturn will conſequently be leſs than at his own ſurface, in the ratio of the ſquare of 2082 or 4334724 to 1: If Saturn therefore reflected all the light that falls upon him, he would be leſs luminous in that ſame proportion. And beſides, his apparent diameter, in the oppoſition, being but about the 105th part of that of the ſun, the quantity of light which we receive from him muſt be again diminiſhed in the ratio of the ſquare of 105 or 11025 to 1. Conſequently, by multiplying theſe two numbers together, we ſhall have the whole of the light of the ſun to that of Saturn, as the ſquare nearly of 220,000 or 48,400,000,000 to 1. Hence, removing the ſun to 220,000 times his preſent diſtance, he would ſtill appear at leaſt as bright as Saturn, and his whole parallax upon the diameter of the earth's orbit would be leſs than 2 ſeconds: and this muſt be aſſumed for the parallax of the brighteſt of the fixed Stars, upon the ſuppoſition that their light does not exceed that of Saturn.

By a like computation it may be found, that the diſtance, at which the ſun would afford us as much light as we receive from Jupiter, is not leſs than 46,000 times his preſent diſtance, and his whole parallax in that caſe, upon the diameter of the earth's orbit, would not be more than 9 ſeconds; the light of Jupiter and Saturn, as ſeen from the earth, being in the ratio of about 22 to 1, when they are both in oppoſition, and ſuppoſing them to reflect equally in proportion to the whole of the light that falls upon them. But if Jupiter and Saturn, inſtead of reflecting the whole of the light that falls upon them, ſhould really reflect only a part of it, as a 4th, or a 6th, which may be the caſe, the above diſtances muſt be increaſed in the ratio of 2 or $2\frac{1}{2}$ to 1, to make the ſun's light no more than equal to theirs; and his parallax would be leſs in the ſame proportion. Suppoſing then that the fixed Stars are of the ſame magnitude and brightneſs with the ſun, it is no wonder that their parallax ſhould hitherto have eſcaped obſervation; ſince in this caſe it could hardly amount to 2 ſeconds, and probably not more than one in Sirius himſelf, though he had been placed in the pole of the ecliptic; and in thoſe that appear much leſs luminous, as γ Draconis, which is only of the 3d magnitude, it could hardly be expected to be ſenſible with ſuch inſtruments as have hitherto been uſed. However, Mr. Michell

chell ſuggeſts, that it is not impracticable to conſtruct inſtruments capable of diſtinguiſhing even to the 20th part of a ſecond, provided the air will admit of that degree of exactneſs. This ingenious writer apprehends that the quantity of light which we receive from Sirius, does not exceed the light we receive from the leaſt fixed Star of the 6th magnitude, in a greater ratio than that of 1000 to 1, nor leſs than that of 400 to 1; and the ſmaller Stars of the 2d magnitude ſeem to be about a mean proportional between the other two. Hence the whole parallax of the leaſt fixed Stars of the 6th magnitude, ſuppoſing them of the ſame ſize and native brightneſs with the ſun, ſhould be from about $2'''$ to $3'''$, and their diſtance from about 8 to 12 million times that of the ſun: and the parallax of the ſmaller Stars of the 2d magnitude, upon the ſame ſuppoſition, ſhould be about $12'''$, and their diſtance about 2 million times that of the ſun.

This author farther ſuggeſts, that, from the apparent ſituation of the Stars in the heavens, there is the greateſt probability that the Stars are collected together in cluſters in ſome places, where they form a kind of ſyſtems, whilſt in others there are either few or none of them; whether this diſpoſition be owing to their mutual gravitation, or to ſome other law or appointment of the Creator. Hence it may be inferred, that ſuch double Stars, &c. as appear to conſiſt of two or more Stars placed very near together, do really conſiſt of Stars placed near together, and under the influence of ſome general law: and he proceeds to inquire whether, if the Stars be collected into ſyſtems, the ſun does not likewiſe make one of ſome ſyſtem, and which fixed Stars thoſe are that belong to the ſame ſyſtem with him.

Thoſe Stars, he apprehends, which are found in cluſters, and ſurrounded by many others at a ſmall diſtance from them, belong probably to other ſyſtems, and not to ours. And thoſe Stars, which are ſurrounded with nebulæ, are probably only very large Stars which, on account of their ſuperior magnitude, are ſingly viſible, while the others, which compoſe the remaining parts of the ſame ſyſtem, are ſo ſmall as to eſcape our ſight. And thoſe nebulæ in which we can diſcover either none or only a few Stars, even with the aſſiſtance of the beſt teleſcopes, are probably ſyſtems that are ſtill more diſtant than the reſt. For other particulars of this inquiry, ſee Philoſ. Tranſ. vol. 57, pa. 234 &c.

As the diſtance of the fixed Stars is beſt determined by their parallax, various methods have been purſued, though hitherto without ſucceſs, for inveſtigating it; the reſult of the moſt accurate obſervations having given us little more than a diſtant approximation; from which however we may conclude, that the neareſt of the fixed Stars cannot be leſs than 40 thouſand diameters of the whole annual orbit of the earth diſtant from us.

The method pointed out by Galileo, and attempted by Hook, Flamſteed, Molyneux, and Bradley, of taking the diſtances of ſuch Stars from the zenith as paſs very near it, has given us a much juſter idea of the immenſe diſtance of the Stars, and furniſhed an approximation to their parallax, much nearer the truth, than any we had before.

Dr. Bradley aſſures us (Philoſ. Tranſ. num. 406, or Abr. vol. 6, pa. 162), that had the parallax amounted to a ſingle ſecond, or two at moſt, he ſhould have perceived it in the great number of obſervations which he made, eſpecially upon γ Draconis; and that it ſeemed to him very probable, that the annual parallax of this Star does not amount to a ſingle ſecond, and conſequently that it is above 400 thouſand times farther from us than the ſun.

But Dr. Herſchel, to whoſe induſtry and ingenuity, in exploring the heavens, aſtronomy is already much indebted, remarks, that the inſtrument uſed on this occaſion, being the ſame with the preſent zenith ſectors, can hardly be allowed capable of ſhewing an angle of one or even two ſeconds, with accuracy: and beſides, the Star on which the obſervations were made, is only a bright Star of the 3d magnitude, or a ſmall Star of the 2d; and that therefore its parallax is probably much leſs than that of a Star of the firſt magnitude. So that we are not warranted in inferring, that the parallax of the Stars in general does not exceed $1''$, whereas thoſe of the firſt magnitude may have, notwithſtanding the reſult of Dr. Bradley's obſervations, a parallax of ſeveral ſeconds.

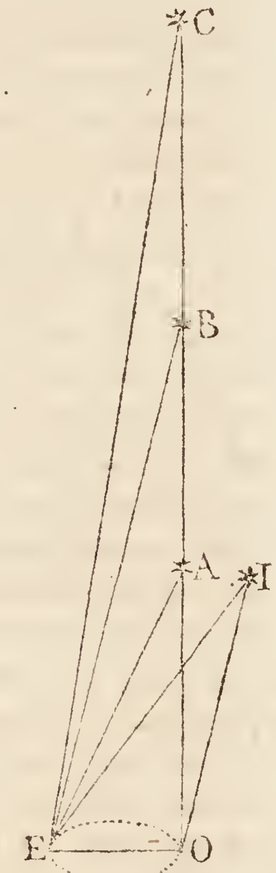
As to the method of zenith diſtances, it is liable to conſiderable errors, on account of refraction, the change of poſition of the earth's axis, ariſing from nutation, preceſſion of the equinoxes, or other cauſes, and the aberration of light.

Dr. Herſchel has propoſed another method, by means of double Stars, which is free from theſe errors, and of ſuch a nature, that the annual parallax, even if it ſhould not exceed the 10th part of a ſecond, may ſtill become viſible, and be aſcertained at leaſt much nearer than heretofore. This method, which was firſt propoſed in an imperfect manner by Galileo, and has been alſo mentioned by other authors, is capable of every improvement which the teleſcope and mechaniſm of micrometers can furniſh. To give a general idea of it, let O and E be two oppoſite points of the annual orbit, taken in the ſame plane with two ſtars A, B, of unequal magnitudes. Let the angle AOB be obſerved when the earth is at O, and AEB be obſerved when the earth is at E. From the difference of theſe angles, when there is any, the parallax of the Stars may be computed, according to the theory ſubjoined. Theſe two Stars ought to be as near as poſſible to each other, and alſo to differ as much in magnitude as we can find them.

This theory of the annual parallax of double Stars, with the method of computing from thence what is uſually called the parallax of the fixed Stars, or of ſingle Stars of the firſt magnitude, ſuch as are neareſt to us, ſuppoſes 1ſt, that the Stars are all about the ſize of the ſun; and 2dly, that the difference in their apparent magnitudes, is owing to their different diſtances, ſo as that a Star of the 2d, 3d, or 4th magnitude, is 2, 3, or 4 times as far off as one of the firſt. Theſe principles, which Dr. Herſchel premiſes as poſtulata, have ſo great a probability in their favour, that they will ſcarcely

ſcarcely be objected to by thoſe who are in the leaſt acquainted with the doctrine of chances. See Mr. Michell's Inquiry, &c. already cited. And Philoſ. Tranſ. vol. 57, pa. 234 - - - - 240. Alſo Dr. Halley, on the Number, Order, and Light of the fixed Stars, in the Philoſ. Tranſ. vol. 31, or Abr. vol. 6, pa. 148.

Therefore, let EO be the whole diameter of the earth's annual orbit; and let A, B, C be three Stars ſituated in the ecliptic, in ſuch a manner, that they may appear all in one line OABC when the earth is at O. Now if OA, AB, BC be equal to each other, A will be a Star of the firſt magnitude, B of the ſecond, and C of the third. Let us next ſuppoſe the angle OAE, or parallax of the whole orbit of the earth, to be 1″ of a degree; then, becauſe very ſmall angles, having the ſame ſubtenſe EO, may be taken to be in the inverſe ratio of the lines OA, OB, OC, &c, we ſhall have EBO $= \frac{1}{2}''$, and ECO $= \frac{1}{3}''$, &c, alſo becauſe EA = AB nearly, the angle AEB = ABE $= \frac{1}{2}''$; and becauſe BC $= \frac{1}{2}$ BO $= \frac{1}{2}$ BE nearly, the angle BEC $= \frac{1}{2}$ BCE $= \frac{1}{6}''$, and hence AEC $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}''$; from all which it follows that, when the earth is at E,



the Stars A and B appear at $\frac{1}{2}''$ diſtant from one another,
the Stars A and C at $\frac{2}{3}''$ diſtant, and
the Stars B and C only $\frac{1}{6}''$ diſtant. In like manner may be deduced a general expreſſion for the parallax that will become viſible in the change of diſtance between the two Stars, by the removal of the earth from one extreme of her orbit to the other. Let P denote the total parallax of a fixed Star of the magnitude of the M order, and m the number of the order of a ſmaller Star, p denoting the partial parallax to be obſerved by the change in the diſtance of a double Star;

then is $p = \frac{m-M}{mM}P$, or $P = \frac{mMp}{m-M}$, which gives P, when p is found by obſervation.

For Ex. Suppoſe a Star of the 1ſt magnitude ſhould have a ſmall Star of the 12th magnitude near it; then will the partial parallax we are to expect to ſee be $\frac{12-1}{12\times1}P = \frac{11}{12}P$, or $\frac{11}{12}$ of the total parallax of the larger Star; and if we ſhould, by obſervation, find the partial parallax between two ſuch Stars to amount to 1″, then will the total parallax $P = \frac{12}{11}p = 1''\frac{1}{11}$. Again, if the Stars be of the 3d and 24th magnitude, the total parallax will be $P = \frac{24\times3}{24-3}p = \frac{72}{21}p = \frac{24}{7}p$; ſo that if by obſervation p be found to be $\frac{1}{10}$ of a ſecond, the whole parallax P will come out $\frac{24}{70}'' = 0{\cdot}3428''$.

Farther, the Stars being ſtill in the ecliptic, ſuppoſe they ſhould appear in one line, when the earth is in ſome other part of her orbit between E and O; then will the parallax be ſtill expreſſed by the ſame algebraic formula, and one of the maxima will ſtill lie at E, the other at O; but the whole effect will be divided into two parts, which will be in proportion to each other, as radius — ſine to radius + ſine of the Star's diſtance from the neareſt conjunction or oppoſition.

When the Stars are any where out of the ecliptic, ſituated ſo as to appear in one line OABC perpendicular to EO, the maximum of parallax will ſtill be expreſſed by $\frac{m-M}{mM}P$; but there will ariſe another additional parallax in the conjunction and oppoſition, which will be to that which is found 90° before or after the ſun, as the ſine (s) of the latitude of the Stars ſeen at O, is to radius (1); and the effect of this parallax will be divided into two parts; half of it lying on one ſide of the large Star, the other half on the other ſide of it. This latter parallax will alſo be compounded with the former, ſo that the diſtance of the Stars in the conjunction and oppoſition will then be repreſented by the diagonal of a parallelogram, whoſe ſides are the two ſemiparallaxes; a general expreſſion for which will be

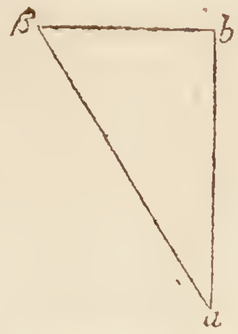
$$\frac{m-M}{2mM}P\sqrt{1+s^2} \text{ or } \tfrac{1}{2}p\sqrt{1+s^2}.$$

When the Stars are in the pole of the ecliptic, s will be $= 1$, and the laſt formula becomes $\frac{1}{2}p\sqrt{2} = {\cdot}7071p$.

Again, let the Stars be at ſome diſtance, as 5″, from each other, and let them be both in the ecliptic. This caſe is reſolvable into the firſt; for imagine the Star A to ſtand at I; then the angle AEI may be accounted equal to AOI; and as the foregoing formula, $p = \frac{m-M}{mM}P$, gives us the angles AEB, AEC, we are to add AEI or 5″ to AEB, which will give IEB. In general, let the diſtance of the Stars be d, and let the obſerved diſtance at E be D; then will $D = d + p$, and therefore the whole parallax of the annual orbit will be expreſſed by $\frac{D-d}{m-M}Dd = P$.

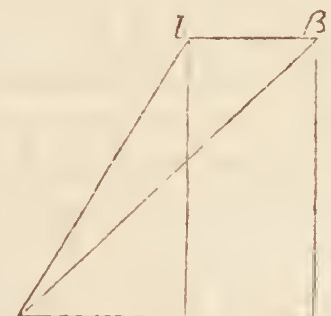
Suppoſe now the Stars to differ only in latitude, one being in the ecliptic, the other at ſome diſtance as 5″ north, when ſeen at O. This caſe may alſo be reſolved by the former; for imagine the Stars B and C to be elevated at right angles above the plane of the figure, ſo that AOB, or AOC, may make an angle of 5″ at O; then inſtead of the lines OABC, EA, EB, EC, imagine them all to be planes at right angles to the figure; and it will appear that the parallax of the Stars in longitude, muſt be the ſame as if the ſmall Star had been without latitude. And ſince the Stars B, C, by the motion of the earth from O to E, will not change their latitude, we ſhall have the following conſtruction for finding the diſtance of the Stars AB and AC at E, and from thence the parallax P.

Let



Let the triangle $ab\beta$ repreſent the ſituation of the Stars; ab is the ſubtenſe of $5''$, the angle under which they are ſuppoſed to be ſeen at O. The quantity $b\beta$ by the former theorem is found $= \frac{m-M}{mM}P$, which is the partial parallax, that would have been ſeen by the earth's moving from O to E, if both Stars had been in the ecliptic; but, on account of the difference in latitude, it will now be repreſented by $a\beta$, the hypotenuſe of the triangle $ab\beta$: therefore in general, putting $ab = d$, $a\beta = D$, we have $\frac{mM}{m-M}\sqrt{D^2-d^2} = P$. Hence, D being found by obſervation, and the three d, m, M given, the total parallax is obtained.

When the Stars differ in longitude as well as latitude, this caſe may be reſolved in the following manner.



Let the triangle $ab\beta$ repreſent the ſituation of the Stars, $ab = d$ being their diſtance ſeen at O, $a\beta = D$ their diſtance ſeen at E. That the change $l\beta$, which is produced by the earth's motion, will be truly expreſſed by $\frac{m-M}{mM}P$, may be proved as before, by ſuppoſing the Star a to have been placed at α. Now let the angle of poſition $ba\alpha$ be taken by a micrometer, or by any other method ſufficiently exact; then, by reſolving the triangle $ab\alpha$, we obtain the longitudinal and latitudinal differences $a\alpha$ and $b\alpha$ of the two ſtars. Put $a\alpha = x$, $b\alpha = y$, and it will be $x + l\beta = aq$, whence

$$D = \sqrt{(x + \frac{m-M}{mM}P)^2 + y^2}; \text{ and } P = \frac{\sqrt{D^2-y^2} - x}{m-M}mM.$$

If neither of the Stars ſhould be in the ecliptic, nor have the ſame longitude or latitude, the laſt theorem will ſtill ſerve to calculate the total parallax, whoſe maximum will lie in E. There will alſo ariſe another parallax, whoſe maximum will be in the conjunction and oppoſition, which will be divided, and lie on different ſides of the large Star; but as the whole parallax is extremely ſmall, it is not neceſſary to inveſtigate every particular caſe of this kind; for by reaſon of the diviſion of the parallax, which renders obſervations taken at any other time, except where it is greateſt, very unfavourable, the formulæ would be of little uſe.

Dr. Herſchel cloſes his account of this theory, with a general obſervation on the time and place where the maxima of parallax will happen. Thus, when two unequal Stars are both in the ecliptic, or, not being in the ecliptic, have equal latitudes, north or ſouth, and the larger Star has moſt longitude, the maximum of the apparent diſtance will be when the ſun's longitude is 90° more than the Star's, or when obſerved in the morning: and the minimum, when the longitude of the ſun is 90° leſs than that of the Star, or when obſerved in the evening. But when the ſmall Star has moſt longitude, the maximum and minimum, as well as the time of obſervation, will be the reverſe of the former. And when the Stars differ in latitude, this makes no alteration in the place of the maximum or minimum, nor in the time of obſervation; that is, it is immaterial which of the two Stars has the greater latitude. Philoſ. Tranſ. vol. 72, art. 11.

The diſtance of the Star γ Draconis appears, by Bradley's obſervations, already recited, to be at leaſt 400,000 times that of the ſun, and the diſtance of the neareſt fixed Star, not leſs than 40,000 diameters of the earth's annual orbit: that is, the diſtance from the earth, of the former at leaſt 38,000,000,000,000 miles, and the latter not leſs than 7,600,000,000,000 miles. As theſe diſtances are immenſely great, it may both be amuſing, and help to a clearer and more familiar idea, to compare them with the velocity of ſome moving body, by which they may be meaſured.

The ſwifteſt motion we know of, is that of light, which paſſes from the ſun to the earth in about 8 minutes; and yet this would be above 6 years traverſing the firſt ſpace, and near a year and a quarter in paſſing from the neareſt fixed Star to the earth. But a cannon ball, moving on a medium at the rate of about 20 miles in a minute, would be 3 million 8 hundred thouſand years in paſſing from γ Draconis to the earth, and 760 thouſand years paſſing from the neareſt fixed Star. Sound, which moves at the rate of about 13 miles in a minute, would be 5 million 600 thouſand years traverſing the former diſtance, and 1 million 128 thouſand, in paſſing through the latter.

The celebrated Huygens purſued ſpeculations of this kind ſo far, as to believe it not impoſſible, that there may be Stars at ſuch inconceivable diſtances, that their light has not yet reached the earth ſince its creation.

Dr. Halley has alſo advanced, what he ſays ſeems to be a metaphyſical paradox (Philoſ. Tranſ. number 364, or Abr. vol. 6, pa. 148), viz, that the number of fixed Stars muſt be more than finite, and ſome of them more than at a finite diſtance from others: and Addiſon has juſtly obſerved, that this thought is far from being extravagant, when we conſider that the univerſe is the work of infinite power, prompted by infinite goodneſs, and having an infinite ſpace to exert itſelf in; ſo that our imagination can ſet no bounds to it.

Magnitude of the fixed STARS. The magnitudes of the Stars appear to be very different from one another; which difference may probably ariſe, partly from a diverſity in their real magnitude, but principally from their diſtances, which are different.

To the bare eye, the Stars appear of ſome ſenſible magnitude, owing to the glare of light ariſing from the numberleſs reflections from the aërial particles &c about the eye: this makes us imagine the Stars to be much larger than they would appear, if we ſaw them only by the few rays which come directly from them, ſo as to enter our eyes without being intermixed with others.

Any perſon may be ſenſible of this, by looking at a Star of the firſt magnitude through a long narrow tube; which, though it takes in as much of the ſky as would hold a thouſand ſuch ſtars, ſcarce renders that one viſible.

The

The Stars, on account of their apparently various fizes, have been diftributed into feveral claffes, called *magnitudes*. The 1ft clafs, or Stars of the firft magnitude, are thofe that appear largeft, and may probably be neareft to us. Next to thefe, are thofe of the 2d magnitude; and fo on to the 6th, which comprehends the fmalleft Stars vifible to the naked eye. All beyond thefe, that can be perceived by the help of telefcopes, are called *telefcopic* ftars. Not that all the Stars of each clafs appear juftly of the fame magnitude; there being great latitude in this refpect; and thofe of the firft magnitude appearing almoft all different in luftre and fize. There are alfo other Stars, of intermediate magnitudes, which aftronomers cannot refer to one clafs rather than another, and therefore they place them between the two. Procyon, for inftance, which Ptolomy makes of the firft magnitude, and Tycho of the 2d, Flamfteed lays down as between the 1ft and 2d. So that, inftead of 6 magnitudes, we may fay there are almoft as many orders of Stars, as there are Stars; fo great variations being obfervable in the magnitude, colour, and brightnefs of them.

There feems to be little chance of difcovering with certainty the real fize of any of the fixed Stars; we muft therefore be content with an approximation, deduced from their parallax, if this fhould ever be found; and the quantity of light they afford us, compared with that of the fun. And to this purpofe, Dr. Herfchel informs us, that with a magnifying power of 6450, and by means of his new micrometer, he found the apparent diameter of α Lyræ to be $0''\cdot355$.

The Stars are alfo diftinguifhed, with regard to their fituation, into *afterifms*, or *conftellations*; which are nothing but affemblages of feveral neighbouring Stars, confidered as conftituting fome determinate figure, as of an animal, &c, from which it is therefore denominated: a divifion as ancient as the book of Job, in which mention is made of Orion, the Pleiades, &c.

Befides the Stars thus diftinguifhed into magnitudes and conftellations, there are others not reduced to either. Thofe not reduced into conftellations, are called *informes*, or *unformed* Stars; of which kind feveral, fo left at large by the ancients, have fince been formed into new conftellations by the modern aftronomers, and efpecially by Hevelius.

Thofe not reduced to claffes or magnitudes, are called nebulous Stars; but fuch as only appear faintly in clufters, in form of little lucid fpots, nebulæ, or clouds.

Ptolomy fets down five of fuch nebulæ, viz, one at the extremity of the right hand of Perfeus, which appears through the telefcope, thick fet with Stars; one in the middle of the crab, called *Præfepe*, or the Manger, in which Galileo counted above 40 Stars; one unformed near the fting of the Scorpion; another in the eye of Sagittarius, in which two Stars may be feen in a clear fky with the naked eye, and feveral more with the telefcope; and the fifth in the head of Orion, in which Galileo counted 21 Stars.

Flamfteed obferved a cloudy Star before the bow of Sagittarius, which confifts of a great number of fmall Stars; and the Star *d* above the right fhoulder of this conftellation is encompaffed with feveral more. Flamfteed and Caffini alfo difcovered one between the great and little dog, which is very full of Stars, that are vifible only by the telefcope.

But the moft remarkable of all the cloudy Stars, is that in the middle of Orion's fword, in which Huygens and Dr. Long obferved 12 Stars, 7 of which (3 of them, now known to be 4, being very clofe together) feem to fhine through a cloud, very lucid near the middle, but faint and ill defined about the edges. But the greateft difcoveries of nebulæ and clufters of Stars, we owe to the powerful telefcopes of Dr. Herfchel, who has given accounts of fome thoufands of fuch nebulæ, in many of which the Stars feem to be innumerable, like grains of fand. See Philof. Tranf. 1784, 1785, 1786, 1789. See Galaxy, and Magellanic *clouds*, and *lucid* Spots.

Caffini is of opinion, that the brightnefs of thefe proceeds from Stars fo minute, as not to be diftinguifhed by the beft glaffes: and this opinion is fully confirmed by the obfervations of Dr. Herfchel, whofe powerful telefcopes fhew thofe lucid fpecks to be compofed entirely of maffes of fmall Stars, like heaps of fand.

There are alfo many Stars which, though they appear fingle to the naked eye, are yet difcovered by the telefcope to be double, triple, &c. Of thefe, feveral have been obferved by Caffini, Hooke, Long, Mafkelyne, Hornfby, Pigott, Mayer, &c; but Dr. Herfchel has been much the moft fuccefsful in obfervations of this kind; and his fuccefs has been chiefly owing to the very extraordinary magnifying powers of the Newtonian 7 feet reflector which he has ufed, and the advantage of an excellent micrometer of his own conftruction. The powers which he has ufed, have been 146, 227, 278, 460, 754, 932, 1159, 1536, 2010, 3168, and even 6450. He has already formed a catalogue, containing 269 double Stars, 227 of which, as far as he knows, have not been noticed by any other perfon. Among thefe there are alfo fome Stars that are treble, double-double, quadruple, double-treble, and multiple. His catalogue comprehends the names of the Stars, and the number in Flamfteed's catalogue, or fuch a defcription of thofe that are not contained in it, as will be found fufficient to diftinguifh them; alfo the comparative fize of the Stars; their colours as they appeared to his view; their diftances determined in feveral different ways; their angle of pofition with regard to the parallel of declination; and the dates when he firft perceived the Stars to be double, treble, &c. His obfervations appear to commence with the year 1776, but almoft all of them were made in the years 1779, 1780, 1781.

Dr. Herfchel has diftributed the double Stars contained in his catalogue, into 6 different claffes. In the firft he has placed all thofe which require a very fuperior telefcope, with the utmoft clearnefs of air, and every other favourable circumftance, to be feen at all, or well enough to judge of them; and there are 24 of thefe. To the 2d clafs belong all thofe double Stars that are proper for eftimations by the eye, and very delicate meafures by the micrometer; the number being 38. The 3d clafs comprehends all thofe double Stars, that are between $5''$ and $15''$ afunder; the number of them being 46. The 4th, 5th, and 6th claffes contain double

double Stars that are from 15″ to 30″, and from 30″ to 1′, and from 1′ to 2′ or more asunder; of which there are 44 in the 4th class, 51 in the 5th class, and 66 in the 6th class: the last of this class is α Tauri, number 87 of Flamsteed, whose apparent diameter, upon the meridian measured with a power of 460 at a mean of two observations 1″ 46‴, and with a power of 932 at a mean of two observations 1″ 12‴. See the list at large, Philosoph. Trans. vol. 72, art. 12.

The Stars are also distinguished, in each constellation, by numbers, or by the letters of the alphabet. This sort of distinction was introduced by John Bayer, in his Uranometria, 1654; where he denotes the Stars, in each constellation, by the letters of the Greek alphabet, α, β, γ, δ, ε, &c, viz, the most remarkable Star of each by α, the 2d by β, the 3d by γ, &c; and when there are more Stars in a constellation than the characters in the Greek alphabet, he denotes the rest, in their order, by the Roman letters A, b, c, d, &c. But as the number of the Stars, that have been observed and registered in catalogues, since Bayer's time, is greatly increased, as by Flamsteed and others, the additional ones have been marked by the ordinal numbers 1, 2, 3, 4, 5, &c.

The *Number of* STARS. The number of the Stars appears to be immensely great, almost infinite; yet have astronomers long since ascertained the number of such as are visible to the eye, which are much fewer than at first sight could be imagined. See CATALOGUE *of the Stars*.

Of the 3000 contained in Flamsteed's catalogue, there are many that are only visible through a telescope; and a good eye scarce ever sees more than a thousand at the same time in the clearest heaven; the appearance of innumerable more, that are frequent in clear winter nights, arising from our sight's being deceived by their twinkling, and from our viewing them confusedly, and not reducing them to any order. But nevertheless we cannot but think the Stars are almost, if not altogether, infinite. See Halley, on the number, order, and light of the fixed Stars, Philos. Trans. number 364, or Abr. vol. 6, pa. 148.

Riccioli, in his New Almagest, affirms, that a man who shall say there are above 20 thousand times 20 thousand, would say nothing improbable. For a good telescope, directed indifferently to almost any point of the heavens, discovers multitudes that are lost to the naked eye; particularly in the milky way, which some take to be an assemblage of Stars, too remote to be seen singly, but so closely disposed as to give a luminous appearance to that part of the heavens where they are. And this fact has been confirmed by Herschel's observations: though it is disputed by others, who contend that the milky way must be owing to some other cause.

In the single constellation of the Pleiades, instead of 6, 7, or 8 Stars seen by the best eye; Dr. Hook, with a telescope 12 feet long, told 78, and with larger glasses many more, of different magnitudes. And F. de Rheita affirms, that he has observed above 2000 Stars in the single constellation of Orion. The same author found above 188 in the Pleiades. And Huygens, looking at the Star in the middle of Orion's sword, instead of one, found it to be 12. Galileo found 80 in the space of the belt of Orion's sword, 21 in the nebulous Star of his head, and above 500 in another part of him, within the compass of one or two degrees space, and more than 40 in the nebulous Star Præsepe.

The Changes that have happened in the STARS are very considerable. The first change that is upon record, was about 120 years before Christ; when Hipparchus, discovering a new Star to appear, was first induced to make a catalogue of the Stars, that posterity might perceive any future changes of the like nature.

In the year 1572, Cornelius Gemma and Tycho Brahe observed another new Star in the constellation Cassiopeia, which was likewise the occasion of Tycho's making a new catalogue. At first its magnitude and brightness exceeded the largest of the Stars, Sirius and Lyra; and even equalled the planet Venus when nearest the earth, and was seen in fair day-light. It continued 16 months; towards the latter end of which it began to dwindle, and at length, in March 1574, it totally disappeared, without any change of place in all that time.

Leovicius tells us of another Star appearing in the same constellation, about the year 945, which resembled that of 1572; and he quotes another ancient observation, by which it appears that a new Star was seen about the same place in 1264. Dr. Keil thinks these were all the same Star; and indeed the periodical intervals, or distance of time between these appearances, were nearly equal, being from 318 to 319 years; and if so, its next appearance may be expected about 1890.

Fabricius, in 1596, discovered another new Star, called the *stella mira*, or *wonderful Star*, in the neck of the whale, which has since been found to appear and disappear periodically, 7 times in 6 years, continuing in its greatest lustre for 15 days together; and is never quite extinguished. Its course and motion are described by Bulliald, in a treatise printed at Paris in 1667. Dr. Herschel has lately, viz, in the years 1777, 1778, 1779, and 1780, made several observations on this Star, an account of which may be seen in the Philos. Trans. vol. 70, art. 21.

In the year 1600, William Jansen discovered a changeable Star in the neck of the Swan, which gradually decreased till it became so small as to be thought to disappear entirely, till the years 1657, 1658, and 1659, when it regained its former lustre and magnitude; but soon decayed again, and is now of the smallest size.

In the year 1604, a new Star was seen by Kepler, and several of his friends, near the heel of the right foot of Serpentarius, which was particularly bright and sparkling; and it was observed to be every moment changing into some of the colours of the rainbow, except when it is near the horizon, at which time it was generally white. It surpassed Jupiter in magnitude, but was easily distinguished from him, by the steady light of the planet. It disappeared about the end of the year 1605, and has not been seen since that time.

Simon Marius discovered another in Andromeda's girdle,

girdle, in 1612 and 1613; though Bulliald fays it had been feen before, in the 15th century.

In July 1670, Hevelius difcovered a fecond changeable Star in the Swan, which was fo diminifhed in October as to be fcarce perceptible. In April following it regained its former luftre, but wholly difappeared in Auguft. In March 1672 it was feen again, but appeared very fmall, and has not been vifible fince.

In 1686 a third changeable Star was difcovered by Kirchius in the Swan, viz, the Star χ of that conftellation, which returned periodically in about 405 days.

In 1672 Caffini faw a Star in the neck of the Bull, which he thought was not vifible in Tycho's time, nor when Bayer made his figures.

It is certain, from the old catalogues, that many of the ancient Stars are not now vifible. This has been particularly remarked with regard to the Pleiades.

M. Montanari, in his letter to the Royal Society in 1670, obferves that there are now wanting in the heavens two Stars of the 2d magnitude, in the ftern of the fhip Argo, and its yard, which had been feen till the year 1664. When they firft difappeared is not known; but he affures us there was not the leaft glimpfe of them in 1668. He adds, he has obferved many more changes in the fixed Stars, even to the number of a hundred. And many other changes of the Stars have been noticed by Caffini, Maraldi, and other obfervers. See Gregory's Aftron. lib. 2, prop. 30.

But the greateft numbers of variable Stars have been obferved of late years, and the moft accurate obfervations made on their periods, &c, by Herfchel, Goodricke, Pigott, &c, in the late volumes of the Philof. Tranf. particularly in the vol. for 1786, where the laft of thefe gentlemen has given a catalogue of all that have been hitherto obferved, with accounts of the obfervations that have been made upon them.

Various hypothefes have been devifed to account for fuch changes and appearances in the Stars. It is not probable they could be comets, as they had no parallax, even when largeft and brighteft. It has been fuppofed that the periodical Stars have vaft dark fpots, or dark fides, and very flow rotations on their axes, by which means they muft difappear when the darker fide is turned towards us. And as for thofe which break out fuddenly with fuch luftre, thefe may perhaps be funs whofe fuel is almoft fpent, and again fupplied by fome of their comets falling upon them, and occafioning an uncommon blaze and fplendor for fome time; which it is conjectured may be one ufe of the cometary part of our fyftem.

Maupertuis, in his Differtation on the figures of the Celeftial Bodies (pa. 61—63), is of opinion that fome Stars, by their prodigious fwift rotation on their axes, may not only affume the figures of oblate fpheroids, but that by the great centrifugal force arifing from fuch rotations, they may become of the figures of mill-ftones, or be reduced to flat circular planes, fo thin as to be quite invifible when their edges are turned towards us, as Saturn's ring is in fuch pofition. But when very eccentric planets or comets go round any flat Star in orbits much inclined to its equator, the attraction of the planets or comets in their perihelions muft alter the inclination of the axis of that Star; on which account it will appear more or lefs large and luminous, as its broad fide is more or lefs turned towards us. And thus he imagines we may account for the apparent changes of magnitude and luftre of thofe Stars, and alfo for their appearing and difappearing.

Hevelius apprehends (Cometograph. pa. 380), that the Sun and Stars are furrounded with atmofpheres, and that by whirling round their axes with great rapidity, they throw off great quantities of matter into thofe atmofpheres, and fo caufe great changes in them; and that thus it may come to pafs that a Star, which, when its atmofphere is clear, fhines out with great luftre, may at another time, when it is full of clouds and thick vapours, appear greatly diminifhed in brightnefs and magnitude, or even become quite invifible.

Nature of the fixed STARS. The immenfe diftance of the Stars leaves us greatly at a lofs about the nature of them. What we can gather for certain from their phenomena, is as follows:

1ft, That the fixed Stars are greater than our earth: becaufe if that was not the cafe, they could not be vifible at fuch an immenfe diftance.

2nd, The fixed Stars are farther diftant from the earth than the fartheft of the planets. For we frequently find the fixed Stars hid behind the body of the planets: and befides, they have no parallax, which the planets have.

3rd, The fixed Stars fhine with their own light; for they are much farther from the Sun than Saturn, and appear much fmaller than Saturn; but fince, notwithftanding this, they are found to fhine much brighter than that planet, it is evident they cannot borrow their light from the fame fource as Saturn does, viz, the Sun; but fince we know of no other luminous body befide the Sun, whence they might derive their light, it follows that they fhine with their own native light.

Befides, it is known, that the more a telefcope magnifies, the lefs is the aperture through which the Star is feen; and confequently, the fewer rays it admits into the eye. Now fince the Stars appear lefs in a telefcope which magnifies two hundred times, than they do to the naked eye, infomuch that they feem to be only indivifible points, it proves at once that the Stars are at immenfe diftances from us, and that they fhine by their own proper light. If they fhone by borrowed light, they would be as invifible without telefcopes as the fatellites of Jupiter are; for the fatellites appear larger when viewed with a good telefcope than the largeft fixed Stars do.

Hence,

1. We deduce, that the fixed Stars are fo many funs; for they have all the characters of funs.

2. That in all probability the Stars are not fmaller than our fun.

3. That it is highly probable each Star is the centre of a fyftem, and has planets or earths revolving round it, in the fame manner as round our fun, i. e. it has opake bodies illuminated, warmed, and cherifhed by its light and heat. As we have incomparably more light from the moon than from all the Stars together, it is abfurd to imagine that the Stars were made for no other purpofe than to caft a faint light upon the earth; efpecially

especially since many more require the assistance of a good telescope to find them out, than are visible without that instrument. Our sun is surrounded by a system of planets and comets, all which would be invisible from the nearest fixed Star; and from what we already know of the immense distance of the Stars, it is easy to prove, that the sun, seen from such a distance, would appear no larger than a Star of the first magnitude.

From all this it is highly probable, that each Star is a sun to a system of worlds moving round it, though unseen by us; especially as the doctrine of a plurality of worlds is rational, and greatly manifests the power, the wisdom, and the goodness of the great creator.

How immense, then, does the universe appear! Indeed, it must either be infinite, or infinitely near it.

Kepler, it is true, denies that each Star can have its system of planets as ours has; and takes them all to be fixed in the same surface or sphere; urging, that were one twice or thrice as remote as another, it would be twice or thrice as small, supposing their real magnitudes equal; whereas there is no difference in their apparent magnitudes, justly observed, at all. But to this it is opposed, that Huygens has not only shewn, that fires and flames are visible at distances where other bodies, comprehended under equal angles, disappear; but it should likewise seem, that the optic theorem about the apparent diameters of objects, being reciprocally proportional to their distances from the eye, does only hold while the object has some sensible ratio to its distance.

As for periodical Stars, &c. see CHANGES, *&c of Stars*, supra.

Motion of the STARS. The fixed Stars have two kinds of apparent motion; one called the *first*, *common*, or *diurnal motion*, arising from the earth's motion round its axis: by this they seem to be carried along with the sphere or firmament, in which they appear fixed, round the earth, from east to west, in the space of 24 hours.

The other, called the *second*, or *proper motion*, is that by which they appear to go backwards from west to east, round the poles of the ecliptic, with an exceeding slow motion, as describing a degree of their circle only in the space of 71½ years, or 50⅓ seconds in a year. This apparent motion is owing to the recession of the equinoctial points, which is 50⅓ seconds of a degree in a year backward, or contrary to the order of the signs of the zodiac.

In consequence of this second motion, the longitude of the Stars will be always increasing. Thus, for example, the longitude of Cor Leonis was found at different periods, to be as follows: viz,

	Year.		*Long.*
By Ptolomy, in - - -	138	to be	2° 30′
By the Persians, in -	1115	- - -	17 30
By Alphonsus, in - -	1364	- - -	20 40
By Prince of Hesse, in	1586	- - -	24 11
By Tycho, in - - -	1601	- - -	24 17
By Flamsteed, in - -	1690	- - -	25 31⅓

Whence the proper motion of the Stars, according to the order of the signs, in circles parallel to the ecliptic, is easily inferred.

It was Hipparchus who first suspected this motion, upon comparing his own observations with those of Timocharis and Aristyllus. Ptolomy, who lived three centuries after Hipparchus, demonstrated the same by undeniable arguments.

The increase of longitude in a century, as stated by different astronomers, is as follows:

By Tycho Brahe - - - -	1°	25′	0″
Copernicus - - - - -	1	23	40½
Flamsteed and Riccioli	1	23	20
Bulliald - - - - - - -	1	24	54
Hevelius - - - - - -	1	24	46½
Dr. Bradley, &c. - -	1	23	55

which is at the rate of 50⅓ seconds per year.

From these data, the increase in the longitude of a Star for any given time, is easily had, and thence its longitude at any time: ex. gr. the longitude of Sirius, in Flamsteed's tables, for the year 1690, being 9° 49′ 1″, its longitude for the year 1800, is found by multiplying the interval of time, viz, 110 years, by 50⅓, the product 5537″, or 1° 32′ 17″, added to the given longitude - - - - 9 49 1

gives the longitude - - 11 21 18 for the year 1800.

The chief phenomena of the fixed Stars, arising from their common and proper motion, besides their longitude, are their altitudes, right ascensions, declinations, occultations, culminations, risings, and settings.

Some have supposed that the latitudes of the Stars are invariable. But this supposition is founded on two assumptions, which are both controverted among astronomers. The one of these is, that the orbit of the earth continues unalterably in the same plane, and consequently that the ecliptic is invariable; the contrary of which is now very generally allowed.

The other assumption is, that the Stars are so fixed as to keep their places immoveably. Ptolomy, Tycho, and others, comparing their observations with those of the ancient astronomers, have adopted this opinion. But from the result of the comparison of our best modern observations, with such as were formerly made with any tolerable degree of exactness, there appears to have been a real change in the position of some of the fixed Stars, with respect to each other; and several Stars of the first magnitude have already been observed, and others suspected to have a proper motion of their own.

Dr. Halley (Philos. Trans. number 355, or Abr. vol. 4, p. 225) has observed, that the three following Stars, the Bull's eye, Sirius, and Arcturus, are now found to be above half a degree more southerly than the ancients reckoned them: that this difference cannot arise from the errors of the transcribers, because the declinations of the Stars, set down by Ptolomy, as observed by Timocharis, Hipparchus, and himself, shew their latitudes given by him are such as those authors intended: and it is scarce to be believed that those three observers could be deceived in so plain a matter. To this he adds, that the bright Star in the shoulder of Orion has, in Ptolomy, almost a whole degree more southerly latitude than at present: that an ancient observation, made at Athens in the year 509, as Bulliald supposes, of an appulse of the moon to the Bull's eye, shews that Star to have had less latitude at that time than it now has: that as to Sirius, it appears by Tycho's observations, that he found him 4½′ more northerly than

than he is at this time. All theſe obſervations, compared together, ſeem to favour an opinion, that ſome of the Stars have a proper motion of their own, which changes their places in the ſphere of heaven: this change of place, as Dr. Halley obſerves, may ſhew itſelf in ſo long a time as 1800 years, though it be entirely imperceptible in the ſpace of one ſingle century; and it is likely to be ſooneſt diſcovered in ſuch Stars as thoſe juſt now mentioned; becauſe they are all of the firſt magnitude, and may, therefore, probably be ſome of the neareſt to our ſolar Syſtem. Arcturus, in particular, affords a ſtrong proof of this: for if its preſent declination be compared with its place, as determined either by Tycho or Flamſteed, the difference will be found to be much greater than what can be ſuſpected to ariſe from the uncertainty of their obſervations. See Arcturus, and Mr. Hornſby's enquiry into the quantity and direction of the proper motion of Arcturus, Phil. Tranſ. vol. 63, part 1, pa. 93, &c.

For an account of Dr. Bradley's obſervations, ſee the ſequel of this article.

Dr. Herſchel has alſo lately obſerved, that the diſtance of the two Stars forming the double Star γ Draconis, is 54″ 48‴, and their poſition 44° 19′ N. preceding. Whereas, from the right aſcenſion and declination of theſe Stars in Flamſteed's catalogue, their diſtance, in his time, appears to have been 1′ 11″ ·418, and their poſition 44° 23′ N. preceding. Hence he infers, that as the difference in the diſtance of theſe two Stars is ſo conſiderable, we can hardly account for it, otherwiſe than by admitting a proper motion in one or the other of the Stars, or in our ſolar ſyſtem: moſt probably he ſays, neither of the three is at reſt. He alſo ſuſpects a proper motion in one of the double Stars, in Cauda Lyncis Media, and in o Ceti. Phil. Tranſ. vol. 72, part 1, p. 117, 143, 150.

It is reaſonable to expect, that other inſtances of the like kind muſt alſo occur among the great number of viſible Stars, becauſe their relative poſitions may be altered by various means. For if our own ſolar ſyſtem be conceived to change its place with reſpect to abſolute ſpace, this might, in proceſs of time, occaſion an apparent change in the angular diſtances of the fixed Stars; and in ſuch a caſe, the places of the neareſt Stars being more affected than of thoſe that are very remote, their relative poſition might ſeem to alter, though the Stars themſelves were really immoveable; and vice verſa, we may ſurmiſe, from the obſerved motion of the Stars, that our ſun, with all its planets and comets, may have a motion towards ſome particular part of the heavens, on account of a greater quantity of matter collected in a number of Stars and their ſurrounding planets there ſituated, which may perhaps occaſion a gravitation of our whole ſolar ſyſtem towards it. If this ſurmiſe ſhould have any foundation, as Dr. Herſchel obſerves, ubi ſupra, p. 103, it will ſhew itſelf in a ſeries of ſome years; ſince from that motion there will ariſe another kind of hitherto unknown parallax (ſuggeſted by Mr. Michell, Philoſ. Tranſ. vol. 57, p. 252), the inveſtigation of which may account for ſome part of the motions already obſerved in ſome of the principal Stars; and for the purpoſe of determining the direction and quantity of ſuch a motion, accurate obſervations of the diſtance of Stars, that are near enough to be meaſured with a micrometer, and a very high power of teleſcopes, may be of conſiderable uſe, as they will undoubtedly give us the relative places of thoſe Stars to a much greater degree of accuracy than they can be had by inſtruments or ſectors, and thereby much ſooner enable us to diſcover any apparent change in their ſituation, occaſioned by this new kind of ſecular or ſyſtematical parallax, if we may ſo expreſs the change ariſing from the motion of the whole ſolar ſyſtem.

And, on the other hand, if our ſyſtem be at reſt, and any of the Stars really in motion, this might likewiſe vary their apparent poſitions; and the more ſo, the nearer they are to us, or the ſwifter their motions are; or the more proper the direction of the motion is to be rendered perceptible by us. Since then the relative places of the Stars may be changed from ſuch a variety of cauſes, conſidering the amazing diſtance at which it is certain ſome of them are placed, it may require the obſervations of many ages to determine the laws of the apparent changes, even of a ſingle Star; much more difficult, therefore, muſt it be to ſettle the laws relating to all the moſt remarkable Stars.

When the cauſes which affect the places of all the Stars in general are known; ſuch as the preceſſion, aberration, and nutation, it may be of ſingular uſe to examine nicely the relative ſituations of particular Stars, and eſpecially of thoſe of the greateſt luſtre, which, it may be preſumed, lie neareſt to us, and may therefore be ſubject to more ſenſible changes, either from their own motion, or from that of our ſyſtem. And if, at the ſame time the brighter Stars are compared with each other, we likewiſe determine the relative poſitions of ſome of the ſmalleſt that appear near them, whoſe places can be aſcertained with ſufficient exactneſs, we may perhaps be able to judge to what cauſe the change, if any be obſervable, is owing. The uncertainty that we are at preſent under, with reſpect to the degree of accuracy with which former aſtronomers could obſerve, makes us unable to determine ſeveral things relating to this ſubject; but the improvements, which have of late years been made in the methods of taking the places of the heavenly bodies, are ſo great, that a few years may hereafter be ſufficient to ſettle ſome points, which cannot now be ſettled; by comparing even the earlieſt obſervations with thoſe of the preſent age.

Dr. Hook communicated ſeveral obſervations on the apparent motions of the fixed Stars; and as this was a matter of great importance in aſtronomy, ſeveral of the learned were deſirous of verifying and confirming his obſervations. An inſtrument was accordingly contrived by Mr. George Graham, and executed with ſurpriſing exactneſs.

With this inſtrument the Star γ, in the conſtellation Draco, was frequently obſerved by Meſſrs. Molyneux, Bradley, and Graham, in the years 1725, 1726; and the obſervations were afterwards repeated by Dr. Bradley with an inſtrument contrived by the ſame ingenious perſon, Mr. Graham, and ſo exact, that it might be depended on to half a ſecond. The reſult of theſe obſervations was, that the Star did not always appear in the ſame place, but that its diſtance from the zenith varied, and that the difference of the apparent places amounted to 21 or 22 ſeconds. Similar obſervations were made on other Stars, and a like apparent motion was

was found in them, proportional to the latitude of the Star. This motion was by no means fuch as was to have been expected, as the effect of a parallax, and it was fome time before any way could be found of accounting for this new phenomenon. At length Dr. Bradley refolved all its variety, in a fatisfactory manner, by the motion of light and the motion of the earth compounded together. See LIGHT, and Phil. Tranf. No. 406, p. 364, or Abr. vol. vi, p. 149, &c.

Our excellent aftronomer, Dr. Bradley, had no fooner difcovered the caufe, and fettled the laws of aberration of the fixed Stars, than his attention was again excited by another new phenomenon, viz, an annual change of declination in fome of the fixed Stars, which appeared to be fenfibly greater than a preceffion of the equinoctial points of 50″ in a year, the mean quantity now ufually allowed by aftronomers, would have occafioned.

This apparent change of declination was obferved in the Stars near the equinoctial colure, and there appearing at the fame time an effect of a quite contrary nature, in fome Stars near the folftitial colure, which feemed to alter their declination lefs than a preceffion of 50″ required, Dr. Bradley was thereby convinced, that all the phenomena in the different Stars could not be accounted for merely by fuppofing that he had affumed a wrong quantity for the preceffion of the equinoctial points. He had alfo, after many trials, fufficient reafon to conclude, that thefe fecond unexpected deviations of the Stars were not owing to any imperfection of his inftruments. At length, from repeated obfervations he began to guefs at the real caufe of thefe phenomena.

It appeared from the Doctor's obfervations, during his refidence at Wanfted, from the year 1727 to 1732, that fome of the Stars near the folftitial colure had changed their declinations 9″ or 10″ lefs than a preceffion of 50″ would have produced; and, at the fame time, that others near the equinoctial colure had altered theirs about the fame quantity more than a like preceffion would have occafioned: the north pole of the equator feeming to have approached the Stars, which come to the meridian with the fun about the vernal equinox, and the winter folftice; and to have receded from thofe, which come to the meridian with the fun about the autumnal equinox and the fummer folftice.

From the confideration of thefe circumftances, and the fituation of the afcending node of the moon's orbit when he firft began to make his obfervations, he fufpected that the moon's action upon the equatorial parts of the earth might produce thefe effects.

For if the preceffion of the equinox be, according to Sir Ifaac Newton's principles, caufed by the actions of the fun and moon upon thofe parts; the plane of the moon's orbit being, at one time, above 10 degrees more inclined to the plane of the equator than at another, it was reafonable to conclude, that the part of the whole annual preceffion, which arifes from her action, would, in different years, be varied in its quantity; whereas the plane of the ecliptic, in which the fun appears, keeping always nearly the fame inclination to the equator, that part of the preceffion, which is owing to the fun's action, may be the fame every year; and from hence it would follow, that although the mean annual preceffion, proceeding from the joint actions of the fun and moon, were 50″; yet the apparent annual preceffion might fometimes exceed, and fometimes fall fhort of that mean quantity, according to the various fituations of the nodes of the moon's orbit.

In the year 1727, the moon's afcending node was near the beginning of Aries, and confequently her orbit was as much inclined to the equator as it can at any time be; and then the apparent annual preceffion was found, by the Doctor's firft year's obfervations, to be greater than the mean; which proved, that the Stars near the equinoctial colure, whofe declinations are moft of all affected by the preceffion, had changed theirs, above a tenth part more than a preceffion of 50″ would have caufed. The fucceeding year's obfervations proved the fame thing; and, in three or four years' time, the difference became fo confiderable as to leave no room to fufpect it was owing to any imperfection either of the inftrument or obfervation.

But fome of the Stars, that were near the folftitial colure, having appeared to move, during the fame time, in a manner contrary to what they ought to have done, by an increafe of the preceffion; and the deviations in them being as remarkable as in the others, it was evident that fomething more than a mere change in the quantity of the preceffion would be requifite to folve this part of the phenomenon. Upon comparing the obfervations of Stars near the folftitial colure, that were almoft oppofite to each other in right afcenfion, they were found to be equally affected by this caufe. For whilft γ Draconis appeared to have moved northward, the fmall Star, which is the 35th Camelopardali Hevelii, in the Britifh catalogue, feemed to have gone as much towards the fouth; which fhewed, that this apparent motion in both thofe Stars might proceed from a nutation of the earth's axis; whereas the comparifon of the Doctor's obfervations of the fame Stars formerly enabled him to draw a different conclufion, with refpect to the caufe of the annual aberrations arifing from the motion of light. For the apparent alteration in γ Draconis, from that caufe, being as large again as in the other fmall Star, proved, that that did not proceed from a nutation of the earth's axis; as, on the contrary, this may.

Upon making the like comparifon between the obfervations of other Stars, that lie nearly oppofite in right afcenfion, whatever their fituations were with refpect to the cardinal points of the equator, it appeared, that their change of declination was nearly equal, but contrary; and fuch as a nutation or motion of the earth's axis would effect.

The moon's afcending node being got back towards the beginning of Capricorn in the year 1732, the Stars near the equinoctial colure appeared about that time to change their declinations no more than a preceffion of 50″ required; whilft fome of thofe near the folftitial colure altered theirs above 2″ in a year lefs than they ought. Soon after the annual change of declination of the former was perceived to be diminifhed, fo as to become lefs than 50″ of preceffion would caufe; and it continued to diminifh till the year 1736, when the moon's afcending node was about the beginning of Libra, and her orbit had the leaft inclination to the equator. But by this time, fome of the Stars near the folftitial colure had altered their declinations 18″ lefs

lefs fince the year 1727, than they ought to have done from a preceffion of 50″. For γ Draconis, which in thofe 9 years would have gone about 8″ more foutherly, was obferved, in 1736, to appear 10″ more northerly than it did in the year 1727.

As this appearance in γ Draconis indicated a diminution of the inclination of the earth's axis to the plane of the ecliptic, and as feveral aftronomers have fuppofed that inclination to diminifh regularly; if this phenomenon depend upon fuch a caufe and amounted to 18″ in 9 years, the obliquity of the ecliptic would, at that rate, alter a whole minute in 30 years; which is much fafter than any obfervations before made would allow. The Doctor had therefore reafon to think, that fome part of this motion at leaft, if not the whole, was owing to the moon's action on the equatorial parts of the earth, which he conceived might caufe a libratory motion of the earth's axis. But as he was unable to judge, from only 9 years obfervation, whether the axis would entirely recover the fame pofition that it had in the year 1727, he found it neceffary to continue his obfervations through a whole period of the moon's nodes; at the end of which he had the fatisfaction to fee, that the Stars returned into the fame pofitions again, as if there had been no alteration at all in the inclination of the earth's axis; which fully convinced him, that he had gueffed rightly as to the caufe of the phenomenon. This circumftance proves likewife, that if there be a gradual diminution of the obliquity of the ecliptic, it does not arife only from an alteration in the pofition of the earth's axis, but rather from fome change in the plane of the ecliptic itfelf; becaufe the Stars, at the end of the period of the moon's nodes, appeared in the fame places, with refpect to the equator, as they ought to have done if the earth's axis had retained the fame inclination to an invariable plane.

The Doctor having communicated thefe obfervations, and his fufpicion of their caufe, to the late Mr. Machin, that excellent geometrician foon after fent him a table, containing the quantity of the annual preceffion in the various pofitions of the moon's nodes, as alfo the correfponding nutations of the earth's axis; which was computed upon the fuppofition that the mean annual preceffion is 50″, and that the whole is governed by the pole of the moon's orbit only; and therefore Mr. Machin imagined, that the numbers in the table would be too large, as, in fact, they were found to be. But it appeared that the changes which Dr. Bradley had obferved, both in the annual preceffion and nutation, kept the fame law, as to increafing and decreafing, with the numbers of Mr. Machin's table. Thofe were calculated on the fuppofition, that the pole of the equator, during a period of the moon's nodes, moved round in the periphery of a little circle, whofe centre was 23° 29′ diftant from the pole of the ecliptic; having itfelf alfo an angular motion of 50″ in a year about the fame pole. The north pole of the equator was conceived to be in that part of the fmall circle which is fartheft from the north pole of the ecliptic at the fame time when the moon's afcending node is in the beginning of Aries; and in the oppofite point of it, when the fame node is in Libra.

If the diameter of the little circle, in which the pole of the equator moves, be fuppofed equal to 18″, which is the whole quantity of the nutation, as collected from Dr. Bradley's obfervations of the Star γ Draconis, then all the phenomena of the feveral Stars which he obferved will be very nearly folved by this hypothefis. But for the particulars of his folution, and the application of his theory to the practice of aftronomy, we muft refer to the excellent author himfelf; our intention being only to give the hiftory of the invention.

The corrections arifing from the aberration of light, and from the nutation of the earth's axis, muft not be neglected in aftronomical obfervations; fince fuch neglects might produce errors of near a minute in the polar diftance of fome Stars.

As to the allowance to be made for the aberration of light, Dr. Bradley affures us, that having again examined thofe of his own obfervations, which were moft proper to determine the tranfverfe axis of the ellipfis, which each Star feems to defcribe, he found it to be neareft to 40″; and this is the number he makes ufe of in his computations relating to the nutation.

Dr. Bradley fays, in general, that experience has taught him, that the obfervations of fuch Stars as lie neareft the zenith, generally agree beft with one another, and are therefore fitteft to prove the truth of any hypothefis. Phil. Tranf. N°. 485, vol. 45, p. 1, &c.

Monfieur d'Alembert has publifhed a treatife, entitled, Recherches fur la Preceffion des Equinoxes, et fur la Nutation de la Terre dans le Syfteme Newtonien, 4to. Paris, 1749. The calculations of this learned gentleman agree in general with Dr. Bradley's obfervations. But Monfieur d'Alembert finds, that the pole of the equator defcribes an ellipfis in the heavens, the ratio of whofe axes is that of 4 to 3; whereas, according to Dr. Bradley, the curve defcribed is either a circle or an ellipfis, the ratio of whofe axes is as 9 to 8.

The feveral Stars in each conftellation, as in Taurus, Bootes, Hercules, &c, fee under the proper article of each conftellation, TAURUS, BOOTES, HERCULES, &c.

To learn to know the feveral fixed Stars by the globe, fee GLOBE.

The parallax and diftance of the fixed Stars, fee under PARALLAX and DISTANCE.

Circumpolar STARS. See CIRCUMPOLAR.

Morning STAR. See MORNING.

Place of a STAR. See PLACE.

Pole STAR. See POLE.

Twinkling of the STARS. See TWINKLING.

Unformed STARS. See INFORMES.

The following two catalogues of Stars are taken from Dr. Zach's Tabulæ Motuum Solis &c, and are adapted to the beginning of the year 1800. The former contains 381 Stars, fhewing their names and Bayer's mark, their magnitude, declination, and right afcenfion, both in time and in arcs or degrees of a great circle, with the annual variations of the fame. And the latter contains 162 principal Stars, fhewing their declinations to feconds of a degree, with their annual variations. The explanations are fufficiently clear from the titles of the columns.

A CATALOGUE *of the most remarkable* FIXED STARS, *with their Magnitudes, Right Ascensions, Declinations and Annual Variations, for the Beginning of the Year* 1800.

No. of Stars	Names and Characters of the Stars.	Magnitude.	Right Ascens. in time.	Annual Variat. in ditto.	Right Ascension in degrees &c.	Annual Variat. in ditto.	Declination North and South.	Annual Variat. in ditto.
			h. m. s. 1/100	s 1/1000	° ′ ″ 1/100	″ 1/100	° ′ ″	
				+		+		
1	γ Pegasi	2	0 2 56·79	3·063	0 44 11·85	45·95	14 4 N	
2	ι Ceti	3	0 5 13·51	3·059	2 18 22·66	45·89	9 57 S	
3	κ Cassiopeæ	4	0 21 45·12	3·301	5 26 16·75	49·51	61 50 N	
4	ζ Cassiopeæ	4	0 25 53·93	3·262	6 28 29·01	48·93	52 49 N	
5	δ Andromedæ	3	0 28 39·02	3·161	7 9 45·31	47·42	29 45 N	
6	α Cassiopeæ	3	0 29 14·40	3·311	7 18 35·95	49·66	55 26 N	
7	β Ceti	2 3	0 33 31·83	3·001	8 22 57·40	45·01	19 5 S	
8	η Cassiopeæ	4	0 37 1·44	3·389	9 15 21·64	50·83	56 46 N	
9	δ Piscium	4	0 38 19·08	3·093	9 34 46·15	46·39	6 30 N	
10	γ Cassiopeæ	3	0 44 44·75	3·505	11 11 11·29	52·58	59 38 N	
11	ε Piscium	4	0 52 33·95	3·103	13 8 29·20	46·55	6 49 N	
12	β Andromedæ	2	0 58 34·23	3·297	14 38 33·38	49·46	34 33 N	
13	ϑ Cassiopeæ	4	0 59 0·22	3·531	14 45 3·33	52·96	53 35 N	
14	ζ Piscium	4	1 3 16·99	3·109	15 49 14·80	46·63	6 31 N	
15	δ Cassiopeæ	3	1 12 50·58	3·761	18 12 38·70	56·42	59 11 N	
10	μ Piscium	5	1 19 42·07	3·108	19 55 31·07	46·62	5 7 N	
17	π Piscium	5	1 30 30·70	3·164	22 37 40·56	47·46	11 7 N	
18	ν Piscium	4 5	1 31 1·77	3·107	22 45 26·62	46·61	4 28 N	
19	ο Piscium	4 5	1 34 50·72	3·144	23 42 40·84	47·16	8 9 N	
20	ε Cassiopeæ	3	1 40 10·01	4·155	25 2 30·13	62·33	62 41 N	
21	ζ Ceti	3	1 41 36·69	2·953	25 24 10·33	44·30	11 20 S	
22	α Triang. Bor.	3 4	1 41 42·74	3·379	25 25 41·15	50·68	28 36 N	
23	γ′ Arietis	4	1 42 34·52	3·258	25 38 37·73	48·87	18 19 N	
24	β Arietis	3	1 43 36·77	3·277	25 54 11·48	49·15	19 50 N	
25	λ Arietis	5	1 46 48·86	3·315	26 42 12·83	49·73	22 37 N	
26	γ Andromedæ	2	1 51 41·05	3·615	27 55 15·76	54·23	41 22 N	
27	✳ preced. α ♈	·	1 50 26·15	· · · ·	27 36 32·25	· · ·	· · · ·	
28	α Arietis	2	1 55 55·27	3·335	28 58 49·05	50·02	22 31 N	
29	✳ seq. α ♈	·	1 59 38·13	· · ·	29 54 31·95	· · ·	· · · ·	
30	ξ^1 Arietis	5 6	2 7 1·64	3·308	31 45 24·55	49·62	18 58 N	
31	ο Ceti (Variab.)	2	2 9 14·70	3·019	32 18 40·50	45·29	3 54 S	
32	π^0 Arietis	6	2 38 9·29	3·321	39 32 19·32	49·81	16 38 N	
33	σ Arietis	6	2 40 28·09	3·285	40 7 1·39	49·28	14 15 N	
34	δ Ceti	3	2 29 14·17	3·060	37 18 32·54	45·90	0 33 S	
35	ε Ceti	3	2 29 53·42	2·884	37 23 21·37	43·27	12 44 S	
36	γ Ceti	3	2 32 57·18	3·102	38 14 17·76	46·53	2 23 N	
37	π Ceti	3	2 34 36·02	2·849	38 39 0·29	42·74	14 43 S	
38	♈ Lilii Bor.	4	2 35 57·70	3·521	38 59 25·49	52·81	28 25 N	
39	♈ Lilii Aust.	4	2 38 14·43	3·489	39 33 36·49	52·34	26 26 N	
40	ρ^2 Arietis	6	2 44 35·65	3·344	41 8 54·72	50·16	17 31 N	
41	ρ^3 Arietis	5 6	2 45 9·35	3·340	41 17 20·19	50·10	17 13 N	
42	η Eridani	3	2 46 39·72	2·917	41 39 55·78	43·75	9 42 S	
43	ε Arietis	5	2 47 48·01	3·401	41 57 1·80	51·01	20 32 N	
44	γ Persei	3	2 50 24·42	4·250	42 36 6·25	63·75	52 43 N	
45	α Ceti	2	2 51 50·07	3·119	42 57 31·06	46·66	3 18 N	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Afcenf. in time.	Annual Variat. in ditto.	Right Afcenf. in degrees. &c.	Annual Variat. in ditto.	Declination North and South.	Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″	
				+		+		
46	✱ feq. α Ceti	.	2 51 54·61	. . .	42 58 39·15	. . .	. . . . .	
47	β Perfei	2 3	2 55 12·07	3·846	43 48 1·04	57·69	40 11 N	
48	δ Arietis	4	3 0 12·71	3·393	45 3 10·59	50·89	18 58 N	
49	ζ Arietis	5	3 3 25·98	3·422	45 51 29·77	51·33	20 18 N	
50	ζ Eridani	3	3 6 7·48	2·904	46 31 52·20	43·56	9 34 S	
51	τ^1 Arietis	7	3 9 42·36	3·433	47 25 35·39	51·49	20 25 N	
52	α Perfei	2	3 10 6·85	4·203	47 31 42·77	63·05	49 8 N	
53	τ^2 Arietis	6	3 11 16·37	3·428	47 49 5·48	51·42	20 1 N	
54	65 Arietis	7	3 12 55·52	3·430	48 13 52·74	51·45	20 5 N	
55	*f* Tauri	5	3 19 54·65	3·289	49 57 39·78	49·33	12 15 N	
56	ε Eridani	3 4	3 23 31·52	2·883	50 52 52·84	43·24	10 9 S	
57	δ Perfei	3	3 28 44·93	4·203	52 11 13·91	63·05	47 8 N	
58	η Lucida Plei.	3	3 35 37·17	3·535	53 54 17·56	53·03	23 29 N	
59	ζ Perfei	3	3 41 35·20	3·734	55 23 48·00	56·01	31 17 N	
60	ε Perfei	3	3 44 29·19	3·977	56 7 17·87	59·66	39 25 N	
61	γ Eridani	2 3	3 48 42·25	2·786	57 10 33·77	41·79	14 5 S	
62	*A* Tauri	5	3 52 53·46	3·515	58 13 21·83	52·72	21 32 N	
63	γ Tauri	3	4 8 25·23	3·387	62 6 18·48	50·80	15 8 N	
64	δ^1 Tauri	3 4	4 11 24·68	3·432	62 51 10·26	51·48	17 4 N	
65	δ^2 Tauri	4	4 12 34·93	3·431	63 8 43·89	51·46	16 58 N	
66	κ^1 Tauri	5	4 13 27·66	3·545	63 21 54·93	53·17	21 50 N	
67	κ^2 Tauri	4 5	4 13 31·13	3·543	63 22 47·02	53·14	21 44 N	
68	ε Tauri	3 4	4 16 56·98	3·475	64 14 14·76	52·12	18 44 N	
69	ϑ^1 Tauri	5	4 17 9·30	3·401	64 17 19·53	51·02	15 31 N	
70	ϑ^2 Tauri	5	4 17 19·53	3·399	64 19 52·91	50·99	15 25 N	
71	✱ præced. α ♉	.	4 22 12·56	. . .	65 33 8·40	. . .	. . . . .	
72	Aldebaran	1	4 24 27·29	3·421	66 6 49·38	51·31	16 6 N	
73	✱ fequ. α ♉	.	4 26 43·44	. . .	66 40 51·60	. . .	. . . . .	
74	σ^2 Tauri	6	4 27 44·78	3·406	66 56 11·77	51·09	15 24 N	
75	υ^2 Eridani	3 4	4 27 47·26	2·329	66 56 48·83	34·94	30 59 S	
76	σ^2 Tauri	6	4 27 50·67	3·409	66 57 40·87	51·13	15 31 N	
77	Eridani	3 4	4 31 43·15	2·615	67 55 47·21	39·23	20 4 S	
78	ι Tauri	4	4 51 9·38	3·565	72 47 20·75	53·47	21 18 N	
79	β Eridani	3	4 58 2·38	2·948	74 30 35·74	44·22	5 21 S	
80	λ Eridani	4	4 59 34·73	2·863	74 53 40·95	42·95	9 1 S	
81	✱ præc. α Aurig.	.	5 1 39·44	. . .	75 24 51·60	. . .	. . . . .	
82	Capella	1	5 1 56·16	4·414	75 29 2·40	66·21	45 47 N	
83	✱ feq. α Aurig.	.	5 3 14·28	. . .	75 48 34·20	. . .	. . . . .	
84	✱ præc. β Orio.	.	5 3 56·39	. . .	75 59 5·85	. . .	. . . . .	
85	Rigel	1	5 4 55 54	2·867	76 13 53·10	43·01	8 27 S	
86	✱ feq. β Orionis	.	5 8 24·57	. . .	77 6 8·55	. . .	. . . . .	
87	β Tauri	2	5 13 39·38	3·778	78 24 50·70	56·67	28 26 N	
88	γ Orionis	2	5 14 24·54	3·209	78 36 8·16	48·13	6 9 N	
89	β Leporis	3 4	5 19 41·09	2·565	79 55 16·40	38·47	20 56 S	
90	δ Orionis	2	5 21 47·38	3·057	80 26 53·69	45·86	0 28 S	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Ascens. in time.	Annual Variat. in ditto.	Right Ascension in degrees, &c.	Annual Variat. in ditto	Declination North and South.	Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″	
				+		+		
91	α Leporis	3	5 23 54·94	2·639	80 58 44·13	39·59	17 59 S	
92	ζ Tauri	3	5 25 42·41	3·575	81 25 36·14	53·62	21 0 N	
93	ε Orionis	2	5 26 4·15	3·037	81 31 2·19	45·55	1 20 S	
94	ζ Orionis	2	5 30 40·57	3·020	82 40 8·57	45·30	2 4 S	
95	α Columbæ	2	5 32 25·03	2·167	83 6 15·45	32·50	34 11 S	
96	γ Leporis	3 4	5 36 9·02	2·517	84 2 15·34	37·75	22 31 S	
97	κ Orionis	4	5 38 16·28	2·839	84 34 4·21	42·59	9 45 S	
98	* præc. α Orio.	.	5 41 27·74	. . .	85 21 56·10	. . .	. . .	
99	α Orionis	1	5 44 20·57	3·239	86 5 8·55	48·59	7 21 N	
100	* ſeq. α Orionis	.	5 47 35·59	. . .	86 58 23·85	. . .	. . .	
101	β Aurigæ	2 3	5 44 51·68	4·398	86 12 55·22	65·97	44 55 N	
102	H Gemin. (prop.)	4 5	5 51 57·72	3·642	87 59 25·77	54·63	23 16 N	
103	η Geminorum	3 4	6 2 48·29	3·623	90 42 4·34	54·34	22 33 N	
104	μ Geminorum	3	6 10 51·44	3·624	92 42 51·64	54·36	22 36 N	
105	ζ Canis majoris	3	6 12 39·08	2·298	93 9 46·24	34·47	29 59 S	
106	β Canis majoris	2 3	6 13 53·76	2·638	93 28 26·33	39·57	17 52 S	
107	ν Geminorum	4	6 17 5·48	3·562	94 16 22·22	53·43	20 20 N	
108	γ Geminorum	2 3	6 26 9·35	3·463	96 32 20·32	51·95	16 34 N	
109	ε Geminorum	3	6 31 37·34	3·695	97 54 20·05	55·42	25 19 N	
110	* præc. α Can. maj.	.	6 29 41·80	. . .	97 25 27·00	. . .	. . .	
111	Sirius	1	6 36 19·91	2·647	99 4 58·65	39·71	16 26 S	
112	* ſeq. α Can. maj.	.	6 41 26·68	. . .	100 21 40·20	. . .	. . .	
113	ε Canis majoris	2 3	6 50 46·21	2·354	102 41 33·20	35·31	28 43 S	
114	ζ Geminorum	3 4	6 52 14·55	3·567	103 3 38·24	53·47	20 51 N	
115	δ Canis majoris	2 3	7 0 15·39	2·436	105 3 53·85	36·54	26 5 S	
116	δ Geminorum	3	7 8 10·06	3·594	107 2 30·94	52·91	22 20 N	
117	β Canis minoris	3	7 16 18·01	3·261	109 4 30·21	48·92	8 41 N	
118	* præc. α Gemin.	.	7 16 13·66	. . .	109 3 24·90	. . .	. . .	
119	Caſtor	1 2	7 21 48·81	3·855	110 27 12·15	57·83	32 19 N	
120	* ſeq. α Gemin.	.	7 27 4·84	. . .	111 46 12·60	. . .	. . .	
121	υ Geminorum	4 5	7 23 34·54	3·715	110 53 38·04	55·72	27 21 N	
122	* præc. α Can. min.	.	7 26 40·27	. . .	111 40 4·05	. . .	. . .	
123	Procyon	1 2	7 28 49·10	3·137	112 12 16·50	47·06	5 44 N	
124	* ſeq. α Can. min.	.	7 30 27·12	. . .	112 36 46·80	. . .	. . .	
125	Pollux	2	7 33 3·18	3·687	113 15 47·70	55·31	28 30 N	
126	* ſeq. β Gemin.	.	7 35 28·78	. . .	113 52 11·70	. . .	. . .	
127	μ^2 Cancri	5	7 55 57·64	3·545	118 59 24·55	53·18	22 9 N	
128	ψ^2 Cancri	4	7 58 23·25	3·639	119 35 48·73	54·58	26 7 N	
129	β Cancri	3 4	8 5 39·37	3·266	121 24 50·61	48·99	9 48 N	
130	ϑ Cancri	5 6	8 20 10·35	3·441	125 2 35·31	51·61	18 46 N	
131	η Cancri	6 7	8 21 7·79	3·491	125 16 56·87	52·36	21 6 N	
132	δ Hydræ	4	8 27 3·04	3·189	126 45 45·53	47·83	6 23 N	
133	γ Cancri	4	8 31 41·86	3·499	127 55 27·85	52·49	22 10 N	
134	δ Cancri	4	8 33 18·11	3·428	128 19 31·70	51·42	18 53 N	
135	ε Hydræ	4	8 36 10·14	3·199	129 2 32·03	47·98	7 8 N	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Afcenf. in time.	Annual Variat. in ditto.	Right Afcenfion in degrees, &c.	Annual Variat. in ditto.	Declination North and South.		Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″		
				+		+			
136	ζ Hydræ	4 5	8 44 48·86	3·187	131 12 12·84	47·81	6 42	N	
137	α^1 Cancri	4 5	8 44 59·39	3·290	131 14 50·81	49·35	12 24	N	
138	α^2 Cancri	3 4	8 47 31·82	3·292	131 52 57·26	49·38	12 37	N	
139	κ Cancri	4 5	8 56 54·33	3·263	134 13 34·92	48·95	11 28	N	
140	ξ^1 Cancri	5 6	8 58 30·37	3·472	134 27 35·48	52·08	22 51	N	
141	ϑ Hydræ	4	9 3 54·80	3·120	135 58 42·03	46·80	3 10	N	
142	κ Leonis	4	9 12 58·32	3·524	138 14 34·77	52·86	27 2	N	
143	Alphard	2	9 17 44·97	2·935	139 26 14·55	44·03	7 48	S	
144	* feq. α Hydræ	.	9 23 9·19	. . .	140 47 17·85	. . .	. .	.	
145	ξ Leonis	4	9 21 9·26	3·253	140 17 18·97	48·80	12 11	N	
146	ο Leonis	4	9 30 27·65	3·224	142 36 54·82	48·36	10 48	N	
147	ε Leonis	3	9 34 28·29	3·434	143 37 4·31	51·51	24 41	N	
148	μ Leonis	3	9 41 21·84	3·457	145 20 27·54	51·85	26 57	N	
149	ν Leonis	4	9 47 26·92	3·243	146 51 43·76	48·65	13 24	N	
150	π Leonis	4	9 49 37·99	3·183	147 24 29·89	47·75	9 0	N	
151	η Leonis	3 4	9 52 24·60	3·289	149 6 9·04	49·33	17 44	N	
152	Regulus	1	9 57 42·02	3·204	149 25 30·30	48·06	12 56	N	
153	* feq. α Leonis	.	10 4 28·58	. . .	151 7 8·70	. . .	. .	.	
154	ζ Leonis	3	10 5 32·34	3·361	151 23 5·16	50·42	24 25	N	
155	γ^2 Leonis	2 3	10 8 55·22	3·306	152 13 48·23	49·60	20 51	N	
156	μ Urfæ majoris	3	10 10 21·35	3·635	152 35 23·32	54·52	42 30	N	
157	ρ Leonis	4	10 22 15·77	3·170	155 33 56·49	47·55	10 20	N	
158	β Urfæ majoris	2	10 49 39·53	3·709	162 24 54·93	55·63	57 27	N	
159	α Crateris	4	10 50 4·55	2·943	162 31 8·25	44·14	17 14	S	
160	α Urfæ majoris	1 2	10 51 15·84	3 847	162 48 57·61	57·70	62 50	N	
161	β Crateris	3 4	11 1 50·06	2·933	165 27 30·97	44·02	31 44	S	
162	δ Leonis	2 3	11 3 26·39	3·199	165 51 35·91	47·98	21 37	N	
163	ϑ Leonis	3	11 3 44·23	3·165	165 56 3·49	47·48	16 31	N	
164	λ Crateris	5 6	11 13 28·15	2·981	168 22 2·21	44·72	17 17	S	
165	ι Leonis	4	11 13 28·32	3·125	168 22 4·85	46·87	11 38	N	
166	τ Leonis	4	11 17 39·49	3 085	169 24 52·74	46·28	3 57	N	
167	υ Leonis	4	11 26 42·82	3·069	171 40 42·29	46·04	0 17	N	
168	ν Virginis	5	11 35 34·11	3·087	173 53 51·70	46·31	7 39	N	
169	* præc. β Leonis	.	11 38 19·46	. . .	174 34 51·90	. . .	. .	.	
170	Denebola	1 2	11 38 50·49	3·062	174 42 37·35	45·93	15 41	N	
171	β Virginis	3	11 40 16·38	3·122	175 4 5·70	46·83	2 54	N	
172	γ Urfæ majoris	2	11 43 14·22	3·212	175 48 33·33	48·18	54 48	N	
173	α Corvi	4	11 58 6 94	3·062	179 31 44·10	45·93	23 37	S	
174	ε Corvi	4	11 59 51·63	3·067	179 57 54·47	46·00	21 30	S	
175	δ Urfæ majoris	3	12 5 27·23	3·021	181 21 48·42	45·32	58 9	N	
176	γ Corvi	3	12 5 32·31	3·077	181 23 4·58	46·16	16 26	S	
177	η Virginis	3	12 9 40·74	3·067	182 25 11·13	46·01	0 27	N	
178	β Corvi	3	12 23 54·39	3·124	185 58 35·92	46·86	22 17	S	
179	κ Draconis	3	12 24 47·65	2·661	186 11 54·72	39·91	70 53	N	
180	γ^1 Virginis	3	12 31 33·85	3·069	187 53 27·72	46·03	0 21	S	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Mag-ni-tude.	Right Aſcenſ. in time.	Annual Variat. in ditto.	Right Aſcenſ. in degrees. &c.	Annual Variat. in ditto.	Declination North and South.		Annual Variat in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″		
				+		+			
181	ε Urſæ majoris	2 3	12 45 12·58	2·746	191 18 8·67	41·19	57 3	N	
182	δ Virginis	3	12 45 33·66	3·047	191 23 24·93	45·71	4 29	N	
183	ε Virginis	3	12 52 13·33	3·004	193 3 20·01	45·06	12 2	N	
184	ϑ Virginis	3 4	12 59 36·46	3·095	194 54 6·97	46·42	4 28	S	
185	γ Hydræ	3	13 8 4·31	3·225	197 1 4·64	48·38	22 7	S	
186	✳ præc. a Virg.	.	13 9 13·00	. . .	197 18 15·00	. . .	. .	.	
187	Spica	1	13 14 40·11	3·137	198 40 1·66	47·06	10 7	S	
188	ζ Urſæ majoris	3	13 15 49·62	2·425	198 57 24·26	36·37	55 59	N	
189	ι Virginis	4	13 16 10·79	3·129	199 2 41·83	46·93	11 40	S	
190	ζ Virginis	3	13 24 30·65	3·064	201 7 39·68	45·96	0 26	N	
191	τ Bootis	4	13 37 46·36	2·884	204 26 35·35	43·26	18 27	N	
192	η Urſæ majoris	2 3	13 39 38·85	2·355	204 54 42·80	35·88	50 19	N	
193	η Bootis	3	13 45 9·21	2·860	206 17 18·12	42·90	19 25	N	
194	α Draconis	2 3	13 58 58·88	1·628	209 44 43·24	24·42	65 20	N	
195	κ Virginis	4	14 2 14·87	3·179	210 33 43·04	47·68	9 20	S	
196	Arcturus	1	14 6 32·21	2·722	211 38 3·16	40·83	20 15	N	
197	✳ ſeq. α Bootis	.	14 6 36·46	. . .	211 39 6·90	. . .	. .	.	
198	λ Virginis	4	14 8 19·14	3·223	212 4 47·16	48·35	12 27	S	
199	γ Bootis	3	14 24 1·50	2·428	216 0 22·54	36·42	39 11	N	
200	ζ Bootis	3	14 31 35·56	2·854	217 53 53·42	42·81	14 36	N	
201	ε Bootis	3	14 36 14·99	2·612	219 3 44·80	39·33	27 56	N	
202	μ Libræ	5	14 38 22·95	3·268	219 35 44·22	49·02	13 18	S	
203	α^1 Libræ	6	14 39 38·74	3·299	219 54 41·10	49·49	15 9	S	
204	✳ præc. α^2 ♎	.	14 39 38·77	. . .	219 54 41·55	. . .	. .	.	
205	α^2 Libræ	2 3	14 39 49·97	3·289	219 57 29·55	49·34	15 12	S	
206	β Urſæ minoris	3	14 51 27·55	−0·329	222 51 53·19	−4·94	74 59	N	
207	γ Scorpii	3	14 52 24·35	3·482	223 6 5·22	52·23	24 29	S	
208	β Bootis	3	14 54 24·99	2·262	223 36 14·85	33·93	41 11	N	
209	ψ Bootis	5	14 55 52·50	2·580	223 58 7·57	38·70	27 44	N	
210	β Libræ	2 3	15 6 15·61	3·215	226 33 54·21	48·22	8 38	S	
211	δ Bootis	3	15 7 26·62	2·409	226 51 39·28	36·13	34 4	N	
212	ο Coron. bor.	6	15 11 51·97	2·487	227 57 59·55	37·30	30 21	N	
213	η Coron. bor.	5	15 14 56·05	2·465	228 44 0·78	36·97	31 1	N	
214	β Coron. bor.	4	15 19 34·88	2·483	229 53 43·13	37·24	29 48	N	
215	γ^2 Urſæ minoris	2 3	15 21 11·76	−0·209	230 17 56·39	−3·14	72 33	N	
216	ζ^4 Libræ	3 4	15 21 38·42	3·365	230 24 36·26	50·48	16 10	S	
217	γ Libræ	4	15 24 21·22	3·328	231 5 18·34	49·92	14 7	S	
218	δ Serpentis	3	15 25 15·84	2·861	231 18 57·61	42·91	11 13	N	
219	Gemma	2	15 26 13·29	2·543	231 33 19·35	38·15	27 24	N	
220	κ Libræ	4	15 30 27·12	3·433	232 36 46·77	51·49	19 1	S	
221	α Serpentis	2	15 34 25·21	2·936	223 36 18·00	44·04	7 4	N	
222	✳ ſeq. α Serpentis	.	15 36 23·53	. . .	234 5 53·05	. . .	. .	.	
223	β Serpentis	3	15 36 57·70	2·756	234 14 25·47	41·34	16 4	N	
224	μ Serpentis	4	15 39 10·30	3·023	234 47 34·55	45·35	2 48	S	
225	ε Serpentis	3 4	15 40 50·97	2·969	235 12 44·51	44·54	5 6	N	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Ascens. in time.	Annual Variat. in ditto.	Right Ascension in degrees, &c.	Annual Variat. in ditto.	Declination North and South.		Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″		
				+		+			
226	δ Coron. bor.	4	15 41 12·45	2·515	235 18 6·71	37·73	26 42	N	
227	λ Libræ	4	15 41 44·86	3·457	235 26 12·93	51·86	19 33	S	
228	ρ Scorpii	3 4	15 44 33·34	3·671	236 8 20·06	55·06	28 37	S	
229	π Scorpii	3	15 46 46·43	3·600	236 41 36·48	54·00	25 31	S	
230	ψ Libræ	4	15 47 0·91	3·339	236 45 13·63	50·09	13 41	S	
231	γ Serpentis	3	15 47 12·93	2·740	236 48 13·98	41·10	16 21	N	
232	δ Scorpii	3	15 48 31·89	3·521	237 7 58·38	52·82	22 2	S	
233	ε Coron. bor.	4 5	15 49 18·54	2·483	237 19 38·04	37·24	27 28	N	
234	π Serpentis	4	15 53 41·18	2·576	238 25 17·72	38·64	23 21	N	
235	β Scorpii	2	15 53 49·71	3·465	238 27 25·65	51·97	19 15	S	
236	θ Draconis	3 4	15 58 8·28	1·142	239 32 4·27	17·13	59 6	N	
237	ν Scorpii	4	16 0 23·31	3·465	240 5 49·60	51·96	18 56	S	
238	δ Ophiuchi	3	16 3 52·80	3·132	240 58 11·95	46·98	3 10	S	
239	ε Ophiuchi	3 4	16 7 45·09	3·154	241 56 16·31	47·30	4 12	S	
240	γ Herculis	3	16 13 5·83	2·642	243 16 27·41	39·63	19 38	N	
241	Antares	1	16 17 9·69	3·645	244 17 25·35	54·68	25 58	S	
242	* α Scorpii	·	16 19 6·66	. . .	244 46 39·90	. . .	. .	.	
243	φ Ophiuchi	4 5	16 19 43·03	3·418	244 55 45·42	51·27	16 10	S	
244	η Draconis	3 4	16 21 18·33	0·785	245 19 34·92	11·78	61 58	N	
245	β Herculis	3	16 21 37·87	2·579	245 24 28·08	38·68	21 56	N	
246	τ Scorpii	4	16 23 26·96	3·709	245 51 44·36	55·64	27 47	S	
247	ζ Ophiuchi	2 3	16 26 9·55	3·287	246 32 23·2	49·30	10 9	S	
248	ζ Herculis	3 4	16 33 45·64	2·292	248 26 24·67	34·38	32 1	N	
249	η Herculis	3 4	16 36 3·14	2·046	249 0 47·15	30·69	39 19	N	
250	ε Herculis	3	16 52 38·68	2·292	253 9 40·16	34·38	31 16	N	
251	η Ophiuchi	2 3	16 58 55·15	3·424	254 43 47·26	51·36	15 28	S	
252	* præc. α Herc.	·	17 5 12·70	. . .	256 18 10·50	. . .	. .	.	
253	α Herculis	2 3	17 5 31·76	2·726	256 22 56·40	40·89	14 38	N	
254	δ Herculis	3 4	17 6 49·41	2·459	256 42 21·17	36·88	25 5	N	
255	θ Ophiuchi	3	17 9 44·25	3·669	257 26 3·68	55·04	24 47	S	
256	λ Scorpii	3	17 20 2·64	4·057	260 0 39·58	60·85	36 57	S	
257	* præc. α Ophi.	·	17 24 45·90	. . .	261 11 28·50	. . .	. .	.	
258	α Ophiuchi	2	17 25 38·97	2·768	261 24 44·55	41·52	17 43	N	
259	* seq. α Ophi.	·	17 29 11·02	. . .	262 17 45·32	. . .	. .	.	
260	β Draconis	3	17 25 55·99	1·348	261 28 59·82	20·22	52 27	N	
261	β Ophiuchi	3	17 33 35·77	2·959	263 23 56·54	44·39	4 40	N	
262	γ Ophiuchi	3	17 37 52·04	3·003	264 28 0·56	45·05	2 48	N	
263	ζ Serpentis	3 4	17 49 54·59	3·153	267 28 38·87	47·30	3 40	S	
264	ο Ophiuchi	4	17 50 37·47	2·999	267 39 21·99	44·99	2 57	N	
265	γ Draconis	2 3	17 51 57·79	1·389	267 59 26·85	20·83	51 31	N	
266	γ Sagittarii	3 4	17 52 58·05	3·851	268 14 30·76	57·77	30 25	S	
267	b Taur. Poniat.	·	18 0 41·10	2·993	270 10 16·50	44·90	3 19	N	
268	μ^1 Sagittarii	4	18 1 48·37	3·584	270 27 5·63	53·76	21 6	S	
269	μ^2 Sagittarii	4 6	18 3 16·90	3·575	270 49 13·57	53·62	20 46	S	
270	ε Sagittarii	2 3	18 10 53·67	3·984	272 43 25·11	59·76	34 28	S	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Afcenf. in time.	Annual Variat. in ditto	Right Afcenf. in degrees, &c.	Annual Variat. in ditto.	Declination North and South.	Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″	
				+		+		
271	λ Sagittarii	4	18 15 37·66	3·705	273 54 24·92	55·57	25 31 S	
272	* præc. α Lyræ	.	18 28 40·12	. . .	277 10 1·88	. .	. . .	
273	Wega	1	18 30 9·89	1·994	277 32 28·35	29·91	38 36 N	
274	* feq. α Lyræ	.	18 31 40·00	. . .	277 54 50·00	. .	. . .	
275	φ Sagittarii	3 4	18 33 9·39	3·747	278 17 20·8[illegible]	56·21	27 11 S	
276	ε Lyræ	5	18 37 42·87	1·983	279 25 43·03	29·74	39 28 N	
277	ν¹ Sagittarii	4 5	18 42 5·41	3·025	280 31 21·22	54·38	22 59 S	
278	β Lyræ	3	18 42 41·86	2·211	280 40 27·89	33·16	33 9 N	
279	σ Sagittarii	3	18 42 51·40	3·724	280 42 50·99	55·86	26 32 S	
280	ν² Sagittarii	4 5	18 43 1·13	3·623	280 45 16·99	54·35	22 54 S	
281	ϑ Serpentis	3	18 46 16·82 / 18·32	2·977	281 34 12·35 / 34·84	44·66	3 57 N	
282	δ Lyræ	3 4	18 47 31·16	2·095	281 52 47·43	31·42	36 39 N	
283	ο Draconis	4	18 48 14·15	0·810	282 3 32·20	13·21	59 9 N	
284	γ Lyræ	3	18 51 27·04	2·241	282 51 45·55	33·61	32 26 N	
285	ο Sagittarii	4	18 52 41·18	3·595	283 10 17·72	53·92	22 1 S	
286	τ Sagittarii	4	18 54 26·45	3·758	283 36 36·82	56·37	27 57 S	
287	λ Antinoi	3 4	18 55 38·10	3·186	283 54 31·55	47·79	5 10 S	
288	ζ Aquilæ	3	18 56 12·69	2·755	284 3 10·37	41·33	13 35 N	
289	π Sagittarii	3 4	18 57 51·37	3·574	284 27 50·57	53·61	21 20 S	
290	ψ Sagittarii	4 5	19 3 15·27	3·685	285 48 49·04	55·27	25 35 S	
291	d Sagittarii	4 6	19 5 55·86	3·517	286 28 57·90	52·76	19 18 S	
292	δ Draconis	3	19 12 27·95	0·033	288 6 59·21	0·49	67 19 N	
293	κ Cygni	4	19 12 28·[illegible]	1·583	288 7 4·19	20·75	52 58 N	
294	δ Aquilæ	3	19 15 24·12	3·008	288 51 1·79	45·12	2 44 N	
295	β Cygni	3	19 22 38·53	2·415	290 39 37·97	36·23	27 33 N	
296	ι Cygni	4 6	19 24 39·61	1·511	291 9 54·19	22·67	51 19 N	
297	ι Antinoi	3 4	19 26 22·16	·106	291 35 32·37	46·59	1 43 S	
298	ϑ Cygni	4	19 31 5·16	1·64[illegible]	292 46 17·40	24·68	49 46 N	
299	α Sagittæ	4	19 31 9·25	2·678	292 47 18·74	40·17	17 34 N	
300	f Sagittarii	6	19 34 41·43	3·520	293 40 21·39	52·80	20 14 S	
301	* præc. γ Aquilæ	.	19 35 12·50	. . .	293 48 7·50	. . .	. . .	
302	γ Aquilæ	3	19 36 44·50	2·837	294 11 7·50	42·59	10 8 N	
303	* feq. γ Aquilæ	.	19 39 0·81	. . .	294 45 12·16	. . .	. . .	
304	δ Cygni	3	19 38 43·09	1·869	294 40 46·34	28·02	44 39 N	
305	* præc. α Aquilæ	.	19 38 35·8[illegible]	. . .	294 38 8·05	. . .	. . .	
306	Atair	1 2	19 41 1·02	2·918	295 15 15·30	43·78	8 21 N	
307	* feq. α Aquilæ	.	19 42 52·37	. . .	295 43 5·55	. . .	. . .	
308	η Antinoi	3 4	19 42 17·14	3·058	295 34 17·08	45·87	0 30 N	
309	b Sagittarii	4 5	19 44 39·56	3·699	296 9 53·46	55·48	27 41 S	
310	β Aquilæ	3 4	19 45 28·97	2·939	296 22 14·55	44·08	5 55 N	
311	ϑ Aquilæ	3	20 0 58·52	3·097	300 14 37·75	46·45	1 24 S	
312	α¹ Capricorni	3 4	20 6 32·79	3·330	301 38 11·88	49·95	13 7 S	
313	* præc. α² Capri.	.	20 5 17·48	. . .	301 19 22·20	. . .	. . .	
314	α² Capricorni	3	20 6 56·48	3·331	301 44 7·20	49·96	13 9 S	
315	* feq. α² Capri.	.	20 9 33·32	. . .	302 23 19·80	. . .	. . .	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Ascens. in time.	Annual Variat. in ditto.	Right Ascens. in degrees, &c.	Annual Variat. in ditto.	Declination North and South.		Annual Variat. in ditto.
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″		
				+					
316	β Capricorni	.	20 9 31·35	2·380	302 22 50·19	50·70	15 24	S	
317	ν Capricorni	6	20 9 33·69	3·337	302 23 25·39	50·06	13 23	S	
318	β Capricorni	3	20 9 45·50	3·380	302 26 22·54	50·70	15 24	S	
319	γ Cygni	3	20 15 2·63	2·148	303 45 39·39	32·22	39 38	N	
320	ϱ Capricorni	6	20 17 26·45	3·438	304 21 36·72	51·57	18 28	S	
321	ζ Delphini	4 5	20 25 57·50	2·801	306 29 22·44	42·01	14 0	N	
322	β Delphini	3	20 28 10·32	2·804	307 2 34·74	42·06	13 55	N	
323	α Delphini	3	20 30 20·76	2·780	307 35 11·37	41·70	15 13	N	
324	Deneb.	1 2	20 34 36·68	2·034	308 39 10·20	30·51	44 34	N	
325	✱ seq. α Cygni	.	20 40 28·55	. . .	310 7 8·25	. . .	. .	.	
326	ε Aquarii	4 5	20 36 50·38	3·255	309 12 35·72	48·83	10 13	S	
327	✱ præc. γ Delphini	.	20 37 21·94	. . .	309 20 29·08	. . .	. .	.	
328	γ Delphini	3	20 37 22·96	2·783	309 20 44·34	41·75	15 25	N	
329	ε Cygni	3	20 38 6·70	2·393	309 31 40·56	35·89	33 13	N	
330	μ Aquarii	4 5	20 41 51·17	3·233	310 27 47·61	48·65	9 43	S	
331	Aquarii	6	20 46 4·59	3·255	311 31 8·80	48·82	10 28	S	
332	ϑ Capricorni	5 4	20 54 39·75	3·384	313 39 56·25	50·76	18 1	S	
333	ν Aquarii	5	20 58 41·24	3·274	314 40 18·64	49·11	12 10	S	
334	α Equulei	4	21 5 48·78	2·997	316 27 11·73	44·96	4 26	N	
335	ι Capricorni	5	21 11 5·67	3·355	317 46 25·00	50·33	17 41	S	
336	β Equulei	6	21 12 57·71	2·981	318 14 25·62	44·72	5 58	N	
337	Aquarii	6	21 13 14·77	3·286	318 18 41·53	49·29	13 44	S	
338	α Cephei	3	21 13 46·85	1·427	318 26 42·69	21·40	61 45	N	
339	β Aquarii	3	21 21 1·12	3·165	320 15 16·74	47·48	6 27	S	
340	ε Capricorni	4	21 25 52·29	3·379	321 28 4·34	50·68	20 21	S	
341	β Cephei	3	21 26 1·18	0·821	321 30 17·76	12·32	69 41	N	
342	γ Capricorni	3 4	21 28 59·14	3·329	322 14 47·15	49·93	17 33	S	
343	κ Capricorni	5	21 31 27·98	3·360	322 51 59·74	50·40	19 46	S	
344	ε Pegasi	3	21 34 21·53	2·943	323 35 22·99	44·15	8 58	N	
345	π¹ Cygni	4	21 34 59·63	2·116	323 44 54·38	31·74	50 17	N	
346	δ Capricorni	3	21 35 58·68	3·310	323 59 40·25	49·65	17 1	S	
347	✱ præc. α Aquarii	.	21 55 8·21	. . .	328 47 3·15	. . .	. .	.	
348	α Aquarii	3	21 55 29·75	3·067	328 52 26·25	46·00	1 17	S	
349	γ Aquarii	3	22 11 18·89	3·094	332 49 43·39	46·41	2 23	S	
350	π Aquarii	4 5	22 15 4·00	3·065	333 45 59·98	45·97	0 22	N	
351	ζ Aquarii	4	22 18 31·68	3·079	334 37 55·26	46·18	1 2	S	
352	σ Aquarii	5	22 20 2·99	3·186	335 0 44·84	47·79	11 42	S	
353	Lacertæ	4	22 23 7·61	2·431	335 46 54·14	36·46	49 16	N	
354	η Aquarii	4	22 25 4·59	3·079	336 16 8·78	46·19	1 9	S	
355	κ Aquarii	5	22 27 23·38	3·117	336 50 50·67	46·76	5 14	S	
356	ζ Pegasi	3	22 31 29·06	2·981	337 52 15·89	44·72	9 48	N	
357	η Pegasi	3	22 33 37·87	3·792	338 24 27·91	41·88	29 11	N	
358	τ¹ Aquarii	5	22 37 4·70	3·197	339 16 10·57	47·96	15 7	S	
359	τ² Aquarii	5 6	22 38 59·29	3·190	339 44 49·41	47·85	14 39	S	
360	λ Aquarii	4	22 42 10·52	3·137	340 32 37·87	47·05	8 38	S	

Catalogue of the principal Fixed Stars for the Beginning of the Year 1800.

No. of Stars.	Names and Characters of the Stars.	Magnitude.	Right Aſcenſ. in time.	Annual Variat. in ditto.	Right Aſcenſ. in degrees, &c.	Annual Variat. in ditto.	Declination North and South.		Annual Variat. in ditto
			h. m. s. $\frac{1}{100}$	s. $\frac{1}{1000}$	° ′ ″ $\frac{1}{100}$	″ $\frac{1}{100}$	° ′ ″		
				+		+			
361	ι Cephei	4	22 42 35·33	2·109	340 38 49·95	31·63	65 9	N	
362	δ Aquarii	3	22 44 1·80	3·201	341 0 27·06	48·02	16 53	S	
363	⁎ præc. α Piſc. auſt.	.	22 40 17·35	. . .	340 4 20·25	. . .	. .	.	
364	Fomalhaut	1	22 46 33·60	3·330	341 38 24·00	49·95	30 41	S	
365	⁎ ſeq. α Piſc. auſt.	.	22 48 38·04	. . .	342 9 30·60	. . .	. .	.	
366	β Pegaſi	2	22 54 5·50	2·874	343 31 22·47	43·11	27 0	N	
367	Markab	2	22 54 47·99	2·964	343 41 59·85	44·46	14 8	N	
368	⁎ ſeq. α Pegaſi	.	22 55 35·64	. . .	343 53 54·60	. . .	. .	.	
369	φ Aquarii	4 5	23 3 57·39	3·109	345 59 20·79	46·64	7 7	S	
370	ψ¹ Aquarii	5	23 5 23·05	3·125	346 20 45·68	46·88	10 10	S	
371	γ Piſcium	5	23 6 46·42	3·057	346 41 36·37	45·85	2 12	N	
372	ψ³ Aquarii	3	23 8 32·63	3·125	347 8 9·48	46·88	10 42	S	
373	Piſcium	6	23 26 11·40	3·065	351 32 50·96	45·97	1 0	N	
374	λ Piſcium	5	23 31 51·01	3·066	352 57 45·08	45·99	0 40	N	
375	Piſcium	5 6	23 36 10·88	3·062	354 2 43·18	45·93	2 23	N	
376	ω Piſcium	5	23 49 2·88	3·061	357 15 43·16	45·92	5 46	N	
377	⁎ præc. α Androm.	.	23 55 45·47	. . .	358 56 22·05	. . .	. .	.	
378	⁎ præc. α Androm.	.	23 56 15·28	3·060	359 3 49·19	45·90			
379	α Andromedæ	2	23 58 4·32	3·065	359 31 4·95	45·97	27 59	N	
380	⁎ ſeq. α Androm.	.	0 1 32·98	. . .	0 23 14·70	. . .	. .	.	
381	β Caſſiopeiæ	2 3	23 58 34·32	3·051	359 38 34·75	45·76	58 3	N	

Another CATALOGUE *of* 162 PRINCIPAL STARS, *ſhewing their Mean Declinations to Beginning of the Year* 1800.

No.	Stars Names.	Mean Declin. north.	Annual Variation.	No	Stars Names.	Mean Declin. north.	Annual Variation.
		° ′ ″	″			° ′ ″	″
1	Polaris	88 14 25	+ 19·57	11	α Lyræ	38 36 15	+ 2·59
2	Polaris	88 14 26		12	α Lyræ	38 36 10	
3	η Urſæ majoris	50 19 4	− 18·20	13	ζ Herculis	32 58 19	− 7·40
4	α Perſei	49 8 10	+ 13·59	14	Caſtor	32 18 54	− 6·95
5	ι Urſæ majoris	48 49 9	− 13·21	15	Caſtor	32 18 41	
6	δ Perſei	47 8 14	+ 12·35	16	Pollux	28 29 47	− 7·46
7	Capella	45 46 50	+ 5·09	17	β Tauri	28 25 25	+ 4·08
8	α Cygni	44 34 20	+ 12·52	18	β Tauri	28 25 30	
9	α Cygni	44 34 19		19	ε Bootis	27 55 32	− 15·59
10	β Bootis	41 11 11	− 14·54	20	α Andromedæ	27 59 15	+ 20·25

The Mean Declinations of 162 principal Stars for the Beginning of the Year 1800.

No.	Stars Names.	Mean Declin. north.	Annual Variation.	No.	Stars Names.	Mean Declin. north.	Annual Variation.
		° ′ ″	″			° ′ ″	″
21	α Andromedæ	27 59 11	+ 20·25	66	γ Geminorum	16 33 28	− 2·22
22	β Cygni	27 32 51	+ 7·04	67	γ Serpentis	16 19 40	− 11·01
23	Gemma	27 23 48	} − 12·50	68	β Serpentis	16 3 21	− 11·75
24	Gemma	27 23 49		69	Aldebaran	16 5 43	} + 8·16
25	μ Leonis	26 56 37	− 16·46	70	Aldebaran	16 5 45	
26	β Pegasi	26 59 58	+ 19·21	71	β Leonis	15 41 30	} − 19·96
27	ε Geminorum	25 18 55	} − 2·72	72	β Leonis	15 41 27	
28	ε Geminorum	25 18 56		73	γ Delphini	15 24 40	+ 12·68
29	δ Herculis	25 5 4	− 4·56	74	α Delphini	15 12 48	+ 12·21
30	ε Leonis	24 41 17	− 16·10	75	γ Tauri	15 8 1	+ 9·42
31	ζ Leonis	24 24 27	− 17·56	76	ζ Bootis	14 35 34	− 15·85
32	Alcione	23 28 34	+ 11·88	77	α Herculis	14 37 38	− 4·75
33	Electra	23 28 27	+ 12·04	78	α Pegasi	14 7 49	} + 19·22
34	Atlas	23 25 54	+ 11·74	79	α Pegasi	14 7 57	
35	Propus	23 15 40	+ 0·75	80	γ Pegasi	14 4 16	+ 20·04
36	τ Pegasi	22 38 51	+ 19·57	81	γ Pegasi	14 4 15	+ 20·04
37	μ Geminorum	22 36 12	− 0·89	82	β Delphini	13 54 31	+ 12·05
38	η Geminorum	22 32 59	− 0·19	83	ζ Aquilæ	13 34 32	+ 4·83
39	η Geminorum	22 33 5	− 0·19	84	Regulus	12 56 23	} − 17·24
40	α Arietis	22 30 37	+ 17·55	85	Regulus	12 56 20	
41	α Arietis	22 30 40	+ 17·55	86	α Cancri	12 37 30	− 13·18
42	δ Geminorum	22 20 19	− 5·83	87	α Ophiuchi	12 42 55	} − 3·05
43	γ Cancri	22 10 43	− 12·28	88	α Ophiuchi	12 43 7	
44	μ Cancri	22 9 9	− 9·67	89	ε Virginis	12 2 11	− 19·54
45	β Herculis	21 56 2	− 8·38	90	δ Serpentis	11 12 56	− 12·57
46	δ Leonis	21 37 1	− 19·43	91	ο Leonis	10 47 40	− 15·94
47	ζ Tauri	21 0 32	+ 3·05	92	ε Delphini	10 37 50	+ 11·73
48	γ Leonis	20 50 54	− 17·72	93	ρ Leonis	19 19 52	− 18·24
49	ζ Geminorum	20 51 0	} − 4·48	94	γ Aquilæ	10 8 6	} + 8·17
50	ζ Geminorum	20 51 5		95	γ Aquilæ	10 8 10	
51	ν Geminorum	20 19 34	− 1·44	96	ε Pegasi	8 57 43	+ 16·10
52	Arcturus	20 13 45	− 19·10	97	β Canis minoris	8 40 51	− 6·51
53	Arcturus	20 13 45	* − 19·10	98	α Aquilæ	8 20 58	} + 8·51
54	γ Herculis	19 37 52	− 9·05	99	α Aquilæ	8 20 48	
55	η Bootis	19 24 19	− 18·00	100	α Orionis	7 21 27	+ 1·42
56	δ Cancri	18 52 52	− 12·40	101	α Orionis	7 21 27	+ 1·42
57	e Pegasi	18 57 14	+ 14·91	102	ε Hydræ	7 8 37	− 12·60
58	β Arietis	18 49 30	+ 18·03	103	α Serpentis	7 3 55	} − 11·94
59	γ Arietis	18 18 29	+ 18·09	104	α Serpentis	7 3 50	
60	δ Sagittæ	18 3 15	+ 7·73	105	δ Hydræ	6 23 22	− 11·97
61	η Leonis	17 43 57	− 17·18	106	β Aquilæ	5 55 4	+ 8·86
62	α Sagittæ	17 33 48	+ 7·73	107	β Aquilæ	5 55 19	+ 8·86
63	δ¹ Tauri	17 3 38	+ 9·19	108	Procyon	5 44 11	− 7·51
64	ϑ Leonis	16 31 13	− 19·43	109	β Ophiuchi	4 39 41	− 2·35
65	γ Geminorum	16 33 27	− 2·22	110	δ Virginis	4 29 13	− 19·66

The Mean Declinations of 162 PRINCIPAL STARS *for the Beginning of the Year* 1800.

No.	Stars Names.	Mean Declin. north.	Annual Variation.	No.	Stars Names.	Mean Declin. south.	Annual Variation.
		° ′ ″	″			° ′ ″	″
111	ϑ Serpentis	3 57 12	+ 3·97	156	γ Eridani	14 5 3	− 10·90
112	α Ceti	3 17 49	+ 14·70	157	α Libræ	15 12 0	+ 15·40
113	α Ceti	3 18 0		158	α Libræ	15 12 0	
114	β Virginis	2 53 35	− 19·97	159	δ Corvi	15 24 5	+ 19·98
115	β Virginis	2 53 38		160	β Capricorni	15 24 10	− 10·71
116	γ Ophiuchi	2 47 42	− 1·97	161	γ Canis majoris	15 21 6	+ 4·69
117	δ Aquilæ	2 43 36	+ 6·44	162	η Ophiuchi	15 27 58	+ 5·33
118	γ Ceti	2 23 17	+ 15·77	163	ι Aquarii	15 48 54	− 17·14
119	α Piſcium	1 47 40	+ 17·73	164	γ Corvi	16 25 47	+ 20·04
120	η Antinoi	0 30 12	+ 8·63	165	Sirius	16 27 7	+ 4·43
121	δ Orionis	0 27 17	− 3·38	166	Sirius	16 27 5	+ 4·33
122	ζ Virginis	0 25 48	− 18·72	167	δ Aquarii	16 52 59	− 18·85
		South Decl.					
123	ι Hydræ	0 8 18	− 15·86	168	δ Capricorni	17 1 38	− 16·19
124	γ Virginis	0 21 4	+ 19·86	169	α Crateris	17 14 11	+ 19·11
125	δ Ceti	0 32 18	− 15·97	170	ι Capricorni	17 33 22	− 15·82
126	α Aquarii	1 17 7	− 17·15	171	γ Capricorni	17 39 58	− 14·97
127	α Aquarii	1 17 7		172	β Canis majoris	17 51 55	+ 1·18
128	ε Orionis	1 20 24	− 3·02	173	α Leporis	17 58 22	− 3·18
129	ϑ Antinoi	1 24 10	− 10·05	174	ϑ Capricorni	18 1 3	− 13·81
130	ζ Orionis	2 3 33	− 2·60	175	ν Scorpii	18 55 46	+ 10·03
131	γ Aquarii	2 23 31	− 17·81	176	β Ceti	19 5 9	− 19·84
132	δ Ophiuchi	3 10 8	+ 9·77	177	β Scorpii	19 14 46	+ 10·52
133	ζ Serpentis	3 39 32	− 0·93	178	μ Sagittarii	21 5 50	− 0·09
134	ε Ophiuchi	4 11 37	+ 9·47	179	π Sagittarii	21 19 45	− 4·95
135	ϑ Virginis	4 28 4	+ 19·39	180	ε Corvi	21 30 32	+ 20·05
136	β Eridani	5 21 16	− 5·41	181	δ Scorpii	22 2 30	+ 10·92
137	ι Orionis	6 3 0	− 3·04	182	ο Sagittarii	22 1 21	− 4·51
138	β Aquarii	6 26 37	− 15·39	183	β Corvi	22 17 17	+ 19·94
139	φ Aquarii	7 7 25	− 19·44	184	γ Leporis	22 31 15	− 2·11
140	α Hydræ	7 47 56	+ 15·21	185	α Corvi	23 36 50	+ 20·04
141	α Hydræ	7 47 53	+ 15·21	186	γ Scorpii	24 29 10	+ 14·67
142	Rigel	8 26 35	− 4·81	187	ϑ Ophiuchi	24 47 17	+ 4·43
143	β Libræ	8 38 14	+ 13·82	188	σ Scorpii	25 6 8	+ 9·38
144	λ Aquarii	8 38 29	− 18·89	189	π Scorpii	25 31 36	+ 11·06
145	α Spica	10 6 46	+ 19·01	190	Antares	25 58 38	+ 8·75
146	Spicæ	10 6 45	+ 19·01	191	Antares	25 58 23	+ 8·75
147	ζ Ophiuchi	10 9 1	+ 8·02	192	δ Canis majoris	26 5 10	+ 5·18
148	δ Eridani	10 27 5	− 11·99	193	ε Canis majoris	28 42 29	+ 4·38
149	μ Ceti	11 22 48	+ 15·71	194	ζ Canis majoris	29 58 50	+ 1·07
150	λ Virginis	12 26 26	+ 17·01	195	Fomalhaut	30 40 38	− 19·01
151	α¹ Capricorni	13 6 58	− 10·47				
152	α¹ Capricorni	13 7 0					
153	α² Capricorni	13 9 15	− 10·50				
154	α² Capricorni	13 9 17					
155	γ Libræ	14 6 42	+ 12·63				

STAR, in Electricity, denotes the appearance of the electric matter on a point into which it enters. Beccaria supposes that the Star is occasioned by the difficulty with which the electric fluid is extricated from the air, which is an electric substance. See BRUSH.

STAR, in Fortification, denotes a small fort, having 5 or more points, or saliant and re-entering angles, flanking one another, and their faces 90 or 100 feet long.

STAR, in Pyrotechny, a composition of combustible matters; which being borne, or thrown aloft into the air, exhibits the appearance of a real Star.—Stars are chiefly used as appendages to rockets, a number of them being usually inclosed in a conical cap, or cover, at the head of the rocket, and carried up with it to its utmost height, where the Stars, taking fire, are spread around, and exhibit an agreeable spectacle.

To make Stars.—Mix 3lbs of saltpetre, 11 ounces of sulphur, one of antimony, and 3 of gunpowder dust: or, 12 ounces of sulphur, 6 of saltpetre, 5½ of gunpowder dust, 4 of olibanum, one of mastic, camphor, sublimate of mercury, and half an ounce of antimony and orpiment. Moisten the mass with gumwater, and make it into little balls, of the size of a chesnut; which dry either in the sun, or in the oven. These being set on fire in the air, will represent Stars.

STAR-*Board* denotes the right hand side of a ship, when a person on board stands with the face looking forward towards the head or fore part of the ship. In contradistinction from *Larboard*, which denotes the left hand side of the ship in the same circumstances.—They say, *Starboard the helm*, or *helm a Starboard*, when the man at the helm should put the helm to the right hand side of the ship.

Falling STAR, or *Shooting* STAR, a luminous meteor darting rapidly through the air, and resembling a Star falling.—The explication of this phenomenon has puzzled all philosophers, till the modern discoveries in electricity have led to the most probable account of it. Signior Beccaria makes it pretty evident, that it is an electrical appearance, and recites the following fact in proof of it. About an hour after sunset, he and some friends that were with him, observed a falling Star directing its course towards them, and apparently growing larger and larger, but it disappeared not far from them; when it left their faces, hands, and clothes, with the earth, and all the neighbouring objects, suddenly illuminated with a diffused and lambent light, not attended with any noise at all. During their surprize at this appearance, a servant informed them that he had seen a light shine suddenly in the garden, and especially upon the streams which he was throwing to water it. All these appearances were evidently electrical; and Beccaria was confirmed in his conjecture, that electricity was the cause of them, by the quantity of electric matter which he had seen gradually advancing towards his kite, which had very much the appearance of a falling Star. Sometimes also he saw a kind of glory round the kite, which followed it when it changed its place, but left some light, for a small space of time, in the place it had quitted. Priestley's Elect. vol. 1, pa. 434, 8vo. See IGNIS *Fatuus*.

STAR-*fort*, or *Redoubt*, in Fortification. See STAR, REDOUBT, and FORT.

STARLINGS, or STERLINGS, or *Jettees*, a kind of case made about a pier of stilts, &c, to secure it. See STILTS.

STATICS, a branch of mathematics which considers weight or gravity, and the motion of bodies resulting from it.

Those who define mechanics, the science of motion, make Statics a part of it; viz, that part which considers the motion of bodies arising from gravity.

Others make them two distinct doctrines; restraining mechanics to the doctrine of motion and weight, as depending on, or connected with, the power of machines; and Statics to the doctrine of motion, considered merely as arising from the weight of bodies, without any immediate respect to machines. In this way, Statics should be the doctrine or theory of motion; and mechanics, the application of it to machines.

For the laws of Statics, see GRAVITY, DESCENT, &c.

STATION, or STATIONARY, in Astronomy, the position or appearance of a planet in the same point of the zodiac, for several days. This happens from the observer being situated on the earth, which is far out of the centre of their orbits, by which they seem to proceed irregularly; being sometimes seen to go forwards, or from west to east, which is their natural *direction*; sometimes to go backwards, or from east to west, which is their *retrogradation*; and between these two states there must be an intermediate one, where the planet appears neither to go forwards nor backwards, but to stand still, and keep the same place in the heavens, which is called her *Station*, and the planet is then said to be *Stationary*.

Apollonius Pergæus has shewn how to find the Stationary point of a planet, according to the old theory of the planets, which supposes them to move in epicycles; which was followed by Ptolomy in his Almag. lib. 12, cap. 1, and others, till the time of Copernicus. Concerning this, see Regiomontanus in Epitome Almagesti, lib. 12, prop. 1; Copernicus's Revolutiones Cœlest. lib. 5, cap. 35 and 36; Kepler in Tabulis Rudolphinis, cap. 24; Riccioli's Almag. lib. 7, sect. 5, cap. 2: Harman in Miscellan. Berolinens, pa. 197. Dr. Halley, Mr. Facio, Mr. De Moivre, Dr. Keil, and others have treated on this subject. See also the articles RETROGRADE and STATIONARY in this Dictionary.

STATION, in Practical Geometry &c, is a place pitched upon to make an observation, or take an angle, or such like, as in surveying, measuring heights and distances, levelling, &c.

An accessible height is taken from one Station; but an inaccessible height or distance is only to be taken by making two Stations, from two places whose distance asunder is known. In making maps of counties, provinces, &c, Stations are fixed upon certain eminencies &c of the country, and angles taken from thence to the several towns, villages, &c.—In surveying, the instrument is to be adjusted by the needle, or otherwise, to answer the points of the horizon at every Station; the distance from hence to the last Station is to be measured, and an angle is to be taken to the next Station; which process repeated includes the chief practice of surveying.

surveying.—In levelling, the instrument is rectified, or placed level at each Station, and observations made forwards and backwards.

There is a method of measuring distances at one Station, in the Philos. Trans. numb. 7, by means of a telescope. I have heard of another, by Mr. Ramsden; and have seen a third ingenious way by Mr. Green of Deptford, not yet published; this consists of a permanent scale of divisions, placed at any point whose distance is required; then the number of divisions seen through the telescope, gives the distance sought.

STATION-*Line*, in Surveying, and *Line of Station*, in Perspective. See LINE.

STATIONARY, in Astronomy, the state of a planet when, to an observer on the earth, it appears for some time to stand still, or remain immoveable in the same place in the heavens. For as the planets, to such an observer, have sometimes a progressive motion, and sometimes a retrograde one, there must be some point between the two where they must appear Stationary. Now a planet will be seen Stationary, when the line that joins the centres of the earth and planet is constantly directed to the same point in the heavens, which is when it keeps parallel to itself. For all right lines drawn from any point of the earth's orbit, parallel to one another, do all point to the same star; the distance of these lines being insensible, in comparison of that of the fixed stars.

The planet Herschel is seen Stationary at the distance of   from the sun; Saturn at somewhat more than 90°; Jupiter at the distance of 52°; and Mars at a much greater distance; Venus at 47°, and Mercury at 28°.

Herschel is Stationary   days, Saturn 8, Jupiter 4, Mars 2, Venus $1\frac{1}{2}$, and Mercury $\frac{1}{2}$ a day: though the several stations are not always equal; because the orbits of the planets are not circles which have the sun in their centre.

STEAM, the smoke or vapour arising from water, or any other liquid or moist body, when considerably heated. Subterranean Steams often affect the surface of the earth in a remarkable manner, and promote or prevent vegetation more than any thing else. It has been imagined that Steams may be the generative cause of both minerals and metals, and of all the peculiarities of springs. See Philos. Trans. vol. 5, pa. 1154, or Abr. vol. 2, pa. 833.—Of the use of the air to elevate the Steams of bodies, see pa. 2048 and 297 ib.—Concerning the warm and fertilizing temperature and Steams of the earth, see Phil. Trans. vol. 10, pa. 307 and 357. See also Dr. Hamilton "On the Ascent of Vapours."

The Steam raised from hot water is an elastic fluid, which, like elastic air, has its elasticity proportional to its density when the heat is the same, or proportional to the heat when the density is the same. The Steam raised with the ordinary heat of boiling water, is almost 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, and has its elasticity about equal to that of the common air of the atmosphere. And by great heat it has been found that the Steam may be expanded into 14000 times the space of water, or may be made about 5 times stronger than the atmosphere. But from some accidents that have happened, it appears that Steam, suddenly raised from water, or moist substances, by the immediate application of strong heat, is vastly stronger than the atmosphere, or even than gunpowder itself. Witness the accident that happened to a foundery of cannon at Moorfields, when upon the hot metal first running into the mould in the earth, some small quantity of water in the bottom of it was suddenly changed into Steam, which by its explosion, blew the foundery all to pieces. I remember another such accident at a foundery at Newcastle; the founder having purchased, among some old brass, a hollow brass ball that had been used for many years as a valve in a pump, withinside of which it would seem some water had got insinuated; and having put it into his fire to melt, when it had become very hot, it suddenly burst with a prodigious noise, and blew the adjacent parts of the furnace in pieces.

Steam may be applied to many purposes useful in life, but one of its chief uses is in the Steam-engine described in the following article.

STEAM *Engine*, an engine for raising water by the force of Steam produced from boiling water; and often called the *Fire-engine*, on account of the fire employed in boiling the water to produce the Steam. This is one of the most curious and useful machines, which modern art can boast, for raising water from ponds, wells, or pits, for draining mines, &c. Were it not for the use of this most important invention, it is probable we should not now have the benefit of coal fires in England; as our forefathers had, before the present century, excavated all the mines of coal as deep as it could be worked, without the benefit of this engine to draw the water from greater depths.

This engine is commonly a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight or pressure of the atmosphere upon a piston, at the other end, a temporary vacuum being made below it, by suddenly condensing the Steam, that had been let into the cylinder in which this piston works, by a jet of cold water thrown into it. A partial vacuum being thus made, the weight of the atmosphere presses down the piston, and raises the other end of the straight lever with the water from the well &c. Then immediately a hole is uncovered in the bottom of the cylinder, by which a fresh fill of hot Steam rushes in from a boiler of water below it, which proves a counterbalance for the atmosphere above the piston, upon which the weight of the pump rods at the other end of the lever carries that end down, and raises the piston of the Steam cylinder. Immediately the Steam hole is shut, and the cock opened for injecting the cold water into the cylinder of Steam, which condenses it to water again, and thus making another vacuum below the piston, the atmosphere above it presses it down, and raises the pump rods with another lift of water; and so on continually. This is the common principle: but there are also other modes of applying the force of the Steam, as we shall see in the following short history of this invention and its various improvements.

The earliest account to be met with of the invention of this engine, is in the marquis of Worcester's small book intitled a Century of Inventions (being a description of 100 notable discoveries), published in the year 1663, where he proposed the raising of great quantities of

of water by the force of Steam, raifed from water by means of fire; and he mentions an engine of that kind, of his own contrivance, which could raife a continual ftream like a fountain 40 feet high, by means of two cocks which were alternately and fucceffively turned by a man to admit the Steam, and to re-fill the veffel with cold water, the fire being continually kept up.

However, this invention not meeting with encouragement, probably owing to the confufed ftate of public affairs at that time, it was neglected, and lay dormant feveral years, until one Captain Thomas Savery, having read the marquis of Worcefter's books, feveral years afterwards, tried many experiments upon the force and power of Steam; and at laft hit upon a method of applying it to raife water. He then bought up and deftroyed all the marquis's books that could be got, and claimed the honour of the invention to himfelf, and obtained a patent for it, pretending that he had difcovered this fecret of nature by accident. He contrived an engine which, after many experiments, he brought to fome degree of perfection, fo as to raife water in fmall quantities: but he could not fucceed in raifing it to any great height, or in large quantities, for the draining of mines; to effect which by his method, the Steam was required to be fo ftrong as would have burft all his veffels; fo that he was obliged to limit himfelf to raifing the water only to a fmall height, or in fmall quantities. The largeft engine he erected, was for the York-buildings Company in London, for fupplying the inhabitants in the Strand and that neighbourhood with water.

The principle of this machine was as follows: H (fig. 3, pl. 27) reprefents a copper boiler placed on a furnace. E is a ftrong iron veffel, communicating with the boiler by means of a pipe at top, and with the main pipe AB by means of a pipe I at bottom; AB is the main pipe immerfed in the water at B; D and C are two fixed valves, both opening upwards, one being placed above, and the other below the pipe of communication I. Laftly, at G is a cock that ferves occafionally to wet and cool the veffel E, by water from the main pipe, and F is a cock in the pipe of communication between the veffel E and the boiler.

The engine is fet to work, by filling the copper in part with water, and alfo the upper part of the main pipe above the valve C, the fire in the furnace being lighted at the fame time. When the water boils ftrongly, the cock F is opened, the Steam rufhes into the veffel E, and expels the air from thence through the valve C. The veffel E thus filled, and violently heated by the Steam, is fuddenly cooled by the water which falls upon it by turning the cock C; the cock F being at the fame time fhut, to prevent any frefh acceffion of Steam from the boiler. Hence, the Steam in E becoming condenfed, it leaves the cavity within almoft intirely a vacuum; and therefore the preffure of the atmofphere at B forces the water through the valve D till the veffel E is nearly filled. The condenfing cock G is then fhut, and the Steam cock F again opened; hence the Steam, rufhing into E, expels the water through the valve C, as it before did the air. Thus E becomes again filled with hot Steam, which is again cooled and condenfed by the water from G, the fupply of Steam being cut off by fhutting F, as in the former operation: the water confequently rufhes through D, by the preffure of the atmofphere at B, and E is again filled. This water is forced up the main pipe through C, by opening F and fhutting G, as before. And thus it is eafy to conceive, that by this alternate opening and fhutting the cocks, water will be continually raifed, as long as the boiler continues to fupply the Steam.

For the fake of perfpicuity, the drawing is divefted of the apparatus that ferves to turn the two cocks at once, and of the contrivances for filling the copper to the proper quantity. But it may be found complete, with a full account of its ufes and application, in Mr. Savery's book intituled the *Miner's Friend*. The engines of this conftruction were ufually made to work with two receivers or Steam veffels, one to receive the Steam, while the other was raifing water by the condenfation. This engine has been fince improved, by admitting the end of the condenfing pipe G into the veffel E, by which means the Steam is more fuddenly and effectually condenfed than by water on the outfide of the veffel.

The advantages of this engine are, that it may be erected in almoft any fituation, that it requires but little room, and is fubject to very little friction in its parts.—Its difadvantages are, that great part of the Steam is condenfed and lofes its force upon coming into contact with the water in the veffel E, and that the heat and elafticity of the Steam muft be increafed in proportion to the height that the water is required to be raifed to. On both thefe accounts a large fire is required, and the copper muft be very ftrong, when the height is confiderable, otherwife there is danger of its burfting.

While captain Savery was employed in perfecting his engine, Dr. Papin of Marburg was contriving one on the fame principles, which he defcribes in a fmall book publifhed in 1707, intitled *Ars Nova ad Aquam Ignis adminiculo efficaciffimè elevandam*. Capt. Savery's engine however was much completer than that propofed by Dr. Papin.

About the fame time alfo one Monf. Amontons of Paris was engaged in the fame purfuit: but his method of applying the force of Steam was different from thofe before-mentioned; for he intended it to drive or turn a wheel, which he called a *fire-mill*, which was to work pumps for raifing water; but he never brought it to perfection. Each of thefe three gentlemen claimed the originality of the invention; but it is moft probable they all took the hint from the book publifhed by the marquis of Worcefter, as before-mentioned.

In this imperfect ftate it continued, without farther improvements, till the year 1705, when Mr. Newcomen, an iron-monger, and Mr. John Cowley, a glazier, both of Dartmouth, contrived another way to raife water by Steam, bringing the engine to work with a beam and pifton, and where the Steam, even at the greateft depths of mines, is not required to be greater than the preffure of the atmofphere: and this is the ftructure of the engine as it has fince been chiefly ufed. Thefe gentlemen obtained a patent for the fole ufe of this invention, for 14 years. The firft propofal they made for draining of mines by this engine, was in the year 1711; but they were very coldly received by many

many perſons in the ſouth of England, who did not underſtand the nature of it. In 1712 they came to an agreement with the owners of a colliery at Griff in Warwickſhire, where they erected an engine with a cylinder of 22 inches diameter. At firſt they were under great difficulties in many things; but by the aſſiſtance of ſome good workmen they got all the parts put together in ſuch a manner, as to anſwer their intention tolerably well: and this was the firſt engine of the kind erected in England. There was at firſt one man to attend the Steam-cock, and another to attend the injection cock; but they afterwards contrived a method of opening and ſhutting them by ſome ſmall machinery connected with the working beam. The next engine erected by theſe patentees, was at a colliery in the county of Durham, about the year 1718, where was concerned, as an agent, Mr. Henry Beighton, F. R. S. and conductor of the Ladies' Diary from the year 1714 to the year 1744: this gentleman, not approving of the intricate manner of opening and ſhutting the cocks by ſtrings and catches, as in the former engine, ſubſtituted the hanging beam for that purpoſe as at preſent uſed, and likewiſe made improvements in the pipes, valves, and ſome other parts of the engine.

In a few years afterwards, theſe engines came to be better underſtood than they had been; and their advantages, eſpecially in draining of mines, became more apparent: and from the great number of them erected, they received additional improvements from different perſons, till they arrived at their preſent degree of perfection: as will appear in the ſequel, after we have a little conſidered the general principles of this engine, which are as follow.

Principles of the Steam Engine.

The principles on which this engine acts, are truly philoſophical; and when all the parts of the machine are proportioned to each other according to theſe principles, it never fails to anſwer the intention of the engineer.

1. It has been proved in pneumatics, that the preſſure of the atmoſphere upon a ſquare inch at the earth's ſurface, is about $14\frac{3}{4}$lb avoirdupois at a medium, or $11\frac{1}{2}$lb on a circular inch, that is on a circle of an inch diameter. And,

2. If a vacuum be made by any means in a cylinder, which has a moveable piſton ſuſpended at one end of a lever equally divided, the air will endeavour to ruſh in, and will preſs down the piſton, with a force proportionable to the area of the ſurface, and will raiſe an equal weight at the other end of the lever.

3. Water may be rarefied near 14000 times by being reduced into Steam, and violently heated: the particles of it are ſo ſtrongly repellent, as to drive away air of the common denſity, only by a heat ſufficient to keep the water in a boiling ſtate, when the Steam is almoſt 3000 times rarer than water, or $3\frac{1}{2}$ times rarer than air, as appears by an experiment of Mr. Beighton's: by increaſing the heat, the Steam may be rendered much ſtronger; but this requires great ſtrength in the veſſels. This Steam may be again condenſed into its former ſtate by a jet of cold water diſperſed through it; ſo that 14000 cubic inches of Steam admitted into a cylinder, may be reduced into the ſpace of one cubic inch of water only, by which means a partial vacuum is obtained.

4. Though the preſſure of the atmoſphere be about $14\frac{3}{4}$ pounds upon every ſquare inch, or $11\frac{1}{2}$ pounds upon a circular inch; yet, on account of the friction of the ſeveral parts, the reſiſtance from ſome air which is unavoidably admitted with the jet of cold water, and from ſome remainder of Steam in the cylinder, the vacuum is very imperfect, and the piſton does not deſcend with a force exceeding 8 or 9 pounds upon every ſquare inch of its ſurface.

5. The gallon of water of 282 cubic inches weighs $10\frac{1}{5}$ pounds avoirdupois, or a cubic foot $62\frac{1}{2}$ pounds, or 1000 ounces. The piſton being preſſed by the atmoſphere with a force proportional to its area in inches, multiplied by about 8 or 9 pounds, depreſſes that end of the lever, and raiſes a column of water in the pumps of equal weight at the other end, by means of the pump-rods ſuſpended to it. When the Steam is again admitted, the pump-rods ſink by their ſuperior weight, and the piſton riſes; and when that Steam is condenſed, the piſton deſcends, and the pump-rods lift; and ſo on alternately as long as the piſton works.

It has been obſerved above, that the piſton does not deſcend with a force exceeding 8 or 9 pounds upon every ſquare inch of its ſurface; but by reaſon of accidental frictions, and alterations in the denſity of the air, it will be ſafeſt, in calculating the power of the cylinder, to allow ſomething leſs than 8 pounds for the preſſure of the atmoſphere, upon every ſquare inch, viz 7lb. 10 oz. = 7·64lb, or juſt 6lb. upon every circular inch; and it being allowed that the gallon of water, of 282 cubic inches, weighs $10\frac{1}{5}$lb, from theſe premiſes the dimenſions of the cylinder, pumps, &c, for any Steam-engine, may be deduced as follows:—Suppoſe

c = the cylinder's diameter in inches,
p = the pump's ditto,
f = the depth of the pit in fathoms,
g = gallons drawn by a ſtroke of 6 feet or a fathom,
h = the hogſheads drawn per hour,
s = the number of ſtrokes per minute.

Then c^2 is the area of the cylinder in circular inches, theref. $6c^2$ is the power of the cylinder in pounds.

And $\frac{p^2 \times \cdot 7854 \times 72}{282}$ or $\frac{1}{5}p^2$ is $= g$ the gallons contained in one fathom or 72 inches of any pump; which multiplied by f fathoms, gives $\frac{1}{5}p^2f$ for the gallons contained in f fathoms of any pump whoſe diameter is p.

Hence $\frac{1}{5}p^2f \times 10\frac{1}{5}$lb. gives $2p^2f$ nearly, for the weight in pounds of the column of water which is to be equal to the power of the cylinder, which was before found equal to $6c^2$. Hence then we have the 2d equation,

viz, $6c^2 = 2p^2f$, or $3c^2 = p^2f$;
the firſt equation being $\frac{1}{5}p^2 = g$, or $p^2 = 5g$.

From which two equations, any particular circumſtance may be determined.

Or if, inſtead of 6lb, for the preſſure of the air on each circular inch of the cylinder, that force be ſuppoſed

posed any number as a pounds; then will the power of the cyclinder be ac^2, and the 2d equation becomes $ac^2 = 2p^2f = 10fg$, by substituting $5g$ instead of p^2.

And farther, $63h = 60gs$, or $21h = 20gs$.

From a comparison of these equations, the following theorems are derived, which will determine the size of the cylinder and pumps of any Steam-engine capable of drawing a certain quantity of water from any assigned depth, with the pressure of the atmosphere on each circular inch of the cylinder's area.

These theorems are more particularly adapted to one pump in a pit. But it often happens in practice, that an engine has to draw several pumps of different diameters from different depths; and in this case, the square of the diameter of each pump must be multiplied by its depth, and double the sum of all the products will be the weight of water drawn at each stroke, which is to be used instead of $2p^2f$ for the power of the cylinder.

The following is a Table, calculated from the foregoing theorems, of the powers of cylinders from 30 to 70 inches diameter; and the diameter and lengths of pumps which those cylinders are capable of working, from a 6 inch bore to that of 20 inches, together with the quantity of water drawn per stroke and per hour, allowing the engine to make 12 strokes of 6 feet per minute, and the pressure of the atmosphere at the rate of 7 lb 10 oz per square inch, or 6 lb per circular inch.

A Table *of* Theorems *for the readier computing the Powers of a* Steam-Engine.

1	$a = \dfrac{2fp^2}{c^2} = \dfrac{10fg}{c^2} = \dfrac{21fh}{2c^2s}$
2	$c = \sqrt{\dfrac{2fp^2}{a}} = \sqrt{\dfrac{10fg}{a}} = \sqrt{\dfrac{21fh}{2as}}$
3	$f = \dfrac{ac^2}{2p^2} = \dfrac{ac^2}{10g} = \dfrac{2ac^2s}{21h}$
4	$g = \dfrac{p^2}{5} = \dfrac{ac^2}{10f} = \dfrac{21h}{20s}$
5	$h = \dfrac{4p^2s}{21} = \dfrac{20gs}{21} = \dfrac{2ac^2s}{21f}$
6	$p = \sqrt{5g} = \sqrt{\dfrac{ac^2}{2g}} = \sqrt{\dfrac{21h}{4s}}$
7	$s = \dfrac{21h}{4p^2} = \dfrac{21h}{20g} = \dfrac{21fh}{2ac^2}$

Table

TABLE *of the Power and Effects of* STEAM-ENGINES, *allowing* 12 *Strokes, of* 6 *Feet long each, per Minute, and the pressure of the Air* 7*lb* 10*oz per Square Inch, or* 6*lb per Circular Inch.*

The Diameters of the Cylinders in Inches.	The Diameters of the Pumps in Inches.															Power of the cylindeisand weight of water in pounds.
	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
30	75	55	42	33	27	22	19	16	14	12	10	·	·	·	·	5400
31	80	58	45	35	29	24	20	17	15	13	11	10	·	·	·	5766
32	83	61	47	37	30	25	21	18	16	13	12	10	·	·	·	6144
33	90	67	51	40	3	27	22	19	17	14	13	11	10	·	·	6534
34	94	70	53	42	34	28	23	20	18	15	14	12	10	·	·	693
35	102	75	57	45	37	30	26	22	19	16	14	13	11	·	·	7350
36	·	79	61	48	39	32	27	23	20	17	15	14	12	10	·	7776
37	·	84	64	51	41	34	29	24	21	18	16	14	12	11	10	8214
38	·	88	68	53	43	35	30	26	22	19	17	15	13	12	10	8664
39	·	93	71	56	45	37	32	27	23	20	18	16	14	12	11	9126
40	·	98	75	59	48	39	34	28	24	21	19	17	15	13	12	9600
42	·	108	83	65	53	43	38	31	27	23	21	18	16	14	13	10584
44	·	·	90	71	58	48	41	34	30	26	23	20	18	16	14	11616
46	·	·	99	78	63	52	45	37	33	29	25	21	19	17	16	12696
48	·	·	·	85	69	57	49	41	35	31	27	24	21	19	17	13824
50	·	·	·	92	75	62	53	44	38	34	29	26	23	21	19	15000
52	·	·	·	100	81	67	57	48	41	36	31	28	25	22	20	16224
54	·	·	·	·	87	72	61	52	44	38	34	30	27	24	22	17496
56	·	·	·	·	94	78	66	56	48	42	37	32	29	26	23	18816
58	·	·	·	·	101	83	70	59	51	44	39	34	31	28	25	20184
60	·	·	·	·	·	89	75	63	55	48	42	37	33	30	27	21600
62	·	·	·	·	·	95	80	68	58	51	45	39	35	32	28	23064
64	·	·	·	·	·	·	85	72	62	54	48	42	38	34	30	24546
66	·	·	·	·	·	·	90	77	66	57	51	45	40	36	32	26676
68	·	·	·	·	·	·	96	82	70	61	54	48	42	38	34	27744
70	·	·	·	·	·	·	·	86	75	64	57	50	45	40	36	29400
Quan. drawn at one stroke in gallons.	7·2	10	13	16·2	20	24·2	28·8	33·8	39·2	45	51·2	57·8	64·8	72·2	80	
Quan. drawn in one hour in hogsheads.	82	114	148	184	228	276	328	385	447	513	583	659	738	[illegible]	912	
Diameter of pumps.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

Let-

Let us now defcribe the feveral parts of an engine, and exemplify the application of the foregoing principles, in the conftruction of one of the completeft of the modern engines. See fig. 4. pl. 27.

A reprefents the fire-place under the boiler, for the boiling of the water, and the afh-hole below it.

B, the boiler, filled with water about three feet above the bottom, made of iron plates.

C, the Steam pipe, through which the Steam paffes from the boiler into the receiver.

D, the receiver, a clofe iron veffel, in which is the regulator or Steam-cock, which opens and fhuts the hole of communication at each ftroke.

E, the communication pipe between the receiver and the cylinder; it rifes 5 or 6 inches up, in the infide of the cylinder bottom, to prevent the injected water from defcending into the receiver.

F, the cylinder, of caft iron, about 10 feet long, bored fmooth in the infide; it has a broad flanch in the middle on the outfide, by which it is fupported when hung in the cylinder-beams.

G, the pifton, made to fit the cylinder exactly: it has a flanch rifing 4 or 5 inches upon its upper furface, between which and the fide of the cylinder a quantity of junk or oakum is ftuffed, and kept down by weights, to prevent the entrance of air or water and the efcaping of Steam.

H, the chain and pifton fhank, by which it is connected to the working beam.

I I, the working-beam or lever: it is made of two or more large logs of timber, bent together at each end, and kept at the diftance of 8 or 9 inches from each other in the middle by the gudgeon, as reprefented in the Plate. The arch-heads, II, at the ends, are for giving a perpendicular direction to the chains of the pifton and pump-rods.

K, the pump-rod which works in the fucking pump.

L, and draws the water from the bottom of the pit to the furface.

M, a ciftern, into which the water drawn out of the pit is conducted by a trough, fo as to keep it always full: and the fuperfluous water is carried off by another trough.

N, the jack-head pump, which is a fucking-pump wrought by a fmall lever or working-beam, by means of a chain connected to the great beam or lever near the arch *g* at the inner end, and the pump-rod at the outer end. This pump commonly ftands near the corner of the front of the houfe, and raifes the column of water up to the ciftern O, into which it is conducted by a trough.

O, the jack-head ciftern for fupplying the injection, which is always kept full by the pump N: it is fixed fo high as to give the jet a fufficient velocity into the cylinder when the cock is opened. This ciftern has a pipe on the oppofite fide for conveying away the fuperfluous water.

P P, the injection-pipe, of 3 or 4 inches diameter, which turns up in a curve at the lower end, and enters the cylinder bottom: it has a thin plate of iron upon the end *a*, with 3 or 4 adjutage holes in it, to prevent the jet of cold water of the jack-head ciftern from flying up againft the pifton, and yet to condenfe the Steam each ftroke, when the injection-cock is open.

e, a valve upon the upper end of the injection pipe within the ciftern, which is fhut when the engine is not working, to prevent any wafte of the water.

f, a fmall pipe which branches off from the injection-pipe, and has a fmall cock to fupply the pifton with a little water to keep it air-tight.

Q, the working plug, fufpended by a chain to the arch *g* of the working beam. It is ufually a heavy piece of timber, with a flit vertically down its middle, and holes bored horizontally through it, to receive pins for the purpofe of opening and fhutting the injection and Steam cocks, as it afcends and defcends by the motion of the working beam.

h, the handle of the fteam-cock or regulator. It is fixed to the regulator by a fpindle which comes up through the top of the receiver. The regulator is a circular plate of brafs or caft iron, which is moved horizontally by the handle *h*, and opens or fhuts the communication at the lower end of the pipe E within the receiver. It is reprefented in the plate by a circular dotted line.

i i, the fpanner, which is a long rod or plate of iron for communicating motion to the handle of the regulator: to which it is fixed by means of a flit in the latter, and fome pins put through to faften it.

k l, the vibrating lever, called the Y, having the weight *k* at one end and two legs at the other end. It is fixed to an horizontal axis, moveable about its centre-pins or pivots *mn*, by means of the two fhanks *op* fixed to the fame axis, which are alternately thrown backwards and forwards by means of two pins in the working plug; one pin on the outfide depreffing the fhank *o*, throws the loaded end *k* of the Y from the cylinder into the pofition reprefented in the plate, and caufes the leg *l* to ftrike againft the end of the fpanner; which forcing back the handle of the regulator or fteam cock, opens the communication, and permits the fteam to fly into the cylinder. The pifton immediately rifing by the admiffion of the Steam, the working beam II rifes; which alfo raifes the working-plug, and another pin which goes through the flit raifes the fhank *p*, which throws the end *k* of the Z towards the cylinder, and, ftriking the end of the fpanner, forces it forward, and fhuts the regulator Steam-cock.

qr, the lever for opening and fhutting the injection cock, called the F. It has two toes from its centre, which take between them the key of the injection cock. When the working-plug has afcended nearly to its greateft height, and fhut the regulator, a pin catches the end *q* of the F and raifes it up, which opens the injection-cock, admits a jet of cold water to fly into the cylinder, and, condenfing the Steam, makes a vacuum; then the preffure of the atmofphere bringing down the pifton in the cylinder, and alfo the plug-frame, another pin fixed in it catches the end of the lever in its defcent, and, by preffing it down, fhuts the injection-cock, at the fame time the regulator is opened to admit Steam, and fo on alternately; when the regulator is fhut the injection is open, and when the former is open the latter is fhut.

R, the

R, the hot-well, a ſmall ciſtern made of planks, which receives all the waſte water from the cylinder.

S, the ſink-pit to convey away the water which is injected into the cylinder at each ſtroke. Its upper end is even with the inſide of the cylinder bottom, its lower end has a lid or cover moveable on a hinge which ſerves as a valve to let out the injected water, and ſhuts cloſe each ſtroke of the engine, to prevent the water being forced up again when the vacuum is made.

T, the feeding pipe, to ſupply the boiler with water from the hot-well. It has a cock to let in a large or ſmall quantity of water as occaſion requires, to make up for what is evaporated ; it goes nearly down to the boiler bottom.

U, two gage cocks, the one larger than the other, to try when a proper quantity of water is in the boiler: upon opening the cocks, if one give Steam and the other water, it is right ; if they both give Steam, there is too little water in the boiler ; and if they both give water, there is too much.

W, a plate which is ſcrewed on to a hole on the ſide of the boiler, to allow a paſſage into the boiler for the convenience of cleaning or repairing it.

X, the Steam-clack or puppet valve, which is a braſs valve on the top of a pipe opening into the boiler, to let off the Steam when it is too ſtrong. It is loaded with lead, at the rate of one pound to an inch ſquare ; and when the Steam is nearly ſtrong enough to keep it open, it will do for the working of the engine.

f, the ſnifting valve, by which the air is diſcharged from the cylinder each ſtroke, which was admitted with the injection, and would otherwiſe obſtruct the due operation of the engine.

tt, the cylinder-beams ; which are ſtrong joiſts going through the houſe for ſupporting the cylinder.

v, the cylinder cap of lead, ſoldered on the top of the cylinder, to prevent the water upon the piſton from flaſhing over when it riſes too high.

w, the waſte-pipe, which conducts the ſuperfluous water from the top of the cylinder to the hot-well.

xx, iron bars, called the catch-pins, fixed horizontally through each arch head, to prevent the beam deſcending too low in caſe the chain ſhould break.

yy, two ſtrong wooden ſprings, to weaken the blow given by the catch pins when the ſtroke is too long.

zz, two friction-wheels, on which the gudgeon or centre of the great beam is hung ; they are the third or fourth part of a circle, and move a little each way as the beam vibrates. Their uſe is to diminiſh the friction of the axis, which, in ſo heavy a lever, would otherwiſe be very great.

When this engine is to be ſet to work, the boiler muſt be filled about three or four feet deep with water, and a large fire made under it ; and when the Steam is found to be of a ſufficient ſtrength by the puppet-clack, then by thruſting back the ſpanner, which opens the regulator or Steam-cock, the Steam is admitted into the cylinder, which raiſes the piſton to the top of the cylinder, and forces out all the air at the ſnifting valve ; then by turning the key of the injection-cock, a jet of cold water is admitted into the cylinder, which condenſes the Steam and makes a vacuum ; and the atmoſphere then preſſing upon the piſton, forces it down to the lower part of the cylinder, and makes a ſtroke by raiſing the column of water at the other end of the beam. After two or three ſtrokes are made in this manner, by a man opening and ſhutting the cocks to try if they be right, then the pins may be put into the pin-holes in the working plug, and the engine left to turn the cocks of itſelf ; which it will do with greater exactneſs than any man can do.

There are in ſome engines, methods of ſhutting and opening the cocks different from the one above deſcribed, but perhaps none better adapted to the purpoſe ; and as the principles on which they all act are originally the ſame, any difference in the mechanical conſtruction of the ſmall machinery will have no influence of conſequence upon the total effect of the grand machine.

The furnace or fire-place ſhould not have the bars ſo cloſe as to prevent the free admiſſion of freſh air to the fire, nor ſo open as to permit the coals to fall through them ; for which purpoſe two inches or thereabouts is ſufficient for the diſtance betwixt the bars. The ſize of the furnace depends upon the ſize of the boiler ; but in every caſe the aſh-hole ought to be capacious to admit the air, and the greater its height the better. If the flame is conducted in a flue or chimney round the outſide of the boiler, or in a pipe round the inſide of it, it ought to be gradually diminiſhed from the entrance at the furnace to its egreſs at the chimney ; and the ſection of the chimney at that place ſhould not exceed the ſection of the flue or pipe, and ſhould alſo be ſomewhat leſs at the chimney-top.

The boiler or veſſel in which the water is rarefied by the force of fire, may be made of iron plates, or caſt iron, or ſuch other materials as can withſtand the effects of the fire, and the elaſtic force of the Steam. It may be conſidered as conſiſting of two parts ; the upper part which is expoſed to the Steam, and the under part which is expoſed to the fire. The form of the latter ſhould be ſuch as to receive the full force of the fire in the moſt advantageous manner, ſo that a certain quantity of fuel may have the greateſt poſſible effect in heating and evaporating the water ; which is beſt done by making the ſides cylindrical, and the bottom a little concave, and then conducting the flame by an iron flue or pipe round the inſide of the boiler beneath the ſurface of the water, before it reach the chimney. For, by this means, after the fire in the furnace has heated the water by its effect on the bottom, the flame heats it again by the pipe being wholly included in the water, and having every part of its ſurface in contact with it ; which is preferable to carrying it in a flue or chimney round the outſide of the boiler, as a third or a half of the ſurface of the flame only could be in contact with the boiler, the other being ſpent upon the brick-work. This cylindric lower part may be leſs in its diameter than the upper part, and may contain from four to ſix feet perpendicular height of water in it.

The upper part of the boiler is beſt made hemiſpherical, for reſiſting the elaſticity of the Steam; yet any other form may do, provided it be of ſufficient ſtrength for the purpoſe. The quick going of the engine depends much on the capaciouſneſs of the boiler-top; for if it be too ſmall, it requires the Steam to be heated to a great degree, to increaſe its elaſtic force ſo much as to work the engine. If the top is ſo capacious as to contain eight or ten times the quantity of Steam uſed each ſtroke, it will require no more fire to preſerve its elaſticity than is ſufficient to keep the water in a proper ſtate of boiling; this, therefore, is the beſt ſize for a boiler top. If the diameter of the cylinder be c, and works a ſix-foot ſtroke, and the diameter of the boiler be ſuppoſed b, then

$$200c^2 = b^3, \text{ or } b = \sqrt[3]{200c^2}.$$

The effect of the injection in condenſing the Steam in the cylinder, depends upon the height of the reſervoir and the diameter of the adjutage. If the engine makes a 6 feet ſtroke, then the jackhead ciſtern ſhould be 12 feet perpendicular above the bottom of the cylinder or the adjutage. The ſize of the adjutage may be from 1 to 2 inches in diameter; or if the cylinder be very large, it is proper to have three or four holes rather than one large one, in order that the jet may be diſperſed the more effectually over the whole area of the cylinder. The injection pipe, or pipe of conduct, ſhould be ſo large as to ſupply the injection freely with water; if the diameter of the injection pipe be called p, and the diameter of the adjutage, a, then $4a^2 = p^2$, and $a^2 = \frac{1}{4}p^2$, or $a = \frac{1}{2}p$.

For a further account of theſe engines, ſee Deſaguliers's Exp. Philoſ. vol. 2, ſect. 14, pa. 465, &c.; or for an abſtract, Martin's Phil. Brit. number 461, or Nicholſon's Nat. Philoſ. p. 83 &c. And for an account of the improvement made in the fire-engine by Mr. Payne, ſee Philoſ. Tranſ. number 461, or Martin's Philoſ. Brit. p. 87 &c.

Mr. Blakey communicated to the Royal Society, in 1752, remarks on the beſt proportions for Steam-engine cylinders of a given content: and Mr. Smeaton deſcribes an engine of this kind, invented by Mr. De Moura of Portugal, being an improvement of Savery's conſtruction, to render it capable of working itſelf: for both which accounts, ſee Philoſ. Tranſ. vol. 47 art. 29 and 72.

We are informed in the new edit. of the Biograph. Brit. in the article Brindley, that in 1756 this gentleman, ſo well known for his concern in our inland navigations, undertook to erect a Steam-engine near Newcaſtle-under-Line, upon a new plan. The boiler of it was made with brick and ſtone, inſtead of iron plates, and the water was heated by iron flues of a peculiar conſtruction; by which contrivances the conſumption of fuel, neceſſary for working a Steam engine, was reduced one half. He introduced alſo in his engine, wooden cylinders, made in the manner of cooper's ware, inſtead of iron ones; the former being both cheaper and more eaſily managed in the ſhafts: and he likewiſe ſubſtituted wood for iron in the chains which worked at the end of the beam. He had formed deſigns of introducing other improvements into the conſtruction of this uſeful engine; but was diſcouraged by obſtacles that were thrown in his way.

Mr. Blakey, ſome years ago, obtained a patent for his improvement of Savery's Steam-engine, by which it is excellently adapted for raiſing water out of ponds, rivers, wells, &c, and for forcing it up to any height wanted for ſupplying houſes, gardens, and other places; though it has not power ſufficient to drain off the water from a deep mine. The principles of his conſtruction are explained by Mr. Ferguſon, in the Supplement to his Lectures, pa. 19; and a more particular deſcription of it, accompanied with a drawing, is given by the patentee himſelf in the Gentleman's Magazine for 1769, p. 392.

Mr. Blakey, it is ſaid, is the firſt perſon who ever thought of making uſe of air as an intermediate body between Steam and water; by which means the Steam is always kept from touching the water, and conſequently from being condenſed by it: and on this new principle he has obtained a patent. The engine may be built at a trifling expence, in compariſon of the common fire-engine now in uſe; it will ſeldom need repairs, and will not conſume half ſo much fuel. And as it has no pumps with piſtons, it is clear of all their friction; and the effect is equal to the whole ſtrength or compreſſive force of the Steam; which the effect of the common fire-engine never is, on account of the great friction of the piſtons in their pumps.

Ever ſince Mr. Newcomen's invention of the Steam fire engine, the great conſumption of fuel with which it is attended, has been complained of as an immenſe drawback upon the profits of our mines. It is a known fact, that every fire-engine of conſiderable ſize conſumes to the amount of three thouſand pounds worth of coals in every year. Hence many of our engineers have endeavoured, in the conſtruction of theſe engines, to ſave fuel. For this purpoſe, the fire-place has been diminiſhed, the flame has been carried round from the bottom of the boiler in a ſpiral direction, and conveyed through the body of the water in a tube before its arrival at the chimney; ſome have uſed a double boiler, ſo that fire might act in every poſſible point of contact; and ſome have built a moor-ſtone boiler, heated by three tubes of flame paſſing through it. But the moſt important improvements which have been made in the Steam-engine for more than thirty years paſt, we owe to the ſkill of Mr. James Watt; of which we ſhall give ſome account: premiſing, that the internal ſtructure of his new engines ſo much reſembles that of the common ones, that thoſe who are acquainted with them will not fail to underſtand the mechaniſm of his from the following deſcription: he has contrived to obſerve an uniform heat in the cylinder of his engines, by ſuffering no cold water to touch it, and by protecting it from the air, or other cold bodies, by a ſurrounding caſe filled with Steam, or with hot air or water, and by coating it over with ſubſtances that tranſmit heat ſlowly. He makes his vacuum to approach nearly to that of the barometer, by condenſing the Steam in a ſeparate veſſel, called the condenſer, which may be cooled at pleaſure without cooling the cylinder, either by an injection of cold water, or by ſurrounding the

the condenser with it, and generally by both. He extracts the injection water, and detached air, from the cylinder or condenser by pumps, which are wrought by the engine itself, or blows them out by the Steam. As the entrance of air into the cylinder would stop the operation of the engines, and as it is hardly to be expected that such enormous pistons as those of Steam-engines can move up and down, and yet be absolutely tight in the common engines; a stream of water is kept always running upon the piston, which prevents the entry of the air: but this mode of securing the piston, though not hurtful in the common ones, would be highly prejudicial to the new engines. Their piston is therefore made more accurately; and the outer cylinder, having a lid, covers it, the Steam is introduced above the piston; and when a vacuum is produced under it, acts upon it by its elasticity, as the atmosphere does upon common engines by its gravity. This way of working effectually excludes the air from the inner cylinder, and gives the advantage of adding to the power, by increasing the elasticity of the Steam.

In Mr. Watt's engines, the cylinder, the great beams, the pumps, &c, stand in their usual positions. The cylinder is smaller than usual, in proportion to the load, and is very accurately bored.

In the most complete engines, it is surrounded at a small distance, with another cylinder, furnished with a bottom and a lid. The interstice between the cylinders communicates with the boilers by a large pipe, open at both ends: so that it is always filled with Steam, and thereby maintains the inner cylinder always of the same heat with the Steam, and prevents any condensation within it, which would be more detrimental than an equal condensation in the outer one. The inner cylinder has a bottom and piston as usual: and as it does not reach up quite to the lid of the outer cylinder, the Steam in the interstice has always free access to the upper side of the piston. The lid of the outer cylinder has a hole in its middle; and the piston rod, which is truly cylindrical, moves up and down through that hole, which is kept Steam-tight by a collar of oakum screwed down upon it. At the bottom of the inner cylinder, there are two regulating valves, one of which admits the Steam to pass from the interstice into the inner cylinder below the piston, or shuts it out at pleasure: the other opens or shuts the end of a pipe, which leads to the condenser. The condenser consists of one or more pumps furnished with clacks and buckets (nearly the same as in common pumps) which are wrought by chains fastened to the great working beam of the engine. The pipe, which comes from the cylinder, is joined to the bottom of these pumps, and the whole condenser stands immersed in a cistern of cold water supplied by the engine. The place of this cistern is either within the house or under the floor, between the cylinder and the lever wall; or without the house between that wall and the engine shaft, as conveniency may require. The condenser being exhausted of air by blowing, and both the cylinders being filled with Steam, the regulating valve which admits the Steam into the inner cylinder is shut, and the other regulator which communicates with the condenser is opened, and the Steam rushes into the vacuum of the condenser with violence: but there it comes into contact with the cold sides of the pumps and pipes, and meets a jet of cold water, which was opened at the same time with the exhaustion regulator; these instantly deprive it of its heat, and reduce it to water; and the vacuum remaining perfect, more Steam continues to rush in, and be condensed until the inner cylinder be exhausted. Then the Steam which is above the piston, ceasing to be counteracted by that which was below it, acts upon the piston with its whole elasticity, and forces it to descend to the bottom of the cylinder, and so raises the buckets of the pumps which are hung to the other end of the beam. The exhaustion regulator is now shut, and the Steam one opened again, which, by letting in the Steam, allows the piston to be pulled up by the superior weight of the pump rods; and so the engine is ready for another stroke.

But the nature of Mr. Watt's improvement will be perhaps better understood from the following description of it as referred to a figure.—The cylinder or Steam vessel A, of this engine (fig. 5, pl. 27), is shut at bottom and opened at top as usual; and is included in an outer cylinder or case BB, of wood or metal, covered with materials which transmit heat slowly. This case is at a small distance from the cylinder, and close at both ends. The cover C has a hole in it, through which the piston rod E slides; and near the bottom is another hole F, by which the Steam from the boiler has always free entrance into this case or outer cylinder, and by the interstice GG between the two cylinders has access to the upper side of the piston HH. To the bottom of the inner cylinder A is joined a pipe I, with a cock or valve K, which is opened and shut when necessary, and forms a passage to another vessel L called a *Condenser*, made of thin metal. This vessel is immersed in a cistern M full of cold water, and it is contrived so as to expose a very great surface externally to the water, and internally to the Steam. It is also made air-tight, and has pumps N wrought by the engine, which keep it always exhausted of air and water.

Both the cylinders A and BB being filled with Steam, the passage K is opened from the inner one to the condenser L, into which the Steam violently rushes by its elasticity, because that vessel is exhausted; but as soon as it enters it, coming into contact with the cold matter of the condenser, it is reduced to water, and, the vacuum still remaining, the Steam continues to rush in till the inner cylinder A below the piston is left empty. The Steam which is above the piston, ceasing to be counteracted by that which is below it, acts upon the piston HH, and forces it to descend to the bottom of the cylinder, and so raises the bucket of the pump by means of the lever. The passage K between the inner cylinder and the condenser is then shut, and another passage O is opened, which permits the Steam to pass from the outer cylinder, or from the boiler into the inner cylinder under the piston; and then the superior weight of the bucket and pump rods pulls down the outer end of the lever or great beam, and raises the piston, which is suspended to the inner end of the same beam.

The advantages that accrue from this construction are, first, that the cylinder being surrounded with the Steam from the boiler, it is kept always uniformly as hot as the Steam itself, and is therefore incapable of destroying

ing any part of the Steam, which should fill it, as the common engines do. Secondly, the condenser being kept always as cold as water can be procured, and colder than the point at which it boils in vacuo, the Steam is perfectly condensed, and does not oppose the descent of the piston; which is therefore forced down by the full power of the Steam from the boiler, which is somewhat greater than that of the atmosphere.

In the common fire-engines, when they are loaded to 7 pounds upon the inch, and are of a middle size, the quantity of Steam which is condensed in restoring to the cylinder the heat which it had been deprived of by the former injection of cold water, is about one full of the cylinder, besides what it really required to fill that vessel; so that twice the full of the cylinder is employed to make it raise a column of water equal to about 7 pounds for each square inch of the piston: or, to take it more simply, a cubic foot of Steam raises a cubic foot of water about 8 feet high, besides overcoming the friction of the engine, and the resistance of the water to motion.

In the improved engine, about one full and a fourth of the cylinder is required to fill it, because the Steam is one-fourth more dense than in the common engine. This engine raises a load equal to 12 pounds and a half upon the square inch of the piston; and each cubic foot of Steam of the density of the atmosphere, raises one cubic foot of water 22 feet high.

The working of these engines is more regular and steady than the common ones, and from what has been said, their other advantages seem to be very considerable.

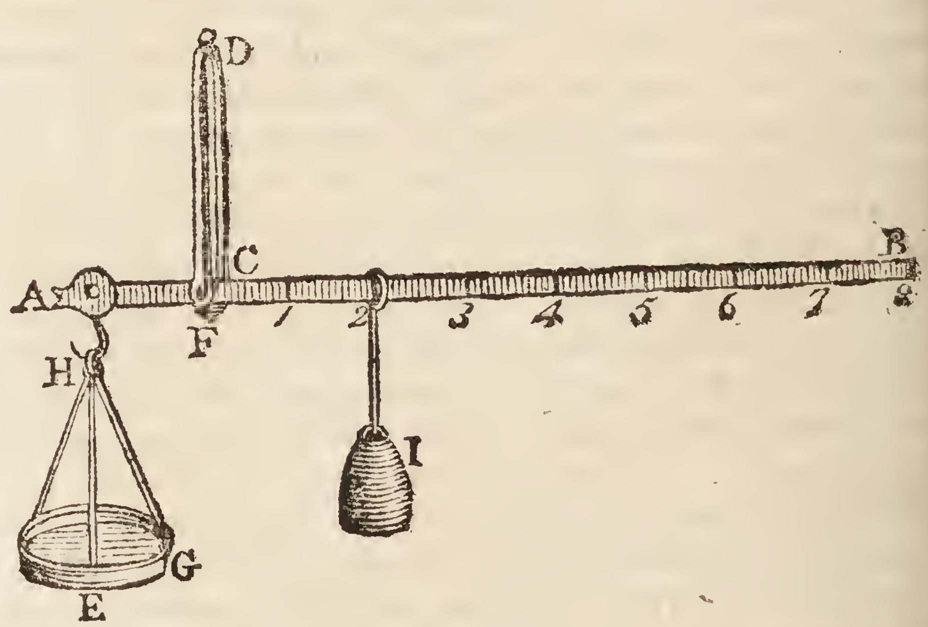
It is said, that the savings amount at least to two thirds of the fuel, which is an important object, especially where coals are dear. The new engines will raise from twenty thousand to twenty-four thousand cubic feet of water, to the height of twenty-four feet by one hundred weight of good pit coal: and Mr. Watt has proposed to produce engines upon the same principles, though somewhat differing in construction, which will require still much less fuel, and be more convenient for the purposes of mining, than any kind of engine yet used. Mr. Watt has also contrived a kind of mill wheel, which turns round by the power of Steam exerted within it.

The improvements above recited were invented by Mr. James Watt, at Glasgow, in Scotland, in 1764: he obtained the king's letters patent for the sole use of his invention in 1768; but meeting with difficulties in the execution of a large machine, and being otherwise employed, he laid aside the undertaking till the year 1774, when, in conjunction with Mr. Boulton near Birmingham, he completed both a reciprocating and rotative or wheel engine. He then applied to parliament for a prolongation of the term of his patent, which was granted by an act passed in 1775. Since that time, Mr. Watt and Mr. Boulton have erected several engines in Staffordshire, Shropshire, and Warwickshire, and a small one at Stratford near London. They have also lately finished another at Hawkesbury colliery near Coventry, which is justly supposed to be the most powerful engine in England. It has a cylinder 58 inches in diameter, which works a pump 14 inches in diameter, 65 fathoms high, and makes regularly twelve strokes, each 8 feet long, in a minute. They have also erected several engines in Cornwall; one of which has a cylinder 30 inches in diameter, that works a pump $6\frac{1}{2}$ inches in diameter in two shafts, by flat rods with great friction, 300 feet distant from each other, 45 fathoms high in each shaft, equal in all to 90 fathoms, and can make 14 strokes, 8 feet long, in a minute, with a consumption of coals less than 20 bushels in 24 hours. The terms they offer to the public are, to take in lieu of all profits, one third part of the annual savings in fuel, which their engine makes when compared with a common engine of the same dimensions in the neighbourhood. The engines are built at the expence of those who use them, and Messrs. Boulton and Watt furnish such drawings, directions, and attendance, as may be necessary to enable a resident engineer to complete the machine. See the appendix to Pryce's Mineralogia, &c, 1778.

It has been said that some useful improvements have been made in the Steam engine by Mr. William Powel, who had lately the direction and care of an engine of this kind at a colliery near Swansea, in Glamorganshire.

It is hardly necessary to add, that Dr. Falck, in 1776, published an account and description of an improved Steam-engine, which, as he says, will, with the same quantity of fuel, and in an equal space of time, raise above double the quantity of water raised by any lever engine of the same dimensions; as he does not seem to have constructed even a working model of his proposed engine. The principal improvement, however, which he suggests, is to use two cylinders; into which the Steam is let alternately to ascend, by a common regulator, which always opens the communication of the Steam to one, whilst it shuts up the opening of the other: the piston rods are kept (by means of a wheel fixed to an arbour) in a continual ascending and descending motion, by which they move the common arbour, to which is affixed another wheel, moving the pump rods, in the same alternate direction as the piston rods, by which continual motion the pumps are kept in constant action.

STEELYARD, or STILYARD, in Mechanics, a kind of balance, called also, *Statera Romana*, or the *Roman Balance*, by means of which the weights of different bodies are discovered by using one single weight only.



The common Steelyard consists of an iron beam AB, in

in which is assumed a point at pleasure, as C, on which is raised a perpendicular CD. On the shorter arm AC is hung a scale or bason to receive the bodies weighed: the moveable weight I is shifted backward and forward on the beam, till it be a counterbalance to 1, 2, 3, 4, &c pounds placed in the scale; and the points are noted where the constant weight I weighs, as 1, 2, 3, 4, &c pounds. From this construction of the Steelyard, the manner of using it is evident. But the instrument is very liable to deceit, and therefore is not much used in ordinary commerce.

Chinese Steelyard. The Chinese carry this Statera about them to weigh their gems, and other things of value. The beam or yard is a small rod of wood or ivory, about a foot in length: upon this are three rules of measure, made of a fine silver-studded work; they all begin from the end of the beam, whence the first is extended 8 inches, the second $6\frac{1}{2}$, the third $8\frac{1}{2}$. The first is the European measure, the other two seem to be Chinese measures. At the other end of the yard hangs a round scale, and at three several distances from this end are fastened so many slender strings, as different points of suspension. The first distance makes $1\frac{3}{5}$ or $\frac{8}{5}$ of an inch, the second $3\frac{1}{5}$ or double the first, and the third $4\frac{4}{5}$ or triple of the first. When they weigh any thing, they hold up the yard by some one of these strings, and hang a sealed weight, of about $1\frac{1}{4}$oz troy weight, upon the respective divisions of the rule, as the thing requires. Grew's Museum, pa. 369.

Spring Steelyard, is a kind of portable balance, serving to weigh any matter, from 1 to about 40 pounds.

It is composed of a brass or iron tube, into which goes a rod, and about that is wound a spring of tempered steel in a spiral form. On this rod are the divisions of pounds and parts of pounds, which are made by successively hanging on, to a hook fastened to the other end, 1, 2, 3, 4, &c, pounds.

Now the spring being fastened by a screw to the bottom of the rod; the greater the weight is that is hung upon the hook, the more will the spring be contracted, and consequently a greater part of the rod will come out of the tube; the proportions or quantities of which greater weights are indicated by the figures appearing against the extremity of the tube.

Steelyard-*Swing*. In the Philos. Trans. (no. 462, sect. 5) is given an account of a Steelyard swing, proposed as a mechanical method for assisting children labouring under deformities, owing to the contraction of the muscles on one side of the body. The crooked person is suspended with cords under his arm, and these are placed at equal distances from the centre of the beam. It is supposed that the gravity of the body will affect the contracted side, so as to put the muscles upon the stretch; and hence by degrees the defect may be remedied.

STEEPLE, an appendage usually raised on the western end of a church to contain the bells.—Steeples are denominated from their form, either *spires*, or *towers*. The first are such as rise continually diminishing like a cone or other pyramid. The latter are mere parallelopipedons, or some other prism, and are covered at top platform-like.—In each kind there is usually a sort of windows, or loop-holes, to let out the sound, and so contrived as to throw it downward.

Masius, in his treatise on bells, treats likewise of Steeples. The most remarkable in the world, it is said, is that at Pisa, which leans so much to one side, that you fear every moment it will fall; yet is in no danger. This odd disposition, he observes, is not owing to a shock of an earthquake, as is generally imagined; but was contrived so at first by the architect; as is evident from the cielings, windows, doors, &c, which are all in the bevel.

STEERAGE, in a ship, that part next below the quarter-deck, before the bulk-head of the great cabin, where the steersman stands in most ships of war. In large ships of war it is used as a hall, through which it is necessary to pass to or from the great cabin. In merchant ships it is mostly the habitation of the lower officers and ship's crew.

Steerage, in Sea-language, is also used to express the effort of the helm: and hence

Steerage-*way* is that degree of progressive motion communicated to a ship, by which she becomes susceptible of the effect of the helm to govern her course.

STEERING, in Navigation, the art of directing the ship's way by the movements of the helm; or of applying its efforts to regulate her course when she advances.

The perfection of Steering consists in a vigilant attention to the motion of the ship's head, so as to check every deviation from the line of her course in the first instant of its motion; and in applying as little of the power of the helm as possible. By this means she will run more uniformly in a straight path, as declining less to the right and left; whereas, if a greater effort of the helm be employed, it will produce a greater declination from the course, and not only increase the difficulty of Steering, but also make a crooked and irregular path through the water.

The helmsman, or steersman, should diligently watch the movements of the head by the land, clouds, moon, or stars; because, although the course is in general regulated by the compass, yet the vibrations of the needle are not so quickly perceived, as the sallies of the ship's head to the right or left, which, if not immediately restrained, will acquire additional velocity in every instant of their motion, and require a more powerful impulse of the helm to reduce them; the application of which will operate to turn her head as far on the contrary side of her course.

The phrases used in Steering a ship, vary according to the relation of the wind to her course. Thus, when the wind is large or fair, the phrases used by the pilot or officer who superintends the Steerage, are *port*, *starboard*, and *steady*: the first of which is intended to direct the ship's course farther to the right; the second to the left; and the last is designed to keep her exactly in the line on which she advances, according to the intended course. The excess of the first and second movement is called *hard-a port*, and *hard-a-starboard*: the former of which gives her the greatest possible inclination to the right, and the latter an equal tendency to the left.—If, on the contrary, the wind be scant or foul, the phrases are *luff*, *thus*, and *no nearer*: the first of which is the order to keep her close to the wind; the second, to retain her

her in her present situation; and the third, to keep her sails full.

STELLA. See STAR.

STENTOROPHONIC *Tube*, a *Speaking Trumpet*, or tube employed to speak to a person at a great distance. It has been so called from Stentor, a person mentioned in the 5th book of the Iliad, who, as Homer tells us, could call out louder than 50 men. The Stentorophonic horn of Alexander the Great is famous; with this it is said he could give orders to his army at the distance of 100 stadia, which is about 12 English miles.

The present speaking trumpet it is said was invented by Sir Samuel Moreland. But Derham, in his Physico-Theology, lib. 4, ch. 3, says, that Kircher found out this instrument 20 years before Moreland, and published it in his Mesurgia; and it is farther said that Gaspar Schottus had seen one at the Jesuits' College at Rome. Also one Conyers, in the Philos. Trans. number 141, gives a description of an instrument of this kind, different from those commonly made. Gravesande, in his Philosophy, disapproves of the usual figures of these instruments; he would have them to be parabolic conoids, with the focus of one of its parabolic sections at the mouth.—Concerning this instrument, see Sturmy's Collegium Curiosum, Pt. 2, Tentam. 8; also Philos. Trans. vol. 6, pa. 3056, vol. 12, pa. 1027, or Abridg. vol. 1, pa. 505.

STEREOGRAPHIC *Projection of the Sphere*, is that in which the eye is supposed to be placed in the surface of the sphere. Or it is the projection of the circles of the sphere on the plane of some one great circle, when the eye, or a luminous point, is placed in the pole of that circle.—For the fundamental principles and chief properties of this kind of projection, see PROJECTION.

STEREOGRAPHY, is the art of drawing the forms of solids upon a plane.

STEVIN, STEVINUS (SIMON), a Flemish mathematician of Bruges, who died in 1633. He was master of mathematics to prince Maurice of Nassau, and inspector of the dykes in Holland. It is said he was the inventor of the sailing chariots, sometimes made use of in Holland. He was a good practical mathematician and mechanist, and was author of several useful works: as, treatises on Arithmetic, Algebra, Geometry, Statics, Optics, Trigonometry, Geography, Astronomy, Fortification, and many others, in the Dutch language, which were translated into Latin, by Snellius, and printed in 2 volumes folio. There are also two editions in the French language, in folio, both printed at Leyden, the one in 1608, and the other in 1634, with curious notes and additions, by Albert Girard.—For a particular account of Stevin's inventions and improvements in Algebra, which were many and ingenious, see our article Algebra, vol. 1, pa. 82 and 83.

STEWART (the Rev. Dr. MATTHEW), late professor of mathematics in the university of Edinburgh, was the son of the reverend Mr. Dugald Stewart, minister of Rothsay in the Isle of Bute, and was born at that place in the year 1717. After having finished his course at the grammar school, being intended by his father for the church, he was sent to the university of Glasgow, and was entered there as a student in 1734. His academical studies were prosecuted with diligence and success; and he was so happy as to be particularly distinguished by the friendship of Dr. Hutcheson, and Dr. Simson the celebrated geometrician, under whom he made great progress in that science.

Mr. Stewart's views made it necessary for him to attend the lectures in the university of Edinburgh in 1741; and that his mathematical studies might suffer no interruption, he was introduced by Dr. Simson to Mr. Maclaurin, who was then teaching with so much success, both the geometry and the philosophy of Newton, and under whom Mr. Stewart made that proficiency which was to be expected from the abilities of such a pupil, directed by those of so great a master. But the modern analysis, even when thus powerfully recommended, was not able to withdraw his attention from the relish of the ancient geometry, which he had imbibed under Dr. Simson. He still kept up a regular correspondence with this gentleman, giving him an account of his progress, and of his discoveries in geometry, which were now both numerous and important, and receiving in return many curious communications with respect to the *Loci Plani*, and the Porisms of Euclid. Mr. Stewart pursued this latter subject in a different, and new direction. In doing so, he was led to the discovery of those curious and interesting propositions, which were published, under the title of *General Theorems*, in 1746. They were given without the demonstrations; but they did not fail to place their discoverer at once among the geometricians of the first rank. They are, for the most part, Porisms, though Mr. Stewart, careful not to anticipate the discoveries of his friend, gave them only the name of Theorems. They are among the most beautiful, as well as most general propositions, known in the whole compass of geometry, and are perhaps only equalled by the remarkable locus to the circle in the second book of Apollonius, or by the celebrated theorem of Mr. Cotes.

Such is the history of the invention of these propositions; and the occasion of the publication of them was as follows. Mr. Stewart, while engaged in them, had entered into the church, and become minister of Roseneath. It was in that retired and romantic situation, that he discovered the greater part of those theorems. In the summer of 1746, the mathematical chair in the university of Edinburgh became vacant, by the death of Mr. Maclaurin. The General Theorems had not yet appeared; Mr. Stewart was known only to his friends; and the eyes of the public were naturally turned on Mr. Stirling, who then resided at Leadhills, and who was well known in the mathematical world. He however declined appearing as a candidate for the vacant chair; and several others were named, among whom was Mr. Stewart. Upon this occasion he printed the *General Theorems*, which gave their author a decided superiority above all the other candidates. He was accordingly elected professor of mathematics in the university of Edinburgh, in September 1747.

The duties of this office gave a turn somewhat different to his mathematical pursuits, and led him to think of the most simple and elegant means of explaining those difficult propositions, which were hitherto only accessible to men deeply versed in the modern analysis. In doing this, he was pursuing the object which, of

of all others, he most ardently wished to attain, viz, the application of geometry to such problems as the algebraic calculus alone had been thought able to resolve. His solution of Kepler's problem was the first specimen of this kind which he gave to the world; and it was perhaps impossible to have produced one more to the credit of the method he followed, or of the abilities with which he applied it. Among the excellent solutions hitherto given of this famous problem, there were none of them at once direct in its method, and simple in its principles. Mr. Stewart was so happy as to attain both these objects. He founds his solution on a general property of curves, which, though very simple, had perhaps never been observed; and by a most ingenious application of that property, he shows how the approximation may be continued to any degree of accuracy, in a series of results which converge with great rapidity.

This solution appeared in the second volume of the Essays of the Philosophical Society of Edinburgh, for the year 1756. In the first volume of the same collection, there are some other propositions of Mr. Stewart's, which are an extension of a curious theorem in the 4th book of Pappus. They have a relation to the subject of Porisms, and one of them forms the 91st of Dr. Simson's Restoration.

It has been already mentioned, that Mr. Stewart had formed the plan of introducing into the higher parts of mixed mathematics, the strict and simple form of ancient demonstration. The prosecution of this plan produced the *Tracts Physical and Mathematical*, which were published in 1761. In the first of these, Mr. Stewart lays down the doctrine of centripetal forces, in a series of propositions, demonstrated (if we admit the quadrature of curves) with the utmost rigour, and requiring no previous knowledge of the mathematics, except the elements of plane Geometry, and of Conic Sections. The good order of these propositions, added to the clearness and simplicity of the demonstrations, renders this Tract perhaps the best elementary treatise of Physical Astronomy that is any where to be found.

In the three remaining Tracts, our author had it in view to determine, by the same rigorous method, the effect of those forces which disturb the motions of a secondary planet. From this he proposed to deduce, not only a theory of the moon, but a determination of the sun's distance from the earth. The former, it is well known, is the most difficult subject to which mathematics have been applied, and the resolution required and merited all the clearness and simplicity which our author possessed in so eminent a degree. It must be regretted therefore, that the decline of Dr. Stewart's health, which began soon after the publication of the Tracts, did not permit him to pursue this investigation.

The other object of the Tracts was, to determine the distance of the sun, from his effect in disturbing the motions of the moon; and his enquiries into the lunar irregularities had furnished him with the means of accomplishing it.

The theory of the composition and resolution of forces enables us to determine what part of the solar force is employed in disturbing the motions of the moon; and therefore, could we measure the instantaneous effect of that force, or the number of feet by which it accelerates or retards the moon's motion in a second, we should be able to determine how many feet the whole force of the sun would make a body, at the distance of the moon, or of the earth, descend in a second of time, and consequently how much the earth is, in every instant, turned out of its rectilineal course. Thus the curvature of the earth's orbit, or, which is the same thing, the radius of that orbit, that is, the distance of the sun from the earth, would be determined. But the fact is, that the instantaneous effects of the sun's disturbing force are too minute to be measured; and that it is only the effect of that force, continued for an entire revolution, or some considerable portion of a revolution, which astronomers are able to observe.

There is yet a greater difficulty which embarrasses the solution of this problem. For as it is only by the difference of the forces exerted by the sun on the earth and on the moon, that the motions of the latter are disturbed, the farther off the sun is supposed, the less will be the force by which he disturbs the moon's motions; yet that force will not diminish beyond a fixed limit, and a certain disturbance would obtain, even if the distance of the sun were infinite. Now the sun is actually placed at so great a distance, that all the disturbances, which he produces on the lunar motions, are very near to this limit, and therefore a small mistake in estimating their quantity, or in reasoning about them, may give the distance of the sun infinite, or even impossible. But all this did not deter Dr. Stewart from undertaking the solution of the problem, with no other assistance than that which geometry could afford. Indeed the idea of such a problem had first occurred to Mr. Machin, who, in his book on the laws of the moon's motion, has just mentioned it, and given the result of a rude calculation (the method of which he does not explain), which assigns 8″ for the parallax of the sun. He made use of the motion of the nodes; but Dr. Stewart considered the motion of the apogee, or of the longer axis of the moon's orbit, as the irregularity best adapted to his purpose. It is well known that the orbit of the moon is not immoveable; but that, in consequence of the disturbing force of the sun, the longer axis of that orbit has an angular motion, by which it goes back about 3 degrees in every lunation, and completes an entire revolution in 9 years nearly. This motion, though very remarkable and easily determined, has the same fault, in respect of the present problem, that was ascribed to the other irregularities of the moon: for a very small part of it only depends on the parallax of the sun; and of this Dr. Stewart seems not to have been perfectly aware.

The propositions however which defined the relation between the sun's distance and the mean motion of the apogee, were published among the Tracts, in 1761. The transit of Venus happened in that same year: the astronomers returned, who had viewed that curious phenomenon, from the most distant stations; and no very satisfactory result was obtained from a comparison of their observations. Dr. Stewart then resolved to apply the principles he had already laid down; and, in 1763, he published his essay on the Sun's Distance, where the computation being actually made, the parallax of the sun was found to be no more than 6″·9,

and

and consequently his distance almost 29875 semidiameters of the earth, or nearly 119 millions of miles.

A determination of the sun's distance, that so far exceeded all former estimations of it, was received with surprise, and the reasoning on which it was founded was likely to undergo a severe examination. But, even among astronomers, it was not every one who could judge in a matter of such difficult discussion. Accordingly, it was not till about 5 years after the publication of the sun's distance, that there appeared a pamphlet, under the title of *Four Propositions*, intended to point out certain errors in Dr. Stewart's investigation, which had given a result much greater than the truth. From his desire of simplifying, and of employing only the geometrical method of reasoning, he was reduced to the necessity of rejecting quantities, which were considerable enough to have a great effect on the last result. An error was thus introduced, which, had it not been for certain compensations, would have become immediately obvious, by giving the sun's distance near three times as great as that which has been mentioned.

The author of the pamphlet, referred to above, was the first who remarked the dangerous nature of these simplifications, and who attempted to estimate the error to which they had given rise. This author remarked what produced the compensation above mentioned, viz, the immense variation of the sun's distance, which corresponds to a very small variation of the motion of the moon's apogee. And it is but justice to acknowledge that, besides being just in the points already mentioned, they are very ingenious, and written with much modesty and good temper. The author, who at first concealed his name, but has now consented to its being made public, was Mr. Dawson, a surgeon at Sudbury in Yorkshire, and one of the most ingenious mathematicians and philosophers this country now possesses.

A second attack was soon after this made on the Sun's Distance, by Mr. Landen; but by no means with the same good temper which has been remarked in the former. He fancied to himself errors in Dr. Stewart's investigation, which have no existence; he exaggerated those that were real, and seemed to triumph in the discovery of them with unbecoming exultation. If there are any subjects on which men may be expected to reason dispassionately, they are certainly the properties of number and extension; and whatever pretexts moralists or divines may have for abusing one another, mathematicians can lay claim to no such indulgence. The asperity of Mr. Landen's animadversions ought not therefore to pass uncensured, though it be united with sound reasoning and accurate discussion. The error into which Dr. Stewart had fallen, though first taken notice of by Mr. Dawson, whose pamphlet was sent by me to Mr. Landen as soon as it was printed (for I had the care of the edition of it) yet this gentleman extended his remarks upon it to greater exactness. But Mr. Landen, in the zeal of correction, brings many other charges against Dr. Stewart, the greater part of which seem to have no good foundation. Such are his objections to the second part of the investigation, where Dr. Stewart finds the relation between the disturbing force of the sun, and the motion of the apses of the lunar orbit. For this part, instead of being liable to objection, is deserving of the greatest praise, since it resolves, by geometry alone, a problem which had eluded the efforts of some of the ablest mathematicians, even when they availed themselves of the utmost resources of the integral calculus. Sir Isaac Newton, though he assumed the disturbing force very near the truth, computed the motion of the apses from thence only at one half of what it really amounts to; so that, had he been required, like Dr. Stewart, to invert the problem, he would have committed an error, not merely of a few thousandth parts, as the latter is alleged to have done, but would have brought out a result double of the truth. (*Princip. Math. lib.* 3, *prop.* 3.) Machin and Callendrini, when commenting on this part of the Principia, found a like inconsistency between their theory and observation. Three other celebrated mathematicians, Clairaut, D'Alembert, and Euler, severally experienced the same difficulties, and were led into an error of the same magnitude. It is true, that, on resuming their computations, they found that they had not carried their approximations to a sufficient length, which when they had at last accomplished, their results agreed exactly with observation. Mr. Walmsley and Dr. Stewart were, I think, the first mathematicians who, employing in the solution of this difficult problem, the one the algebraic calculus, and the other the geometrical method, were led immediately to the truth; a circumstance so much for the honour of both, that it ought not to be forgotten. It was the business of an impartial critic, while he examined our author's reasonings, to have remarked and to have weighed these considerations.

The *Sun's Distance* was the last work which Dr. Stewart published; and though he lived to see the animadversions made on it, that have been taken notice of above, he declined entering into any controversy. His disposition was far from polemical; and he knew the value of that quiet, which a literary man should rarely suffer his antagonists to interrupt. He used to say, that the decision of the point in question was now before the public; that if his investigation was right, it would never be overturned, and that if it was wrong, it ought not to be defended.

A few months before he published the Essay just mentioned, he gave to the world another work, entitled, *Propositiones More Veterum Demonstratæ*. It consists of a series of geometrical theorems, mostly new; investigated, first by an analysis, and afterwards synthetically demonstrated by the inversion of the same analysis. This method made an important part in the analysis of the ancient geometricians; but few examples of it have been preserved in their writings, and those in the *Propositiones Geometricæ* are therefore the more valuable.

Doctor Stewart's constant use of the geometrical analysis had put him in possession of many valuable propositions, which did not enter into the plan of any of the works that have been enumerated. Of these, not a few have found a place in the writings of Dr. Simson, where they will for ever remain, to mark the friendship of these two mathematicians, and to evince the esteem which Dr. Simson entertained for the abilities of his pupil. Many of these are in the work upon the Porisms, and others in the Conic Sections, viz, marked with the letter *x*; also a theorem in the edition of Euclid's Data.

Soon

Soon after the publication of the *Sun's Distance*, Dr. Stewart's health began to decline, and the duties of his office became burdensome to him. In the year 1772, he retired to the country, where he afterwards spent the greater part of his life, and never resumed his labours in the university. He was however so fortunate as to have a son to whom, though very young, he could commit the care of them with the greatest confidence. Mr. Dugald Stewart, having begun to give lectures for his father from the period above mentioned, was elected joint professor with him in 1775, and gave an early specimen of those abilities, which have not been confined to a single science.

After mathematical studies (on account of the bad state of health into which Dr. Stewart was falling) had ceased to be his business, they continued to be his amusement. The analogy between the circle and hyperbola had been an early object of his admiration. The extensive views which that analogy is continually opening; the alternate appearance and disappearance of resemblance in the midst of so much dissimilitude, make it an object that astonishes the experienced, as well as the young geometrician. To the consideration of this analogy therefore the mind of Dr. Stewart very naturally returned, when disengaged from other speculations. His usual success still attended his investigations; and he has left among his papers some curious approximations to the areas, both of the circle and hyperbola. For some years toward the end of his life, his health scarcely allowed him to prosecute study even as an amusement. He died the 23d of January 1785, at 68 years of age.

The habits of study, in a man of original genius, are objects of curiosity, and deserve to be remembered. Concerning those of Dr. Stewart, his writings have made it unnecessary to remark, that from his youth he had been accustomed to the most intense and continued application. In consequence of this application, added to the natural vigour of his mind, he retained the memory of his discoveries in a manner that will hardly be believed. He seldom wrote down any of his investigations, till it became necessary to do so for the purpose of publication. When he discovered any proposition, he would set down the enunciation with great accuracy, and on the same piece of paper would construct very neatly the figure to which it referred. To these he trusted for recalling to his mind, at any future period, the demonstration, or the analysis, however complicated it might be. Experience had taught him that he might place this confidence in himself without any danger of disappointment; and for this singular power, he was probably more indebted to the activity of his invention, than to the mere tenaciousness of his memory.

Though Dr. Stewart was extremely studious, he read but few books, and thus verified the observation of D'Alembert, that, of all the men of letters, mathematicians read least of the writings of one another. Our author's own investigations occupied him sufficiently; and indeed the world would have had reason to regret the misapplication of his talents, had he employed, in the mere acquisition of knowledge, that time which he could dedicate to works of invention.

It was Dr. Stewart's custom to spend the summer at a delightful retreat in Ayrshire, where, after the academical labours of the winter were ended, he found the leisure necessary for the prosecution of his researches. In his way thither, he often made a visit to Dr. Simson of Glasgow, with whom he had lived from his youth in the most cordial and uninterrupted friendship. It was pleasing to observe, in these two excellent mathematicians, the most perfect esteem and affection for each other, and the most entire absence of jealousy, though no two men ever trode more nearly in the same path. The similitude of their pursuits served only to endear them to each other, as it will ever do with men superior to envy. Their sentiments and views of the science they cultivated, were nearly the same: they were both profound geometricians; they equally admired the ancient mathematicians, and were equally versed in their methods of investigation; and they were both apprehensive that the beauty of their favourite science would be forgotten, for the less elegant methods of algebraic computation. This innovation they endeavoured to oppose; the one, by reviving those books of the ancient geometry which were lost; the other, by extending that geometry to the most difficult enquiries of the moderns. Dr. Stewart, in particular, had remarked the intricacies, in which many of the greatest of the modern mathematicians had involved themselves in the application of the calculus, which a little attention to the ancient geometry would certainly have enabled them to avoid. He had observed too the elegant synthetical demonstrations that, on many occasions, may be given of the most difficult propositions, investigated by the inverse method of fluxions. These circumstances had perhaps made a stronger impression than they ought, on a mind already filled with admiration of the ancient geometry, and produced too unfavourable an opinion of the modern analysis. But if it be confessed that Dr. Stewart rated, in any respect too high, the merit of the former of these sciences, this may well be excused in the man whom it had conducted to the discovery of the *General Theorems*, to the *solution of Kepler's Problem*, and to an *accurate* determination of the *Sun's disturbing force*. His great modesty made him ascribe to the method he used, that success which he owed to his own abilities.

The foregoing account of Dr. Stewart and his writings, is chiefly extracted from the learned history of them, by Mr. Playfair, in the 1st volume of the Edinburgh Philosophical Transactions, pa. 57, &c.

STIFELS, Stifelius (Michael), a Protestant minister, and very skilful mathematician, was born at Eslingen a town in Germany; and died at Jena in Thuringia, in the year 1567, at 58 years of age according to Vossius, but some others say 80. Stifels was one of the best mathematicians of his time. He published, in the German language, a treatise on Algebra, and another on the Calendar or Ecclesiastical computation. But his chief work, is the *Arithmetica Integra*, a complete and excellent treatise, in Latin, on Arithmetic and Algebra, printed in 4to at Norimberg 1544. In this work there are a number of ingenious inventions, both in common arithmetic and in algebra; of which, those relating to the latter are amply explained under the article *Algebra* in this dictionary, vol. 1, pa. 77 &c; to which may be added some particulars

ticulars concerning the arithmetic, from my volume of *Tracts* printed in 1786, pa. 68. In this original work are contained many curious things, some of which have mistakenly been ascribed to a much later date. He here treats pretty fully and ably, of progressional and figurate numbers, and in particular of the triangular table, for constructing both them and the coefficients of the terms of all powers of a binomial; which has been so often used since his time for these and other purposes, and which more than a century after was, by Pascal, otherwise called the Arithmetical Triangle, and who only mentioned some additional properties of the table. Stifels shews, that the horizontal lines of the table furnish the coefficients of the terms of the corresponding powers of a binomial; and teaches how to make use of them in the extraction of roots of all powers whatever. Cardan seems to ascribe the invention of that table to Stifelius; but I apprehend that is only to be understood of its application to the extraction of roots.

It is remarkable too, how our author, at p. 35 &c of the same book, treats of the nature and use of logarithms; not under that name indeed, but under the idea of a series of arithmeticals, adapted to a series of geometricals. He there explains all their uses; such as, that the addition of them answers to the multiplication of their geometricals; subtraction to division; multiplication of exponents, to involution; and dividing of exponents to evolution. He also exemplifies the use of them in cases of the Rule-of-three, and in finding mean proportionals between given terms, and such like, exactly as is done in logarithms. So that he seems to have been in the full possession of the idea of logarithms, and wanted only the necessity of troublesome calculations to induce him to make a table of such numbers.

Stifels was a zealous, though weak disciple of Luther. He took it into his head to become a prophet, and he predicted that the end of the world would happen on a certain day in the year 1553, by which he terrified many people. When the proposed day arrived, he repaired early, with multitudes of his followers, to a particular place in the open air, spending the whole day in the most fervent prayers and praises, in vain looking for the coming of the Lord, and the universal conflagration of the elements, &c.

STILE. See STYLE.

STILYARD. See STEELYARD.

STOFLER (JOHN), a German mathematician, was born at Justingen in Suabia, in 1452, and died in 1531, at 79 years of age. He taught mathematics at Tubinga, where he acquired a great reputation, which however he in a great measure lost again, by intermeddling with the prediction of future events. He announced a great deluge, which he said would happen in the year 1524, a prediction with which he terrified all Germany, where many persons prepared vessels proper to escape with from the floods. But happily the prediction failing, it enraged the astrologer, though it served to convince him of the vanity of his prognostications.—He was author of several works in mathematics, and astrology, full of foolish and chimerical ideas; such as,

1. Elucidatio Fabric. Ususque Astrolabii; fol. 1513.
2. Procli Sphæram Comment. fol. 154.
3. Cosmographicæ aliquot Descriptiones; 4to, 1537.

STONE, (EDMUND), a good Scotch mathematician, who was author of several ingenious works. I know not the particular place or date of his birth, but it was probably in the shire of Argyle, and about the beginning of the present century, or conclusion of the last. Nor have we any memoirs of his life, except a letter from the Chevalier de Ramsay, author of the Travels of Cyrus, in a letter to father Castel, a Jesuit at Paris, and published in the Memoires de Trevoux, p. 109, as follows: "True genius overcomes all the disadvantages of birth, fortune, and education; of which Mr. Stone is a rare example. Born a son of a gardener of the duke of Argyle, he arrived at 8 years of age before he learnt to read.—By chance a servant having taught young Stone the letters of the alphabet, there needed nothing more to discover and expand his genius. He applied himself to study, and he arrived at the knowledge of the most sublime geometry and analysis, without a master, without a conductor, without any other guide but pure genius."

"At 18 years of age he had made these considerable advances without being known, and without knowing himself the prodigies of his acquisitions. The duke of Argyle, who joined to his military talents, a general knowledge of every science that adorns the mind of a man of his rank, walking one day in his garden, saw lying on the grass a Latin copy of Sir Isaac Newton's celebrated Principia. He called some one to him to take and carry it back to his library. Our young gardener told him that the book belonged to him. *To you?* replied the Duke. *Do you understand geometry, Latin, Newton?* I know a little of them, replied the young man with an air of simplicity arising from a profound ignorance of his own knowledge and talents. The Duke was surprised; and having a taste for the sciences, he entered into conversation with the young mathematician: he asked him several questions, and was astonished at the force, the accuracy, and the candour of his answers. *But how,* said the Duke, *came you by the knowledge of all these things?* Stone replied, *A servant taught me, ten years since, to read: does one need to know any thing more than the 24 letters in order to learn every thing else that one wishes?* The Duke's curiosity redoubled—he sat down upon a bank, and requested a detail of all his proceedings in becoming so learned."

"*I first learned to read,* said Stone: *the masons were then at work upon your house: I went near them one day, and I saw that the architect used a rule, compasses, and that he made calculations. I enquired what might be the meaning of and use of these things; and I was informed that there was a science called Arithmetic; I purchased a book of arithmetic, and I learned it.—I was told there was another science called Geometry: I bought the books, and I learnt geometry. By reading I found that there were good books in these two sciences in Latin: I bought a dictionary, and I learned Latin. I understood also that there were good books of the same kind in French: I bought a dictionary, and I learned French. And this, my lord, is what I have done: it seems to me that we may learn every thing when we know the 24 letters of the alphabet.*

This account charmed the Duke. He drew this wonderful genius out of his obscurity; and he provided him with an employment which left him plenty of

of time to apply himſelf to the ſciences. He diſcovered in him alſo the ſame genius for muſic, for painting, for architecture, for all the ſciences which depend on calculations and proportions."

"I have ſeen Mr. Stone. He is a man of great ſimplicity. He is at preſent ſenſible of his own knowledge: but he is not puffed up with it. He is poſſeſſed with a pure and diſtintereſted love for the mathematics; though he is not ſolicitous to paſs for a mathematician; vanity having no part in the great labour he ſuſtains to excel in that ſcience. He deſpiſes fortune alſo: and he has ſolicited me twenty times to requeſt the duke to give him leſs employment, which may not be worth the half of that he now has, in order to be more retired, and leſs taken off from his favourite ſtudies. He diſcovers ſometimes, by methods of his own, truths which others have diſcovered before him. He is charmed to find on theſe occaſions that he is not a firſt inventor, and that others have made a greater progreſs than he thought. Far from being a plagiary, he attributes ingenious ſolutions, which he gives to certain problems, to the hints he has found in others, although the connection is but very diſtant," &c.

Mr. Stone was author and tranſlator of ſeveral uſeful works; viz.

1. A New Mathematical Dictionary, in 1 vol. 8vo, firſt printed in 1726.

2. Fluxions, in 1 vol. 8vo, 1730. The Direct Method is a tranſlation from the French, of Hoſpital's Analyſe des Infiniments Petits; and the Inverſe Method was ſupplied by Stone himſelf.

3. The Elements of Euclid, in 2 vols. 8vo, 1731. A neat and uſeful edition of thoſe Elements, with an account of the life and writings of Euclid, and a defence of his elements againſt modern objectors.

Beſide other ſmaller works.

Stone was a fellow of the Royal Society, and had inſerted in the Philoſ. Tranſactions (vol. 41, pa. 218) an "Account of two ſpecies of lines of the 3d order, not mentioned by Sir Iſaac Newton, or Mr. Stirling."

STRABO, a celebrated Greek geographer, philoſopher, and hiſtorian, was born at Amaſia, and was deſcended from a family ſettled at Gnoſſus in Crete. He was the diſciple of Xenarchus, a Peripatetic philoſopher, but at length attached himſelf to the Stoics. He contracted a ſtrict friendſhip with Cornelius Gallus, governor of Egypt; and travelled into ſeveral countries, to obſerve the ſituation of places, and the cuſtoms of nations.

Strabo flouriſhed under Auguſtus; and died under Tiberius about the year 25, in a very advanced age.—He compoſed ſeveral works; all of which are loſt, except his *Geography*, in 17 books; which are juſtly eſteemed very precious remains of antiquity. The firſt two books are employed in ſhowing, that the ſtudy of geography is not only worthy of a philoſopher, but even neceſſary to him; the 3d deſcribes Spain; the 4th, Gaul and the Britannic iſles; the 5th and 6th, Italy and the adjacent iſles; the 7th, which is imperfect at the end, Germany, the countries of the Getæ and Illyrii, Taurica, Cherſoneſus, and Epirus; the 8th, 9th, and 10th, Greece with the neighbouring iſles; the four following, Aſia within Mount Taurus; the 15th and 16th, Aſia without Taurus, India, Perſia, Syria, Arabia; and the 17th, Egypt, Ethiopia, Carthage, and other parts of Africa.

Strabo's work was publiſhed with a Latin verſion by Xylander, and notes by Iſaac Caſaubon, at Paris 1620, in folio; but the beſt edition is that of Amſterdam in 1707, in 2 volumes folio, by the learned Theodore Janſon of Almelooveen, with the entire notes of Xylander, Caſaubon, Meurſius, Cluver, Holſten, Salmaſius, Bochart, Ez. Spanheim, Cellar, and others. To this edition is ſubjoined the *Chreſtomathiæ*, or Epitome of Strabo; which, according to Mr. Dodſwell, who has written a very elaborate and learned diſſertation about it, was made by ſome unknown perſon, between the years of Chriſt 676 and 996. It has been found of ſome uſe, not only in helping to correct the original, but in ſupplying in ſome meaſure the defect in the 7th book. Mr. Dodſwell's diſſertation is prefixed to this edition.

STRAIT, or STRAIGHT, or STREIGHT, in Hydrography, is a narrow channel or arm of the ſea, ſhut up between lands on either ſide, and uſually affording a paſſage out of one great ſea into another. As the Straits of Magellan, of Le Maire, of Gibraltar, &c.

STRAIT is alſo ſometimes uſed, in Geography, for an iſthmus, or neck of land between two ſeas, preventing their communication.

STRENGTH, *vis*, force, power.

Some authors make the Strength of animals, of the ſame kind, to depend on the quantity of blood; but moſt on the ſize of the bones, joints, and muſcles; though we find by daily experience, that the animal ſpirits contribute greatly to Strength at different times.

Emerſon has moſt particularly treated of the Strength of bodies depending on their dimenſions and weight. In the General Scholium after his propoſitions on this ſubject, he adds; If a certain beam of timber be able to ſupport a given weight; another beam, of the ſame timber, ſimilar to the former, may be taken ſo great, as to be able but juſt to bear its own weight: while any larger beam cannot ſupport itſelf, but muſt break by its own weight; but any leſs beam will bear ſomething more. For the Strength being as the cube of the depth; and the ſtreſs, being as the length and quantity of matter, is as the 4th power of the depth; it is plain therefore, that the ſtreſs increaſes in a greater ratio than the Strength. Whence it follows, that a beam may be taken ſo large, that the ſtreſs may far exceed the Strength: and that, of all ſimilar beams, there is but one that will juſt ſupport itſelf, and nothing more. And the like holds true in all machines, and in all animal bodies. And hence there is a certain limit, in regard to magnitude, not only in all machines and artificial ſtructures, but alſo in natural ones, which neither art nor nature can go beyond; ſuppoſing them made of the ſame matter, and in the ſame proportion of parts.

Hence it is impoſſible that mechanic engines can be increaſed to any magnitude at pleaſure. For when they arrive at a particular ſize, their ſeveral parts will break and fall aſunder by their weight. Neither can any buildings of vaſt magnitudes be made to ſtand, but muſt fall to pieces by their great weight, and go to ruin.

It is likewiſe impoſſible for nature to produce animals of any vaſt ſize at pleaſure: except ſome ſort of matter can be found, to make the bones of, which may be ſo much harder and ſtronger than any hitherto known: or elſe that the proportion of the parts be ſo much altered, and the bones and muſcles made thicker in proportion; which will make the animal diſtorted, and of a monſtrous figure, and not capable of performing any proper actions. And being made ſimilar and of common matter, they will not be able to ſtand or move; but, being burthened with their own weight, muſt fall down. Thus, it is impoſſible that there can be any animal ſo large as to carry a caſtle upon his back; or any man ſo ſtrong as to remove a mountain, or pull up a large oak by the roots: nature will not admit of theſe things; and it is impoſſible that there can be animals of any ſort beyond a determinate ſize.

Fiſh may indeed be produced to a larger ſize than land animals; becauſe their weight is ſupported by the water. But yet even theſe cannot be increaſed to immenſity, becauſe the internal parts will preſs upon one another by their weight, and deſtroy their fabric.

On the contrary, when the ſize of animals is diminiſhed, their Strength is not diminiſhed in the ſame proportion as the weight. For which reaſon a ſmall animal will carry far more than a weight equal to its own, whilſt a great one cannot carry ſo much as its weight. And hence it is that ſmall animals are more active, will run faſter, jump farther, or perform any motion quicker, for their weight, than large animals: for the leſs the animal, the greater the proportion of the Strength to the ſtreſs. And nature ſeems to know no bounds as to the ſmallneſs of animals, at leaſt in regard to their weight.

Neither can any two unequal and ſimilar machines reſiſt any violence alike, or in the ſame proportion; but the greater will be more hurt than the leſs. And the ſame is true of animals; for large animals by falling break their bones, while leſſer ones, falling higher, receive no damage. Thus a cat may fall two or three yards high, and be no worſe, and an ant from the top of a tower.

It is likewiſe impoſſible in the nature of things, that there can be any trees of immenſe ſize; if there were any ſuch, their limbs, boughs, and branches, muſt break off and fall down by their own weight. Thus it is impoſſible there can be an oak a quarter of a mile high; ſuch a tree cannot grow or ſtand, but its limbs will drop off by their weight. And hence alſo ſmaller plants can better ſuſtain themſelves than large ones.

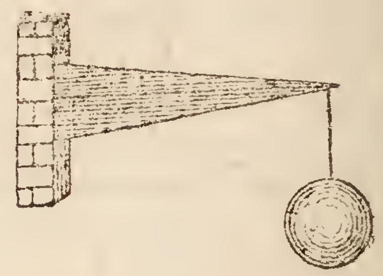
As to the due proportion of Strength in ſeveral bodies, according to their particular poſitions, and the weights they are to bear; he farther obſerves that, If a piece of timber is to be pierced with a mortiſe-hole, the beam will be ſtronger when it is taken out of the middle, than when taken out of either ſide. And in a beam ſupported at both ends, it is ſtronger when the hole is made in the upper ſide than when made in the under, provided a piece of wood is driven hard in to fill up the hole.

If a piece is to be ſpliced upon the end of a beam, to be ſupported at both ends; it will be the ſtronger when ſpliced on the under ſide of a beam: but if the piece is ſupported only at one end, to bear a weight on the other; it is ſtronger when ſpliced on the upper ſide.

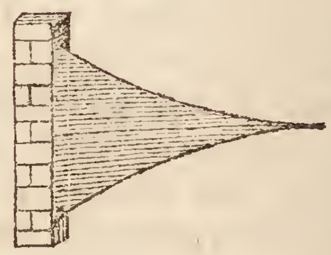
When a ſmall lever, &c, is nailed to a body, to move it or ſuſpend it by; the ſtrain is greater upon the nail neareſt the hand, or point where the power is applied.

If a beam be ſupported at both ends; and the two ends reach over the props, and be fixed down immoveable; it will bear twice as much weight, as when the ends only lie looſe or free upon the ſupporters.

When a ſlender cylinder is to be ſupported by two pieces; the diſtance of the pins ought to be nearly $\frac{3}{5}$ of the length of the cylinder, and the pins equidiſtant from its ends; and then the cylinder will endure the leaſt bending or ſtrain by its weight.



A beam fixed at one end, and bearing a weight at the other; if it be cut in the form of a wedge, and placed with its parallel ſides parallel to the horizon; it will be equally ſtrong every where; and no ſooner break in one place than another.



When a beam has all its ſides cut in form of a concave parabola, having the vertex at the end, and its abſciſs perpendicular to the axis of the ſolid, and the baſe a ſquare, or a circle, or any regular polygon; ſuch a beam fixed horizontally, at one end, is equally ſtrong throughout for ſupporting its own weight.

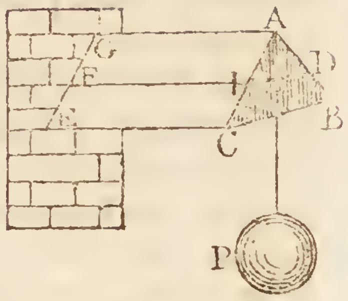
Alſo when a wall faces the wind, and if the vertical ſection of it be a right-angled triangle; or if the fore part next the wind &c be perpendicular to the horizon, and the back part a ſloping plane; ſuch a wall will be equally ſtrong in all its parts to reſiſt the wind, if the parts of the wall cohere ſtrongly together; but when it is built of looſe materials, it is better to be convex on the back part in form of a parabola.

When a wall is to ſupport a bank of earth or any fluid body, it ought to be built concave in form of a ſemicubical parabola, whoſe vertex is at the top of the wall, provided the parts of the wall adhere firmly together. But if the parts be looſe, then a right line or ſloping plane ought to be its figure. Such walls will be equally ſtrong throughout

All ſpires of churches in the form of cones or pyramids, are equally ſtrong in all parts to reſiſt the wind. But when the parts do not cohere together, then they ought to be parabolic conoids, to be equally ſtrong throughout.

Likewiſe if there be a pillar erected in form of the logarithmic curve, the aſymptote being the axis; it cannot be cruſhed to pieces in one part ſooner than in another, by its own weight. And if ſuch a pillar be turned upſide down, and ſuſpended by the thick end, it will not be more liable to ſeparate in one part than another, by its own weight.

Moreover,



Moreover, if AE be a beam in form of a triangular priſm; and if $AD = \frac{1}{9}AB$, and $AI = \frac{1}{9}AC$, and the edge or ſmall ſimilar priſm ADIF be cut away parallel to the baſe; the remaining beam DIBEF will bear a greater weight P, than the whole ABCEG, or the part will be ſtronger than the whole; which is a paradox in Mechanics.

As to the Strength of ſeveral ſorts of wood, drawn from experiments, he ſays, On a medium, a piece of good oak, an inch ſquare, and a yard long, ſupported at both ends, will bear in the middle, for a very ſhort time, about 330lb averdupois, but will break with more than that weight. But ſuch a piece of wood ſhould not, in practice, be truſted for any length of time, with more than a third or a fourth part of that weight. And the proportion of the Strength of ſeveral ſorts of wood, he found to be as follows:

Box, oak, plumbtree, yew - - - - - - - - - -	11
Aſh, elm - - - - - - - - - - - - - - - - -	$8\frac{1}{2}$
Thorn, walnut - - - - - - - - - - - - - - -	$7\frac{1}{2}$
Apple tree, elder, red fir, holly, plane - -	7
Beech, cherry, hazle - - - - - - - - - - -	$6\frac{2}{3}$
Alder, aſp, Birch, white-fir, willow - - -	6
Iron - - - - - - - - - - - - - - - - - - -	107
Braſs - - - - - - - - - - - - - - - - - - -	50
Bone - - - - - - - - - - - - - - - - - - -	22
Lead - - - - - - - - - - - - - - - - - - -	$6\frac{1}{2}$
Fine free ſtone - - - - - - - - - - - - - -	1

As to the Strength of bodies in direction of the fibres, he obſerves, A cylindric rod of good clean fir, of an inch circumference, drawn in length, will bear at extremity 400lb; and a ſpear of fir 2 inches diameter, will bear about 7 ton.—A rod of good iron, of an inch circumference, will bear near 3 ton weight. And a good hempen rope of an inch circumference, will bear 1000lb. at extremity.

All this ſuppoſes theſe bodies to be found and good throughout; but none of them ſhould be put to bear more than a third or a fourth part of that weight, eſpecially for any length of time. From what has been ſaid; if a ſpear of fir, or a rope, or a ſpear of iron, of d inches diameter, were to lift $\frac{1}{4}$ the extreme weight; then

The fir would bear $8\frac{4}{5}dd$ hundred weight.
The rope would bear $22dd$ hundred weight.
The iron would bear $6\frac{3}{4}dd$ ton weight.

As to Animals; Men may apply their Strength ſeveral ways, in working a machine. A man of ordinary Strength turning a roller by the handle, can act for a whole day againſt a reſiſtance equal to 30lb. weight; and if he works 10 hours a day, he will raiſe a weight of 30lb. through $3\frac{1}{2}$ feet in a ſecond of time; or if the weight be greater, he will raiſe it ſo much leſs in proportion. But a man may act, for a ſmall time, againſt a reſiſtance of 50lb. or more.

If two men work at a windlaſs, or roller, they can more eaſily draw up 70lb, than one man can 30lb, provided the elbow of one of the handles be at right angles to that of the other. And with a fly, or heavy wheel, applied to it, a man may do $\frac{1}{7}$ part more work; and for a little while he can act with a force, or overcome a continual reſiſtance, of 80l; and work a whole day when the reſiſtance is but 40lb.

Men uſed to bear loads, ſuch as porters, will carry, ſome 150lb, others 200 or 250lb. according to their Strength.

A man can draw but about 70 or 80lb. horizontally; for he can but apply about half his weight.

If the weight of a man be 140lb, he can act with no greater a force in thruſting horizontally, at the height of his ſhoulders, than 27lb.

As to Horſes: A horſe is, generally ſpeaking, as ſtrong as 5 men. A horſe will carry 240 or 270lb. A horſe draws to greateſt advantage, when the line of direction is a little elevated above the horizon, and the power acts againſt his breaſt: and he can draw 200lb. for 8 hours a day, at $2\frac{1}{2}$ miles an hour. If he draw 240lb, he can work but 6 hours, and not go quite ſo faſt. And in both caſes, if he carries ſome weight, he will draw the better for it. And this is the weight a horſe is ſuppoſed to be able to draw over a pulley out of a well. But in a cart, a horſe may draw 1000lb, or even double that weight, or a ton weight, or more.

As the moſt force a horſe can exert, is when he draws a little above the horizontal poſition: ſo the worſt way of applying the ſtrength of a horſe, is to make him carry or draw uphill: And three men in a ſteep hill, carrying each 100lb, will climb up faſter than a horſe with 300l. Alſo, though a horſe may draw in a round walk of 18 feet diameter; yet ſuch a walk ſhould not be leſs than 25 or 30 feet diameter. Emerſon's Mechan. pa. 111 and 177.

STRIKE, or STRYKE, a meaſure, containing 4 buſhels, or half a quarter.

STRIKING-*wheel*, in a clock, the ſame as that by ſome called the *pin-wheel*, becauſe of the pins which are placed on the round or rim, the number of which is the quotient of the pinion divided by the pinion of the detent-wheel. In ſixteen-day clocks, the firſt or great wheel is uſually the pin-wheel; but in ſuch as go 8 days, the ſecond wheel is the pin-wheel, or ſtriking-wheel.

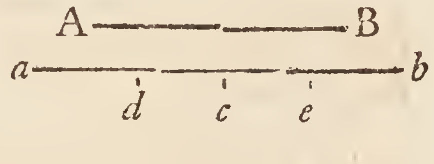
STRING, in Muſic. See CHORD.

If two Strings or chords of a muſical inſtrument only differ in length; their tones, or the number of vibrations they make in the ſame time, are in the inverſe ratio of their lengths. If they differ only in thickneſs, their tones are in the inverſe ratio of their diameters.

As to the tenſion of Strings, to meaſure it regularly, they muſt be conceived ſtretched or drawn by weights; and then, cæteris paribus, the tones of two Strings are in a direct ratio of the ſquare roots of the weights that ſtretch them; that is, ex. gr. the tone of a String ſtretched by a weight 4, is an octave above the tone of a String ſtretched by the weight 1.

It is an obſervation of very old ſtanding, that if a viol or lute-ſtring be touched with the bow, or the hand, another String on the ſame inſtrument, or even on another, not far from it, if in uniſon with it, or in octave, or the like, will at the ſame time tremble of its

its own accord. But it is now found, that it is not the whole of that other String that thus trembles, but only the parts, severally, according as they are unisons to the whole, or the parts, of the String so struck. Thus, supposing AB to be an upper octave to *ab*, and therefore an unison to each half of it, stopped at *c*; if while *ab* is open, AB be



struck, the two halves of this other, that is, *ac*, and *cb*, will both tremble; but the middle point will be at rest; as will be easily perceived, by wrapping a bit of paper lightly about the string *ab*, and moving it successively from one end of the string to the other. In like manner, if AB were an upper 12th to *ab*, and consequently an unison to its three parts *ad*, *de*, *eb*; then, *ab* being open, if AB be struck, the three parts of the other, *ad*, *de*, *eb* will severally tremble; but the points *d* and *e* remain at rest.

This, Dr. Wallis tells us, was first discovered by Mr. William Noble of Merton college; and after him by Mr. T. Pigot of Wadham college, without knowing that Mr. Noble had observed it before. To which may be added, that M. Sauveur, long afterwards, proposed it to the Royal Academy at Paris, as his own discovery, which in reality it might be; but upon his being informed, by some of the members then present, that Dr. Wallis had published it before, he immediately resigned all the honour of it. Philos. Trans. Abridg. vol. I, pa. 606.

STURM, Sturmius (John Christopher), a noted German mathematician and philospher, was born at Hippolstein in 1635. He became professor of philosophy and mathematics at Altdorf, where he died in 1703, at 68 years of age.

He was author of several useful works on mathematics and philosophy, the most esteemed of which are,

1. His *Mathesis enucleata*, in 1 vol. 8vo.

2. *Mathesis Juvenilis*, in 2 large volumes 8vo.

3. *Collegium Experimentale, sive Curiosum, in quo primaria Seculi superioris Inventa & Experimenta Physico-Mathematica, Speciatim Campanæ Urinatoriæ, Cameræ obscuræ, Tubi Torricelliani, seu Baroscopii, Antliæ Pneumaticæ, Thermometrorum Phænomena & Effecta; partim ac aliis jampridem exhibita, partim noviter istis superaddita, &c.* in one large vol. 4to, Norimberg, 1701.

This is a very curious work, containing a multitude of interesting experiments, neatly illustrated by copperplate figures printed upon almost every page, by the side of the letter-press. Of these, the 10th experiment is an improvement on father Lana's project for navigating a small vessel suspended in the atmosphere by several globes exhausted of air.

STYLE, in Chronology, a particular manner of counting time; as the *Old Style*, the *New Style*. See Calendar.

Old Syyle, is the Julian manner of computing, as instituted by Julius Cæsar, in which the mean year consists of $365\frac{1}{4}$ days.

New Style, is the Gregorian manner of computation, instituted by pope Gregory the 13th, in the year 1582, and is used by most catholic countries, and many other states of Europe.

The Gregorian, or new Style, agrees with the true solar year, which contains only 365 days 5 hours 49 minutes. In the year of Christ 200, there was no difference of Styles. In the year 1582, when the new Style was first introduced, there was a difference of 10 days. At present there is 11 days difference, and accordingly at the diet of Ratisbon, in the year 1700, it was decreed by the body of protestants of the empire, that 11 days should be retrenched from the old Style, to accommodate it for the future to the new. And the same regulation has since passed into Sweden, Denmark, and into England, where it was established in the year 1752, when it was enacted, that in all dominions belonging to the crown of Great Britain, the supputation, according to which the year of our lord begins on the 25th day of March, shall not be used from and after the last day of December 1751; and that from thenceforth, the 1st day of January every year shall be reckoned to be the first day of the year; and that the natural day next immediately following the 2d day of September 1752, shall be accounted the 14th day of September, omitting the 11 intermediate nominal days of the common calendar. It is farther enacted, that all kinds of writings, &c, shall bear date according to the new method of computation, and that all courts and meetings &c, feasts, fasts, &c, shall be held and observed accordingly. And for preserving the calendar in the same regular course for the future, it is enacted, that the several years of our lord 1800, 1900, 2100, 2200, 2300, &c, except only every 400th year, of which the year 2000 shall be the first, shall be common years of 365 days, and that the years 2000, 2400, 2800, &c, and every other 400th year from the year 2000 inclusive, shall be leap years, consisting of 366 days. See Bissextile and Calendar.

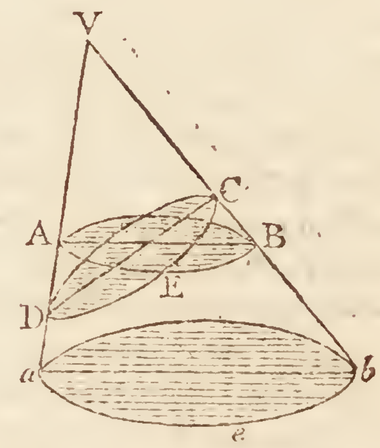
The following table shews by what number of days the new style differs from the old, from 5900 years before the birth of Christ, to 5900 years after it. The days under the sign — (viz from 6000 years before to 200 years after Christ) are to be subtracted from the old Style, to reduce it to the new; and the days under the sign + (viz from 200 to 5900 years after Christ) are to be added to the old Style, to reduce it to the new.—N.B. All the years mentioned in the table are leap years in the old Style; but those only that are marked with an L are leap years in the new.

Years before Christ. New Style.	Days diff. —	Years after Christ. New Style.	Days diff. ∓
5900	46	L 0	−2
5800	45	100	−1
5700	44	200	0
L 5600	44	300	+1
5500	43	L 400	1
5400	42	500	2
5300	41	600	3
L 5200	41	700	4
5100	40	L 800	4
5000	39	900	5
4900	38	1000	6
L 4800	38	1100	7
4700	37	L 1200	7
4600	36	1300	8
4500	35	1400	9
L 4400	35	1500	10
4300	34	L 1600	10
4200	33	1700	11
4100	32	1800	12
L 4000	32	1900	13
3900	31	L 2000	13
3800	30	2100	14
3700	29	2200	15
L 3600	29	2300	16
3500	28	L 2400	16
3400	27	2500	17
3300	26	2600	18
L 3200	26	2700	19
3100	25	L 2800	19
3000	24	2900	20
2900	23	3000	21
L 2800	23	3100	22
2700	22	L 3200	22
2600	21	3300	23
2500	20	3400	24
L 2400	20	3500	25
2300	19	L 3600	25
2200	18	3700	26
2100	17	3800	27
L 2000	17	3900	28
1900	16	L 4000	28
1800	15	4100	29
1700	14	4200	30
L 1600	14	4300	31
1500	13	L 4400	31
1400	12	4500	32
1300	11	4600	33
L 1200	11	4700	34
1100	10	L 4800	34
1000	9	4900	35
900	8	5000	36
L 800	8	5100	37
700	7	L 5200	37
600	6	5300	38
500	5	5400	39
L 400	5	5500	40
300	4	L 5600	40
200	3	5700	41
100	2	5800	42
L 0	2	5900	43

The French nation has lately commenced another new Style, or computation of time, viz, in the year 1792; according to which, the year commences usually on our 22d of September. The year is divided into 12 months of 30 days each; and each month into 3 decades of 10 days each. For the names and computations of which, see the article CALENDAR.

STYLE, in Dialling, denotes the cock or gnomon, raised above the plane of the dial, to project a Shadow.—The edge of the Style, which by its shadow marks the hours on the face of the dial, is to be set according to the latitude, always parallel to the axis of the world.

STYLOBATA, or STYLOBATON, in Architecture, the same with the pedestal of a column. It is sometimes taken for the trunk of the pedestal, between the cornice and the base, and is then called *truncus*. It is also otherwise named *abacus*.



SUBCONTRARY position, in Geometry, is when two equiangular triangles, as VAB and VCD are so placed as to have one common angle V at the vertex, and yet their bases not parallel. Consequently the angles at the bases are equal, but on the contrary sides; viz, the $\angle A = \angle C$, and the $\angle B = \angle D$.

If the oblique cone VAB or V*ab*, having the circular base AEB, or *aeb*, be so cut by a plane DEC, that the angle D be = the $\angle B$, or the $\angle C = \angle A$, then the cone is said to be cut, by this plane, in a Subcontrary position to the base AEB, or *aeb*; and in this case the section DEC is always a circle, as well as the base AEB or *aeb*.

SUBDUCTION, in Arithmetic, the same as Subtraction.

SUBDUPLE *Ratio*, is when any number or quantity is the half of another, or contained twice in it. Thus, 3 is said to be subduple of 6, as 3 is the half of 6, or is twice contained in it.

SUBDUPLICATE *Ratio*, of any two quantities, is the ratio of their square roots, being the opposite to duplicate ratio, which is the ratio of the squares. Thus, of the quantities, a and b, the subduplicate ratio is that of $\surd a$ to $\surd b$ or $a^{\frac{1}{2}}$ to $b^{\frac{1}{2}}$, as the duplicate ratio is that of a^2 to b^2.

SUBLIME *Geometry*, the higher geometry, or that of curve lines. See GEOMETRY.

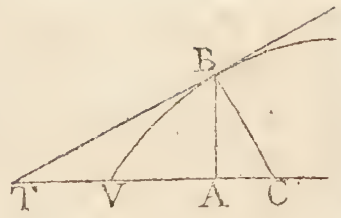
SUBLUNARY, is said of all things below the moon; as all things on the earth, or in its atmosphere, &c.

SUBMULTIPLE, the contrary of a multiple, being a number or quantity which is contained exactly a certain number of times in another of the same kind; or it is the same as an aliquot part of it. Thus, 3 is a Submultiple of 21, or an aliquot part of it, because 21 is a multiple of 3.

SUBMULTIPLE *Ratio*, is the ratio of a Submultiple or aliquot part, to its multiple; as the ratio of 3 to 21.

SUBNORMAL, in Geometry, is the subperpendicular AC, or line under the perpendicular to the curve BC, a term used in curve lines to denote the distance AC in the axis, between the ordinate AB, and the perpendicular

pendicular BC to the curve or to the tangent. And the said perpendicular BC is the normal.



In all curves, the Subnormal AC is a 3d proportional to the subtangent TA and the ordinate AB; and in the parabola, it is equal to half the parameter of the axis.

SUBSTITUTION, in Algebra, is the putting and using, in an equation, one quantity instead of another which is equal to it, but expressed after another manner. See REDUCTION of Equations.

SUBSTRACTION, or SUBTRACTION, in Arithmetic, is the taking of one number or quantity from another, to find the remainder or difference between them; and is usually made the second rule in arithmetic.

The greater number or quantity is called the *minuend*, the less is the *subtrahend*, and the *remainder* is the *difference*. Also the sign of Subtraction is −, or minus.

SUBTRACTION *of Whole Numbers*, is performed by setting the less number below the greater, as in addition, units under units, tens under tens, &c; and then, proceeding from the right hand towards the left, subtract or take each lower figure from that just above, and set down the several remainders or differences underneath; and these will compose the whole remainder or difference of the two given numbers. But when any one of the figures of the under number is greater than that of the upper, from which it is to be taken, you must add 10 (in your mind) to that upper figure, then take the under one from this sum, and set the difference underneath, carrying or adding 1 to the next under figure to be subtracted. Thus, for example, to subtract 2904821 from 37409732

Minuend	37409732
Subtrahend	2904821
Difference	34504911
Proof	37409732

To prove Subtraction: Add the remainder or difference to the less number, and the sum will be equal to the greater when the work is right.

SUBTRACTION *of Decimals*, is performed in the same manner as in whole numbers, by observing only to set the figures or places of the same kind under each other. Thus:

From	351·04	·479	27
Take	72·71	·0573	0·936
Diff.	278·33	·4217	26·064

To Subtract Vulgar Fractions. Reduce the two fractions to a common denominator, if they have different ones; then take the less numerator from the greater, and set the remainder over the common denominator, for the difference sought.—N. B. It is best to set the less fraction after the greater, with the sign (−) of subtraction between them, and the mark of equality (=) after them.

Thus, $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$.

And $\frac{3}{5} - \frac{4}{7} = \frac{21}{35} - \frac{20}{35} = \frac{1}{35}$.

SUBTRACTION, *in Algebra*, is performed by changing the signs of all the terms of the subtrahend, to their contrary signs, viz, + into −, and − into +; and then uniting the terms with those of the minuend after the manner of addition of Algebra.

Ex.	From	$+ 6a$
	Take	$+ 2a$
	Rem.	$6a - 2a = 4a$.
	From	$+ 6a$
	Take	$- 2a$
	Rem.	$6a + 2a = 8a$.
	From	$- 6a$
	Take	$+ 2a$
	Rem.	$- 6a - 2a = - 8a$.
	From	$- 6a$
	Take	$- 4a$
	Rem.	$- 6a + 4a = - 2a$.
	From	$2a - 3x + 5z - 6$
	Take	$6a + 4x + 5z + 4$
	Rem.	$-4a - 7x \quad 0 - 10$

SUBSTILE, or SUBSTILAR *Line*, in Dialling, a right line upon which the stile or gnomon of a dial is erected, being the common section of the face of the dial and a plane perpendicular to it passing through the stile.

The angle included between this line and the stile, is called the elevation or height of the stile.

In polar, horizontal, meridional, and northern dials, the Substilar line is the meridional line, or line of 12 o'clock; or the intersection of the plane of the dial with that of the meridian.—In all declining dials, the Substile makes an angle with the hour line of 12, and this angle is called the distance of the Substile from the meridian.—In easterly and westerly dials, the substilar line is the line of 6 o'clock, or the intersection of the dial plane with the prime vertical.

SUBSUPERPARTICULAR. } See RATIO.
SUBSUPERPARTICUS. }

SUBTANGENT *of a curve*, is the line TA in the axis below the tangent TB, or limited between the tangent and ordinate to the point of contact. (See the last figure above).

The tangent, subtangent, and ordinate, make a right-angled triangle.

In all paraboliform and hyperboliform figures, the Subtangent is equal to the abscis multiplied by the exponent of the power of the ordinate in the equation of the curve. Thus, in the common parabola, whose property or equation is $px = y^2$, the Subtangent is equal to $2x$, double the abscis. And if $ax^2 = y^3$, or $px =$

$px = y^{\frac{3}{2}}$, then the Subtangent is $= \frac{3}{2}x$. Also if $a^m x^n = y^{m+n}$, or $px = y^{\frac{m+n}{n}}$, the Subtangent is $= \frac{m+n}{n}x$. See *Method of* TANGENTS.

SUBTENSE, in Geometry, of an arc, is the same as the chord of the arc; but of an angle, it is a line drawn across from the one leg of the angle to the other, or between the two extremes of the arc that measures the angle.

SUBTRACTION. See SUBSTRACTION.

SUBTRIPLE, is when one quantity is the 3d part of another; as 2 is Subtriple of 6. And SUBTRIPLE *Ratio*, is the ratio of 1 to 3.

SUBTRIPLICATE *Ratio*, is the ratio of the cube roots. So the Subtriplicate ratio of a to b, is the ratio of $\sqrt[3]{a}$ to $\sqrt[3]{b}$, or of $a^{\frac{1}{3}}$ to $b^{\frac{1}{3}}$.

SUCCESSION *of Signs*, in Astronomy, is the order in which they are reckoned, or follow one another, and according to which the sun enters them; called also *consequentia*. As Aries, Taurus, Gemini, Cancer, &c.

When a planet goes according to the order and succession of the signs, or *in consequentia*, it is said to be direct; but retrograde when contrary to the succession of the signs, or *in antecedentia*, as from Gemini to Taurus, then to Aries, &c.

SUCCULA, in Mechanics, a bare axis or cylinder with staves in it to move it round; but without any tympanum, or peritrochium.

SUCKER, in Mechanics, a name by which sometimes is called the piston or bucket, in a sucking pump; and sometimes the pump itself is so called.

SUCKING-*Pump*, the common pump, working by two valves opening upwards. See PUMP.

SUMMER, the name of one of the seasons of the year, being one of the quarters when the year is divided into 4 quarters, or one half when the year is divided only into two, Summer and winter. In the former case, Summer is the quarter during which, in northern climates, the sun is passing through the three signs Cancer, Leo, Virgo, or from the time of the greatest declination, till the sun come to the equinoctial again, or have no declination; which is from about the 21st of June, till about the 22d of September. In the latter case, Summer contains the 6 warmer months, while the sun is on one side of the equinoctial; and winter the other 6 months, when the sun is on the other side of it.

It is said, that a frosty winter produces a dry Summer; and a mild winter, a wet Summer. See Philos. Trans. no. 458, sect. 10.

SUMMER *Solstice*, the time or point when the sun comes to his greatest declination, and nearest the zenith of the place. See SOLSTICE.

SUM, the quantity produced by addition, or by adding two or more numbers or quantities together. So the Sum of 6 and 4 is 10, and the Sum of a and b is $a + b$.

SUN, SOL, ☉, in Astronomy, the great luminary which enlightens the world, and by his presence constitutes day.

The Sun, which was reckoned among the planets in the infancy of astronomy, should rather be counted among the fixed stars. He only appears brighter and larger than they do, because we keep constantly near the Sun; whereas we are immensely farther from the stars. But a spectator, placed as near to any star as we are to the Sun, would probably see that star a body as large and as bright as the Sun appears to us; and, on the other hand, a spectator as far distant from the Sun as we are from the stars, would see the Sun as small as we see a star, divested of all his circumvolving planets; and he would reckon it one of the stars in numbering them.

According to the Pythagorean and Copernican hypothesis, which is now generally received, and has been demonstrated to be the true system, the Sun is the common centre of all the planetary and cometary system; around which all the planets and comets, and our earth among the rest, revolve, in different periods, according to their different distances from the Sun.

But the Sun, though thus eased of that prodigious motion by which the Ancients imagined he revolved daily round our earth, yet is he not a perfectly quiescent body. For, from the phenomena of his maculæ or spots, it evidently appears, that he has a rotation round his axis, like that of the earth by which our natural day is measured, but only slower. For, some of these spots have made their first appearance near the edge or margin of the Sun, from thence they have seemed gradually to pass over the Sun's face to the opposite edge, then disappear; and hence, after an absence of about 14 days, they have reappeared in their first place, and have taken the same course over again; finishing their entire circuit in 27 days 12^h 20^m; which is hence inferred to be the period of the Sun's rotation round his axis: and therefore the periodical time of the Sun's revolution to a fixed star is 25^d 15^h 16^m; because in 27^d 12^h 20^m of the month of May, when the observations were made, the earth describes an angle about the Sun's centre of $26° 22'$, and therefore as the angular motion

$$360° + 26° 22' : 360° :: 27^d\ 12^h\ 20^m : 25^d\ 15^h\ 16^m.$$

This motion of the spots is from west to east: whence we conclude the motion of the Sun, to which the other is owing, to be from east to west.

Beside this motion round his axis, the Sun, on account of the various attractions of the surrounding planets, is agitated by a small motion round the centre of gravity of the system.—Whether the Sun and stars have any proper motion of their own in the immensity of space, however small, is not absolutely certain. Though some very accurate observers have intimated conjectures of this kind, and have made such a general motion not improbable. See STARS.

As for the apparent annual motion of the SUN *round the earth*; it is easily shewn, by astronomers, that the real annual motion of the earth, about the Sun, will cause such an appearance. A spectator in the Sun would see the earth move from west to east, for the same reason as we see the Sun move from east to west: and all the phenomena resulting from this annual motion in whichsoever of the bodies it be, will appear the same from

from either. And hence arifes that apparent motion of the Sun, by which he is feen to advance infenfibly towards the eaftern ftars; in fo much that, if any ftar, near the ecliptic, rife at any time with the Sun; after a few days the Sun will be got more to the eaft of the ftar, and the ftar will rife and fet before him.

Nature, Properties, Figure, &c, of the Sun.

Thofe who have maintained that the fubftance of the Sun is fire, argue in the following manner: The Sun fhines, and his rays, collected by concave mirrors, or convex lenfes, do burn, confume, and melt the moft folid bodies, or elfe convert them into afhes, or glafs: therefore, as the force of the folar rays is diminifhed, by their divergency, in a duplicate ratio of the diftances reciprocally taken; it is evident that their force and effect are the fame, when collected by a burning lens, or mirror, as if we were at fuch diftance from the fun, where they were equally denfe. The Sun's rays therefore, in the neighbourhood of the Sun, produce the fame effects, as might be expected from the moft vehement fire: confequently the Sun is of a fiery fubftance.

Hence it follows, that its furface is probably every where fluid; that being the condition of flame. Indeed, whether the whole body of the Sun be fluid, as fome think; or folid, as others; they do not prefume to determine: but as there are no other marks, by which to diftinguifh fire from other bodies, but light, heat, a power of burning, confuming, melting, calcining, and vitrifying; they do not fee what fhould hinder but that the Sun may be a globe of fire, like our fires, invefted with flame: and, fuppofing that the maculæ are formed out of the folar exhalations, they infer that the Sun is not pure fire; but that there are heterogeneous parts mixed along with it.

Philofophers have been much divided in opinion with refpect to the nature of fire, light, and heat, and the caufes that produce them: and they have given very different accounts of the agency of the Sun, with which, whether we confider them as fubftances or qualities, they are intimately connected, and on which they feem primarily to depend. Some, among whom we may reckon Sir Ifaac Newton, confider the rays of light as compofed of fmall particles, which are emitted from fhining bodies, and move with uniform velocities in uniform mediums, but with variable velocities in mediums of variable denfities. Thefe particles, fay they, act upon the minute conftituent parts of bodies, not by impact, but at fome indefinitely fmall diftance; they attract and are attracted; and in being reflected or refracted, they excite a vibratory motion in the component particles. This motion increafes the diftance between the particles, and thus occafions an augmentation of bulk, or an expanfion in every dimenfion, which is the moft certain characteriftic of fire. This expanfion, which is the beginning of a difunion of the parts, being increafed by the increafing magnitude of the vibrations proceeding from the continued agency of light, it may eafily be apprehended, that the particles will at length vibrate beyond their fphere of mutual attraction, and thus the texture of the body will be altered or deftroyed; from folid it may become fluid, as in melted gold; or from being fluid, it may be difperfed in vapour, as in boiling water.

Others, as Boerhaave, reprefent fire as a fubftance *fui generis*, unalterable in its nature, and incapable of being produced or deftroyed; naturally exifting in equal quantities in all places, imperceptible to our fenfes, and only difcoverable by its effects, when, by various caufes, it is collected for a time into a lefs fpace than that which it would otherwife occupy. The matter of this fire is not in any wife fuppofed to be derived from the Sun: the folar rays, whether direct or reflected, are of ufe only as they impel the particles of fire in parallel directions: that parallelifm being deftroyed, by intercepting the folar rays, the fire inftantly affumes its natural ftate of uniform diffufion. According to this explication, which attributes heat to the matter of fire, when driven in parallel directions, a much greater degree muft be given it when the quantity, fo collected, is amaffed into a focus; and yet the focus of the largeft fpeculum does not heat the air or medium in which it is is found, but only bodies of denfities different from that medium.

M. de Luc (Lettres Phyfiques) is of opinion, that the folar rays are the principal caufe of heat; but that they heat fuch bodies only as do not allow them a free paffage. In this remark he agrees with Newton; but then he differs totally from him, as well as from Boerhaave, concerning the nature of the rays of the Sun. He does not admit the emanation of any luminous corpufcles from the Sun, or other felf-fhining fubftances, but fuppofes all fpace to be filled with an ether of great elafticity and fmall denfity, and that light confifts in the vibrations of this ether, as found confifts in the vibrations of the air. "Upon Newton's fuppofition, fays an excellent writer, the caufe by which the particles of light, and the corpufcles conftituting other bodies are mutually attracted and repelled, is uncertain. The reafon of the uniform diffufion of fire, of its vibration, and repercuffion, as ftated in Boerhaave's opinion, is equally inexplicable. And in the laft mentioned hypothefis, we may add to the other difficulties attending the fuppofition of an univerfal ether, the want of a firft mover to make the Sun vibrate. Of thefe feveral opinions concerning elementary fire, it may be faid, as Cicero remarked upon the opinions of philofophers concerning the nature of the foul: *Harum fententiarum quæ vera fit, Deus aliquis viderit; quæ verifimillima, magna queftio eft.*" Watfon's Chem. Eff. vol. 1, pa. 164.

As to the Figure of the Sun; this, like the planets, is not perfectly globular, but fpheroidical, being higher about the equator than at the poles. The reafon of which is this: the Sun has a motion about his own axis; and therefore the folar matter will have an endeavour to recede from the axis, and that with the greater force as their diftances from it, or the circles they move in, are greater: but the equator is the greateft circle; and the reft, towards the poles, continually decreafe; therefore the folar matter, though at firft in a fpherical form, will endeavour to recede from the centre of the equator farther than from the centres of the parallels. Confequently, fince the gravity, by which it is retained in its place, is fuppofed to be uniform throughout the whole Sun, it will really recede from the centre more at the

the equator, than at any of the parallels; and hence the Sun's diameter will be greater through the equator, than through the poles; that is, the Sun's figure is not perfectly spherical, but spheroidical.

Several particulars of the Sun, related by Newton, in his Principia, are as follow:

1. That the density of the Sun's heat, which is proportional to his light, is 7 times as great at Mercury as with us; and therefore our water there would be all carried off, and boil away: for he found by experiments of the thermometer, that a heat but 7 times greater than that of the Sun beams in summer, will serve to make water boil.

2. That the quantity of matter in the Sun is to that in Jupiter, nearly as 1100 to 1; and that the distance of that planet from the Sun, is in the same ratio to the Sun's semidiameter.

3. That the matter in the Sun is to that in Saturn, as 2360 to 1; and the distance of Saturn from the Sun is in a ratio but little less than that of the Sun's semidiameter. And hence, that the common centre of gravity of the Sun and Jupiter is nearly in the superficies of the Sun; of the Sun and Saturn, a little within it.

4. And by the same mode of calculation it will be found, that the common centre of gravity of all the planets, cannot be more than the length of the solar diameter distant from the centre of the Sun. This common centre of gravity he proves is at rest; and therefore though the Sun, by reason of the various positions of the planets, may be moved every way, yet it cannot recede far from the common centre of gravity, and this, he thinks, ought to be accounted the centre of our world. Book 3, prop. 12.

5. By means of the solar spots it hath been discovered, that the Sun revolves round his own axis, without moving considerably out of his place, in about 25 days, and that the axis of this motion is inclined to the ecliptic in an angle of 87° 30′ nearly. The Sun's apparent diameter being sensibly longer in December than in June, the Sun must be proportionably nearer to the earth in winter than in Summer; in the former of which seasons therefore will be the perihelion, in the latter the aphelion: and this is also confirmed by the earth's motion being quicker in December than in June, as it is by about $\frac{1}{15}$ part. For since the earth always describes equal areas in equal times, whenever it moves swifter, it must needs be nearer to the Sun: and for this reason there are about 8 days more from the sun's vernal equinox to the autumnal, than from the autumnal to the vernal.

6. That the Sun's diameter is equal to 100 diameters of the earth; and therefore the body of the Sun must be 1.000000 times greater than that of the earth.—Mr. Azout assures us, that he observed, by a very exact method, the Sun's diameter to be no less than 21′ 45″ in his apogee, and not greater than 32′ 45″ in his perigee.

7. According to Newton, in his theory of the moon, the mean apparent diameter of the Sun is 32′ 12″.—The Sun's horizontal parallax is now fixed at 8″$\frac{6}{10}$.

8. If you divide 360 degrees (the whole ecliptic) by the quantity of the solar year, it will give 59′ 8″ &c, which therefore is the medium quantity of the Sun's daily motion: and if this 59′ 8″ be divided by 24, you have the Sun's horary motion equal to 2′ 28″: and if this last be divided by 60, it will give his motion in a minute, &c. And in this way are the tables of the Sun's mean motion constructed, as placed in books of Astronomical tables and calculations.

SUNDAY, the first day of the week; thus called by our idolatrous ancestors, because set apart for the worship of the sun.

It is sometimes called the *Lord's Day*, because kept as a feast in memory of our Lord's resurrection on this day: and also *Sabbath-day*, because substituted under the new law instead of the Sabbath in the old law.

It was Constantine the Great who first made a law for the proper observation of Sunday; and who, according to Eusebius, appointed that it should be regularly celebrated throughout the Roman empire.

SUNDAY *Letter*. See DOMINICAL *Letter*.

SUPERFICIAL, relating to Superficies.

SUPERFICIES, or SURFACE, in Geometry, the outside or exterior face of any body. This is considered as having the two dimensions of length and breadth only, but no thickness; and therefore it makes no part of the substance or solid content or matter of the body.

The terms or bounds or extremities of a Superficies, are lines; and Superficies may be considered as generated by the motions of lines.

Superficies are either rectilinear, curvilinear, plane, concave, or convex. A

Rectilinear SUPERFICIES, is that which is bounded by right lines.

Curvilinear SUPERFICIES, is bounded by curve lines.

Plane SUPERFICIES is that which has no inequality in it, nor risings, nor sinkings, but lies evenly and straight throughout, so that a right line may wholly coincide with it in all parts and directions.

Convex SUPERFICIES, is that which is curved and rises outwards.

Concave SUPERFICIES, is curved and sinks inward.

The measure or quantity of a Surface, is called the *area* of it. And the finding of this measure or area, is sometimes called the *quadrature* of it, meaning the reducing it to an equal square, or to a certain number of smaller squares. For all plane figures, and the Surfaces of all bodies, are measured by squares; as square inches, or square feet, or square yards, &c; that is, squares whose sides are inches, or feet, or yards, &c. Our least superficial measure is the square inch, and other squares are taken from it according to the proportion in the following Table of superficial or square measure.

Table of Superficial or Square Measure.

144	square inches	=	1 square foot
9	square feet	=	1 square yard
$30\frac{1}{4}$	square yards	=	1 square pole
16	square poles	=	1 square chain
10	square chains	=	1 acre
640	acres - -	=	1 square mile.

The Superficial measure of all bodies and figures depends entirely on that of a rectangle; and this is found by drawing or multiplying the length by the breadth of it;

it; as is proved from plane geometry only, in my Menſuration, pt. 2, ſect. 1, prob. 1. From the area of the rectangle we obtain that of any oblique parallelogram, which, by geometry, is equal to a rectangle of equal baſe and altitude; thence a triangle, which is the half of ſuch a parallelogram or rectangle; and hence, by compoſition, we obtain the Superficies of all other figures whatever, as theſe may be conſidered as made up of triangles only.

Beſide this way of deriving the Superficies of all figures, which is the moſt ſimple and natural, as proceeding on common geometry alone, there are certain other methods; ſuch as the methods of exhauſtions, of fluxions, &c. See theſe articles in their places, as alſo QUADRATURES.

Line of SUPERFICIES, a line uſually found on the ſector, and Gunter's ſcale. The deſcription and uſe of which, ſee under SECTOR and GUNTER'S *Scale.*

SUPERPARTICULAR *Proportion*, or *Ratio*, is that in which the greater term exceeds the leſs by unit or 1. As the ratio of 1 to 2, or 2 to 3, or 3 to 4, &c.

SUPERPARTIENT *Proportion*, or *Ratio*, is when the greater term contains the leſs term, once, and leaves ſome number greater than 1 remaining. As the ratio

of 3 to 5, which is equal to that of 1 to $1\frac{2}{3}$;
of 7 to 10, which is equal to that of 1 to $1\frac{3}{7}$; &c.

SUPPLEMENT, of an arch, or angle, in Geometry or Trigonometry, is what it wants of a ſemicircle, or of 180 degrees; as the *complement* is what it wants of a quadrant, or of 90 degrees. So, the Supplement of 50° is 130°; as the complement of it is 40°.

SURD, in Arithmetic, denotes a number or quantity that is incommenſurate to unity; or that is inexpreſſible in rational numbers by any known way of notation, otherwiſe than by its radical ſign or index.—This is otherwiſe called an *irrational* or *incommenſurable number*, as alſo an *imperfect power.*

Theſe Surds ariſe in this manner: when it is propoſed to extract a certain root of ſome number or quantity, which is not a complete power or a true figurate number of that kind; as, if its ſquare root be demanded, and it is not a true ſquare; or if its cube root be required, and it is not a true cube, &c; then it is impoſſible to aſſign, either in whole numbers, or in fractions, the exact root of ſuch propoſed number. And whenever this happens, it is uſual to denote the root by ſetting before it the proper mark of radicality, which is $\surd$, and placing above this radical ſign the number that ſhews what kind of root is required. Thus, $\sqrt[2]{2}$ or $\sqrt{2}$ ſignifies the ſquare root of 2, and $\sqrt[3]{10}$ ſignifies the cube root of 10; which roots, becauſe it is impoſſible to expreſs them in numbers exactly, are properly called *Surd roots.*

There is alſo another way of notation, now much in uſe, by which roots are expreſſed by fractional indices, without the radical ſign: thus, like as x^2, x^3, x^4, &c, denote the ſquare, cube, 4th power, &c, of x; ſo $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{1}{4}}$, &c, denote the ſquare root, cube root, 4th root, &c, of the ſame quantity x.—The reaſon of this is plain enough; for ſince $\sqrt{x}$ is a geometrical mean proportional between 1 and x, ſo $\frac{1}{2}$ is an arithmetical mean between 0 and 1; and therefore, as 2 is the index of the ſquare of x, $\frac{1}{2}$ will be the proper index of its ſquare root, &c.

It may be obſerved that, for convenience, or the ſake of brevity, quantities which are not naturally Surds, are often expreſſed in the form of Surd roots. Thus $\sqrt{4}$, $\sqrt{\frac{9}{4}}$, $\sqrt[3]{27}$, are the ſame as 2, $\frac{3}{2}$, 3.

Surds are either *ſimple* or *compound.*

Simple SURDS, are ſuch as are expreſſed by one ſingle term; as $\sqrt{2}$, or $\sqrt[3]{a}$, &c.

Compound SURDS, are ſuch as conſiſt of two or more ſimple Surds connected together by the ſigns + or −; as $\sqrt{3} + \sqrt{2}$, or $\sqrt{3} - \sqrt{2}$, or $\sqrt[3]{5 + \sqrt{2}}$: which laſt is called an *univerſal* root, and denotes the cubic root of the ſum ariſing by adding 5 and the root of 2 together.

Of certain Operations by Surds.

1. Such Surds as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, &c. though they are themſelves incommenſurable with unity, according to the definition, are commenſurable in power with it, becauſe their powers are integers, which are multiples of unity. They may alſo be ſometimes commenſurable with one another; as $\sqrt{8}$ and $\sqrt{2}$, which are to one another as 2 to 1, as is found by dividing them by their greateſt common meaſure, which is $\sqrt{2}$, for then thoſe two become $\sqrt{4} = 2$, and 1 the ratio.

2. *To reduce Rational Quantities to the form of any propoſed Surd Roots.*—Involve the rational quantity according to the index of the power of the Surd, and then prefix before that power the propoſed radical ſign.

Thus, $a = \sqrt{a^2} = \sqrt[3]{a^3} = \sqrt[4]{a^4} = \sqrt[n]{a^n}$, &c.
and $4 = \sqrt{16} = \sqrt[3]{64} = \sqrt[4]{256} = \sqrt[n]{4^n}$, &c.

And in this way may a ſimple Surd fraction, whoſe radical ſign refers to only one of its terms, be changed into another, which ſhall include both numerator and denominator. Thus, $\frac{\sqrt{2}}{5}$ is reduced to $\sqrt{\frac{2}{25}}$, and $\frac{5}{\sqrt[3]{4}}$ to $\sqrt[3]{\frac{125}{4}}$: thus alſo the quantity a reduced to the form of $x^{\frac{1}{n}}$ or $\sqrt[n]{x}$, is $\overline{a^n}|^{\frac{1}{n}}$ or $\sqrt[n]{a^n}$. And thus may roots with rational coefficients be reduced ſo as to be wholly affected by the radical ſign; as $a\sqrt[n]{x} = \sqrt[n]{a^n x}$.

3. *To reduce Simple Surds, having different radical ſigns (which are called heterogeneal Surds) to others that may have one common radical ſign, or which are homogeneal: Or to reduce roots of different names to roots of the ſame name.*—Involve the powers reciprocally, each according to the index of the other, for new powers; and multiply their indices together, for the common index. Otherwiſe, as Surds may be conſidered as powers with fractional exponents, reduce theſe fractional exponents to fractions having the ſame value and a common denominator.

Thus, by the 1ſt way,
$\sqrt[n]{a}$ and $\sqrt[m]{x}$ become $\sqrt[mn]{a^m}$ and $\sqrt[mn]{x^n}$;

and,

and, by the 2d way,

$a^{\frac{1}{n}}$ and $x^{\frac{1}{m}}$ become $\overline{a^{m}}|^{\frac{1}{mn}}$ and $\overline{x^{n}}|^{\frac{1}{mn}}$.

Alſo $\sqrt{3}$ and $\sqrt[3]{2}$ are reduced to $\sqrt[6]{27}$ and $\sqrt[6]{4}$, which are equal to them, and have a common radical ſign.

4. *To reduce Surds to their moſt ſimple expreſſions, or to the loweſt terms poſſible.*—Divide the Surd by the greateſt power, of the ſame name with that of the root, which you can diſcover is contained in it, and which will meaſure or divide it without a remainder; then extract the root of that power, and place it before the quotient or Surd ſo divided; this will produce a new Surd of the ſame value with the former, but in more ſimple terms. Thus, $\sqrt{16a^2x}$, by dividing by $16a^2$, and prefixing its root $4a$, before the quotient $\sqrt{x}$, becomes $4a\sqrt{x}$; in like manner, $\sqrt{12}$ or $\sqrt{4 \times 3}$, becomes $2\sqrt{3}$;

And $\sqrt[3]{ab^3x}$ reduces to $b\sqrt[3]{ax}$.

Alſo $\sqrt[3]{81} = \sqrt[3]{27 \times 3} = \sqrt[3]{3^3 \times 3} = 3\sqrt[3]{3}$.

And $\sqrt{288} = \sqrt{144 \times 2} = 12\sqrt{2}$.

5. *To Add and Subtract Surds.*—When they are reduced to their loweſt terms, if they have the ſame irrational part, add or ſubtract their rational coefficients, and to the ſum or difference ſubjoin the common irrational part.

Thus, $\sqrt{75} + \sqrt{48} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$;
and $\sqrt{150} - \sqrt{54} = 5\sqrt{6} - 3\sqrt{6} = 2\sqrt{6}$;
alſo $\sqrt{a^2x} + \sqrt{c^2x} = a\sqrt{x} + c\sqrt{x} = \overline{a+c}.\sqrt{x}$.

Or ſuch Surds may be added and ſubtracted, by firſt ſquaring them (by uniting the ſquare of each part with double their product), and then extracting the root univerſal of the whole. Thus, for the firſt example above,

$\sqrt{75} + \sqrt{48} = \sqrt{75 + 48 + 2\sqrt{75 \times 48}} = \sqrt{123 + 2\sqrt{3600}} = \sqrt{123 + 120} = \sqrt{243} = 9\sqrt{3}$.

If the quantities cannot be reduced to the ſame irrational part, they may juſt be connected by the ſigns + or −.

6. *To Multiply and Divide Surds.*—If the terms have the ſame radical, they will be multiplied and divided like powers, viz, by adding their indices for multiplication, and ſubtracting them for diviſion.

Thus,

$\sqrt{a} \times \sqrt[3]{a} = a^{\frac{1}{2}} \times a^{\frac{1}{3}} = a^{\frac{3}{6}} \times a^{\frac{2}{6}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$;

and $\sqrt{2} \times \sqrt[3]{2} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}$;

alſo $\sqrt{a} \div \sqrt[3]{a} = a^{\frac{1}{2}} \div a^{\frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a}$;

and $\sqrt{2} \div \sqrt[3]{2} = 2^{\frac{1}{6}} = \sqrt[6]{2}$.

If the quantities be different, but under the ſame radical ſign; multiply or divide the quantities, and place the radical ſign to the product or quotient.

Thus, $\sqrt{2} \times \sqrt{5} = \sqrt{10}$;
and $\sqrt[3]{a^2} \times \sqrt[3]{c} = \sqrt[3]{a^2c}$;
alſo $\sqrt[3]{20} \div \sqrt[3]{4} = \sqrt[3]{5}$.

But if the Surds have not the ſame radical ſign, reduce them to ſuch as ſhall have the ſame radical ſign, and proceed as before.

Thus, $\sqrt[m]{a} \times \sqrt[n]{b} = \sqrt[mn]{a^n} \times \sqrt[mn]{b^m} = \sqrt[mn]{a^nb^m}$;
and $\sqrt{2} \times \sqrt[3]{4} = \sqrt[6]{2^3} \times \sqrt[6]{4^2} = \sqrt[6]{8 \times 16} = \sqrt[6]{128}$.

If the Surds have any rational coefficients, their product or quotient muſt be prefixed.

Thus, $5\sqrt{6} \times 2\sqrt{3} = 10\sqrt{18} = 30\sqrt{2}$;
and $8\sqrt{5} \div 2\sqrt{6} = 4\sqrt{\frac{5}{6}}$.

7. *Involution and Evolution of Surds.*——Surds are involved, or raiſed to any power, by multiplying their indices by the index of the power; and they are evolved or extracted, by dividing their indices by the index of the root.

Thus, the ſquare of $\sqrt[3]{2}$ or of $2^{\frac{1}{3}}$ is $2^{\frac{2}{3}} = \sqrt[3]{4}$;

and the cube of $\sqrt{5}$ or of $5^{\frac{1}{2}}$, is $5^{\frac{3}{2}} = \sqrt{125}$;

alſo the ſquare root of $\sqrt[3]{4}$ or $4^{\frac{1}{3}}$, is $4^{\frac{1}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$.

Or thus: involve or extract the quantity under the radical ſign according to the power or root required, continuing the ſame radical ſign.

So the ſquare of $\sqrt[3]{2}$ is $\sqrt[3]{4}$;
and the ſquare root of $\sqrt[3]{4}$, is $\sqrt[3]{2}$.

Unleſs the index of the power is equal to the name of the Surd, or a multiple of it, for in that caſe the power of the Surd becomes rational. Thus, the ſquare of $\sqrt{3}$ is 3, and the cube of $\sqrt[3]{a^2}$ is a^2.

Simple Surds are commenſurable in power, and by being multiplied by themſelves give, at length, rational quantities: but compound Surds, multiplied by themſelves, commonly give irrational products. Yet, in this caſe, when any compound Surd is propoſed, there is another compound Surd, which, multiplied by it, gives a rational product.

Thus, $\sqrt{a} + \sqrt{b}$ multiplied by $\sqrt{a} - \sqrt{b}$ gives $a - b$;
and $\sqrt[3]{a} - \sqrt[3]{b}$ mult. by $\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}$ gives $a - b$.

The finding of ſuch a Surd as multiplying the propoſed Surd gives a rational product, is made eaſy by three theorems, delivered by Maclaurin, in his Algebra, pa. 109 &c.

This operation is of uſe in reducing Surd expreſſions to more ſimple forms. Thus, ſuppoſe a binomial Surd divided by another, as $\sqrt{20} + \sqrt{12}$ by $\sqrt{5} - \sqrt{3}$, the quotient might be expreſſed by

$\frac{\sqrt{20} + \sqrt{12}}{\sqrt{5} - \sqrt{3}} = \frac{2\sqrt{5} + 2\sqrt{3}}{\sqrt{5} - \sqrt{3}}$; but this will be expreſſed in a more ſimple form, by multiplying both numerator and denominator by ſuch a Surd as makes the product of the denominator become a rational quantity: thus, multiplying them by $\sqrt{5} + \sqrt{3}$, the fraction or quotient becomes

$2 \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = 2 \times \frac{\overline{\sqrt{5} + \sqrt{3}}|^2}{5 - 3 = 2} = \overline{\sqrt{5} + \sqrt{3}}|^2 = 8 + 2\sqrt{15}$.

To do this generally, ſee Maclaurin's Alg. p. 113.

When the ſquare root of a Surd is required, it may be found nearly, by extracting the root of a rational quantity that approximates to its value. Thus, to find the ſquare

ſquare root of $3 + 2\sqrt{2}$; firſt calculate $\sqrt{2} = 1{\cdot}41421$; hence $3 + 2\sqrt{2} = 5{\cdot}82842$, the root of which is nearly $2{\cdot}41421$.

In like manner we may proceed with any other propoſed root. And if the index of the root be very high, a table of logarithms may be uſed to advantage: thus, to extract the root $\sqrt[7]{5 + \sqrt[13]{17}}$; take the logarithm of 17, divide it by 13, find the number anſwering to the quotient, add this number to 5, find the log. of the ſum, and divide it by 7, and the number anſwering to this quotient will be nearly equal to $\sqrt[7]{5 + \sqrt[13]{17}}$.

But it is ſometimes requiſite to expreſs the roots of Surds exactly by other Surds. Thus, in the firſt example, the ſquare root of $3 + 2\sqrt{2}$ is $1 + \sqrt{2}$, for $\overline{1 + \sqrt{2}}|^2 = 1 + 2\sqrt{2} + 2 = 3 + \sqrt{2}$. For the method of performing this, the curious may conſult Maclaurin's Algeb. p. 115, where alſo rules for trinomials &c may be found. See alſo the article Binomial *Roots*, in this Dictionary.

For extracting the higher roots of a binomial, whoſe two members when ſquared are commenſurable numbers, we have a rule in Newton's Arith. pa. 59, but without demonſtration. This is ſupplied by Maclaurin, in his Alg. p. 120: as alſo by Graveſande, in his Matheſeos Univerſ. Elem. p. 211.

It ſometimes happens, in the reſolution of cubic equations, that binomials of this form $a \pm b\sqrt{-1}$ occur, the cube roots of which muſt be found; and to theſe Newton's rule cannot always be applied, becauſe of the impoſſible or imaginary factor $\sqrt{-1}$; yet if the root be expreſſible in rational numbers, the rule will often yield to it in a ſhort way, not merely tentative, the trials being confined to known limits. See Maclaurin's Alg. p. 127. It may be farther obſerved, that ſuch roots, whether expreſſible in rational numbers or not, may be found by evolving the quantity $a + b\sqrt{-1}$ by Newton's binomial theorem, and ſumming up the alternate terms. Maclaurin, p. 130.

Thoſe who are deſirous of a general and elegant ſolution of the problem, *to extract any root of an impoſſible binomial* $a + b\sqrt{-1}$, *or of a poſſible binomial* $a + \sqrt{b}$, may have recourſe to the appendix to Saunderſon's Algebra, and to the Philoſ. Tranſ. number 451, or Abridg. vol. 8, p. 1. On the management of Surds, ſee alſo the numerous authors upon Algebra.

SURDESOLID. See Sursolid.

SURFACE, in Geometry. See Superficies.

A *mathematical* Surface is the mere exterior face of a body, but is not any part of it, being of no thickneſs, but only the bare figure or termination of the body.

A *Phyſical* Surface is conſidered as of ſome very ſmall thickneſs.

SURSOLID, or Surdesolid, in Arithmetic, the 5th power of a number, conſidered as a root. The number 2, for inſtance, conſidered as a root, produces the powers thus:

$2 = 2$ the root or 1ſt power,
$2 \times 2 = 4$ the ſquare or 2d power,
$2 \times 4 = 8$ the cube or 3d power,
$2 \times 8 = 16$ the biquadratic or 4th power,
$2 \times 16 = 32$ the Surſolid or 5th power.

6

Sursolid Problem, is that which cannot be reſolved but by curves of a higher kind than the conic ſections.

SURVEYING, the art, or act, of meaſuring land. This compriſes the three following parts; viz, taking the dimenſions of any tract or piece of ground; the delineating or laying the ſame down in a map or draught; and finding the ſuperficial content or area of the ſame; beſide the dividing and laying out of lands.

The firſt of theſe is what is properly called *Surveying*; the ſecond is called *plotting*, or *protracting*, or *mapping*; and the third *caſting up*, or *computing the contents*.

The firſt again conſiſts of two parts, the making of obſervations for the angles, and the taking of lineal meaſures for the diſtances.

The former of theſe is performed by ſome of the following inſtruments; the theodolite, circumferentor, ſemicircle, plain-table, or compaſs, or even by the chain itſelf: the latter is performed by means either of the chain, or the perambulator. The deſcription and manner of uſing each of theſe, ſee under its reſpective article or name.

It is uſeful in Surveying, to take the angles which the bounding lines form with the magnetic needle, in order to check the angles of the figure, and to plot them conveniently afterwards. But, as the difference between the true and magnetic meridian perpetually varies in all places, and at all times; it is impoſſible to compare two ſurveys of the ſame place, taken at diſtant times, by magnetic inſtruments, without making due allowance for this variation. See obſervations on this ſubject, by Mr. Molineux, Philoſ. Tranſ. number 230, p. 625, or Abr. vol. 1, p. 125.

The ſecond branch of Surveying is performed by means of the protractor, and plotting ſcale. The deſcription of which, ſee under their proper names.

If the lands in the ſurvey are hilly, and not in any one plane, the meaſured lines cannot be truly laid down on paper, till they are reduced to one plane, which muſt be the horizontal one, becauſe angles are taken in that plane. And in this caſe, when obſerving diſtant objects, for their elevation or depreſſion, the following table ſhews the links or parts to be ſubtracted from each chain in the hypothenuſal line, when the angle is the correſponding number of degrees.

A Table *of the links to be ſubtracted out of every chain in hypothenuſal lines, of ſeveral degrees of altitude or depreſſion, for reducing them to horizontal.*

	links		links
4° 3′	$\frac{1}{4}$	19° 57′	6
5 44	$\frac{1}{2}$	21 34	7
7 1	$\frac{3}{4}$	23 4	8
8 7	1	24 30	9
11 29	2	24 50	10
14 4	3	27 8	11
16 16	4	28 22	12
18 12	5	29 32	13

For

For example, if a ſtation line meaſure 1250 links, or $12\frac{1}{2}$ chains, on an aſcent, or a deſcent, of 11°; here it is after the rate of almoſt two links per chain, and it will be exact enough to take only the 12 chains at that rate, which make 24 links in all, to be deducted from 1250, which leaves 1226 links, for the length to be laid down.

Practical ſurveyors ſay, it is beſt to make this deduction at the end of every chain-length while meaſuring, by drawing the chain forward every time as much as the deduction is; viz, in the preſent inſtance, drawing the chain on 2 links at each chain-length.

The third branch of Surveying, namely computing or caſting-up, is performed by reducing the ſeveral incloſures and diviſions into triangles, trapeziums, and parallelograms, but eſpecially the two former; then finding the areas or contents of theſe ſeveral figures, and adding them all together.

The Practice of Surveying.

1. Land is meaſured with a chain, called Gunter's chain, of 4 poles or 22 yards in length, which conſiſts of 100 equal links, each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, or 7·92 inches long, that is nearly 8 inches or $\frac{2}{3}$ of a foot.

An acre of land is equal to 10 ſquare chains, that is, 10 chains in length and 1 chain in breadth.

Or it is 40 × 4 or 160 ſquare poles.
Or it is 220 × 22 or 4840 ſquare yards.
Or it is 1000 × 100 or 100000 ſquare links.

Theſe being all the ſame quantity.

Alſo, an acre is divided into 4 parts called roods, and a rood into 40 parts called perches, which are ſquare poles, or the ſquare of a pole of $5\frac{1}{2}$ yards long, or the ſquare of $\frac{1}{4}$ of a chain, or of 25 links, which is 625 ſquare links. So that the diviſions of land meaſure will be thus:

625 ſq. links = 1 pole or perch
40 perches = 1 rood
4 roods = 1 acre

The length of lines, meaſured with a chain, are ſet down in links as integers, every chain in length being 100 links; and not in chains and decimals. Therefore, after the content is found, it will be in ſquare links; then cut off five of the figures on the right-hand for decimals, and the reſt will be acres. Thoſe decimals are then multiplied by 4 for roods, and the decimals of theſe again by 40 for perches.

Ex. Suppoſe the length of a rectangular piece of ground be 792 links, and its breadth 385: to find the area in acres, roods, and perches.

```
      792
      385
   ------
     3960
    6336
   2376
   ------        ac. ro. p.
  3·04920   Anſ.  3  0  7
        4
   ------
   ·19680
       40
   ------
  7·87200
```

2. Among the various inſtruments for ſurveying, the plain-table is the eaſieſt and moſt generally uſeful, eſpecially in crooked difficult places, as in a town among houſes, &c. But although the plain-table be the moſt generally uſeful inſtrument, it is not *always* ſo; there being many caſes in which ſometimes one inſtrument is the propereſt, and ſometimes another; nor is that ſurveyor maſter of his buſineſs who cannot in any caſe diſtinguiſh which is the fitteſt inſtrument or method, and uſe it accordingly: nay, ſometimes no inſtrument at all, but barely the chain itſelf, is the beſt method, particularly in regular open fields lying together; and even when you are uſing the plain-table, it is often of advantage to meaſure ſuch large open parts with the chain only, and from thoſe meaſures lay them down upon the table.

The perambulator is uſed for meaſuring roads, and other great diſtances on level ground, and by the ſides of rivers. It has a wheel of $8\frac{1}{4}$ feet, or half a pole, in circumference, upon which the machine turns; and the diſtance meaſured is pointed out by an index, which is moved round by clock work.

Levels, with teleſcopic or other ſights, are uſed to find the level between place and place, or how much one place is higher or lower than another.

An offset-ſtaff is a very uſeful and neceſſary inſtrument, for meaſuring the offsets and other ſhort diſtances. It is 10 links in length, being divided and marked at each of the 10 links.

Ten ſmall arrows, or rods of iron or wood, are uſed to mark the end of every chain length, in meaſuring lines. And ſometimes pickets, or ſtaves with flags, are ſet up as marks or objects of direction.

Various ſcales are alſo uſed in protracting and meaſuring on the plan or paper; ſuch as plane ſcales, line of chords, protractor, compaſſes, reducing ſcales, parallel and perpendicular rulers, &c. Of plane ſcales, there ſhould be ſeveral ſizes, as a chain in 1 inch, a chain in $\frac{3}{4}$ of an inch, a chain in $\frac{1}{2}$ of an inch, &c. And of theſe, the beſt for uſe are thoſe that are laid on the very edges of the ivory ſcale, to prick off diſtances by, without compaſſes.

3. *The Field Book.*

In ſurveying with the plain-table, a field-book is not uſed, as every thing is drawn on the table immediately when it is meaſured. But in ſurveying with the theodolite, or any other inſtrument, ſome ſort of a field-book muſt be uſed, to write down in it a regiſter or account of all that is done and occurs relative to the ſurvey in hand.

This book every one contrives and rules as he thinks fitteſt for himſelf. The following is a ſpecimen of a form that has formerly been much uſed. It is ruled into 3 columns: the middle, or principal column, is for the ſtations, angles, bearings, diſtances meaſured, &c; and thoſe on the right and left are for the offsets on the right and left, which are ſet againſt their correſponding diſtances in the middle column; as alſo for ſuch remarks as may occur, and be proper to note for drawing the plan, &c.

Here ⊙ 1 is the firſt ſtation, where the angle or bearing is 105° 25′. On the left, at 73 links in the diſtance

diſtance or principal line, is an offset of 92; and at 610 an offset of 24 to a croſs hedge. On the right, at 0, or the beginning, an offset 25 to the corner of the field; at 248 Brown's boundary hedge commences; at 610 an offset 35; and at 954, the end of the firſt line, the 0 denotes its terminating in the hedge. And ſo on for the other ſtations.

Draw a line under the work, at the end of every ſtation line, to prevent confuſion.

Offsets and Remarks on the left.	Stations, Bearings, and Diſtances.	Offsets and Remarks on the right.
	⊙1	
	105°25′	
	00	25 corner
92	73	
	248	Brown's hedge
croſs a hedge 24	610	35
	954	00
	⊙2	
	53°10′	
	00	00
houſe corner 51	25	21
	120	29 a tree
34	734	40 a ſtile
	⊙3	
	67°20′	
	61	35
a brook 30	248	
	639	16 a ſpring
foot path 16	810	
croſs hedge 18	973	20 a pond

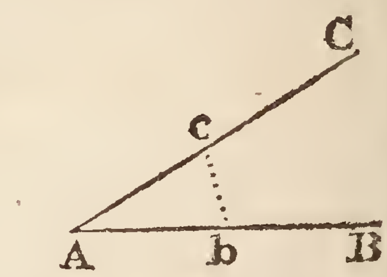
But a few ſkilful ſurveyers now make uſe of a different method for the field book, namely, beginning at the bottom of the page and writing upwards; by which they ſketch a neat boundary on either hand, as they paſs it; an example of which will be given below, with the plan of the ground to accompany it.

In ſmaller ſurveys and meaſurements, a very good way of ſetting down the work, is, to draw, by the eye, on a piece of paper, a figure reſembling that which is to be meaſured; and ſo write the dimenſions, as they are found, againſt the correſponding parts of the figure. And this method may be practiſed to a conſiderable extent, even in the larger ſurveys.

4. *To meaſure a line on the ground with the chain:* Having provided a chain, with 10 ſmall arrows, or rods, to ſtick one into the ground, as a mark, at the end of every chain; two perſons take hold of the chain, one at each end of it, and all the 10 arrows are taken by one of them who goes foremoſt, and is called the leader; the other being called the follower, for diſtinction's ſake.

A picket, or ſtation ſtaff, being ſet up in the direction of the line to be meaſured, if there do not appear ſome marks naturally in that direction; the follower ſtands at the beginning of the line, holding the ring at the end of the chain in his hand, while the leader drags forward the chain by the other end of it, till it is ſtretched ſtraight, and laid or held level, and the leader directed, by the follower waving his hand, to the right or left, till the follower ſee him exactly in a line with the mark or direction to be meaſured to; then both of them ſtretching the chain ſtraight, and ſtooping and holding it level, the leader having the head of one of his arrows in the ſame hand by which he holds the end of the chain, he there ſticks one of them down with it, while he holds the chain ſtretched. This done, he leaves the arrow in the ground, as a mark for the follower to come to, and advances another chain forward, being directed in his poſition by the follower ſtanding at the arrow, as before; as alſo by himſelf now, and at every ſucceeding chain's length, by moving himſelf from ſide to ſide, till he brings the follower and the back mark into a line. Having then ſtretched the chain, and ſtuck down an arrow, as before, the follower takes up his arrow, and they advance again in the ſame manner another chain-length. And thus they proceed till all the 10 arrows are employed, and are in the hands of the follower; and the leader, without an arrow, is arrived at the end of the 11th chain length. The follower then ſends or brings the 10 arrows to the leader, who puts one of them down at the end of his chain, and advances with the chain as before. And thus the arrows are changed from the one to the other at every 10 chains' length, till the whole line is finiſhed; the number of changes of the arrows ſhews the number of tens, to which the follower adds the arrows he holds in his hand, and the number of links of another chain over to the mark or end of the line. So if there have been 3 changes of the arrows, and the follower hold 6 arrows, and the end of the line cut off 45 links more, the whole length of the line is ſet down in links thus, 3645.

5. *To take Angles and Bearings.*



Let B and C be two objects, or two pickets ſet up perpendicular; and let it be required to take their bearings, or the angle formed between them at any ſtation A.

1ſt. *With the Plain Table.* The table being covered with a paper, and fixed on its ſtand; plant it at the ſtation A, and fix a fine pin, or a point of the compaſſes, in a proper part of the paper, to repreſent the point A: Cloſe by the ſide of this pin lay the fiducial edge of the index, and turn it about, ſtill touching the pin, till one object B can be ſeen through the ſights: then by the fiducial edge of the index draw a line. In the very ſame manner draw another line in the direction of the other object C. And it is done.

2d. *With the Theodolite, &c.* Direct the fixed ſights along one of the lines, as AB, by turning the inſtrument about till you ſee the mark B through theſe ſights; and there ſcrew the inſtrument faſt. Then turn

turn the moveable index about till, through its fights, you fee the other mark C. Then the degrees cut by the index, upon the graduated limb or ring of the inftrument, fhew the quantity of the angle.

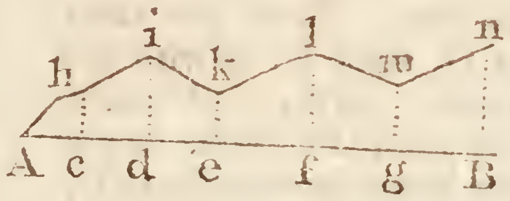
3*d*. *With the Magnetic Needle and Compafs*. Turn the inftrument, or compafs, fo, that the north end of the needle point to the flower-de-luce. Then direct the fights to one mark, as B; and note the degrees cut by the needle. Then direct the fights to the other mark C, and note again the degrees cut by the needle. Then their fum or difference, as the cafe is, will give the quantity of the angle BAC.

4*th*. *By Meafurement with the Chain*, &c. Meafure one chain length, or any other length, along both directions, as to b and c. Then meafure the diftance b c, and it is done.—This is eafily transferred to paper, by making a triangle A b c with thefe three lengths, and then meafuring the angle A as in Practical Geometry.

6. *To Meafure the Offsets.*

A h i k l m n being a crooked hedge, or river, &c. From A meafure in a ftraight direction along the fide of it to B. And in meafuring along this line AB obferve when you are directly oppofite any bends or corners of the hedge, as at c d, e, &c; and from thence meafure the perpendicular offsets, ch, di, &c, with the offset-ftaff, if they are not very large, otherwife with the chain itfelf; and the work is done. And the regifter, or field-book, may be as follows:

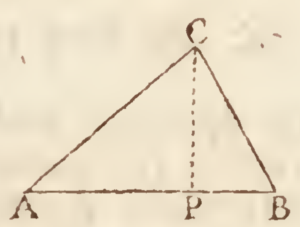
Offs. left.		Bafe line AB	
	0	⊙	A
ch	62	45	Ac
di	84	220	Ad
ek	70	340	Ae
fl	98	510	Af
gm	57	634	Ag
Bn	91	785	AB



7. *To Survey a triangular Field* ABC.

1*ft*. *By the Chain.*

AP	794
AB	1321
PC	826



Having fet up marks at the corners, which is to be done in all cafes where there are not marks naturally; meafure with the chain from A to P, where a perpendicular would fall from the angle C, and there meafure from P to C; then complete the diftance AB by meafuring from P to B; fetting down each of thefe meafured diftances. And thus, having the bafe and perpendicular, the area from them is eafily found. Or having the place P of the perpendicular, the triangle is eafily conftructed,

Or, meafure all the three fides with the chain, and note them down. From which the content is eafily found, or the figure conftructed.

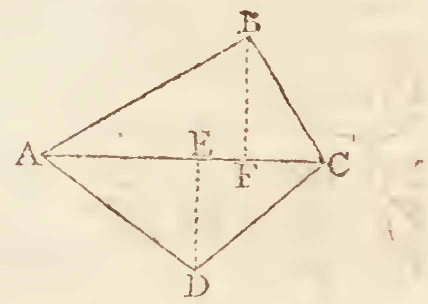
2*d*. *By taking one or more of the Angles.*

Meafure two fides AB, AC, and the angle A between them. Or meafure one fide AB, and the two adjacent angles A and B. From either of thefe ways the figure is eafily planned: then by meafuring the perpendicular CP on the plan, and multiplying it by half AB, you have the content.

8. *To meafure a Four-fided Field.*

1*ft*. *By the Chain.*

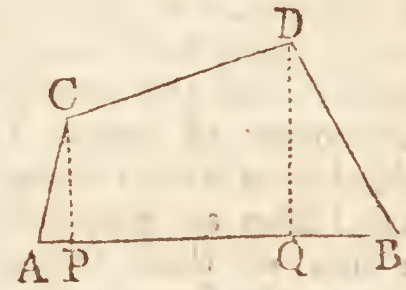
AE	214	210	DE
AF	362	306	BF
AC	592		



Meafure along either of the diagonals, as AC; and either the two perpendiculars DE, BF, as in the laft problem; or elfe the fides AB, BC, CD, DA. From either of which the figure may be planned and computed as before directed.

Otherwife by the Chain.

AP	110	352	PC
AQ	74	595	QD
AB	1110		



Meafure on the longeft fide, the diftances AP, AQ, AB; and the perpendiculars PC, QD.

2*d*. *By taking one or more of the Angles.*

Meafure the diagonal AC (fee the firft fig. above), and the angles CAB, CAD, ACB, ACD.—Or meafure the four fides, and any one of the angles as BAD.

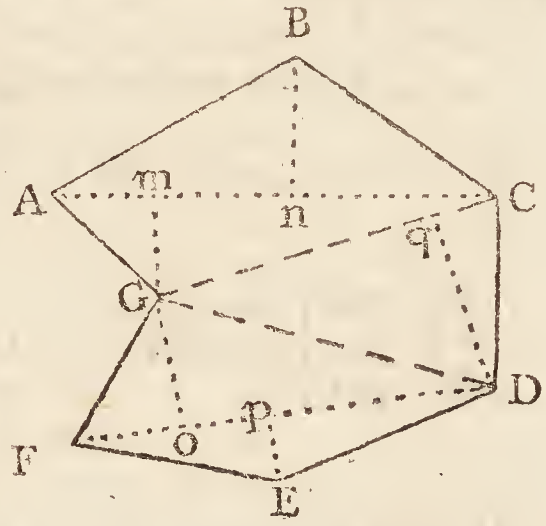
Thus		Or thus	
AC	591	AB	486
CAB	37°20′	BC	394
CAD	41 15	CD	410
ACB	72 25	DA	462
ACD	54 40	BAD	78°35′

9. *To Survey any Field by the Chain only.*

Having fet up marks at the corners, where neceffary, of the propofed field ABCDEFG. Walk over the ground, and confider how it can beft be divided into triangles and trapeziums; and meafure them feparately as in the laft two problems. And in this way it will be proper to divide it into as few feparate triangles, and as many trapeziums as may be, by drawing diagonals

nals from corner to corner: and ſo, as that all the perpendiculars may fall within the figure. Thus, the following figure is divided into the two trapeziums ABCG, GDEF, and the triangle GCD. Then, in the firſt, beginning at A, meaſure the diagonal AC, and the two perpendiculars Gm, Bn. Then the baſe GC and the perpendicular Dq. Laſtly the diagonal DF, and the two perpendiculars pE, oG. All which meaſures write againſt the correſponding parts of a rough figure drawn to reſemble the figure to be ſurveyed, or ſet them down in any other form you chooſe.

Am 135	130 mG
An 410	180 nB
Ac 550	
Cq 152	230 qD
CG 440	
FO 206	120 oG
FP 288	80 pE
FD 520	



Or thus:

Meaſure all the ſides AB, BC, CD, DE, EF, FG, and GA; and the diagonals AC, CG, GD, DF.

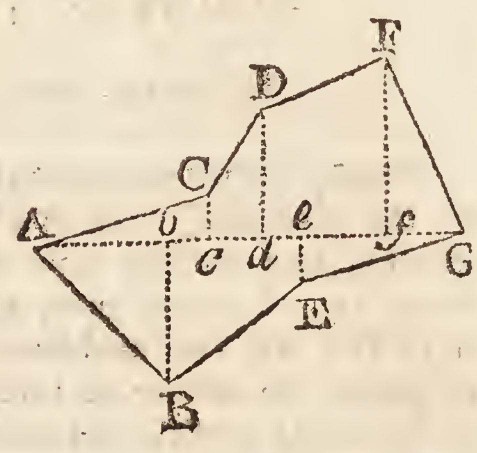
Otherwiſe,

Many pieces of land may be very well ſurveyed, by meaſuring any baſe line, either within or without them, together with the perpendiculars let fall upon it from every corner of them. For they are by theſe means divided into ſeveral triangles and trapezoids, all whoſe parallel ſides are perpendicular to the baſe line; and the ſum of theſe triangles and trapeziums will be equal to the figure propoſed if the baſe line fall within it; if not, the ſum of the parts which are without being taken from the ſum of the whole which are both within and without, will leave the area of the figure propoſed.

In pieces that are not very large, it will be ſufficiently exact to find the points, in the baſe line, where the ſeveral perpendiculars will fall, by means of the *croſs*, and from thence meaſuring to the corners for the lengths of the perpendiculars.—And it will be moſt convenient to draw the line ſo as that all the perpendiculars may fall within the figure.

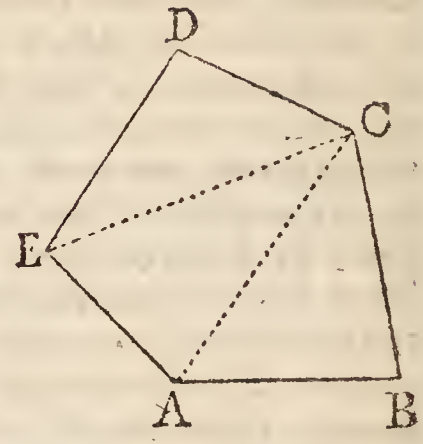
Thus, in the following figure, beginning at A, and meaſuring along the line AG, the diſtances and perpendiculars, on the right and left, are as below.

Ab	315	350 bB
Ac	440	70 cC
Ad	585	320 dD
Ae	610	50 eE
Af	990	470 fF
AG	1020	0



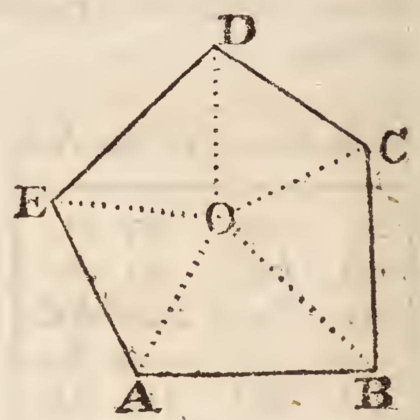
10. *To Survey any Field with the Plain Table.*

1ſt. From one Station.



Plant the table at any angle, as C, from whence all the other angles, or marks ſet up, can be ſeen; and turn the table about till the needle point to the flower-de-luce: and there ſcrew it faſt. Make a point for C on the paper on the table, and lay the edge of the index to C, turning it about there till through the ſights you ſee the mark D; and by the edge of the index draw a dry or obſcure line: then meaſure the diſtance CD, and lay that diſtance down on the line CD. Then turn the index about the point C, till the mark E be ſeen through the ſights, by which draw a line, and meaſure the diſtance to E, laying it on the line from C to E. In like manner determine the poſitions of CA and CB, by turning the ſights ſucceſſively to A and B; and lay the lengths of thoſe lines down. Then connect the points with the boundaries of the field, by drawing the black lines CD, DE, EA, AB, BC.

2d. From a Station within the Field.



When all the other parts cannot be ſeen from one angle, chooſe ſome place O within; or even without, if more convenient, from whence the other parts can be ſeen. Plant the table at O, then fix it with the needle north, and mark the point O upon it. Apply the index ſucceſſively to O, turning it round with the ſights to each angle A, B, C, D, E, drawing dry lines to them by the edge of the index, then meaſuring the diſtances OA, OB, &c, and laying them down upon thoſe lines. Laſtly draw the boundaries AB, BC, CD, DE, EA.

3d. By going round the Figure.

When the figure is a wood or water, or from ſome other obſtruction you cannot meaſure lines acroſs it; begin at any point A, and meaſure round it, either within or without the figure, and draw the directions of all the ſides thus: Plant the table at A, turn it with the needle to the north or flower-de-luce, fix it and mark the point A. Apply the index to A, turning it till you can ſee the point E, there draw a line; and then the point B, and there draw a line: then meaſure theſe lines, and lay them down from A to E and B. Next move the table to B, lay the index along the line AB, and turn the table about till you can ſee the mark A, and ſcrew faſt the table; in which poſition alſo the needle will again point to the flower-de-luce, as it will do.

do indeed at every ſtation when the table is in the right poſition. Here turn the index about B till through the ſights you ſee the mark C; there draw a line, meaſure BC, and lay the diſtance upon that line after you have ſet down the table at C. Turn it then again into its proper poſition, and in like manner find the next line CD. And ſo on quite round by E to A again. Then the proof of the work will be the joining at A: for if the work is all right, the laſt direction EA on the ground, will paſs exactly through the point A on the paper; and the meaſured diſtance will alſo reach exactly to A. If theſe do not coincide, or nearly ſo, ſome error has been committed, and the work muſt be examined over again.

11. *To Survey a Field with the Theodolite, &c.*

1ſt. From one Point or Station.

When all the angles can be ſeen from one point, as the angle C (laſt fig. but one), place the inſtrument at C, and turn it about till, through the fixed ſights, you ſee the mark B, and there fix it. Then turn the moveable index about till the mark A is ſeen through the ſights, and note the degrees cut on the inſtrument. Next turn the index ſucceſſively to E and D, noting the degrees cut off at each; which gives all the angles BCA, BCE, BCD. Laſtly, meaſure the lines CB, CA, CE, CD; and enter the meaſures in a field-book, or rather againſt the correſponding parts of a rough figure drawn by gueſs to reſemble the field.

2d. From a Point within or without.

Plant the inſtrument at O, (laſt fig.) and turn it about till the fixed ſights point to any object, as A; and there ſcrew it faſt. Then turn the moveable index round till the ſights point ſucceſſively to the other points E, D, C, B, noting the degrees cut off at each of them; which gives all the angles round the point O. Laſtly, meaſure the diſtances OA, OB, OC, OD, OE, noting them down as before, and the work is done.

3d. By going round the Field.

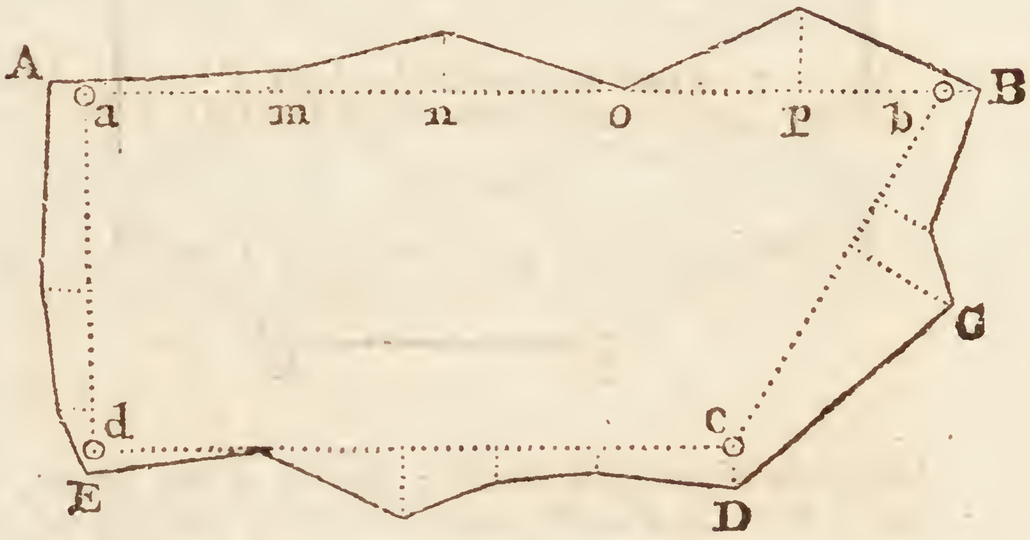
By meaſuring round, either within or without the field, proceed thus. Having ſet up marks at B, C, &c. near the corners as uſual, plant the inſtrument at any point A, and turn it till the fixed index be in the direction AB, and there ſcrew it faſt: then turn the moveable index to the direction AF; and the degrees cut off will be the angle A. Meaſure the line AB, and plant the inſtrument at B, and there in the ſame manner obſerve the angle A. Then meaſure BC, and obſerve the angle C. Then meaſure the diſtance CD, and take the angle D. Then meaſure DE, and take the angle E. Then meaſure EF, and take the angle F. And laſtly meaſure the diſtance FA.

To prove the work; add all the inward angles, A, B, C, &c, together, and when the work is right, their ſum will be equal to twice as many right angles as the figure has ſides, wanting 4 right angles. And when there is an angle, as F, that bends inwards, and you meaſure the external angle, which is leſs than two right angles, ſubtract it from 4 right angles, or 360 degrees, to give the internal angle greater than a ſemicircle or 180 degrees.

Otherwiſe. Inſtead of obſerving the internal angles, you may take the external angles, formed without the figure by producing the ſides further out. And in this caſe, when the work is right, their ſum altogether will be equal to 360 degrees. But when one of them, as F, runs inwards, ſubtract it from the ſum of the reſt, to leave 360 degrees.

12. *To Survey a Field with crooked Hedges, &c.*

With any of the inſtruments meaſure the lengths and poſitions of imaginary lines running as near the ſides of the field as you can; and in going along them meaſure the offſets in the manner before taught; and you will have the plan on the paper in uſing the plain table, drawing the crooked hedges through the ends of the offſets; but in ſurveying with the theodolite, or other inſtrument, ſet down the meaſures properly in a field-book, or memorandum-book, and plan them after returning from the field, by laying down all the lines and angles.



So, in ſurveying the piece ABCDE, ſet up marks a, b, c, d, dividing it into as few ſides as may be. Then begin at any ſtation a, and meaſure the lines ab, bc, cd, da, and take their poſitions, or the angles a, b, c, d; and in going along the lines meaſure all the offſets, as at m, n, o, p, &c, along every ſtation line.

And this is done either within the field, or without, as may be moſt convenient. When there are obſtructions within, as wood, water, hills, &c; then meaſure without, as in the figure here below.

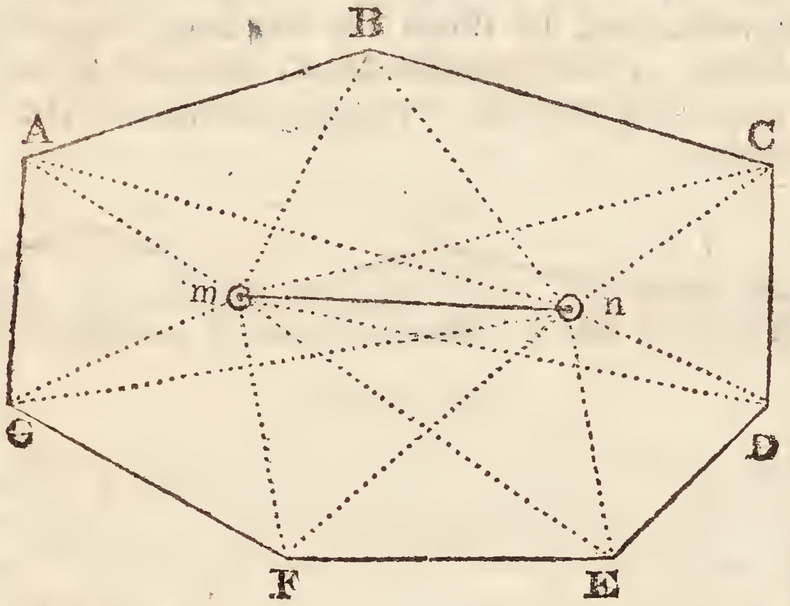
 13. To

13. *To Survey a Field or any other Thing, by Two Stations.*

This is performed by choosing two stations, from whence all the marks and objects can be seen, then measuring the distance between the stations, and at each station taking the angles formed by every object, from the station line or distance.

The two stations may be taken either within the bounds, or in one of the sides, or in the direction of two of the objects, or quite at a distance, and without the bounds of the objects, or part to be surveyed.

In this manner, not only grounds may be surveyed, without even entering them, but a map may be taken of the principal parts of a country, or the chief places of a town, or any part of a river or coast surveyed, or any other inaccessible objects; by taking two stations, on two towers, or two hills, or such like.



When the plain table is used; plant it at one station m, draw a line m n on it, along which lay the edge of the index, and turn the table about till the sights point directly to the other station; and there screw it fast. Then turn the sights round m successively to all the objects ABC, &c, drawing a dry line by the edge of the index at each, as mA, mB, mC, &c. Then measure the distance to the other station, there plant the table, and lay that distance down on the station line from m to n. Next lay the index by the line nm, and turn the table about till the sights point to the other station m, and there screw it fast. Then direct the sights successively to all the objects A, B, C, &c, as before, drawing lines each time, as nA, nB, nC, &c: and their intersection with the former lines will give the places of all the objects, or corners, A, B, C, &c.

When the theodolite, or any other instrument for taking angles, is used; proceed in the same way, measuring the station distance mn, planting the instrument first at one station, and then at another; then placing the fixed sights in the direction mn, and directing the moveable sights to every object, noting the degrees cut off at each time. Then, these observations being planned, the intersections of the lines will give the objects as before.

When all the objects, to be surveyed, cannot be seen from two stations; then three stations may be used, or four, or as many as is necessary; measuring always the distance from one station to another; placing the instrument in the same position at every station, by means described before; and from each station observing or setting every object that can be seen from it, by taking its direction or angular position, till every object be determined by the intersection of two or more lines of direction, the more the better. And thus may very extensive surveys be taken, as of large commons, rivers, coasts, countries, hilly grounds, and such like.

14. *To Survey a Large Estate.*

If the estate be very large, and contain a great number of fields, it cannot well be done by surveying all the fields singly, and then putting them together; nor can it be done by taking all the angles and boundaries that inclose it. For in these cases, any small errors will be so multiplied, as to render it very much distorted.

1st. Walk over the estate two or three times, in order to get a perfect idea of it, and till you can carry the map of it tolerably in your head. And to help your memory, draw an eye draught of it on paper, or at least, of the principal parts of it, to guide you.

2d. Choose two or more eminent places in the estate, for your stations, from whence you can see all the principal parts of it: and let these stations be as far distant from one another as possible; as the fewer stations you have to command the whole, the more exact your work will be: and they will be fitter for your purpose, if these station lines be in or near the boundaries of the ground, and especially if two or more lines proceed from one station.

3d. Take angles, between the stations, such as you think necessary, and measure the distances from station to station, always in a right line: these things must be done, till you get as many angles and lines as are sufficient for determining all your points of station. And in measuring any of these station distances, mark accurately where these lines meet with any hedges, ditches, roads, lanes, paths, rivulets, &c, and where any remarkable object is placed, by measuring its distance from the station line, and where a perpendicular from it cuts that line; and always mind, in any of these observations, that you be in a right line, which you will know by taking backsight and foresight, along your station line. And thus as you go along any main station line, take offsets to the ends of all hedges, and to any pond, house, mill, bridge, &c, omitting nothing that is remarkable. And all these things must be noted down; for these are your data, by which the places of such objects are to be determined upon your plan. And be sure to set marks up at the intersections of all hedges with the station line, that you may know where to measure from, when you come to survey these particular fields, which must immediately be done, as soon as you have measured that station line, whilst they are fresh in memory. In this way all your station lines are to be measured, and the situation of all places adjoining to them determined, which is the first grand point to be obtained. It will be proper for you to lay down your work upon paper every night, when you go home, that you may see how you go on.

4th. As to the inner parts of the estate, they must be deter-

determined in like manner, by new ſtation lines: for, after the main ſtations are determined, and every thing adjoining to them, then the eſtate muſt be ſubdivided into two or three parts by new ſtation lines; taking inner ſtations at proper places, where you can have the beſt view. Meaſure theſe ſtation lines as you did the firſt, and all their interſections with hedges, and all offsets to ſuch objects as appear. Then you may proceed to ſurvey the adjoining fields, by taking the angles that the ſides make with the ſtation line, at the interſections, and meaſuring the diſtances to each corner, from the interſections. For every ſtation line will be a baſis to all the future operations; the ſituation of all parts being entirely dependant upon them; and therefore they ſhould be taken of as great a length as poſſible; and it is beſt for them to run along ſome of the hedges or boundaries of one or more fields, or to paſs through ſome of their angles. All things being determined for theſe ſtations, you muſt take more inner ones, and ſo continue to divide and ſubdivide, till at laſt you come to ſingle fields; repeating the ſame work for the inner ſtations, as for the outer ones, till all be done: and cloſe the work as often as you can, and in as few lines as poſſible. And that you may chooſe ſtations the moſt conveniently, ſo as to cauſe the leaſt labour, let the ſtation lines run as far as you can along ſome hedges, and through as many corners of the fields, and other remarkable points, as you can. And take notice how one field lies by another; that you may not miſplace them in the draught.

5th. An eſtate may be ſo ſituated, that the whole cannot be ſurveyed together; becauſe one part of the eſtate cannot be ſeen from another. In this caſe, you may divide it into three or four parts, and ſurvey the parts ſeparately, as if they were lands belonging to different perſons; and at laſt join them together.

6th. As it is neceſſary to protract or lay down your work as you proceed in it, you muſt have a ſcale of a due length to do it by. To get ſuch a ſcale, you muſt meaſure the whole length of the eſtate in chains; then you muſt conſider how many inches in length the map is to be; and from theſe you will know how many chains you muſt have in an inch; then make your ſcale, or chooſe one already made, accordingly.

7th. The trees in every hedge row muſt be placed in their proper ſituation, which is ſoon done by the plain table; but may be done by the eye without an inſtrument; and being thus taken by gueſs, in a rough draught, they will be exact enough, being only to look at; except it be ſuch as are at any remarkable places, as at the ends of hedges, at ſtiles, gates, &c, and theſe muſt be meaſured. But all this need not be done till the draught is finiſhed. And obſerve in all the hedges, what ſide the gutter or ditch is on, and conſequently to whom the fences belong.

8th. When you have long ſtations, you ought to have a good inſtrument to take angles with; and the plain table may very properly be made uſe of, to take the ſeveral ſmall internal parts, and ſuch as cannot be taken from the main ſtations, as it is a very quick and ready inſtrument.

15. Inſtead of the foregoing method, an ingenious friend (Mr. Abraham Crocker), after mentioning the new and improved method of keeping the field book by writing from bottom to top of the pages, obſerves that "In the former method of meaſuring a large eſtate, the accuracy of it depends on the correctneſs of the inſtruments uſed in taking the angles. To avoid the errors incident to ſuch a multitude of angles, other methods have of late years been uſed by ſome few ſkilful ſurveyors; the moſt practical, expeditious, and correct, ſeems to be the following.

"As was adviſed in the foregoing method, ſo in this, chooſe two or more eminences, as grand ſtations, and meaſure a principal baſe line from one ſtation to the other, noting every hedge, brook, or other remarkable object as you paſs by it; meaſuring alſo ſuch ſhort perpendicular lines to ſuch bends of hedges as may be near at hand. From the extremities of this baſe line, or from any convenient parts of the ſame, go off with other lines to ſome remarkable object ſituated towards the ſides of the eſtate, without regarding the angles they make with the baſe line or with one another; ſtill remembering to note every hedge, brook or other object that you paſs by. Theſe lines, when laid down by interſections, will with the baſe line form a grand triangle upon the eſtate; ſeveral of which, if need be, being thus laid down, you may proceed to form other ſmaller triangles and trapezoids on the ſides of the former: and ſo on, until you finiſh with the encloſures individually.

"To illuſtrate this excellent method, let us take AB (in the plan of an eſtate, fig. 1, pl. 28) for the principal baſe line. From B go off to the tree at C; noting down, in the field-book, every croſs hedge, as you meaſure on; and from C meaſure back to the firſt ſtation at A, noting down every thing as before directed.

"This grand triangle being completed, and laid down on the rough-plan paper, the parts, exterior as well as interior, are to be completed by ſmaller triangles and trapezoids.

"When the whole plan is laid down on paper, the contents of each field might be calculated by the methods laid down below, at article 20.

"In countries where the lands are encloſed with high hedges, and where many lanes paſs through an eſtate, a theodolite may be uſed to advantage, in meaſuring the angles of ſuch lands; by which means, a kind of ſkeleton of the eſtate may be obtained, and the lane-lines ſerve as the baſes of ſuch triangles and trapezoids as are neceſſary to fill up the interior parts."

The method of meaſuring the other croſs lines, offſets and interior parts and encloſures, appears in the plan, fig. 1, laſt referred to.

16. Another ingenious correſpondent (Mr. John Rodham of Richmond, Yorkſhire) has alſo communicated the following example of the new method of ſurveying, accompanied by the field-book, and its correſponding plan. His account of the method is as follows.

The field-book is ruled into three columns. In the middle one are ſet down the diſtances on the chain line at which any mark, offset, or other obſervation is made; and in the right and left hand columns are entered, the offsets and obſervations made on the right and left hand reſpectively of the chain line.

It is of great advantage, both for brevity and perſpicuity,

ſpicuity, to begin at the bottom of the leaf and write upwards; denoting the croſſing of fences, by lines drawn acroſs the middle column, or only a part of ſuch a line on the right and left oppoſite the figures, to avoid confuſion, and the corners of fields, and other remarkable turns in the fences where offsets are taken to, by lines joining in the manner the fences do, as will be beſt ſeen by comparing the book with the plan annexed, fig. 2, pl. 28.

The marks called, *a*, *b*, *c*, &c, are beſt made in the fields, by making a ſmall hole with a ſpade, and a chip or ſmall bit of wood, with the particular letter upon it, may be put in, to prevent one mark being taken for another, on any return to it. But in general, the name of a mark is very eaſily had by referring in the book to the line it was made in. After the ſmall alphabet is gone through, the capitals may be next, the print letters afterwards, and ſo on, which anſwer the purpoſe of ſo many different letters; or the marks may be numbered.

The letter in the left hand corner at the beginning of every line, is the mark or place meaſured *from*; and, that at the right hand corner at the end, is the mark meaſured *to*: But when it is not convenient to go exactly from a mark, the place meaſured from, is deſcribed *ſuch a diſtance* from *one mark* towards *another*; and where a mark is not meaſured to, the exact place is aſcertained by ſaying, turn to the right or left hand, *ſuch a diſtance* to *ſuch a mark*, it being always underſtood that thoſe diſtances are taken in the chain line.

The characters uſed, are ⌐ for *turn to the right hand*, ¬ for turn to the left hand, and ^ placed over an offset, to ſhew that it is not taken at right angles with the chain line, but in the line with ſome ſtraight fence; being chiefly uſed when croſſing their directions, and is a better way of obtaining their true places than by offsets at right angles.

When a line is meaſured whoſe poſition is determined, either by former work (as in the caſe of producing a given line or meaſuring from one known place or mark to another) or by itſelf (as in the third ſide of a triangle) it is called a *faſt line*, and a double line acroſs the book is drawn at the concluſion of it; but if its poſition is not determined (as in the ſecond ſide of a triangle) it is called a *looſe line*, and a ſingle line is drawn acroſs the book. When a line becomes determined in poſition, and is afterwards continued, a double line half through the book is drawn.

When a looſe line is meaſured, it becomes abſolutely neceſſary to meaſure ſome line that will determine its poſition. Thus, the firſt line *ab*, being the baſe of a triangle, is always determined; but the poſition of the ſecond ſide *hj*, does not become determined, till the third ſide *jb* is meaſured; then the triangle may be conſtructed, and the poſition of both is determined.

At the beginning of a line, to fix a looſe line to the mark or place meaſured from, the ſign of turning to the right or left hand muſt be added (as at *j* in the third line); otherwiſe a ſtranger, when laying down the work may as eaſily conſtruct the triangle *hjb* on the wrong ſide of the line *ab*, as on the right one: but this error cannot be fallen into, if the ſign above named be carefully obſerved.

In chooſing a line to fix a looſe one, care muſt be taken that it does not make a very acute or obtuſe angle; as in the triangle *p*B*r*, by the angle at B being very obtuſe, a ſmall deviation from truth, even the breadth of a point at *p* or *r*, would make the error at B when conſtructed very conſiderable; but by conſtructing the triangle *p*B*q*, ſuch a deviation is of no conſequence.

Where the words *leave off* are written in the field-book, it is to ſignify that the taking of offsets is from thence diſcontinued; and of courſe ſomething is wanting between that and the next offset.

The field-book above referred to, is engraved on plate 29, in parts, repreſenting ſo many pages, each of which is ſuppoſed to begin at the bottom, and end at top. And the map or plan belonging to it, in fig. 2, pl. 28.

17. *To Survey a County, or Large Tract of Land.*

1ſt. Chooſe two, three, or four eminent places for ſtations; ſuch as the tops of high hills or mountains, towers, or church ſteeples, which may be ſeen from one another; and from which moſt of the towns, and other places of note, may alſo be ſeen. And let them be as far diſtant from one another as poſſible. Upon theſe place raiſe beacons, or long poles, with flags of different colours flying at them; ſo as to be viſible from all the other ſtations.

2d. At all the places, which you would ſet down in the map, plant long poles with flags at them of ſeveral colours, to diſtinguiſh the places from one another; fixing them upon the tops of church ſteeples, or the tops of houſes, or in the centres of leſſer towns.

But you need not have theſe marks at many places at once, as ſuppoſe half a ſcore at a time. For when the angles have been taken, at the two ſtations, to all theſe places, the marks may be moved to new ones; and ſo ſucceſſively to all the places you want. Theſe marks then being ſet up at a convenient number of places, and ſuch as may be ſeen from both ſtations; go to one of theſe ſtations, and with an inſtrument to take angles, ſtanding at that ſtation, take all the angles between the other ſtation, and each of theſe marks, obſerving which is blue, which red, &c, and which hand they lie on; and ſet all down with their colours. Then go to the other ſtation, and take all the angles between the firſt ſtation, and each of the former marks, and ſet them down with the others, each againſt his fellow with the ſame colour. You may, if you can, alſo take the angles at ſome third ſtation, which may ſerve to prove the work, if the three lines interſect in that point, where any mark ſtands. The marks muſt ſtand till the obſervations are finiſhed at both ſtations; and then they muſt be taken down, and ſet up at freſh places. And the ſame operations muſt be performed, at both ſtations, for theſe freſh places; and the like for others. Your inſtrument for taking angles muſt be an exceeding good one, made on purpoſe with teleſcopic ſights; and of three, four, or five feet radius. A circumferentor is reckoned a good inſtrument for this purpoſe.

3d. And though it is not abſolutely neceſſary to meaſure any diſtance, becauſe a ſtationary line being laid down from any ſcale, all the other lines will be proportional

proportional to it; yet it is better to meaſure ſome of the lines, to aſcertain the diſtances of places in miles; and to know how many geometrical miles there are in any length; and from thence to make a ſcale to meaſure any diſtance in miles. In meaſuring any diſtance, it will not be exact enough to go along the high roads; by reaſon of their turnings and windings, and hardly ever lying in a right line between the ſtations, which would cauſe endleſs reductions, and create trouble to make it a right line; for which reaſon it can never be exact. But a better way is to meaſure in a right line with a chain, between ſtation and ſtation, over hills and dales or level fields, and all obſtacles. Only in caſe of water, woods, towns, rocks, banks, &c, where one cannot paſs, ſuch parts of the line muſt be meaſured by the methods of inacceſſible diſtances; and beſides, allowing for aſcents and deſcents, when we meet with them. And a good compaſs that ſhews the bearing of the two ſtations, will always direct you to go ſtraight, when you do not ſee the two ſtations; and in your progreſs, if you can go ſtraight, you may take offſets to any remarkable places, likewiſe noting the interſection of the ſtationary line with all roads, rivers, &c.

4th. And from all the ſtations, and in the whole progreſs, be very particular in obſerving ſea coaſts, river mouths, towns, caſtles, houſes, churches, windmills, watermills, trees, rocks, ſands, roads, bridges, fords, ferries, woods, hills, mountains, rills, brooks, parks, beacons, ſluices, floodgates, locks, &c; and in general all things that are remarkable.

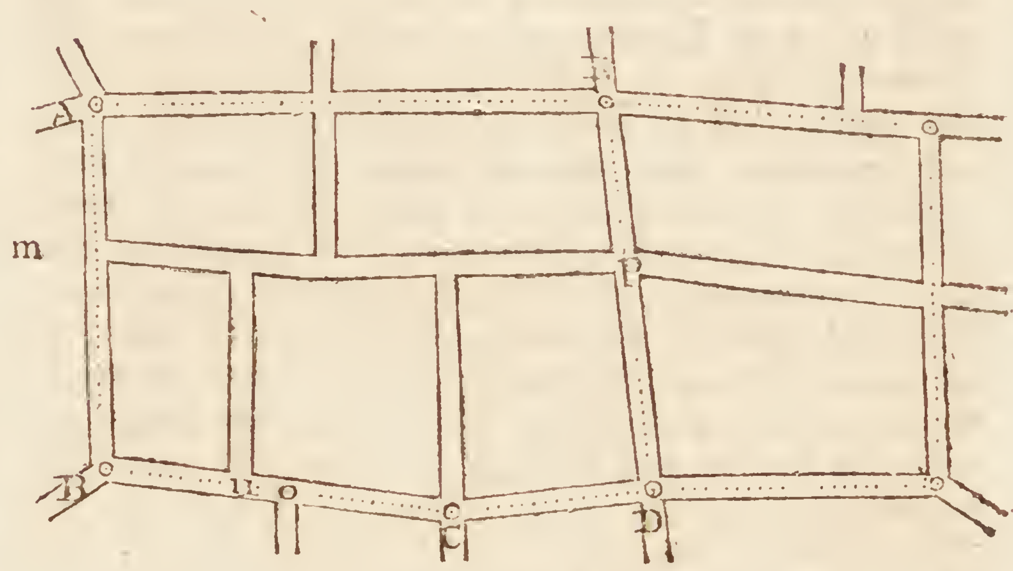
5th. After you have done with the firſt and main ſtation lines, which command the whole county; you muſt then take inner ſtations, at ſome places already determined; which will divide the whole into ſeveral partitions: and from theſe ſtations you muſt determine the places of as many of the remaining towns as you can. And if any remain in that part, you muſt take more ſtations, at ſome places already determined; from which you may determine the reſt. And thus proceed through all the parts of the country, taking ſtation after ſtation, till we have determined all we want. And in general the ſtation diſtances muſt always paſs through ſuch remarkable points as have been determined before, by the former ſtations.

6th. Laſtly, the poſition of the ſtation line you meaſure, or the point of the compaſs it lies on, muſt be determined by aſtronomical obſervation. Hang up a thread and plummet in the ſun, over ſome part of the ſtation line, and obſerve when the ſhadow runs along that line, and at that moment take the ſun's altitude; then having his declination, and the latitude, the azimuth will be found by ſpherical trigonometry. And the azimuth is the angle the ſtation line makes with the meridian; and therefore a meridian may eaſily be drawn through the map: Or a meridian may be drawn through it by hanging up two threads in a line with the pole ſtar, when he is juſt north, which may be known from aſtronomical tables. Or thus; obſerve the ſtar Alioth, or that in the rump of the great bear, being that next the ſquare; or elſe Caſſiopeia's hip; I ſay, obſerve by a line and plummet when either of theſe ſtars and the pole ſtar come into a perpendicular; and at that time they are due north. Therefore two perpendicular lines being fixed at that moment, towards theſe two ſtars, will give the poſition of the meridian.

18. *To Survey a Town or City.*

This may be done with any of the inſtruments for taking angles, but beſt of all with the plain table, where every minute part is drawn while in ſight. It is proper alſo to have a chain of 50 feet long, divided into 50 links, and an offſet-ſtaff of 10 feet long.

Begin at the meeting of two or more of the principal ſtreets, through which you can have the longeſt proſpects, to get the longeſt ſtation lines. There having fixed the inſtrument, draw lines of direction along thoſe ſtreets, uſing two men as marks, or poles ſet in wooden pedeſtals, or perhaps ſome remarkable places in the houſes at the farther ends, as windows, doors, corners, &c. Meaſure theſe lines with the chain, taking offſets with the ſtaff, at all corners of ſtreets, bendings, or windings, and to all remarkable things, as churches, markets, halls, colleges, eminent houſes, &c. Then remove the inſtrument to another ſtation along one of theſe lines; and there repeat the ſame proceſs as before. And ſo on till the whole is finiſhed.



Thus, fix the inſtrument at A, and draw lines in the direction of all the ſtreets meeting there; and meaſure AB, noting the ſtreet on the left at m. At the ſecond ſtation B, draw the directions of the ſtreets meeting there; meaſure from B to C, noting the places of the ſtreets at n and o as you paſs by them. At the 3d ſtation C, take the direction of all the ſtreets meeting there, and meaſure CD. At D do the ſame, and meaſure DE, noting the place of the croſs ſtreets at p. And in this manner go through all the principal ſtreets. This done, proceed to the ſmaller and intermediate ſtreets; and laſtly to the lanes, alleys, courts, yards, and every part that it may be thought proper to repreſent.

Of Planning, Computing, and Dividing.

19. *To Lay down the Plan of any Survey.*

If the ſurvey was taken with a plain table, you have a rough plan of it already on the paper which covered the

the table. But if the survey was with any other instrument, a plan of it is to be drawn from the measures that were taken in the survey, and first of all a rough plan upon paper.

To do this, you must have a set of proper instruments, for laying down both lines and angles, &c; as scales of various sizes, the more of them, and the more accurate, the better; scales of chords, protractors, perpendicular and parallel rulers, &c. Diagonal scales are best for the lines, because they extend to three figures, or chains and links, which are hundredth parts of chains. And in using the diagonal scale, a pair of compasses must be employed to take off the lengths of the principal lines very accurately. But a scale with a thin edge divided, is much readier for laying down the perpendicular offsets to crooked hedges, and for marking the places of those offsets upon the station line; which is done at only one application of the edge of the scale to that line, and then pricking off all at once the distances along it. Angles are to be laid down either with a good scale of chords, which is perhaps the most accurate way; or with a large protractor, which is much readier when many angles are to be laid down at one point, as they are pricked off all at once round the edge of the protractor.

Very particular directions for laying down all sorts of figures cannot be necessary in this place, to any person who has learned practical geometry, or the construction of figures, and the use of his instruments. It may therefore be sufficient to observe, that all lines and angles must be laid down on the plan in the same order in which they were measured in the field, and in which they are written in the field-book; laying down first the angles for the position of lines, then the lengths of the lines, with the places of the offsets, and then the lengths of the offsets themselves, all with dry or obscure lines; then a black line drawn through the extremities of all the offsets, will be the hedge or bounding line of the field, &c. After the principal bounds and lines are laid down, and made to fit or close properly, proceed next to the smaller objects, till you have entered every thing that ought to appear in the plan, as houses, brooks, trees, hills, gates, stiles, roads, lanes, mills, bridges, woodlands, &c, &c.

The north side of a map or plan is commonly placed uppermost, and a meridian somewhere drawn, with the compass or flower-de-luce pointing north. Also, in a vacant place, a scale of equal parts or chains is drawn, and the title of the map in conspicuous characters, and embellished with a compartment. All hills must be shadowed, to distinguish them in the map. Colour the hedges with different colours; represent hilly grounds by broken hills and valleys; draw single dotted lines for foot-paths, and double ones for horse or carriage roads. Write the name of each field and remarkable place within it, and, if you choose, its content in acres, roods, and perches.

In a very large estate, or a county, draw vertical and horizontal lines through the map, denoting the spaces between them by letters, placed at the top, and bottom, and sides, for readily finding any field or other object, mentioned in a table.

In mapping counties, and estates that have uneven grounds of hills and valleys, reduce all oblique lines, measured up hill and down hill, to horizontal straight lines, if that was not done during the survey, before they were entered in the field-book, by making a proper allowance to shorten them. For which purpose, there is commonly a small table engraven on some of the instruments for Surveying.

20. *To Compute the Contents of Fields.*

1st. Compute the contents of the figures, whether triangles, or trapeziums, &c, by the proper rules for the several figures laid down in measuring; multiplying the lengths by the breadths, both in links; the product is acres after you have cut off five figures on the right, for decimals; then bring these decimals to roods and perches, by multiplying first by 4, and then by 40. An example of which was given in the description of the chain, art. 1.

2d. In small and separate pieces, it is usual to cast up their contents from the measures of the lines taken in surveying them, without making a correct plan of them.

Thus, in the triangle in art. 7, where we had AP = 794, and AB = 1321

```
            PC = 826
                 ----
                 7926
                2642
               10568
              -------
          2 ) 10·91146
               5·45573        ac r p
                     4   Ans. 32 1 33 nearly
              -------
               1·82292
                    40
              -------
              32·91680
              -------
```

Or the first example to art. 8, thus:

```
AE 214 | 210 ED
AF 362 | 306 FB
AC 592 | ----
         516 sum of perp.
         592 AC
         ----
         1032
        4644
       2580
       -------
       3·05472            ac r p
             4       Ans. 3 0 8
       -------
        .21888
            40
       -------
       8·75520
       -------
```


Or

Or the 2d example to the same article, thus:

AP	110	352	PC
AQ	745	595	QD
AB	1110		

PC 352	PC 352	QD 595
AP 110	QD 595	QB 365
2 APC 38720	sum 947	2975
	PQ 635	3570
	4735	1785
	2841	217175 = 2QDB
	5682	601345 = 2PCDQ
		38720 = 2APC
2PCDQ 601345		2) 8·57240 = dou. the whole
		4·2862
		4
ac r p		1·1448
Ans. 4 1 5		40
		5·7920

3d. In pieces bounded by very crooked and winding hedges, measured by offsets, all the parts between the offsets are most accurately measured separately as small trapezoids. Thus, for the example to art. 6, where

Ac	45	62	ch
Ad	220	84	di
Ae	340	70	ek
Af	510	98	fl
Ag	634	57	gm
AB	785	91	Bn

Then

Ac 45	ch 62	di 84	ek 70	fl 98	gm 57
ch 62	di 84	ek 70	fl 98	gm 57	Bn 91
90	146	154	168	145	148
270	cd 175	de 120	ef 170	fg 124	gB 151
2790	730	18480	11760	580	148
	1022		168	290	740
	146			145	148
			28560		
	25550			17980	22348

2790
25550
18480
28560
17980
22348

2) 1·15708		ac r p
·57854	Content	0 2 12
4		
2·31416		
40		
12·56640		

4th. Sometimes such pieces as that above, are computed by finding a mean breadth, by dividing the sum of the offsets by the number of them, accounting that for one of them where the boundary meets the station line, as at A; then multiply the length AB by that mean breadth.

Thus:

00	785	AB
62	66	mean breadth
84		
70	4710	ac r p
98	4710	Content 0 2 2 by this method, which is 10 perches too little.
57		
91	·51810	
	4	
7) 462		
66	2·07240	
	40	
	2·89600	

But this method is always erroneous, except when the offsets stand at equal distances from one another.

5th. But in larger pieces, and whole estates, consisting of many fields, it is the common practice to make a rough plan of the whole, and from it compute the contents quite independent of the measures of the lines and angles that were taken in Surveying. For then new lines are drawn in the fields in the plan, so as to divide them into trapeziums and triangles, the bases and perpendiculars of which are measured on the plan by means of the scale from which it was drawn, and so multiplied together for the contents. In this way the work is very expeditiously done, and sufficiently correct; for such dimensions are taken, as afford the most easy method of calculation; and, among a number of parts, thus taken and applied to a scale, it is likely that some of the parts will be taken a small matter too little, and others too great; so that they will, upon the whole, in all probability, very nearly balance one another. After all the fields, and particular parts, are thus computed separately, and added all together into one sum, calculate the whole estate independent of the fields, by dividing it into large and arbitrary triangles and trapeziums, and add these also together. Then if this sum be equal to the former, or nearly so, the work is right; but if the sums have any considerable difference, it is wrong, and they must be examined, and recomputed, till they nearly agree.

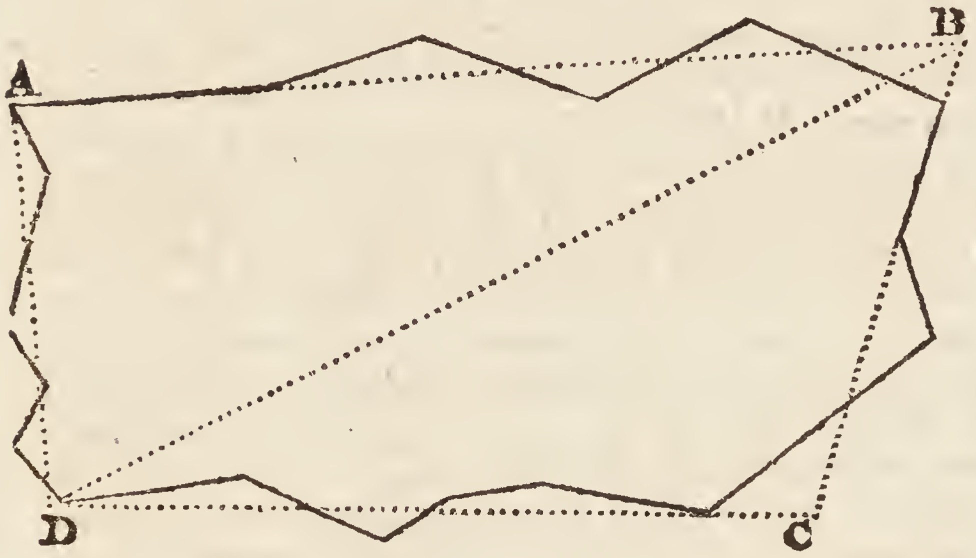
A specimen of dividing into one triangle, or one trapezium, which will do for most single fields, may be seen in the examples to the last article; and a specimen of dividing a large tract into several such trapeziums and triangles, in article 9, where a piece is so divided, and its dimensions taken and set down; and again in articles 15, 16.

6th. But the chief secret in casting up, consists in finding the contents of pieces bounded by curved, or very irregular lines, or in reducing such crooked sides of fields or boundaries to straight lines, that shall inclose the same or equal area with those crooked sides, and so obtain the area of the curved figure by means of the right-lined one, which will commonly be a trapezium.

zium. Now this reducing the crooked ſides to ſtraight ones, is very eaſily and accurately performed thus: Apply the ſtraight edge of a thin, clear piece of lanthorn-horn to the crooked line, which is to be reduced, in ſuch a manner, that the ſmall parts cut off from the crooked figure by it, may be equal to thoſe which are taken in: which equality of the parts included and excluded, you will preſently be able to judge of very nicely by a little practice: then with a pencil draw a line by the ſtraight edge of the horn. Do the ſame by the other ſides of the field or figure. So ſhall you have a ſtraight-ſided figure equal to the curved one; the contents of which, being computed as before directed, will be the content of the curved figure propoſed.

Or, inſtead of the ſtraight edge of the horn, a horſe-hair may be applied acroſs the crooked ſides in the ſame manner; and the eaſieſt way of uſing the hair, is to ſtring a ſmall ſlender bow with it, either of wire, or cane, or whale-bone, or ſuch like ſlender ſpringy matter; for, the bow keeping it always ſtretched, it can be eaſily and neatly applied with one hand, while the other is at liberty to make two marks by the ſide of it, to draw the ſtraight line by.

Ex. Thus, let it be required to find the contents of the ſame figure as in art. 12, to a ſcale of 4 chains to an inch.



Draw the four dotted ſtraight lines AB, BC, CD, DA, cutting off equal quantities on both ſides of them, which they do as near as the eye can judge: ſo is the crooked figure reduced to an equivalent right-lined one of four ſides ABCD. Then draw the diagonal BD, which, by applying a proper ſcale to it, meaſures 1256. Alſo the perpendicular, or neareſt diſtance, from A to this diagonal, meaſures 456; and the diſtance of C from it, is 428. Then

$$\begin{array}{r}456\\428\\\hline 884\\1256\\\hline 5024\\10048\\10048\;\;\\\hline 1110304\end{array}\qquad\qquad\begin{array}{r}2\;)\;11\cdot10304\\5\cdot55152\\4\\\hline 2\cdot20608\\40\\\hline 8\cdot24320\\\hline\end{array}$$

And thus the content of the trapezium, and conſequently of the irregular figure, to which it is equal, is eaſily found to be 5 acres, 2 roods, 8 perches.

21. *To Transfer a Plan to another Paper, &c.*

After the rough plan is completed, and a fair one is wanted; this may be done, either on paper or vellum, by any of the following Methods.

Firſt Method.—Lay the rough plan upon the clean paper, and keep them always preſſed flat and cloſe together, by weights laid upon them. Then, with the point of a fine pin or pricker, prick through all the corners of the plan to be copied. Take them aſunder, and connect the pricked points on the clean paper, with lines; and it is done. This method is only to be practiſed in plans of ſuch figures as are ſmall and tolerably regular, or bounded by right lines.

Second Method.—Rub the back of the rough plan over with black lead powder; and lay the ſaid black part upon the clean paper, upon which the plan is to be copied, and in the proper poſition. Then, with the blunt point of ſome hard ſubſtance, as braſs, or ſuch like, trace over the lines of the whole plan; preſſing the tracer ſo much as that the black lead under the lines may be transferred to the clean paper; after which take off the rough plan, and trace over the leaden marks with common ink, or with Indian ink, &c.—Or, inſtead of blacking the rough plan, you may keep conſtantly a blacked paper to lay between the plans.

Third Method.—Another way of copying plans, is by means of ſquares. This is performed by dividing both ends and ſides of the plan, which is to be copied, into any convenient number of equal parts, and connecting the correſponding points of diviſion with lines; which will divide the plan into a number of ſmall ſquares. Then divide the paper, upon which the plan is to be copied, into the ſame number of ſquares, each equal to the former when the plan is to be copied of the ſame ſize, but greater or leſs than the others, in the proportion in which the plan is to be increaſed or diminiſhed, when of a different ſize. Laſtly, copy into the clean ſquares, the parts contained in the correſponding ſquares of the old plan; and you will have the copy either of the ſame ſize, or greater or leſs in any proportion.

Fourth Method.—A fourth way is by the inſtrument called a pentagraph, which alſo copies the plan in any ſize required.

Fifth Method.—But the neateſt method of any is this. Procure a copying frame or glaſs, made in this manner; namely, a large ſquare of the beſt window glaſs, ſet in a broad frame of wood, which can be raiſed up to any angle, when the lower ſide of it reſts on a table. Set this frame up to any angle before you, facing a ſtrong light; fix the old plan and clean paper together with ſeveral pins quite around, to keep them together, the clean paper being laid uppermoſt, and upon

upon the face of the plan to be copied. Lay them, with the back of the old plan, upon the glaſs, namely, that part which you intend to begin at to copy firſt; and, by means of the light ſhining through the papers, you will very diſtinctly perceive every line of the plan through the clean paper. In this ſtate then trace all the lines on the paper with a pencil. Having drawn that part which covers the glaſs, ſlide another part over the glaſs, and copy it in the ſame manner. And then another part. And ſo on till the whole be copied.

Then, take them aſunder, and trace all the pencil-lines over with a fine pen and Indian ink, or with common ink.

And thus you may copy the fineſt plan, without injuring it in the leaſt.

When the lines, &c, are copied upon the clean paper or vellum, the next buſineſs is to write ſuch names, remarks, or explanations as may be judged neceſſary; laying down the ſcale for taking the lengths of any parts, a flower-de-luce to point out the direction, and the proper title ornamented with a compartment; and illuſtrating or colouring every part in ſuch manner as ſhall ſeem moſt natural, ſuch as ſhading rivers or brooks with crooked lines, drawing the repreſentations of trees, buſhes, hills, woods, hedges, houſes, gates, roads, &c, in their proper places; running a ſingle dotted line for a foot path, and a double one for a carriage road; and either repreſenting the baſes or the elevations of buildings, &c.

22. *Of the Diviſion of Lands.*

In the diviſion of commons, after the whole is ſurveyed and caſt up, and the proper quantities to be allowed for roads, &c, deducted, divide the net quantity remaining among the ſeveral proprietors, by the rule of Fellowſhip, in proportion to the real value of their eſtates, and you will thereby obtain their proportional quantities of the land. But as this diviſion ſuppoſes the land, which is to be divided, to be all of an equal goodneſs, you muſt obſerve that if the part in which any one's ſhare is to be marked off, be better or worſe than the general mean quality of the land, then you muſt diminiſh or augment the quantity of his ſhare in the ſame proportion.

Or, which comes to the ſame thing, divide the ground among the claimants in the direct ratio of the value of their claims, and the inverſe ratio of the quality of the ground allotted to each; that is, in proportion to the quotients ariſing from the diviſion of the value of each perſon's eſtate, by the number which expreſſes the quality of the ground in his ſhare.

But theſe regular methods cannot always be put in practice; ſo that, in the diviſion of commons, the uſual way is, to meaſure ſeparately all the land that is of different values, and add into two ſums the contents and the values; then, the value of every claimant's ſhare is found, by dividing the whole value among them in proportion to their eſtates; and, laſtly, by the 24th article, a quantity is laid out for each perſon, that ſhall be of the value of his ſhare before found.

23. *It is required to divide any given Quantity of Ground, or its Value, into any given Number of Parts, and in Proportion as any given Numbers.*

Divide the given piece, or its value, as in the rule of Fellowſhip, by dividing the whole content or value by the ſum of the numbers expreſſing the proportions of the ſeveral ſhares, and multiplying the quotient ſeverally by the ſaid proportional numbers for the reſpective ſhares required, when the land is all of the ſame quality. But if the ſhares be of different qualities, then divide the numbers expreſſing the proportions or values of the ſhares, by the numbers which expreſs the qualities of the land in each ſhare; and uſe the quotients inſtead of the former proportional numbers.

Ex. 1. If the total value of a common be 2500 pounds, it is required to determine the values of the ſhares of the three claimants A, B, C, whoſe eſtates are of theſe values, 10000, and 15000, and 25000 pounds.

The eſtates being in proportion as the numbers 2, 3, 5, whoſe ſum is 10, we ſhall have $2500 \div 10 = 250$; which being ſeverally multiplied by 2, 3, 5, the products 500, 750, 1250, are the values of the ſhares required.

Ex. 2. It is required to divide 300 acres of land among A, B, C, D, E, F, G, and H, whoſe claims upon it are reſpectively in proportion as the numbers 1, 2, 3, 5, 8, 10, 15, 20.

The ſum of theſe proportional numbers is 64, by which dividing 300, the quotient is 4 ac. 2 r. 30 p. which being multiplied by each of the numbers, 1, 2, 3, 5, &c, we obtain for the ſeveral ſhares as below:

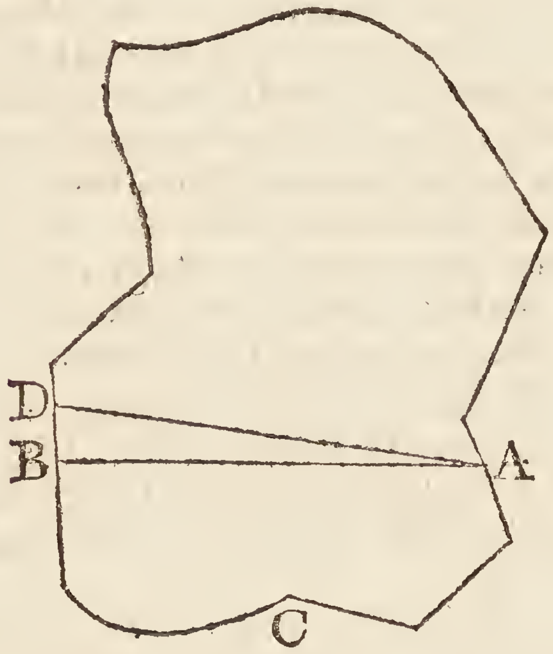
	Ac.	R.	P.
A =	4	2	30
B =	9	1	20
C =	14	0	10
D =	23	1	30
E =	37	2	00
F =	46	3	20
G =	70	1	10
H =	93	3	00
Sum =	300	0	00

Ex. 3. It is required to divide 780 acres among A, B, and C, whoſe eſtates are 1000, 3000, and 4000 pounds a year; the ground in their ſhares being worth 5, 8, and 10 ſhillings the acre reſpectively.

Here their claims are as 1, 3, 4; and the qualities of their land are as 5, 8, 10; therefore their quantities muſt be as $\frac{1}{5}$, $\frac{3}{8}$, $\frac{4}{10}$, or, by reduction, as 8, 15, 16. Now the ſum of theſe numbers is 39; by which dividing the 780 acres, the quotient is 20; which being multiplied ſeverally by the three numbers 8, 15, 16, the three products are 160, 300, 320, for the ſhares of A, B, C, reſpectively.

24. *To Cut off from a Plan a Given Number of Acres, &c, by a Line drawn from any Point in the Side of it.*

Let A be the given point in the annexed plan, from which a line is to be drawn cutting off suppose 5 ac. 2 r. 14 p.



Draw AB cutting off the part ABC as near as can be judged equal to the quantity proposed; and let the true quantity of ABC, when calculated, be only 4 ac. 3 r. 20 p. which is less than 5 ac. 2 r. 14 p. the true quantity, by 0 ac. 2 r. 34 p. or 71250 square links. Then measure AB, which suppose = 1234 links, and divide 71250, by 617 the half of it, and the quotient 115 links will be the altitude of the triangle to be added, and whose base is AB. Therefore if upon the centre B, with the radius 115, an arc be described; and a line be drawn parallel to AB, touching the arc, and cutting BD in D; and if AD be drawn, it will be the line cutting off the required quantity ADCA.

Note. If the first piece had been too much, then D must have been set below B.

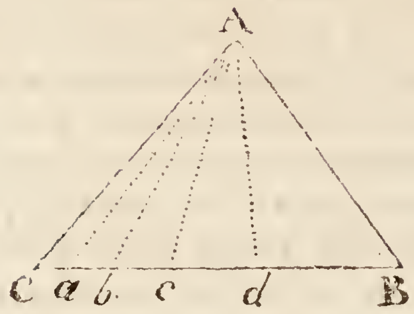
In this manner the several shares of commons, to be divided, may be laid down upon the plan, and transferred from thence to the ground itself.

Also for the greater ease and perfection in this business, the following problems may be added.

25. *From an Angle in a Given Triangle, to draw Lines to the opposite Side, dividing the Triangle into any Number of Parts, which shall be in any assigned Proportion to each other.*

Divide the base into the same number of parts, and in the same proportion, by article 22; then from the several points of division draw lines to the proposed angle, and they will divide the triangle as required.—For, the several parts are triangles of the same altitude, and which therefore are as their bases, which bases are taken in the assigned proportion.

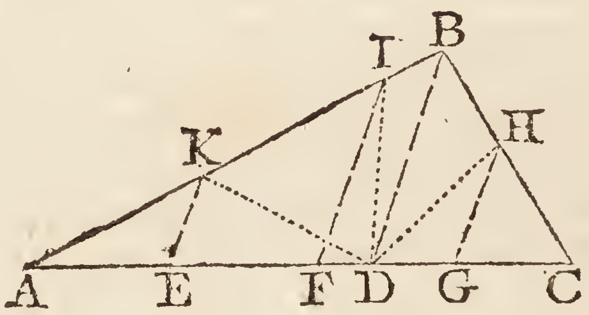
Ex. Let the triangle ABC, of 20 acres, be divided into five parts, which shall be in proportion to the numbers 1, 2, 3, 5, 9; the lines of division to be drawn from A to CB, whose length is 1600 links.



Here $1 + 2 + 3 + 5 + 9 = 20$, and $1600 \div 20 = 80$; which being multiplied by each of the proportional numbers, we have 80, 160, 240, 400, and 720. Therefore make $Ca = 80$, $ab = 160$, $bc = 240$, $cd = 400$, and $dB = 720$; then by drawing the lines Aa, Ab, Ac, Ad, the triangle is divided as required.

26. *From any Point in one side of a Given Triangle, to draw Lines to the other two Sides, dividing the Triangle into any Number of Parts which shall be in any assigned Ratio.*

From the given point D, draw DB to the angle opposite the side AC in which the point is taken; then divide the same side AC into as many parts AE, EF, FG, GC, and in the same proportion with the required parts of the triangle, like as was done in the last problem; and from the points of division draw lines EK, FI, GH, parallel to the line BD, and meeting the other sides of the triangle in K, I, H; lastly, draw KD, ID, HD, so shall ADK, KDI, IDHB, HDC be the parts required.—The example to this will be done exactly as the last.



For, the triangles ADK, KDI, IDB, being of the same height, are as their bases AK, KI, IB; which, by means of the parallels EK, FI, DB, are as AE, EF, FD; in like manner, the triangles CDH, HDB, are to each other as CG, GD: but the two triangles IDB, BDH, having the same base BD, are to each other as the distances of I and H from BD, or as FD to DG; consequently the parts DAK, DKI, DIBH, DHC, are to each other as AE, EF, FG, GC.

Surveying of Harbours.

The method of Surveying harbours, and of forming maps of them, as also of the adjacent coasts, sands, &c, depends on the same principles, and is chiefly conducted like that of common Surveying. The operation is indeed more complicated and laborious; as it is necessary to erect a number of signals, and to mark a variety of objects along the coast, with different bearings from one another, and the several parts of the harbour; and likewise to measure a great number of angles at

at different ftations, whether on the land or the water. For this purpofe, the beft inftrument is Hadley's quadrant, as all thefe operations may be performed by it, not only with greater eafe, but alfo with much more precifion, than can be hoped for by any other means, as it is the only inftrument in ufe, in which neither the exactnefs of the obfervations, nor the eafe with which they may be made, are fenfibly affected by the motion of a veffel: and hence a fingle obferver, in a boat, may generally determine the fituation of any place at pleafure, with a fufficient degree of exactnefs, by taking the angles fubtended by feveral pairs of objects properly chofen upon fhores round about him; but it will be ftill better to have two obfervers, or the fame obferver at different ftations, to take the like angles to the feveral objects, and alfo to the ftations. By this means, two angles and one fide are given, in every triangle, from whence the fituation of every part of them will be known. By fuch obfervations, when carefully made with good inftruments, the fituation of places may be eafily determined to 20 or 30 feet, or lefs, upon every 3 or 4 miles. See Philof. Tranf. vol. 55, pa. 70; alfo Mackenzie's Maritime Surveying.

SURVEYING *Crofs*. See CROSS.

SURVEYING *Quadrant*. See QUADRANT.

SURVEYING *Scale*, the fame with Reducing Scale.

SURVEYING *Wheel*. See PERAMBULATOR.

SURVIVORSHIP, the doctrine of reverfionary payments that depend upon certain contingencies, or contingent circumftances.

Payments which are not to be made till fome future period, are termed *reverfions*, to diftinguifh them from payments that are to be made immediately.

Reverfions are either *certain* or *contingent*. Of the former fort, are all fums or annuities, payable certainly or abfolutely at the expiration of any terms, or on the extinction of any lives. And of the latter fort, are all fuch reverfions as depend on any contingency; and particularly the Survivorfhip of any lives beyond or after other lives. An account of the former may be found under the articles Affurance, Annuities, and Life-annuities. But the latter form the moft intricate and difficult part of the doctrine of reverfions and life-annuities; and the books in which this fubject is treated moft at large, and at the fame time with the moft precifion, are Mr. Simpfon's Select Exercifes; Dr. Price's Reverfionary Payments; and Mr. Morgan's Annuities and Affurances on Lives and Survivorfhips. The whole likewife of the 3d volume of Dodfon's Mathematical Repofitory is on this fubject; but his inveftigations are founded on De Moivre's falfe hypothefis, viz of an equal decrement of life through all its ftages, and which is explained under Life-annuities: but as this hypothefis does not agree near enough to fact and experience, the rules deduced from it cannot be fufficiently correct. For this reafon, Dr. Price, and alfo the ingenious Mr. Maferes, curfitor baron of the exchequer (in two volumes lately publifhed, entitled the Principles of the Doctrine of Life Annuities), have difcarded the valuations of lives grounded upon it; and the former in particular, in order to obviate all occafion for ufing them, has fubftituted in their ftead, a great variety of new tables of the probabilities and values of lives, at every age and in every fituation; calculated, not upon any hypothefis, but in ftrict conformity to the beft obfervations. Thefe tables, added to other new tables of the fame kind, in Mr. Baron Maferes's work juft mentioned, form a complete fet of tables, by which all queftions relating to annuities on lives and Survivorfhips, may be anfwered with as much correctnefs as the nature of the fubject allows.

Rules for calculating correctly, in moft cafes, the values of reverfions depending on Survivorfhips, may be found in the three treatifes juft mentioned. Mr. Morgan, in particular, has gone a good way towards exhaufting this fubject, as far as any queftions can include in them any Survivorfhips between two or three lives, either for terms, or the whole duration of the lives.

There is, however, one circumftance neceffary to be attended to in calculating fuch values, to which no regard could be paid till lately. This circumftance is the fhorter duration of the lives of males than of females; and the confequent advantage in favour of females in all cafes of Survivorfhip. In the 4th edition of Dr. Price's Treatife on Reverfionary Payments, this fact is not only afcertained, but feparate tables of the duration and values of lives are given for males and females.

SUSPENSION, in Mechanics, as in a balance, are thofe points in the axis or beam where the weights are applied, or from which they are fufpended.

SUTTON's *Quadrant*. See QUADRANT.

SWAN, in Aftronomy. See CYGNUS.

SWALLOW's-TAIL, in Fortification, is a fingle Tenaille, which is narrower towards the place than towards the country.

SWING-*Wheel*, in a royal pendulum, is that wheel which drives the pendulum. In a watch, or balance clock, it is called the *crown-wheel*.

SYDEREAL *Day*, or *Year*. See SIDEREAL.

SYMMETRY, the relation of parity, both in refpect of length, breadth, and height, of the parts neceffary to compofe a beautiful whole.

Symmetry arifes from that proportion which the Greeks call *analogy*, which is the relation of conformity of all the parts of a building, and of the whole, to fome certain meafure; upon which depends the nature of Symmetry.

According to Vitruvius, Symmetry confifts in the union and conformity of the feveral members of a work to their whole, and of the beauty of each of the feparate parts to that of the intire work; regard being had to fome certain meafure: fo the body, for inftance, is framed with Symmetry, by the due relation which the arm, elbow, hand, fingers, &c, have to each other, and to their whole.

SYMPHONY, is a confonance or concert of feveral founds agreeable to the ear; whether they be vocal or inftrumental, or both; called alfo *harmony*.

The Symphony of the Ancients went no farther than to two or more voices or inftruments fet to unifon; for they had no fuch thing as mufic in parts; as is very well proved by Perrault: at leaft, if ever they knew fuch a thing, it muft have been early loft.

It is to Guido Aretine, about the year 1022, that moft writers agree in afcribing the invention of compofition: it was he, they fay, who firft joined in one harmony feveral diftinct melodies; and brought it even to the

the length of 4 parts, viz. bass, tenor, counter-tenor, and treble.

The term Symphony is now applied to instrumental music, both that of pieces designed only for instruments, as sonatas and concertos, and that in which the instruments are accompanied with the voice, as in operas, &c.

A piece is said to be in grand Symphony, when, besides the bass and treble, it has also two other instrumental parts, viz, tenor and 5th of the violin.

SYNCHRONISM, the being or happening of several things together, at or in the same time.

The happening or performing of several things in equal times, as the vibrations of pendulums, &c, is more properly called *isochronism:* though some authors confound the two.

SYNCOPATION, in Music, denotes a striking or breaking of the time; by which the distinctness of the several times or parts of the measure is interrupted.

Syncopation, or Syncope, is more particularly used for the connecting the last note of one measure or bar with the first of the following measure; so as to make only one note of both.

Syncopation is also used when a note of one part ends on the middle of a note of the other part. This is otherwise called *binding*.

SYNODICAL *Month*, is the period or interval of time in which the moon passes from one conjunction with the sun to another. This period is also called a *Lunation*, since in this period the moon puts on all her phases, or appearances, as to increase and decrease. —Kepler found the quantity of the mean Synodical month to be 29 days, 12 hrs, 44 min. 3 sec. 11 thirds.

SYNTHESIS denotes a method of composition, as opposed to analysis.

In the Synthesis, or synthetic method, we pursue the truth by reasons drawn from principles before established, or assumed, and propositions formerly proved; thus proceeding by a regular chain till we come to the conclusion; and hence called also the *direct* method, and *composition*, in opposition to analysis or resolution.

Such is the method in Euclid's Elements, and most demonstrations of the ancient mathematicians, which proceed from definitions and axioms, to prove theorems &c, and from those theorems proved, to demonstrate others. See Analysis.

SYNTHETICAL *Method*, the method by Synthesis, or composition, or the direct method. See Synthesis.

SYPHON. See Siphon.

SYRINGE, in Hydraulics, a small simple machine, serving first to imbibe or suck in a quantity of water, or other fluid, and then to squirt or expel the same with violence in a small jet.

The Syringe is just a small single sucking pump, without a valve, the water ascending in it on the same principle. It consists, like the pump, of a small cylinder, with an embolus or sucker, moving up and down in it by means of a handle, and fitting it very close within. At the lower end is either a small hole, or a smaller tube fixed to it than the body of the instrument, through which the fluid or the water is drawn up, and squirted out again.

Thus, the embolus being first pushed close down, introduce the lower end of the pipe into the fluid, then draw up, by the handle, the sucker, and the fluid will immediately follow, so as to fill the whole tube of the Syringe, and will remain there, even when the pipe is taken out of the fluid; but by thrusting forward the embolus, it will drive the water before it; and, being partly impeded by the smallness of the hole, or pipe, it will hence be expelled in a smart jet or squirt, and to the greater distance, as the sucker is pushed down with the greater force, or the greater velocity.

This ascent of the water the Ancients, who supposed a plenum, attributed to Nature's abhorrence of a vacuum; but the Moderns, more reasonably, as well as more intelligibly, attribute it to the pressure of the atmosphere on the exterior surface of the fluid. For, by drawing up the embolus, the cavity of the cylinder would become a vacuum, or the air left there extremely rarefied; so that being no longer a counterbalance to the air incumbent on the surface of the fluid, this prevails, and forces the water through the little tube, or hole, up into the body of the Syringe.

SYSTEM, in a general Sense, denotes an assemblage or chain of principles and conclusions: or the whole of any doctrine, the several parts of which are bound together, and follow or depend on each other. As a System of astronomy, a System of planets, a System of philosophy, a System of motion, &c.

System, in Astronomy, denotes an hypothesis or a supposition of a certain order and arrangement of the several parts of the universe; by which astronomers explain all the phenomena or appearances of the heavenly bodies, their motions, changes, &c.

This is more peculiarly called the *System of the world*, and sometimes the *Solar System*.

System and hypothesis have much the same signification; unless perhaps hypothesis be a more particular System, and System a more general hypothesis.

Some late authors indeed make another distinction: an hypothesis, say they, is a mere supposition or fiction, founded rather on imagination than reason; while a System is built on the firmest ground, and raised by the severest rules; it is founded on astronomical observations, and physical causes, and confirmed by geometrical demonstrations.

The most celebrated Systems of the world, are the Ptolomaic, the Copernican or Pythagorean, and the Tychonic: the economy of each of which is as follows.

Ptolomaic System is so called from the celebrated astronomer Ptolomy. In this System, the earth is placed at rest, in the centre of the universe, while the heavens are considered as revolving about it, from east to west, and carrying along with them all the heavenly bodies, the stars and planets, in the space of 24 hours.

The principal assertors of this System, are Aristotle, Hipparchus, Ptolomy, and many of the old philosophers, followed by the whole world, for a great number of ages, and long adhered to in many universities, and other places. But the late improvements in philosophy and reasoning, have utterly exploded this erroneous System from the place it so long held in the minds of men.

Copernican System, is that System of the world which

which places the Sun at reſt, in the centre of the world, and the earth and planets all revolving round him, in their ſeveral orbits. See this more particularly explained under the article COPERNICAN *Syſtem*.

Solar or *Planetary* SYSTEM, is uſually confined to narrower bounds; the ſtars, by their immenſe diſtance, and the little relation they ſeem to bear to us, being accounted no part of it. It is highly probable that each fixed ſtar is itſelf a Sun, and the centre of a particular Syſtem, ſurrounded with a company of planets &c, which, in different periods, and at different diſtances, perform their courſes round their reſpective ſun, which enlightens, warms, and cheriſhes them. Hence we have a very magnificent idea of the world, and the immenſity of it. Hence alſo ariſes a kind of Syſtem of Syſtems.

The Planetary Syſtem, deſcribed under the article COPERNICAN, is the moſt ancient in the world. It was firſt of all, as far as we know, introduced into Greece and Italy by Pythagoras; from whom it was called the Pythagorean Syſtem. It was followed by Philolaus, Plato, Archimedes, &c: but it was loſt under the reign of the Peripatetic philoſophy; till happily retrieved about the year 1500 by Nic. Copernicus.

Tychonic SYSTEM, was taught by Tycho, a Dane; who was born An. Dom. 1546. It ſuppoſes that the earth is fixed in the centre of the univerſe or firmament of ſtars, and that all the ſtars and planets revolve round the earth in 24 hours; but it differs from the Ptolomaic Syſtem, as it not only allows a menſtrual motion to the moon round the earth, and that of the ſatellites about Jupiter and Saturn, in their proper periods, but it makes the ſun to be the centre of the orbits of the primary planets Mercury, Venus, Mars, Jupiter, &c, in which they are carried round the ſun in their reſpective years, as the ſun revolves round the earth in a ſolar year; and all theſe planets, together with the ſun, are ſuppoſed to revolve round the earth in 24 hours. This hypotheſis was ſo embarraſſed and perplexed, that very few perſons embraced it. It was afterwards altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the ſun. This hypotheſis, partly true and partly falſe, is called the *Semi-Tychonic Syſtem*. See the figure and economy of theſe Syſtems, in plates 30, 31, 32, 33.

SYSTEM, in Muſic, denotes a compound interval; or an interval compoſed, or conceived to be compoſed of ſeveral leſs intervals. Such is the octave, &c.

SYSTYLE, in Architecture, the manner of placing columns, where the ſpace between the two ſuſts conſiſts of 2 diameters, or 4 modules.

SYZYGY, a term equally uſed for the conjunction and oppoſition of a planet with the ſun.

On the phenomena and circumſtances of the Syzygies, a great part of the lunar theory depends. See MOON. For,

1. It is ſhewn in the phyſical aſtronomy, that the force which diminiſhes the gravity of the moon in the Syzygies, is double that which increaſes it in the quadratures; ſo that, in the Syzygies, the gravity of the moon is diminiſhed by a part which is to the whole gravity, as 1 to 89·36; for in the quadratures, the addition of gravity is to the whole gravity, as 1 to 178·73.

2. In the Syzygies, the diſturbing force is directly as the diſtance of the moon from the earth, and inverſely as the cube of the diſtance of the earth from the ſun. And at the Syzygies, the gravity of the moon towards the earth receding from its centre, is more diminiſhed than according to the inverſe ratio of the ſquare of the diſtance from that centre.—Hence, in the moon's motion from the Syzygies to the quadratures, the gravity of the moon towards the earth is continually increaſed, and the moon is continually retarded in her motion; but in the moon's motion from the quadratures to the Syzygies, her gravity is continually diminiſhed, and the motion in her orbit is accelerated.

3. Farther, in the Syzygies, the moon's orbit, or circuit round the earth, is more convex than in the quadratures; for which reaſon ſhe is leſs diſtant from the earth at the former than the latter.—Alſo, when the moon is in the Syzygies, her apſes go backward, or are retrograde.—Moreover, when the moon is in the Syzygies, the nodes move in antecedentia faſteſt; then ſlower and ſlower, till they become at reſt when the moon is in the quadratures.—Laſtly, when the nodes are come to the Syzygies, the inclination of the plane of the orbit is the leaſt of all.

However, theſe ſeveral irregularities are not equal in each Syzygy, being all ſomewhat greater in the conjunction than in the oppoſition.

T.

TABLE, in Architecture, a ſmooth, ſimple member or ornament, of various forms, but moſt commonly in that of a parallelogram.

TABLE, in Perſpective, is ſometimes uſed for the perſpective plane, or the tranſparent plane upon which the objects are formed in their reſpective appearance.

TABLE *of Pythagoras*, is the ſame as the MULTIPLICATION

CATION Table; which see; as also PYTHAGORAS's Table.

TABLES *of Houses*, among astrologers, are certain Tables, ready drawn up, for the assistance of practitioners in that art, for the erecting or drawing of figures or schemes. See HOUSE.

TABLES, in Mathematics, are systems or series of numbers, calculated to be ready at hand for expediting any sort of calculations in the various branches of mathematics.

Astronomical TABLES, are computations of the motions, places, and other phenomena of the planets, both primary and secondary.

The oldest astronomical Tables, now extant, are those of Ptolomy, found in his Almagest. These however are not now of much use, as they no longer agree with the motions of the heavens.

In 1252, Alphonso XI, king of Castile, undertook the correcting of them, chiefly by the assistance of Isaac Hazen, a learned Jew; and spent 400,000 crowns on the business. Thus arose the *Alphonsine Tables*, to which that prince himself prefixed a preface. But the deficiency of these also was soon perceived by Purbach and Muller, or Regiomontanus; upon which the latter, and after him Walther Warner, applied themselves to celestial observations, for farther improving them; but death, or various difficulties, prevented the effect of these good designs.

Copernicus, in his books of the celestial revolutions, gives other Tables, calculated by himself, partly from his own observations, and partly from the Alphonsine Tables.

From Copernicus's observations and theorems, Erasmus Reinhold afterwards compiled the *Prutenic Tables*, which have been printed several times, and in several places.

Tycho Brahe, even in his youth, became sensible of the deficiency of the Prutenic Tables: which determined him to apply himself with so much vigour to celestial observations. From these he adjusted the motions of the sun and moon; and Longomontanus, from the same observations, made out Tables of the motions of the planets, which he added to the Theories of the same, published in his Astronomia Danica; those being called the *Danish Tables*. And Kepler also, from the same observations, published in 1627 his *Rudolphine Tables*, which are much esteemed.

These were afterwards, viz in 1650, changed into another form, by Maria Cunitia, whose Astronomical Tables, comprehending the effect of Kepler's physical hypothesis, are very easy, satisfying all the phenomena without any mention of logarithms, and with little or no trouble of calculation. So that the Rudolphine calculus is here greatly improved.

Mercator made a like attempt in his Astronomical Institution, published in 1676. And the like did J. Bap. Morini, whose abridgment of the Rudolphine Tables was prefixed to a Latin version of Street's Astronomia Carolina, published in 1705.

Lansbergius indeed endeavoured to discredit the Rudolphine Tables, and framed *Perpetual Tables*, as he calls them, of the heavenly motions. But his attempt was never much regarded by the astronomers; and our countryman Horrox warmly attacked him, in his defence of the Keplerian astronomy.

Since the Rudolphine Tables, many others have been framed, and published: as the *Philolaic Tables* of Bulliald; the *Britannic Tables* of Vincent Wing, calculated on Bulliald's hypothesis; the *Britannic Tables* of John Newton; the French ones of the Count Pagan; the *Caroline Tables* of Street, all calculated on Ward's hypothesis; and the *Novalmajestic Tables* of Riccioli. Among these, however, the Philolaic and Caroline Tables are esteemed the best; insomuch that Mr. Whiston, by the advice of Mr. Flamsteed, thought fit to subjoin the Caroline Tables to his astronomical lectures.

The *Ludovician Tables*, published in 1702, by De la Hire, were constructed wholly from his own observations, and without the assistance of any hypothesis; which, before the invention of the micrometer telescope and the pendulum clock, was held impossible.

Dr. Halley also long laboured to perfect another set of Tables; which were printed in 1719, but not published till 1752.

M. Monnier, in 1746, published, in his Institutions Astronomiques, Tables of the motions of the sun and moon, with the satellites, as also of refractions, and the places of the fixed stars. La Hire also published Tables of the planets, and La Caille Tables of the sun: Gael Morris published Tables of the sun and moon, and Mayer constructed Tables of the moon, which were published by the Board of Longitude. Tables of the same have also been computed by Charles Mason, from the principles of the Newtonian philosophy, which are found to be very accurate, and are employed in computing the Nautical Ephemeris. Many other sets of astronomical Tables have also been published by various persons and academies; and divers sets of them may be found in the modern books of astronomy, navigation, &c, of which those are esteemed the best and most complete, that are printed in Lalande's Astronomy. For an account of several, and especially of those published annually under the direction of the Commissioners of Longitude, see ALMANAC, EPHEMERIS, and LONGITUDE.

For TABLES *of the Stars*, see CATALOGUE.

TABLES of *Sines*, *Tangents*, and *Secants*, used in trigonometry, &c, are usually called CANONS. See SINE.

TABLES of *Logarithms*, *Rhumbs*, *&c*, used in geometry, navigation, &c, see LOGARITHM, and RHUMB.

TABLES, *Loxodromic*, and *of Difference of Latitude and Departure*, are Tables used in computing the way and reckoning of a ship on a voyage, and are published in most books of navigation.

TACQUET (ANDREW), a Jesuit of Antwerp, who died in 1660. He was a most laborious and voluminous writer in mathematics. His works were collected, and printed at Antwerp in one large volume in folio, 1669.

TACTION, in Geometry, the same as tangency, or touching. See TANGENT.

TALUS, or TALUD, in Architecture, the inclination or slope of a work; as of the outside of a wall, when its thickness is diminished by degrees, as it rises in height, to make it the firmer.

Talus, in Fortification, means alſo the ſlope of a work, whether of earth or maſonry.

The *Exterior Talus* of a work, is its ſlope on the ſide outwards or towards the country; which is always made as little as poſſible, to prevent the enemy's eſcalade, unleſs the earth be bad, for then it is neceſſary to allow a conſiderable Talus for its parapet, and ſometimes to ſupport the earth with a ſlight wall, called a revetement.

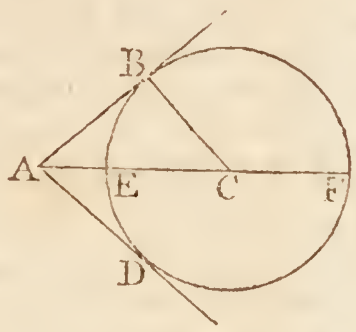
The *Interior Talus* of a work, is its ſlope on the inſide, towards the place. This is larger than the former, and it has, at the angles of the gorge, and ſometimes in the middle of the curtains, ramps, or ſloping roads for mounting upon the terreplain of the rampart.

Superior Talus *of the Parapet*, is a ſlope on the top of the parapet, that allows of the ſoldiers defending the covert-way with ſmall-ſhot, which they could not do if it were level.

TAMBOUR, in Architecture, a term applied to the Corinthian and Compoſite capitals, as bearing ſome reſemblance to a tambour or drum.

TAMUZ, in Chronology, the 4th month of the Jewiſh eccleſiaſtical year, anſwering to part of our June and July. The 17th day of this month is obſerved by the Jews as a faſt, in memory of the deſtruction of Jeruſalem by Nebuchadnezzar, in the 11th year of Zedekiah, and the 588th before Chriſt.

TANGENT, in Geometry, is a line that touches a curve, &c, that is, which meets it in a point without cutting it there, though it be produced both ways; as the Tangent AB of the circle BD. The point B, where the Tangent touches the curve, is called the *point of contact.*



The direction of a curve at the point of contact, is the ſame as the direction of the Tangent.

It is demonſtrated in Geometry;

1. That a Tangent to a circle, as AB, is perpendicular to the radius BC drawn to the point of contact.

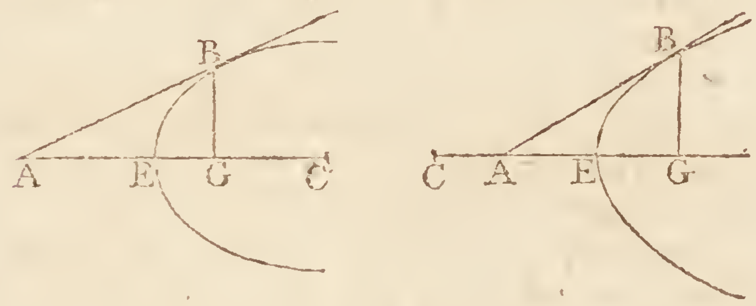
2. The Tangent AB is a mean proportional between AF and AE, the whole ſecant and the external part of it; and the ſame for any other ſecant drawn from the ſame point A.

3. The two Tangents AB and AD, drawn from the ſame point A, are always equal to one another. And therefore alſo, if a number of Tangents be drawn to different points of the curve quite around, and an equal length BA be ſet off upon each of them from the points of contact, the locus of all the points A will be a circle having the ſame centre C.

4. The angle of contact ABE, formed at the point of contact, between the Tangent AB and the arc BE, is leſs than any rectilineal angle.

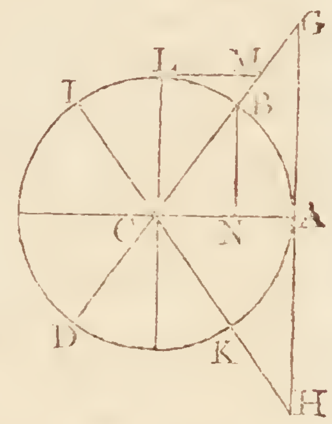
5. The Tangent of an arc is the right line that limits the poſition of all the ſecants that can paſs through the point of contact; though ſtrictly ſpeaking it is not one of the ſecants, but only the limit of them.

6. As a right line is the Tangent of a circle, when it touches the circle ſo cloſely, that no right line can be drawn through the point of contact between it and the arc, or within the angle of contact that is formed by them; ſo, in general, when any right line touches an arc of any curve, in ſuch a manner, that no right line can be drawn through the point of contact, between the right line and the arc, or within the angle of contact that is formed by them, then is that line the Tangent of the curve at the ſaid point; as AB.



7. In all the conic ſections; if C be the centre of the figure, and BG an ordinate drawn from the point of contact and perpendicular to the axis; then is CG : CE :: CE : CA, or the ſemiaxis CE is a mean proportional between CG and CA.

Tangent, in Trigonometry. A Tangent *of an arc*, is a right line drawn touching one extremity of the arc, and limited by a ſecant or line drawn through the centre and the other extremity of the arc.



So, AG is the Tangent of the arc AB, or of the arc ABD; and AH is the Tangent of the arc AI, or of the arc AIDK.

The ſame are alſo the Tangents of the angles that are ſubtended or meaſured by the arcs.

Hence, 1. The Tangents in the 1ſt and 3d quadrants are poſitive, in the 2d and 4th negative, or drawn the contrary way. But of 0 or 180° the ſemicircle, the Tangent is 0 or nothing; while thoſe of 90° or a quadrant, and 270° or 3 quadrants, are both infinite; the former infinitely poſitive, and the latter infinitely negative. That is,

Between 0 and 90°, or bet. 180° and 270°, the Tangents are poſitive.
Bet. 90° and 180°, or bet. 270° and 360°, the Tangents are negative.

2. The Tangent of an arc and the Tangent of its ſupplement, are equal, but of contrary affections, the one being poſitive, and the other negative;

as of a and $180° - a$, where a is any arc.

$\left.\begin{array}{l}\text{Alſo } 180° + a \\ \text{and } \quad\quad\;\; a\end{array}\right\}$ have the ſame Tangent, and of the ſame affection.

$\left.\begin{array}{l}\text{Or } 180° + a \\ \text{and } 180° - a\end{array}\right\}$ have the ſame Tangent, but of different affections.

3. The Tangent of an arc is a 4th proportional to the coſine the ſine and the radius; that is, CN : NB :: CA : AG. Hence, a canon of ſines being made or given, the canon of Tangents is eaſily conſtructed from them.

Co-Tangent, contracted from complement-tangent, is the Tangent of the complement of the arc or angle, or of what it wants of a quadrant or 90°. So LM is the Cotangent of the arc AB, being the Tangent of its complement BL.

The Tangent is reciprocally as the cotangent; or the

Tangent and cotangent are reciprocally proportional with the radius. That is Tang. is as $\frac{1}{\text{cotan.}}$, or Tang. : radius : : radius : cotan. And the rectangle of the Tangent and cotangent is equal to the square of the radius; that is, Tan. × cot. = radius^2.

Artificial TANGENTS, or *logarithmic* TANGENTS, are the logarithms of the tangents of arcs; so called, in contradistinction from the natural Tangents, or the Tangents expressed by the natural numbers.

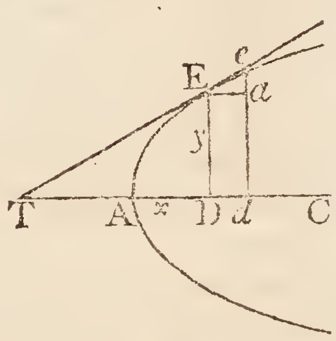
Line of TANGENTS, is a line usually placed on the sector, and Gunter's scale; the description and uses of which see under the article SECTOR.

Sub-TANGENT, a line lying beneath the Tangent, being the part of the axis intercepted by the Tangent and the ordinate to the point of contact; as the line AG in the 2d and 3d figures above.

Method of TANGENTS, is a method of determining the quantity of the Tangent and subtangent of any algebraic curve; the equation of the curve being given.

This method is one of the great results of the doctrine of fluxions. It is of great use in Geometry; because that in determining the Tangents of curves, we determine at the same time the quadrature of the curvilinear spaces: on which account it deserves to be here particularly treated on.

To Draw the Tangent, or to find the Subtangent, of a curve.



If AE be any curve, and E any point in it, to which it is required to draw a Tangent TE. Draw the ordinate DE: then if we can determine the subtangent TD, by joining the points T and E, the line TE will be the Tangent sought.

Let *dae* be another ordinate indefinitely near to DE, meeting the curve, or Tangent produced, in *e*; and let E*a* be parallel to the axis AD. Then is the elementary triangle E*ae* similar to the triangle TDE;

and therefore - *ea* : *a*E : : ED : DS;
but - - - - - - - *ea* : *a*E : : flux. ED : flux. AD;
therefore - - - - flux. ED : flux. AD : : DE : DT;

that is - - - - - $\dot{y} : \dot{x} :: y : \frac{y\dot{x}}{\dot{y}} = \text{DT}$,

which is therefore the value of the subtangent sought; where x is the absciss AD, and y the ordinate DE.

Hence we have this general rule: By means of the given equation of the curve, find the value either of $\dot{x}$ or $\dot{y}$, or of $\frac{\dot{x}}{\dot{y}}$, which value substitute for it in the expression $\text{DT} = \frac{y\dot{x}}{\dot{y}}$, and, when reduced to its simplest terms, it will be the value of the subtangent sought. This we may illustrate in the following examples.

Ex. 1. The equation defining a circle is $2ax - xx = y^2$, where a is the radius; and the fluxion of this is $2a\dot{x} - 2x\dot{x} = 2y\dot{y}$; hence $\frac{\dot{x}}{\dot{y}} = \frac{y}{a-x}$; this multiplied by y, gives $\frac{y\dot{x}}{\dot{y}} = \frac{y^2}{a-x} = \frac{\text{DE}^2}{\text{CD}}$ = the subtangent TD, or CD : DE : : DE : TD, which is a property of the circle we also know from common geometry.

Ex. 2. The equation defining the common parabola is $ax = y^2$, a being the parameter, and x and y the abscifs and ordinate in all cases. The fluxion of this is $a\dot{x} = 2y\dot{y}$; hence $\frac{\dot{x}}{\dot{y}} = \frac{2y}{a}$; conseq. $\frac{y\dot{x}}{\dot{y}} = \frac{2y^2}{a} = \frac{2ax}{a} = 2x = \text{TD}$; that is, the subtangent TD is double the abscifs AD, or TA is = AD, which is a well-known property of the parabola.

Ex. 3. The equation defining an ellipsis is $c^2.\overline{2ax - x^2} = a^2y^2$, where a and c are the semiaxes. The fluxion of it is $c^2.\overline{2a\dot{x} - 2x\dot{x}} = 2a^2y\dot{y}$; hence

$$\frac{y\dot{x}}{\dot{y}} = \frac{a^2y^2}{c^2(a-x)} = \frac{c^2(2ax - x^2)}{c^2(a-x)} = \frac{2a-x}{a-x}x = \text{TD}$$

the subtangent; or by adding CD which is $= a - x$, it becomes $\text{CT} = \frac{2ax - x^2}{a-x} + a - x = \frac{a^2}{a-x} = \frac{\text{CA}^2}{\text{CD}}$, or CD : CA : : CA : CT, a well-known property of the ellipse.

Ex. 4. The equation defining the hyperbola is $c^2.\overline{2ax + x^2} = a^2y^2$, which is similar to that for the ellipse, having only $+ x^2$ for $- x^2$; hence the conclusion is exactly similar also, viz,

$\frac{2a + x}{a + x}x$ or $\frac{2ax + xx}{a + x}$ = TD, which taken from CD or $a + x$, gives $\text{CT} = \frac{\text{CA}^2}{\text{CD}}$, or CD : CA : : CA : CT.

And so on, for the Tangents to other curves.

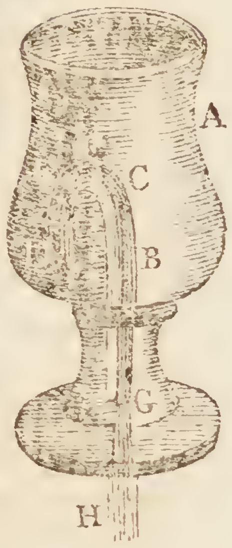
The Inverse Method of TANGENTS. This is the reverse of the foregoing, and consists in finding the nature of the curve that has a given subtangent. The method of solution is to put the given subtangent equal to the general expression $\frac{y\dot{x}}{\dot{y}}$, which serves for all sorts of curves; then the equation reduced, and the fluents taken, will give the fluential equation of the curve sought.

Ex. 1. To find the curve line whose subtangent is $= \frac{2y^2}{a}$. Here $\frac{2y^2}{a} = \frac{y\dot{x}}{\dot{y}}$; hence $2y\dot{y} = a\dot{x}$, and the fluents of this give $y^2 = ax$, the equation to a parabola, which therefore is the curve sought.

Ex. 2. To find the curve whose subtangent is $= \frac{yy}{2a - x}$, or a third proportional to $2a - x$ and y. Here $\frac{yy}{2a-x} = \frac{y\dot{x}}{\dot{y}}$; hence $y\dot{y} = 2a\dot{x} - x\dot{x}$, the fluents of which give $y^2 = ax - x^2$, the equation to a circle, which therefore is the curve sought.

TAN-

TANTALUS's *Cup*, in Hydraulics, is a cup, as A, with a hole in the bottom, and the longer leg of a syphon BCED cemented into the hole; so that the end D of the shorter leg DE may always touch the bottom of the cup within. Then, if water be poured into this cup, it will rise in the shorter leg by its upward pressure, extruding the air before it through the longer leg, and when the cup is filled above the bend of the syphon at E, the pressure of the water in the cup will force it over the bend; from whence it will descend in the longer leg EB, and through the bottom at G, till the cup be quite emptied. The legs of this syphon are almost close together, and it is sometimes concealed by a small hollow statue, or figure of a man placed over it; the bend E being within the neck of the figure as high as the chin. So that poor thirsty Tantalus stands up to the chin in water, according to the fable, imagining it will rise a little higher, as more water is poured in, and he may drink; but instead of that, when the water comes up to his chin, it immediately begins to descend, and therefore, as he cannot stoop to follow it, he is left as much tormented with thirst as ever. Ferguson's Lect. p. 72, 4to.



TARRANTIUS (Lucius), surnamed *Firmanus*, because he was a native of Firmum, a town in Italy, flourished at the same time with Cicero, and was one of his friends. He was a mathematical philosopher, and therefore was thought to have great skill in judicial astrology. He was particularly famous by two horoscopes which he drew, the one the horoscope of Romulus, and the other of Rome. Plutarch says, "Varro, who was the most learned of the Romans in history, had a particular friend named Tarrantius, who, out of curiosity, applied himself to draw horoscopes, by means of astronomical tables, and was esteemed the most eminent in his time." Historians controvert some particular circumstances of his calculations; but all agree in conferring on him the honorary title *Prince of astrologers*.

TARTAGLIA, or Tartalea (Nicholas), a noted mathematician who was born at Brescia in Italy, probably towards the conclusion of the 15th century, as we find he was a considerable master or preceptor in mathematics in the year 1521, when the first of his collection of questions and answers was written, which he afterwards published in the year 1546, under the title of *Quesiti et Inventioni diverse*, at Venice, where he then resided as a public lecturer on mathematics, he having removed to this place about the year 1534. This work consists of 9 chapters, containing answers to a number of questions on all the different branches of mathematics and philosophy then in vogue. The last or 9th of these, contains the questions in Algebra, among which are those celebrated letters and communications between Tartalea and Cardan, by which our author put the latter in possession of the rules for cubic equations, which he first discovered in the year 1530.

But the first work of Tartalea's that was published, was his *Nova Scientia inventa*, in 4to, at Venice in 1537. This is a treatise on the theory and practice of gunnery, and the first of the kind, he being the first writer on the flight and path of balls and shells. This work was translated into English, by Lucar, and printed at London in 1588, in folio, with many notes and additions by the translator.

Tartalea published at Venice, in folio, 1543, the whole books of Euclid, accompanied with many curious notes and commentaries.

But the last and chief work of Tartalea, was his *Trattato di Numeri et Misure*, in folio, 1556 and 1560. This is an universal treatise on arithmetic, algebra, geometry, mensuration, &c. It contains many other curious particulars of the disputes between our author and Cardan, which ended only with the death of Tartalea, before the last part of this work was published, or about the year 1558.

For many other circumstances concerning Tartalea and his writings, see the article Algebra, vol. 1, pa. 73.

TATIUS (Achilles), an ancient Greek writer of Alexandria; but the age he lived in is uncertain. According to Suidas, who calls him Statius, he was at first a Heathen, then a Christian, and afterwards a bishop. He wrote a book upon the Sphere, which seems to have been nothing more than a commentary upon Aratus. Part of it is extant, and was translated into Latin by father Petavius, under the title of *Isagoges in Phænomena Arati*. He wrote also, *Of the Loves of Cliptophon and Leucippe*, in 8 books. He is well spoken of by Photius.

TAURUS, *the Bull*, in Astronomy, one of the 12 signs in the zodiac, and the second in order.

The Greeks fabled that this was the bull which carried Europa safe across the seas to Crete; and that Jupiter, in reward for so signal a service, placed the creature, whose form he had assumed on that occasion, among the stars, and that this is the constellation formed of it. But it is probable that the Egyptians, or Babylonians, or whoever invented the constellations of the zodiac, placed this figure in that part of it which the sun entered about the time of the bringing forth of calves; like as they placed the ram in the first part of spring, as the lambs appear before them, and the two kids (for that was the original figure of the sign Gemini), afterward, to denote the time of the goats bringing forth their young.

In the constellation Taurus there are some remarkable stars that have names; as Aldebaran in the south or right eye of the bull, the cluster called the Pleiades in the neck, and the cluster called Hyades in the face.

The stars in the constellation Taurus, in Ptolomy's catalogue are 44, in Tycho's catalogue 43, in Hevelius's catalogue 51, and in the Britannic catalogue 141.

TEBET, or Thevet, the 4th month of the civil year of the Hebrews, and the 10th of their ecclesiastical year. It answered to part of our December and January, and had only 29 days.

TEETH, of various sorts of machines, as of mill wheels, &c. These are often called cogs by the workmen; and by working in the pinions, rounds, or trundles, the wheels are made to turn one another.

Mr. Emerson (in his Mechanics, prop. 25), treats of the theory of Teeth, and shews that they ought to

 have

have the figure of epicycloids, for properly working in one another. Camus too (in his Cours de Mathematique, tom. 2, p. 349, &c, Edit. 1767) treats more fully on the same subject; and demonstrates that the Teeth of the two wheels should have the figures of epicycloids, but that the generating circles of these epicycloids should have their diameters only the half of what Mr. Emerson makes them.

Mr. Emerson observes, that the Teeth ought not to act upon one another before they arrive at the line which joins their centres. And though the inner or under sides of the Teeth may be of any form; yet it is better to make them both sides alike, which will serve to make the wheels turn backwards. Also a part may be cut away on the back of every Tooth, to make way for those of the other wheel. And the more Teeth that work together, the better; at least one Tooth should always begin before the other hath done working. The Teeth ought to be disposed in such manner as not to trouble or hinder one another, before they begin to work; and there should be a convenient length, depth and thickness given to them, as well for strength, as that they may more easily disengage themselves.

TELEGRAPH, a machine brought into use by the French nation, in the year 1793, contrived to communicate words or signals from one person to another at a great distance, in a very small space of time.

The Telegraph it seems was originally the invention of William Amontons, an ingenious philosopher, born in Normandy in the year 1663. See his life in this Dictionary, vol. 1, pa. 105; where it is related that he pointed out a method to acquaint people at a great distance, and in a very little time, with whatever one pleased. This method was as follows: let persons be placed in several stations, at such distances from each other, that, by the help of a telescope, a man in one station may see a signal made by the next before him: this person immediately repeats the same signal to the third man; and this again to a fourth, and so on through all the stations to the last.

This, with considerable improvements, it seems has lately been brought into use by the French, and called a Telegraph. It is said they have availed themselves of this contrivance to good purpose, in the present war; and from the utility of the invention, it has also just been brought into use in this country.

The following account of this curious instrument is copied from Barrere's report in the sitting of the French Convention of August 15, 1794.—"The new-invented telegraphic language of signals is an artful contrivance to transmit thoughts, in a peculiar language, from one distance to another, by means of machines, which are placed at different distances, of from 12 to 15 miles from one another, so that the expression reaches a very distant place in the space of a few minutes. Last year an experiment of this invention was tried in the presence of several Commissioners of the Convention. From the favourable report which the latter made of the efficacy of the contrivance, the Committee of Public Welfare tried every effort to establish, by this means, a correspondence between Paris and the frontier places, beginning with Lisle. Almost a whole twelvemonth has been spent in collecting the necessary instruments for the machines, and to teach the people employed how to use them. At present, the telegraphic language of signals is prepared in such a manner, that a correspondence may be conducted with Lisle upon every subject, and that every thing, nay even proper names, may be expressed; an answer may be received, and the correspondence thus be renewed several times a day. The machines are the invention of Citizen Chappe, and were constructed under his own eye; he also directs their establishment at Paris. They have the advantage of resisting the changes in the atmosphere, and the inclemencies of the seasons. The only thing which can interrupt their effect is, if the weather is so very bad and turbid that the objects and signals cannot be distinguished. By this invention, remoteness and distance almost disappear; and all the communications of correspondence are effected with the rapidity of the twinkling of an eye. The operations of Government can be very much facilitated by this contrivance, and the unity of the Republic can be the more consolidated by the speedy communication with all its parts. The greatest advantage which can be derived from this correspondence is, that, if one chooses, its object shall only be known to certain individuals, or to one individual alone, or to the extremities of any distance; so that the Committee of Public Welfare may now correspond with the Representative of the People at Lisle without any other persons getting acquainted with the object of the correspondence. Hence it follows that, were Lisle even besieged, we should know every thing at Paris that might happen in that place, and could send thither the Decrees of the Convention without the enemy's being able to discover or to prevent it."—The description and figure of the French machine, as given in some English prints, are as follow.

Explanation of the Machine (Telegraph) placed on the Mountain of Bellville, near Paris, for the purpose of communicating Intelligence.

AA is a beam or mast of wood, placed upright on a rising ground (fig. 3, pl. 28) which is about 15 or 16 feet high. BB is a beam or balance, moving upon the centre AA. This balance-beam may be placed vertically, or horizontally, or any how inclined, by means of strong cords, which are fixed to the wheel D, on the edge of which is a double groove, to receive the two chords. This balance is about 11 or 12 feet long, and 9 inches broad, having at the ends two pieces of wood CC, which likewise turn upon angles by means of four other cords that pass through the axis of the main balance, otherwise the balance would derange the cords; the pieces C are each about 3 feet long, and may be placed either to the right or left, straight or square with the balance-beam. By means of these three, the combination of movement is said to be very extensive, remarkably simple, and easy to perform. Below is a small wooden gouge or hut, in which a person is employed to observe the movements of the machine. In the mountain nearest to this, another person is to repeat these movements, and a third to write them down. The time taken up for each movement is 20 seconds; of which the motion alone is 4 seconds, the other 16 the machine is stationary. The stations of this machine are about 3 or 4 leagues distance; and there is an observatory near the Committee of Public Safety

Safety to obferve the motions of the laft, which is at Bellville. The figns are fometimes made in words, and fometimes in letters; when in words, a fmall flag is hoifted, and, as the alphabet may be changed at pleafure, it is only the correfponding perfon who knows the meaning of the figns. In general, news are given every day, about 11 or 12 o'clock; but the people in the wooden gouge obferve from time to time, and, as foon as a certain fignal is given and anfwered, they begin, from one end to the other, to move the machine. It is painted of a dark brown colour.

Such is the account given of the French invention. Various improved contrivances have been fince made in England, and a pamphlet has lately been publifhed, giving an account of fome of them, by the Rev. J. Gamble, under the title of, *Obfervations and Telegraphic Experiments*, from whence the following remarks are extracted.

The object propofed is, to obtain an intelligible figurative language, which may be diftinguifhed at a diftance, and by which the obvious delay in the difpatch of orders or information by meffenger may be avoided.

On firft reflection we find the practical modes of fuch diftant communication muft be confined to Sound and Vifion. Each of which is in a great degree fubject to the ftate of the atmofphere: as, independent of the wind's direction, it is known that the air is fometimes fo far deprived of its elafticity, or whatever other quality the conveyance of found depends on, that the heavieft ordnance is fcarce heard farther than the fhot flies; it is alfo well known, that in thick hazy weather the largeft objects become totally obfcured at a fhort diftance. No inftrument therefore defigned for the purpofe can be perfect. We can only endeavour to diminifh thefe irremediable defects as much as may be.

It feems the Romans had a method in their walled cities, either by a hollow formed in the mafonry, or by tubes affixed to it, fo to confine and augment found as to convey information to any part they wifhed; and in lofty houfes it is now fometimes the cuftom to have a pipe, by way of fpeaking trumpet, to give orders from the upper apartments to the lower: by this mode of confining found its volume may be carried to a very great diftance; but beyond a certain extent the found, lofing articulation, would only convey alarm, not give directions.

Every city among the antients had its watch-towers; and the caftra ftativa of the Romans, had always fome fpot, elevated either by nature or art, from whence fignals were given to the troops cantoned or foraging in the neighbourhood. But I believe they had not arrived to greater refinement than that on feeing a certain fignal they were immediately to repair to their appointed ftations.

A beacon or bonfire made of the firft inflammable materials that offered, as the moft obvious, is perhaps the moft antient mode of general alarm; and by being previoufly concerted, the number or point where the fires appeared might have its particular intelligence affixed. The fame obfervations may be referred to the throwing up of rockets, whofe number or point from whence thrown may have its affixed fignification.

Flags or enfigns with their various devices are of earlieft invention, efpecially at fea; where, from the firft idea, which moft probably was that of a vane to fhew the direction of the wind, they have been long adopted as the diftinguifhing mark of nations, and are now fo neatly combined by the ingenuity of a great naval commander, that by his fyftem every requifite order and queftion is received and anfwered by the moft diftant fhips of a fleet.

To the adopting this or a fimilar mode in land fervice, the following are objections: That in the latter cafe, the variety of matter neceffary to be conveyed, is fo infinitely greater, that the combinations would become too complicated. And if the perfon for whom the information is intended fhould be in the direction of the wind, the flag would then prefent a ftraight line only, and at a little diftance be fcarce vifible. The Romans were fo well aware of this inconvenience of flags, that many of their ftandards were folid, and the name manipulus denotes the rudeft of their modes, which was a trufs of hay fixed on a pole.

The principle of water always keeping its own level has been fuggefted, as a mode of conveying intelligence, by Mr. Daniel Brent, of Rotherhithe, and put in practice on a fmall fcale. As for example, fuppofe a pipe AB to reach from London to Dover, and to have a per-

pendicular tube connected to each extremity, as AC and BD. Then, if the pipe be conftantly filled with water to a certain height, as AE, it will alfo rife to its level in the oppofite perpendicular tube BF; and if one inch of water be added in the tube AC, it will almoft inftantly produce a fimilar elevation of the tube BD; fo that by correfponding letters being adapted to the tubes AC and BD, at different heights, intelligence might be conveyed. But the method is liable to fuch objections, that it is not likely it can ever be adopted to facilitate the object of very diftant communication.

Full as many, if not greater objections, will perhaps operate againft every mode of electricity being ufed as the vehicle of information.—And the requifite magnitude of painted or illuminated letters offers an unfurmountable obftacle; befides, in them one object would be loft, that of the language being figurative.

As to the French machine, it is evident that to every angular change of the greater beam or of the leffer end arms, a different letter or figure may be annexed. But where the whole difference confifts in the variation of the angle of the greater or leffer pieces, much error may be expected, from the inaccuracy either of the operator or the obferver: befides other inconveniences arifing from the great magnitude of the machinery.

Another idea is perfectly numerical; which is to raife and deprefs a flag or curtain a certain number of times for each letter, according to a previoufly concerted fyftem: as, fuppofe one elevation to mean A, two to mean B, and fo on through the alphabet. But in this cafe, the leaft inaccuracy in giving or noting the number

number changes the letter; and besides, the last letters of the alphabet would be a tedious operation.

Another method that has been proposed, is an ingenious combination of the magnetical experiment of Comus, and the telescopic micrometer. But as this is only an imperfect idea of Mr. Garnet's very ingenious machine, described in the latter part of this article, no farther notice need be taken of it here.

Mr. Gamble then proposes one on a new idea of his own. The principle of it is simply that of a Venetian blind, or rather what are called the lever boards of a brewhouse, which, when horizontal, present so small a surface to the distant observer, as to be lost to his view, but are capable of being in an instant converted into a screen of a magnitude adapted to the required distance of vision.—Let AB and CD (fig. 4. pl. 28), two upright posts fixed in the ground, and joined by the braces BD and EF, be considered as the frame work for 9 lever boards working upon centres in EB and DF, and opening in three divisions by iron rods connected with each three of the lever boards. Let *abcd* and *efgh* be two lesser frames fixed to the great one, having also three lever boards in each, and moving by iron rods, in the same manner as the others. If all these rods be brought so near the ground as to be in the management of the operator, he will then have five, of what may be called, keys to play on. Now as each of the handles *iklmn* commands three lever boards, by raising any one of them, and fixing it in its place by a catch or hook, it will give a different appearance to the machine; and by the proper variation of these five movements, there will be more than 25 of what may be called mutations, in each of which the machine exhibits a different appearance, and to which any letter or figure may be annexed at pleasure.

Should it be required to give intelligence in more than one direction, the whole machine may be easily made to turn to different points on a strong centre, after the manner of a single-post windmill.—To use this machine by night, another frame must be connected with the back part of the Telegraph, for raising five lamps, of different colours, behind the openings of the lever boards; these lamps by night answer for the openings by day.

M. Gamble gives also particular directions for placing and using the machine, and for writing down the several figures or movements.

I shall now conclude this article with a short idea of Mr. John Garnet's most simple and ingenious contrivance. This is merely a bar or plank turning upon a centre, like the sail of a windmill, and being moved into any position, the distant observer turns the tube of a telescope into the same position, by bringing a fixed wire within it to coincide with or parallel to the bar, which is a thing extremely easy to do. The centre of motion of the bar has a small circle about it, with letters and figures around the circumference, and an index moving round with the bar, pointing to any letter or mark that the operator wishes to set the bar to, or to communicate to the observer. The eye end of the telescope without has a like index and circle, with the corresponding letters or other marks. The consequence is obvious: the telescope being turned round till its wire cover or become parallel to the bar, the index of the former necessarily points out the same letter or mark in its circle, as that of the latter, and the communication of sentiment is immediate and perfect. The use of this machine is so easy, that I have seen it put into the hands of two common labouring men, who had never seen it before, and they have immediately held a quick and distant conversation together.

The more particular description and figure of this machine, take as follows. ABDE (fig. 5, pl. 28), is the Telegraph, on whose centre of gravity C, about which it revolves, is a fixed pin, which goes through a hole or socket in the firm upright post G, and on the opposite side of which is fixed an index CI. Concentric to C, on the same post, is fixed a wooden or brass circle, of 6 or 8 inches diameter, divided into 48 equal parts, 24 of which represent the letters of the alphabet, and between the letters, numbers. So that the index, by means of the arm AB, may be moved to any letter or number. The length of the arm should be $2\frac{1}{2}$ or 3 feet for every mile of distance. Two revolving lamps of different colours suspended occasionally at A and B, the ends of the arm, would serve equally at night.

Let *ss* (fig. 6, pl. 28) represent the section of the outward tube of a telescope perpendicular to its axis, and *xx* the like section of the sliding or adjusting tube, on which is fixed an index II. On the part of the outward tube next to the observer, there is fixed a circle of letters and numbers, similarly divided and situated to the circle in figure 3; then the index II, by means of the sliding or adjusting tube, may be turned to any letter or number.—Now there being a cross hair, or fine silver wire *fg*, fixed in the focus of the eye glass, in the same direction as the index II; so that when the arm AB (fig. 5) of the Telegraph is viewed at a distance through the telescope, the cross hair may be turned, by means of the sliding tube, to the same direction of the arm AB; then the index II (fig. 6) will point to the same letter or number on its own circle, as the index I (fig. 5) points to on the Telegraphic circle.

If, instead of using the letters and numbers to form words at length, they be used as signals, three motions of the arm will give above a hundred thousand different signals.

TELESCOPE, an optical instrument which serves for discovering and viewing distant objects, either directly by glasses, or by reflection, by means of specula, or mirrors. Accordingly,

Telescopes are either refracting or reflecting; the former consisting of different lenses, through which the objects are seen by rays refracted through them to the eye; and the latter of specula, from which the rays are reflected and passed to the eye. The lens or glass turned towards the object, is called the *object-glass*; and that next the eye, the *eye-glass*; and when the Telescope consists of more than two lenses, all but that next the object are called *eye-glasses*.

The invention of the Telescope is one of the noblest and most useful these ages have to boast of: by means of it, the wonders of the heavens are discovered to us, and astronomy is brought to a degree of perfection which former ages could have no idea of.

The

The diſcovery indeed was owing rather to chance than deſign; ſo that it is the good fortune of the diſcoverer, rather than his ſkill or ability, we are indebted to: on this account it concerns us the leſs to know, who it was that firſt hit upon this admirable invention. Be that as it may, it is certain it muſt have been caſual, ſince the theory it depends upon was not then known.

John Baptiſta Porta, a Neapolitan, according to Wolfius, firſt made a Teleſcope, which he infers from this paſſage in the *Magia Naturalis* of that author, printed in 1560: "If you do but know how to join "the two (viz, the concave and convex glaſſes) rightly "together, you will ſee both remote and near objects, "much larger than they otherwiſe appear, and withal "very diſtinct. In this we have been of good help "to many of our friends, who either ſaw remote "things dimly, or near ones confuſedly; and have "made them ſee every thing perfectly."

But it is certain, that Porta did not underſtand his own invention, and therefore neither troubled himſelf to bring it to a greater perfection, nor ever applied it to celeſtial obſervation. Beſides, the account given by Porta of his concave and convex lenſes, is ſo dark and indiſtinct, that Kepler, who examined it by deſire of the emperor Rudolph, declared to that prince, that it was perfectly unintelligible.

Thirty years afterwards, or in 1590, a Teleſcope 16 inches long was made, and preſented to prince Maurice of Naſſau, by a ſpectacle maker of Middleburg: but authors are divided about his name. Sirturus, in a treatiſe on the Teleſcope, printed in 1618, will have it to be John Lipperſheim: and Borelli, in a volume expreſsly on the inventor of the Teleſcope, publiſhed in 1655, ſhews that it was Zacharias Janſen, or, as Wolfius writes it, Hanſen.

Now the invention of Lipperſheim is fixed by ſome in the year 1609, and by others in 1605: Fontana, in his *Novæ Obſervationes Cæleſtium et Terreſtrium Rerum*, printed at Naples in 1646, claims the invention in the year 1608. But Borelli's account of the diſcovery of Teleſcopes is ſo circumſtantial, and ſo well authenticated, as to render it very probable that Janſen was the original inventor.

In 1620, James Metius of Alcmaer, brother of Adrian Metius who was profeſſor of mathematics at Franeker, came with Drebel to Middleburg, and there bought Teleſcopes of Janſen's children, who had made them public; and yet this Adr. Metius has given his brother the honour of the invention, in which too he is miſtakenly followed by Deſcartes.

But none of theſe artificers made Teleſcopes of above a foot and a half: Simon Marius in Germany, and Galileo in Italy, it is ſaid, firſt made long ones fit for celeſtial obſervations; though, from the recently diſcovered aſtronomical papers of the celebrated Harriot, author of the Algebra, it appears that he muſt have made uſe of Teleſcopes in viewing the ſolar maculæ, which he did quite as early as they were obſerved by Galileo. Whether Harriot made his own Teleſcopes, or whether he had them from Holland, does not appear: it ſeems however that Galileo's were made by himſelf; for Le Roſſi relates, that Galileo, being then at Venice, was told of a ſort of optic glaſs made in Holland, which brought objects nearer: upon which, ſetting himſelf to think how it ſhould be, he ground two pieces of glaſs into form as well as he could, and fitted them to the two ends of an organ-pipe; and with theſe he ſhewed at once all the wonders of the invention to the Venetians, on the top of the tower of St. Mark. The ſame author adds, that from this time Galileo devoted himſelf wholly to the improving and perfecting the Teleſcope; and that he hence almoſt deſerved all the honour uſually done him, of being reputed the inventor of the inſtrument, and of its being from him called *Galileo's tube*. Galileo himſelf, in his *Nuncius Sidereus*, publiſhed in 1610, acknowledges that he firſt heard of the inſtrument from a German; and that, being merely informed of its effects, firſt by common report, and a few days after by letter from a French gentleman, James Badovere, at Paris, he himſelf diſcovered the conſtruction by conſidering the nature of refraction. He adds in his *Saggiatore*, that he was at Venice when he heard of the effects of prince Maurice's inſtrument, but nothing of its conſtruction; that the firſt night after his return to Padua, he ſolved the problem, and made his inſtrument the next day, and ſoon after preſented it to the Doge of Venice, who, in honour of his grand invention, gave him the ducal letters, which ſettled him for life in his lectureſhip, at Padua, and doubled his ſalary, which then became treble of what any of his predeceſſors had enjoyed before. And thus Galileo may be conſidered as an inventor of the Teleſcope, though not the firſt inventor.

F. Mabillon indeed relates, in his travels through Italy, that in a monaſtery of his own order, he ſaw a manuſcript copy of the works of Commeſtor, written by one Conradus, who lived in the 13th century; in the 3d page of which was ſeen a portrait of Ptolomy, viewing the ſtars through a tube of 4 joints or draws: but that father does not ſay that the tube had glaſſes in it. Indeed it is more than probable, that ſuch tubes were then uſed for no other purpoſe but to defend and direct the ſight, or to render it more diſtinct, by ſingling out the particular object looked at, and ſhutting out all the foreign rays reflected from others, whoſe proximity might have rendered the image leſs preciſe. And this conjecture is verified by experience; for we have often obſerved that without a tube, by only looking through the hand, or even the fingers, or a pinhole in a paper, the objects appear more clear and diſtinct than otherwiſe.

Be this as it may, it is certain that the optical principles, upon which Teleſcopes are founded, are contained in Euclid, and were well known to the ancient geometricians; and it has been for want of attention to them, that the world was ſo long without that admirable invention; as doubtleſs there are many others lying hid in the ſame principles, only waiting for reflection or accident to bring them forth.

To the foregoing abſtract of the hiſtory of the invention of the Teleſcope, it may be proper to add ſome particulars relating to the claims of our own celebrated countryman, friar Bacon, who died in 1294. Mr. W. Molyneux, in his Dioptrica Nova, pa. 256, declares his opinion, that Bacon did perfectly well underſtand all ſorts of optic glaſſes, and knew likewiſe the way

way of combining them, ſo as to compoſe ſome ſuch inſtrument as our Teleſcope: and his ſon, Samuel Molyneux, aſſerts more poſitively, that the invention of Teleſcopes, in its firſt original, was certainly put in practice by an Engliſhman, friar Bacon; although its firſt application to aſtronomical purpoſes may probably be aſcribed to Galileo. The paſſages to which Mr. Molyneux refers, in ſupport of Bacon's claims, occur in his Opus Majus, pa. 348 and 357 of Jebb's edit. 1773. The firſt is as follows: *Si vero non ſint corpora plana, per quæ viſus videt, ſed ſphæria, tunc eſt magna diverſitas; nam vel concavitas corporis eſt verſus oculum vel convexitas:* whence it is inferred, that he knew what a concave and a convex glaſs was. The ſecond is compriſed in a whole chapter, where he ſays, *De viſione fracta majora ſunt; nam de facili patet per canones ſupra dictos, quod maxima poſſunt apparere minima, et e contra, et longe diſtantia videbuntur propinquiſſime, et e converſo. Nam poſſumus ſic figurare perſpicua, et taliter ea ordinare reſpectu noſtri viſus et rerum, quod frangentur radii, et flectentur quorſumcunque voluerimus, ut ſub quocunque angulo voluerimus, videbimus rem prope vel longe, &c." Sic etiam faceremus ſolem et lunam et ſtellas deſcendere ſecundum apparentiam hic inferius, &c:* that is, Greater things than theſe may be performed by refracted viſion; for it is eaſy to underſtand by the canons above mentioned, that the greateſt things may appear exceeding ſmall, and the contrary; alſo that the moſt remote objects may appear juſt at hand, and the converſe; for we can give ſuch figures to tranſparent bodies, and diſpoſe them in ſuch order with reſpect to the eye and the objects, that the rays ſhall be refracted and bent towards any place we pleaſe; ſo that we ſhall ſee the object near at hand or at a diſtance, under any angle we pleaſe, &c. So that thus the ſun, moon, and ſtars may be made to deſcend hither in appearance, &c. Mr. Molyneux has alſo cited another paſſage out of Bacon's Epiſtle ad Pariſienſem, of the Secrets of Art and Nature, cap. 5, to this purpoſe, *Poſſunt etiam ſic figurari perſpicua, ut longiſſime poſita appareant propinqua, et è contrario; ita quod ex incredibili diſtantia legeremus literas minutiſſimas, et numeraremus res quantumquo parvas, et ſtellas faceremus apparere quo vellemus:* that is, Glaſſes, or diaphanous bodies may be ſo formed, that the moſt remote objects may appear juſt at hand, and the contrary; ſo that we may read the ſmalleſt letters at an incredible diſtance, and may number things though never ſo ſmall, and may make the ſtars appear as near as we pleaſe.

Moreover, Doctor Jebb, in the dedication of his edition of the Opus Majus, produces a paſſage from a manuſcript, to ſhew that Bacon actually applied Teleſcopes to aſtronomical purpoſes: *Sed longe magis quam hæc,* ſays he, *oporteret homines haberi, qui bene, immo optime, ſcirent perſpectivam et inſtrumenta ejus—quia inſtrumenta aſtronomia non vadunt niſi per viſionem ſecundum leges iſtius ſcientiæ.*

From theſe paſſages, it is not unreaſonable to conclude, that Bacon had actually combined glaſſes ſo as to have produced the effects which he mentions, though he did not complete the conſtruction of Teleſcopes. Dr. Smith, however, to whoſe judgment particular deference is due, is of opinion that the celebrated friar wrote hypothetically, without having made any actual trial of the things he mentions: to which purpoſe he obſerves, that this author does not aſſert one ſingle trial or obſervation upon the ſun or moon, or any thing elſe, though he mentions them both: on the other hand, he imagines ſome effects of Teleſcopes that cannot poſſibly be performed by them. He adds, that perſons unexperienced in looking through Teleſcopes expect, in viewing any object, as for inſtance the face of a man, at the diſtance of one hundred yards, through a Teleſcope that magnifies one hundred times, that it will appear much larger than when they are cloſe to it: this he is ſatisfied was Bacon's notion of the matter; and hence he concludes that he had never looked through a Teleſcope.

It is remarkable that there is a paſſage in Thomas Digges's Stratioticos, pa. 359, where he affirms that his father, Leonard Digges, among other curious practices, had a method of diſcovering, by perſpective glaſſes ſet at due angles, all objects pretty far diſtant that the ſun ſhone upon, which lay in the country round about; and that this was by the help of a manuſcript book of Roger Bacon of Oxford, who he conceived was the only man beſides his father (ſince Archimedes) who knew it. This is the more remarkable, becauſe the Stratioticos was firſt printed in 1579, more than 30 years before Metius or Galileo made their diſcovery of thoſe glaſſes; and therefore it has hence been thought that Roger Bacon was the firſt inventor of Teleſcopes, and Leonard Digges the next reviver of them. But from what Thomas Digges ſays of this matter, it would ſeem that the inſtrument of Bacon, and of his father, was ſomething of the nature of a camera obſcura, or, if it were a Teleſcope, that it was of the reflecting kind; although the term *perſpective* glaſs ſeems to favour a contrary opinion.

There is alſo another paſſage to the ſame effect in the preface to the Pantometria of Leonard Digges, but publiſhed by his ſon Thomas Digges, ſome time before the Stratioticos, and a ſecond time in the year 1591. The paſſage runs thus: *My father by his continuall painfull practiſes, aſſiſted with demonſtrations mathematical, was able, and ſundrie times hath by Proportional Glaſſes duely ſituate in convenient angles, not only diſcovered things farre off, read letters, numbered peeces of money with the very coyne and ſuperſcription thereof, caſt by ſome of his freends of purpoſe upon downes in open fields, but alſo ſeven myles off declared what hath beene doone at that inſtant in private places: He hath alſo ſundrie times by the ſunne beames fixed* (ſhould be *fired*) *powder, and diſchargde ordinance halfe a mile and more diſtante,* &c.

But to whomſoever we aſcribe the honour of firſt inventing the Teleſcope, the rationale of this admirable inſtrument, depending on the refraction of light in paſſing through mediums of different forms, was firſt explained by the celebrated Kepler, who alſo pointed out methods of conſtructing others, of ſuperior powers, and more commodious application, than that firſt uſed: though ſomething of the ſame kind, it is ſaid, was alſo done by Maurolycus, whoſe treatiſe De Lumine et Umbra was publiſhed in 1575.

The Principal Effects of TELESCOPES, depend upon this plain maxim, viz, that objects appear larger in proportion to the angles which they ſubtend at the eye;

eye; and the effect is the same, whether the pencils of rays, by which objects are visible to us, come directly from the objects themselves, or from any place nearer to the eye, where they may have been united, so as to form an image of the object; because they issue again from those points in certain directions, in the same manner as they did from the corresponding points in the objects themselves. In fact therefore, all that is effected by a Telescope, is first to make such an image of a distant object, by means of a lens or mirror, and then to give the eye some assistance for viewing that image as near as possible; so that the angle, which it shall subtend at the eye, may be very large, compared with the angle which the object itself would subtend in the same situation. This is done by means of an eye-glass, which so refracts the pencils of rays, as that they may afterwards be brought to their several foci, by the natural humours of the eye. But if the eye had been so formed as to be able to see the image, with sufficient distinctness, at the same distance, without an eye-glass, it would appear to him as much magnified, as it does to another person who makes use of a glass for that purpose, though he would not in all cases have so large a field of view.

Although no image be actually formed by the foci of the pencil without the eye, yet if, by the help of an eye-glass, the pencils of rays shall enter the pupil, just as they would have done from any place without the eye, the visual angle will be the same as if an image had been actually formed in that place. Priestley's History of Light &c, pa. 69, &c.

As to the Grinding of Telescopic Glasses, the first persons who distinguished themselves in that way, were two Italians, Eustachio Divini at Rome, and Campani at Bologna, whose fame was much superior to that of Divini, or that of any other person of his time; though Divini himself pretended, that in all the trials that were made with their glasses, his of a great focal distance performed better than those of Campani, and that his rival was not willing to try them fairly, viz, with equal eye-glasses. It is however generally supposed, that Campani really excelled Divini, both in the goodness and the focal length of his object-glasses.

It was with Campani's Telescopes that Cassini discovered the nearest satellites of Saturn. They were made at the express desire of Lewis XIV, and were of 86, 100, and 136, Paris feet focal length.

Campani's laboratory was purchased, after his death, by pope Benedict XIV, who made a present of it to the academy at Bologna called the Institute; and by the account which Fougeroux has given, we learn that (except a machine which Campani constructed, to work the basons on which he ground his glasses) the goodness of his lenses depended upon the clearness of his glass, his Venetian tripoli, the paper with which he polished his glasses, and his great skill and address as a workman. It does not appear that he made many lenses of a very great focal distance. Accordingly Dr. Hook, who probably speaks with the partiality of an Englishman, says that some glasses, made by Divini and Campani, of 36 and 50 feet focal distance, did not excel Telescopes of 12 or 15 feet made in England. He adds, that Sir Paul Neilli made Telescopes of 36 feet, pretty good; and one of 50, but not of proportionable goodness.

Afterwards, Mr. Reive first, and then Mr. Cox, who were the most celebrated in England, as grinders of optic glasses, made some good Telescopes of 50 and 60 feet focal distance; and Mr. Cox made one of 100, but how good Dr. Hook could not assert. Borelli also in Italy, made object-glasses of a great focal length, one of which he presented to the Royal Society. But, with respect to the focal length of Telescopes, these and all others were far exceeded by those of Auzout, who made one object-glass of 600 feet focus; but he was never able to manage it, so as to make any use of it. And Hartsoeker, it is said, made some of a still greater focal length. Philos. Trans. Abr. vol. i, p. 193. Hook's Exper. by Derham, p. 261. Priestley as above, p. 211. See GRINDING.

Telescopes are of several kinds, distinguished by the number and form of their lenses, or glasses, and denominated from their particular uses &c: such are the *terrestrial* or *land Telescope*, the *celestial* or *astronomical Telescope*; to which may be added, the *Galilean* or *Dutch Telescope*, the *reflecting Telescope*, the *refracting Telescope*, the *aërial Telescope*, *achromatic Telescope*, &c.

Galileo's, or the *Dutch Telescope*, is one consisting of a convex object-glass, and a concave eye-glass.

This is the most ancient form of any, being the only kind made by the inventors, Galileo, &c. or known, before Huygens. The first Telescope, constructed by Galileo, magnified only 3 times; but he soon made another, which magnified 18 times: and afterwards, with great trouble and expence, he constructed one that magnified 33 times; with which he discovered the satellites of Jupiter, and the spots of the sun. The construction, properties, &c, of it, are as follow:

Construction of Galileo's, or the *Dutch* TELESCOPE. In a tube prepared for the purpose, at one end is fitted a convex object lens, either a plain convex, or convex on both sides, but a segment of a very large sphere: at the other end is fitted an eye-glass, concave on both sides, and the segment of a less sphere, so disposed as to be at the distance of the virtual focus before the image of the convex lens.

Let AB (fig. 10, pl. 23) be a distant object, from every point of which pencils of rays issue, and falling upon the convex glass DE, tend to their foci at FSG. But a concave lens HI (the focus of which is at FG) being interposed, the converging rays of each pencil are made parallel when they reach the pupil; so that by the refractive humours of the eye, they can easily be brought to a focus on the retina at PRQ. Also the pencils themselves diverging, as if they came from X, MXO is the angle under which the image will appear, which is much larger than the angle under which the object itself would have appeared. Such then is the Telescope that was at first discovered and used by philosophers: the great inconvenience of which is, that the field of view, which depends, not on the breadth of the eye-glass, as in the astronomical Telescope, but upon the breadth of the pupil of the eye, is exceedingly small: for since the pencils of the rays enter the eye very much diverging from one another, but few of them can be intercepted by the pupil; and this inconvenience increases with the magnifying power of the Telescope, so that philosophers may now well wonder at

at the patience and addrefs with which Galileo and others, with fuch an inftrument, made the difcoveries they did. And yet no other Telefcope was thought of for many years after the difcovery. Defcartes, who wrote 30 years after, mentions no other as actually conftructed, though Kepler had fuggefted fome. Hence,

1. In an inftrument thus framed, all people, except myopes, or fhort-fighted perfons, muft fee objects diftinctly in an erect fituation, and increafed in the ratio of the diftance of the virtual focus of the eye-glafs, to the diftance of the focus of the object glafs.

2. But for myopes to fee objects diftinctly through fuch an inftrument, the eye-glafs muft be fet nearer the object-glafs, fo that the rays of each pencil may not emerge parallel, but may fall diverging upon the eye; in which cafe the apparent magnitude will be altered a little, though fcarce fenfibly.

3. Since the focus of a plano-convex object lens, and the vertical focus of a plano-concave eye-lens, are at the diftance of the diameter; and the focus of an object-glafs convex on both fides, and the vertical focus of an eye-glafs concave on both fides, are at the diftance of a femidiameter; if the object-glafs be plano-convex, and the eye-glafs plano-concave, the Telefcope will increafe the diameter of the object, in the ratio of the diameter of the concavity to that of the convexity: if the object-glafs be convex on both fides, and the eye-glafs concave on both fides, it will magnify in the ratio of the femidiameter of the concavity to that of the convexity: if the object-glafs be plano-convex, and the eye-glafs concave on both fides, the femidiameter of the object will be increafed in the ratio of the diameter of the convexity to the femidiameter of the concavity: and laftly, if the object-glafs be convex on both fides, and the eye-glafs plano-concave, the increafe will be in the ratio of the diameter of the concavity to the femidiameter of the convexity.

4. Since the ratio of the femidiameters is the fame as that of the diameters, Telefcopes magnify the object in the fame manner, whether the object-glafs be plano-convex, and the eye-glafs plano-concave; or whether the one be convex on both fides, and the other concave on both.

5. Since the femidiameter of the concavity has a lefs ratio to the diameter of the convexity than its diameter has, a Telefcope magnifies more if the object-glafs be plano-convex, than if it be convex on both fides. The cafe is the fame if the eye-glafs be concave on both fides, and not plano-concave.

6. The greater the diameter of the object-glafs, and the lefs that of the eye-glafs, the lefs ratio has the diameter of the object, viewed with the naked eye, to its femidiameter when viewed with a Telefcope, and confequently the more is the object magnified by it.

7. Since a Telefcope exhibits fo much a lefs part of the object, as it increafes its diameter more, for this reafon, mathematicians were determined to look out for another Telefcope, after having clearly found the imperfection of the firft, which was difcovered by chance. Nor were their endeavours vain, as appears from the aftronomical Telefcope defcribed below.

If the femidiameter of the eye-glafs have too fmall a ratio to that of the object-glafs, an object through the Telefcope will not appear fufficiently clear, becaufe the great divergency of the rays will occafion the feveral pencils reprefenting the feveral points of the object on the retina, to confift of too few rays.

It is alfo found that equal object-lenfes will not bear the fame eye-lenfes, if they be differently tranfparent, or if there be a difference in their polifh; a lefs tranfparent object-glafs, or one lefs accurately ground, requiring a more fpherical eye-glafs than another more tranfparent, &c.

Hevelius recommends an object-glafs convex on both fides, whofe diameter is 4 feet; and an eye-glafs concave on both fides, whofe diameter is $4\frac{1}{2}$ tenths of a foot. An object-glafs, equally convex on both fides, whofe diameter is 5 feet, he obferves, will require an eye-glafs of $5\frac{1}{2}$ tenths; and adds, that the fame eye-glafs will alfo ferve an object-glafs of 8 or 10 feet.

Hence, as the diftance between the object-glafs and eye-glafs is the difference between the diftance of the vertical focus of the eye-glafs, and the diftance of the focus of the object glafs; the length of the telefcope is had by fubtracting that from this. That is, the length of the Telefcope is the difference between the diameters of the object-glafs and eye-glafs, if the former be plano-convex, and the latter plano-concave; or the difference between the femidiameters of the object-glafs and eye-glafs, if the former be convex on both fides, and the latter concave on both; or the difference between the femidiameter of the object-glafs and the diameter of the eye-glafs, if the former be convex on both fides, and the latter plano-concave; or laftly the difference between the diameter of the object-glafs and the femidiameter of the eye-glafs, if the former be plano-convex, and the latter concave on both fides. Thus, for inftance, if the diameter of an object-glafs, convex on both fides, be 4 feet, and that of an eye glafs, concave on both fides, be $4\frac{1}{2}$ tenths of a foot; then the length of the Telefcope will be 1 foot and $7\frac{1}{2}$ tenths.

Aftronomical TELESCOPE; this is one that confifts of an object-glafs, and an eye-glafs, both convex. It is fo called from being wholly ufed in aftronomical obfervations.

It was Kepler who firft fuggefted the idea of this Telefcope; having explained the rationale, and pointed out the advantages of it in his Catoptrics, in 1611. But the firft perfon who actually made an inftrument of this conftruction, was father Scheiner, who has given a defcription of it in his Rofa Urfina, publifhed in 1630. To this purpofe he fays, If you infert two fimilar convex lenfes in a tube, and place your eye at a convenient diftance, you will fee all terreftrial objects, inverted indeed, but magnified and very diftinct, with a confiderable extent of view. He afterwards fubjoined an account of a Telefcope of a different conftruction, with two convex eye-glaffes, which again reverfes the images, and makes them appear in their natural pofition. Father Reita however foon after propofed a better conftruction, ufing three eye-glaffes inftead of two.

Conftruction of the Aftronomical TELESCOPE. The tube being prepared, an object-glafs, either plano-convex,

vex, or convex on both ſides, but a ſegment of a large ſphere, is fitted in at one end; and an eye-glaſs, convex on both ſides, which is the ſegment of a ſmall ſphere, is fitted into the other end; at the common diſtance of the foci.

Thus the rays of each pencil iſſuing from every point of the object ABC, (fig. 3 pl. 30) paſſing through the object-glaſs DEF, become converging, and meet in their foci at IHG, where an image of the object will be formed. If then another convex lens KM, of a ſhorter focal length, be ſo placed, as that its focus ſhall be in IHG, the rays of each pencil, after paſſing through it, will become nearly parallel, ſo as to meet upon the retina, and form an enlarged image of the object at RST. If the proceſs of the rays be traced, it will preſently be perceived that this image muſt be inverted. For the pencil that iſſues from A, has its focus in G, and again in R, on the ſame ſide with A. But as there is always one inverſion in ſimple viſion, this want of inverſion produces juſt the reverſe of the natural appearance. The field of view in this Teleſcope will be large, becauſe all the pencils that can be received on the ſurface of the lens KM, being converging after paſſing through it, are thrown into the pupil of the eye, placed in the common interſection of the pencils at P.

Theory of the Aſtronomical TELESCOPE.—An eye placed near the focus of the eye-glaſs, of ſuch a Teleſcope, will ſee objects diſtinctly, but inverted, and magnified in the ratio of the diſtance of the focus of the eye-glaſs to the diſtance of the focus of the object-glaſs.

If the ſphere of concavity in the eye glaſs of the Galilean Teleſcope, be equal to the ſphere of convexity in the eye-glaſs of another Teleſcope, their magnifying power will be the ſame. The concave glaſs however being placed between the object-glaſs and its focus, the Galilean Teleſcope will be ſhorter than the other, by twice the focal length of the eye-glaſs. Conſequently, if the length of the Teleſcopes be the ſame; the Galilean will have the greater magnifying power. Viſion is alſo more diſtinct in theſe Teleſcopes, owing in part perhaps to there being no intermediate image between the eye and the object. Beſides, the eye-glaſs being very thin in the centre, the rays will be leſs liable to be diſtorted by irregularities in the ſubſtance of the glaſs. Whatever be the cauſe, we can ſometimes ſee Jupiter's ſatellites very clearly in a Galilean Teleſcope, of 20 inches or 2 feet long, when one of 4 or 5 feet, of the common ſort, will hardly make them viſible.

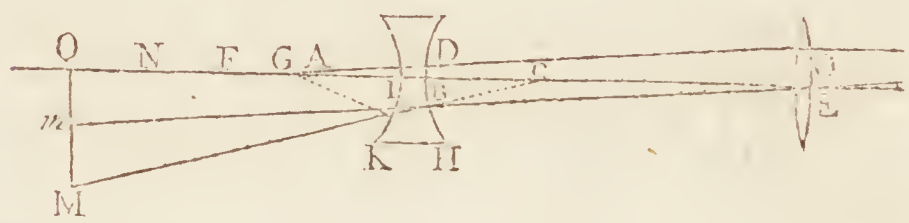
As the aſtronomical Teleſcope exhibits objects inverted, it ſerves commodiouſly enough for obſerving the ſtars, as it is not material whether *they* be ſeen erect or inverted; but for terreſtrial objects it is much leſs proper, as the inverting often prevents them from being known. But if a plane well-poliſhed metal ſpeculum, of an oval figure, and about an inch long, and inclined to the axis in an angle of 45°, be placed bebehind the eye-glaſs; then the eye, conveniently placed, will ſee the image, hence reflected, in the ſame magnitude as before, but in an erect ſituation; and therefore, by the addition of ſuch a ſpeculum, the aſtronomical Teleſcope is thus rendered fit to obſerve terreſtrial objects.

Since the focus of the glaſs convex on both ſides is diſtant from the glaſs itſelf a ſemidiameter, and that of a plano-convex glaſs, a diameter; if the object-glaſs be convex on both ſides, the Teleſcope will magnify the ſemidiameter of the object, in the ratio of the diameter of the eye-glaſs to the diameter of the object-glaſs; but if the object-glaſs be a plano-convex, in the ratio of the ſemidiameter of the eye-glaſs to the diameter of the object glaſs. And therefore a Teleſcope magnifies more if the object-glaſs be a plano-convex, than if convex on both ſides. And for the ſame reaſon, a Teleſcope magnifies more when the eye-glaſs is convex on both ſides, than when it is plano-convex.

A Teleſcope magnifies the more, as the object-glaſs is a ſegment of a great ſphere, and the eye-glaſs of a leſs one. And yet the eye-glaſs muſt not be too ſmall in reſpect of the object-glaſs; for if it be, it will not refract rays enough to the eye from each point of the object; nor will it ſeparate ſufficiently thoſe that come from different points; by which means the viſion will be rendered obſcure and confuſed.—De Chales obſerves, that an object-lens of $2\frac{1}{4}$ feet will require an eye-glaſs of $1\frac{1}{2}$ tenth of a foot; and an object glaſs of 8 or 10 feet, an eye-glaſs of 4 tenths; in which he is confirmed by Euſtachio Divini.

To ſhorten the Aſtronomical TELESCOPE; that is, to conſtruct a Teleſcope ſo, as that, though ſhorter than the common one, it ſhall magnify as much.

Having provided a drawing tube, fit in it an object-lens EO which is a ſegment of a moderate ſphere;



let the firſt eye-glaſs BD be concave on both ſides, and ſo placed in the tube, as that the focus of the object-glaſs A may be behind it, but nearer to the centre of the concavity G: then will the image be thrown in Q, ſo as that GA : GI : : AB : QI. Laſtly, fit in another object-glaſs, convex on both ſides, and a ſegment of a ſmaller ſphere, ſo as that its focus may be in Q.

This Teleſcope will magnify the diameter of the object more than if the object-glaſs were to repreſent its image at the ſame diſtance EQ; and conſequently a ſhorter Teleſcope, conſtructed this way, is equivalent to a longer in the common way. See Wolfius Elem. Math. vol. 3, p. 245.

Sir Iſaac Newton furniſhes us with another method of conſtructing the Teleſcope, in his catoptrical or reflecting Teleſcope, the conſtruction of which is given below. See *Achromatic* TELESCOPE.

Aërial TELESCOPE, a kind of aſtronomical Teleſcope, the lenſes of which are uſed without a tube. In ſtrictneſs however, the aërial Teleſcope is rather a particular manner of mounting and managing long

4 D 2 Teleſcopes

Telefcopes for celeftial obfervation in the night-time, by which the trouble of long unwieldy tubes is faved, than a particular kind of Telefcope; and the contrivance was one of Huygens's. This invention was fuccefsfully practifed by the inventor himfelf and others, particularly with us by Dr. Pound and Dr. Bradley, with an object-glafs of 123 feet focal diftance, and an apparatus belonging to it, made and prefented by Huygens to the Royal Society, and defcribed in his Aftrofcopia Compendiaria Tubi Optici Molimine Liberata, printed at the Hague in 1684.

The principal parts of this Telefcope may be comprehended from a view of fig. 4, pl. 30, where AB is a long pole, or a maft, or a high tree, &c, in a groove of which flides a piece that carries a fmall tube LK in which is fixed an object glafs; which tube is connected by a fine line, with another fmall tube OQ, which contains the eye-glafs, &c.

La Hire contrived a little machine for managing the object-glafs which is defcribed Mem. de l'Acad. 1715. See Smith's Optics, book 3, chap. 10.

Hartfoeker, who made Telefcopes of a very confiderable focal length, contrived a method of ufing them without a tube, by fixing them to the top of a tree, a high wall, or the roof of a houfe. Mifcel. Berol. vol. 1, p. 261.

Huygens's great Telefcope, with which Saturn's true face, and one of his fatellites were firft difcovered, confifts of an object-glafs of 12 feet, and an eye-glafs of a little more than 3 inches; though he frequently ufed a Telefcope of 23 feet long, with two eye-glaffes joined together, each $1\frac{1}{2}$ inch diameter; fo that the two were equal to one of 3 inches.

The fame author obferves, that an object-glafs of 30 feet requires an eye-glafs of $3\frac{3}{10}$ inches; and has given a table of proportions for conftructing aftronomical Telefcopes, an abridgment of which is as follows:

Dift. of Foc. of Obj. Glaff.	Diameter of Apert.	Dift. of Foc. of Eye-glafs.	Power or Magnitude of Diam.
Feet.	Inches and Decim.	Inches and Decim.	
1	0·55	0·61	20
2	0·77	0·85	28
3	0·95	1·05	34
4	1·09	1·20	40
5	1·23	1·35	44
6	1·34	1·47	49
7	1·45	1·60	53
8	1·55	1·71	56
9	1·64	1·80	60
10	1·73	1·90	63
15	2·12	2·33	77
20	2·45	2·70	89
25	2·74	3·01	100
30	3·00	3·30	109
40	3·46	3·81	120

Dift. of Foc. of Object-glafs.	Diameter of Apert.	Dift. of Foc. of Eye-glafs.	Power or Magnitude of Diam.
Feet.	Inches and Decim.	Inches and Decim.	
50	3·87	4·26	141
60	4·24	4·66	154
70	4·58	5·04	166
80	4·90	5·39	178
90	5·20	5·72	189
100	5·49	6·03	200
120	6·00	6·60	218
140	6·48	7·12	235
160	6·93	7·62	252
180	7·35	8·09	267
200	7·75	8·53	281
220	8·12	8·93	295
240	8·48	9·33	308
260	8·83	9·71	321
280	9·16	10·08	333
300	9·49	10·44	345
400	10·95	12·05	400
500	12·25	13·47	445
600	13·42	14·76	488

Dr. Smith (Rem. p. 78) obferves, that the magnifying powers of this table are not fo great as Huygens himfelf intended, or as the beft object-glaffes now made will admit of. For the author, in his Aftrofcopia Compendiaria, mentions an object-glafs of 34 feet focal diftance, which, in aftronomical obfervations, bore an eye-glafs of $2\frac{1}{2}$ inches focal diftance, and confequently magnified 163 times. According to this ftandard, a Telefcope of 35 feet ought to magnify 166 times, and of 1 foot 28 times; whereas the table allows but 118 times to the former, and but 20 to the latter. Now $\frac{166}{118}$ or $\frac{28}{20} = 1·4$; by which if we multiply the numbers in the given column of magnifying powers, we fhall gain a new column, fhewing how much thofe object-glaffes ought to magnify if wrought up to the perfection of this ftandard.

The new apertures and eye glaffes muft alfo be taken in the fame proportions to one another, as the old ones have in the table; or the eye-glaffes may be found by dividing the length of each Telefcope by its magnifying power. And thus a new table may be eafily made for this or any other more perfect ftandard when offered.

The rule for computing this table depends on the following theorem, viz, that in refracting Telefcopes of different lengths, a given object will appear equally bright and equally diftinct, when their linear apertures and the focal diftances of their eye-glaffes are feverally in a fubduplicate ratio of their lengths, or focal diftances of their object-glaffes; and then alfo the breadth of their apertures will be in the fubduplicate ratio of their lengths.

The rule is this: Multiply the number of feet in the focal diftance of any propofed object-glafs by 3000, and the fquare-root of the product will give the breadth of its aperture in centefms, or 100th parts of an inch; that

that is, $\sqrt{3000F}$ is the breadth of the aperture in centesms of an inch, where F is the focal diſtance of the object-glaſs in feet. Alſo, the ſame breadth of the aperture increaſed by the 10th part of itſelf, gives the focal diſtance of the eye-glaſs in centesms of an inch. And the magnifying powers are as the breadths of the apertures.

If, in different Teleſcopes, the ratio between the object-glaſs and eye-glaſs be the ſame, the object will be magnified the ſame in both. Hence ſome may conclude the making of large Teleſcopes a needleſs trouble. But it muſt be remembered, that an eye-glaſs may be in a leſs ratio to a greater object-glaſs than to a ſmaller: thus, for example, in Huygens's Teleſcope of 25 feet, the eye-glaſs is 3 inches: now, keeping this proportion in a Teleſcope of 50 feet, the eye-glaſs ſhould be 6 inches; but the table ſhews that $4\frac{1}{2}$ are ſufficient. Hence, from the ſame table it appears, that a Teleſcope of 50 feet magnifies in the ratio of 1 to 141; whereas that of 25 feet only magnifies in the ratio of 1 to 100.

Since the diſtance of the lens is equal to the aggregate of the diſtances of the foci of the object and eye-glaſſes; and ſince the focus of a glaſs convex on each ſide is a ſemidiameter's diſtance from the lens, and that of a plano-convex at a diameter's diſtance from the ſame; the length of a Teleſcope is equal to the aggregate of the ſemidiameters of the lenſes, if the object-glaſs be convex on both ſides; and to the ſum of the ſemidiameter of the eye-glaſs and the whole diameter of the object glaſs, if the object-glaſs be a plano-convex.

But as the diameter of the eye-glaſs is very ſmall in reſpect of that of the object-glaſs, the length of the Teleſcope is uſually eſtimated from the diſtance of the object-glaſs; i. e. from its ſemidiameter if it be convex on both ſides, or its whole diameter if plano-convex. Thus, a Teleſcope is ſaid to be 12 feet, if the ſemidiameter of the object-glaſs, convex on both ſides, be 12 feet, &c.

Since myopes ſee near objects beſt; for them, the eye-glaſs is to be removed nearer to the object-glaſs, that the rays refracted through it may be the more diverging.

To take in the larger field at one view, ſome make uſe of two eye-glaſſes, the foremoſt of which is a ſegment of a larger ſphere than that behind; to this it muſt be added, that if two lenſes be joined immediately together, ſo as the one may touch the other, the focus is removed to double the diſtance which that of one of them would be at.

Land Telescope, or *Day* Telescope, is one adapted for viewing objects in the day-time, on or about the earth. This contains more than two lenſes, uſually it has a convex object-glaſs, and three convex eye-glaſſes; exhibiting objects erect, yet different from that of Galileo.

In this Teleſcope, after the rays have paſſed the firſt eye-glaſs HI (fig. 2, pl. 30), as in the former conſtruction, inſtead of being there received by the eye, they paſs on to another equally convex lens, ſituated at twice its focal diſtance from the other, ſo that the rays of each pencil, being parallel in that whole interval, thoſe pencils croſs one another in the common focus, and the rays conſtituting them are tranſmitted parallel to the ſecond eye-glaſs LM; after which the rays of each pencil converge to other foci at NO, where a ſecond image of the object is formed, but inverted with reſpect to the former image in EF. This image then being viewed by a third eye-glaſs QR, is painted upon the retina at XYZ, exactly as before, only in a contrary poſition.

Father Reita was the author of this conſtruction; which is effected by fitting in at one end of a tube an object-glaſs, which is either convex on both ſides, or plano-convex, and a ſegment of a large ſphere; to this add three eye-glaſſes, all convex on both ſides, and ſegments of equal ſpheres; diſpoſing them in ſuch a manner as that the diſtance between any two may be the aggregate of the diſtances of their foci. Then will an eye applied to the laſt lens, at the diſtance of its focus, ſee objects very diſtinctly, erect, and magnified in the ratio of the diſtance of the focus of one eye-glaſs, to the diſtance of the focus of the object-glaſs.

Hence, 1. An aſtronomical Teleſcope is eaſily converted into a Land Teleſcope, by uſing three eye-glaſſes for one; and the Land Teleſcope, on the contrary, into an aſtronomical one, by taking away two eye-glaſſes, the faculty of magnifying ſtill remaining the ſame.

2. Since the diſtance of the eye-glaſſes is very ſmall, the length of the Teleſcope is much the ſame as if you only uſed one.

3. The length of the Teleſcope is found by adding five times the ſemidiamer of the eye-glaſſes, to the diameter of the object-glaſs when this is a plano-convex, or to its ſemidiameter when convex on both ſides.

Huygens firſt obſerved, both in the aſtronomical and Land Teleſcope, that it contributes conſiderably to the perfection of the inſtrument, to have a ring of wood or metal, with an aperture, a little leſs than the breadth of the eye-glaſs, fixed in the place where the image is found to radiate upon the lens next the eye: by means of which, the colours, which are apt to diſturb the clearneſs and diſtinctneſs of the object, are prevented, and the whole compaſs taken in at one view, perfectly defined.

Some make Land Teleſcopes of three lenſes, which yet repreſent objects erect, and magnified as much as the former. But ſuch Teleſcopes are ſubject to very great inconveniences, both as the objects in them are tinged with falſe colours, and as they are diſtorted about the margin.

Some again uſe five lenſes, and even more; but as ſome parts of the rays are intercepted in paſſing every lens, objects are thus exhibited dim and feeble.

Teleſcopes of this kind, longer than 20 feet, will be of hardly any uſe in obſerving terreſtrial objects, on account of the continual motion of the particles of the atmoſphere, which theſe powerful Teleſcopes render viſible, and give a tremulous motion to the objects themſelves.

The great length of dioptric Teleſcopes, adapted to any important aſtronomical purpoſe, rendered them extremely inconvenient for uſe; as it was neceſſary to increaſe their length in no leſs a proportion than the duplicate

duplicate of the increafe of their magnifying power: fo that, in order to magnify twice as much as before, with the fame light and diftinctnefs, the Telefcope required to be lengthened 4 times; and to magnify thrice as much, 9 times the length, and fo on. This unwieldinefs of refracting Telefcopes, poffeffing any confiderable magnifying power, was one caufe, why the attention of aftronomers, &c, was directed to the difcovery and conftruction of reflecting Telefcopes. And indeed a refracting Telefcope, even of 1000 feet focus, fuppofing it poffible to make ufe of fuch an inftrument, could not be made to magnify with diftinctnefs more than 1000 times; whereas a reflecting Telefcope, of 9 or 10 feet, will magnify 12 hundred times. The perfection of refracting Telefcopes, it is well known, is very much limited by the aberration of the rays of light from the geometrical focus: and this arifes from two different caufes, viz, from the different degrees of refrangibility of light, and from the figure of the fphere, which is not of a proper curvature for collecting the rays in a fingle point. Till the time of Newton, no optician had imagined that the object glaffes of Telefcopes were fubject to any other error befide that which arofe from their fpherical figure, and therefore all their efforts were directed to the conftruction of them, with other kinds of curvature: but that author had no fooner demonftrated the different refrangibility of the rays of light, than he difcovered in this circumftance a new and a much greater caufe of error in Telefcopes. Thus, fince the pencils of each kind of light have their foci in different places, fome nearer and fome farther from the lens, it is evident that the whole beam cannot be brought into any one point, but that it will be drawn the neareft to a point in the middle place between the focus of the moft and leaft refrangible rays; fo that the focus will be a circular fpace of a confiderable diameter. Newton fhews that this fpace is about the 55th part of the aperture of the Telefcope, and that the focus of the moft refrangible rays is nearer to the object-glafs than the focus of the leaft refrangible ones, by about the $27\frac{1}{2}$ part of the diftance between the object-glafs, and the focus of the mean refrangible rays. But he fays, that if the rays flow from a lucid point, as far from the lens on one fide as their foci are on the other, the focus of the moft refrangible rays will be nearer to the lens than that of the leaft refrangible, by more than the 14th part of the whole diftance. Hence, he concludes, that if all the rays of light were equally refrangible, the error in Telefcopes, arifing from the fpherical figure of the glafs, would be many hundred times lefs than it now is; becaufe the error arifing from the fpherical figure of the glafs, is to that arifing from the different refrangibility of the rays of light, as 1 to 5449. See Aberration.

Upon the whole he obferves, that it is a wonder that Telefcopes reprefent objects fo diftinctly as they do. The reafon of which is, that the difperfed rays are not fcattered uniformly over all the circular fpace above-mentioned, but are infinitely more denfe in the centre than in any other part of the circle; and that in the way from the centre to the circumference they grow continually rarer and rarer, till at the circumference they become infinitely rare: for which reafon, thefe difperfed rays are not copious enough to be vifible, except about the centre of the circle. He alfo mentions another argument to prove, that the different refrangibility of the rays of light is the true caufe of the imperfection of Telefcopes. For the difperfions of the rays arifing from the fpherical figures of object-glaffes, are as the cubes of their apertures; and therefore, to caufe Telefcopes of different lengths to magnify with equal diftinctnefs, the apertures of the object-glaffes, and the charges or magnifying powers ought to be as the cubes of the fquare roots of their lengths, which does not anfwer to experience. But the errors of the rays, arifing from the different refrangibility, are as the apertures of the object-glaffes; and thence, to make Telefcopes of different lengths to magnify with equal diftinctnefs, their apertures and charges ought to be as the fquare roots of their lengths; and this anfwers to experience.

Were it not for this different refrangibility of the rays, Telefcopes might be brought to a fufficient degree of perfection, by compofing the object-glafs of two glaffes with water between them. For by this means, the refractions on the concave fides of the glaffes will very much correct the errors of the refractions on the convex fides, fo far as they arife from their fpherical figure: but on account of the different refrangibility of different kinds of rays, Newton did not fee any other means of improving Telefcopes by refraction only, except by increafing their length. Newton's Optics, pa. 73, 83, 89, 3d edition.

This important defideratum in the conftruction of dioptric Telefcopes, has been fince difcovered by the ingenious Mr. Dollond; an account of which is given below.

Achromatic Telescope, is a name given to the refracting Telefcope, invented by Mr. John Dollond, and fo contrived as to remedy the aberration arifing from colours, or the different refrangibility of the rays of light. See Achromatic.

The principles of Mr. Dollond's difcovery and conftruction, have been already explained under the articles Aberration, and Achromatic. The improvement made by Mr. Dollond in his Telefcopes, by making two object-glaffes of crown-glafs, and one of flint, which was tried with fuccefs when concave eye-glaffes were ufed, was completed by his fon Peter Dollond; who, conceiving that the fame method might be practifed with fuccefs with convex eye-glaffes, found, after a few trials, that it might be done. Accordingly he finifhed an object-glafs of 5 feet focal length, with an aperture of $3\frac{3}{4}$ inches, compofed of two convex lenfes of crown-glafs, and one concave of white flint glafs. But apprehending afterward that the apertures might be admitted ftill larger, he completed one of $3\frac{1}{2}$ feet focal length, with the fame aperture of $3\frac{3}{4}$ inches. Philof. Tranf. vol. 55, p. 56.

But befide the obligation we are under to Mr. Dollond, for correcting the aberration of the rays of light in the focus of object-glaffes, arifing from their different refrangibility, he made another confiderable improvement in Telefcopes, viz, by correcting, in a great meafure, both this kind of aberration, and alfo that which arifes from the fpherical form of lenfes, by an expedient of a very different nature, viz, increafing the

the number of eye-glaſſes. If any perſon, ſays he, would have the viſual angle of a Teleſcope to contain 20 degrees, the extreme pencils of the field muſt be bent or refracted in an angle of 10 degrees; which, if it be performed by one eye-glaſs, will cauſe an aberration from the figure, in proportion to the cube of that angle: but if two glaſſes be ſo proportioned and ſituated, as that the refraction may be equally divided between them, they will each of them produce a refraction equal to half the required angle; and therefore, the aberration being in this caſe proportional to double the cube of half the angle, will be but a 4th part of that which is in proportion to the cube of the whole angle; becauſe twice the cube of 1 is but $\frac{1}{4}$ of the cube of 2: ſo that the aberration from the figure, where two eye-glaſſes are rightly proportioned, is but a 4th part of what it muſt unavoidably be, where the whole is performed by a ſingle eye-glaſs. By the ſame way of reaſoning, when the refraction is divided among three glaſſes, the aberration will be found to be but the 9th part of what would be produced from a ſingle glaſs; becauſe 3 times the cube of 1 is but the 9th part of the cube of 3. Whence it appears, that by increaſing the number of eye-glaſſes, the indiſtinctneſs, near the borders of the field of a Teleſcope, may be very much diminiſhed, though not entirely taken away.

The method of correcting the errors ariſing from the different refrangibility of light, is of a different conſideration from the former: for, whereas the errors from the figure can only be diminiſhed in a certain proportion to the number of glaſſes, in this they may be entirely corrected, by the addition of only one glaſs; as we find in the aſtronomical Teleſcope, that two eye-glaſſes, rightly proportioned, will cauſe the edges of objects to appear free from colours quite to the borders of the field. Alſo, in the day teleſcope, where no more than two eye glaſſes are abſolutely neceſſary for erecting the object, we find, by the addition of a third rightly ſituated, that the colours, which would otherwiſe confuſe the image, are entirely removed: but this muſt be underſtood with ſome limitation; for though the different colours, which the extreme pencils muſt neceſſarily be divided into by the edges of the eye-glaſſes, may in this manner be brought to the eye in a direction parallel to each other, ſo as, by its humours, to be converged to a point in the retina, yet if the glaſſes exceed a certain length, the colours may be ſpread too wide to be capable of being admitted through the pupil or aperture of the eye; which is the reaſon that, in long Teleſcopes, conſtructed in the common way, with three eye-glaſſes, the field is always very much contracted.

Theſe conſiderations firſt ſet Mr. Dollond upon contriving how to enlarge the field, by increaſing the number of eye-glaſſes, without any hindrance to the diſtinctneſs or brightneſs of the image: and though others had been about the ſame work before, yet obſerving that the five-glaſs Teleſcopes, ſold in the ſhops, would admit of farther improvement, he endeavoured to conſtruct one with the ſame number of glaſſes in a better manner; which ſo far anſwered his expectations, as to be allowed by the beſt judges to be a conſiderable improvement on the former. Encouraged by this ſucceſs, he reſolved to try if he could not make ſome farther enlargement of the field, by the addition of another glaſs, and by placing and proportioning the glaſſes in ſuch a manner, as to correct the aberrations as much as poſſible, without any detriment to the diſtinctneſs: and at laſt he obtained as large a field as is convenient or neceſſary, and that even in the longeſt Teleſcopes that can be made. Theſe Teleſcopes, with 6 glaſſes, having been well received both at home and abroad, the author has ſettled the date of the invention in a letter addreſſed to Mr. Short, and read at the Royal Society, March 1, 1753. Philoſ. Tranſ. vol. 48, art. 14.

Of the Achromatic Teleſcopes, invented by Mr. Dollond, there are ſeveral different ſizes, from one foot to 8 feet in length, made and ſold by his ſons P. and J. Dollond. In the 17-inch improved Achromatic Teleſcope, the object-glaſs is compoſed of three glaſſes, viz, two convex of crown-glaſs, and one concave of white flint-glaſs: the focal diſtance of this combined object-glaſs is about 17 inches, and the diameter of the aperture 2 inches. There are 4 eye-glaſſes contained in the tube, to be uſed for land objects; the magnifying power with theſe is near 50 times; and they are adjuſted to different ſights, and to different diſtances of the object, by turning a finger ſcrew at the end of the outer tube. There is another tube, containing two eye-glaſſes that magnify about 70 times, for aſtronomical purpoſes. The Teleſcope may be directed to any object by turning two ſcrews in the ſtand on which it is fixed, the one giving a vertical motion, and the other a horizontal one. The ſtand may be incloſed in the inſide of the braſs tube.

The object-glaſs of the $2\frac{1}{2}$ and $3\frac{1}{2}$ feet Teleſcopes is compoſed of two glaſſes, one convex of crown glaſs, and the other concave of white flint glaſs; and the diameters of their apertures are 2 inches and $2\frac{3}{4}$ inches. Each of them is furniſhed with two tubes; one for land objects, containing four eye-glaſſes, and another with two eye-glaſſes for aſtronomical uſes. They are adjuſted by buttons on the outſide of the wooden tube; and the vertical and horizontal motions are given by joints in the ſtands. The magnifying power of the leaſt of theſe Teleſcopes, with the eye-glaſs for land objects, is near 50 times, and with thoſe for aſtronomical purpoſes, 80 times; and that of the greateſt for land objects is near 70 times, but for aſtronomical obſervations 80 and 130 times; for this has two tubes, either of which may be uſed as occaſion requires. This Teleſcope is alſo moved by a ſcrew and rackwork, and the ſcrew is turned by means of a Hook's joint.

Theſe opticians alſo conſtruct an Achromatic pocket perſpective glaſs, or Galilean Teleſcope; ſo contrived, that all the different parts are put together and contained in one piece $4\frac{1}{2}$ inches long. This ſmall Teleſcope is furniſhed with 4 concave eye-glaſſes, the magnifying powers of which are 6, 12, 18, and 28 times. With the greateſt power of this Teleſcope, the ſatellites of Jupiter and the ring of Saturn may be eaſily ſeen. They have alſo contrived an Achromatic Teleſcope, the ſliding tubes of which are made of very thin braſs, which paſs through ſprings or tubes; the outſide tube being either of mahogany or braſs. Theſe Teleſcopes, which from their convenience for gentlemen in the army are called military Teleſcopes, have 4 convex eye-glaſſes,

glaſſes, whoſe ſurfaces and focal lengths are ſo proportioned, as to render the field of view very large. They are of 4 different lengths and ſizes, uſually called one foot, 2, 3, and 4 feet: the firſt is 14 inches when in uſe, and 5 inches when ſhut up, having the aperture of the object-glaſs $1\frac{1}{10}$ inch, and magnifying 22 times: the ſecond 28 inches for uſe, 9 inches ſhut up, the aperture $1\frac{5}{10}$ inch, and magnifying 35 times; the third 40 inches, and 10 inches ſhut, with the aperture 2 inches, and magnifying 45 times; and the fourth 52 inches, and 14 inches ſhut, with the aperture $2\frac{1}{4}$ inches, and magnifying 55 times.

Mr. Euler, who, in a memoir of the Academy of Berlin for the year 1757, p. 323, had calculated the effects of all poſſible combinations of lenſes in Teleſcopes and microſcopes, publiſhed another long memoir on the ſubject of theſe Teleſcopes, ſhewing with preciſion of what advantages they are naturally capable. See Miſcel. Taurin. vol. 3, par. 2, pag. 92.

Mr. Caleb Smith, having paid much attention to the ſubject of ſhortening and improving Teleſcopes, thought he had found it poſſible to rectify the errors which ariſe from the different degrees of refrangibility, on the principle that the ſines of refraction of rays differently refrangible, are to one another in a given proportion, when their ſines of incidence are equal; and the method he propoſed for this purpoſe, was to make the ſpecula of glaſs, inſtead of metal, the two ſurfaces having different degrees of concavity. But it does not appear that this ſcheme was ever carried into practice. See Philoſ. Tranſ. number 456, pa. 326, or Abr. vol. 8, pa. 113.

The ingenious Mr. Ramſden has lately deſcribed a new conſtruction of eye-glaſſes for ſuch Teleſcopes as may be applied to mathematical inſtruments. The conſtruction which he propoſes, is that of two plano-convex lenſes, both of them placed between the eye and the obſerved image formed by the object-glaſs of the inſtrument, and thus correcting not only the aberration ariſing from the ſpherical figure of the lenſes, but alſo that ariſing from the different refrangibility of light. For a more particular account of this conſtruction, its principle, and its effects, ſee Philoſ. Tranſ. vol. 73, art. 5.

A conſtruction, ſimilar at leaſt in its principle to that above, is aſcribed, in the Synopſis Optica Honorati Fabri, to Euſtachio Divini, who placed two equal narrow plano-convex lenſes, inſtead of one eye lens, to his Teleſcopes, which touched at their vertices; the focus of the object-glaſs coinciding with the centre of the plano-convex lens next it. And this, it is ſaid, was done at once both to make the rays that come parallel from the object fall parallel upon the eye, to exclude the colours of the rainbow from it, to augment the angle of ſight, the field of view, the brightneſs of the object, &c. This was alſo known to Huygens, who ſometimes made uſe of the ſame conſtruction, and gives the theory of it in his Dioptrics. See Hugenii Opera Varia, vol. 4, ed. 1728.

Telescope, *Reflecting*, or *Catoptric*, or *Catadioptric*, is a Teleſcope which, inſtead of lenſes, conſiſts chiefly of mirrors, and exhibits remote objects by reflection inſtead of refraction.

A brief account of the hiſtory of the invention of this important and uſeful Teleſcope, is as follows. The ingenious Mr. James Gregory, of Aberdeen, has been commonly conſidered as the firſt inventor of this Teleſcope — But it ſeems the firſt thought of a reflector had been ſuggeſted by Merſenne, about 20 years before the date of Gregory's invention: a hint to this purpoſe occurs in the 7th propoſition of his Catoptrics, which was printed in 1651: and it appears from the 3d and 29th letters of Deſcartes, in vol. 2 of his Letters, which it is ſaid were written in 1639, though they were not publiſhed till the year 1666, that Merſenne propoſed a Teleſcope with ſpecula to Deſcartes in that correſpondence; though indeed in a manner ſo very unſatisfactory, that Deſcartes, who had given particular attention to the improvement of the Teleſcope, was ſo far from approving the propoſal, that he endeavoured to convince Merſenne of its fallacy. This point has been largely diſcuſſed by Le Roi in the Encyclopedia, art. Teleſcope, and by Montucla in his Hiſt. des Mathem. tom. 2, p. 643.

Whether Gregory had ſeen Merſenne's treatiſe on optics and catoptrics, and whether he availed himſelf of the hint there ſuggeſted, or not, perhaps cannot now be determined. He was led however to the invention by ſeeking to correct two imperfections in the common Teleſcope: the firſt of theſe was its too great length, which made it troubleſome to manage; and the ſecond was the incorrectneſs of the image. It had been already demonſtrated, that a pencil of rays could not be collected in a ſingle point by a ſpherical lens; and alſo, that the image tranſmitted by ſuch a lens would be in ſome degree incurvated. Theſe inconveniences he thought might be obviated by ſubſtituting for the object-glaſs a metallic ſpeculum, of a parabolical figure, to receive the image, and to reflect it towards a ſmall ſpeculum of the ſame metal; this again was to return the image to an eye-glaſs placed behind the great ſpeculum, which was, for that purpoſe, to be perforated in its centre. This conſtruction he publiſhed in 1663, in his Optica Promota. But as Gregory, according to his own account, poſſeſſed no mechanical ſkill, and could not find a workman capable of realizing his invention, after ſome fruitleſs trials, he was obliged to give up the thoughts of bringing Teleſcopes of this kind into uſe.

Sir Iſaac Newton however interpoſed, to ſave this excellent invention from periſhing, and to bring it forward to maturity. Having applied himſelf to the improvement of the Teleſcope, and imagining that Gregory's ſpecula were neither very neceſſary, nor likely to be executed, he began with proſecuting the views of Deſcartes; who aimed at making a more perfect image of an object, by grinding lenſes, not to the figure of a ſphere, but to that formed from one of the conic ſections. But, in the year 1666, having diſcovered the different refrangibility of the rays of light, and finding that the errors of Teleſcopes, ariſing from that cauſe alone, were much more conſiderable than ſuch as were occaſioned by the ſpherical figure of lenſes, he was conſtrained to turn his thoughts to reflectors. The plague however interrupted his progreſs in this buſineſs; ſo that it was towards the end of 1668, or in the beginning of 1669, when, deſpairing of perfecting Teleſcopes by means of refracted light,

and

and recurring to the conſtruction of reflectors, he ſet about making his own ſpecula, and early in the year 1672 completed two ſmall reflecting Teleſcopes. In theſe he ground the large ſpeculum into a ſpherical concave, being unable to accompliſh the parabolic form propoſed by Gregory; but though he then deſpaired of performing that work by geometrical rules, yet (as he writes in a letter that accompanied one of theſe inſtruments, which he preſented to the Royal Society) he doubted not but that the thing might in ſome meaſure be accompliſhed by mechanical devices. With a perſeverance equal to his ingenuity, he, in a great meaſure, overcame another difficulty, which was to find a metallic ſubſtance that would be of a proper hardneſs, have the feweſt pores, and receive the ſmootheſt poliſh: this difficulty he deemed almoſt inſurmountable, when he conſidered that every irregularity in a reflecting ſurface would make the rays of light deviate 5 or 6 times more out of their due courſe, than the like irregularities in a refracting ſurface. After repeated trials, he at laſt found a compoſition that anſwered in ſome degree, leaving it to thoſe who ſhould come after him to find a better. Theſe difficulties have accordingly been ſince obviated by other artiſts, particularly by Dr. Mudge, the rev. Mr. Edwards, and Dr. Herſchel, &c. Newton having ſucceeded ſo far, he communicated to the Royal Society a full and ſatisfactory account of the conſtruction and performance of his Teleſcope. The Society, by their ſecretary Mr. Oldenburgh, tranſmitted an account of the diſcovery to Mr. Huygens, celebrated as a diſtinguiſhed improver of the refractor; who not only replied to the Society in terms expreſſing his high approbation of the invention, but drew up a favourable account of the new Teleſcope, which he cauſed to be publiſhed in the Journal des Sçavans of the year 1672, and by this mode of communication it was ſoon known over Europe. See Huygenii Opera Varia, tom. 4.

Notwithſtanding the excellence and utility of this contrivance, and the honourable manner in which it was announced to the world, it ſeems to have been greatly neglected for nearly half a century. Indeed when Newton had publiſhed an account of his Teleſcopes in the Philoſ. Tranſ. M. Caſſegrain, a Frenchman, in the Journal des Sçavans of 1672, claimed the honour of a ſimilar invention, and ſaid, that, before he heard of Newton's improvement, he had hit upon a better conſtruction, by uſing a ſmall convex mirror inſtead of the reflecting priſm. This Teleſcope, which was the Gregorian one diſguiſed, the large mirror being perforated, and which it is ſaid was never executed by the author, is much ſhorter than the Newtonian; and the convex mirror, by diſperſing the rays, ſerves greatly to increaſe the image made by the large concave mirror.

Newton made many objections to Caſſegrain's conſtruction, but ſeveral of them equally affect that of Gregory, which has been found to anſwer remarkably well in the hands of good artiſts.

Dr. Smith took the pains to make many calculations of the magnifying power, both of Newton's and Caſſegrain's Teleſcopes, in order to their farther improvement, which may be ſeen in his Optics, Rem. p. 97.

Mr. Short, it is alſo ſaid, made ſeveral Teleſcopes on the plan of Caſſegrain.

Dr. Hook conſtructed a Reflecting Teleſcope (mentioned by Dr. Birch in his Hiſt. of the Royal Soc. vol. 3, p. 122) in which the great mirror was perforated, ſo that the ſpectator looked directly towards the object, and it was produced before the Royal Society in 1674. On this occaſion it was ſaid that this conſtruction was firſt propoſed by Merſenne, and afterwards repeated by Gregory, but that it never had been actually executed before it was done by Hook. A deſcription of this inſtrument may be ſeen in Hook's Experiments, by Derham, p. 269.

The Society alſo made an unſucceſsful attempt, by employing an artificer to imitate the Newtonian conſtruction; however, about half a century after the invention of Newton, a Reflecting Teleſcope was produced to the world, of the Newtonian conſtruction, which the venerable author, ere yet he had finiſhed his very diſtinguiſhed courſe, had the ſatisfaction to find executed in ſuch a manner, as left no room to fear that the invention would longer continue in obſcurity. This effectual ſervice to ſcience was accompliſhed by Mr. John Hadley, who, in the year 1723, preſented to the Royal Society a Teleſcope, which he had conſtructed upon Newton's plan. The two Teleſcopes which Newton had made, were but 6 inches long, were held in the hand for viewing objects, and in power were compared to a 6-feet refractor: but the radius of the ſphere, to which the principal ſpeculum of Hadley's was ground, was 10 feet $5\frac{1}{4}$ inches, and conſequently its focal length was $62\frac{5}{8}$ inches. In the Philoſ. Tranſ. Abr. vol. 6, p. 133, may be ſeen a drawing and deſcription of this Teleſcope, and alſo of a very ingenious but complex apparatus, by which it was managed. One of theſe Teleſcopes, in which the focal length of the large mirror was not quite $5\frac{1}{4}$ feet, was compared with the celebrated Huygenian Teleſcope, which had the focal length of its object-glaſs 123 feet; and it was found that the former would bear ſuch a charge, as to make it magnify the object as many times as the latter with its due charge; and that it repreſented objects as diſtinctly, though not altogether ſo clear and bright. With this Reflecting Teleſcope might be ſeen whatever had been hitherto diſcovered by the Huygenian, particularly the tranſits of Jupiter's ſatellites, and their ſhades over the diſk of Jupiter, the black liſt in Saturn's ring, and the edge of the ſhade of Saturn caſt upon his ring. Five ſatellites of Saturn were alſo obſerved with this Teleſcope, and it afforded other obſervations on Jupiter and Saturn, which confirmed the good opinion which had been conceived of it by Pound and Bradley.

Mr. Hadley, after finiſhing two Teleſcopes of the Newtonian conſtruction, applied himſelf to make them in the Gregorian form, in which the large mirror is perforated. This ſcheme he completed in the year 1726.

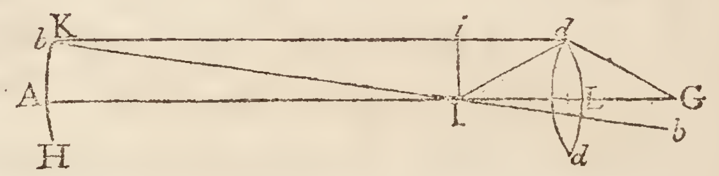
Dr. Smith prefers the Newtonian conſtruction to that of Gregory; but if long experience be admitted as a final judge in ſuch matters, the ſuperiority muſt be adjudged to the latter; as it is now, and has been for many years paſt, the only inſtrument in requeſt.

Mr. Hadley ſpared no pains, after having completed his conſtruction, to inſtruct Mr. Molyneux and Dr. Bradley; and when theſe gentlemen had made a good proficiency in the art, being deſirous that theſe Teleſcopes ſhould become more public, they liberally communicated to ſome of the chief inſtrument-makers of London, the knowledge they had acquired from him: and thus, as it is reaſonable to imagine, reflectors were completed by other and better methods than even thoſe in which they had been inſtructed. Mr. James Short in particular ſignalized himſelf as early as the year 1734, by his work in this way. He at firſt made his ſpecula of glaſs; but finding that the light reflected from the beſt glaſs ſpecula was much leſs than the light reflected from metallic ones, and that glaſs was very liable to change its form by its own weight, he applied himſelf to improve metallic ſpecula; and, by giving particular attention to the curvature of them, he was able to give them greater apertures than other workmen could do; and by a more accurate adjuſtment of the ſpecula, &c, he greatly improved the whole inſtrument. By ſome which he made, in which the larger mirror was 15 inches focal diſtance, he and ſome other perſons were able to read in the Philoſ. Tranſ. at the diſtance of 500 feet; and they ſeveral times ſaw the five ſatellites of Saturn together, which greatly ſurpriſed Mr. Maclaurin, who gave this account of it, till he found that Caſſini had ſometimes ſeen them all with a 17 feet refractor. Short's Teleſcopes were all of the Gregorian conſtruction. It is ſuppoſed that he diſcovered a method of giving the parabolic figure to his great ſpeculum; a degree of perfection which Gregory and Newton deſpaired of attaining, and which Hadley it ſeems had never attempted in either of his Teleſcopes. However, the ſecret of working that configuration, whatever it was, it ſeems died with that ingenious artiſt. Though lately in ſome degree diſcovered by Dr. Mudge and others.

On the Hiſtory of Reflecting Teleſcopes, ſee Dr. David Gregory's Elem. of Catopt. and Dioptr. Appendix by Deſaguliers: Smith's Optics, book 3, c. 2, Rem. on art. 489: and Sir John Pringle's excellent Diſcourſe on the Invention &c of the Reflecting Teleſcope.

Conſtruction of the Reflecting Teleſcope of the Newtonian form.—Let ABCD (fig. 2, pl. 32) be a large tube, open at AD, and cloſed at BC, and its length at leaſt equal to the diſtance of the focus from the metallic ſpherical concave ſpeculum GH placed at the end BC. The rays EG, FH, &c, proceeding from a remote object PR, interſect one another ſomewhere before they enter the tube, ſo that EG and *eg* are thoſe that come from the lower part of the object, and *fh* FH from its upper part: theſe rays, after falling on the ſpeculum GH, will be reflected ſo as to converge and meet in *mn*, where they will form a perfect image of the object. But as this image cannot be ſeen by the ſpectator, they are intercepted by a ſmall plane metallic ſpeculum KK, interſecting the axis at an angle of 45°, by which the rays tending to *m*, *n*, will be reflected towards a hole LL in the ſide of the tube, and the image of the object will be thus formed in *q*S; which image will be leſs diſtinct, becauſe ſome of the rays which would otherwiſe fall on the concave ſpeculum GH, are intercepted by the plane ſpeculum: it will nevertheleſs appear pretty diſtinct, becauſe the aperture AD of the tube, and the ſpeculum GH, are large. In the lateral hole LL is fixed a convex lens, whoſe focus is at S*q*; and therefore this lens will refract the rays that proceed from any point of the image, ſo as at their exit they will appear parallel, and thoſe that proceed from the extreme points S, *q*, will converge after refraction, and form an angle at O, where the eye is placed; which will ſee the image S*q*, as if it were an object, through the lens LL: conſequently the object will appear enlarged, inverted, bright, and diſtinct. In LL may be placed lenſes of different convexities, which, by being moved nearer to the image and farther from it, will repreſent the object more or leſs magnified, if the ſurface of the ſpeculum GH be of a figure truly ſpherical. If, inſtead of one lens LL, three lenſes be diſpoſed in the ſame manner with the three eye-glaſſes of the refracting Teleſcope, the object will appear erect, but leſs diſtinct than when it is obſerved with one lens. On account of the poſition of the eye in this Teleſcope, it is extremely difficult to direct the inſtrument towards any object: Huygens therefore firſt thought of adding to it a ſmall refracting Teleſcope, having its axis parallel to that of the reflector: this is called a *finder* or *director*. The Newtonian Teleſcope is alſo furniſhed with a ſuitable apparatus for the commodious uſe of it.

To determine the magnifying power of this Teleſcope, it is to be conſidered that the plane ſpeculum KK is of no uſe in this reſpect: let us then ſuppoſe that one ray proceeding from the object coincides with the axis GLIA of the lens and ſpeculum; let *bb* be



another ray proceeding from the lower extremity of the object, and paſſing through the focus I of the ſpeculum KH; this will be reflected in the direction *bid*, parallel to the axis GLA, and falling on the lens *d*L*d*, will be refracted to G, ſo that GL will be equal to LI, and $dG = dI$. To the naked eye the object would appear under the angle $Ibi = bIA$; but by means of the Teleſcope it appears under the angle $dGL = dIL = Idi$: and the angle I*di* is to the angle I*bi* as *b*I to I*d*; conſequently the apparent magnitude by the Teleſcope, is to that with the naked eye, as the diſtance of the focus of the ſpeculum from the ſpeculum, to the diſtance of the focus of the lens from the lens.

Conſtruction of the Gregorian Reflecting Teleſcope.—Let TYYT (fig. 3, pl. 32) be a braſs tube, in which L*ld*D is a metallic concave ſpeculum, perforated in the middle at X; and EF a leſs concave mirror, ſo fixed by the arm or ſtrong wire RT, which is moveable by means of a long ſcrew on the outſide of the tube, as to be moved nearer to, or farther from the larger ſpeculum L*ld*D; its axis being kept in the ſame line with that of the great one. Let AB repreſent a very remote object, from each part of which iſſue pencils of rays, as *cd*, CD, from A the upper extremity of the object,

object, and IL, il, from the lower part B; the rays IL, CD, from the extremities, crossing one another before they enter the tube. These rays, falling upon the larger mirror LD, are reflected from it into the focus KH, where they form an inverted image of the object AB, as in the Newtonian Telescope. From this image the rays, issuing as from an object, fall upon the small mirror EF, the centre of which is at e, so that after reflection they would meet in their foci at QQ, and there form an erect image. But since an eye at that place could see but a small part of an object, in order to bring rays from more distant parts of it into the pupil, they are intercepted by the plano-convex lens MN, by which means a smaller erect image is formed at PV, which is viewed through the meniscus SS, by an eye at O. This meniscus both makes the rays of each pencil parallel, and magnifies the image PV. At the place of this image all the foreign rays are intercepted by the perforated partition ZZ. For the same reason the hole near the eye O is very narrow. When nearer objects are viewed by this Telescope, the small speculum EF is removed to a greater distance from the larger LD, so that the second image may be always formed in PV: and this distance is to be adjusted (by means of the screw on the outside of the great tube) according to the form of the eye of the spectator. It is also necessary that the axis of the Telescope should pass through the middle of the speculum EF, and its centre, the centre of the speculum LL, and the middle of the hole X, the centres of the lenses MN, SS, and the hole near O. As the hole X in the speculum LL can reflect none of the rays issuing from the object, that part of the image which corresponds to the middle of the object, must appear to the observer more dark and confused than the extreme parts of it. Besides, the speculum EF will also intercept many rays proceeding from the object; and therefore, unless the aperture TT be large, the object must appear in some degree obscure.

The magnifying power of this Telescope is estimated in the following manner. Let LD be the larger mirror (fig. 3, pl. 31), having its focus at G, and aperture in A; and FF the small mirror with the focus of parallel rays in I, and the axis of both the specula and lenses MN, SS, be in the right line DIGAOK. Let bb be a ray of light coming from the lower extremity of a very distant visible object, passing through the focus G, and falling upon the point b of the speculum LD; which, after being reflected from b to F in a direction parallel to the axis of the mirror DAK, is reflected by the speculum F so as to pass through the focus I in the direction FIN to N, at the extremity of the lens MN, by which it would have been refracted to K; but by the interposition of another lens SS is brought to O, so that the eye in O sees half the object under the angle TOS. The angle GbF, or AGb, under which the object is viewed by the naked eye, is to SOT under which it is viewed by the Telescope, in the ratio of GbF to IFi = nIN, of nIN to NKn, and of NKn to SOT.

But GbF : IFi :: DI : GA,
and nIN : nKN :: nK : nI,
and nKN : SOT :: TO : TK;

theref. GbF : SOT :: DI × nK × TO : GA × nI × TK. Musschenbroek's Introd. vol. 2, p. 819.

In Reflecting Telescopes of different lengths, a given object will appear equally bright and equally distinct, when their linear apertures, and also their linear breadths, are as the 4th roots of the cubes of their lengths; and consequently when the focal distances of their eye-glasses are also as the 4th roots of their lengths. See the demonstration of this proposition in Smith's Optics, art. 361.

Hence he has deduced a rule, by which he has computed the following table for Telescopes of different lengths, taking, for a standard, the middle eye-glass and aperture of Hadley's Reflecting Telescope, described in Philos. Trans. number 376 and 378: the focal distances and linear apertures being given in 1000th parts of an inch.

Table for Telescopes of different Lengths.

Length of the Tel. or focal dist. of the conc.	Focal dist. of the Eye-glass.	Linear amplifying or magnifying power.	Linear aperture of the concave metal.
feet	inches	- - -	inches
½	0·167	36	0·864
1	0·199	60	1·440
2	0·236	102	2·448
3	0·261	138	3·312
4	0·281	171	4·104
5	0·297	202	4·843
6	0·311	232	5·568
7	0·323	260	6·240
8	0·334	287	6·888
9	0·344	314	7·536
10	0·353	340	8·160
11	0·362	365	8·760
12	0·367	390	9·360
13	0·377	414	9·936
14	0·384	437	10·488
15	0·391	460	11·040
16	0·397	483	11·592
17	0·403	506	12·143

Mr. Hadley's Telescope, above mentioned, magnified 228 or 230 times; but we are informed that an object-metal of 3¼ feet focal distance was wrought by Mr. Hauksbee to so great a perfection, as to magnify 226 times, and therefore it was scarcely inferior to Hadley's of 5½ feet. If Hauksbee's Telescope be taken for a new standard, it follows that a speculum of one foot focal distance ought to magnify 93 times, whereas the above table allows it but 60. Now $\frac{93}{60} = 1·55$, and the given column of magnifying powers multiplied by this number, gives a new column, shewing how much the object-metals ought to magnify if wrought up to the perfection of Hauksbee's. And thus a new table may be easily made for this or any other more perfect standard, taking also the new eye-glasses and apertures in the same ratio to one another as the old ones have in this table. Smith's Optics, Rem. p. 79.

 The

The magnifying power of any Telefcope may be eafily found by experiment, viz, by looking with one eye through the Telefcope upon an object of known dimenfions, and at a given diftance, and throwing the image upon another object feen with the naked eye. Dr. Smith has given a particular account of the procefs, Rem. p. 79.

But the eafieft method of all, is to meafure the diameter of the aperture of the object-glafs, and that of the little image of it, which is formed at the place of the eye. For the proportion between thefe gives the ratio of the magnifying power, provided no part of the original pencil be intercepted by the bad conftruction of the Telefcope. For in all cafes the magnifying power of Telefcopes, or microfcopes, is meafured by the proportion of the diameter of the original pencil, to that of the pencil which enters the eye. Prieftley's Hift. of Light, p. 747.

But the moft confiderable, and indeed truly aftonifhing magnifying powers, that have ever been ufed, are thofe of Dr. Herfchel's Reflecting Telefcopes. Some account of thefe, and of the difcoveries made by them, has been already introduced under the article Star. For his method of afcertaining them, fee Philof. Tranf. vol. 72, pa. 173 &c. See alfo feveral of the other late volumes of the Philof. Tranf.

Dr. Herfchel obferves, that though opticians have proved, that two eye-glaffes will give a more correct image than one, he has always (from experience) perfifted in refufing the affiftance of a fecond glafs, which is fure to introduce errors greater than thofe he would correct. "Let us refign, fays he, the double eye-glafs to thofe who view objects merely for entertainment, and who muft have an exorbitant field of view. To a philofopher, this is an unpardonable indulgence. I have tried both the fingle and double eye-glafs of equal powers, and always found that the fingle eye-glafs had much the fuperiority in point of light and diftinctnefs. With the double eye-glafs I could not fee the belts in Saturn, which I very plainly faw with the fingle one. I would however except all thofe cafes where a large field is abfolutely neceffary, and where power joined to diftinctnefs is not the fole object of our view." Philof. Tranf. vol. 72, p. 95.

Mr. Green of Deptford has lately added both to the reflecting and refracting Telefcope an apparatus, which fits it for the purpofes of furveying, levelling, meafuring angles and diftances, &c. See his Defcription and Ufe of the improved Reflecting and Refracting Telefcopes, and Scale of Surveying &c, 1778.—Mr. Ramfden too has lately adapted Telefcopes to the like purpofe of meafuring diftances from one ftation, &c.

Meridian TELESCOPE, is one that is fixed at right angles to an axis, and turned about it in the plane of the meridian; and is otherwife called a *tranfit inftrument*.—The common ufe of it is to correct the motion of a clock or watch, by daily obferving the exact time when the fun or a ftar comes to the meridian. It ferves alfo for a variety of other ufes. The tranfverfe axis is placed horizontal by a fpirit level. For the farther defcription and method of fixing this inftrument by means of its levels &c, fee Smith's Optics, p. 321. See alfo TRANSIT *Inftrument*.

TELESCOPICAL *Stars*, are fuch as are not vifible to the naked eye, being only difcernible by means of a telefcope. See STAR.

All ftars lefs than thofe of the 6th magnitude, are Telefcopic to an ordinary eye.

TEMPERAMENT, in Mufic, ufually denotes a rectifying or amending the falfe or imperfect concords, by transferring to them part of the beauty of the perfect ones.

TENACITY, in Natural Philofophy, is that quality of bodies by which they fuftain a confiderable preffure or force without breaking; and is the oppofite quality to fragility or brittlenefs. Mem. Acad. Berlin. 1745, p. 47.

TENAILLE, in Fortification, a kind of outwork, confifting of two parallel fides, with a front, having a re-entering angle. In fact, that angle, and the faces which compofe it, are the Tenaille.

The Tenaille is of two kinds, *fimple* and *double*.

Simple or *Single* TENAILLE, is a large outwork, confifting of two faces or fides, including a re-entering angle.

Double, or *Flanked* TENAILLE, is a large outwork, confifting of two fimple Tenailles, or three faliant and two re-entering angles.

The great defects of Tenailles are, that they take up too much room, and on that account are advantageous to the enemy; that the re-entering angle is not defended; the height of the parapet preventing the feeing down into it, fo that the enemy can lodge there under cover; and the fides are not fufficiently flanked. For thefe reafons, Tenailles are now moftly excluded out of fortification by the beft engineers, and never made but where time does not ferve to form a hornwork.

TENAILLE *of the Place*, is the front of the place, comprehended between the points of two neighbouring baftions; including the curtain, the two flanks raifed on the curtain, and the two fides of the baftions which face one another. So that the Tenaille, in this fenfe, is the fame with what is otherwife called the *face of a fortrefs*.

TENAILLE *of the Ditch*, is a low work raifed before the curtain, in the middle of the fofs or ditch; the parapet of which is only 2 or 3 feet higher than the level ground of the ravelin.

The ufe of Tenailles in general, is to defend the bottom of the ditch by a grazing fire, and likewife the level ground of the ravelin, which cannot be fo conveniently defended from any other place. The firft fort do not defend the ditch fo well as the others, becaufe they are too oblique a defence; but as they are not fubject to be enfiladed, Vauban has generally preferred them in the fortifying of places. Thofe of the fecond fort defend the ditch much better than the firft, and add a low flank to thofe of the baftions; but as thefe flanks are liable to be enfiladed, they have not been much ufed. This defect however might be remedied, by making them fo as to be covered by the extremities of the parapets of the oppofite ravelins, or by fome other work. And the fame thing may be faid of the third fort as of the fecond.

The *Ram's-horn* is a curved Tenaille, raifed in the fofs before the flanks, and prefenting its convexity to the

the covered way. This work ſeems preferable to either of the other Tenailles, both on account of its ſimplicity, and the defence for which it is conſtructed.

TENAILLONS, in Fortification, are works conſtructed on each ſide of the ravelin, much like the lunettes. They differ, as one of the faces of a Tenaillon is in the direction of the ravelin, whereas that of the lunette is perpendicular to it.

TENOR, in Muſic, the firſt mean or middle part, or that which is the ordinary pitch, or Tenor, of the voice, when not either raiſed to the treble, or lowered to the baſs.

TENSION, the ſtate of a thing tight, or ſtretched. Thus, animals ſuſtain and move themſelves by the Tenſion of their muſcles and nerves. A chord, or ſtring, gives an acuter or deeper ſound, as it is in a greater or leſs degree of Tenſion, that is, more or leſs ſtretched or tightened.

TERM, in Geometry, is the extreme of any magnitude, or that which bounds and limits its extent. So the Terms of a line, are points; of a ſuperficies, lines; of a ſolid, ſuperficies.

TERMS, of an equation, or of any quantity, in Algebra, are the ſeveral names or members of which it is compoſed, ſeparated from one another by the ſigns $+$ or $-$. So, the quantity $ax + 2bc - 3ax^2$, conſiſts of the three Terms ax and $2bc$ and $3ax^2$.

In an equation, the Terms are the parts which contain the ſeveral powers of the ſame unknown letter or quantity: for if the ſame unknown quantity be found in ſeveral members in the ſame degree or power, they ſhall paſs but for one Term, which is called a compound one, in diſtinction from a ſimple or ſingle Term. Thus, in the equation $x^3 + \overline{a - 3b} . x^2 - acx = b^3$, the four terms are x^3 and $\overline{a - 3b} . x^2$ and acx and b^3; of which the ſecond Term $\overline{a - 3b} . x^2$ is compound, and the other three are ſimple Terms.

TERMS, of a Product, or of a Fraction, or of a Ratio, or of a Proportion, &c, are the ſeveral quantities employed in forming or compoſing them. Thus, the Terms

of the product ab, are a and b;
of the fraction $\frac{5}{8}$, are 5 and 8;
of the ratio 6 to 7, are 6 and 7;
of the proportion $a : b :: 5 : 9$, are a, b, 5, 9.

TERMS are alſo uſed for the ſeveral times or ſeaſons of the year in which the public colleges or univerſities, or courts of law, are open, or ſit. Such are the Oxford and Cambridge Terms; alſo the Terms for the courts of King's-Bench, Common Pleas, and the Exchequer, which are the high courts of common law. But the high court of Parliament, the Chancery, and inferior courts, do not obſerve the Terms.—The reſt of the year, out of Term-time, is called *vacation*.

There are four law Terms in the year; viz,

Hilary-Term, which, at London, begins the 23d day of January, and ends the 12th of February.

Eaſter-Term, which begins the 3d Wedneſday after Eaſter-day, and ends on the Monday next after Aſcenſion-day.

Trinity-Term, which begins the Friday next after Trinity-Sunday, and ends the 4th Wedneſday after Trinity-Sunday.

Michaelmas-Term, which begins the 6th of November, and ends the 28th of November.

All theſe terms have alſo their returns, the days of which are expreſſed in the following table or ſynopſis.

Table of the Law Terms, and their Returns.

Term	Begin.	1ſt Return	2d Return	3d Return	4th Return	5th Return	End.
Hilary	January 23	January 20	January 27	February 3	February 9	- - - -	February 12
Eaſter	3 Wed. af. Eaſt.	2 Wks. af. Eaſt.	3 Wks. af. Eaſt.	4 Wks. af. Eaſt.	5 Wks. af. Eaſt.	Aſcenſ. day	Mond. af. Aſcenſ.
Trinity	Frid. af. Trin. S.	Trinity Mond.	1 Wk. af. Trin	2 Wks. af. Trin	3 Wks. af. Trin.	- - - -	4th Wed. af. Trin. S.
Mich.	November 6	November 3	November 12	November 18	November 25	- - - -	November 28

N. B. When the beginning or ending of any of theſe Terms happens on a Sunday, it is held on the Monday after.

Oxford TERMS. Theſe are four; which begin and end as below:

Terms	Begin.	End.
Lent Term	January 14	Sat. bef. Palm-Sund.
Eaſter Term	Wed. af. Low-Sund	Thurſ. bef. Whitſun.
Trinity Term	Wed. af. Trin. Sund.	Sat. after the Act
Michaelmas T.	October 10	December 17

N. B. The *Act* is 1ſt Monday after the 6th of July. —When the day of the beginning or ending happens on a Sunday, the Terms begin or end the day after.

Cambridge-TERMS. Theſe are three, as below:

Terms	Begin.	End.
Lent Term	January 13	Frid. bef. Palm-Sund.
Eaſter Term	Wed. aft. Low-Sund.	Frid. aft. Commence.
Michaelmas T.	October 10	December 16

N. B. The *Commencement* is the 1ſt Tueſday in July. —There is no difference on account of the beginning or ending being Sunday.

Scottiſh

Scottiſh TERMS. In Scotland, *Candlemas Term* begins January 23d, and ends February the 12th. *Whitſuntide-Term* begins May 25th, and ends June 15th. *Lammas-Term* begins July the 20th, and ends Auguſt the 8th. *Martinmas-Term* begins November the 3d, and ends November the 29th.

Iriſh TERMS. In Ireland the Terms are the ſame as at London, except *Michaelmas-Term*, which begins October the 13th, and adjourns to November the 3d, and thence to the 6th.

TERMINATOR, in Aſtronomy, a name ſometimes given to the circle of illumination, from its property of terminating the boundaries of light and darkneſs.

TERRA, in Geography. See EARTH.

TERRA-*firma*, in Geography, is ſometimes uſed for a continent, in contradiſtinction to iſlands. Thus, Aſia, the Indies, and South America, are uſually diſtinguiſhed into Terra-firmas and iſlands.

TERRAQUEOUS, in Geography, an epithet given to our globe or earth, conſidered as conſiſting of land and water, which together conſtitute one maſs.

TERRE-PLEIN, or TERRE-PLAIN, in Fortification, the top, platform, or horizontal ſurface of the rampart, upon which the cannon are placed, and where the defenders perform their office. It is ſo called, becauſe it lies level, having only a little ſlope outwardly to counteract the recoil of the cannon. Its breadth is from 24 to 30 feet; being terminated by the parapet on the outer ſide, and inwardly by the inner talus.

TERRELLA, or little earth, is a magnet turned of a ſpherical figure, and placed ſo as that its poles, equator, &c, do exactly correſpond with thoſe of the world. It was ſo firſt called by Gilbert, as being a juſt repreſentation of the great magnetic globe we inhabit. Such a Terrella, it was ſuppoſed, if nicely poiſed, and hung in a meridian like a globe, would be turned round like the earth in 24 hours by the magnetic particles pervading it; but experience has ſhewn that this is a miſtake.

TERRESTRIAL, ſomething relating to the earth. As Terreſtrial globe, Terreſtrial line, &c.

TERTIAN; denotes an old meaſure, containing 84 gallons, ſo called becauſe it is the 3d part of a tun.

TERTIATE, in Gunnery. To Tertiate a great gun, is to examine the thickneſs of the metal at the muzzle, by which to judge of the ſtrength of the piece, and whether it be ſufficiently fortified or not.

TETRACHORD, in Muſic, called by the moderns a *fourth*, is a concord or interval of four tones.—The Tetrachord of the ancients, was a rank of four ſtrings, accounting the Tetrachord for one tone, as it is often taken in muſic.

TETRADIAPASON, or *quadruple diapaſon*, is a muſical chord, otherwiſe called a quadruple eighth, or a nine and-twentieth.

TETRAEDRON, or TETRAHEDRON, in Geometry, is one of the five Platonic or regular bodies or ſolids, comprehended under four equilateral and equal triangles. Or it is a triangular pyramid of four equal and equilateral faces.

It is demonſtrated in geometry, that the ſide of a Tetraedron is to the diameter of its circumſcribing ſphere, as $\sqrt{2}$ to $\sqrt{3}$; conſequently they are incommenſurable.

If a denote the linear edge or ſide of a Tetraedron, b its whole ſuperficies, c its ſolidity, r the radius of its inſcribed ſphere, and R the radius of its circumſcribing ſphere; then the general relation among all theſe is expreſſed by the following equations, viz,

$$
\begin{aligned}
a &= 2r\sqrt{6} = \tfrac{2}{3}R\sqrt{6} = \sqrt{\tfrac{1}{3}b\sqrt{3}} = \sqrt[3]{6c\sqrt{2}}.\\
b &= 24r^2\sqrt{3} = \tfrac{8}{3}R^2\sqrt{3} = a^2\sqrt{3} = 6\sqrt[3]{c^2\sqrt{3}}.\\
c &= 8r^3\sqrt{3} = \tfrac{8}{27}R^3\sqrt{3} = \tfrac{1}{12}a^3\sqrt{2} = \tfrac{1}{36}b\sqrt{2b\sqrt{3}}.\\
R &= 3r = \tfrac{1}{4}a\sqrt{6} = \tfrac{1}{4}\sqrt{2b\sqrt{3}} = \tfrac{3}{2}\sqrt[3]{\tfrac{1}{3}c\sqrt{3}}.\\
r &= \tfrac{1}{3}R = \tfrac{1}{12}a\sqrt{6} = \tfrac{1}{12}\sqrt{2b\sqrt{3}} = \tfrac{1}{2}\sqrt[3]{\tfrac{1}{3}c\sqrt{3}}.
\end{aligned}
$$

See my Menſuration, pa. 248 &c, 2d ed. See alſo the articles REGULAR and BODIES.

TETRAGON, in Geometry, a quadrangle, or a figure having 4 angles. Such as a ſquare, a parallelogram, a rhombus, and a trapezium. It ſometimes alſo means peculiarly a ſquare.

TETRAGON, in Aſtrology, denotes an aſpect of two planets with regard to the earth, when they are diſtant from each other a 4th part of a circle, or 90 degrees. The Tetragon is expreſſed by the character □, and is otherwiſe called a ſquare or quartile aſpect.

TETRAGONIAS, a meteor, whoſe head is of a quadrangular figure, and its tail or train is long, thick, and uniform. It does not differ much from the meteor called *Trabs* or beam.

TETRAGONISM, a term which ſome authors uſe to expreſs the quadrature of the circle, becauſe the quadrature is the finding a ſquare equal to it.

TETRASPASTON, in Mechanics, a machine in which are four pulleys.

TETRASTYLE, in the Ancient Architecture, a building, and particularly a temple, with four columns in front.

THALES, a celebrated Greek philoſopher, and the firſt of the ſeven wiſemen of Greece, was born at Miletum, about 640 years before Chriſt. After acquiring the uſual learning of his own country, he travelled into Egypt and ſeveral parts of Aſia, to learn aſtronomy, geometry, myſtical divinity, natural knowledge or philoſophy, &c. In Egypt he met for ſome time great favour from the king, Amaſis; but he loſt it again, by the freedom of his remarks on the conduct of kings, which it is ſaid occaſioned his return to his own country, where he communicated the knowledge he had acquired to many diſciples, among the principal of whom were Anaximander, Anaximenes, and Pythagoras, and was the author of the Ionian ſect of philoſophers. He always however lived very retired, and refuſed the proffered favours of many great men. He was often viſited by Solon; and it is ſaid he took great pleaſure in the converſation of Thraſybulus, whoſe excellent wit made him forget that he was Tyrant of Miletum.

Laertius, and ſeveral other writers, agree, that he was the father of the Greek philoſophy; being the firſt that made any reſearches into natural knowledge, and mathematics. His doctrine was, that water was the principle of which all the bodies in the univerſe are compoſed; that the world was the work of God; and that God ſees the moſt ſecret thoughts in the heart of man. He ſaid, that in order to live well, we ought to abſtain from what we find fault with in others: that

bodily felicity consists in health; and that of the mind in knowledge. That the most ancient of beings is God, because he is uncreated: that nothing is more beautiful than the world, because it is the work of God; nothing more extensive than space, quicker than spirit, stronger than necessity, wiser than time. He used to observe, that we ought never to say that to any one which may be turned to our prejudice; and that we should live with our friends as with persons that may become our enemies.

In Geometry, it has been said, he was a considerable inventor, as well as an improver; particularly in triangles. And all the writers agree, that he was the first, even in Egypt, who took the height of the pyramids by the shadow.

His knowledge and improvements in astronomy were very considerable. He divided the celestial sphere into five circles or zones, the arctic and antarctic circles, the two tropical circles, and the equator. He observed the apparent diameter of the sun, which he made equal to half a degree; and formed the constellation of the Little Bear. He observed the nature and course of eclipses, and calculated them exactly; one in particular, memorably recorded by Herodotus, as it happened on a day of battle between the Medes and Lydians, which, Laertius says, he had foretold to the Ionians. And the same author informs us, that he divided the year into 365 days. Plutarch not only confirms his general knowledge of eclipses, but that his doctrine was, that an eclipse of the sun is occasioned by the intervention of the moon, and that an eclipse of the moon is caused by the intervention of the earth.

His morals were as just, as his mathematics well grounded, and his judgment in civil affairs equal to either. He was very averse to tyranny, and esteemed monarchy little better in any shape.—Diogenes Laertius relates, that walking to contemplate the stars, he fell into a ditch; on which a good old woman, that attended him, exclaimed, "How canst thou know what is doing in the heavens, when thou seest not what is at thy feet?"—He went to visit Crœsus, who was marching a powerful army into Cappadocia, and enabled him to pass the river Halys without making a bridge. Thales died soon after, at above 90 years of age, it is said, at the Olympic games, where, oppressed with heat, thirst, and a load of years, he, in public view, sunk into the arms of his friends.

Concerning his writings, it remains doubtful whether he left any behind him; at least none have come down to us. Augustine mentions some books of Natural Philosophy; Simplicius, some written on Nautic Astrology; Laertius, two treatises on the Tropics and Equinoxes; and Suidas, a treatise on Meteors, written in verse.

THAMMUZ, in Chronology, the 10th month of the year of the Jews, containing 29 days, and answering to our June.

THEMIS, in Astronomy, a name given by some to the 3d satellite of Jupiter.

THEODOLITE, an instrument much used in surveying, for taking angles, distances, altitudes, &c.

This instrument is variously made; different persons having their several ways of contriving it, each attempting to make it more simple and portable, more accurate and expeditious, than others. It usually consists of a brass circle, about a foot diameter, cut in form of fig. 5, pl. 31; having its limb divided into 360 degrees, and each degree subdivided either diagonally, or otherwise, into minutes. Underneath, at *cc*, are fixed two little pillars *bb* (fig. 6), which support an axis, bearing a telescope, for viewing remote objects.

On the centre of the circle moves the index C, which is a circular plate, having a compass in the middle, the meridian line of which answers to the fiducial line *aa*; at *bb* are fixed two pillars to support an axis, bearing a telescope like the former, whose line of collimation answers to the fiducial line *aa*. At each end of either telescope is, or may be, fixed a plain sight, for the viewing of nearer objects.

The ends of the index *aa* are cut circularly, to fit the divisions of the limb B; and when that limb is diagonally divided, the fiducial line at one end of the index shews the degrees and minutes upon the limb. It is also furnished with cross spirit levels, for setting the plane of the circle truly horizontal; and a vertical arch, divided into degrees, for taking angles of elevation and depression. The whole instrument is mounted with a ball and socket, upon a three-legged staff.

Many Theodolites however have no telescopes, but only four plain sights, two of them fastened on the limb, and two on the ends of the index. Two different ones, mounted on their stand, are represented in fig. 2 and 3, plate 33.

The use of the Theodolite is abundantly shewn in that of the semicircle, which is only half a Theodolite. And the index and compass of the Theodolite serve also for a circumferentor, and are used as such.

The ingenious Mr. Ramsden has lately made a most excellent Theodolite, for the use of the military survey now carrying on in England.

THEODOSIUS, a celebrated mathematician, who flourished in the times of Cicero and Pompey; but the time and place of his death are unknown. This Theodosius, the Tripolite, as mentioned by Suidas, is probably the same with Theodosius the philosopher of Bythinia, who Strabo says excelled in the mathematical sciences, as also his sons; for the same person might have travelled from the one of those places to the other, and spent part of his life in each of them; like as Hipparchus was called by Strabo the Bythinian; but by Ptolomy and others the Rhodian.

Theodosius chiefly cultivated that part of geometry which relates to the doctrine of the sphere, concerning which he published three books. The first of these contains 22 propositions; the second 23; and the third 14; all demonstrated in the pure geometrical manner of the Ancients. Ptolomy made great use of these propositions, as well as all succeeding writers. These books were translated by the Arabians, out of the original Greek, into their own language. From the Arabic, the work was again translated into Latin, and printed at Venice. But the Arabic version being very defective, a more complete edition was published in Greek and Latin, at Paris 1558, by John Pena, Regius Professor of Astronomy. And Vitello acquired reputation by translating Theodosius into Latin. This author's works were also commented on and illustrated by Clavius, Heleganius, and Guarinus, and lastly by

De Chales, in his Curfus Mathematicus. But that edition of Theodofius's Spherics which is now moft in ufe, was tranflated, and publifhed, by our countryman the learned Dr. Barrow, in the year 1675, illuftrated and demonftrated in a new and concife method. By this author's account, Theodofius appears not only to be a great mafter in this more difficult part of geometry, but the firft confiderable author of antiquity who has written on that fubject.

Theodofius too wrote concerning the Celeftial Houfes; alfo of Days and Nights; copies of which, in Greek, are in the king's library at Paris. Of which there was a Latin edition, publifhed by Peter Dafypody, in the year 1572.

THEON, of Alexandria, a celebrated Greek philofopher and mathematician, who flourifhed in the 4th century, about the year 380, in the time of Theodofius the Great; but the time and manner of his death are unknown. His genius and difpofition for the ftudy of philofophy were very early improved by a clofe application to ftudy; fo that he acquired fuch a proficiency in the fciences, as to render his name venerable in hiftory; and to procure him the honour of being prefident of the famous Alexandrian fchool. One of his pupils was the admirable Hypatia, his daughter, who fucceeded him in the prefidency of the fchool; a truft, which, like himfelf, fhe difcharged with the greateft honour and ufefulnefs. [See her life in its place in the firft volume of this Dictionary.]

The ftudy of nature led Theon to many juft conceptions concerning God, and to many ufeful reflections in the fcience of moral philofophy; hence, it is faid, he wrote with great accuracy on divine providence. And he feems to have made it his ftanding rule, to judge the truth of certain principles, or fentiments, from their natural or neceffary tendency. Thus, he fays, that a full perfuafion, that the Deity fees every thing we do, is the ftrongeft incentive to virtue; for he infifts, that the moft profligate have power to refrain their hands, and hold their tongues, when they think they are obferved, or overheard, by fome perfon whom they fear or refpect. With how much more reafon then, fays he, fhould the apprehenfion and belief, that God fees all things, reftrain men from fin, and conftantly excite them to their duty? He alfo reprefents this belief, concerning the Deity, as productive of the greateft pleafure imaginable, efpecially to the virtuous, who might depend with greater confidence on the favour and protection of Providence. For this reafon, he recommends nothing fo much as meditation on the prefence of God: and he recommended it to the civil magiftrate, as a reftraint on fuch as were profane and wicked, to have the following infcription written, in large characters, at the corner of every ftreet; GOD SEES THEE, O SINNER.

Theon wrote notes and commentaries on fome of the ancient mathematicians. He compofed alfo a book, entitled *Progymnafmata*, a rhetorical work, written with great judgment and elegance; in which he criticifed on the writings of fome illuftrious orators and hiftorians; pointing out, with great propriety and judgment, their beauties and imperfections; and laying down proper rules for propriety of ftyle. He recommends concifenefs of expreffion, and perfpicuity, as the principal ornaments. This book was printed at Bafle, in the year 1541; but the beft edition is that of Leyden, in 1626, in 8vo.

THEOPHRASTUS, a celebrated Greek philofopher, was the fon of Melanthus, and was born at Eretus in Bœotia. He was at firft the difciple of Lucippus, then of Plato, and laftly of Ariftotle; whom he fucceeded in his fchool, about the 322d year before the Chriftian era, and taught philofophy at Athens with great applaufe.

He faid of an orator without judgment, "that he was a horfe without a bridle." He ufed alfo to fay, "There is nothing fo valuable as time, and thofe who lavifh it are the moft inexcufable of all prodigals."—He died at about 100 years of age.

Theophraftus wrote many works, the principal of which are the following—1. An excellent moral treatife entitled, *Characters*, which, he fays in the preface, he compofed at 99 years of age. Ifaac Cafaubon has written learned commentaries on this fmall treatife. It has been tranflated from the Greek into French, by Bruyere; and it has alfo been tranflated into Englifh.—2. A curious treatife on Plants.—3. A treatife on foffils or ftones; of which Dr. Hill has given a good edition, with an Englifh tranflation, and learned notes, in 8vo.

THEOREM, a propofition which terminates in theory, and which confiders the properties of things already made or done. Or, a Theorem is a fpeculative propofition, deduced from feveral definitions compared together. Thus, if a triangle be compared with a parallelogram ftanding on the fame bafe, and of the fame altitude, and partly from their immediate definitions, and partly from other of their properties already determined, it is inferred that the parallelogram is double the triangle; that propofition is a Theorem.

Theorem ftands contradiftinguifhed from problem, which denotes fomething to be done or conftructed, as a Theorem propofes fomething to be proved or demonftrated.

There are two things to be chiefly regarded in every Theorem, viz, the propofition, and the demonftration. In the firft is expreffed what agrees to fome certain thing, under certain conditions, and what does not. In the latter, the reafons are laid down by which the underftanding comes to conceive that it does or does not agree to it.

Theorems are of various kinds: as,

Univerfal THEOREM, is that which extends to any quantity without reftriction, univerfally. As this, that the rectangle or product of the fum and difference of any two quantities, is equal to the difference of their fquares.

Particular THEOREM, is that which extends only to a particular quantity. As this, in an equilateral rectilinear triangle, each angle is equal to 60 degrees.

Negative THEOREM, is that which expreffes the impoffibility of any affertion. As, that the fum of two biquadrate numbers cannot make a fquare number.

Local THEOREM is that which relates to a furface. As, that triangles of the fame bafe and altitude are equal.

Plane THEOREM, is that which relates to a furface that is either rectilinear or bounded by the circumference of a circle. As, that all angles in the fame fegment of a circle are equal.

Solid THEOREM, is that which confiders a fpace terminated

minated by a solid line; that is, by any of the three conic sections. As this, that if a right line cut two asymptotic parabolas, its two parts terminated by them shall be equal.

Reciprocal THEOREM, is one whose converse is true. As, that if a triangle have two sides equal, it has also two angles equal: the converse of which is likewise true, viz, that if the triangle have two angles equal, it has also two sides equal.

THEORY, a doctrine which terminates in the sole speculation or consideration of its object, without any view to the practice or application of it.

To be learned in an art, &c, the Theory is sufficient; to be a master of it, both the Theory and practice are requisite.—Machines often promise very well in Theory, but fail in the practice.

We say Theory of the moon, Theory of the rainbow, of the microscope, of the camera obscura, &c.

THEORIES *of the Planets*, &c, are systems or hypotheses, according to which the astronomers explain the reasons of the phenomena or appearances of them.

THERMOMETER, an instrument for measuring the temperature of the air, &c, as to heat and cold.

The Thermometer and thermoscope are usually accounted the same thing. But Wolfius makes a difference; and he also shews that what we call Thermometers, are really no more than thermoscopes.

The invention of the Thermometer is attributed to several persons by different authors, viz, to Sanctorio, Galileo, father Paul, and to Drebbel. Thus, the invention is ascribed to Cornelius Drebbel of Alcmar, about the beginning of the 17th century, by his countrymen Boerhaave (Chem. 1, pa. 152, 156), and Musschenbroeck (Introd. ad Phil. Nat. vol. 2, pa. 625).—Fulgenzio, in his Life of father Paul, gives him the honour of the first discovery.—Vincenzio Viviani (Vit. de l'Galil. pa. 67; also Oper. di Galil. pref. pa. 47) speaks of Galileo as the inventor of Thermometers.—But Sanctorino (Com. in Galen. Art. Med. pa. 736, 842, Com. in Avicen. Can. Fen. 1, pa. 22, 78, 219) expressly assumes to himself this invention: and Borelli (De Mot. Animal. 2, prop. 175) and Malpighi (Oper. Posth. pa. 30) ascribe it to him without reserve. Upon which Dr. Martine remarks, that these Florentine academicians are not to be suspected of partiality in favour of one of the Patavinian school.

But whoever was the first inventor of this instrument, it was at first very rude and imperfect; and as the various degrees of heat were indicated by the different contraction or expansion of air, it was afterwards found to be an uncertain and sometimes a deceiving measure of heat, because the bulk of the air was affected, not only by the difference of heat, but also by the variable weight of the atmosphere.

There are various kinds of Thermometers, the construction, defects, theory, &c, of which, are as follow.

The Air THERMOMETER.—This instrument depends on the rarefaction of the air. It consists of a glass tube BE (fig. 1, pl. 34) connected at one end with a large glass ball A, and at the other end immersed in an open vessel, or terminating in a ball DE, with a narrow orifice at D; which vessel, or ball, contains any coloured liquor that will not easily freeze. Aquafortis tinged of a fine blue colour with solution of vitriol or copper, or spirit of wine tinged with cochineal, will answer this purpose. But the ball A must be first moderately warmed, so that a part of the air contained in it may be expelled through the orifice D; and then the liquor pressed by the weight of the atmosphere, will enter the ball DE, and rise, for example, to the middle of the tube at C, at a mean temperature of the weather; and in this state the liquor by its weight, and the air included in the ball and tube ABC, by its elasticity, will counterbalance the weight of the atmosphere. As the surrounding air becomes warmer, the air in the ball and the upper part of the tube, expanding by heat, will drive the liquor into the lower ball, and consequently its surface will descend; on the contrary, as the ambient air becomes colder, that in the ball is condensed, and the liquor, pressed by the weight of the atmosphere, will ascend: so that the liquor in the tube will ascend or descend more or less, according to the state of the air contiguous to the instrument. To the tube is affixed a scale of the same length, divided upwards and downwards, from the middle C, into 100 equal parts, by means of which may be observed the ascent and descent of the liquor in the tube, and consequently the variations also in the temperature of the atmosphere.

A similar Thermometer may be constructed by putting a small quantity of mercury, not exceeding the bulk of a pea, into the tube BC (fig. 4, pl. 33), bent into wreaths, that taking up the less height, it may be the more manageable, and less liable to harm; divide this tube into any number of equal parts to serve for a scale. Here the approaches of the mercury towards the ball A will shew the increase of the degree of heat. The reason of which is the same as in the former.

The defect of both these instruments consists in this, that they are liable to be acted on by a double cause: for, not only a decrease of heat, but also an increase of weight of the atmosphere, will make the liquor rise in the one, and the mercury in the other; and, on the contrary, either an increase of heat, or decrease of the weight of the atmosphere, will cause them to descend.

For these, and other reasons, Thermometers of this kind have been long disused. However, M. Amontons, in 1702, with a view of perfecting the aërial Thermometer, contrived his *Universal Thermometer*. Finding that the changes produced by heat and cold in the bulk of the air were subject to invincible irregularities, he substituted for these the variations produced by heat in the elastic force of this fluid. This Thermometer consisted of a long tube of glass (fig. 3, pl. 34) open at one end, and recurved at the other end, which terminated in a ball. A certain quantity of air was compressed into this ball by the weight of a column of mercury, and also by the weight of the atmosphere. The effect of heat on this included air was to make it sustain a greater or less weight; and this effect was measured by the variation of the column of mercury in the tube, corrected by that of the barometer, with respect to the changes of the weight of the external air. This instrument, though much more perfect than the former, is nevertheless subject to very considerable defects and incon-

inconveniences. Its length of 4 feet renders it unfit for a variety of experiments, and its construction is difficult and complex: it is extremely inconvenient for carriage, as a very small inclination of the tube would suffer the included air to escape: and the friction of the mercury in the tube, and the compressibility of the air, contribute to render the indications of this instrument extremely uncertain. Besides, the dilatation of the air is not so regularly proportional to its heat, nor is its dilatation by a given heat nearly so uniform as he supposed. This depends much on its moisture; for dry air does not expand near so much by a given heat, as air stored with watery particles. For these, and other reasons, enumerated by De Luc (Recherches sur les Mod. de l'Atmo. tom. 1, pa. 278 &c), this instrument was imitated by very few, and never came into general use.

Of the Florentine THERMOMETER.—The academists del Cimento, about the middle of the 17th century, considering the inconveniencies of the air Thermometers above described, attempted another, that should measure heat and cold by the rarefaction and condensation of spirit of wine; though much less than those of air, and consequently the alterations in the degree of heat likely to be much less sensible.

The spirit of wine coloured, was included in a very fine and cylindrical glass tube (fig. 2, pl. 34), exhausted of its air, having a hollow ball at one end A, and hermetically sealed at the other end D. The ball and tube are filled with rectified spirit of wine to a convenient height, as to C, when the weather is of a mean temperature, which may be done by inverting the tube into a vessel of stagnant coloured spirit, under a receiver of the air-pump, or in any other way. When the thermometer is properly filled, the end D is heated red hot by a lamp, and then hermetically sealed, leaving the included air of about $\frac{1}{3}$ of its natural density, to prevent the air which is in the spirit from dividing it in its expansion. To the tube is applied a scale, divided from the middle, into 100 equal parts, upwards and downwards.

Now spirit of wine rarefying and condensing very considerably; as the heat of the ambient atmosphere increases, the spirit will dilate, and so ascend in the tube; and as the heat decreases, the spirit will descend; and the degree or quantity of the motion will be shewn by the attached scale.

These Thermometers could not be subject to any inconvenience by an evaporation of the liquor, or a variable gravity of the incumbent atmosphere. Instruments of this kind were first introduced into England by Mr. Boyle, and they soon came into general use among philosophers in other countries. They are however subject to considerable inconveniences, from the weight of the liquor itself, and from the elasticity of the air above it in the tube, both which prevent the freedom of its ascent; besides, the rarefactions are not exactly proportional to the surrounding heat. Moreover spirit of wine is incapable of bearing very great heat or very great cold: it boils sooner than any other liquor; and therefore the degrees of heat of boiling fluids cannot be determined by this Thermometer. And though it retains its fluidity in pretty severe cold, yet it seems not to condense very regularly in them: and at Torneao, near the polar circle, the winter cold was so severe, as Maupertuis informs us, that the spirits were frozen in all their Thermometers. So that the degrees of heat and cold, which spirit of wine is capable of indicating, is much too limited to be of very great or general use.

Another great defect of these, and other Thermometers, is, that their degrees cannot be compared with each other. It is true they mark the variations of heat and cold; but each marks for itself, and after its own manner; because they do not proceed from any point of temperature that is common to all of them.

From these and various other imperfections in these Thermometers, it happens, that the comparisons of them become so precarious and defective: and yet the most curious and interesting use of them, is what ought to arise from such comparison. It is by this we should know the heat or cold of another season, of another year, another climate, &c; and what is the greatest degree of heat or cold that men and other animals can subsist in.

Reaumur contrived a new Thermometer, in which the inconveniences of the former are proposed to be remedied. He took a large ball and tube, the content or dimensions of which are known in every part; he graduated the tube, so that the space from one division to another might contain a 1000th part of the liquor, which liquor would contain 1000 parts when it stood at the freezing point: then putting the ball of his Thermometer and part of the tube into boiling water, he observed whether it rose 80 divisions: if it exceeded these, he changed his liquor, and by adding water lowered it, till upon trial it should just rise 80 divisions; or if the liquor, being too low, fell short of 80 divisions, he raised it by adding rectified spirit to it. The liquor thus prepared suited his purpose, and served for making a Thermometer of any size, whose scale would agree with his standard. Such liquor, or spirits, being about the strength of common brandy, may easily be had any where, or made of a proper degree of density by raising or lowering it.

The abbé Nollet made many excellent Thermometers upon Reaumur's principle. Dr. Martine however expresses his apprehensions that Thermometers of this kind cannot admit of such accuracy as might be wished. The balls or bulbs, being large, as 3 or 4 inches in diameter, are neither heated nor cooled soon enough to shew the variations of heat. Small bulbs and small tubes, he says, are much more convenient, and may be constructed with sufficient accuracy. Though it must be allowed that Reaumur, by his excellent scale, and by depriving the spirit of its air, and expelling the air by means of heat from the ball and tube of his Thermometer, has brought it to as much perfection as may be; yet it is liable to some of the inconveniences of spirit Thermometers, and is much inferior to mercurial ones. These two kinds do not agree together in indicating the same degrees of intense cold; for when the mercury has stood at 22° below 0, the spirit indicated only 18°, and when the mercury stood at 28° or 37° below 0, the spirit rested at 25° or 29°. See the description of Reaumur's Thermometer at large in Mem. de l'Acad. des Scienc. an. 1730, pa. 645, Hist. pa. 15. Ib. an. 1731, pa. 354, Hist. pa. 7.

Mercurial

Mercurial THERMOMETER.——It is a most important circumstance in the construction of Thermometers, to procure a fluid that measures equal variations of heat by corresponding equal variations in its own bulk: and the fluid which possesses this essential requisite in the most perfect degree, is mercury: the variations in its bulk approaching nearer to a proportion with the corresponding variations of its heat, than any other fluid. Besides, it is the most easy to purge of its air; and is also the most proper for measuring very considerable variations of heat and cold, as it will bear more cold before freezing, and more heat before boiling, than any other fluid. Mercury is also more sensible than any other fluid, air excepted, or conforms more speedily to the several variations of heat. Moreover, as mercury is an homogeneous fluid, it will in every Thermometer exhibit the same dilatation or condensation by the same variations of heat..

Dr. Halley, though apprized only of some of the remarkable properties of mercury above recited, seems to have been the first who suggested the application of this fluid to the construction of Thermometers. Philos. Trans. Abr. vol. 2, pa. 34.

Boerhaave (Chem. 1, pa. 720) says, these mercurial Thermometers were first contrived by Olaus Roemer; but the claims of Fahrenheit of Amsterdam, who gave an account of his invention to the Royal Society in 1724, (Philos. Trans. num. 381, or Abr. vol. 7, pa. 49) have been generally allowed. And though Prius and others, in England, Holland, France, and other countries, have made this instrument as well as Fahrenheit, most of the mercurial Thermometers are graduated according to his scale, and are called *Fahrenheit's Thermometers*.

The cone or cylinder, which these Thermometers are often made with, instead of the ball, is made of glass of a moderate thickness, lest, when the exhausted tube is hermetically sealed, its internal capacity should be diminished by the weight of the ambient atmosphere. When the mercury is thoroughly purged of its air and moisture by boiling, the Thermometer is filled with a sufficient quantity of it; and before the tube is hermetically sealed, the air is wholly expelled from it by heating the mercury, so that it may be rarefied and ascend to the top of the tube. To the side of the tube is annexed a scale (fig. 3, pl. 34), which Fahrenheit divided into 600 parts, beginning with that of the severe cold which he had observed in Iceland in 1709, or that produced by surrounding the bulb of the Thermometer with a mixture of snow or beaten ice and sal ammoniac or sea salt. This he apprehended to be the greatest degree of cold, and accordingly he marked this, as the beginning of his scale, with 0; the point at which mercury begins to boil, he conceived to shew the greatest degree of heat, and this he made the limit of his scale. The distance between these two points he divided into 600 equal parts or degrees; and by trials he found at the freezing point, when water just begins to freeze, or snow or ice just begins to thaw, that the mercury stood at 32 of these divisions, therefore called the degree of the freezing point; and when the tube was immersed in boiling water, the mercury rose to 212, which therefore is the boiling point, and is just 180 degrees above the former or freezing point. But the present method of making the scale of these Thermometers, which is the sort in most common use, is first to immerge the bulb of the Thermometer in ice or snow just beginning to thaw, and mark the place where the mercury stands with a 32; then immerge it in boiling water, and again mark the place where the mercury stands in the tube, which mark with the num. 212, exceeding the former by 180; dividing therefore the intermediate space into 180 equal parts, will give the scale of the Thermometer, and which may afterwards be continued upwards and downwards at pleasure.

Other Thermometers of a similar construction have been accommodated to common use, having but a portion of the above scale. They have been made of a small size and portable form, and adapted with appendages to particular purposes; and the tube with its annexed scale has often been enclosed in another thicker glass tube, also hermetically sealed, to preserve the Thermometer from injury. And all these are called *Fahrenheit's Thermometers.*

In 1733, M. De l'Isle of Petersburgh constructed a mercurial Thermometer (see fig. 3, pl. 34), on the principles of Reaumur's spirit Thermometer. In his Thermometer, the whole bulk of quicksilver, when immerged in boiling water, is conceived to be divided into 100,000 parts; and from this one fixed point the various degrees of heat, either above or below it, are marked in these parts on the tube or scale, by the various expansion or contraction of the quicksilver in all imaginable varieties of heat.—Dr. Martine apprehends it would have been better if De l'Isle had made the integer 100,000 parts, or fixed point, at freezing water, and from thence computed the dilatations or condensations of the quicksilver in those parts; as all the common observations of the weather, &c, would have been expressed by numbers increasing as the heat increased, instead of decreasing, or counting the contrary way. However, in practice it will not be very easy to determine exactly all the divisions from the alteration of the bulk of the contained fluid. And besides, as glass itself is dilated by heat, though in a less proportion than quicksilver, it is only the excess of the dilatation of the contained fluid above that of the glass that is observed; and therefore if different kinds of glass be differently affected by a given degree of heat, this will make a seeming difference in the dilatations of the quicksilver in the Thermometers constructed in the Newtonian method, either by Reaumur's rules or De l'Isle's. Accordingly it has been found, that the quicksilver in De l'Isle's Thermometers, has stood at different degrees of the scale when immerged in thawing snow: having stood in some at 154°, while in others it has been at 156 or even 158°.

Metallic THERMOMETER.—This is a name given to a machine composed of two metals, which, whilst it indicates the variations of heat, serves to correct the errors hence resulting in the going of pendulum clocks and watches. Instruments of this kind have been contrived by Graham, Le Roy, Ellicot, Harrison, and other eminent artificers. See the Philos. Trans. vol. 44, pa. 689, and vol. 45, pa. 129, and vol. 51, pa. 823, where the particular descriptions &c may be seen.

M. De Luc has likewise described two Thermometers

of

of metal, which he uses for correcting the effects of heat upon a barometer, and an hygrometer of his construction connected with them. See Philos. Trans. vol. 68, p. 437.

Oil Thermometers.—To this class belongs Newton's Thermometer, constructed in 1701, with linseed oil, instead of spirit of wine. This fluid has the advantage of being sufficiently homogeneous, and capable of a considerable rarefaction, not less than 15 times greater than that of spirit of wine. It has not been observed to freeze even in very great colds; and it sustains a great heat, about 4 times that of water, before it boils. With these advantages it was made use of by Sir I. Newton, who discovered by it the comparative degree of heat for boiling water, melting wax, boiling spirit of wine, and melting tin; beyond which it does not appear that this Thermometer was applied. The method he used for adjusting the scale of this oil Thermometer, was as follows: supposing the bulb, when immerged in thawing snow, to contain 10,000 parts, he found the oil expanded by the heat of the human body so as to take up a 39th more space, or 10256 such parts; and by the heat of water boiling strongly 10725; and by the heat of melting tin 11516. So that, reckoning the freezing point as a common limit between heat and cold, he began his scale there, marking it 0, and the heat of the human body he made 12°; and consequently, the degrees of heat being proportional to the degrees of rarefaction, or 256 : 725 :: 12 : 34, this number 34 will express the heat of boiling water; and, by the same rule, 72 that of melting tin. Philos. Trans. number 270, or Abridg. vol. 4, par. 2, p. 3.

There is an insuperable inconvenience attending all Thermometers made with oil, or any other viscid fluid, viz, that such liquor adheres too much to the sides of the tube, and so inevitably disturbs the regularity and uniformity of the Thermometer.

Of the fixed points of Thermometers.—Various methods have been proposed by different authors, for finding a fixed point or degree of heat, from which to reckon the other degrees, and adjust the scale; so that different observations and instruments might be compared together. Mr. Boyle was very sensible of the inconveniences arising from the want of a universal scale and mode of graduation; and he proposed either the freezing of the essential oil of aniseeds, or of distilled water, as a term to begin the numbers at, and from thence to graduate them according to the proportional dilatations or contractions of the included spirits.

Dr. Halley (Philos. Trans. Abr. vol. 2, p. 36) seems to have been fully apprized of the bad effects of the indefinite method of constructing Thermometers, and wished to have them adjusted to some determined points. What he seems to prefer, for this purpose, is the degree of temperature found in subterranean places, where the heat in summer or cold in winter appears to have no influence. But this degree of temperature, Dr. Martine shews, is a term for the universal construction of Thermometers, both inconvenient and precarious, as it cannot be easily ascertained, and as the difference of soils and depths may occasion a considerable variation. Another term of heat, which he thought might be of use in a general graduation of Thermometers, is that of boiling spirit of wine that has been highly rectified.

The first trace that occurs of the method of actually applying fixed points or terms to the Thermometer, and of graduating it, so that the unequal divisions of it might correspond to equal degrees of heat, is the project of Renaldini, professor at Padua, in 1694: it is thus described in the Acta Erud. Lips. "Take a slender tube, about 4 palms long, with a ball fastened to the same; pour into it spirit of wine, enough just to fill the ball, when surrounded with ice, and not a drop over: in this state seal the orifice of the tube hermetically, and provide 12 vessels, each capable of containing a pound of water, and somewhat more; and into the first pour 11 ounces of cold water, into the second 10 ounces, into the third 9, &c; this done, immerge the Thermometer in the first vessel, and pour into it one ounce of hot water, observing how high the spirit rises in the tube, and noting the point with unity; then remove the Thermometer into the second vessel, into which are to be poured 2 ounces of hot water, and note the place the spirit rises to with 2: by thus proceeding till the whole pound of water is spent, the instrument will be found divided into 12 parts, denoting so many terms or degrees of heat; so that at 2 the heat is double to that at 1, at 3 triple, &c."

But this method, though plausible, Wolfius shews, is deceitful, and built on false suppositions; for it takes for granted, that we have one degree of heat, by adding one ounce of hot water to 11 of cold; two degrees by adding 2 ounces to 10, &c: it supposes also, that a single degree of heat acts on the spirit of wine, in the ball, with a single force; a double with a double force, &c: lastly it supposes, that if the effect be produced in the Thermometer by the heat of the ambient air, which is here produced by the hot water, the air has the same degree of heat with the water.

Soon after this project of Renaldini, viz, in 1701, Newton constructed his oil Thermometer, and placed the base or lowest fixed point of his scale at the temperature of thawing snow, and 12 at that of the human body, &c, as above explained.—De Luc observes, that the 2d term of this scale should have been at a greater distance from the first, and that the heat of boiling water would have answered the purpose better than that of the human body.

In 1702, Amontons contrived his universal Thermometer, the scale of which was graduated in the folling manner. He chose for the first term, the weight that counterbalanced the air included in his Thermometer, when it was heated by boiling water: and in this state he so adjusted the quantity of mercury contained in it, till the sum of its height in the tube, and of its height in the barometer at the moment of observation, was equal to 73 inches. Fixing this number at the point to which the mercury in the tube rose by plunging it in boiling water, it is evident that if the barometer at this time was at 28 inches, the height of the column of mercury in the Thermometer, above the level of that in the ball, was 45 inches; but if the height of the barometer was less by a certain quantity, the column of the Thermometer ought to be greater by the same quantity, and reciprocally. He formed his scale on the supposition, that the weight of the atmosphere was always equal to that of a column of mercury of 28 inches; and he divided it into inches from

from the point 73 downward, marking the divifions with 72, 71, 70, &c, and fubdividing the inches into lines. But as the weight of the atmofphere is variable, the barometer muft be obferved at the fame time with the Thermometer, that the number indicated by this laft inftrument may be properly corrected, by adding or fubtracting the quantity which the mercury is below or above 28 inches in the barometer. In this fcale then, the freezing point is at $51\frac{1}{2}$ inches, correfponding to 32 degrees of Fahrenheit, and the heat of boiling water at 73 inches, anfwering to 212 of Fahrenheit's; and thus they may be eafily compared together.

The fixed points of Fahrenheit's Thermometer, as has been already obferved, are the congelation produced by fal ammoniac and the heat of boiling water. The interval between thefe points is divided into 212 equal parts; the former of thefe points being marked 0, and the other 212.

Reaumur in his Thermometer, the conftruction of which he publifhed in 1730, begins his fcale at an artificial congelation of water in warm weather, which, as he ufes large bulbs for his glaffes, gives the freezing point much higher than it fhould be, and at boiling water he marks 80 degrees, which point Dr. Martine thinks is more vague and uncertain than his freezing point. In order to determine the correfpondence of his fcale with that of Fahrenheit, it is to be confidered that his boiling water heat, is really only the boiling heat of weakened fpirit of wine, coinciding nearly, as Dr. Martine apprehends, with Fahrenheit's 180 degrees. And as his $10\frac{1}{4}$ degrees is the conftant heat of the cave of the obfervatory at Paris, or Fahrenheit's 53°, he thence finds his freezing point, inftead of anfwering juft to 32°, to be fomewhat above 34°.

De l'Ifle's Thermometer, an account of which he prefented to the Peterfburgh Academy in 1733, has only one fixed point, which is the heat of boiling water, and, contrary to the common order, the feveral degrees are marked from this point downward, according to the condenfations of the contained quickfilver, and confequently by numbers increafing as the heat decreafes. The freezing point of De l'Ifle's fcale, Dr. Martine makes near to his 150°, correfponding to Fahrenheit's 32, by means of which they may be compared; but Ducreft fays, that this point ought to be marked at leaft at 154°.

Ducreft, in his fpirit Thermometer, conftructed in 1740, made ufe of two fixed points; the firft, or 0, indicated the temperature of the earth, and was marked on his fcale in the cave of the Paris Obfervatory; and the other was the heat of boiling water, which that fpirit in his Thermometer was made to endure, by leaving the upper part of the tube full of air. He divided the interval between thefe points into 100 equal parts; calling the divifions upward, degrees of heat, and thofe below 0, degrees of cold.—It is faid that he has fince regulated his Thermometer by the degree of cold indicated by melting ice, which he found to be $10\frac{2}{5}$.

The Florentine Thermometers were of two forts. In one fort the freezing point, determined by the degree at which the fpirit ftood in the ordinary cold of ice or fnow (probably in a thawing ftate) and coinciding with 32° of Fahrenheit, fell at 20°; and in the other fort at $13\frac{1}{2}$. And the natural heat of the vifcera of cows and deer, &c, raifed the fpirit in the latter, or lefs fort, to about 40°, coinciding with their fummer heat, and nearly with 102° in Fahrenheit's; and in their other or long Thermometer, the fpirit, when expofed to the great midfummer heat in their country, rofe to the point at which they marked 80°.

In the Thermometer of the Paris Obfervatory, made of fpirit of wine by De la Hire, the fpirit always ftands at 48° in the cave of the obfervatory, correfponding to 53 degrees in Fahrenheit's; and his 28° correfponded with 51 inches 6 lines in Amontons' Thermometer, and confequently with the freezing point, or 32° of Fahrenheit's.

In Poleni's Thermometer, made after the manner of Amontons', but with lefs mercury, 47 inches correfponded, according to Dr. Martine, with 51 in that of Amontons, and 53 with $59\frac{1}{2}$.

In the ftandard Thermometer of the Royal Society of London, according to which Thermometers were for a long time conftructed in England, Dr. Martine found that $34\frac{1}{2}$ degrees anfwered to 64° in Fahrenheit, and 0 to 89.

In the Thermometers graduated for adjufting the degrees of heat proper for exotic plants, &c, in ftoves and greenhoufes, the middle temperature of the air is marked at 0, and the degrees of heat and cold are numbered both above and below. Many of thefe are made on no regular and fixed principles. But in that formerly much ufed, called Fowler's regulator, the fpirit fell, in melting fnow, to about 34° under 0; and Dr. Martine found that his 16° above 0, nearly coincided with 64° of Fahrenheit.

Dr. Hales (Statical Effays, vol. 1, p. 58), in his Thermometer, made with fpirit of wine, and ufed in experiments on vegetation, began his fcale with the loweft degree of freezing, or 32° of Fahrenheit, and carried it up to 100°, which he marked where the fpirit ftood when the ball was heated in hot water, upon which fome wax floating firft began to coagulate, and this point Dr. Martine found to correfpond with 142° of Fahrenheit. But by experience it is found that Hales's 100 falls confiderably above our 142.

In the Edinburgh Thermometer, made with fpirit of wine, and ufed in the meteorological obfervations publifhed in the Medical Effays, the fcale is divided into inches and tenths. In melting fnow the fpirit ftood at $8\frac{2}{10}$, and the heat of the human fkin raifed it to $22\frac{2}{10}$. Dr. Martine found that the heat of the perfon who graduated it, was 97 of Fahrenheit.

As it is often of ufe to compare different Thermometers, in order to judge of the refult of former obfervations, I have annexed from Dr. Martine's Effays, the table by which he compared 15 different thermometers. See Plate 34, fig. 3.

There is a Thermometer which has often been ufed in London, called the Thermometer of Lyons, becaufe M. Criftin

M. Criftin brought it there into ufe, which is made of mercury: the freezing point is marked 0, and the interval from that point to the heat of boiling water is divided into 100 equal degrees.

From the abftract of the hiftory of the conftruction of Thermometers it appears, that freezing and boiling water have furnifhed the diftinguifhing points that have been marked upon almoft all Thermometers. The inferior fixed point is that of freezing, which fome have determined by the freezing of water, and others by the melting of ice, plunging the ball of the Thermometer into the water and ice, while melting, which is the beft way. The fuperior fixed point of almoft all Thermometers, is the heat of boiling water. But this point cannot be confidered as fixed and certain, unlefs the heat be produced by the fame degree of boiling, and under the fame weight of the atmofphere; for it is found that the higher the barometer, or the heavier the atmofphere, the greater is the heat when the water boils. It is now agreed therefore that the operation of plunging the ball of the Thermometer in the boiling water, or fufpending it in the fteam of the fame in an inclofed veffel, be performed when the water boils violently, and when the barometer ftands at 30 Englifh inches, in a temperature of 55° of the atmofphere, marking the height of the Thermometer then for the degree of 212 of Fahrenheit; the point of melting ice being 32 of the fame; thus having 180 degrees between thofe two fixed points, fo determined. This was Mr. Bird's method, who it is apprehended firft attended to the ftate of the barometer, in the making of Thermometers. But thefe inftruments may be made equally true under any preffure of the atmofphere, by making a proper allowance for the difference in the height of the barometer from 30 inches. M. De Luc, in his Recherches fur les Mod. de l'Atmofphere, from a feries of experiments, has given an equation for the allowance on account of this difference, in Paris meafure, which has been verified by Sir George Schuckburgh, Philof. Tranf. 1775 and 1778; alfo Dr. Horfley, Dr. Mafkelyne, and Sir George Shuckburgh have adapted the equation and rules, to Englifh meafures, and have reduced the allowances into tables for the ufe of the artift. Dr. Horfley's rule, deduced from De Luc's, is this:

$$\frac{99}{8990000} \log. z - 92{\cdot}804 = h,$$

where h denotes the height of a Thermometer plunged in boiling water, above the point of melting ice, in degrees of Bird's Fahrenheit, and z the height of the barometer in 10ths of an inch. From this rule he has computed the following table, for finding the heights, to which a good Bird's Fahrenheit will rife, when plunged in boiling water, in all ftates of the barometer, from 27 to 31 Englifh inches; which will ferve, among other ufes, to direct inftrument makers in making a true allowance for the effect of the variation of the barometer, if they fhould be obliged to finifh a Thermometer at a time when the barometer is above or below 30 inches; though it is beft to fix the boiling point when the barometer is at that height.

Equation of the Boiling Point.

Barometer.	Equation.	Difference.
31·0	+ 1·57	
30·5	+ 0·79	0·78
30·0	0·00	0·79
29·5	− 0·80	0·80
29·0	− 1·62	9·82
28·5	− 2·45	0·83
28·0	− 3·31	0·85
27·5	− 4·16	0·86
27·0	− 5·04	0·88

The numbers in the firft column of this table exprefs heights of the quickfilver in the barometer in Englifh inches and decimal parts: the 2d column fhews the equation to be applied, according to the fign prefixed, to 212° of Bird's Fahrenheit, to find the true boiling point for every fuch ftate of the barometer. The boiling point for all intermediate ftates of the barometer may be had with fufficient accuracy by taking proportional parts, by means of the 3d column of differences of the equations. See Philof. Tranf. vol. 64, art. 30; alfo Dr. Mafkelyne's paper, vol. 64, art. 20.

Sir Geo. Shuckburgh (Philof. Tranf. vol. 69, pa. 362) has alfo given feveral tables and rules relating to the boiling point, both from his own obfervations and De Luc's, form whence is extracted the following table, for the ufe of artifts in conftructing the Thermometer.

Height of the Barometer.	Corr. of the Boil. Point.	Differences.	Correct. accord. to De Luc.	Differences.
26·0	− 7·09		− 6·83	
26·5	− 6·18	0·91	− 5·93	0·90
27·0	− 5·27	0·91	− 5·04	0·89
27·5	− 4·37	0·90	− 4·16	0·88
28·0	− 3·48	0·89	− 3·31	0·87
28·5	− 2·59	0·89	− 2·45	0·86
29·0	− 1·72	0·87	− 1·62	0·83
29·5	− 0·85	0·87	− 0·80	0·82
30·0	0·00	0·85	0·00	0·80
30·5	+ 0·85	0·85	+ 0·79	0·79
31·0	+ 1·60	0·84	+ 1·57	0·78

The Royal Society too, fully fenfible of the importance of adjufting the fixed points of Thermometers, appointed a committee of feven gentlemen to confider of the beft method for this purpofe; and their report may be feen in the Philof. Tranf. vol. 67, art. 37.

They obferve, that although the boiling point be placed fo much higher on fome of the Thermometers now made, than on others, yet this does not produce any confiderable error in the obfervations of the weather, at leaft in this climate; for an error of 1½ degree in the pofition of the boiling point, will make an error only of half a degree in the pofition of 92°, and of not more than

than a quarter of a degree in the point of 62°. It is only in nice experiments, or in trying the heat of hot liquors, that this error in the boiling point can be of much ſignification.

In adjuſting the freezing, as well as the boiling point, the quickſilver in the tube ought to be kept of the ſame heat as that in the ball. When the freezing point is placed at a conſiderable diſtance from the ball, the pounded ice ſhould be piled up very near to it; if it be not ſo piled, then the obſerved point, to be very accurate, ſhould be corrected, according to the following table.

Heat of the Air.	Correction.
42°	·00087
52	·00174
62	·00261
72	·00348
82	·00435

The correction in this table is expreſſed in 10000th parts of the diſtance between the freezing point and the ſurface of the ice: ex. gr. if the freezing point ſtand 6 inches above the ſurface of the ice, and the heat of the room be 62, then the point of 32 ſhould be placed 6 × ·00261, or ·01566 of an inch lower down than the obſerved point.

The committee farther obſerve, that in trying the heat of liquors, care ſhould be taken that the quickſilver in the tube of the Thermometer be heated to the ſame degree as that in the ball; or if this cannot be done conveniently, the obſerved heat ſhould be corrected on that account; for the manner of doing which, and a table calculated for that purpoſe, ſee Philoſ. Tranſ. vol. 67, art. 37.

It was for ſome time thought, eſpecially from the experiments at Peterſburgh, that quickſilver ſuffered a cold of ſeveral hundred degrees below 0 before it congealed and became fixed and malleable; but later experiments have ſhewn that this perſuaſion was merely owing to a deception in the experiments, and later ones have made it appear that its point of congelation is no lower than − 40°, or rather − 39°, of Fahrenheit's ſcale. But that it will bear however to be cooled a few degrees below that point, to which it leaps up again on beginning to congeal; and that its rapid deſcent in a Thermometer, through many hundred degrees, when it has once paſſed the above-mentioned limit, proceeds merely from its great contraction in the act of freezing. See Philoſ. Tranſ. vol. 73, art. *20, 20, 21.

Miſcellaneous Obſervations.

It is abſolutely neceſſary that thoſe who would derive any advantage from theſe inſtruments, ſhould agree in uſing the ſame liquor, and in determining, according to the ſame method, the two fundamental points. If they agree in theſe fixed points, it is of no great importance whether they divide the interval between them into a greater or a leſs number of equal parts. The ſcale of Fahrenheit, in which the fundamental interval between 212°, the point of boiling water, and 32° that of melting ice, is divided into 180 parts, ſhould be retained in the northern countries, where Fahrenheit's Thermometer is uſed: and the ſcale in which the fundamental interval is divided into 80 parts, will ſerve for thoſe countries where Reaumur's Thermometer is adopted. But no inconvenience is to be apprehended from varying the ſcale for particular uſes, provided care be taken to ſignify into what number of parts the fundamental interval is divided, and the point where 0 is placed.

With regard to the choice of tubes, it is beſt to have them exactly cylindrical through their whole length. The capillary tubes are preferable to others, becauſe they require ſmaller bulbs, and they are alſo more ſenſible, and leſs brittle. The moſt convenient ſize for common experiments has the internal diameter about the 40th or 50th of an inch, about 9 inches long, and made of thin glaſs, that the riſe and fall of the mercury may be better ſeen.

For the whole proceſs of filling, marking, and graduating, ſee De Luc's Recherches &c, tom. 1, p. 393, &c.

Experiments with Thermometers.

The following is a table of ſome obſervations made with Fahrenheit's Thermometer, the barometer ſtanding at 29 inches, or little higher.

At 600° Mercury boils.
546 Oil of vitriol boils.
242 Spirit of nitre boils.
240 Lixivium of tartar boils.
213 Cow's milk boils.
212 Water boils.
206 Human urine boils.
190 Brandy boils.
175 Alcohol boils.
156 Serum of blood and white of eggs harden.
146 Kills animals in a few minutes.
108 to 99, Hens hatch eggs.
107 } Heat of ſkin in ducks, geeſe, hens, pi-
103 } geons, partridges, and ſwallows.
106 Heat of ſkin in a common ague and fever.
103 } Heat of ſkin in dogs, cats, ſheep, oxen,
100 } ſwine, and moſt other quadrupeds.
99 to 92, Heat of the human ſkin in health.
97 Heat of a ſwarm of bees.
96 A perch died in 3 minutes in water ſo warm.
80 Heat of air in the ſhade, in very hot weather.
74 Butter begins to melt.
64 Heat of air in the ſhade, in warm weather.
55 Mean temperature of air in England.
43 Oil of olives begins to ſtiffen and grow opake.
32 { Water juſt freezing, or ſnow and ice juſt melting.
30 Milk freezes.
28 Urine and common vinegar freezes.
25 Blood out of the body freezes.
20 Burgundy, Claret, and Madeira freeze.
5 { Greateſt cold in Pennſylvania in 1731-2, lat. 40°.
4 Greateſt cold at Utrecht in 1728-9.
0 { A mixture of ſnow and ſalt, which can freeze oil of tartar per deliquium, but not brandy.
−39 Mercury freezes.

Martine's Eſſays, p. 284, &c.

On

On the general subject of Thermometers also see Martine's Essays, Medical and Philosophical. Desaguliers's Exp. Phil. vol. 2, p. 289. Musschenbroeck's Int. ad Phil. Nat. vol. 2, p. 625, ed. 1762. De Luc's Recherches sur les Modif. &c, tom. 1, part 2, ch. 2. Nollet's Leçons de Physique, tom. 4, p. 375.

THERMOMETERS *for particular uses*.—In 1757, lord Cavendish presented to the Royal Society an account of a curious construction of Thermometers, of two different forms; one contrived to shew the greatest degree of heat, and the other the greatest cold, that may happen at any time in a person's absence. Philos. Trans. vol. 50, p. 300.

Since the publication of Mr. Canton's discovery of the compressibility of spirits of wine and other fluids, there are two corrections necessary to be made in the result given by lord Cavendish's Thermometer. For in estimating, for instance, the temperature of the sea at any depth, the Thermometer will appear to have been colder than it really was: and besides, the expansion of spirits of wine by any given number of degrees of Fahrenheit's Thermometer, is greater in the higher degrees than in the lower. For the method of making these two corrections by Mr. Cavendish, see Phipps's Voyage to the North Pole, p. 145.

Instruments of this kind, for determining the degree of heat or cold in the absence of the observer, have been invented and described by others. Van Swinden (Diss. sur la Comparaison du Therm. p. 253 &c) describes one, which he says was the first of the kind, made on a plan communicated by Bernoulli to Leibnitz. Mr. Kraft, he also tells us, made one nearly like it. Mr. Six has lately, viz, in 1782, proposed another construction of a Thermometer of the same kind, described in the Philos. Trans. vol. 72, p. 72 &c.

M. De Luc has described the best method of constructing a Thermometer, fit for determining the temperature of the air, in the measuring of heights by the barometer. He has also shewn how to divide the scale of a Thermometer, so as to adapt it for astronomical purposes in the observation of refractions. See Recherches &c, tom. 2, p. 35 and 265.

Mr. Cavallo, in 1781, proposed the construction of a *Thermometrical Barometer*, which, by means of boiling water, might indicate the various gravity of the atmosphere, or the height of the barometer. This Thermometer, he says, with its apparatus, might be packed up into a small portable box, and serve for determining the heights of mountains &c, with greater facility, than with the common portable barometer. The instrument, in its present state, consists of a cylindrical tin vessel, about 2 inches in diameter, and 5 inches high, in which vessel the water is contained, which may be made to boil by the flame of a large wax-candle. The Thermometer is fastened to the tin vessel in such a manner, as that its bulb may be about an inch above the bottom. The scale of this Thermometer, which is of brass, exhibits on one side of the glass tube a few degrees of Fahrenheit's scale, viz, from 200° to 216°. On the other side of the tube are marked the various barometrical heights, at which the boiling water shews those particular degrees of heat which are set down in Sir Geo. Shuckburgh's table. With this instrument the barometrical height is shewn within one 10th of an inch. The degrees of this Thermometer are rather longer than one 9th of an inch, and therefore may be divided into many parts, especially by a Nonius. But a considerable imperfection arises from the smallness of the tin vessel, which does not admit a sufficient quantity of water; but when the quantity of water shall be sufficiently large, as for instance 10 or 12 ounces, and is kept boiling in a proper vessel, its degree of heat under the same pressure of the atmosphere is very settled; whereas when a Thermometer is kept in a small quantity of boiling water, the mercury in its stem does not stand very steady, sometimes rising or falling so much as half a degree. Mr. Cavallo proposes a farther improvement of this instrument, in the Philos. Trans. vol. 71, p. 524.

The ingenious Mr. Wedgwood, so well known for his various improvements in the different sorts of pottery ware, has contrived to make a Thermometer for measuring the higher degrees of heat, by means of a distinguishing property of argillaceous bodies, viz, the diminution of their bulk by fire. This diminution commences in a low red heat, and proceeds regularly, as the heat increases, till the clay becomes vitrified. The total contraction of some good clays which he has examined in the strongest of his own fires, is considerably more than one-fourth part in every dimension. By measuring the contraction of such substances then, Mr. Wedgwood contrived to measure the most intense heats of ovens, furnaces, &c. For the curious particulars of which, see Philos. Trans. vol. 72, p. 305 &c.

THERMOSCOPE, an instrument shewing the changes happening in the air with respect to heat and cold.

The word Thermoscope is often used indifferently with that of thermometer. There is some difference however in the literal import of the two; the first signifying an instrument that shews or exhibits the changes of heat &c to the eye; and the latter an instrument that measures those changes; so that a thermometer should be a more accurate Thermoscope.

THIR, in Chronology, the name of the 5th month of the Ethiopians, which corresponds, according to Ludolf, to the month of January.

THIRD, in Music, a concord resulting from a mixture of two sounds containing an interval of 2 degrees: being called a third, because containing 3 terms, or sounds, between the extremes.

There is a greater and a less Third. The former takes its form from the sesquiquarta ratio, 4 to 5. The logarithm or measure of the octave $\frac{2}{1}$ being 1·00000, the measure of the greater Third $\frac{5}{4}$ will be 0·32193.—The *greater Third* is by practitioners often taken for the third part of an octave; which is an error, since three greater Thirds fall short of the octave by a diesis; for $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{128}{125} = \frac{2}{1}$.

The *lesser Third* takes its form from the sesquiquinta ratio 5 to 6; the measure or logarithm of this lesser Third $\frac{6}{5}$, being 0·26303, that of the octave $\frac{2}{1}$ being 1·00000.

Both these Thirds are of great use in melody; making as it were the foundation and life of harmony.

THIRD-*Point*, or *Tierce-point*, in Architecture, the point of section in the vertex of an equilateral triangle.—Arches or vaults of the Third Point, are those con-

fifting of two arches of a circle, meeting in an angle at top.

THREE-*legged-ftaff*, an inftrument confifting of three wooden legs, made with joints, fo as to fhut all together, and to take off in the middle for the better carriage. It has ufually a ball and focket on the top; and its ufe is to fupport and adjuft inftruments for aftronomy, furveying, &c.

THUNDER, a noife in the lower region of the air, excited by a fudden explofion of electrical clouds; which are therefore called Thunder-clouds.

The phenomenon of Thunder is varioufly accounted for. Seneca, Rohault, and fome other authors, both ancient and modern, account for Thunder, by fuppofing two clouds impending over one another, the upper and rarer of which, becoming condenfed by a frefh acceffion of air raifed by warmth from the lower parts of the atmofphere, or driven upon it by the wind, immediately falls forcibly down upon the lower and denfer cloud; by which fall, the air interpofed between the two being compreffed, that next the extremities of the two clouds is fqueezed out, and leaves room for the extremity of the upper cloud to clofe tight upon the under; thus a great quantity of the air is enclofed, which at length efcaping through fome winding irregular vent or paffage, occafions the noife called Thunder.

But this lame device could only reach at moft to the cafe of Thunder heard without lightning; and therefore recourfe has been had to other modes of folution. Thus, it has been faid that Thunder is not occafioned by the falling of clouds, but by the kindling of fulphurous exhalations, in the fame manner as the noife of the aurum fulminans. "There are fulphurous exhalations, fays Sir I. Newton, always afcending into the air when the earth is dry; there they ferment with the nitrous acids, and, fometimes taking fire, generate Thunder, lightning, &c."

The effects of Thunder are fo like thofe of fired gunpowder, that Dr. Wallis thinks we need not fcruple to afcribe them to the fame caufe; and the principal ingredients in gunpowder, we know, are nitre and fulphur; charcoal only ferving to keep the parts feparate, for their better kindling. Hence, if we conceive in the air a convenient mixture of nitrous and fulphurous particles; and thofe, by any caufe, to be fet on fire, fuch explofion may well follow, and with fuch noife and light as attend the firing of gunpowder; and being once kindled, it will run from place to place, different ways, as the exhalations happen to lead it; much as is found in a train of gunpowder.

But a third, and moft probable opinion is, that Thunder is the report or noife produced by an electrical explofion in the clouds. Ever fince the year 1752, in which the identity of the matter of lightning and of the electrical fluid has been afcertained, philofophers have generally agreed in confidering Thunder as a concuffion produced in the air by an explofion of electricity. For the illuftration and proof of this theory, fee ELECTRICITY, and LIGHTNING.

It may here be obferved, that Mr. Henry Eeles, in a letter written in 1751, and read before the Royal Society in 1752, confiders the electrical fire as the caufe of Thunder, and accounts for it on this hypothefis; and he tells us, that he did not know of any other perfon's having made the fame conjecture. Philof. Tranf. vol. 47, p. 524 &c.

That rattling in the noife of Thunder, which makes it feem as if it paffed through arches, or were varioufly broken, is probably owing to the found being excited among clouds hanging over one another, and the agitated air paffing irregularly between them.

The explofion, if high in the air, and remote from us, will do no mifchief; but when near, it may deftroy trees, animals, &c.

This proximity, or fmall diftance, may be eftimated nearly by the interval of time between feeing the flafh of lightning, and hearing the report of the Thunder, eftimating the diftance, after the rate of 1142 feet per fecond of time, or $3\frac{4}{5}$ feconds to the mile. Dr. Wallis obferves, that commonly the difference between the two is about 7 feconds, which, at the rate above mentioned, gives the diftance almoft 2 miles. But fometimes it comes in a fecond or two, which argues the explofion very near us, and even among us. And in fuch cafes, the doctor affures us, he has fometimes foretold the mifchiefs that happened.

The noife of Thunder, and the flame of lightning, are eafily made by art. If a mixture of oil or fpirit of vitriol be made with water, and fome filings of fteel added to it, there will immediately arife a thick fmoke, or vapour, out of the mouth of the veffel; and if a lighted candle be applied to this, it will take fire, and the flame will immediately defcend into the veffel, which will be burft to pieces with a noife like that of a cannon.

This is fo far analogous to Thunder and lightning, that a great explofion and fire are occafioned by it; but in this they differ, that this matter when once fired is deftroyed, and can give no more explofions; whereas, in the heavens, one clap of Thunder ufually follows another, and there is a continued fucceffion of them for a long time. Mr. Homberg explained this by the lightnefs of the air above us, in comparifon of that near, which therefore would not fuffer all the matter fo kindled to be diffipated at once, but keeps it for feveral returns.

THUNDERBOLT. When lightning acts with extraordinary violence, and breaks or fhatters any thing, it is called a *Thunderbolt*, which the vulgar, to fit it for fuch effects, fuppofe to be a hard body, and even a ftone.—But that we need not have recourfe to a hard folid body to account for the effects commonly attributed to the Thunderbolt, will be evident to any one, who confiders thofe of the pulvis fulminans, and of gunpowder; but more efpecially the aftonifhing powers of electricity, when only collected and employed by human art, and much more when directed and exercifed in the courfe of nature.

When we confider the known effects of electrical explofions, and thofe produced by lightning, we fhall be at no lofs to account for the extraordinary operations vulgarly afcribed to Thunderbolts. As ftones and bricks ftruck by lightning are often found in a vitrified ftate, we may reafonably fuppofe, with Beccaria, that fome ftones in the earth, having been ftruck in this manner, gave occafion to the vulgar opinion of the Thunderbolt.

THUNDER-*clouds*, in Phyfiology, are thofe clouds which

which are in a ſtate fit for producing lightning and thunder.

From Beccaria's exact and circumſtantial account of the external appearances of Thunder-clouds, the following particulars are extracted.

The firſt appearance of a Thunder ſtorm, which uſually happens when there is little or no wind, is one denſe cloud, or more, increaſing very faſt in ſize, and riſing into the higher regions of the air. The lower ſurface is black and nearly level; but the upper finely arched, and well defined. Many of theſe clouds often ſeem piled upon one another, all arched in the ſame manner; but they are continually uniting, ſwelling, and extending their arches.

At the time of the riſing of this cloud, the atmoſphere is commonly full of a great many ſeparate clouds, that are motionleſs, and of odd whimſical ſhapes. All theſe, upon the appearance of the Thunder-cloud, draw towards it, and become more uniform in their ſhapes as they approach; till, coming very near the Thunder-cloud, their limbs mutually ſtretch toward one another, and they immediately coaleſce into one uniform maſs. Theſe he calls adſcititious clouds, from their coming in, to enlarge the ſize of the Thunder-cloud. But ſometimes the Thunder-cloud will ſwell, and increaſe very faſt, without the conjunction of any adſcititious clouds; the vapours in the atmoſphere forming themſelves into clouds wherever it paſſes. Some of the adſcititious clouds appear like white fringes, at the ſkirts of the Thunder-cloud, or under the body of it, but they keep continually growing darker and darker, as they approach to unite with it.

When the Thunder-cloud is grown to a great ſize, its lower ſurface is often ragged, particular parts being detached towards the earth, but ſtill connected with the reſt. Sometimes the lower ſurface ſwells into various large protuberances bending uniformly downward; and ſometimes one whole ſide of the cloud will have an inclination to the earth, and the extremity of it nearly touch the ground. When the eye is under the Thunder-cloud, after it is grown larger, and well formed, it is ſeen to ſink lower, and to darken prodigiouſly; at the ſame time that a number of ſmall adſcititious clouds (the origin of which can never be perceived) are ſeen in a rapid motion, driving about in very uncertain directions under it. While theſe clouds are agitated with the moſt rapid motions, the rain commonly falls in the greateſt plenty, and if the agitation be exceedingly great, it commonly hails.

While the Thunder-cloud is ſwelling, and extending its branches over a large tract of country, the lightning is ſeen to dart from one part of it to another, and often to illuminate its whole maſs. When the cloud has acquired a ſufficient extent, the lightning ſtrikes between the cloud and the earth, in two oppoſite places, the path of the lightning lying through the whole body of the cloud and its branches. The longer this lightning continues, the leſs denſe does the cloud become, and the leſs dark its appearance; till at length it breaks in different places, and ſhews a clear ſky.

Theſe Thunder-clouds were ſometimes in a poſitive as well as a negative ſtate of electricity. The electricity continued longer of the ſame kind, in proportion as the Thunder-cloud was ſimple, and uniform in its direction; but when the lightning changed its place, there commonly happened a change in the electricity of the apparatus, over which the clouds paſſed. It would change ſuddenly after a very violent flaſh of lightning, but the change would be gradual when the lightning was moderate, and the progreſs of the Thunder-cloud ſlow. Beccar. Lettere dell' Elettricifmo pa. 107; or Prieſtley's Hiſt. Elec. vol. 1, p. 397. See alſo LIGHTNING.

THUNDER-*Houſe*, in Electricity, is an inſtrument invented by Dr. James Lind, for illuſtrating the manner in which buildings receive damage from lightning, and to evince the utility of metallic conductors in preſerving them from it.

A (fig. 1, pl. 35), is a board about ¾ of an inch thick, and ſhaped like the gable end of a houſe. This board is fixed perpendicularly upon the bottom board B, upon which the perpendicular glaſs pillar CD is alſo fixed in a hole about 8 inches diſtant from the baſis of the board A. A ſquare hole ILMK, about a quarter of an inch deep, and nearly one inch wide, is made in the board A, and is filled with a ſquare piece of wood, nearly of the ſame dimenſions. It is nearly of the ſame dimenſions, becauſe it muſt go ſo eaſily into the hole, that it may drop off, by the leaſt ſhaking of the inſtrument. A wire LK is faſtened diagonally to this ſquare piece of wood. Another wire IH of the ſame thickneſs, having a braſs ball H, ſcrewed on its pointed extremity, is faſtened upon the board A: ſo alſo is the wire MN, which is ſhaped in a ring at O. From the upper extremity of the glaſs pillar CD, a crooked wire proceeds, having a ſpring ſocket F, through which a double knobbed wire ſlips perpendicularly, the lower knob G of which falls juſt above the knob H. The glaſs pillar DC muſt not be made very faſt into the bottom board; but it muſt be fixed ſo that it may be pretty eaſily moved round its own axis, by which means the braſs ball G may be brought nearer to or farther from the ball H, without touching the part EFG. Now when the ſquare piece of wood LMIK (which may repreſent the ſhutter of a window or the like) is fixed into the hole ſo that the wire LK ſtands in the dotted repreſentation IM, then the metallic communication from H to O is complete, and the inſtrument repreſents a houſe furniſhed with a proper metallic conductor; but if the ſquare piece of wood LMIK be fixed ſo that the wire LK ſtands in the direction LK, as repreſented in the figure, then the metallic conductor HO, from the top of the houſe to its bottom, is interrupted at IM, in which caſe the houſe is not properly ſecured.

Fix the piece of wood LMIK, ſo that its wire may be as repreſented in the figure, in which caſe the metallic conductor HO is diſcontinued. Let the ball G be fixed at about half an inch perpendicular diſtance from the ball H; then, by turning the glaſs pillar DC, remove the former ball from the latter; by a wire or chain connect the wire EF with the wire Q of the jar P; and let another wire or chain, faſtened to the hook O, touch the outſide coating of the jar. Connect the wire Q with the prime conductor, and charge the jar; then, by turning the glaſs pillar DC, let the ball G come gradually near the ball H, and when they are arrived ſufficiently near one another, you will obſerve, that the jar explodes and the piece of wood LMIK is puſhed

pushed out of the hole to a considerable distance from the Thunder-house.

Now the ball G, in this experiment, represents an electrified cloud, which, when it is arrived sufficiently near the top of the house A, the electricity strikes it; and as this house is not secured with a proper conductor, the explosion breaks part of it, i. e. knocks off the piece of wood IM.

Repeat the experiment with only this variation, viz, that this piece of wood IM be situated so that the wire LK may stand in the situation IM; in which case the conductor HO is not discontinued; and you will observe that the explosion will have no effect upon the piece of wood LM; this remaining in the hole unmoved; which shews the usefulness of the metallic conductor.

Farther, unscrew the brass ball H from the wire HI, so that this may remain pointed, and with this difference only in the apparatus repeat both the above experiments, and you will find that the piece of wood IM is in neither case moved from its place, nor will any explosion be heard; which not only demonstrates the preference of conductors with pointed terminations to those with blunted ones, but also shews that a house, furnished with sharp terminations, although not furnished with a regular conductor, is almost sufficiently guarded against the effects of lightning.

Mr. Henley, having connected a jar containing 509 square inches of coated surface with his prime conductor, observed that if it was so charged as to raise the index of his electrometer to 60°, by bringing the ball on the wire of the Thunder-house, to the distance of half an inch from that connected with the prime conductor, the jar would be discharged, and the piece in the Thunder-house thrown out to a considerable distance. Using a pointed wire for a conductor to the Thunder-house, instead of the knob, the charge being the same as before, the jar was discharged silently, though suddenly; and the piece was not thrown out of the Thunder-house. In another experiment, having made a double circuit to the Thunder-house, the first by the knob, the second by a sharp-pointed wire, at an inch and a quarter distance from each other, but of exactly the same height (as in fig. 2) the charge being the same; although the knob was brought first under that connected with the prime conductor, which was raised half an inch above it, and followed by the point, yet no explosion could fall upon the knob; the point drew off the whole charge silently, and the piece in the Thunder-house remained unmoved.

Phil. Transf. vol. 64, p. 136. See Points in Electricity.

THURSDAY, the 5th day of the Christian's week, but the 6th of the Jews. The name is from Thor, one of the Saxon Gods.

THUS, in Sea-Language, a word used by the pilot in directing the helmsman or steersman to keep the ship in her present situation when sailing with a scant wind, so that she may not approach too near the direction of the wind, which would shiver her sails, nor fall to leeward, and run farther out of her course.

TIDES, two periodical motions of the waters of the sea; called also the *flux* and *reflux*, or the *ebb* and *flow*.

The Tides are found to follow periodically the course of the sun and moon, both as to time and quantity. And hence it has been suspected, in all ages, that the Tides were somehow produced by the influence of these luminaries. Thus, several of the ancients, and among others, Pliny, Ptolomy, and Macrobius, were acquainted with the influence of the sun and moon upon the Tides; and Pliny says expressly, that the cause of the ebb and flow is in the sun, which attracts the waters of the ocean; and adds, that the waters rise in proportion to the proximity of the moon to the earth. It is indeed now well known, from the discoveries of Sir Isaac Newton, that the Tides are caused by the gravitation of the earth towards the sun and moon. Indeed the sagacious Kepler, long ago, conjectured this to be the cause of the Tides: "If, says he, the earth ceased to attract its waters towards itself, all the water in the ocean would rise and flow into the moon: the sphere of the moon's attraction extends to our earth, and draws up the water." Thus thought Kepler, in his Introd. ad Theor. Mart. This surmise, for it was then no more, is now abundantly verified in the theory laid down by Newton, and by Halley, from his principles.

As to the Phenomena of the Tides: 1. The sea is observed to flow, for about 6 hours, from south towards north; the sea gradually swelling; so that, entering the mouths of rivers, it drives back the river-waters towards their heads, or springs. After a continual flux of 6 hours, the sea seems to rest for about a quarter of an hour; after which it begins to ebb, or retire back again, from north to south, for 6 hours more; in which time, the water sinking, the rivers resume their natural course. Then, after a seeming pause of a quarter of an hour, the sea again begins to flow, as before: and so on alternately.

2. Hence, the sea ebbs and flows twice a day, but falling every day gradually later and later, by about 48 minutes, the period of a flux and reflux being on an average about 12 hours 24 minutes, and the double of each 24 hours 48 minutes; which is the period of a lunar day, or the time between the moon's passing a meridian, and coming to it again. So that the sea flows as often as the moon passes the meridian, both the arch above the horizon, and that below it; and ebbs as often as she passes the horizon, both on the eastern and western side.

Other phenomena of the Tides are as below; and the reasons of them will be noticed in the Theory of the Tides that follows.

3. The elevation towards the moon a little exceeds the opposite one. And the quantity of the ascent of the water is diminished from the equator towards the poles.

4. From the sun, every natural day, the sea is twice elevated, and twice depressed, the same as for the moon. But the solar Tides are much less than the lunar ones, on account of the immense distance of the sun; yet they are both subject to the same laws.

5. The Tides which depend upon the actions of the sun and moon, are not distinguished, but compounded, and so forming as to sense one united Tide, increasing and decreasing, and thus making neap and spring Tides: for, by the action of the sun, the

 lunar

lunar Tide is only changed; which change varies every day, by reafon of the inequality between the natural and lunar day.

6. In the fyzygies the elevations from the action of both luminaries concur, and the fea is more elevated. But the fea afcends lefs in the quadratures; for where the water is elevated by the action of the moon, it is depreffed by the action of the fun; and vice verfa. Therefore, while the moon paffes from the fyzygy to the quadrature, the daily elevations are continually diminifhed: on the contrary, they are increafed while the moon moves from the quadrature to the fyzygy. At a new moon alfo, *cæteris paribus*, the elevations are greater; and thofe that follow one another the fame day, are more different than at full moon.

7. The greateft elevations and depreffions are not obferved till the 2d or 3d day after the new or full moon. And if we confider the luminaries receding from the plane of the equator, we fhall perceive that the agitation is diminifhed, and becomes lefs, according as the declination of the luminaries becomes greater.

8. In the fyzygies, and near the equinoxes, the Tides are obferved to be the greateft, both luminaries being in or near the equator.

9. The actions of the fun and moon are greater, the nearer thofe bodies are to the earth; and the lefs, as they are farther off. Alfo the greateft Tides happen near the equinoxes, or rather when the fun is a little to the fouth of the equator, that is, a little before the vernal, and after the autumnal equinox. But yet this does not happen regularly every year, becaufe fome variation may arife from the fituation of the moon's orbit, and the diftance of the fyzygy from the equinox.

10. All thefe phenomena obtain, as defcribed, in the open fea, where the ocean is extended enough to be fubject to thefe motions. But the particular fituations of places, as to fhores, capes, ftraits, &c, difturb thefe general rules. Yet it is plain, from the moft common and univerfal obfervations, that the Tides follow the laws above laid down.

11. The mean force of the moon to move the fea, is to that of the fun, nearly as $4\frac{1}{2}$ to 1. And therefore, if the action of the fun alone produce a Tide of 2 feet, which it has been ftated to do, that of the moon will be 9 feet; from which it follows, that the fpring Tides will be 11 feet, and the neap Tides 7 feet high. But as to fuch elevations as far exceed thefe, they happen from the motion of the waters againft fome obftacles, and from the fea violently entering into ftraits or gulphs where the force is not broken till the water rifes higher.

Theory of the Tides.

1. If the earth were entirely fluid, and quiefcent, it is evident that its particles, by their mutual gravity towards each other, would form the whole mafs into the figure of an exact fphere. Then fuppofe fome power to act on all the particles of this fphere with an equal force, and in parallel directions; by fuch a power the whole mafs will be moved together, but its figure will fuffer no alteration by it, being ftill the fame perfect fphere, whofe centre will have the fame motion as each particle.

Upon this fuppofition, if the motion of the earth round the common centre of gravity of the earth and moon were deftroyed, and the earth left to the influence of its gravitation towards the moon, as the acting power above mentioned; then the earth would fall or move ftraight towards the moon, but ftill retaining its true fpherical figure.

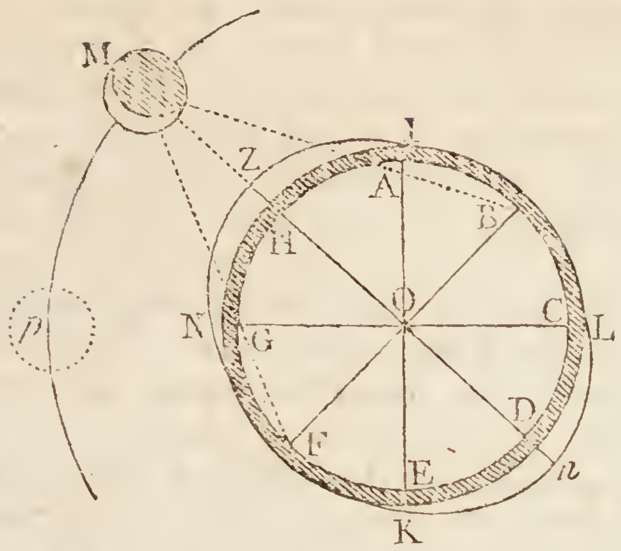
But the fact is, that the effects of the moon's action, as well as the action itfelf, on different parts of the earth, are not equal: thofe parts, by the general law of gravity, being moft attracted that are neareft the moon, and thofe being leaft attracted that are fartheft from her, while the parts that are at a middle diftance are attracted by a mean degree of force: befides, all the parts are not acted on in parallel lines, but in lines directed towards the centre of the moon: on both which accounts the fpherical figure of the fluid earth muft fuffer fome change from the action of the moon. So that, in falling, as above, the nearer parts, being moft attracted, would fall quickeft; the farther parts, being leaft attracted, would fall floweft; and the fluid mafs would be lengthened out, and take a kind of fpheroidical form.

Hence it appears, and what muft be carefully obferved, that it is not the action of the moon itfelf, but the inequalities in that action, that caufe any variation from the fpherical figure; and that, if this action were the fame in all the particles as in the central parts, and operating in the fame direction, no fuch change would enfue.

Let us now admit the parts of the earth to gravitate toward its centre: then, as this gravitation far exceeds the action of the moon, and much more exceeds the differences of her actions on different parts of the earth, the effect that refults from the inequalities of thefe actions of the moon, will be only a fmall diminution of the gravity of thofe parts of the earth which it endeavoured in the former fuppofition to feparate from its centre; that is, thofe parts of the earth which are neareft to the moon, and thofe that are fartheft from her, will have their gravity toward the earth fomewhat abated; to fay nothing of the lateral parts. So that fuppofing the earth fluid, the columns from the centre to the neareft, and to the fartheft parts, muft rife, till by their greater height they be able to balance the other columns, whofe gravity is lefs altered by the inequalities of the moon's action. And thus the figure of the earth muft ftill be an oblong fpheroid.

Let us now confider the earth, inftead of falling toward the moon by its gravity, as projected in any direction, fo as to move round the centre of gravity of the earth and moon: it is evident that in this cafe, the feveral parts of the fluid earth will ftill preferve their relative pofitions; and the figure of the earth will remain the fame as if it fell freely toward the moon; that is, the earth will ftill affume a fpheroidal form, having its longeft diameter directed toward the moon.

From



From the above reaſoning it appears, that the parts of the earth directly under the moon, as at H, and alſo the oppoſite parts at D, will have the flood or highwater at the ſame time; while the parts, at B and F, at 90° diſtance, or where the moon appears in the horizon, will have the ebbs or loweſt waters at that time.

Hence, as the earth turns round its axis from the moon to the moon again in 24 hours 48 minutes, this oval of water muſt ſhift with it; and thus there will be two Tides of flood and two of ebb in that time.

But it is further evident that, by the motion of the earth on her axis, the moſt elevated part of the water is carried beyond the moon in the direction of the rotation. So that the water continues to riſe after it has paſſed directly under the moon, though the immediate action of the moon there begins to decreaſe, and comes not to its greateſt elevation till it has got about half a quadrant farther. It continues alſo to deſcend after it has paſſed at 90° diſtance from the point below the moon, to a like diſtance of about half a quadrant. The greateſt elevation therefore is not in the line drawn through the centres of the earth and moon, nor the loweſt points where the moon appears in the horizon, but all theſe about half a quadrant removed eaſtward from theſe points, in the direction of the motion of rotation. Thus in open ſeas, where the water flows freely, the moon M is generally paſt the north and ſouth meridian, as at *p*, when the high water is at Z and at *n*: the reaſon of which is plain, becauſe the moon acts with the ſame force after ſhe has paſſed the meridian, and thus adds to the libratory or waving motion, which the water acquired when ſhe was in the meridian; and therefore the time of high water is not preciſely at the time of her coming to the meridian, but ſome time after, &c.

Beſides, the Tides anſwer not always to the ſame diſtance of the moon, from the meridian, at the ſame places; but are variouſly affected by the action of the ſun, which brings them on ſooner when the moon is in her firſt and third quarters, and keeps them back later when ſhe is in her 2d and 4th; becauſe, in the former caſe the Tide raiſed by the ſun alone would be earlier than the Tide raiſed by the moon, and in the latter caſe later.

2. We have hitherto adverted only to the action of the moon in producing Tides; but it is manifeſt that, for the ſame reaſons, the inequality of the ſun's action on different parts of the earth, would produce a like effect, and a like variation from the exact ſpherical figure of a fluid earth. So that in reality there are two Tides every natural day from the action of the ſun, as there are in the lunar day from that of the moon, ſubject to the ſame laws; and the lunar Tide, as we have obſerved, is ſomewhat changed by the action of the ſun, and the change varies every day on account of the inequality between the natural and the lunar day. Indeed the effect of the ſun in producing Tides, becauſe of his immenſe diſtance, muſt be conſiderably leſs than that of the moon, though the gravity toward the ſun be much greater: for it is not the action of the ſun or moon itſelf, but the inequalities in that action, that have any effect: the ſun's diſtance is ſo great, that the diameter of the earth is but as a point in compariſon with it, and therefore the difference between the ſun's actions on the neareſt and fartheſt parts, becomes vaſtly leſs than it would be if the ſun were as near as the moon. However the immenſe bulk of the ſun makes the effect ſtill ſenſible, even at ſo great a diſtance; and therefore, though the action of the moon has the greateſt ſhare in producing the Tides, the action of the ſun adds ſenſibly to it when they conſpire together, as in the full and change of the moon, when they are nearly in the ſame line with the centre of the earth, and therefore unite their forces: conſequently, in the ſyzygies, or at new and full moon, the Tides are the greateſt, being what are called the *Spring-Tides*. But the action of the ſun diminiſhes the effect of the moon's action in the quarters, becauſe the one raiſes the water in that caſe where the other depreſſes it; therefore the Tides are the leaſt in the quadratures, and are called *Neap-Tides*.

Newton has calculated the effects of the ſun and moon reſpectively upon the Tides, from their attractive powers. The former he finds to be to the force of gravity, as 1 to 12868200, and to the centrifugal force at the equator as 1 to 44527. The elevation of the waters by this force is conſidered by Newton as an effect ſimilar to the elevation of the equatorial parts above the polar parts of the earth, ariſing from the centrifugal force at the equator; and as it is 44527 times leſs, he finds it to be $24\frac{1}{2}$ inches, or 2 feet and $\frac{1}{2}$ an inch.

To find the force of the moon upon the water, Newton compares the ſpring-tides at the mouth of the river Avon, below Briſtol, with the neap-tides there, and finds the proportion as 9 to 5; whence, after ſeveral neceſſary corrections, he concludes that the force of the moon to that of the ſun, in raiſing the waters of the ocean, is as 4·4815 to 1; ſo that the force of the moon is able of itſelf to produce an elevation of 9 feet $1\frac{3}{4}$ inch, and the ſun and moon together may produce an elevation of about 11 feet 2 inches, when at their mean diſtances from the earth, or an elevation of about $12\frac{1}{4}$ feet, when the moon is neareſt the earth. The height to which the water is found to riſe, upon coaſts of the open and deep ocean, is agreeable enough to this computation.

Dr. Horſley eſtimates the force of the moon to that of the ſun, as 5·0469 to 1, in his edit. of Newton's Princip. See the Princip. lib. 3, ſect. 3, pr. 36 and 37; alſo Maclaurin's Diſſert. de Cauſa Phyſica Fluxus et Refluxus Maris apud Phil. Nat. Princ. Math. Comment.

ment. le Seur & Jacquier, tom. 3, p. 272. And other calculators make the proportion still more different.

3. It must be observed, that the spring-tides do not happen precisely at new and full moon, nor the neap-tides at the quarters, but a day or two after; because, as in other cases, so in this, the effect is not greatest or least when the immediate influence of the cause is greatest or least. As, for example, the greatest heat is not on the day of the solstice, when the immediate action of the sun is greatest, but some time after; so likewise, if the actions of the sun and moon should suddenly cease, yet the Tides would continue to have their course for some time; and like also as the waves of the sea continue after a storm.

4. The different distances of the moon from the earth produce a sensible variation in the Tides. When the moon approaches toward the earth, her action on every part increases, and the differences of that action, on which the Tides depend, also increase; and as the moon approaches, her action on the nearest parts increases more quickly than that on the remote parts, so that the Tides increase in a higher proportion as the moon's distances decrease. In fact, it is shewn by Newton, that the Tides increase in proportion as the cubes of the distances decrease; so that the moon at half her distance would produce a Tide 8 times greater.

The moon describes an oval about the earth, and at her nearest distance produces a Tide sensibly greater than at her greatest distance from the earth: and hence it is that two great spring-tides never succeed each other immediately; for if the moon be at her least distance from the earth at the change, she must be at her greatest distance at the full, having made half a revolution in the intervening time, and therefore the spring-tide then will be much less than that at the last change was; and for the same reason, if a great spring-tide happen at the time of full moon, the Tide at the ensuing change will be less.

5. The spring-tides are highest, and the neap-tides lowest, about the time of the equinoxes, or the latter end of March and September; and, on the contrary, the spring-tides are the lowest, and the neap-tides the highest, at the solstices, or about the latter end of June and December: so that the difference between the spring and neap Tides, is much more considerable about the equinoctial than the solstitial seasons of the year. To illustrate and evince the truth of this observation, let us consider the effect of the luminaries upon the Tides, when in and out of the plane of the equator. Now it is manifest, that if either the sun or moon were in the pole, they could not have any effect on the Tides; for their action would raise all the water at the equator, or at any parallel, quite around, to a uniform height; and therefore any place of the earth, in describing its parallel to the equator, would not meet, in its course, with any part of the water more elevated than another; so that there could be no Tide in any place, that is, no alteration in the height of the waters.

On the other hand, the effect of the sun or moon is greatest when in the equinoctial; for then the axis of the spheroidal figure, arising from their action, moves in the greatest circle, and the water is put into the greatest agitation; and hence it is that the spring-tides produced when the sun and moon are both in the equinoctial, are the greatest of any, and the neaptides the least of any about that time. And when the luminary is any where between the equinoctial and the pole, the Tides are the smaller.

6. The highest spring tides are after the autumnal and before the vernal equinox: the reason of which is, because the sun is nearer the earth in winter than in summer.

7. Since the greatest of the two Tides happening in every diurnal revolution of the moon, is that in which the moon is nearest the zenith, or nadyr: for this reason, while the sun is in the northern signs, the greater of the two diurnal Tides in our climates, is that arising from the moon above the horizon; when the sun is in the southern signs, the greatest is that arising from the moon below the horizon. Thus it is found by observation that the evening Tides in the summer exceed the morning Tides, and in winter the morning Tides exceed the evening Tides. The difference is found at Bristol to amount to 15 inches, and at Plymouth to 12. It would be still greater, but that a fluid always retains an impressed motion for some time; so that the preceding Tides affect always those that follow them. Upon the whole, while the moon has a north declination, the greatest Tides in the northern hemisphere are when she is above the horizon, and the reverse while her declination is south.

8. Such would the Tides regularly be, if the earth were all over covered with the sea very deep, so that the water might freely follow the influence of the sun and moon; but, by reason of the shoalness of some places, and the narrowness of the straits in others, through which the Tides are propagated, there arises a great diversity in the effect according to the various circumstances of the places. Thus, a very slow and imperceptible motion of the whole body of water, where it is very deep, as 2 miles for instance, will suffice to raise its surface 10 or 12 feet in a Tide's time: whereas, if the same quantity of water were to be conveyed through a channel of 40 fathoms deep, it would require a very rapid stream to effect it in so large inlets as are the English channel, and the German ocean; whence the Tide is found to set strongest in those places where the sea grows narrowest, the same quantity of water being in that case to pass through a smaller passage. This is particularly observable in the straits between Portland and Cape la Hogue in Normandy, where the Tide runs like a sluice: and would be yet more so between Dover and Calais, if the Tide coming round the island did not check it.

This force, when once impressed, continues to carry the water above the ordinary height in the ocean, especially where the water meets a direct obstacle, as it does in St. Maloes; and where it enters into a long channel which, running far into the land, grows very strait at its extremity, as it does into the Severn sea at Chepstow and Bristol.

This shoalness of the sea, and the intercurrent continents, are the reasons that in the open ocean the Tides rise but to very small heights in proportion to what they do in wide-mouthed rivers, opening in the direction

tion of the ſtream of the Tide; and that high water is not ſoon after the moon's appulſe to the meridian, but ſome hours after it, as it is obſerved upon all the weſtern coaſt of Europe and Africa, from Ireland to the Cape of Good Hope; in all which a ſouth-weſt moon makes high water; and the ſame it is ſaid is the caſe on the weſtern ſide of America. So that Tides happen to different places at all diſtances of the moon from the meridian, and conſequently at all hours of the day.

To allow the Tides their full motion, the ocean in which they are produced, ought to be extended from eaſt to weſt 90 degrees at leaſt; becauſe that is the diſtance between the places where the water is moſt raiſed and depreſſed by the moon. Hence it appears that it is only in the great oceans that ſuch Tides can be produced, and why in the larger Pacific ocean they exceed thoſe in the Atlantic ocean. Hence alſo it is obvious, why the Tides are not ſo great in the torrid zone, between Africa and America, where the ocean is narrower, as in the temperate zones on either ſide; and hence we may alſo underſtand why the Tides are ſo ſmall in iſlands that are very far diſtant from the ſhores. It is farther manifeſt that, in the Atlantic ocean, the water cannot riſe on one ſhore but by deſcending on the other; ſo that at the intermediate iſlands it muſt continue at a mean height between its elevations on thoſe two ſhores. But when Tides paſs over ſhoals, and through ſtraits into bays of the ſea, their motion becomes more various, and their height depends on many circumſtances.

To be more particular. The Tide that is produced on the weſtern coaſts of Europe, in the Atlantic, correſponds to the ſituation of the moon already deſcribed. Thus it is high water on the weſtern coaſts of Ireland, Portugal and Spain, about the 3d hour after the moon has paſſed the meridian: from thence it flows into the adjacent channels, as it finds the eaſieſt paſſage. One current from it, for inſtance, runs up by the ſouth of England, and another comes in by the north of Scotland; they take a conſiderable time to move all this way, making always high water ſooner in the places to which they firſt come; and it begins to fall at theſe places while the currents are ſtill going on to others that are farther diſtant in their courſe. As they return, they are not able to raiſe the Tide, becauſe the water runs faſter off than it returns, till, by a new Tide propagated from the open ocean, the return of the current is ſtopped, and the water begins to riſe again. The Tide propagated by the moon in the German ocean, when ſhe is 3 hours paſt the meridian, takes 12 hours to come from thence to London bridge; ſo that when it is high water there, a new Tide is already come to its height in the ocean; and in ſome intermediate place it muſt be low water at the ſame time. Conſequently when the moon has north declination, and we ſhould expect the Tide at London to be the greateſt when the moon is above the horizon, we find it is leaſt; and the contrary when ſhe has ſouth declination.

At ſeveral places it is high water 3 hours before the moon comes to the meridian; but that Tide, which the moon puſhes as it were before her, is only the Tide oppoſite to that which was raiſed by her when ſhe was 9 hours paſt the oppoſite meridian.

It would be endleſs to recount all the particular ſolutions, which are eaſy conſequences from this doctrine: as, why the lakes and ſeas, ſuch as the Caſpian ſea and the Mediterranean ſea, the Black ſea and the Baltic, have little or no ſenſible Tides: for lakes are uſually ſo ſmall, that when the moon is vertical ſhe attracts every part of them alike, ſo that no part of the water can be raiſed higher than another: and having no communication with the ocean, it can neither increaſe nor diminiſh their water, to make it riſe and fall; and ſeas that communicate by ſuch narrow inlets, and are of ſo immenſe an extent, cannot ſpeedily receive and empty water enough to raiſe or ſink their ſurface any thing ſenſibly.

In general; when the time of high water at any place is mentioned, it is to be underſtood on the days of new and full moons.—Among pilots, it is cuſtomary to reckon the time of flood, or high water, by the point of the compaſs the moon bears on, at that time, allowing $\frac{3}{4}$ of an hour for each point. Thus, on the full and change days, in places where it is flood at noon, the Tide is ſaid to flow north and ſouth, or at 12 o'clock: in other places, on the ſame days, where the moon bears 1, 2, 3, 4, or more points to the eaſt or weſt of the meridian, when it is high water, the Tide is ſaid to flow on ſuch point; thus, if the moon bears SE, at flood, it is ſaid to flow SE and NW, or 3 hours before the meridian, that is, at 9 o'clock; if it bears SW, it flows SW and NE, or at 3 hours after the meridian; and in like manner for the other points of the moon's bearing.

The times of high water in any place fall about the ſame hours after a period of about 15 days, or between one ſpring Tide and another; but during that period, the times of high water fall each day later by about 48 minutes.

On the ſubject of this article, ſee Newton Princ. Math. lib. 3, prop. 24, and De Syſtem. Mundi ſect. 38, &c. Apud Opera edit. Horſley, tom. 3, pa. 52 &c. p. 203 &c. Maclaurin's Account of Newton's Diſcoveries, book 4, ch. 7. Ferguſon's Aſtron. ch. 17. Robertſon's Navig. book 6, ſect. 7, 8, 9. Lalande's Aſtron. vol. 4.

Tide *Dial*, an inſtrument contrived by Mr. Ferguſon, for exhibiting and determining the ſtate of the Tides. For the conſtruction and uſe of which ſee his Aſtron. p. 297.

Tide *Tables*, are tables commonly exhibiting the times of high water at ſundry places, as they fall on the days of the full and change of the moon, and ſometimes the height of them alſo. Theſe are common in moſt books on Navigation, particularly Robertſon's, and the 2d ed. of Tables requiſite to be uſed with the Nautical Almanac. See one at High-*water*.

TIERCE, or Teirce, a liquid meaſure, as of wine, oil, &c, containing 42 gallons, or the 3d part of a pipe; whence its name.

TIME, a ſucceſſion of phenomena in the univerſe; or a mode of duration, marked by certain periods and meaſures; chiefly indeed by the motion

and

and revolution of the luminaries, and particularly of the ſun.

The idea of Time in general, Locke obſerves, we acquire by conſidering any part of infinite duration, as ſet out by periodical meaſures: the idea of any particular Time, or length of duration, as a day, an hour, &c, we acquire firſt by obſerving certain appearances at regular and ſeemingly equidiſtant periods. Now, by being able to repeat theſe lengths or meaſures of Time as often as we will, we can imagine duration, where nothing really endures or exiſts; and thus we imagine tomorrow, or next year, &c.

Some of the later ſchool-philoſophers define Time to be the duration of a thing whoſe exiſtence is neither without beginning nor end: by this, Time is diſtinguiſhed from eternity.

Ariſtotle and the Peripatetics define it, *numerus motus ſecundum prius & poſterius*, or a multitude of tranſient parts of motion, ſucceeding each other, in a continual flux, in the relation of priority and poſteriority. Hence it ſhould follow that Time is motion itſelf, or at leaſt the duration of motion, conſidered as having ſeveral parts, ſome of which are continually ſucceeding to others. But on this principle, Time or temporal duration would not agree to bodies at reſt, which yet nobody will deny to exiſt in Time, or to endure for a Time.

To avoid this inconvenience, the Epicureans and Corpuſcularians made Time to be a ſort of flux different from motion, conſiſting of infinite parts, continually and immediately ſucceeding each other, and this from eternity to eternity. But others directly explode this notion, as eſtabliſhing an eternal being, independent of God. For how ſhould there be a flux before any thing exiſted to flow? and what ſhould that flux be, a ſubſtance, or an accident? According to the philoſophic poet,

"Time of itſelf is nothing, but from thought
Receives its riſe; by labouring fancy wrought
From things conſider'd, whilſt we think on ſome
As preſent, ſome as paſt, or yet to come.
No thought can think on Time, that's ſtill confeſt,
But thinks on things in motion or at reſt."

And ſo on. Vide Lucretius, book i.

Time may be diſtinguiſhed, like place, into *abſolute* and *relative*.

Abſolute Time, is Time conſidered in itſelf, and without any relation to bodies, or their motions.

Relative or *Apparent* Time, is the ſenſible meaſure of any duration by means of motion.

Some authors diſtinguiſh Time into *aſtronomical* and *civil*.

Aſtronomical Time, is that which is taken purely from the motion of the heavenly bodies, without any other regard.

Civil Time, is the former Time accommodated to civil uſes, and formed or diſtinguiſhed into years, months, days, &c.

Time makes the ſubject of chronology.

Time, in muſic, is an affection of ſound, by which it is ſaid to be long or ſhort, with regard to its continuance in the ſame tone or degree of tune.

Muſical Time is diſtinguiſhed into *common* or *duple* Time, and *triple* Time.

Double, *duple*, or *common Time*, is when the notes are in a duple duration of each other, viz, a ſemibreve equal to 2 minims, a minim to 2 crotchets, a crotchet to 2 quavers, &c.

Common or double Time is of two kinds. The firſt when every bar or meaſure is equal to a ſemibreve, or its value in any combination of notes of a leſs quantity. The ſecond is where every bar is equal to a minim, or its value in leſs notes. The movements of this kind of meaſure are various, but there are three common diſtinctions; the firſt *ſlow*, denoted at the beginning of the line by the mark C; the 2d *briſk*, marked thus ¢; and the 3d *very briſk*, thus marked Ↄ.

Triple Time is when the durations of the notes are triple of each other, that is, when the ſemibreve is equal to 3 minims, the minim to 3 crotchets, &c. and it is marked T.

Time-*keepers*, in a general ſenſe, denote inſtruments adapted for meaſuring time. See Chronometer.

In a more peculiar and definite ſenſe, Time-keeper is a term firſt applied by Mr. John Harriſon to his watches, conſtructed and uſed for determining the longitude at ſea, and for which he received, at different times, the parliamentary reward of 20 thouſand pounds. And ſeveral other artiſts have ſince received alſo conſiderable ſums for their improvements of Time-keepers; as Arnold, Mudge, &c. See Longitude.

This appellation is now become common among artiſts, to diſtinguiſh ſuch watches as are made with extraordinary care and accuracy for nautical or aſtronomical obſervations.

The principles of Mr. Harriſon's Time-keeper, as they were communicated by himſelf, to the commiſſioners appointed to receive and publiſh the ſame in the year 1765, are as below:

"In this Time-keeper there is the greateſt care taken to avoid friction, as much as can be, by the wheel moving on ſmall pivots, and in ruby-holes, and high numbers in the wheels and pinions.

"The part which meaſures time goes but the eighth part of a minute without winding up; ſo that part is very ſimple, as this winding-up is performed at the wheel next to the balance-wheel; by which means there is always an equal force acting at that wheel, and all the reſt of the work has no more to do in the meaſuring of time than the perſon that winds up once a day.

"There is a ſpring in the inſide of the fuſee, which I will call a ſecondary main ſpring. This ſpring is always kept ſtretched to a certain tenſion by the main ſpring; and during the time of winding-up the Time-keeper, at which time the main-ſpring is not ſuffered to act, this ſecondary-ſpring ſupplies its place.

"In common watches in general, the wheels have about one-third the dominion over the balance, that the balance-ſpring has; that is, if the power which the balance-ſpring has over the balance be called three, that

that from the wheel is one: but in this my Time-keeper, the wheels have only about one-eightieth part of the power over the balance that the balance ſpring has; and it muſt be allowed, the leſs the wheels have to do with the balance, the better. The wheels in a common watch having this great dominion over the balance, they can, when the watch is wound up, and the balance at reſt, ſet the watch a-going; but when my Time-keeper's balance is at reſt, and the ſpring is wound up, the force of the wheels can no more ſet it a-going, than the wheels of a common regulator can, when the weight is wound-up, ſet the pendulum a-vibrating; nor will the force from the wheels move the balance when at reſt, to a greater angle in proportion to the vibration that it is to fetch, than the force of the wheels of a common regulator can move the pendulum from the perpendicular, when it is at reſt.

"My Time-keeper's balance is more than three times the weight of a large ſized common watch balance, and three times its diameter; and a common watch balance goes through about ſix inches of ſpace in a ſecond, but mine goes through about twenty-four inches in that time: ſo that had my Time-keeper only theſe advantages over a common watch, a good performance might be expected from it. But my Time-keeper is not affected by the different degrees of heat and cold, nor agitation of the ſhip; and the force from the wheels is applied to the balance in ſuch a manner, together with the ſhape of the balance-ſpring, and (if I may be allowed the term) an artificial cycloid, which acts at this ſpring; ſo that from theſe contrivances, let the balance vibrate more or leſs, all its vibrations are performed in the ſame time; and therefore if it go at all, it muſt go true. So that it is plain from this, that ſuch a Time-keeper goes entirely from principle, and not from chance."

We muſt refer thoſe who may deſire to ſee a minute account of the conſtruction of Mr. Harriſon's Time-keeper, to the publication by order of the commiſſioners of longitude.

We ſhall here ſubjoin a ſhort view of the improvements in Mr. Harriſon's watch, from the account preſented to the board of longitude by Mr. Ludlam, one of the gentlemen to whom, by order of the commiſſioners, Mr. Harriſon diſcovered and explained the principle upon which his Time-keeper is conſtructed. The defects in common watches which Mr. Harriſon propoſes to remedy, are chiefly theſe: 1. That the main ſpring acts not conſtantly with the ſame force upon the wheels, and through them upon the balance: 2. That the balance, either urged with an unequal force, or meeting with a different reſiſtance from the air, or the oil, or the friction, vibrates through a greater or leſs arch: 3. That theſe unequal vibrations are not performed in equal times: and, 4. That the force of the balance-ſpring is altered by a change of heat.

To remedy the firſt defect, Mr. Harriſon has contrived that his watch ſhall be moved by a very tender ſpring, which never unrolls itſelf more than one-eighth part of a turn, and acts upon the balance through one wheel only. But ſuch a ſpring cannot keep the watch in motion a long time. He has, therefore, joined another, whoſe office is to wind up the firſt ſpring eight times in every minute, and which is itſelf wound up but once a day. To remedy the ſecond defect, he uſes a much ſtronger balance ſpring than in a common watch. For if the force of this ſpring upon the balance remains the ſame, whilſt the force of the other varies, the errors ariſing from that variation will be the leſs, as the fixed force is the greater. But a ſtronger ſpring will require either a heavier or a larger balance. A heavier balance would have a greater friction. Mr. Harriſon, therefore, increaſes the diameter of it. In a common watch it is under an inch, but in Mr. Harriſon's two inches and two tenths. However, the methods already deſcribed only leſſening the errors, and not removing them, Mr. Harriſon uſes two ways to make the times of the vibrations equal, though the arches may be unequal: one is to place a pin, ſo that the balance-ſpring preſſing againſt it, has its force increaſed, but increaſed leſs when the variations are larger: the other to give the pallets ſuch a ſhape, that the wheels preſs them with leſs advantage, when the vibrations are larger. To remedy the laſt defect, Mr. Harriſon uſes a bar compounded of two thin plates of braſs and ſteel, about two inches in length, riveted in ſeveral places together, faſtened at one end and having two pins at the other, between which the balance ſpring paſſes. If this bar be ſtraight in temperate weather (braſs changing its length by heat more than ſteel) the braſs ſide becomes convex when it is heated, and the ſteel ſide when it is cold: and thus the pins lay hold of a different part of the ſpring in different degrees of heat, and lengthen or ſhorten it as the regulator does in a common watch.

The principles, on which Mr. Arnold's Time-keeper is conſtructed, are theſe: The balance is unconnected with the wheel work, except at the time it receives the impulſe to make it continue its motion, which is only whilſt it vibrates 10° out of 380° which is the whole vibration; and during this ſmall interval it has little or no friction, but what is on the pivots, which work in ruby holes on diamonds. It has but one pallet, which is a plane ſurface formed out of a ruby, and has no oil on it. Watches of this conſtruction, ſays Mr. Lyons, go whilſt they are wound up; they keep the ſame rate of going in every poſition, and are not affected by the different forces of the ſpring; and the compenſation for heat and cold is abſolutely adjuſtable. Phipps's Voyage to the North Pole, p. 230. See LONGITUDE.

TISRI, or TIZRI, in chronology, the firſt Hebrew month of the civil year, and the 7th of the eccleſiaſtical or ſacred year. It anſwered to part of our September and October.

TOD *of wool*, is mentioned in the ſtatute 12 Carol. II. c. 32, as a weight containing 2 ſtone, or 28 pounds.

TOISE, a French meaſure, containing 6 of their feet, ſimilar to our fathom.

TONDIN, or TANDINO, in Architecture. See TORE.

TONE, or TUNE, in Muſic, a property of ſound, by which it comes under the relation of grave and acute; or the degree of elevation any ſound has, from the degree of ſwiftneſs of the vibrations of the parts of the ſonorous body.

For the caufe, meafure, degree, difference, &c, of Tones, fee TUNE.

The word Tone is taken in four different fenfes among the ancients. 1, For any found. 2, For a certain interval; as when it is faid the difference between the diapente and diateffaron is a Tone. 3, For a certain locus or compafs of the voice; in which fenfe they ufed the Dorian, Phrygian, Lydian Tones. 4. For tenfion; as when they fpeak of an acute, a grave, or a middle Tone. Wallis's Append. Ptolom. Harm. p. 172.

TONE is more particularly ufed, in mufic, for a certain degree or interval of tune, by which a found may be either raifed or lowered from one extreme of a concord to the other, fo as ftill to produce true melody.

In tempered fcales of mufic, the Tones are made equal, but in a true and accurate practice of finging they are not fo. Pepufch, in Philof. Tranf. No. 481, p. 274.

Befide the concords, or harmonical intervals, muficians admit three lefs kinds of intervals, which are the meafures and component parts of the greater, and are called *degrees*.

Of thefe degrees, two are called Tones, and the third a femitone. Their ratios in numbers are 8 to 9, called a *greater Tone*; 9 to 10, called a *leffer Tone*; and 15 to 16, a *femitone*.

The Tones arife out of the fimple concords, and are equal to their differences. Thus the greater Tone, 8 : 9, is the difference of a 5th and a 4th; the lefs Tone 9 : 10, the difference of a lefs 3d and a 4th, or of a 5th and a greater 6th; and the femitone 15 : 16, the difference of a greater 3d and a 4th.

Of thefe Tones and femitones every concord is compounded, and confequently every one is refolvable into a certain number of them. Thus the lefs 3d confifts of one greater Tone and one femitone: the greater 3d, of one greater Tone and one lefs Tone: the 4th, of one greater Tone, one lefs Tone, and one femitone: and the 5th, of two greater Tones, one lefs Tone, and one femitone.

TONSTALL (CUTHBERT), a learned Englifh divine and mathematician, was born in the year 1476. He entered a ftudent at the univerfity of Oxford about the year 1491; but afterwards, being driven from thence by the plague, he went to Cambridge, and fhortly after to the univerfity of Padua in Italy, which was then in a flourifhing ftate of literature, where his genius and learning acquired him great refpect from every one, particularly for his knowledge in mathematics, philofophy, and jurifprudence.

Upon his return home, he met with great favours from the government, obtaining feveral church preferments, and the office of fecretary to the cabinet of the king, Henry the 8th. This prince, having alfo employed him on feveral foreign embaffies, was fo well fatisfied with his conduct, that he firft gave him the bifhopric of London in 1522, and afterwards that of Durham in 1530.

Tonftall approved at firft of the diffolution of the marriage of his benefactor with Catherine of Spain, and even wrote a book in favour of that diffolution; but he afterwards condemned that work, and experienced a great reverfe of fortune. He was ejected from the fee of Durham for his religion in the time of Edward the 6th, to which however he was reftored again by queen Mary in the beginning of her reign, but was again expelled in 1559 when queen Elizabeth was fettled in her throne, and he died in a prifon a few months after, in the 84th year of his age.

Tonftall was doubtlefs one of the moft learned men of his time. "He was, fays Wood, a very good Grecian and Ebritian, an eloquent rhetorician, a fkilful mathematician, a noted civilian and canonift, and a profound divine. But that which maketh for his greateft commendation, is, that Erafmus was his friend, and he a faft friend to Erafmus, in an epiftle to whom from Sir Thomas More, I find this character of Tonftall, that, "As there was no man more adorned with knowledge and good literature, no man more fevere and of greater integrity for his life and manners; fo there was no man a more fweet and pleafant companion, with whom a man would rather choofe to converfe."

His writings that were publifhed, were chiefly the following:

1. *In Laudem Matrimonii*, Lond. 1518, 4to.—But that for which he is chiefly entitled to a place in this work, was his book upon arithmetic, viz,

2. *De Arte Supputandi*, Lond 1522, 4to, dedicated to Sir Thomas More. This was afterwards feveral times printed abroad.

3. A Sermon on Palm Sunday before king Henry the 8th, &c. Lond. 1539 and 1633, 4to.

4. *De Veritate Corporis & Sanguinis Domini in Eucharistia.* Lutet. 1554, 4to.

5. *Compendium in decem Libros Ethicorum Ariftotelis.* Par. 1554, in 8vo.

6. *Contra impios Blafphematores Dei prædeftinationis opera.* Antw. 1555, 4to.

7. Godly and devout Prayers in Englifh and Latin. 1558, in 8vo.

TOPOGRAPHY, is a defcription or draught of fome particular place, or fmall tract of land; as that of a city or town, manor or tenement, field, garden, houfe, caftle, or the like; fuch as furveyors fet out in their plots, or make draughts of, for the information and fatisfaction of the proprietors.

Topography differs from Chorography, as a particular from a more general.

TORNADO, a fudden and violent guft of wind arifing fuddenly from the fhore, and afterwards veering round all points of the compafs like a hurricane; very frequent on the coaft of Guinea.

TORRENT, in Hydrography, a temporary ftream of water, falling fuddenly from mountains, &c, where there have been great rains, or an extraordinary thaw of fnow; fometimes making great ravages in the plains.

TORRICELLI (EVANGELISTE), an illuftrious mathematician and philofopher of Italy, was born at Faenza in 1608, and trained up in Greek and Latin literature by an uncle, who was a monk. Natural inclination led him to cultivate mathematical knowledge, which he purfued fome time without a mafter; but at about 20 years of age, he went to Rome, where he

continued

continued the purfuit of it under father Benedict Caftelli. Caftelli had been a fcholar of the great Galileo, and had been appointed by the pope profeffor of mathematics at Rome. Torricelli made fuch progrefs under this mafter, that having read Galileo's *Dialogues*, he compofed a *Treatife concerning motion* upon his principles. Caftelli, furprifed at the performance, carried it and read it to Galileo, who heard it with great pleafure, and conceived a high efteem and friendfhip for the author. Upon this, Caftelli propofed to Galileo, that Torricelli fhould come and live with him; recommending him as the moft proper perfon he could have, fince he was the moft capable of comprehending thofe fublime fpeculations, which his own great age, infirmities, and want of fight, prevented him from giving to the world. Galileo accepted the propofal, and Torricelli the employment, as things of all others the moft advantageous to both. Galileo was at Florence, at which place Torricelli arrived in 1641, and began to take down what Galileo dictated, to regulate his papers, and to act in every refpect according to his directions. But he did not long enjoy the advantages of this fituation, as Galileo died at the end of only three months.

Torricelli was then about returning to Rome; but the Grand Duke engaged him to continue at Florence, making him his own mathematician for the prefent, and promifing him the profeffor's chair as foon as it fhould be vacant.

Here he applied himfelf intenfely to the ftudy of mathematics, phyfics, and aftronomy, making many improvements and fome difcoveries. Among others, he greatly improved the art of making microfcopes and telefcopes; and it is generally acknowledged that he firft found out the method of afcertaining the weight of the atmofphere by a proportionate column of quickfilver, the barometer being called from him the *Torricellian tube*, and *Torricellian experiment*. In fhort, great things were expected from him, and great things would probably have been farther performed by him, if he had lived: but he died, after a few days illnefs, in 1647, when he was but juft entered the 40th year of his age.

Torricelli publifhed at Florence in 1644, a volume of ingenious pieces, intitled, *Opera Geometrica*, in 4to. There was alfo publifhed at the fame place, in 1715, *Lezzioni Accademiche*, confifting of 96 pages in 4to. Thefe are difcourfes that had been pronounced by him upon different occafions. The firft of them was to the academy of La Crufca, by way of thanks for admitting him into their body. The reft are upon fubjects of mathematics and phyfics. Prefixed to the whole is a long life of Torricelli by Thomas Buonaventuri, a Florentine gentleman.

TORRICELLIAN, a term very frequent among phyfical writers, ufed in the phrafes, *Torricellian tube*, or *Torricellian experiment*, on account of the inventor Torricelli, a difciple of the great Galileo.

Torricellian *Tube*, is the barometer tube, being a glafs tube, open at one end, and hermetically fealed at the other, about 3 feet long, and $\frac{2}{10}$ of an inch in diameter.

Torricellian *Experiment*, or the filling the barometer tube, is performed by filling the Torricellian tube with mercury, then ftopping the open orifice with the finger, inverting the tube, and plunging that orifice into a veffel of ftagnant mercury. This done, the finger is removed, and the tube fuftained perpendicular to the furface of the mercury in the veffel.

The confequence is, that part of the mercury falls out of the tube into the veffel, and there remains only enough in the tube to fill about 30 inches of its capacity, above the furface of the ftagnant mercury in the veffel; thefe being fuftained in the tube by the preffure of the atmofphere on the furface of the ftagnant mercury; and according as the atmofphere is more or lefs heavy, or as the winds, blowing upward or downward, heave up or deprefs the air, and fo increafe or diminifh its weight and fpring, more or lefs mercury is fuftained, from 28 to 31 inches.

The Torricellian Experiment conftitutes what we now call the *Barometer*.

Torricellian *Vacuum*, is the vacuum produced by filling a tube with mercury, and when inverted allowing it to defcend to fuch a height as is counterbalanced by the preffure of the atmofphere, as in the Torricellian Experiment and Barometer, the vacuum being that part of the tube above the furface of the mercury.

TORRID *Zone*, is that round the middle of the earth, extending to $23\frac{1}{2}$ degrees on both fides of the equator.

TORUS, or Tore, in Architecture, is a large round moulding in the bafes of the columns.

TOUCAN, or *American Goofe*, is one of the modern conftellations of the fouthern hemifphere, confifting of 9 fmall ftars.

TRACTION, or *Drawing*, is the act of a moving power, by which the moveable is brought nearer to the mover, called alfo attraction.

TRACTRIX, in Geometry, a curve line called alfo Catenaria; which fee.

TRAJECTORY, a term often ufed generally for the path of any body moving either in a void, or in a medium that refifts its motion; or even for any curve paffing through a given number of points. Thus Newton, Princip. lib. 1, prob. 22, propofes to defcribe a Trajectory that fhall pafs through five given points.

Trajectory *of a Comet*, is its path or orbit, or the line it defcribes in its motion. This path, Hevelius, in his Cometographia, will have to be very nearly a right line; but Dr. Halley concludes it to be, as it really is, a very excentric ellipfis; though its place may often be well computed on the fuppofition of its being a parabola.—Newton, in prop. 41 of his 3d book, fhews how to determine the Trajectory of a comet from three obfervations; and in his laft prop. how to correct a Trajectory graphically defcribed.

TRAMMELS, in Mechanics, an inftrument ufed by artificers for drawing ovals upon boards, &c. One part of it confifts of a crofs with two grooves at right angles; the other is a beam carrying two pins which flide in thofe grooves, and alfo the defcribing pencil. All the engines for turning ovals are conftructed on the fame principles with the Trammels: the only difference is, that in the Trammels the board is at reft, and the pen-

cil

cil moves upon it: in the turning engine, the tool, which supplies the place of the pencil, is at rest, and the board moves against it. See a demonstration of the chief properties of these instruments by Mr. Ludlam, in the Philos. Trans. vol. 70, pa. 378 &c.

TRANSACTIONS, *Philosophical*, are a collection of the principal papers and matters read before certain philosophical societies, as the Royal Society of London, and the Royal Society of Edinburgh. These Transactions contain the several discoveries and histories of nature and art, either made by the members of those societies, or communicated by them from their correspondents, with the various experiments, observations, &c, made by them, or transmitted to them, &c.

The Philos. Trans. of the Royal Society of London were set on foot in 1665, by Mr. Oldenburg, the then secretary of that Society, and were continued by him till the year 1677. They were then discontinued upon his death, till January 1678, when Dr. Grew resumed the publication of them, and continued it for the months of December 1678, and January and February 1679, after which they were intermitted till January 1683. During this last interval their want was in some measure supplied by Dr. Hook's Philosophical Collections. They were also interrupted for 3 years, from December 1687 to January 1691, beside other smaller interruptions amounting to near a year and a half more, before October 1695, since which time the Transactions have been carried on regularly to the present day, with various degrees of credit and merit.

Till the year 1752 these Transactions were published in numbers quarterly, and the printing of them was always the single act of the respective secretaries till that time; but then the society thought fit that a committee should be appointed to consider the papers read before them, and to select out of them such as they should judge most proper for publication in the future Transactions. For this purpose the members of the council for the time being, constitute a standing committee: they meet on the first Thursday of every month, and no less than seven of the members of the committee (of which number the president, or in his absence a vice president, is always to be one) are allowed to be a *quorum*, capable of acting in relation to such papers; and the question with regard to the publication of any paper, is always decided by the majority of votes taken by ballot.

They are published annually in two parts, at the expence of the society; and each fellow, or member, is entitled to receive one copy *gratis* of every part published after his admission into the society. For many years past, the collection, in two parts, has made one volume in each year; and in the year 1793 the number of the volumes was 83, being 10 less than the number of the year in the century. They were formerly much respected for the great number of excellent papers and discoveries contained in them; but within the last dozen years there has been a great falling off, and the volumes are now considered as of very inferior merit, as well as quantity.

There is also a very useful Abridgment, of those volumes of the Transactions that were published before the year 1752, when the society began to publish the Transactions on their own account. Those to the end of the year 1700 were abridged, in 3 volumes, by Mr. John Lowthorp: those from the year 1700 to 1720 were abridged, in 2 volumes, by Mr. Henry Jones: and those from 1719 to 1733 were abridged, in 2 volumes, by Mr. John Eames and Mr. John Martyn; Mr. Martyn also continued the abridgment of those from 1732 to 1744 in 2 volumes, and of those from 1744 to 1750 in 2 volumes; making in all 11 volumes, of very curious and useful matters in all the arts and sciences.

The Royal Society of Edinburgh, instituted in 1783, have also published 3 volumes of their Philosophical Transactions; which are deservedly held in the highest respect for the importance of their contents.

TRANSCENDENTAL *Quantities*, among Geometricians, are indeterminate ones; or such as cannot be expressed or fixed to any constant equation: such is a transcendental curve, or the like.

M. Leibnitz has a dissertation in the Acta Erud. Lips. in which he endeavours to shew the origin of such quantities; viz, why some problems are neither plain, solid, nor sursolid, nor of any certain degree, but do *transcend* all algebraic equations.

He also shews how it may be demonstrated without calculus, that an algebraic quadratrix for the circle or hyperbola is impossible: for if such a quadratrix could be found, it would follow, that by means of it any angle, ratio, or logarithm, might be divided in a given proportion of one right line to another, and this by one universal construction: and consequently the problem of the section of an angle, or the invention of any number of mean proportionals, would be of a certain finite degree. Whereas the different degrees of algebraic equations, and therefore the problem understood in general of any number of parts of an angle, or mean proportionals, is of an indefinite degree, and *transcends* all algebraical equations.

Others define Transcendental equations, to be such fluxional equations as do not admit of fluents in common finite algebraical equations, but as expressed by means of some curve, or by logarithms, or by infinite series; thus the expression $\dot{y} = \frac{\dot{x}}{\sqrt{aa - xx}}$ is a Transcendental equation, because the fluents cannot both be expressed in finite terms. And the equation which expresses the relation between an arc of a circle and its sine is a Transcendental equation; for Newton has demonstrated that this relation cannot be expressed by any finite algebraic equation, and therefore it can only be by an infinite or a Transcendental equation.

It is also usual to rank exponential equations among Transcendental ones; because such equations, although expressed in finite terms, have variable exponents, which cannot be expunged but by putting the equation into fluxions, or logarithms, &c. Thus, the exponential equation

equation $y = a^{x}$, gives $x \times$ log. a = log. y, or $\dot{x} \times$ log. $a = \frac{\dot{y}}{y}$.

Transcendental *Curve*, in the Higher Geometry, is ſuch a one as cannot be defined by an algebraic equation; or of which, when it is expreſſed by an equation, one of the terms is a variable quantity, or a curve line. And when ſuch curve line is a geometrical one, or one of the firſt degree or kind, then the Tranſcendental curve is ſaid to be of the ſecond degree or kind, &c.

Theſe curves are the ſame with what Des Cartes, and others after him, call mechanical curves, and which they would have excluded out of geometry; contrary however to the opinion of Newton and Leibnitz; for as much as, in the conſtruction of geometrical problems, one curve is not to be preferred to another as it is defined by a more ſimple equation, but as it is more eaſily deſcribed than that other: beſides, ſome of theſe Tranſcendental, or mechanical curves, are found of greater uſe than almoſt all the algebraical ones.

M. Leibnitz, in the Acta Erudit. Lipſ. has given a kind of Tranſcendental equations, by which theſe Tranſcendental curves are actually defined, and which are of an indefinite degree, or are not always the ſame in every point of the curve. Now whereas algebraiſts uſe to aſſume ſome general letters or numbers for the quantities ſought, in theſe Tranſcendental problems Leibnitz aſſumes general or indefinite equations for the lines ſought; thus, for example, putting x and y for the abſciſs and ordinate, the equation he uſes for a line required, is $a + bx + cy + exy + fxx + gyy$ &c $= 0$: by the help of which indefinite equation, he ſeeks for the tangent; and comparing that which reſults with the given property of tangents, he finds the value of the aſſumed letters a, b, c, &c, and thus defines the equation of the line ſought.

If the compariſon abovementioned do not ſucceed, he pronounces the line ſought not to be an algebraical, but a Tranſcendental one.

This ſuppoſed, he proceeds to find the ſpecies of Tranſcendency: for ſome Tranſcendentals depend on the general diviſion or ſection of a ratio, or upon logarithms, others upon circular arcs, &c.

Here then, beſide the ſymbols x and y, he aſſumes a third, as v, to denote the Tranſcendental quantity; and of theſe three he forms a general equation of the line ſought, from which he finds the tangent according to the differential method, which ſucceeds even in Tranſcendental quantities. This found, he compares it with the given properties of the tangents, and ſo diſcovers not only the values of a, b, c, &c, but alſo the particular nature of the Tranſcendental quantity.

Tranſcendental problems are very well managed by the method of fluxions. Thus, for the relation of a circular arc and right line, let a denote the arc, and x the verſed ſine, to the radius 1, then is a = fluent of $\frac{\dot{x}}{\sqrt{2x - xx}}$; and if the ordinate of a cycloid be y, then is $y = \sqrt{2x - xx}$ + fluent of $\frac{\dot{x}}{\sqrt{2x - xx}}$.

Thus is the analytical calculus extended to thoſe lines which have hitherto been excluded, for no other cauſe but that they were thought incapable of it.

TRANSFORMATION, in Geometry, is the changing or reducing of a figure, or of a body, into another of the ſame area, or the ſame ſolidity, but of a different form. As, to Transform or reduce a triangle to a ſquare, or a pyramid to a parallelopipedon.

Transformation *of Equations*, in Algebra, is the changing equations into others of a different form, but of equal value. This operation is often neceſſary, to prepare equations for a more eaſy ſolution, ſome of the principal caſes of which are as follow.—1. The ſigns of the roots of an equation are changed, viz, the poſitive roots into negative, and the negative roots into poſitive ones, by only changing the ſigns of the 2d, 4th, and all the other even terms of the equation. Thus, the roots of the equation

$x^4 - x^3 - 19x^2 + 49x - 30 = 0$, are $+1, +2, +3, -5$;

whereas the roots of the ſame equation having only the ſigns of the 2d and 4th terms changed, viz, of

$x^4 + x^3 - 19x^2 - 49x - 30 = 0$, are $-1, -2, -3, +5$.

2. To Transform an equation into another that ſhall have its roots greater or leſs than the roots of the propoſed equation by ſome given difference, proceed as follows. Let the propoſed equation be the cubic $x^3 - ax^2 + bx - c = 0$; and let it be required to Transform it into another, whoſe roots ſhall be leſs than the roots of this equation by ſome given difference d; if the root y of the new equation muſt be the leſs, take it $y = x - d$, and hence $x = y + d$; then inſtead of x and its powers ſubſtitute $y + d$ and its powers, and there will ariſe this new equation

$$(A)\quad \left.\begin{array}{rrrr} y^3 & + 3dy^2 & + 3d^2y & + d^3 \\ & - ay^2 & - 2ady & - ad^2 \\ & & + by & + bd \\ & & & - c \end{array}\right\} = 0,$$

whoſe roots are leſs than the roots of the former equation by the difference d. If the roots of the new equation had been required to be greater than thoſe of the old one, we muſt then have ſubſtituted $y = x + d$, or $x = y - d$, &c.

3. To take away the 2d or any other particular term out of an equation; or to Transform an equation, ſo as the new equation may want its 2d, or 3d, or 4th, &c term of the given equation $x^3 - ax^2 + bx - c = 0$, which is transformed into the equation (A) in the laſt article. Now to make any term of this equation (A) vaniſh, is only to make the coefficient of that term $= 0$, which will form an equation that will give the value of the aſſumed quantity d, ſo as to produce the deſired effect, viz, to make that term vaniſh. So, to take away the 2d term, make $3d - a = 0$, which makes the aſſumed quantity $d = \frac{1}{3}a$. To take away the 3d term, we muſt put the ſum of the coefficients of that term $= 0$, that is $3d^2 - 2ad + b = 0$, or $3d^2 - 2ad = -b$; then by reſolving this quadratic equation, there is found the aſſumed quantity $d = \frac{1}{3}a \pm \frac{1}{3}\sqrt{a^2 - 3b}$, by the ſubſtitution of which for d, the 3d term will be taken away out of the equation.

In like manner, to take away the 4th term, we muſt make the ſum of its coefficients $d^3 - ad^2 + bd - c = 0$; and

and so on for any other term whatever. And in the same manner we must also proceed when the proposed equation is not a cubic, but of any height whatever, as

$$x^n - ax^{n-1} + bx^{n-2} - cx^{n-3} \text{ \&c} = 0:$$

this is first, by substituting $y + d$ for x, to be Transformed to this new equation

$$\left.\begin{array}{l} y^n + ndy^{n-1} + \frac{1}{2}n.\overline{n-1}.d^2y^{n-2} \text{ \&c} \\ - ay^{n-1} - a.\overline{n-1}.dy^{n-2} \text{ \&c} \\ + by^{n-2} \text{ \&c} \end{array}\right\} = 0;$$

then, to take away the 2d term, we must make $nd - a = 0$, or $d = \frac{a}{n}$; to take away the 3d term, we must make $\frac{1}{2}n.\overline{n-1}.d^2 - a.\overline{n-1}d + b = 0$, or $d^2 - \frac{2a}{n}d = -\frac{2b}{n(n-1)}$; and so on.

From whence it appears that, to take away the 2d term of an equation, we must resolve a simple equation; for the 3d term, a quadratic equation; for the 4th term, a cubic equation, and so on.

4. To multiply or divide the roots of an equation by any quantity; or to Transform a given equation to another, that shall have its roots equal to any multiple or submultiple of those of the proposed equation. This is done by substituting, for x and its powers, $\frac{y}{m}$ or py, and their powers, viz, $\frac{y}{m}$ for x, to multiply the roots by m; and py for x, to divide the roots by p.

Thus, to multiply the roots by m, substituting $\frac{y}{m}$ for x in the proposed equation

$x^n - ax^{n-1} + bx^{n-2}$ &c $= 0$, and it becomes

$$\frac{y^n}{m^n} - \frac{ay^{n-1}}{m^{n-1}} + \frac{by^{n-2}}{m^{n-2}} \text{ \&c} = 0;$$

or multiply all by m^n, then is

$$y^n - amy^{n-1} + bm^2y^{n-2} - cm^3y^{n-3} \text{ \&c} = 0,$$

an equation that hath its roots equal to m times the roots of the proposed equation.

In like manner, substituting py for x, in the proposed equation, &c, it becomes

$$y^n - \frac{ay^{n-1}}{p} + \frac{by^{n-2}}{p^2} - \frac{cy^{n-3}}{p^3} \text{ \&c} = 0,$$

an equation that hath its roots equal to those of the proposed equation divided by p.

From whence it appears, that to multiply the roots of an equation by any quantity m, we must multiply its terms, beginning at the 2d term, respectively by the terms of the geometrical series, m, m^2, m^3, m^4, &c. And to divide the roots of an equation by any quantity p, that we must divide its terms, beginning at the 2d, by the corresponding terms of this series p, p^2, p^3, p^4, &c.

5. And sometimes, by these Transformations, equations are cleared of fractions, or even of surds. Thus the equation $x^3 - ax^2\sqrt{p} + bx - c\sqrt{p} = 0$, by putting $y = x\sqrt{p}$, or multiplying the terms, from the 2d, by the geometricals $\sqrt{p}$, p, $p\sqrt{p}$, is Transformed to

$$y^3 - apy^2 + bpy - cp^2 = 0.$$

6. An equation, as $x^3 - ax^2 + bx - c = 0$, may be Transformed into another, whose roots shall be the reciprocals of the roots of the given equation, by substituting $\frac{1}{y}$ for x; by which it becomes

$\frac{1}{y^3} - \frac{a}{y^2} + \frac{b}{y} - c = 0$, or, multiplying all by y^3, the same becomes $cy^3 - by^2 + ay - 1 = 0$.

On this subject, see Newton's Alg. on the Transmutation of Equations; Maclaurin's Algeb. pt. 2, chap. 3 and 4. Saunderson's Algebra, vol. 2, pa. 687, &c.

TRANSIT, in Astronomy, denotes the passage of any planet, just before or over another planet or star; or the passing of a star or planet over the meridian, or before an astronomical instrument.

Venus and Mercury, in their Transits over the sun, appear like dark specks.

Doctor Halley computed the times of a number of these visible Transits, for the last and present century, and published in the Philos. Trans. numb. 193. See also Abridg. vol. 1, pa. 427 &c. A Synopsis of these Transits is as follows, those of Mercury happening in the months of April and October, and those of Venus in May and November, both old-style; and if 11 days be added to the dates below, the sums will give the times for the new-style. First for Mercury, and then for Venus.

A Series of the Moments when Mercury is seen in Conjunction with the Sun, and within his Disc, with the Distances of the same Planet from the Sun's Centre.

In April, Old-Style.

Years.	Times of Mercury's Conjunction.			Distances from the the Sun's Centre.		
	d.	h.	min.	′	″	
1615	22	21	38*	7	20	N
1628	25	5	15*	9	35	S
1661	23	4	52*	4	27	N
1674	26	12	29	12	28	S
1707	24	12	6	1	34	N
1720	26	19	43*	15	21	S
1740	21	11	43	15	36	N
1758	24	19	20*	1	19	S
1786	22	18	57*	12	43	N
1799	26	2	34*	4	12	S

In October, Old-Style.

Years.	Times of Mercury's Conjunction.			Distances from the Sun's Centre.		
	d.	h.	m.	′	″	
1605	22	8	29	12	48	S
1618	25	2	4*	4	45	S
1631	27	19	37*	3	18	N
1644	30	13	11	11	21	N
1651	23	13	20	11	26	S
1664	25	6	54*	3	23	S
1677	28	0	28**	4	40	N
1690	30	18	2*	12	43	N
1697	23	18	11*	10	4	S
1710	26	11	45	2	1	S
1723	29	5	19*	6	2	N
1730	22	5	28	16	45	S
1736	30	22	53**	13	5	N
1743	24	23	2**	8	42	S
1756	26	16	36	0	38	S
1769	29	10	10	7	24	N
1776	22	10	19	15	23	S
	November					
1782	1	3	44*	15	27	N
	October					
1789	25	3	53*	7	20	S

"Those Transits which have the mark *, are but partly visible at London; but those which are marked **, are totally visible.

"Now it is to be observed, that at the ascending node of Mercury in the month of October, the diameter of the sun takes up 32′ 34″, and therefore the greatest duration of a central Transit is 5^h 29^m. But in the month of April the diameter of the sun is 31′ 54″, whence by reason of the slower motion of the planet, there arises the greatest duration 8^h 1^m. Now if Mercury approaches obliquely, these durations become shorter on account of the distance from the centre of the sun. And that the calculation may be more perfect, I have added the following Tables, in which are exhibited the half durations of these eclipses, to every minute of the distance seen from the centre of the sun. These added to or subtracted from the moment of conjunction found by the foregoing Table, give the beginning and end of the whole phenomenon."

April.

Distance in Min.	Half duration.	
′	h.	m.
0	4	0½
1	4	0
2	3	58½
3	3	56
4	3	53
5	3	48½
6	3	43
7	3	36
8	3	28
9	3	18½
10	3	7
11	2	54
12	2	38
13	2	19
14	1	55
15	1	21½
15½	0	56

October.

Distance in Min.	Half duration.	
′	h.	m.
0	2	44½
1	2	44
2	2	43
3	2	41½
4	2	39½
5	2	36½
6	2	33
7	2	28½
8	2	23
9	2	17
10	2	10
11	2	1
12	1	51
13	1	39
14	1	24
15	1	4
15½	0	50
16	0	30

Of the Visible Conjunction of Venus with the Sun.

"Though Venus is the most beautiful of all the stars, yet (says Dr. Halley) like the rest of her sex, she does not care to appear in sight without her borrowed ornaments, and her assumed splendor. For the confined laws of motion envy this spectacle to the mortals of a whole age, like the secular games of the Ancients; though it be far the most noble among all those that astronomy can pretend to shew. Now it shall be declared hereafter, that by this one observation alone, the distance of the sun from the earth may be determined with the greatest certainty which hitherto has been included within wide limits, because of the parallax which is otherwise insensible. But as to the periods, they cannot be described so accurately as those of Mercury, since Venus has been observed within the sun's disk but once since the beginning of the world, and that by our Horrox." Dr. Halley then exhibits the principles of calculating these Transits, from whence he infers that,

"After 18 years Venus returns to the sun, taking away 2^d 10^h $52\frac{1}{2}^m$, from the moment of the foregoing Transit; and the planet proceeds in a path which is 24′ 41″ more to the south than the former.

"After 235 years adding 2^d 10^h 9^m, Venus may again enter the sun, but in a more northern path by 11′ 33″. But if the foregoing year is bissextile, 3^d 10^h 9^m must be added.

"After 243 years Venus may also pass the sun, only taking away 0^h 43^m from the time of the former; but

but proceeds more southerly by 13′ 8″. Now if the foregoing year be bissextile, add 23^h 17^m.

"And in all these appulses of Venus to the sun, in the month of November, the angle of her path with the ecliptic is 9° 5′, and her horary motion within the sun is 4′ 7″. And since the semidiameter of the sun is 16′ 21″, the greatest duration of the Transit of the centre of Venus comes out 7^h 56^m.

"Then let the sun and Venus be in conjunction at the descending node in the month of May; and by the same numbers the same intervals may be computed. After 8 years let there be taken away 2^d 6^h 55′. And Venus will make her Transit in a more northern path by 19′ 58″.

"After 235 years add 2^d 8^h 18^m, or if the foregoing year be bissextile 3^d 8^h 18^m, and you will have Venus more to the South by 9′ 21″.

"Lastly, after 243 years add 0^d 1^h 23^m, or if the foregoing year be bissextile 1^d 1^h 23^m, and Venus will be found in conjunction with the sun, but in a more northerly path by 10′ 37″.

"In every Transit within the sun at this node, the angle of Venus's path with the ecliptic is 8° 28′, and her horary motion is 4′ 0″; and the semidiameter of the sun subtending 15′ 51″, the greatest duration of the central Transit comes out also 7^h 56^m, exactly the same as at the other node.

"As to the epochs, from that only ingress which Horrox observed, the sun being then just ready to set, it is concluded, that Venus was in conjunction with the sun at London in the year 1639, Nov. 24^d 6^h 37^m, and that she declined towards the south 8′ 30″. But in the month of May no mortal has seen her as yet within the sun. But from my numbers, which I judge to be not very different from the heavens, it appears that Venus for the next time will enter the sun in 1761, May 25^d 17^h 55^m, that being the middle of the eclipse, and then will be distant from his centre 4′ 15″, towards the south. Hence and from the foregoing revolutions all the phenomena of this kind will be easily exhibited for a whole millennium, as I have computed them in the following Table.

In November.

Years.	Times of Conjunction.			Distance from the Sun's Centre.		
	d.	h.	m.	′	″	
918	20	21	53	6	12	N
1161	20	21	10	6	55	S
1396	23	7	20	4	38	N
1631	26	17	29	16	11	N
1639	24	6	37	8	30	S
1874	26	16	46	3	3	N
2109	29	2	57	14	36	N
2117	26	16	3	10	5	S

In May.

Years.	Times of Conjunction.			Distance from the Sun's Centre.		
	d.	h.	m.	′	″	
1048	24	13	45	3	50	N
1283	23	8	14	5	31	S
1291	25	15	9	14	27	N
1518	25	16	32	14	52	S
1526	23	9	37	5	6	N
1761	25	17	55	4	15	S
1769	23	11	0	15	43	N
1996	28	2	13	13	36	S
2004	25	19	18	6	22	N

"As for the durations of these eclipses of Venus, they may be computed after the same manner as those of Mercury in respect of the centre. But since Venus's diameter is pretty large, and since the parallaxes also may bring a very notable difference as to time, a particular calculation must necessarily be made for every place.

"Now the diameter of Venus is so great, that while she adheres to the sun's limb almost 20 minutes of time will be elapsed, that is, when she applies directly to the sun. But when she is incident obliquely, she continues longer in the limb. Now that diameter, according to Horrox's observation, takes up 1′ 18″, when she is in conjunction with the sun at the ascending node, and 1′ 12″ at the other node.

"Now the chief use of these conjunctions is accurately to determine the sun's distance from the earth, or his parallax, which astronomers have in vain attempted to find by various other methods; for the minuteness of the angles required easily eludes the nicest instruments. But in observing the ingress of Venus into the sun, and her egress from the same, the space of time between the moments of the internal contacts, observed to a second of time, that is, to $\frac{1}{15}$ of a second or 4‴ of an arch, may be obtained by the assistance of a moderate telescope and a pendulum clock, that is consistent with itself exactly for 6 or 8 hours. Now from two such observations rightly made in proper places, the distance of the sun within a 500th part may be certainly concluded, &c." See PARALLAX.

TRANSIT *Instrument*, in Astronomy, is a telescope fixed at right angles to a horizontal axis; this axis being so supported that the line of collimation may move in the plane of the meridian.

The axis, to the middle of which the telescope is fixed, should gradually taper toward its ends, and terminate in cylinders well turned and smoothed; and a proper weight or balance is put on the tube, so that it may stand at any elevation when the axis rests on the supporters. Two upright posts of wood or stone, firmly fixed at a proper distance, are to sustain the supporters to this instrument; these supporters are two thick brass plates,

plates, having well ſmoothed angular notches in their upper ends to receive the cylindrical arms of the axis; each of the notched plates is contrived to be moveable by a ſcrew, which ſlides them upon the ſurfaces of two other plates immoveably fixed to the two upright poſts; one plate moving in a vertical direction, and the other horizontally, they adjuſt the teleſcope to the planes of the horizon and meridian; to the plane of the horizon, by a ſpirit level hung in a poſition parallel to the axis, and to the plane of the meridian in the following manner. Obſerve the times by the clock when a circumpolar ſtar, ſeen through this inſtrument, Tranſits both above and below the pole; then if the times of deſcribing the eaſtern and weſtern parts of its circuit be equal, the teleſcope is then in the plane of the meridian; otherwiſe the notched plates muſt be gently moved till the time of the ſtar's revolution is biſected by both the upper and lower Tranſits, taking care at the ſame time that the axis keeps its horizontal poſition.

When the teleſcope is thus adjuſted, a mark muſt be ſet up, or made, at a conſiderable diſtance (the greater the better) in the horizontal direction of the interſection of the croſs wires, and in a place where it can be illuminated in the night-time by a lanthorn near it, which mark, being on a fixed object, will ſerve at all times afterwards to examine the poſition of the teleſcope, by firſt adjuſting the tranverſe axis by the level.

To adjuſt a clock by the ſun's Tranſit over the meridian, note the times by the clock, when the preceding and following edges of the ſun's limb touch the croſs wires: the difference between the middle time and 12 hours, ſhews how much the mean, or clock time, is faſter and ſlower than the apparent or ſolar time, for that day; to which the equation of time being applied, it will ſhew the time of mean noon for that day, by which the clock may be adjuſted.

TRANSMISSION, in Optics, &c, denotes the property of a tranſparent or tranſlucent body, by which it admits the rays of light to paſs through its ſubſtance; in which ſenſe, the word ſtands oppoſed to reflection.

For the cauſe of Tranſmiſſion, or the reaſon why ſome bodies Tranſmit the rays, and others reflect them, ſee Transparency and Opacity.

The rays of light, Newton obſerves, are ſubject to fits of eaſy Tranſmiſſion and reflection. See Light, and Reflection.

TRANSMUTATION, or Transformation, in Geometry, denotes the reduction or change of one figure or body into another of the ſame area or ſolidity; as a triangle into a ſquare, a pyramid into a cube, &c.

Transmutation, in the Higher Geometry, has been uſed for the converting of a figure into another of the ſame kind and order, whoſe reſpective parts riſe to the ſame dimenſions in an equation, and admit the ſame tangents, &c.

If a rectilineal figure be to be Tranſmuted into another, it is ſufficient that the interſections of the lines which compoſe it be transferred, and lines drawn through the ſame in the new figure. But if the figure to be Tranſmuted be curvilinear, the points, tangents, and other right lines, by means of which the curve line is to be defined, muſt be transferred.

TRANSOM, among Builders, the piece that is framed acroſs a double-light window.

Transom, among Mathematicians, denotes the vane of a croſs-ſtaff; being a wooden member fixed acroſs it, with a ſquare upon which it ſlides, &c.

TRANSPARENCY, or Translucency, in Phyſics, a quality in certain bodies, by which they give paſſage to the rays of light.

The Tranſparency of natural bodies, as glaſs, water, air, &c, is aſcribed by ſome, to the great number and ſize of the pores or interſtices between the particles of thoſe bodies. But this account is very defective; for the moſt ſolid and opaque body in nature, that we know of, contains a great deal more pores than it does matter; ſurely a great deal more than is neceſſary for the paſſage of ſo very fine and ſubtle a body as light.

Ariſtotle, Des Cartes, &c, make Tranſparency to conſiſt in ſtraightneſs or rectilineal direction of the pores; by means of which, ſay they, the rays can make their way through, without ſtriking againſt the ſolid parts, and ſo being reflected back again. But this account, Newton ſhews, is imperfect; the quantity of pores in all bodies being ſufficient to tranſmit all the rays that fall upon them, however thoſe pores be ſituated with reſpect to each other.

The reaſon then why all bodies are not Tranſparent, is not to be aſcribed to their want of rectilineal pores; but either to the unequal denſity of the parts, or to the pores being filled with ſome foreign matters, or to their being quite empty, by means of which the rays, in paſſing through, undergoing a great variety of reflections and refractions, are perpetually diverted different ways, till at length falling on ſome of the ſolid parts of the body, they are extinguiſhed and abſorbed.

Thus cork, paper, wood, &c, are opake; while glaſs, diamonds, &c, are Tranſparent; and the reaſon is, that in the neighbourhood of parts equal in denſity with reſpect to each other, as theſe latter bodies, the attraction being equal on every ſide, no reflection or refraction enſues: but the rays which entered the firſt ſurface of the body proceed quite through it without interruption, thoſe few only excepted that chance to meet with the ſolid parts: but in the neighbourhood of parts that differ much in denſity, ſuch as the parts of wood and paper are, both in reſpect of themſelves and of the air, or the empty ſpace in their pores; as the attraction is very unequal, the reflections and refractions muſt be very great; and therefore the rays will not be able to make their way through ſuch bodies, but will be variouſly deflected, and at length quite ſtopped. See Opacity.

TRANSPOSITION, in Algebra, is the bringing any term of an equation over to the other ſide of it. Thus, if $a + x = c$, and you make $x = c - a$, then a is ſaid to be Tranſpoſed.

This operation is to be performed in order to bring all the known terms to one ſide of the equation, and all thoſe that are unknown to the other ſide of it; and every term thus Tranſpoſed muſt always have its ſign changed,

changed, from + to −, or from − to + ; which in fact is no more than subtracting or adding such term on both sides of the equation. See REDUCTION of Equations.

TRANSVERSE-*Axis*, or *Diameter*, in the Conic Sections, is the first or principal diameter, or axis. See AXIS, DIAMETER, and LATUS TRANSVERSUM.

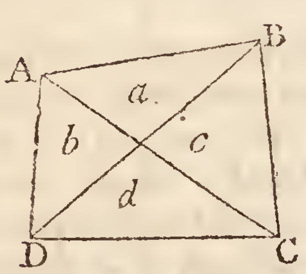
In an ellipse the Transverse is the longest of all the diameters ; but the shortest of all in the hyperbola ; and in the parabola the diameters are all equal, or at least in a ratio of equality.

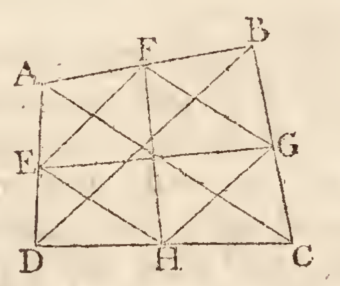
TRAPEZIUM, in Geometry, a plane figure contained under four right lines, of which both the opposite pairs are not parallel.—When this figure has two of its sides parallel to each other, it is sometimes called a *trapezoid*.—The chief properties of the Trapezium are as follow :

1. Any three sides of a Trapezium taken together, are greater than the third side.

2. The two diagonals of any Trapezium divide it into four proportional triangles, a, b, c, d. That is, the triangle $a : b :: c : d$.

3. The sum of all the four inward angles, A, B, C, D, taken together, is equal to 4 right angles, or 360°.



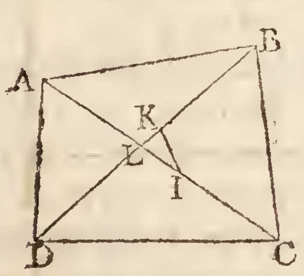


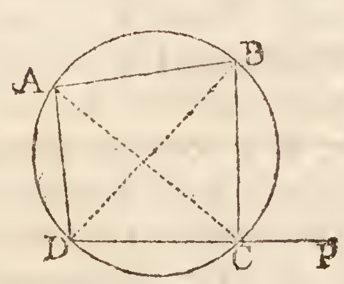
4. In a Trapezium ABCD, if all the sides be bisected, in the points E, F, G, H, the figure EFGH formed by joining the points of bisection will be a parallelogram, having its opposite sides parallel to the corresponding diagonals of the Trapezium, and the area of the said inscribed parallelogram is just equal to half the area of the Trapezium.

5. The sum of the squares of the diagonals of the Trapezium, is equal to twice the sum of the squares of the diagonals of the parallelogram, or of the two lines drawn to bisect the opposite sides of the Trapezium. That is, $AC^2 + BD^2 = 2EG^2 + 2FH^2$.

6. In any Trapezium, the sum of the squares of all the four sides, is equal to the sum of the squares of the two diagonals together with 4 times the square of the line KI joining their middle points. That is, (first fig. below)

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4IK^2.$$





7. In any Trapezium, the sum of the two diagonals, is less than the sum of any four lines that can be drawn, to the four angles, from any point within the figure, beside the intersection of the diagonals.

8. The area of any Trapezium, is equal to half the rectangle or product under either diagonal and the sum of the two perpendiculars drawn upon it from the two opposite angles.

9. The area of any Trapezium may also be found thus : Multiply the two diagonals together, then that product, multiplied by the sine of their angle of intersection, to the radius 1, will be the area. That is,

$$AC \times BD \times \text{fin.} \angle L = \text{area}.$$

10. The same area will be otherwise found thus : Square each side of a Trapezium, add the squares of each pair of opposite sides together, subtract the less sum from the greater, multiply the remainder by the tangent of the angle of intersection of the diagonals (to radius 1), and $\frac{1}{4}$ of the product will be the area. That is, $\overline{(AB^2 + DC^2 - AD^2 + BC^2)} \times \frac{1}{4}\text{tang.} \angle L =$ area.

11. The area of a Trapezoid, or one that has two sides parallel, is equal to the rectangle or product under the sum of the two parallel sides and the perpendicular distance between them.

12. If a Trapezium be inscribed in a circle ; the sum of any two opposite angles is equal to two right angles ; and if the sum of two opposite angles be equal to two right angles, the sum of the other two will also be equal to two right angles, and a circle may be described about it ; and farther, if one side, as DC, be produced out, the external angle will be equal to the interior opposite angle. That is, (last fig. above)

$$\angle A + \angle C = \angle B + \angle D = 2 \text{ right angles},$$
$$\text{and } \angle A = \angle BCP.$$

13. If a Trapezium be inscribed in a circle ; the rectangle of the two diagonals, is equal to the sum of the two rectangles contained under the opposite sides. That is,

$$AC \times BD = AB \times DC + AD \times BC.$$

14. If a Trapezium be inscribed in a circle ; its area may be found thus : Multiply any two adjacent sides together, and the other two sides together ; then add these two products together, and multiply the sum by the sine of the angle included by either of the pairs of sides that are multiplied together, and half this last product will be the area. That is, the area is equal either

$$\text{to } (AB \times AD + CB \times CD) \times \tfrac{1}{2}\text{fin.} \angle A \text{ or } \angle C,$$
$$\text{or } (AB \times BC + AD \times DC) \times \tfrac{1}{2}\text{fin.} < B \text{ or } < D.$$

15. Or, when the Trapezium can be inscribed in a circle, the area may be otherwise found thus : Add all the four sides together, and take half the sum ; then from this half subtract each side severally ; multiply the four remainders continually together, and the square root of the last product will be the area.

16. Lastly, the area of the Trapezium inscribed in a circle may be otherwise found thus :

Put

Put $m = AB \times BC + AD \times DC$,
$n = BA \times AD + BC \times CD$,
$p = AB \times DC + AD \times BC$,
r = radius of the circumfcribing circle,

then $\frac{\sqrt{mnp}}{4r}$ = the area of the Trapezium.

TRAPEZOID, fometimes denotes a trapezium that has two of its fides parallel to each other; and fometimes an irregular folid figure, having four fides not parallel to each other.

TRAVERSE, in Gunnery, is the turning a piece of ordnance about, as upon a centre, to make it point in any particular direction.

TRAVERSE, in Fortification, denotes a trench with a little parapet, fometimes two, one on each fide, to ferve as a cover from the enemy that might come in flank.

TRAVERSE, in a wet fofs, is a fort of gallery, made by throwing fauciffons, joifts, fafcines, ftones, earth, &c, into the fofs, oppofite the place where the miner is to be put, in order to fill up the ditch, and make a paffage over it.

TRAVERSE alfo denotes a wall of earth, or ftone, raifed acrofs a work, to ftop the fhot from rolling along it.

TRAVERSE alfo fometimes fignifies any retrenchment, or line fortified with fafcines, barrels or bags of earth, or gabions.

TRAVERSE, in Navigation, is the variation or alteration of a fhip's courfe, occafioned by the fhifting of the winds, or currents, &c; or a Traverfe is a compound courfe, confifting of feveral different courfes and diftances.

TRAVERSE *Sailing*, is the method of working, or calculating, Traverfes or compound courfes, fo as to bring them into one, &c.

Traverfe Sailing is ufed when a fhip, having failed from one port towards another, whofe courfe and diftance from the former is known, is by reafon of contrary winds, or other accidents, forced to fhift and fail upon feveral courfes, which are to be brought into one courfe, to learn, after fo many turnings and windings, the true courfe and diftance made good, or the true point the fhip is arrived at; and fo to know what muft be the new courfe and diftance to the intended port.

To Conftruct a Traverfe. Affume a convenient point or centre, to begin at, to reprefent the place failed from. From that point as a centre, with the chord of 60°, defcribe a circle, which quarter with two perpendicular lines interfecting in the centre, one to reprefent the meridian, or north-and-fouth line, and the other the eaft-and-weft line. From the interfections of thefe lines with the circle, fet off upon the circumference, the arcs or degrees, taken from the chords, for the feveral courfes that have been failed upon, marking the points they reach to in the circumference with the figures for the order or number of the courfes, 1, 2, 3, 4, &c; and from the centre draw lines to thefe feveral points in the circumference, or conceive them to be drawn. Upon the firft of thefe lines lay off the firft diftance failed; from the extremity of this diftance draw a line parallel to the fecond radius, or line drawn in the circle, upon which lay off the 2d diftance; through the end of this 2d diftance draw a line parallel to the 3d radius, for the direction of the 3d courfe, and upon it lay off the 3d diftance; and fo on, through all the courfes and diftances. This done, draw a line from the centre to the end of the laft diftance, which will be the whole diftance made good, and it will cut the circle in a point fhewing the courfe made good. Laftly, draw a line from the end of the laft diftance to the point reprefenting the port bound to, and it will fhew the diftance and courfe yet to be failed, to gain that port.

To work a Traverfe, or to compute it by the Traverfe Table of Difference of Latitude and Departure.

Make a little tablet with 6 columns; the 1ft for the courfes, the 2d for the diftances, the 3d for the northing, the 4th for the fouthing, the 5th for the eafting, and the 6th for the wefting; firft entering the feveral courfes and diftances, in fo many lines, in the 1ft and 2d columns. Then, from the Traverfe table, take out the quantity of the northings or fouthings, and eaftings or weftings, anfwering to the feveral given courfes and diftances, entering them on their correfponding lines, and in the proper columns of eafting, wefting, northing, and fouthing. This done, add up into one fum the numbers in each of thefe laft four columns, which will give four fums fhewing the whole quantity of eafting, wefting, northing, and fouthing made good; then take the difference between the whole eafting and wefting, and alfo between the northing and fouthing, fo fhall thefe fhew the fpaces made good in thefe two directions, viz, eaft or weft, and north or fouth; which being compared with the given difference of latitude and departure, will fhew thofe yet to be made good in failing to the defired port, and thence the courfe and diftance to it.

Example. A fhip from the latitude 28° 32′ north, bound to a port diftant 100 miles, and bearing NE by N, has run the following courfes and diftances, viz, 1ft, NW by N dift. 20 miles; 2d, SW 40 miles; 3d, NE by E 60 miles; 4th, SE 55 miles; 5th, W by S 41 miles; 6th, ENE 66 miles. Required her prefent latitude, with the direct courfe and diftance made good, and thofe for the port bound to.

The numbers being taken out of the Traverfe table, and entered oppofite the feveral courfes and diftances, the tablet will be as here follows:

Courfes.	Dift.	North.	South.	Eaft.	Weft.
NW by N	20	16·6	.	.	11·1
SW	40	.	28·3	.	28·3
NE by E	60	33·3	.	49·9	.
SE	55	.	38·9	38·9	.
W by S	41	.	8·0	.	40·2
ENE	66	25·3	.	61·0	.
		75·2	75·2	149·8	79·6
		75·2		79·6	
		0		70·2	Dep.

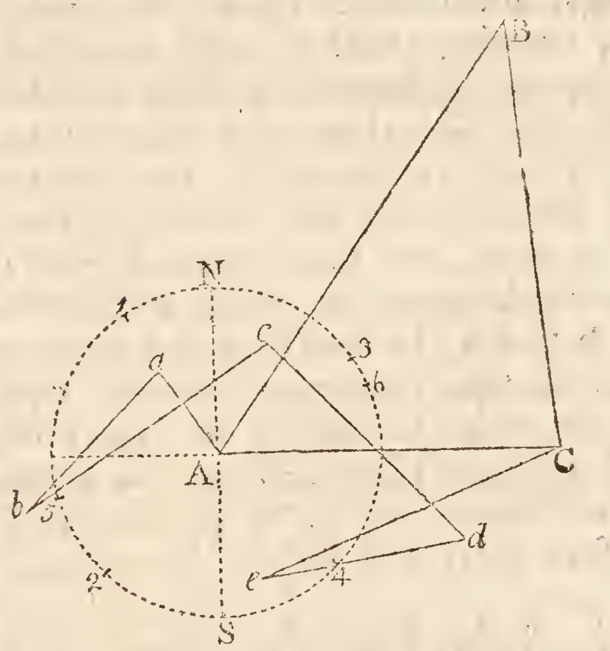
where the fums of the northings and fouthings, being both alike, 75·2, fhews that the fhip is come to the fame parallel of latitude fhe fet out from. And the difference between the fums of the eaftings and weftings, fhews that the fhip is 70·2 miles more to the eaftward, that being the greater. Confequently the courfe made good is due eaft, and the diftance is 70·2 miles.

But, by the Traverfe table, the northing and eafting to the propofed courfe NE by N, and diftance 100,

are thus, viz,	northing	83·1	and eafting	55·6
diff. from made good		0	and eafting	70·2
give - -	northing	83·1	and wefting	14·6

yet to be made good to arrive at the intended port; and therefore, by finding thefe in the Traverfe table, anfwering to them are the intended courfe and diftance, viz, diftance 85, and courfe N 10° W.

The geometrical conftruction, according to the method before defcribed, gives the figure as below: where A is the port fet out from, B is the port bound to,



C is the place come to, by failing the feveral courfes and diftances A*a*, *ab*, *bc*, *cd*, *de*, and *e*C; then CB is the diftance to be failed to arrive at the port B, and its courfe, or direction with the meridian, is nearly 10°, or the angle ACB, made with the eaft-and-weft line, nearly 80°.—*Note*, the radii from the centre to the feveral points in the circumference, are omitted, to prevent a confufion in the figure.

Traverse-*Board*, in a fhip, a fmall round board, hanging up in the fteerage, and pierced full of holes in lines fhewing the points of the compafs: upon which, by moving a fmall peg from hole to hole, the fteerfman keeps an account how many glaffes, that is half hours, the fhip fteers upon any point.

Traverse-*Table*, in Navigation, is the fame with a table of difference of latitude and departure; being the difference of latitude and departure ready calculated to every point, half point, quarter point, degree, &c, of the quadrant; and for every diftance, up to 50 or 100 or 120, &c. Though it may ferve for any greater diftance whatever, by adding two or more together; or by taking their halves, thirds, fourths, &c, and then doubling, tripling, quadrupling, &c, the difference of latitude and departure found to thofe parts of the diftance.

This table is one of the moft neceffary and ufeful things a navigator has occafion for; for by it he can readily reduce all his courfes and diftances, run in the fpace of 24 hours, into one courfe and diftance; whence he finds the latitude he is in, and the departure from the meridian.

One of the beft tables of this kind is in Robertfon's Navigation, at the end of book 7, vol. 1. The diftances are there carried to 120, for the fake of more eafy fubdivifions; and it is divided into two parts; the firft containing the courfes for every quarter point of the compafs, and the 2d adapted to every 15′, or quarter of a degree, in the quadrant. See Traverse *Sailing*.

A fpecimen of fuch a Traverfe Table is the following, otherwife called a Table of Difference of Latitude and Departure. The diftances are placed at top and bottom of the columns, from 1 to 10; but may be extended to any quantity by multiplying the parts, and taking out at feveral times. The courfes, or angles of a right-angled triangle, are in a column, on both fides, each in two parts, the one containing the even points and quarter points, and the other whole degrees, as far as to 45°, or half the quadrant, on the left-hand fide, and the other half quadrant, from 45° to 90°, returned upwards from bottom to top on the right-hand fide. The correfponding Difference of Latitude and Departure are in two columns below or above the diftances, viz, below them when the courfe or angle is within 45°, or found on the left-hand fide; but above them when between 45 and 90°, or found on the right-hand fide.

The fame table ferves alfo to work all cafes of right-angled triangles, for any other purpofes. For example, Suppofe a given courfe be 15°, and diftance 35 miles, to find the correfponding difference of latitude and the departure: Or, in a right-angled triangle, given the hypotenufe 35, and one angle 15°, to find the two legs.

Here, the diftance 3 in the table muft be accounted 30, moving the decimal point proportionally or one place in the other numbers; and thofe numbers taken out at twice, viz, once from the columns under 3 for the 30, and the other from the columns under the diftance 5. Thus, on the line of 15°, and under the

Dift.		*Lat.*		*Dep.*
30	are	28·978	and	7·765
5	are	4·830	and	1·294
theref. for 35	are	33·808	and	9·059

So that the other two legs of the triangle are 33·808 and 9·059.—If the courfe had been 75°, or the complement of the former, which is only the other angle of the fame triangle, and which is found on the fame line of the table, but on the right-hand fide of it; then the numbers in the columns will be the fame as before, and will give the fame fums for the two legs of the triangle, only with the contrary names, as to Latitude and Departure, which change places.

A TABLE *of the Difference of Latitude and Departure, for Degrees and Quarter Points.*

Course		Dist. 1		Dist. 2		Dist. 3		Dist. 4		Dist. 5		Course	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	0·9998	0·0175	1·9997	0·0349	2·9995	0·0524	3·9994	0·0698	4·9992	0·0873	89	
	2	0·9994	0·0349	1·9988	0·0698	2·9982	0·1047	3·9976	0·1396	4·9970	0·1745	88	
0 ¼		0·9988	0·0491	1·9976	0·0981	2·9964	0·1472	3·9952	0·1963	4·9940	0·2453		7 ¾
	3	0·9986	0·0523	1·9973	0·1047	2·9959	0·1570	3·9945	0·2093	4·9931	0·2617	87	
	4	0·9976	0·0698	1·9951	0·1395	2·9927	0·2093	3·9903	0·2790	4·9878	0·3488	86	
	5	0·9962	0·0872	1·9924	0·1743	2·9886	0·2615	3·9848	0·3486	4·9810	0·4358	85	
0 ½		0·9952	0·0980	1·9904	0·1960	2·9856	0·2940	3·9807	0·3921	4·9759	0·4901		7 ½
	6	0·9945	0·1045	1·9890	0·2091	2·9836	0·3136	3·9781	0·4181	4·9726	0·5226	84	
	7	0·9925	0·1219	1·9851	0·2437	2·9776	0·3656	3·9702	0·4875	4·9627	0·6093	83	
	8	0·9903	0·1392	1·9805	0·2783	2·9708	0·4175	3·9611	0·5567	4·9513	0·6959	82	
0 ¾		0·9892	0·1467	1·9784	0·2935	2·9675	0·4402	3·9567	0·5869	4·9459	0·7337		7 ¼
	9	0·9877	0·1564	1·9754	0·3129	2·9631	0·4693	3·9508	0·6257	4·9384	0·7822	81	
	10	0·9848	0·1736	1·9696	0·3473	2·9544	0·5209	3·9392	0·6946	4·9240	0·8682	80	
	11	0·9816	0·1908	1·9633	0·3816	2·9449	0·5724	3·9265	0·7632	4·9081	0·9540	79	
1		0·9808	0·1951	1·9616	0·3902	2·9424	0·5853	3·9231	0·7804	4·9039	0·9754		7
	12	0·9781	0·2079	1·9563	0·4158	2·9344	0·6237	3·9126	0·8316	4·8907	1·0396	78	
	13	0·9744	0·2250	1·9487	0·4499	2·9231	0·6749	3·8975	0·8998	4·8718	1·1248	77	
	14	0·9703	0·2419	1·9406	0·4838	2·9108	0·7258	3·8812	0·9677	4·8515	1·2096	76	
1 ¼		0·9700	0·2430	1·9401	0·4860	2·9101	0·7289	3·8801	0·9719	4·8502	1·2149		6 ¾
	15	0·9659	0·2588	1·9319	0·5176	2·8978	0·7765	3·8637	1·0353	4·8296	1·2941	75	
	16	0·9613	0·2756	1·9225	0·5513	2·8838	0·8269	3·8450	1·1025	4·8063	1·3782	74	
1 ½		0·9569	0·2903	1·9138	0·5806	2·8708	0·8209	3·8278	1·1611	4·7847	1·4514		6 ½
	17	0·9563	0·2924	1·9126	0·5847	2·8689	0·8771	3·8252	1·1695	4·7815	1·4619	73	
	18	0·9511	0·3090	1·9021	0·6180	2·8532	0·9271	3·8042	1·2361	4·7553	1·5451	72	
	19	0·9455	0·3256	1·8910	0·6511	2·8366	0·9767	3·7821	1·3023	4·7276	1·6278	71	
1 ¾		0·9415	0·3369	1·8831	0·6738	2·8246	1·0107	3·7662	1·3476	4·7077	1·6844		6 ¼
	20	0·9397	0·3420	1·8794	0·6840	2·8191	1·0261	3·7588	1·3681	4·6985	1·7101	70	
	21	0·9336	0·3584	1·8672	0·7167	2·8007	1·0751	3·7343	1·4335	4·6679	1·7918	69	
	22	0·9272	0·3746	1·8544	0·7492	2·7816	1·1238	3·7087	1·4984	4·6359	1·8730	68	
2		0·9239	0·3827	1·8478	0·7654	2· 716	1·1480	3·6955	1·5307	4·6194	1·9134		6
	23	0·9205	0·3907	1·8410	0·7815	2·7615	1·1722	3·6820	1·5629	4·6025	1·9537	67	
	24	0·9135	0·4067	1·8270	0·8135	2·7406	1·2202	3·6542	1·6269	4·5677	2·0337	66	
	25	0·9063	0·4226	1·8126	0·8452	2·7189	1·2679	3·6252	1·6905	4·5315	2·1131	65	
2 ¼		0·9040	0·4276	1 8080	0·8551	2·7120	1·2827	3·6160	1·7102	4·5199	2·13 8		5 ¼
	26	0·8988	0·4384	1·7976	0·8767	2·6964	1·3151	3·5952	1·7535	4·4940	2·1919	64	
	27	0·8910	0·4540	1·7820	0·9080	2·6730	1·3620	3·5640	1·8160	4·4550	2·2699	63	
	28	0·8829	0·4695	1·7659	0·9389	2·6488	1·4084	3·5318	1·8779	4·4147	2·3474	62	
2 ½		0·88 9	0·4714	1·7638	0·9428	2·6458	1·4142	3·5277	1·8850	4·4096	2·3570		5 ½
	29	0·8746	0·4848	1·7492	0·9696	2·6239	1·4544	3·4985	1·9392	4·3731	2·4240	61	
	30	0·8660	0·5000	1·7320	1·0000	2·5981	1·5000	3·4641	2·0000	4·3301	2·5000	60	
2 ¾		0·8577	0·5141	1·7155	1·0282	2·5732	1·5423	3·4309	2·0564	4·2886	2·5705		5 ¼
	31	0·8572	0·5150	1·7143	1·0301	2·5715	1·5451	3·4287	2·0602	4·2858	2·5752	59	
	32	0·8480	0·5299	1·6961	1·0598	2·5441	1·5896	3·3922	2·1197	4·2402	2·6496	58	
	33	0·8387	0·5446	1·6773	1·0893	2·5160	1·6339	3·3547	2·1786	4·1934	2·7232	57	
3		0·8315	0·5556	1·6629	1·1111	2·4944	1·6667	3·3259	2·2223	4·1573	2·7778		5
	34	0·8290	0·5592	1·6581	1·1184	2·4871	1·6776	3·3162	2·2368	4·1452	2·7960	56	
	35	0·8192	0·5736	1·6383	1·1472	2·4575	1·7207	3·2766	2·2943	4·0958	2·8679	55	
	36	0·8090	0·5878	1·6180	1·1756	2·4271	1·7634	3·2361	2·3511	4·0451	2·9389	54	
3 ¼		0·8032	0·5957	1·6064	1·1914	2·4096	1·7871	3·2128	2·3828	4·0160	2·9785		4 ¾
	37	0·7986	0·6018	1·5973	1·2036	2·3959	1·8054	3·1945	2·4073	3·9932	3·0091	53	
	38	0·7880	0·6157	1·5760	1·2313	2·3640	1·8470	3·1520	2·4626	3·9401	3·0783	52	
	39	0·7771	0·6293	1·5543	1·2586	2·3314	1·8880	3-1086	2·5173	3·8857	3·1466	51	
3 ½		0·7730	0·6344	1·5460	1·2688	2·3190	1·9032	3·0920	2·5376	3·8650	3·1720		4 ½
	40	0·7660	0·6428	1·5321	1·2856	2·2981	1·9284	3·0642	2·5712	3·8302	3 2139	50	
	41	0·7547	0·6561	1·5094	1·3121	2·2641	1·9682	3·0188	2·6242	3·7736	3·2803	49	
	42	0·7431	0·6691	1·4863	1·3383	2·2294	2·0074	2·9726	2·6765	3·7157	3·3457	48	
3 ¾		0·7410	0·6716	1·4819	1·3431	2·2229	2·0147	2·9638	2·6862	3·7048	3·3578		4 ¼
	43	0·7314	0·6820	1·4628	1·3640	2·1941	2·0460	2·9254	2·7280	3·6568	3·4100	47	
	44	0·7193	0·6947	1·4387	1·3894	2·1580	2·0840	2·8774	2·7786	3·5967	3·4733	46	
4	45	0·7071	0·7071	1·4142	1·4142	2·1213	2·1213	2·8284	2·8284	3·5355	3·5355	45	4
Pts.	Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.	Pts.
		Dist. 1		Dist. 2		Dist. 3		Dist. 4		Dist. 5			

TABLE *of the Difference of Latitude and Departure, for Degrees and Quarter Points.*

Course		Dist. 6		Dist. 7		Dist. 8		Dist. 9		Dist. 10		Course	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	5·9991	0·1047	6·9989	0·1222	7·9988	0·1396	8·9986	0·1571	9·9985	0·1745	89	
	2	5·9963	0·2094	6·9957	0·2443	7·9951	0·2792	8·9945	0·3141	9·9939	0·3490	88	
0 ¼		5·9928	0·2944	6·9916	0·3435	7·9904	0·3925	8·9892	0·4416	9·9880	0·4907		7 ¾
	3	5·9918	0·3140	6·9904	0·3664	7·9890	0·4187	8·9877	0·4710	9·9863	0·5234	87	
	4	5·9854	0·4185	6·9829	0·4883	7·9805	0·5580	8·9781	0·6278	9·9756	0·6976	86	
	5	5·9772	0·5229	6·9734	0·6101	7·9696	0·6972	8·9658	0·7844	9·9619	0·8716	85	
0 ½		5·9711	0·5881	6·9663	0·6861	7·9615	0·7841	8·9567	0·8822	9·9518	0·9802		7 ½
	6	5·9671	0·6272	6·9617	0·7317	7·9562	0·8362	8·9507	0·9408	9·9452	1·0453	84	
	7	5·9553	0·7312	6·9478	0·8531	7·9404	0·9750	8·9329	1·0968	9·9255	1·2187	83	
	8	5·9416	0·8350	6·9319	0·9742	7·9221	1·1134	8·9124	1·2526	9·9037	1·3917	82	
0 ¾		5·9351	0·8804	6·9242	1·0271	7·9134	1·1738	8·9026	1·3206	9·8918	1·4674		7 ¼
	9	5·9261	0·9386	6·9138	1·0950	7·9015	1·2515	8·8892	1·4079	9·8769	1·5643	81	
	10	5·9088	1·0419	6·8937	1·2155	7·8785	1·3892	8·8633	1·5628	9·8481	1·7365	80	
	11	5·8898	1·1449	6·8714	1·3357	7·8530	1·5265	8·8346	1·7173	9·8165	1·9081	79	
1		5·8847	1·1705	6·8655	1·3656	7·8463	1·5607	8·8271	1·7558	9·8079	1·9509		7
	12	5·8689	1·2475	6·8470	1·4554	7·8252	1·6633	8·8033	1·8712	9·7815	2·0791	78	
	13	5·8462	1·3497	6·8206	1·5746	7·7950	1·7996	8·7693	2·0246	9·7437	2·2495	77	
	14	5·8218	1·4515	6·7921	1·6935	7·7624	1·9354	8·7327	2·1773	9·7030	2·4192	76	
1 ¼		5·8202	1·4579	6·7902	1·7009	7·7602	1·9438	8·7303	2·1868	9·7003	2·4298		6 ¾
	15	5·7956	1·5529	6·7615	1·8117	7·7274	2·0706	8·6933	2·3294	9·6593	2·5882	75	
	16	5·7676	1·6538	6·7288	1·9295	7·6901	2·2051	8·6513	2·4807	9·6120	2·7562	74	
1 ½		5·7416	1·7417	6·6986	2·0320	7·6555	2·3223	8·6125	2·6126	9·5694	2·9028		6 ½
	17	5·7378	1·7542	6·6941	2·0466	7·6504	2·3390	8·6067	2·6313	9·5630	2·9237	73	
	18	5·7063	1·8541	6·6574	2·1631	7·6084	2·4721	8·5595	2·7812	9·5106	3·0902	72	
	19	5·6731	1·9534	6·6186	2·2790	7·5642	2·6045	8·5097	2·9301	9·4552	3·2557	71	
1 ¾		5·6493	2·0213	6·5908	2·3582	7·5324	2·6951	8·4739	3·0320	9·4154	3·3689		6 ¼
	20	5·6382	2·0521	6·5779	2·3941	7·5175	2·7362	8·4572	3·0782	9·3969	3·4202	70	
	21	5·6015	2·1502	6·5351	2·5086	7·4686	2·8669	8·4022	3·2253	9·3358	3·5837	69	
	22	5·5631	2·2476	6·4903	2·6222	7·4175	2·9969	8·3447	3·3715	9·2718	3·7461	68	
2		5·5433	2·2961	6·4672	2·6788	7·3910	3·0615	8·3149	3·4441	9·2388	3·8268		6
	23	5·5230	2·3444	6·4435	2·7351	7·3640	3·1258	8·2845	3·5166	9·2050	3·9075	67	
	24	5·4813	2·4404	6·3948	2·8472	7·3084	3·2539	8·2219	3·6606	9·1355	4·0674	66	
	25	5·4378	2·5357	6·3442	2·9583	7·2505	3·3809	8·1568	3·8036	9·0631	4·2262	65	
2 ¼		5·4239	2·5653	6·3279	2·9929	7·2319	3·4204	8·1359	3·8480	9·0399	4·2756		5 ¾
	26	5·3928	2·6302	6·2916	3·0686	7·1904	3·5070	8·0891	3·9453	8·9879	4·3837	64	
	27	5·3460	2·7239	6·2370	3·1779	7·1280	3·6319	8·0191	4·0859	8·9101	4·5399	63	
	28	5·2977	2·8168	6·1806	3·2863	7·0636	3·7558	7·9465	4·2252	8·8295	4·6947	62	
2 ½		5·2915	2·8284	6·1734	3·2998	7·0554	3·7712	7·9373	4·2426	8·8192	4·7140		5 ½
	29	5·2477	2·9089	6·1223	3·3937	6·9970	3·8785	7·8716	4·3633	8·7462	4·8481	61	
	30	5·1961	3·0000	6·0622	3·5000	6·9282	4·0000	7·7942	4·5000	8·6603	5·0000	60	
2 ¾		5·1464	3·0846	6·0041	3·5987	6·8618	4·1128	7·7196	4·6269	8·5773	5·1410		5 ¼
	31	5·1430	3·0902	6·0002	3·6052	6·8573	4·1203	7·7145	4·6353	8·5717	5·1504	59	
	32	5·0883	3·1795	5·9363	3·7094	6·7843	4·2394	7·6324	4·7693	8·4805	5·2992	58	
	33	5·0320	3·2678	5·8707	3·8125	6·7094	4·3571	7·5480	4·9018	8·3867	5·4464	57	
3		4·9888	3·3334	5·8203	3·8890	6·6518	4·4446	7·4832	5·0001	8·3147	5·5557		5
	34	4·9742	3·3552	5·8033	3·9144	6·6323	4·4735	7·4613	5·0327	8·2904	5·5919	56	
	35	4·9149	3·4415	5·7341	4·0150	6·5532	4·5880	7·3724	5·1622	8·1915	5·7358	55	
	36	4·8541	3·5267	5·6631	4·1145	6·4721	4·7023	7·2812	5·2901	8·0902	5·8779	54	
3 ¼		4·8192	3·5742	5·6224	4·1699	6·4257	4·7656	7·2289	5·3613	8·0321	5·9570		4 ¾
	37	4·7918	3·6109	5·5904	4·2127	6·3891	4·8145	7·1877	5·4163	7·9864	6·0182	53	
	38	4·7281	3·6940	5·5161	4·3096	6·3041	4·9253	7·0921	5·5409	7·8801	6·1560	52	
	39	4·6629	3·7759	5·4400	4·4052	6·2172	5·0346	6·9943	5·6639	7·7715	6·2932	51	
3 ½		4·6384	3·8064	5·4111	4·4408	6·1841	5·0751	6·9571	5·7095	7·7301	6·3439		4 ½
	40	4·5963	3·8567	5·3623	4·4995	6·1284	5·1423	6·8944	5·7851	7·6604	6·4279	50	
	41	4·5283	3·9363	5·2830	4·5924	6·0377	5·2485	6·7924	5·9045	7·5471	6·5606	49	
	42	4·4589	4·0148	5·2020	4·6839	5·9452	5·3530	6·6883	6·0222	7·4314	6·6913	48	
3 ¾		4·4457	4·0294	5·1867	4·7009	5·9276	5·3725	6·6680	6·0440	7·4095	6·7156		4 ¼
	43	4·3881	4·0920	5·1195	4·7740	5·8508	5·4560	6·5822	6·1380	7·3135	6·8200	47	
	44	4·3160	4·1679	5·0354	4·8626	5·7547	5·5573	6·4741	6·2519	7·1934	6·9466	46	
4	45	4·2426	4·2426	4·9497	4·9497	5·6569	5·6569	6·3640	6·3640	7·0711	7·0711	45	4
Pts.	Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.	Pts.
		Dist. 6		Dist. 7		Dist. 8		Dist. 9		Dist. 10			

TREBLE, in Muſic, the higheſt or acuteſt of the four parts in ſymphony, or that which is heard the cleareſt and ſhrilleſt in a concert. In the like ſenſe we ſay, a Treble violin, Treble hautboy, &c.

In vocal muſic, the Treble is uſually committed to boys and girls; their proper part being the Treble.

The Treble is divided into firſt or higheſt Treble, and ſecond or baſs Treble. The half Treble is the ſame with the counter-tenor.

TRENCHES, in Fortification, are ditches which the beſiegers cut to approach more ſecurely to the place attacked; whence they are called *lines of approach.* Their breadth is 8 or 10 feet, and depth 6 or 7.

They ſay, *mount the Trenches*, that is, go upon duty in them. To *relieve the Trenches*, is to relieve ſuch as have been upon duty there. The enemy is ſaid to have *cleared the Trenches*, when he has driven away or killed the ſoldiers who guarded them.

Tail of the TRENCH, is the place where it was begun. And the *Head* is the place where it ends.

Opening of the TRENCHES, is when the beſiegers firſt begin to work upon them, or to make them; which is uſually done in the night.

TREPIDATION, in the Ancient Aſtronomy, denotes what they call a libration of the 8th ſphere; or a motion which the Ptolomaic ſyſtem attributed to the firmament, to account for certain almoſt inſenſible changes and motions obſerved in the axis of the world; by means of which the latitudes of the fixed ſtars come to be gradually changed, and the ecliptic ſeems to approach reciprocally, firſt towards one pole, then towards the other.

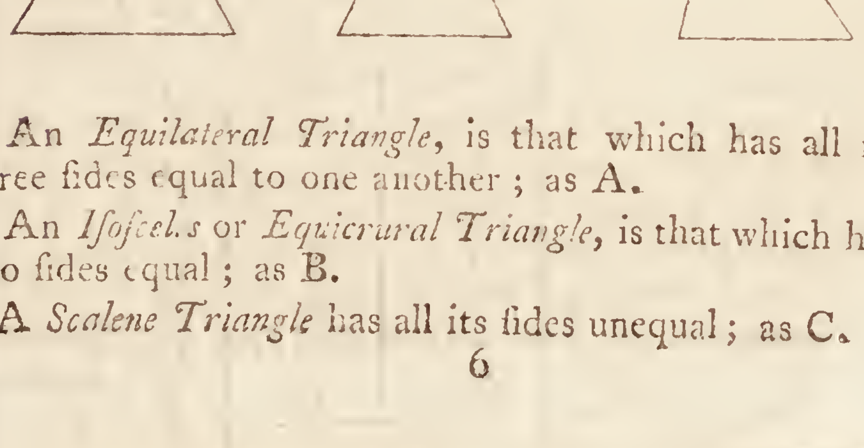
This motion is alſo called the *motion of the firſt libration.*

TRET, in Commerce, is an allowance made for the waſte, or the duſt, that may be mixed with any commodity; which is always 4 pounds on every 104 pounds weight. See TARE.

TRIANGLE, in Geometry, a figure bounded or contained by three lines or ſides, and which conſequently has three angles, from whence the figure takes its name.

Triangles are either plane or ſpherical or curvilinear. Plane when the three ſides of the Triangle are right lines; but ſpherical when ſome or all of them are arcs of great circles on the ſphere.

Plane Triangles take ſeveral denominations, both from the relation of their angles, and of their ſides, as below. And 1ſt with regard to the ſides.



An *Equilateral Triangle*, is that which has all its three ſides equal to one another; as A.

An *Iſoſceles* or *Equicrural Triangle*, is that which has two ſides equal; as B.

A *Scalene Triangle* has all its ſides unequal; as C.

Again, with reſpect to the Angles.

A *Rectangular* or *Right-angled Triangle*, is that which has one right angle; as D.

An *Oblique Triangle* is that which has no right angle, but all oblique ones; as E or F.

An *Acutangular* or *Oxygone Triangle*, is that which has three acute angles; as E.

An *Obtuſangular* or *Amblygone Triangle*, is that which has an obtuſe angle; as F.

A *Curvilinear* or *Curvilineal Triangle*, is one that has all its three ſides curve lines.

A *Mixtilinear Triangle* is one that has its ſides ſome of them curves, and ſome right lines.

A *Spherical Triangle* is one that has its ſides, or at leaſt ſome of them, arcs of great circles of the ſphere.

Similar Triangles are ſuch as have the angles in the one equal to the angles in the other, each to each.

The *Baſe* of a Triangle, is any ſide on which a perpendicular is drawn from the oppoſite angle, called the *vertex;* and the two ſides about the perpendicular, or the vertex, are called the *legs*.

The Chief Properties of Plane Triangles, are as follow, viz, In any plane Triangle,

1. The greateſt ſide is oppoſite to the greateſt angle, and the leaſt ſide to the leaſt angle, &c. Alſo, if two ſides be equal, their oppoſite angles are equal; and if the Triangle be equilateral, or have all its ſides equal, it will alſo be equiangular, or have all its angles equal to one another.

2. Any ſide of a Triangle is leſs than the ſum, but greater than the difference, of the other two ſides.

3. The ſum of all the three angles, taken together, is equal to two right angles.

4. If one ſide of a Triangle be produced out, the external angle, made by it and the adjacent ſide, is equal to the ſum of the two oppoſite internal angles.

5. A line drawn parallel to one ſide of a Triangle, cuts the other two ſides proportionally, the correſponding ſegments being proportional, each to each, and to the whole ſides; and the Triangle cut off is ſimilar to the whole Triangle.

If a perpendicular be let fall from any angle of a Triangle, as a vertical angle, upon the oppoſite ſide as a baſe; then

6. The rectangle of the ſum and difference of the ſides, is equal to twice the rectangle of the baſe and the diſtance of the perpendicular from the middle of the baſe.—Or, which is the ſame thing in other words,

7. The difference of the ſquares of the ſides, is equal to the difference of the ſquares of the ſegments of the baſe. Or, as the baſe is to the ſum of the ſides, ſo is the difference of the ſides, to the difference of the ſegments of the baſe.

8. The rectangle of the legs or ſides, is equal to the rectangle of the perpendicular and the diameter of the circumſcribing circle.

If a line be drawn biſecting any angle, to the baſe or oppoſite ſide; then,

9. The

9. The ſegments of the baſe, made by the line biſecting the oppoſite angle, are proportional to the ſides adjacent to them.

10. The ſquare of the line biſecting the angle, is equal to the difference between the rectangle of the ſides and the rectangle of the ſegments of the baſe.

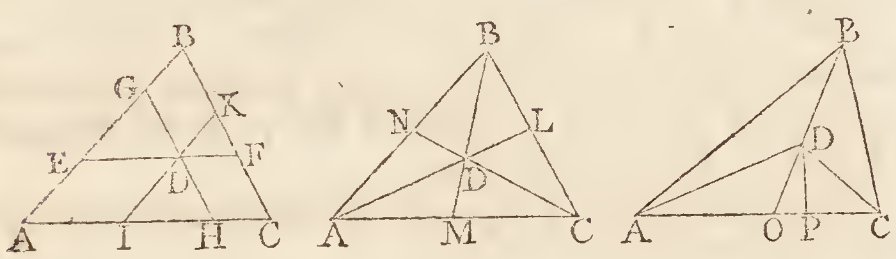
If a line be drawn from any angle to the middle of the oppoſite ſide, or biſecting the baſe; then

11. The ſum of the ſquares of the ſides, is equal to twice the ſum of the ſquares of half the baſe and the line biſecting the baſe.

12. The angle made by the perpendicular from any angle and the line drawn from the ſame angle to the middle of the baſe, is equal to half the difference of the angles at the baſe.

13. If through any point D, within a Triangle ABC, three lines EF, GH, IK, be drawn parallel to the three ſides of the Triangle; the continual products or ſolids made by the alternate ſegments of theſe lines will be equal; viz,

$$DE \times DK \times DH = DG \times DF \times DI.$$



14. If three lines AL, BM, CN, be drawn from the three angles through any point D within a Triangle, to the oppoſite ſides; the ſolid products of the alternate ſegments of the ſides are equal; viz,
$AN \times BL \times CM = AM \times CL \times BN$, (2d fig. above).

15. Three lines drawn from the three angles of a Triangle to biſect the oppoſite ſides, or to the middle of the oppoſite ſides, do all interſect one another in the ſame point D, and that point is the centre of gravity of the Triangle, and the diſtance AD of that point from any angle as D, is equal to double the diſtance DL from the oppoſite ſide; or one ſegment of any of theſe lines is double the other ſegment: moreover the ſum of the ſquares of the three biſecting lines, is $\frac{3}{4}$ of the ſum of the ſquares of the three ſides of the Triangle.

16. Three perpendiculars biſecting the three ſides of a Triangle, all interſect in one point, and that point is the centre of the circumſcribing circle.

17. Three lines biſecting the three angles of a Triangle, all interſect in one point, and that point is the centre of the inſcribed circle.

18. Three perpendiculars drawn from the three angles of a Triangle, upon the oppoſite ſides, all interſect in one point.

19. If the three angles of a Triangle be biſected by the lines AD, BD, CD (3d fig. above), and any one as BD be continued to the oppoſite ſide at O, and DP be drawn perp. to that ſide; then is
$\angle ADO = \angle CDP$, or $\angle ADP = \angle CDO$.

20. Any Triangle may have a circle circumſcribed about it, or touching all its angles, and a circle inſcribed within it, or touching all its ſides.

21. The ſquare of the ſide of an equilateral Triangle, is equal to 3 times the ſquare of the radius of its circumſcribing circle.

22. If the three angles of one Triangle be equal to the three angles of another Triangle, each to each; then thoſe two Triangles are ſimilar, and their like ſides are proportional to one another, and the areas of the two Triangles are to each other as the ſquares of their like ſides.

23. If two Triangles have any three parts of the one (except the three angles), equal to three correſponding parts of the other, each to each; thoſe two Triangles are not only ſimilar, but alſo identical, or having all their ſix correſponding parts equal, and their areas equal.

24. Triangles ſtanding upon the ſame baſe, and between the ſame parallels, are equal; and Triangles upon equal baſes, and having equal altitudes, are equal.

25. Triangles on equal baſes, are to one another as their altitudes: and Triangles of equal altitudes, are to one another as their baſes; alſo equal Triangles have their baſes and altitudes reciprocally proportional.

26. Any Triangle is equal to half its circumſcribing parallelogram, or half the parallelogram on the ſame or an equal baſe, and of the ſame or equal altitude.

27. Therefore the area of any Triangle is found, by multiplying the baſe by the altitude, and taking half the product.

28. The area is alſo found thus: Multiply any two ſides together, and multiply the product by the ſine of their included angle, to radius 1, and divided by 2.

29. The area is alſo otherwiſe found thus, when the three ſides are given: Add the three ſides together, and take half their ſum; then from this half ſum ſubtract each ſide ſeverally, and multiply the three remainders and the half ſum continually together; then the ſquare root of the laſt product will be the area of the Triangle.

30. In a right-angled Triangle, if a perpendicular be let fall from the right angle upon the hypothenuſe, it will divide it into two other Triangles ſimilar to one another, and to the whole Triangle.

31. In a right-angled Triangle, the ſquare of the hypothenuſe is equal to the ſum of the ſquares of the two ſides; and, in general, any figure deſcribed upon the hypothenuſe, is equal to the ſum of two ſimilar figures deſcribed upon the two ſides.

32. In an iſoſceles Triangle, if a line be drawn from the vertex to any point in the baſe; the ſquare of that line together with the rectangle of the ſegments of the baſe, is equal to the ſquare of the ſide.

33. If one angle of a Triangle be equal to 120°; the ſquare of the baſe will be equal to the ſquares of both the ſides, together with the rectangle of thoſe ſides; and if thoſe ſides be equal to each other, then the ſquare of the baſe will be equal to three times the ſquare of one ſide, or equal to 12 times the ſquare of the perpendicular from the angle upon the baſe.

34. In the ſame Triangle, viz, having one angle equal to 120°; the difference of the cubes of the ſides, about that angle, is equal to a ſolid contained by the difference of the ſides and the ſquare of the baſe; and, the ſum of the cubes of the ſides, is equal to a ſolid contained by the ſum of the ſides and the difference between

between the ſquare of the baſe and twice the rectangle of the ſides.

There are many other properties of Triangles to be found among the geometrical writers; ſo Gregory St. Vincent has written a folio volume upon Triangles; there are alſo ſeveral in his Quadrature of the circle. See alſo other properties under the article TRIGONOMETRY.

For the properties of ſpherical Triangles, ſee SPHERICAL *Triangles*.

Solution of TRIANGLES. See TRIGONOMETRY.

TRIANGLE, in Aſtronomy, one of the 48 ancient conſtellations, ſituated in the northern hemiſphere. There is alſo the *Southern Triangle* in the ſouthern hemiſphere, which is a modern conſtellation. The ſtars in the Northern Triangle are, in Ptolomy's catalogue 4, in Tycho's 4, in Hevelius's 12, and in the Britiſh catalogue 16.

The ſtars in the Southern Triangle are, in Sharp's catalogue, 5.

Arithmetical TRIANGLE, a kind of numeral Triangle, or Triangle of numbers, being a table of certain numbers diſpoſed in form of a Triangle. It was ſo called by Paſcal; but he was not the inventor of this table, as ſome writers have imagined, its properties having been treated of by other authors, ſome centuries before him, as is ſhewn in my Mathematical Tracts, vol. 1, pa. 69 &c.

The form of the Triangle is as follows:

1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1
1	6	15	20	&c	
1	7	21	&c		
1	8	&c			
1	9				

And it is conſtructed by adding always the laſt two numbers of the next two preceding columns together, to give the next ſucceeding column of numbers.

The firſt vertical column conſiſts of units; the 2d a ſeries of the natural numbers 1, 2, 3, 4, 5, &c; the 3d a ſeries of Triangular numbers 1, 3, 6, 10, &c; the 4th a ſeries of pyramidal numbers, &c. The oblique diagonal rows, deſcending from left to right, are alſo the ſame as the vertical columns. And the numbers taken on the horizontal lines are the co-efficients of the different powers of a binomial. Many other properties and uſes of theſe numbers have been delivered by various authors, as may be ſeen in the Introduction to my Mathematical Tables, pages 7, 8, 75, 76, 77, 89, 2d edition.

After theſe, Paſcal wrote a treatiſe on the Arithmetical Triangle, which is contained in the 5th volume of his works, publiſhed at Paris and the Hague in 1779, in 5 volumes, 8vo.

In this publication is alſo a deſcription, taken from the 1ſt volume of the French Encyclopedie, art. *Arithmetique Machine*, of that admirable machine invented by Paſcal at the age of 19, furniſhing an eaſy and expeditious method of making all ſorts of arithmetical calculations without any other aſſiſtance than the eye and the hand.

TRIANGULAR, relating to a triangle; as

TRIANGULAR *Canon*, tables relating to trigonometry; as of ſines, tangents, ſecants, &c.

TRIANGULAR *Compaſſes*, are ſuch as have three legs or feet, by which any triangle, or three points, may be taken off at once. Theſe are very uſeful in the conſtruction of maps, globes, &c.

TRIANGULAR *Numbers*, are a kind of polygonal numbers; being the ſums of arithmetical progreſſions, which have 1 for the common difference of their terms.

Thus, from theſe arithmeticals 1 2 3 4 5 6, are formed the Triang. Numb. 1 3 6 10 15 21, or the 3d column of the arithmetical triangle above-mentioned.

The ſum of any number n of the terms of the Triangular numbers, 1, 3, 6, 10, &c, is =

$$\frac{n^3}{6} + \frac{n^2}{2} + \frac{n}{3}, \text{ or } \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3}$$

which is alſo equal to the number of ſhot in a triangular pile of balls, the number of rows, or the number in each ſide of the baſe, being n.

The ſum of the reciprocals of the Triangular ſeries, infinitely continued, is equal to 2; viz,

$$1 + \tfrac{1}{3} + \tfrac{1}{6} + \tfrac{1}{10} + \tfrac{1}{15} \text{ \&c} = 2.$$

For the rationale and management of theſe numbers, ſee Malcolm's Arith. book 5, ch. 2; and Simpſon's Algeb. ſec. 15.

TRIANGULAR *Quadrant*, is a ſector furniſhed with a looſe piece, by which it forms an equilateral triangle. Upon it is graduated and marked the calendar, with the ſun's place, and other uſeful lines; and by the help of a ſtring and a plummet, with the diviſions graduated on the looſe piece, it may be made to ſerve for a quadrant.

TRIBOMETER, in Mechanics, a term applied by Muſſchenbroek to an inſtrument invented by him for meaſuring the friction of metals. It conſiſts of an axis formed of hard ſteel, paſſing through a cylindrical piece of wood: the ends of the axis, which are highly poliſhed, are made to reſt on the poliſhed ſemicircular cheeks of various metals, and the degree of friction is eſtimated by means of a weight ſuſpended by a fine ſilken ſtring or ribband over the wooden cylinder. For a farther deſcription and the figure of this inſtrument, with the reſults of various experiments performed with it, ſee Muſſchenb. Introd. ad Phil. Nat. vol. 1, p. 151.

TRIDENT, is a particular kind of parabola, uſed by Deſcartes in conſtructing equations of 6 dimenſions. See the article *Carteſian* PARABOLA.

TRIGLYPH, in Architecture, is a member of the Doric Frize, placed directly over each column, and at equal diſtances in the intercolumnation, having two entire glyphs or channels engraven in it, meeting in an angle, and ſeparated by three legs from the two demi-channels of the ſides.

TRIGON, a figure of three angles, or a triangle.

TRIGON, in Aſtrology. See TRIPLICITY.

TRIGON, in Aſtronomy, denotes an aſpect of two planets when they are 120 degrees diſtant from each other; called alſo a Trine, being the 3d part of 360 degrees.—The Trigons of Mars and Saturn are by aſtrologers held malific or malignant aſpects.

TRIGON, in Dialling, is an inſtrument of a triangular form.

TRIGON, in Muſic, denoted a muſical inſtrument, uſed among the ancients. It was a kind of triangular lyre, or harp, invented by Ibycus; and was uſed at feaſts, being played on by women, who ſtruck it either with a quill, or beat it with ſmall rods of different lengths and weights, to occaſion a diverſity in the ſounds.

TRIGONAL *Numbers*. See TRIANGULAR *Numbers*.

TRIGONOMETER *Armillary*. See ARMILLARY *Trigonometer*.

TRIGONOMETRY, the art of meaſuring the ſides and angles of triangles, either plane or ſpherical, from whence it is accordingly called either Plane Trigonometry, or Spherical Trigonometry.

Every triangle has 6 parts, 3 ſides, and 3 angles; and it is neceſſary that three of theſe parts be given, to find the other three. In ſpherical Trigonometry, the three parts that are given, may be of any kind, either all ſides, or all angles, or part the one and part the other. But in plane Trigonometry, it is neceſſary that one of the three parts at leaſt be a ſide, ſince from three angles can only be found the proportions of the ſides, but not the real quantities of them.

Trigonometry is an art of the greateſt uſe in the mathematical ſciences, eſpecially in aſtronomy, navigation, ſurveying, dialling, geography, &c, &c. By it, we come to know the magnitude of the earth, the planets and ſtars, their diſtances, motions, eclipſes, and almoſt all other uſeful arts and ſciences. Accordingly we find this art has been cultivated from the earlieſt ages of mathematical knowledge.

Trigonometry, or the reſolution of triangles, is founded on the mutual proportions which ſubſiſt between the ſides and angles of triangles; which proportions are known by finding the relations between the radius of a circle and certain other lines drawn in and about the circle, called *chords*, *ſines*, *tangents*, and *ſecants*. The ancients Menelaus, Hipparchus, Ptolomy, &c, performed their Trigonometry, by means of the chords. As to the ſines, and the common theorems relating to them, they were introduced into Trigonometry by the Moors or Arabians, from whom this art paſſed into Europe, with ſeveral other branches of ſcience. The Europeans have introduced, ſince the 15th century, the tangents and ſecants, with the theorems relating to them. See the hiſtory and improvements at large, in the Introduction to my Mathematical Tables.

The proportion of the ſines, tangents, &c, to their radius, is ſometimes expreſſed in common or natural numbers, which conſtitute what we call the *tables of natural ſines, tangents, and ſecants*. Sometimes it is expreſſed in logarithms, being the logarithms of the ſaid natural ſines, tangents, &c; and theſe conſtitute the table of *artificial ſines*, &c. Laſtly, ſometimes the proportion is not expreſſed in numbers; but the ſeveral ſines, tangents, &c, are actually laid down upon lines of ſcales; whence the *line of ſines*, of *tangents*, &c. See SCALE.

In Trigonometry, as angles are meaſured by arcs of a circle deſcribed about the angular point, ſo the whole circumference of the circle is divided into a great number of parts, as 360 degrees, and each degree into 60 minutes, and each minute into 60 ſeconds, &c; and then any angle is ſaid to conſiſt of ſo many degrees, minutes and ſeconds, as are contained in the arc that meaſures the angle, or that is intercepted between the legs or ſides of the angle.

Now the ſine, tangent, and ſecant, &c, of every degree and minute, &c, of a quadrant, are calculated to the radius 1, and ranged in tables for uſe; as alſo the logarithms of the ſame; forming the triangular canon. And theſe numbers, ſo arranged in tables, form every ſpecies of right-angled triangles, ſo that no ſuch triangle can be propoſed, but one ſimilar to it may be there found, by compariſon with which, the propoſed one may be computed by analogy or proportion.

As to the ſcales of chords, ſines, tangents, &c, uſually placed on inſtruments, the method of conſtructing them is exhibited in the ſcheme annexed to the article SCALE; which, having the names added to each, needs no farther explanation.

There are uſually three methods of reſolving triangles, or the caſes of Trigonometry; viz, geometrical conſtruction, arithmetical computation, and inſtrumental operation. In the 1ſt method, the triangle in queſtion is conſtructed by drawing and laying down the ſeveral parts of their magnitudes given, viz, the ſides from a ſcale of equal parts, and the angles from a ſcale of chords, or other inſtrument; then the unknown parts are meaſured by the ſame ſcales, and ſo they become known.

In the 2d method, having ſtated the terms of the proportion according to rule, which terms conſiſt partly of the numbers of the given ſides, and partly of the ſines, &c, of angles taken from the tables, the proportion is then reſolved like all other proportions, in which a 4th term is to be found from three given terms, by multiplying the 2d and 3d together, and dividing the product by the firſt. Or, in working with the logarithms, adding the log. of the 2d and 3d terms together, and from the ſum ſubtracting the log. of the 1ſt term, then the number anſwering to the remainder is the 4th term ſought.

To work a caſe inſtrumentally, as ſuppoſe by the log. lines on one ſide of the two-foot ſcales: Extend the compaſſes from the 1ſt term to the 2d, or 3d, which happens to be of the ſame kind with it; then that extent will reach from the other term to the 4th. In this operation, for the ſides of triangles, is uſed the line of numbers (marked Num.); and for the angles, the line of ſines or tangents (marked ſin. and tan.) according as the proportion reſpects ſines or tangents.

In every caſe of triangles, as has been hinted before, there

there must be three parts, one at least of which must be a side. And then the different circumstances, as to the three parts that may be given, admit of three cases or varieties only; viz,

1st. When two of the three parts given, are a side and its opposite angle.—2d, When there are given two sides and their contained angle.—3d, And thirdly, when the three sides are given.

To each of these cases there is a particular rule, or proportion, adapted, for resolving it by.

1st. *The Rule for the 1st Case*, or that in which, of the three parts that are given, an angle and its opposite side are two of them, is this, viz, That the sides are proportional to the sines of their opposite angles,

That is,

As one side given :
To the sine of its opposite angle : :
So is another side given :
To the sine of its opposite angle.

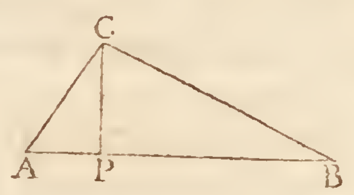
Or,

As the sine of an angle given :
To its opposite side : :
So is the sine of another angle given :
To its opposite side.

So that, to find an angle, we must begin the proportion with a given side that is opposite to a given angle; and to find a side, we must begin with an angle opposite to a given side.

Ex. Suppose, in the triangle ABC, there be given

AB = 365 feet,
AC = 154·33 f.
∠C = 98° 3′
to find the other side, and the angles.



1. *Geometrically, by Construction.*

Draw AC = 154·33 from a scale of equal parts: Make the angle C = 98° 3′, producing CB indefinitely: With centre A, and radius 365 feet, cross CB in B: Then join AB, and the figure is constructed. Then, by measuring the unknown angles and side, the former by the line of cords or otherwise, and the side by the line of equal parts, they will be found, as near as they can be measured, as below, viz,
BC = 310; the ∠A = 57°¼; and ∠B = 24°¾.

2. *Arithmetically, by Tables of Logs.*

As AB	= 365	- - log.	2·5622929
To AC	= 154·33	- -	2·1884504
So sin. ∠C	= 98° 3′ or 81° 57′		9·9956993
To sin. ∠B	= 24° 45′	- -	9·6218568
the sum	122 48		
taken from	180 00		
leaves ∠A	57 12		

Then, again,

As sin. ∠C	= 98° 3′	- log.	9·9956993
To AB	= 365	- - -	2·5622929
So sin. ∠A	= 57° 12′	- -	9·9245721
To BC	= 309·86	- -	2·4911657

3. *Instrumentally, by Gunter's Lines.*

In the first proportion, Extend the compasses from 365 to 154⅓ on the line of numbers; and that extent will reach, upon the line of sines, from 82° to 24¾, which gives the angle B. And, in the second proportion, Extend from 98° to 57¼ on the sines; and that extent will reach, upon the numbers, from 365 to 310, or the side BC nearly.

2d Case, when there are Given two Sides and their contained angle, to find the rest, the rule is this:

As the sum of the two given sides :
Is to the difference of the sides : :
So is the tang. of half the sum of the two opposite angles, or cotangent of half the given angle :
To tang. of half the diff. of those angles.

Then the half diff. added to the half sum, gives the greater of the two unknown angles; and subtracted, leaves the less of the two angles.

Hence, the angles being now all known, the remaining 3d side will be found by the former case.

Ex. Suppose, in the triangle ABC, there be given
the side AC = 154·33
the side BC = 309·86
the included ∠C = 98° 3′
to find the other side and the angles.

1. *Geometrically.*—Draw two indefinite lines making the angle C = 98° 3′: upon these lines set off CA = 154⅓, and CB = 310: Join the points A and B, and the figure is made. Then, by measurement, as before, we find the
∠A = 57¼; ∠B 24¾; and side AB = 365.

2. *By Logarithms.*

As CB + CA	= 464·19	- log.	2·6666958
To CB − CA	= 155·53	- -	2·1918142
So tan. ½A + ½B	= 40° 58½′	- -	9·9387803
To tan. ½A − ½B	= 16 13½	- -	9·4638987
sum gives ∠A	57 12		
diff. gives ∠B	24 45		

Then,

As sin. ∠B	= 24° 45′	- log.	9·6218612
To side AC	= 154·33	-	2·1884504
So sin. ∠C	= 98° 3′, or 81° 57		9·9956993
To side AB	= 365	- -	2·5622885

3. *Instrumentally.*—Extend the compasses from 464 to 155½ upon the line of numbers; then that extent will reach, upon the line of tangents, from 41° to 16°¼. Then, in the 2d proportion, extend the compasses from 24°¾ to 82° on the sines; and that extent

will

will reach, upon the numbers, from $154\frac{1}{3}$ to 365, which is the third fide.

3d *Cafe*, is when the three fides are given, to find the three angles; and the method of refolving this cafe is, to let a perpendicular fall from the greateft angle, upon the oppofite fide or bafe, dividing it into two fegments, and the whole triangle into two fmaller right-angled triangles: then it will be,

As the bafe, or fum of the two fegments :
Is to the fum of the other two fides : :
So is the difference of thofe fides :
To the difference of the fegments of the bafe.

Then half this difference of the two fegments added to the half fum, or half the bafe, gives the greater fegments, and fubtracted, gives the lefs. Hence, in each of the two right-angled triangles, there are given the hypotenufe, and the bafe, befides the right angle, to find the other angles by the 1ft cafe.

Ex. In the trangle ABC, fuppofe there are given the three fides, to find the three angles, viz,

$$\left.\begin{array}{l} AB = 365 \\ AC = 154{\cdot}33 \\ BC = 309{\cdot}86 \end{array}\right\} \text{to find the angles.}$$

1. *Geometrically.*—Draw the bafe AB = 365 : with the radius $154\frac{1}{3}$ and centre A defcribe an arc; and with the radius 310 and centre B defcribe another arc, cutting the former in C; then join AC and BC, and the triangle is conftructed. And by meafuring the angles, they are found, viz.

$$\angle A = 57^\circ\tfrac{1}{4};\ \angle B = 24^\circ\tfrac{3}{4};\ \angle C = 98^\circ \text{ nearly.}$$

2. *Arithmetically.*—Having let fall the perpendicular CP, dividing the bafe into the two fegments AP, PB, and the given triangle ABC into the two right-angled triangles ACP, BCP. Then,

As AB	= 365	log. 2·5622929
To CB + CA	= 464·19	2·6666958
So CB − CA	= 155·53	2·1918142
To BP − PA	= 197·80	2·2962171
its half	= 98·90	
$\frac{1}{2}$AB	= 182·50	
fum BP	= 281·40	
dif. AP	= 83·60	

Then, in the triangle APC, right-angled at P,

As AC	= 154·33	log. 2·1884504
To fin. ∠P	= 90°	10·0000000
So AP	= 83·6	1·9222063
To fin. ∠ACP	= 32° 48′	9·7337559
its comp. ∠A	= 57° 12	

And in the triangle BPC, right-angled at P,

As BC	= 309·86	log. 2·4911655
To fin. ∠P	= 90°	10·0000000
So BP	= 281·4	2·4493241
To fin. ∠BCP	= 65° 15′	9·9581586
its comp. ∠B	= 24 45	
Alfo to ∠ACP	= 32 48′	
add ∠BCP	= 65 15	
makes ∠ACB	= 98 3	

3. *Inftrumentally.*—In the 1ft proportion, Extend the compaffes from 365 to 464 on the line of numbers, and that extent will reach, on the fame line, from $155\frac{1}{2}$ to 197·8 nearly.—In the 2d proportion, Extend the compaffes from $154\frac{1}{3}$ to 83·6 on the line of numbers, and that extent will reach, on the fines, from 90° to $32^\circ\frac{3}{4}$ nearly.—In the 3d proportion, Extend the compaffes from 310 to $281\frac{1}{2}$ on the line of numbers; then that extent will reach, on the fines, from 90° to $65^\circ\frac{1}{4}$.

The foregoing three cafes include all the varieties of plane triangles that can happen, both of right and oblique-angled triangles. But befide thefe, there are fome other theorems that are ufeful upon many occafions, or fuited to fome particular forms of triangles, which are often more expeditious in ufe than the foregoing general ones; one of which, for right-angled triangles, as the cafe for which it ferves fo often occurs, may be here inferted, and is as follows.

Cafe 4. When, in a right-angled triangle, there are given the angles and one leg, to find the other leg, or the hypothenufe. Then it will,

As radius :
To given leg AB : :
So tang. adjacent ∠A :
To the opp. leg BC, and : :
So fec. of fame ∠A :
To hypot. AC

Ex. In the triangle ABC, right-angled at B,

$$\left.\begin{array}{l} \text{Given the leg AB} = 162 \\ \text{and the } \angle A = 53^\circ\ 7'\ 48'' \\ \text{confeq. } \angle C = 36\ 52\ \ 12 \end{array}\right\} \begin{array}{l}\text{to find BC} \\ \text{and AC.}\end{array}$$

1. *Geometrically.*—Draw the leg AB = 162 : Erect the indefinite perpendicular BC : Make the angle A = $53^\circ\frac{1}{8}$, and the fide AC will cut BC in C, and form the triangle ABC. Then, by meafuring, there will be found AC = 270, and BC = 216.

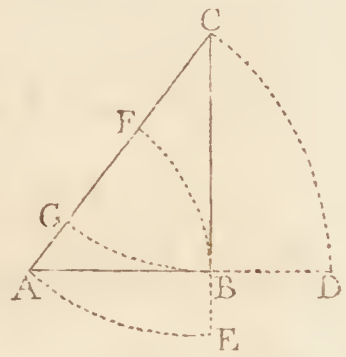
2. *Arithmetically.*

As radius	= 10	log. 10·0000000
To AB	= 162	2·2095150
So tan. ∠A	= 53° 7′ 48″	10·1249372
To BC	= 216	2·3344522
So fec. ∠A	= 53° 7′ 48″	10·2218477
To AC	= 270	2·4313627

3. *Inftrumentally.*

3. *Instrumentally.*—Extend the compasses from 45° at the end of the tangents (the radius) to the tangent of $53°\frac{1}{8}$; then that extent will reach, on the line of numbers, from 162 to 216, for BC. Again, extend the compasses from 36° 52′ to 90 on the sines; then that extent will reach, on the line of numbers, from 162 to 270 for AC.

Note, another method, by making every side radius, is often added by the authors on Trigonometry, which is thus: The given right-angled triangle being ABC, make first the hypotenuse AC radius, that is, with the extent of AC as a radius, and each of the centres A and C, describe arcs CD and AE; then it is evident that each leg will represent the sine of its opposite angle, viz, the leg BC the sine of the arc CD or of the angle A, and the leg AB the sine of the arc AE or of the angle C. Again, making either leg radius, the other leg will represent the tangent of its opposite angle, and the hypotenuse the secant of the same angle; thus, with radius AB and centre A describing the arc BF, BC represents the tangent of that arc, or of the angle A, and the hypotenuse AC the secant of the same; or with the radius BC and centre C describing the arc BG, the other leg AB is the tangent of that arc BG, or of the angle C, and the hypotenuse CA the secant of the same.



And then the general rule for all these cases is this, viz, that the sides bear to each other the same proportions as the parts or things which they represent. And this is called making every side radius.

Spherical TRIGONOMETRY, is the resolution and calculation of the sides and angles of spherical triangles, which are made by three intersecting arcs of great circles on a sphere. Here, any three of the six parts being given, even the three angles, the rest can be found; and the sides are measured or estimated by degrees, minutes, and seconds, as well as the angles.

Spherical Trigonometry is divided into right-angled and oblique-angled, or the resolution of right and oblique-angled spherical triangles. When the spherical triangle has a right angle, it is called a right-angled triangle, as well as in plane triangles; and when a triangle has one of its sides equal to a quadrant of a circle, it is called a quadrantal triangle.

For the resolution of spherical Triangles, there are various theorems and proportions, which are similar to those in plane Trigonometry, by substituting the sines of sides instead of the sides themselves, when the proportion respects sines; or tangents of the sides for the sides, when the proportion respects tangents, &c; some of the principal of which theorems are as follow:

Theor. 1. In any spherical triangle, the sines of the sides are proportional to the sines of their opposite angles.

Theor 2. In any right-angled triangle,

As radius :
To sine of one side : :
So tang. of the adjacent angle :
To tang. of the opposite side.

Theor. 3. If a perpendicular be let fall from any angl upon the base or opposite side of a spherical triangle it will be,

As the sine of the sum of the two sides :
To the sine of their difference : :
So cotan. $\frac{1}{2}$ sum angles at the vertex :
To tang. of half their difference.

Theor. 4.

As tang. half sum of the sides :
To tang. half their difference : :
So tang. $\frac{1}{2}$ sum ∠s at the base :
To tang. half their difference.

Theor. 5.

As cotan. $\frac{1}{2}$ sum of ∠s at the base :
To tang. half their difference : :
So tang. $\frac{1}{2}$ sum of ∠s at the vertex :
To tang. half their difference.

Theor. 6.

As tang. $\frac{1}{2}$ sum segments of base :
To tang. half sum of the sides : :
So tang. half difference of the sides :
To tang. $\frac{1}{2}$ diff. segments of base.

Theor. 7.

As sin. sum of ∠s at the base :
To sine of their difference : :
So tang. $\frac{1}{2}$ sum segments of base :
To tang. of half their difference.

Theor. 8.

As sin. sum of segments of base :
To sine of their difference : :
So sin. sum of angles at the vertex :
To sine of their difference.

Theor. 9.

As sine of the base :
To sine of the vertical angle : :
So sin. of diff. segments of the base :
To sin. diff. ∠s at vertex, when the perp. falls within : :
Or so sin. sum segments of base :
To sin. sum vertical ∠s, where the perp. falls without.

Theor. 10.

As cosin. half sum of the two sides :
To cosine of half their difference : :
So cotang. of half the included angle :
To tang. half sum of opposite angles.

Theor. 11.

As sin. of half sum of two sides :
To sine of half their difference : :
So cotang. half the included angle :
To tang. $\frac{1}{2}$ diff. of the oppos. angles.

Theor.

Theor. 12.

As cosin. half sum of two angles :
To cosine of half their difference : :
So tang. of half the included side :
To tang. $\frac{1}{2}$ sum of the opposite sides.

Theor. 13.

As sin. half sum of two angles :
To sine of half their difference : :
So tang. half the included side :
To tang. $\frac{1}{2}$ diff. of the opposite sides.

Theor. 14. In a right-angled triangle,

As sin. sum of hypot. and one side :
To sin. of their difference : :
So radius squared :
To square of tang. $\frac{1}{2}$ contained angle.

Theor. 15. In any spherical triangle;
The product of the sines of two sides and of the cosine of the included angle, added to the product of the cosines of those sides, is equal to the cosine of the third side; the radius being 1.

Theor. 16. In any spherical triangle;
The product of the sines of two angles and of the cosine of the included side, minus the product of the cosines of those angles, is equal to the cosine of the third angle; the radius being 1.

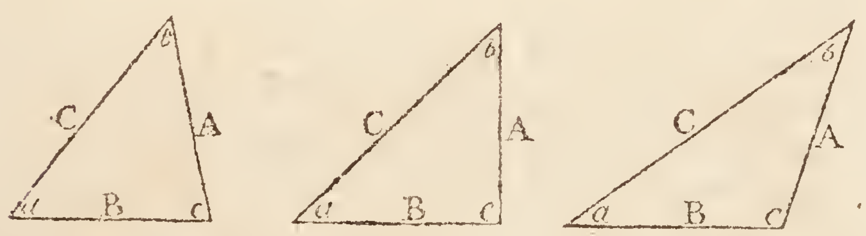
By some or other of these theorems may all the cases of spherical triangles be resolved, both right-angled and oblique: viz, the cases of right-angled triangles by the 1st and 2d theorems, and the oblique triangles by some of the other theorems.

In treatises on Trigonometry are to be found many other theorems, as well as synopses or tables of all the cases, with the theorem that is peculiar or proper to each. See the Introduction to my Mathematical Tables, p. 155 &c; or Robertson's Navigation, vol. 1, p. 162. See also Napier's Catholic or Universal Rule, in this Dictionary.

To the foregoing Theorems may be added the following synopsis of rules for resolving all the cases of plane and spherical triangles, under the title of

Trigonometrical Rules.

I. In a right-lined triangle, whose sides are A, B, C, and their opposite angles a, b, c; having given any three of these, of which one is a side; to find the rest.



Put s for the sine, s′ the cosine, t the tangent, and t′ the cotangent of an arch or angle, to the radius r; also L for a logarithm, and L′ its arithmetical complement. Then

Case 1. When three sides A, B, C, are given.

Put $P = \frac{1}{2}.\overline{A+B+C}$ or semiperimeter.

Then $\text{s.}\ \frac{1}{2}c = r\sqrt{\frac{\overline{P-A} \times \overline{P-B}}{A \times B}}$.

And $\text{s}'\ \frac{1}{2}c = r\sqrt{\frac{P \times \overline{P-C}}{A \times B}}$.

$\text{L.s.}\frac{1}{2}c = \frac{1}{2}(\text{L.}\ \overline{P-A} + \text{L.}\ \overline{P-B} + \text{L}'A + \text{L}'B)$,

$\text{L}'\text{s.}\frac{1}{2}c = \frac{1}{2}(\text{L.}\ P + \text{L.}\ \overline{P-C} + \text{L}'A + \text{L}'B)$.

Note, When $A = B$, then

$\text{s.}\ \frac{1}{2}c = \frac{C}{A} \times \frac{r}{2}$. And $\text{s}'\frac{1}{2}c = r\sqrt{\frac{A^2 - \frac{1}{4}C^2}{A^2}}$.

Case 2. Given two sides A, B, and their included angle c.

Put $\text{s} = 90° - \frac{1}{2}c$, and $\text{t.}\ d = \frac{A-B}{A+B} \times \text{t. s}$; then $a = \text{s} + d$; and $b = \text{s} - d$. And

$$C = \sqrt{\frac{4AB + \text{s}^2\frac{1}{2}c}{rr} + \overline{A-B}|^2}.$$

Or in logarithms, putting $\text{L.}\ Q = 2\,\text{L.}\ \overline{A-B}$, and $\text{L.}\ R = \text{L.}\ 2A + \text{L.}\ 2B + 2\,\text{L. s.}\ \frac{1}{2}c - 20$, we shall have $\text{L.}\ C = \frac{1}{2}\text{L.}\ \overline{Q+R}$.

If the angle c be right, or $= 90°$; then

$\text{t.}\ a = \frac{A}{B}r$; $\text{t.}\ b = \frac{B}{A}r$;

$C = \frac{r}{\text{s.}\ a}A$, or $= \frac{r}{\text{s.}\ b}B$, or $= \sqrt{A^2 + B^2}$.

If $A = B$; we shall have $a = b = 90° - \frac{1}{2}c$, and $\Big\}\ C = \frac{\text{s.}\ \frac{1}{2}c}{r} \times 2A$.

Case 3. When a side and its opposite angle are among the terms given.

Then $\frac{A}{\text{s.}\ a} = \frac{B}{\text{s.}\ b} = \frac{C}{\text{s.}\ c}$; from which equations any term wanted may be found.

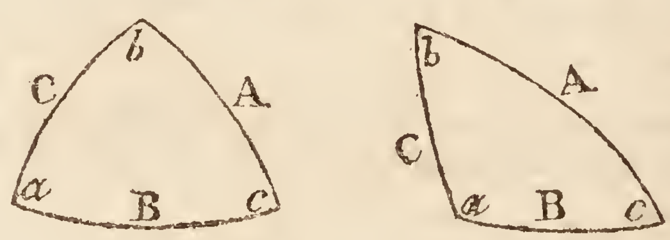
When an angle, as a, is 90°, and A and C are given, then

$B = \sqrt{A^2 - C^2} = \sqrt{\overline{A+C} \times \overline{A-C}}$.

And $\text{L.}\ B = \frac{1}{2}(\text{L.}\ \overline{A+C} + \text{L.}\ \overline{A-C})$.

Note, When two sides A, B, and an angle a opposite to one of them, are given; if A be less than B, then b, c, C have each two values; otherwise, only one value.

II. In a spherical triangle, whose three sides are A, B, C, and their opposite angles a, b, c; any three of these six terms being given, to find the rest.



Case

Case 1. Given the three sides A, B, C.

Calling 2P the perim. or $P = \frac{1}{2} . \overline{A + B + C}$.

Then $s.\frac{1}{2} c = r \sqrt{\frac{s.\overline{P - A} \times s.\overline{P - B}}{s.A \times s.B}}$.

And $s' \frac{1}{2} c = r \sqrt{\frac{s.P \times s.\overline{P - C}}{s.A \times s.B}}$.

$L.s.\frac{1}{2}c = \frac{1}{2}(L.s.\overline{P-A} + L.s.\overline{P-s} + L's.A + L's.B)$

$L.s'c = \frac{1}{2}(L.s.P + L.s.\overline{P-C} + L's.A + L's.B)$.

And the same for the other angles.

Case 2. Given the three angles.

Put $2p = a + b + c$. Then

$s.\frac{1}{2}C = r \sqrt{\frac{s'p \times s'\overline{p - c}}{s.a \times s.b}}$. And

$s' \frac{1}{2}C = r \sqrt{\frac{s'\overline{p - a} \times s'\overline{p - b}}{s.a \times s.b}}$.

$L.s.\frac{1}{2}C = \frac{1}{2}(L.s'p + L s'\overline{p-c} + L's.a + L's.B)$

$L.s'\frac{1}{2}C = \frac{1}{2}(L.s'\overline{p-a} + L.s'\overline{p-b} + L's.a + Ls'.b)$

And the same for the other sides.

Note. The sign $\triangleright$ signifies greater than, and $\triangleleft$ less; also $\backsim$ the difference.

Case 3. Given A, B, and included angle c.

To find an angle a opposite the side A,

let $r : s'c :: t.\ A : t.\ M$, like or unlike A, as c is $\triangleright$ or $\triangleleft$ 90°; also $N = B \backsim M$: then $s.\ N : s.\ M :: t.\ c : t.\ a$, like or unlike c as M is $\triangleright$ or $\triangleleft$ B.

Or let $s'\frac{1}{2}.\overline{A + B} : s'\frac{1}{2}.\overline{A \backsim B} :: t'\frac{1}{2}c : t.\ M$, which is $\triangleright$ or $\triangleleft$ 90° as A + B is $\triangleright$ or $\triangleleft$ 180°;

and $s.\frac{1}{2}.\overline{A + B} : s.\ \overline{A \backsim B} :: t'\frac{1}{2}c : t.\ N$, $\triangleright$ 90°. then $a = M + N$; and $b = M - N$.

Again let $r : s'c :: t.A : t.M$, like or unlike A as c is $\triangleright$ or $\triangleleft$ 90°; and $N = B \backsim M$.

Then $s'\ M : s'\ N :: s'A,\ s'\ C$, like or unlike N as c is $\triangleright$ or $\triangleleft$ 90°. Or,

$s.\frac{1}{2}C = \sqrt{\frac{s\ A \times s.B \times s^2\frac{1}{2}c}{rr}} + s^2\frac{1}{2}.\overline{A \backsim B}$.

In logarithms, put $L.\ Q = 2L.s.\frac{1}{2}\overline{A \backsim B}$; and $L.\ R = L.\ s.\ A + L.\ s.\ B + 2L.\ s.\ \frac{1}{2}c - 20$; then $L.s.\frac{1}{2}C = \frac{1}{2}L.\ \overline{Q + R}$.

Case 4. Given a, b, and included side C.

First, let $r : s'C :: t.\ a : t'm$, like or unlike a as C is $\triangleright$ or $\triangleleft$ 90°; also $n = b \backsim m$.

Then $s'n : s'm :: t.\ C : t.\ A$, like or unlike n as a is $\triangleright$ or $\triangleleft$ 90°.

Or, let $s'\frac{1}{2}\overline{a + b} : s'\frac{1}{2}.\ \overline{a \backsim b} :: t.\frac{1}{2}C : t.\ M$, $\triangleright$ or $\triangleleft$ 90° as $a + b$ is $\triangleright$ or $\triangleleft$ 180°;

and $s.\frac{1}{2}\overline{a + b} : s\ \frac{1}{2}\overline{a \backsim b} :: t\ \frac{1}{2}C : tN$, $\triangleright$ 90°; then $A = M \pm N$; and $B = M \mp N$.

Again, let $r : s'C :: t.\ a : t'm$, like or unlike a as C is $\triangleright$ or $\triangleleft$ 90°; and $n = b \backsim m$:

then $s.m. : s.n. :: s'a : s'c$, like or unlike a as m is $\triangleright$ or $\triangleleft$ b.

Case 5. Given A, B, and an opposite angle a.

1st. $s.\ A : s.\ a. :: s.\ B : s.\ b$, $\triangleright$ or $\triangleleft$ 90°.

2nd. Let $r : s'B :: t.\ a : t'm$, like or unlike s as a is $\triangleright$ or $\triangleleft$ 90°;

and $t.A : t.\ B :: s'm : s'n$, like or unlike A as a is $\triangleright$ or $\triangleright$ 90°:

then $c = m \pm n$, two values also.

3dly. Let $r : s'a :: t.\ B : t.\ M$, like or unlike B as a is $\triangleright$ or $\triangleleft$ 90°;

and $s'B : s'A :: s'M : s'N$, like or unlike A as a is $\triangleright$ or $\triangleleft$ 90°:

then $C = M \pm N$, two values also.

But if A be equal to B, or to its supplement, or between B and its supplement; then is b like to B: also c is $= m \mp n$, and $C = M \pm N$, as B is like or unlike a.

Case 6. Given a, b, and an opposite side A.

1st. $s.a. : s.\ A :: s.b : s.\ B$, $\triangleright$ or $\triangleleft$ 90°.

2nd. Let $r : s'b :: t.\ A :$ to tM, like or unlike b as A is $\triangleright$ or $\triangleleft$ 90°;

and $t.\ a : t.\ b :: s.\ M : s.\ N$, $\triangleright$ or $\triangleleft$ 90°: then $C = M \mp N$, as a is like or unlike b.

3dly. Let $r : s'A :: t.\ b : t'm$, like or unlike b as A $\triangleright$ or $\triangleleft$ 90°;

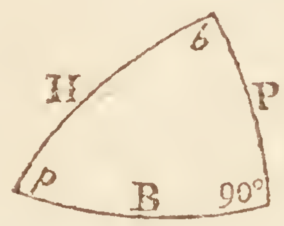
and $s'b : s'a :: s.\ m. : s.\ n$, $\triangleright$ or $\triangleleft$ 90°: then $c = m \pm n$, as a is like or unlike b.

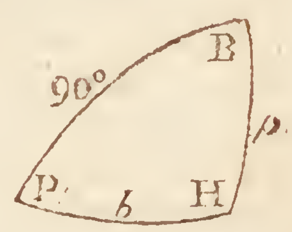
But if A be equal to B, or to its supplement, or between B and its supplement; then B is unlike b, and only the less values of N, n, are possible.

Note, When two sides A, B, and their opposite angles a, b, are known; the third side C, and its opposite angle c, are readily found thus:

$s.\ \frac{1}{2}\ \overline{a \backsim b} : s.\ \frac{1}{2}.\ \overline{a + b} :: t.\ \frac{1}{2}\ \overline{A \backsim B} : t.\ \frac{1}{2}C$.

$s.\ \frac{1}{2}.\ \overline{A \backsim B} : s.\ \frac{1}{2}\ \overline{A + B} :: t.\ \frac{1}{2}.\overline{a \backsim b} : t.\ \frac{1}{2}c$.





III. In a right-angled spheric triangle, where H is the hypotenuse, or side opposite the right angle, B, P the other two sides, and b, p their opposite angles; any two of these five terms being given, to find the rest; the cases, with their solutions, are as in the following Table.

The

The ſame Table will alſo ſerve for the quadrantal triangle, or that which has one ſide = 90°, H being the angle oppoſite that ſide, B, P the other two angles, and *b*, *p* their oppoſite ſides: obſerving, inſtead of H, to take its ſupplement: and mutually change the terms *like* and *unlike* for each other where H is concerned.

Caſe	Given	Req^{d}	SOLUTIONS.
1	H B	*b* *p* P	s. H. : *r* :: s B : s*b*, and is like B *r* : t′H :: t. B : s′*p* } s′B : *r* :: s′H : s′P }, $\rhd$ or $\lhd$ 90° as H is like or unlike B
2	H *b*	B P *p*	*r* : s′H :: s.*b* : s.B, like *b* *r* : s′*b* :: t.H : t.P } *r* : s′H :: t.*b* : t′*p* }, $\rhd$ or $\lhd$ 90° as H is like or unlike *b*
3	B *b*	H P *p*	s.*b* : *r* :: s.B : s.H } *r* : t.B :: t′*b* : s.P } s′B : *r* :: s′*b* : s.*p* }, each $\rhd$ or $\lhd$ 90°; both values true
4	B *p*	H *b* *p*	*r* : t′B :: s′*p* : t′H, $\rhd$ or $\lhd$ 90 as B is like or unlike *p* *r* : s′B :: s.*p* : s′*b*, like B *r* : s.B :: t.*p* : t.P, like *p*
5	B P	H *b* *p*	*r* : s′B :: s′P : s′H, $\rhd$ or $\lhd$ 90° as B is like or unlike P *r* : s.P :: t′B : t′*b*, like B *r* : s.B :: t′P : t′*p*, like P
6	*b* *p*	H B P	*r* : t′*b* :: t′*p* : s′H, $\rhd$ or $\lhd$ 90° as *b* is like or unlike *p* s.*p* : *r* :: s′*b* : s′B like *b* s.*b* : *r* :: s′*p* : s′P like *p*

The following Propoſitions and Remarks, concerning Spherical Triangles, (ſelected and communicated by the reverend Nevil Maſkelyne, D. D. Aſtronomer Royal, F. R. S.) will alſo render the calculation of them perſpicuous, and free from ambiguity.

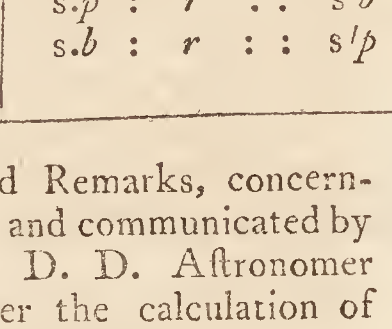
" 1. A ſpherical triangle is equilateral, iſoſcelar, or ſcalene, according as it has its three angles all equal, or two of them equal, or all three unequal; and *vice verſa*.

2. The greateſt ſide is always oppoſite the greateſt angle, and the ſmalleſt ſide oppoſite the ſmalleſt angle.

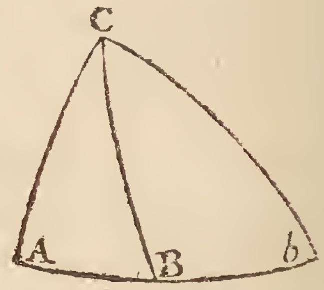
3. Any two ſides taken together, are greater than the third.

4. If the three angles are all acute, or all right, or all obtuſe; the three ſides will be, accordingly, all leſs than 90°, or equal to 90°, or greater than 90°; and *vice verſa*.

5. If from the three angles A, B, C, of a triangle ABC, as poles, there be deſcribed, upon the ſurface of the ſphere, three arches of a great circle DE, DF, FE, forming by their interſections a new ſpherical triangle DEF; each ſide of the new triangle will be the ſupplement of the angle at its pole; and each angle of the ſame triangle, will be the ſupplement of the ſide oppoſite to it in the triangle ABC.



6. In any triangle ABC, or A*b*C, right angled in A, 1ſt, The angles at the hypotenuſe are always of the ſame kind as their oppoſite ſides; 2dly, The hypotenuſe is leſs or greater than a quadrant, according as the ſides including the right angle are of the ſame or different kinds; that is to ſay, according as theſe ſame ſides are either both acute or both obtuſe, or as one is acute and the other obtuſe. And, *vice verſa*, 1ſt, The ſides including the right angle, are always of the ſame kind as their oppoſite angles: 2dly, The ſides including the right angle will be of the ſame or different kinds, according as the hypotenuſe is leſs or more than 90°; but one at leaſt of them will be of 90°, if the hypotenuſe is ſo."



TRILATERAL, three-ſided, a term applied to all figures of three ſides, or triangles.

TRILLION, in Arithmetic, the number of a million of billions, or a million of million of millions.

TRIMMERS, in Architecture, pieces of timber framed at right-angles to the joiſts, againſt the ways for chimneys to ſupport the hearths, and the well-holes for ſtairs.

TRINE *Dimenſion*, or *threefold dimenſion*, includes length, breadth, and thickneſs. The Trine dimenſion is peculiar to bodies or ſolids.

TRINE, in Aſtrology, is the aſpect or ſituation of one planet with reſpect to another, when they are diſtant

tant $\frac{1}{3}$ part of the circle, or 4 signs, or 120 degrees. It is also called trigon, and is denoted by the character △.

TRINITY *Sunday*, is the next after Whitsunday; so called, because on that day was anciently held a festival (as it still continues to be in the Romish Church) in honour of the Holy Trinity.—The observance of this festival was first enjoined by the 6th canon of the council of Arles, in 1260; and John the 22d, who distinguished himself so much by his opinion concerning the beatific vision, it is said, fixed the office for this festival in 1334.

TRINODA, or TRINODIA *Terræ*, in some ancient writers, denotes the quantity of 3 perches of land.

TRINOMIAL, in Algebra, is a quantity, or a root, consisting of three parts or terms, connected together by the signs + or −: as $a + b - c$, or $x + y + z$.

TRIO, in Music, a part of a concert in which three persons sing; or rather a musical composition consisting of 3 parts.—Trios are the finest kind of musical composition, and please most in concerts.

TRIOCTILE, in Astrology, an aspect or situation of two planets, with regard to the earth, when they are 3 octants, or $\frac{3}{8}$ of a circle, which is 135°, distant from each other.—This aspect, which some call the *sesquiquadrans*, is one of the new aspects added to the old ones by Kepler.

TRIONES, in Astronomy, a sort of constellation, or assemblage of 7 stars in the Ursa Major, popularly called *Charles's Wain*.—From the *Septem Triones* the north pole takes the denomination *Septentrio*.

TRIPARTITION, is a division by 3, or the taking of the 3d part of any number or quantity.

TRIPLE, threefold. See RATIO and SUBTRIPLE.

TRIPLE, in Music, is one of the species of measure or time, and is taken from hence, that the whole, or half measure, is divisible into 3 equal parts, and is beaten accordingly.

TRIPLICATE *Ratio*, is the ratio which cubes, or any similar solids, bear to each other; and is the cube of the simple ratio, or this twice multiplied by itself. Thus 1 to 8 is the Triplicate ratio of 1 to 2, and 1 to 27 Triplicate of 1 to 3.

TRIPLICITY, or TRIGON, with Astrologers, is a division of the 12 signs, according to the number of the 4 elements, earth, water, air, fire; each division consisting of 3 signs, making the earthly Triplicity, the watery Triplicity, the airy Triplicity, and the fiery Triplicity.

Triplicity is sometimes confounded with trine aspect; though they are, strictly speaking, very different things; as Triplicity is only used with regard to the signs, and trine with regard to the planets. The signs of Triplicity are those which are of the same nature, and not those that are in trine aspect: thus Aries, Leo, and Sagittary are signs of Triplicity, because those signs are, by these writers, all supposed fiery.

The signs in each of the four Triplicities, are as follow:

Earthly.	*Watery.*	*Airy.*	*Fiery.*
♉ Taurus.	♋ Cancer.	♊ Gemini.	♈ Aries.
♍ Virgo.	♏ Scorpio.	♎ Libra.	♌ Leo.
♑ Capricorn.	♓ Pisces.	♒ Aquarius.	♐ Sagittary.

TRIS-DIAPASON, or *Triple Diapason Chord*, in Music, is what is otherwise called a *triple eighth*.

TRISECTION, the dividing a thing into three equal parts. The term is chiefly used in Geometry, for the division of an angle into three equal parts. The *Trisection of an angle* geometrically, is one of those great problems whose solution has been so much sought for by mathematicians, for 2000 years past; being, in this respect, on a footing with the famous quadrature of the circle, and the duplicature of the cube.

The Ancients Trisected an angle by means of the conic sections, and the book of Inclinations; and Pappus enumerates several ways of doing it, in the 4th book of his Mathematical Collections, prop. 31, 32, 33, 34, 35, &c. He farther observes, that the problem of Trisecting an angle, is a solid problem, or a problem of the 3d degree, being expressed by the resolution of a cubic equation, in which way it has been resolved by Vieta, and others of the Moderns. See his Angular Sections, with those of other authors, and the Trisection in particular by cubic equations, as in Guisne's Application of Algebra to Geometry, in l'Hospital's Conic Sections, and in Emerson's Trigonometry, book 1, sec. 4. The cubic equation by which the problem of Trisection is resolved, is as follows: Let c denote the chord of a given arc, or angle, and x the cord of the 3d part of the same, to the radius 1; then is

$$x^3 - 3x = -c,$$

by the resolution of which cubic equation is found the value of x, or the chord of the 3d part of the given arc or angle, whose chord is c; and the resolution of this equation, by Cardan's rule, gives the chord

$$x = \sqrt[3]{\frac{-c + \sqrt{c^2 - 4}}{2}} + \frac{1}{\sqrt[3]{\frac{-c + \sqrt{c^2 - 4}}{2}}},$$

$$\text{or } x = \sqrt[3]{\frac{-c + \sqrt{c^2 - 4}}{2}} + \sqrt[3]{\frac{-c - \sqrt{c^2 - 4}}{2}}$$

TRISPAST, or TRISPASTON, in Mechanics, a machine with 3 pulleys, or an assemblage of 3 pulleys, for raising great weights; being a lower species of the polyspaston.

TRITE, in Music, the 3d musical chord in the system of the Ancients.

TRITONE, in Music, a false concord, consisting of three tones, or a greater third, and a greater tone. Its ratio or proportion in numbers, is that of 45 to 32.

TROCHILE, in Architecture, is that hollow ring, or cavity, which runs round a column next to the tore.

TROCHLEA, in Mechanics, one of the mechanic powers, more usually called the pulley.

TROCHOID, in the Higher Geometry, a curve defcribed by a point in any part of the radius of a wheel, during its rotatory and progreffive motions. This is the fame curve as what is more ufually called the *Cycloid*, where the conftruction and properties of it are fhewn.

TRONE *Weight*, was the fame with what we now call *Troy Weight*.

TRONE *Pound*, in Scotland, contains 20 Scotch ounces. Or becaufe it is ufual to allow one to the fcore, the Trone-pound is commonly 21 ounces.

TRONE-*Stone*, in Scotland, according to Sir John Skene, contains $19\frac{1}{2}$ pounds.

TROPHY, in Architecture, an ornament which reprefents the trunk of a tree, charged or encompaffed all around with arms or military weapons, both offenfive and defenfive.

TROPICAL, fomething relating to the Tropics. As, TROPICAL-*Winds*. See WIND, and TRADE-*Winds*.

TROPICAL *Year*, the fpace of time during which the fun paffes round from a tropic, till his return to it again. See YEAR.

TROPICS, in Aftronomy, two fixed circles of the fphere, drawn parallel to the equator, through the folftitial points, or at fuch diftance from the equator, as is equal to the fun's greateft recefs or declination, or to the obliquity of the ecliptic.

Of the two Tropics, that on the north fide of the equator, paffes through the firft point of Cancer, and is therefore called the *Tropic of Cancer*. And the other on the fouth fide, paffing through the firft point of Capricorn, is called the *Tropic of Capricorn*.

To determine the diftance between the two Tropics, and thence the fun's greateft declination, or the obliquity of the ecliptic; obferve the fun's meridian altitude, both in the fummer and winter folftice, and fubtract the latter from the former, fo fhall the remainder be the diftance between the two Tropics; and the half of this will be the quantity of the greateft declination, or the obliquity of the ecliptic; the medium of which is now 23° 28′ nearly.

TROPICS, in Geography, are two leffer circles of the globe, drawn parallel to the equator through the beginnings of Cancer and Capricorn, being in the planes of the celeftial Tropics, and confequently at 23° 28′ diftance either way from the equator.

TROY-*Weight*, anciently called *Trone-weight*, is fuppofed to be taken from a weight of the fame name in France, and that from the name of the town of Troyes there.

The original of all weights ufed in England, was a corn or grain of wheat gathered out of the middle of the ear, and, when well dried, 32 of them were to make one pennyweight, 20 pennyweights 1 ounce, and 12 ounces 1 pound Troy. Vide Statutes of 51 Hen. III; 31 Ed. I. and 12 Hen. VII.

But afterward it was thought fufficient to divide the faid pennyweight into 24 equal parts, called grains, being the leaft weight now in common ufe; fo that the divifions of Troy weight now are thefe:

24 grains	=	1 pennyweight	*dwt.*
20 pennyweights	=	1 ounce	*oz.*
12 ounces	=	1 pound	*lb.*

By Troy-weight are weighed jewels, gold, filver, and all liquors.

TRUCKS, among Gunners, are the fmall wooden wheels fixed on the axletrees of gun carriages, efpecially thofe for fhip fervice, to move them about by.

TRUE *Conjunction*, in Aftronomy. See *True* CONJUNCTION.

TRUE *Place of a Planet or Star*, is a point in the heavens fhewn by a right line drawn from the centre of the earth, through the centre of the ftar or planet.

TRUMPET, *Liftening or Hearing*, is an inftrument invented by Jofeph Landini, to affift the hearing of perfons dull of that faculty, or to affift us to hear perfons who fpeak at a great diftance.

Inftruments of this kind are formed of tubes, with a wide mouth, and terminating in a fmall canal, which is applied to the ear. The form of thefe inftruments evidently fhews how they conduce to affift the hearing; for the greater quantity of the weak and languid pulfes of the air being received and collected by the large end of the tube, are reflected to the fmall end, where they are collected and condenfed; thence entering the ear in this condenfed ftate, they ftrike the tympanum with a greater force than they could naturally have done from the ear alone.

Hence it appears, that a fpeaking Trumpet may be applied to the purpofe of a hearing Trumpet, by turning the wide end towards the found, and the narrow end to the ear.

Speaking TRUMPET, is a tube of a confiderable length, from 6 to 15 feet, ufed for fpeaking with to make the voice be heard to a greater diftance.

This tube, which is made of tin, is ftraight throughout its length, but opening to a large aperture outwards, and the other end terminating in a proper fhape and fize to receive both the lips in the act of fpeaking, the fpeaker pufhing his voice or the found outwards, by which means it may be heard at the diftance of a mile or more.

The invention of this Trumpet is held to be modern, and has been afcribed to Sir Samuel Moreland, who called it the *tuba ftentorophonica*, and in a work of the fame name, publifhed at London in 1671, that author gave an account of it, and of feveral experiments made with it. With one of thefe inftruments, of $5\frac{1}{2}$ feet long, 21 inches diameter at the greater end, and 2 inches at the fmaller, tried at Deal-Caftle, the fpeaker was heard to the diftance of 3 miles, the wind blowing from the fhore.

But it feems that Kircher has a better title to the invention; for it is certain that he had fuch an inftrument before ever Moreland thought of his. That author, in his Phonurgia Nova, publifhed in 1673, fays, that the tromba, publifhed laft year in England, he invented 24 years before, and publifhed in his Mefurgia. He adds, that Jac. Albanus Ghibbifius and Fr. Efchinardus afcribe it to him; and that G. Schottus teftifies of him, that he had fuch an inftrument in his chamber in the Roman

Roman college, with which he could call to, and receive anſwers from the porter.

But, conſidering how famous the tube or horn of Alexander the Great was, it is rather ſtrange that the Moderns ſhould pretend to the invention. With his ſtentorophonic horn or tube he uſed to ſpeak to his army, and make himſelf be diſtinctly heard, it is ſaid, 100 ſtadia or furlongs. A figure of this tube is preſerved in the Vatican; and it is nearly the ſame as that now in uſe. See STENTOROPHONIC.

The principle of this inſtrument is obvious; for as ſound is ſtronger in proportion to the denſity of the air, it follows that the voice in paſſing through a tube, or Trumpet, muſt be greatly augmented by the conſtant reflection and agitation of the air through the length of the tube, by which it is condenſed, and its action on the external air greatly increaſed at its exit from the tube.

It has been found, that a man ſpeaking through a tube of 4 feet long, may be underſtood at the diſtance of 500 geometrical paces; with a tube 16½ feet, at the diſtance of 1800 paces; and with a tube 24 feet long, at more than 2500 paces.

Although ſome advantage in heightening the ſound, both in ſpeaking and hearing, be derived from the ſhape of the tube, and the width of the outer end, yet the effect depends chiefly upon its length. As to the form of it, ſome have aſſerted that the beſt figure is that which is formed by the revolution of a parabola about its axis; the mouth-piece being placed in the focus of the parabola, and conſequently the ſonorous rays reflected parallel to the axis of the tube. But Mr. Martin obſerves, that this parallel reflection is by no means eſſential to increaſing the ſound: on the contrary, it prevents the infinite number of reflections and reciprocations of ſound, in which, according to Newton, its augmentation chiefly conſiſts; the augmentation of the impetus of the pulſes of air being proportional to the number of repercuſſions from the ſides of the tube, and therefore to its length, and to ſuch a figure as is moſt productive of them. Hence he infers, that the parabolic Trumpet is the moſt unfit of any for this purpoſe; and he endeavours to ſhew, that the logarithmic or logiſtic curve gives the beſt form, viz, by a revolution about its axis. Martin's Philoſ. Brit. vol. 2, pa. 248, 3d edit.

But Caſſegrain is of opinion that an hyperbola, having the axis of the tube for an aſymptote, is the beſt figure for this inſtrument. Muſſchenb. Intr. ad Phil. Nat. tom. 2, pa. 926, 4to.

For other conſtructions of Speaking Trumpets, by Mr. Conyers, ſee Philoſ. Tranſ. numb. 141, for 1678.

TRUNCATED *Pyramid* or *Cone*, is the fruſtum of one, being the part remaining at the bottom, after the top is cut off by a plane parallel to the baſe. See FRUSTUM.

TRUNNIONS, of a piece of ordnance, are thoſe knobs or ſhort cylinders of metal on the ſides, by which it reſts on the cheeks of the carriage.

TRUNNION-*Ring*, is the ring about a cannon, next before the Trunnions.

TSCHIRNHAUSEN (ERNFROY WALTER), an ingenious mathematician, lord of Killingſwald and of Stolzenberg in Luſatia, where he was born in 1651. After having ſerved as a volunteer in the army of Holland in 1672, he travelled into moſt parts of Europe, as England, Germany, Italy, France, &c. He went to Paris for the third time in 1682; where he communicated to the Academy of Sciences, the diſcovery of the curves called, from him, Tſchirnhauſen's Cauſtics; and the Academy in conſequence elected the inventor one of its foreign members. On returning to Italy, he was deſirous of perfecting the ſcience of optics; for which purpoſe he eſtabliſhed two glaſs-works, from whence reſulted many new improvements in dioptrics and phyſics, particularly the noted burning-glaſs which he preſented to the regent.—It was to him too that Saxony owed its porcelane manufactory.

Content with the enjoyment of literary fame, Tſchirnauſen refuſed all other honours that were offered him. Learning was his ſole delight. He ſearched out men of talents, and gave them encouragement. He was often at the expence of printing the uſeful works of other men, for the benefit of the public; and died, beloved and regretted, the 11th of September 1708.

Tſchirnauſen wrote, *De Medicina Mentis & Corporis*, printed at Amſterdam in 1687. And the following memoirs were printed in the volumes of the Academy of Sciences.

1. Obſervations on Burning Glaſſes of 3 or 4 feet diameter: vol. 1699.
2. Obſervations on the Glaſs of a Teleſcope, convex on both ſides, of 32 feet focal diſtance; 1700.
3. On the Radii of Curvature, with the finding the Tangents, Quadratures, and Rectifications of many curves; 1701.
4. On the Tangents of Mechanical Curves; 1702.
5. On a method of Quadratures; 1702.

TUBE, a pipe, conduit, or canal; being a hollow cylinder, either of metal, wood, glaſs, or other matter, for the conveyance of air, or water, &c.

The term is chiefly applied to thoſe uſed in phyſics, aſtronomy, anatomy, &c. On other ordinary occaſions, we more uſually ſay *pipe*.

In the memoirs of the French Academy of Sciences, Varignon has given a treatiſe on the proportions for the diameters of tubes, to give any particular quantities of water. The reſult of his paper gives theſe two analogies, viz, that the diminutions of the velocity of water, occaſioned by its friction againſt the ſides of Tubes, are as the diameters; the Tubes being ſuppoſed equally long: and the quantities of water iſſuing out at the Tubes, are as the ſquare roots of their diameters, deducting out of them the quantity that each is diminiſhed.

TUBE, in Aſtronomy, is ſometimes uſed for teleſcope; but more properly for that part of it into which the lenſes are fitted, and by which they are directed and uſed.

TUESDAY, the 3d day of the week, fo called from Tuefco, one of the Saxon Gods, fimilar to Mars; for which reafon the aftronomical mark for this day of the week, is ♂.

TUMBREL, is a kind of carriage with two wheels, ufed either in Hufbandry for dung, or in Artillery to carry the tools of the pioneers, &c, and fometimes likewife the money of an army.

TUN, is a meafure for liquids, as wine, oil, &c.

The Englifh Tun contains 2 pipes, or 4 hogfheads, or 252 gallons.

TUNE, or Tone, in Mufic, is that property of founds by which they come under the relation of acute and grave.

If two or more founds be compared together in this relation, they are either equal or unequal in the degree of Tune: fuch as are equal, are called *unifons*. The unequal conftitute what are called *intervals*, which are the differences of Tone between founds.

Sonorous bodies are found to differ in Tone: 1ft, According to the different kinds of matter; thus the found of a piece of gold, is much graver than that of a piece of filver of the fame fhape and dimenfions. 2d, According to the different quantities of the fame matter in bodies of the fame figure; as a folid fphere of brafs of 1 foot diameter, founds acuter than a fphere of brafs of 2 feet diameter.

But the meafures of Tone are only to be fought in the relations of the motions that are the caufe of found, which are moft difcernible in the vibration of chords. Now, in general, we find that in two chords, all things being equal, excepting the tenfion, the thicknefs, or the length, the Tones are different; which difference can only be in the velocity of their vibratory motions, by which they perform a different number of vibrations in the fame time; as it is known that all the fmall vibrations of the fame chord are performed in equal times. Now the frequenter or quicker thofe vibrations are, the more acute is the Tone; and the flower and fewer they are in the fame fpace of time, by fo much the more grave is the Tone. So that any given note of a Tune is made by one certain meafure of velocity of vibrations, that is, fuch a certain number of vibrations of a chord or ftring, in fuch a certain fpace of time, conftitutes a determinate Tone.

This theory is ftrongly fupported by the beft and lateft writers on mufic, Holder, Malcolm, Smith, &c, both from reafon and experience. Dr. Wallis, who owns it very reafonable, adds, that it is evident the degrees of acutenefs are reciprocally as the lengths of the chords; though, he fays, he will not pofitively affirm that the degrees of acutenefs anfwer the number of vibrations, as their only true caufe: but his diffidence arifes from hence, that he doubts whether the thing has been fufficiently confirmed by experiment.

TUNNAGE. See Tonnage.

TURN, is ufed for a circular motion; in which fenfe it agrees with revolution.

Turn, in Clock or Watch-work, particularly denotes the revolution of a wheel or pinion.

In calculation, the number of Turns which the pinion hath, is denoted in common arithmetic thus, 5) 60 (12, where the pinion 5, playing in a wheel of 60, moves round 12 times in one Turns of the wheel. Now by knowing the number of Turns which any pinion hath, in one Turn of the wheel it works in, you may eafily find how many Turns a wheel or pinion has at a greater diftance; as the contrat-wheel, crown-wheel, &c, by multiplying together the quotients, and the number produced is the number of Turns, as in the example here annexed: the firft of thefe three numbers has 11 Turns, the next 9, and the laft 8; if you multiply 11 by 9, it produces 99; that is, in one Turn of the wheel 55, there are 99 Turns of the fecond pinion 5, or the wheel 40, which runs concentrical or on the fame arbor with the fecond pinion 5: and if you again multiply 99 by the laft quotient 8, it produces 792, which is the number of Turns the third pinion 5 hath. See Clock-*work*, and Pinion.

5) 55 (11
5) 45 (9
5) 40 (8

TURNING *to windward*, in Sea Language, denotes that operation in failing when a fhip endeavours to make a progrefs againft the direction of the wind, by a compound courfe, inclined to the place of her deftination.—This method of navigation is otherwife called *plying to windward*.

TUSCAN *Order*, in Architecture, is the firft, the fimpleft, and the ftrongeft or moft maffive of any. Its column has 7 diameters in height; and its capital, bafe, and entablement, have no ornaments, and but few mouldings.

TWELFTH-*Day*, the feftival of the Epiphany, or the manifeftation of Chrift to the Gentiles, fo called, as being the Twelfth day, exclufive, from the nativity or Chriftmas-day; of courfe it falls always on the 6th day of January.

TWILIGHT, in Aftronomy, is that faint light which is perceived before the fun-rifing, and after fun-fetting. The Twilight is occafioned by the earth's atmofphere refracting the rays of the fun, and reflecting them among its particles.

The depreffion of the fun below the horizon, at the beginning of the morning, and end of the evening Twilight, has been varioufly ftated, at different feafons, and by different obfervers: by Alhazen it was obferved to be 19°; by Tycho 17°; by Rothman 24°; by Stevinus 18°; by Caffini 15°; by Riccioli, at the time of the equinox in the morning 16°, in the evening $20\frac{1}{2}°$; in the fummer folftice in the morning 21° 25′, and in the winter 17° 15′. Whence it appears that the caufe of the Twilight is variable; but, on a medium, about 18° of the fun's depreffion will ferve tolerably well for our latitude, for the beginning and end of Twilight, and according to which Dr. Long, (in his Aftronomy, vol. 1, pa. 258) gives the following Table, of the duration of Twilight, in different latitudes, and for feveral different declinations of the fun.

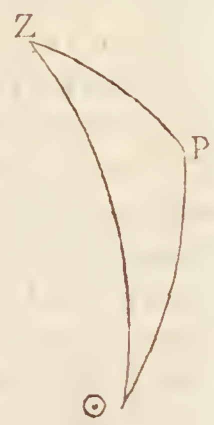
Latitude.

Latitude.	0	10	20	30	40	45	50	52½	55	60	65	70	75	80	85	90
⊙ Enters	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
♋	1 18	1 21	1 28	1 41	2 8	2 39	w n	w n	w n	w n	w n	c d	c d	c d	c d	c d
♊ ♌	1 16	1 19	1 25	1 36	1 58	2 19	3 3	w n	w n	w n	w n	c d	c d	c d	c d	c d
♉ ♍	1 13	1 15	1 20	1 28	1 43	1 55	2 12	2 25	2 41	3 55	w n	w n	w n	c d	c d	c d
♈ ♎	1 12	1 13	1 17	1 24	1 35	1 44	1 55	2 2	2 10	2 33	3 8	4 18	w n	w n	w n	w n
♓ ♏	1 13	1 14	1 18	1 24	1 35	1 43	1 54	2 0	2 8	2 27	2 56	8 41	5 2	17 32	w n	w n
♒ ♐	1 16	1 17	1 21	1 28	1 40	1 49	2 1	2 8	2 18	2 43	3 26	11 38	11 14	10 32	8 38	c n
♑	1 18	1 19	1 23	1 30	1 43	1 53	2 6	2 15	2 26	2 57	4 4	10 24	9 30	7 46	c n	c n

Where *c d* signify that it is then continual day, *c n* continual night, and *w n* that the Twilight lasts the whole night.

Prob.—To find the Beginning or End of Twilight.

In this problem, there are given the sides of an oblique spherical triangle, to find an angle; viz, given the side ZP the colatitude of the place; P⊙ the codeclination, or polar distance; and Z⊙ the zenith distance, which is always equal to 108°, viz, 90° from the zenith to the horizon, and 18° more for the sun's distance below the horizon. For example, suppose the place London in latitude 51° 32′, and the time the 1st of May, when the sun's declination is 15° 12′ north. Here then ZP = 38° 28′ the complement of 51° 32′, and P⊙ = 74° 48′, the complement of 15° 12′. Then the calculation is as follows.



$$P\odot = 74^\circ\ 48'$$
$$PZ = 38\ \ 28$$

$$P\odot - PZ = 36\ \ 20 = D$$
$$Z\odot = 108\ \ 00$$

$$Z\odot + D = 144\ \ 20 \quad | \quad 72^\circ\ 10' = \tfrac{1}{2}\ Z\odot + D$$
$$2\,)\overline{\qquad}$$
$$Z\odot - D = 71\ \ 40 \quad | \quad 35\ \ 50 = \tfrac{1}{2}\ Z\odot - D$$

Then,

Co-ar. sin. polar dist.	= 74° 48′	0·01547
Co-ar. sin. colat.	= 38 28	0·20617
Sine ½ $\overline{Z\odot + D}$	= 72 10	9·97861
Sine ½ $\overline{Z\odot - D}$	= 35 50	9·76747
Sum of these four logs.		19·96772
Half sum gives	74° 28½′	9·98386

Which doubled gives 148 57 for the angle ZPO.

This 148° 57′ reduced to time, at the rate of 15° per hour, gives $9^h\ 55^m\ 48^s$, either before or after noon; that is, the twilight begins at $2^h\ 4^m\ 12^s$ in the morning, and ends at $9^h\ 55^m\ 48^s$ in the evening on the given day at London.

TWINKLING *of the Stars*, denotes that tremulous motion which is observed in the light proceeding from the fixed stars.

This Twinkling in the stars has been variously accounted for. Alhazen, a Moorish philosopher of the 12th century, considers refraction as the cause of this phenomenon.

Vitello, in his Optics, (composed before the year 1270) pa. 449, ascribes the Twinkling of the stars to the motion of the air, in which the light is refracted; and he observes, in confirmation of this hypothesis, that they Twinkle still more when they are viewed in water put into motion.

Dr. Hook (Microgr. pa. 231, &c) ascribes this phenomenon to the inconstant and unequal refraction of the rays of light, occasioned by the trembling motion of the air and interspersed vapours, in consequence of variable degrees of heat and cold in the air, producing corresponding variations in its density, and also of the action of the wind, which must cause the successive rays to fall upon the eye in different directions, and consequently upon different parts of the retina at different times, and also to hit and miss the pupil alternately; and this also is the reason, he says, why the limbs of the sun, moon, and planets appear to wave or dance.

These tremors of the air are manifest to the eye by the tremulous motion of shadows cast from high towers; and by looking at objects through the smoke of a chimney, or through steams of hot water, or at objects situated beyond hot sands, especially if the air be moved transversely over them. But when stars are seen through telescopes that have large apertures; they Twinkle but little, and sometimes not at all. For, as Newton has observed, (Opt. pa. 98) the rays of light which pass through different parts of the aperture, tremble each of them apart, and by means of their various, and contrary tremors, fall at one and the same time

time upon different points in the bottom of the eye, and their trembling motions are too quick and confused to be separately perceived. And all these illuminated points constitute one broad lucid point, composed of those many trembling points confusedly and insensibly mixed with one another by very short and swift tremors, and so cause the star to appear broader than it is, and without any trembling of the whole.

Dr. Jurin, in his Essay upon Distinct and Indistinct Vision, has recourse to Newton's hypothesis of fits of easy refraction and reflection for explaining the Twinkling of the stars: thus, he says, if the middle part of the image of a star be changed from light to dark, and the adjacent ring at the same time be changed from dark to light, as must happen from the least motion of the eye towards or from the star, this will occasion such an appearance as Twinkling.

Mr. Michell (Philos. Trans. vol. 57, pa. 262) supposes that the arrival of fewer or more rays at one time, especially from the smaller or more remote fixed stars, may make such an unequal impression on the eye, as may at least have some share in producing this effect: since it may be supposed that even a single particle of light is sufficient to make a sensible impression on the organs of sight; so that very few particles arriving at the eye in a second of time, perhaps not more than three or four, may be sufficient to make an object constantly visible. See LIGHT.

Hence, he says, it is not improbable that the number of the particles of light which enter the eye in a second of time, even from Syrius himself, may not exceed 3 or 4 thousand, and from stars of the 2d magnitude they may probably not exceed 100. Now the apparent increase and diminution of the light, which we observe in the Twinkling of the stars, seem to be repeated at intervals not very unequal, perhaps about 4 or 5 times in a second. He therefore thought it reasonable to suppose, that the inequalities which will naturally arise from the chance of the rays coming sometimes a little denser, and sometimes a little rarer, in so small a number of them, as must fall upon the eye in the 4th or 5th part of a second, may be sufficient to account for this appearance.

Since these observations were published however, Mr. Michell (as we are informed by Dr. Priestley in his Hist. of Light, pa. 495) has entertained some suspicion, that the unequal density of light does not contribute to this effect in so great a degree as he had imagined; especially as he has observed that even Venus does sometimes Twinkle. This he once observed her to do remarkably when she was about 6 degrees high, though Jupiter, which was then about 16 degrees high, and was sensibly less luminous, did not Twinkle at all. If, notwithstanding the great number of rays which doubtless come to the eye from such a surface as this planet presents, its appearance be liable to be affected in this manner, it must be owing to such undulations in the atmosphere, as will probably render the effect of every other cause altogether insensible.

Musschenbroek suspects (Introd. ad Phil. Nat. vol. 2, sect. 1741, pa. 707) that the Twinkling of the stars arises from some affection of the eye, as well as the state of the atmosphere. For, says he, in Holland, when the weather is frosty, and the sky very clear, the stars Twinkle most manifestly to the naked eye, though not in telescopes; and since he does not suppose there is any great exhalation, or dancing of the vapour, at that time, he questions whether the vivacity of the light, affecting the eye, may not be concerned in the phenomenon.

But this philosopher might have satisfied himself with respect to this hypothesis, by looking at the stars near the zenith, when the light traverses but a small part of the atmosphere, and therefore might be expected to affect the eye most sensibly. For he would have found that they do not Twinkle near so much as they do near the horizon, when much more of their light is intercepted by the atmosphere.

Some astronomers have lately endeavoured to explain the Twinkling of the fixed stars, by the extreme minuteness of their apparent diameter; so that they suppose the sight of them is intercepted by every mote that floats in the air. To this purpose Dr. Long observes (Astron. vol. 1, pa. 170) that our air near the earth is so full of various kinds of particles, which are in continual motion, that some one or other of them is perpetually passing between us and any star we look at, which makes us every moment alternately see it and lose sight of it: and this Twinkling of the stars, he says, is greatest in those that are nearest the horizon, because they are viewed through a great quantity of thick air, where the intercepting particles are most numerous; whereas stars that are near the zenith do not Twinkle so much, because we do not look at them through so much thick air, and therefore the intercepting particles, being fewer, come less frequently before them. With respect to the planets, it is observed that, because they are much nearer to us than the stars, they have a sensible apparent magnitude, so that they are not covered by the small particles floating in the atmosphere, and therefore do not Twinkle, but shine with a steady light.

The fallacy of this hypothesis appears from the observation of Mr. Michell, that no object can hide a star from us that is not large enough to exceed the apparent diameter of the star, by the diameter of the pupil of the eye; so that if a star were even a mathematical point, or of no diameter, the interposing object must still be equal in size to the pupil of the eye; and indeed it must be large enough to hide the star from both eyes at the same time.

The principal cause therefore of the Twinkling of the stars, is now acknowledged to be the unequal refraction of light, in consequence of inequalities and undulations in the atmosphere.

Besides a variation in the quantity of light, it may here be added, that a momentary change of colour has likewise been observed in some of the fixed stars. Mr. Melville (Edinb. Essays, vol. 2, pa. 81) says, that when one looks steadfastly at Sirius, or any bright star, not much elevated above the horizon, its colour seems not to be constantly white, but appears tinctured, at every Twinkling, with red and blue. Mr. Melville could not entirely satisfy himself as to the cause of this phenomenon;

nomenon; observing that the separation of the colours by the refractive power of the atmosphere, is probably too small to be perceived. Mr. Michell's hypothesis above mentioned, though not adequate to the explication of the Twinkling of the stars, may pretty well account for this circumstance. For the red and blue rays being much fewer than those of the intermediate colours, and therefore much more liable to inequalities from the common effect of chance, a small excess or defect in either of them will make a very sensible difference in the colour of the stars.

TYCHONIC *System*, or *Hypothesis*, is an order or arrangement of the heavenly bodies, of an intermediate nature between the Copernican and Ptolomaic; and is so called from its inventor Tycho Brahe. See System.

TYMPAN, or Tympanum, in Architecture, is the area of a pediment, being that part which is on a level with the naked of the frize. Or it is the space included between the three cornices of a triangular pediment, or the two cornices of a circular one.

Tympan is also used for that part of a pedestal called the *trunk* or *dye*.

Tympan, among Joiners, is also applied to the pannels of doors.

Tympan *of an Arch*, is a triangular space or table in the corners of sides of an arch, usually hollowed and enriched, sometimes with branches of laurel, olive-tree, or oak; or with trophies, &c; sometimes with flying figures, as fame, &c; or sitting figures, as the cardinal virtues.

Tympan, in Mechanics, is a kind of wheel placed round an axis, or cylindrical beam, on the top of which are two levers, or fixed staves, for more easily turning the axis about, in order to raise a weight. The Tympanum is much the same with the peritrochium; but that the cylinder of the axis of the peritrochium is much shorter and less than the cylinder of the Tympanum.

Tympanum of a machine, is also used for a hollow wheel, in which people or animals walk, to turn it; such as that of some cranes, calenders, &c.

TYR, in the Ethiopian Calendar, the name of the 5th month of the Ethiopian year. It commences on the 25th of December of the Julian year.

TYSHAS, among the Ethiopians, the name of the 4th month of their year, commencing the 27th of November in the Julian year.

U and V.

V Is a numeral letter, in the Roman numeration, denoting 5 or five. And with a dash over the top thus $\overline{V}$, it denoted 5000.

VACUUM, in Physics, a space empty or devoid of all matter.

Whether there be any such thing in nature as an absolute Vacuum; or whether the universe be completely full, and there be an absolute plenum; is a question that has been agitated by the philosophers of all ages.

The Ancients, in their controversies, distinguished two kinds; a *Vacuum coacervatum*, and a *Vacuum interspersum*, or *disseminatum*.

Vacuum *Coacervatum*, is conceived as a considerably large space destitute of matter; such, for instance, as there would be, should God annihilate all the air, and other bodies, within the walls of a chamber.

The existence of such a Vacuum is maintained by the Pythagoreans, Epicureans, and the Atomists or Corpuscularians; most of whom assert, that such a Vacuum actually exists without the limits of the sensible world. But the modern Corpuscularians, who hold a *Vacuum coacervatum*, deny that appellation; as conceiving that such a Vacuum must be infinite, eternal, and uncreated.

According then to the later philosophers, there is no Vacuum coacervatum without the bounds of the sensible world; nor would there be any other Vacuum, provided God should annihilate divers contiguous bodies, than what amounts to a mere privation, or nothing; the dimensions of such a space, which the Ancients held to be real, being by these held to be mere negations; that is, in such a place there is so much length, breadth, and depth wanting, as a body must have to fill it. To suppose then, that when all the matter in a chamber is annihilated, there should yet be real dimensions, is to suppose, say they, corporeal dimensions without body; which is absurd.

The Cartesians however deny any *Vacuum coacervatum* at all, and assert that if God should immediately annihilate all the matter, for example in a chamber, and prevent the ingress of any other matter, the consequence would be, that the walls would become contiguous, and include no space at all. They add, that if there be no matter in a chamber, the walls cannot be conceived otherwise than as contiguous; those things being

being said to be contiguous, between which there is not any thing intermediate: but if there be no body between, there is, say they, no extension between; extension and body being the same thing: and if there be no extension between, then the walls are contiguous; and where is the Vacuum?——But this reasoning, or rather quibbling, is founded on the mistake, that body and extension are the same thing.

VACUUM *Disseminatum*, or *Interspersum*, is that supposed to be naturally interspersed in and among bodies, in the interstices between different bodies, and in the pores of the same body.

It is this kind of Vacuum which is chiefly contested among the modern philosophers; the Corpuscularians strenuously asserting it; and the Peripatetics and Cartesians as tenaciously denying it. See CARTESIAN and LEIBNITZIAN.

The great argument urged by the Peripatetics against a Vacuum interspersum, is, that there are divers bodies frequently seen to move contrary to their own nature and inclination; and that for no other apparent reason, but to avoid a Vacuum: whence they conclude, that nature abhors a Vacuum; and give us a new class of motions ascribed to the *fuga vacui* or nature's flying a Vacuum. Such, they say, is the rise of water in a syringe, upon the drawing up of the piston; and such is the ascent of water in pumps, and the swelling of the flesh in a cupping glass, &c.—But since the weight, elasticity, &c, of the air have been ascertained by sure experiments, those motions and effects are universally, and justly, ascribed to the gravity and pressure of the atmosphere.

The Cartesians deny, not only the actual existence, but even the possibility of a Vacuum; and that on this principle, that extension being the essence of matter, or body, wherever extension is, there is matter; but mere space, or vacuity, is supposed to be extended; therefore it is material. Whoever asserts an empty space, say they, conceives dimensions in that space, i. e. he conceives an extended substance in it; and therefore he denies a Vacuum, at the same time that he admits it.—But Descartes, if we may believe some accounts, rejected a Vacuum from a complaisance to the taste which prevailed in his time, against his own first sentiments; and among his familiar friends he used to call his system his philosophical romance.

On the other hand, the corpuscular authors prove, not only the possibility, but the actual existence, of a Vacuum, from divers considerations; particularly from the consideration of motion in general; and that of the planets, comets, &c, in particular; as also from the fall of bodies; from the vibration of pendulums; from rarefaction and condensation; from the different specific gravities of bodies; and from the divisibility of matter into parts.

1. First, there could be no linear or progressive motion without a Vacuum; for if all space were full of matter, no body could be moved out of its place, for want of another place unoccupied, to move into. And this argument was stated even by Lucretius.

2. The motions of the planets and comets also prove a Vacuum. Thus, Newton argues, "that there is no such fluid medium as æther," (to fill up the porous parts of all sensible bodies, and so make a plenum), seems probable; because the planets and comets proceed with so regular and lasting a motion, through the celestial spaces; for hence it appears that those celestial spaces are void of all sensible resistance, and consequently of all sensible matter. Consequently if the celestial regions were as dense as water, or as quicksilver, they would resist almost as much as water or quicksilver; but if they were perfectly dense, without any interspersed vacuity, though the matter were ever so fluid and subtle, they would resist more than quicksilver does: a perfectly solid globe, in such a medium, would lose above half its motion, in moving 3 lengths of its diameter; and a globe not perfectly solid, such as the bodies of the planets and comets are, would be stopped still sooner. Therefore, that the motion of the planets and comets may be regular, and lasting, it is necessary that the celestial spaces be void of all matter; except perhaps some few and much rarefied effluvia of the planets and comets, and the passing rays of light."

3. The same great author also deduces a Vacuum from the consideration of the weights of bodies; thus: "All bodies about the earth gravitate towards it; and the weights of all bodies, equally distant from the earth's centre, are as the quantities of matter in those bodies. If the æther therefore, or any other subtile matter, were altogether destitute of gravity, or did gravitate less than in proportion to the quantity of its matter; because (as Aristotle, Descartes, and others, argue) it differs from other bodies only in the form of matter; the same body might, by the change of its form, gradually be converted into a body of the same constitution with those which gravitate most in proportion to the quantity of matter: and, on the other hand, the heaviest bodies might gradually lose their gravity, by gradually changing their form; and so the weights would depend upon the forms of bodies, and might be changed with them; which is contrary to all experiment."

4. The descent of bodies proves, that all space is not equally full; for the same author goes on, "If all spaces were equally full, the specific gravity of that fluid with which the region of the air would, in that case, be filled, would not be less than the specific gravity of quicksilver or gold, or any other the most dense body; and therefore neither gold, nor any other body, could descend in it. For bodies do not descend in a fluid, unless that fluid be specifically lighter than the body. But by the air-pump we can exhaust a vessel, till even a feather shall fall with a velocity equal to that of gold in the open air; and therefore the medium through which this feather falls, must be much rarer than that through which the gold falls in the other case. The quantity of matter therefore in a given space may be diminished by rarefaction: and why may it not be diminished ad infinitum? Add, that we conceive the solid particles of all bodies to be of the same density; and that they are only rarefiable by means of their pores; and hence a Vacuum evidently follows."

5. "That there is a Vacuum, is evident too from the vibrations of pendulums: for since those bodies, in places out of which the air is exhausted, meet with no resistance to retard their motion, or shorten their vibrations; it is evident that there is no sensible matter in those spaces, or in the occult pores of those bodies."

6. That

6. That there are interſperſed vacuities, appears from matter's being actually divided into parts, and from the figures of thoſe parts; for, on ſuppoſition of an abſolute plenum, we do not conceive how any part of matter could be actually divided from that next adjoining, any more than it is poſſible to divide actually the parts of abſolute ſpace from one another: for by the actual diviſion of the parts of a continuum from one another, we conceive nothing elſe underſtood, but the placing of thoſe parts at a diſtance from one another, which in the continuum were at no diſtance from one another: but ſuch diviſions between the parts of matter muſt imply vacuities between them.

7. As for the figures of the parts of bodies, upon the ſuppoſition of a plenum, they muſt either be all rectilinear, or all concavo-convex; otherwiſe they would not adequately fill ſpace; which we do not find to be true in fact.

8. The denying a Vacuum ſuppoſes what it is impoſſible for any one to prove to be true, viz, that the material world has no limits.

However, we are told by ſome, that it is impoſſible to conceive a Vacuum. But this ſurely muſt proceed from their having imbibed Deſcartes's doctrine, that the eſſence of body is conſtituted by extenſion; as it would be contradictory to ſuppoſe ſpace without extenſion. To ſuppoſe that there are fluids penetrating all bodies and repleniſhing ſpace, which neither reſiſt nor act upon bodies, merely in order to avoid admitting a Vacuum, is feigning two ſorts of matter, without any neceſſity or foundation; or is tacitly giving up the queſtion.

Since then the eſſence of matter does not conſiſt in extenſion, but in ſolidity, or impenetrability, the univerſe may be ſaid to conſiſt of ſolid bodies moving in a Vacuum: nor need we at all fear, leſt the phenomena of nature, moſt of which are plauſibly accounted for from a plenum, ſhould become inexplicable when the plenitude is ſet aſide. The principal ones, ſuch as the tides; the ſuſpenſion of the mercury in the barometer; the motion of the heavenly bodies, and of light, &c, are more eaſily and ſatisfactorily accounted for from other principles.

VACUUM *Boileanum*, is uſed to expreſs that approach to a real Vacuum, which we arrive at by means of the air-pump. Thus, any thing put in a receiver ſo exhauſted, is ſaid to be put *in vacuo:* and thus moſt of the experiments with the air-pump are ſaid to be performed *in vacuo*, or *in vacuo Boileano*.

Some of the principal phenomena obſerved of bodies in vacuo, are; that the heavieſt and lighteſt bodies, as a guinea and a feather, fall here with equal velocity:—that fruits, as grapes, cherries, peaches, apples, &c, kept for any time in vacuo, retain their nature, freſhneſs, colour, &c, and thoſe withered in the open air recover their plumpneſs in vacuo:—all light and fire become immediately extinct in vacuo:—little or no ſound is heard from a bell rung in vacuo:—a bladder half full of air, will diſtend the bladder, and lift up 40 pound weight in vacuo:—moſt animals ſoon expire in vacuo.

By experiments made in 1704, Dr. Derham found that animals which have two ventricals, and no foramen ovale, as birds, dogs, cats, mice, &c, die in leſs than half a minute; counting from the firſt exſuction: a mole died in one minute; a bat lived 7 or 8. Inſects, as waſps, bees, graſshoppers, &c, ſeemed dead in two minutes; but after being left in vacuo 24 hours, they came to life again in the open air: ſnails continued 24 hours in vacuo, without appearing much incommoded.—Seeds planted in vacuo do not grow: Small beer dies, and loſes all its taſte, in vacuo: And air ruſhing through mercury into a Vacuum, throws the mercury in a kind of ſhower upon the receiver, and produces a great light in a dark room.

The air-pump can never produce a perfect Vacuum; as is evident from its ſtructure, and the manner of its working: in effect, every exſuction only takes away a part of the air; ſo that there is ſtill ſome left after any finite number of exſuctions. For the air-pump has no longer any effect but while the ſpring of the air remaining in the receiver is able to lift up the valves; and when the rarefaction is come to that degree, you can come no nearer to a Vacuum; unleſs perhaps the air valves can be opened mechanically, independent of the ſpring of the air, as it is ſaid they are in ſome new improved air-pumps.

Torricellian VACUUM, is that made in the barometer tube, between the upper end and the top of the mercury. This is perhaps never a perfect and entire Vacuum; as all fluids are found to yield or to riſe in elaſtic vapours, on the removal of the preſſure of the atmoſphere. See TORRICELLIAN, and BAROMETER.

VALVE, in Hydraulics, Pneumatics, &c, is a kind of lid or cover to a tube or veſſel, contrived to open one way; but which, the more forcibly it is preſſed the other way, the cloſer it ſhuts the aperture: ſo that it either admits the entrance of a fluid into the tube, or veſſel, and prevents its return; or permits it to eſcape, and prevents its re-entrance.

Valves are of great uſe in the air-pump, and other wind machines; in which they are uſually made of pieces of bladder. In hydraulic engines, as the emboli of pumps, they are moſtly of ſtrong leather, of a round figure, and fitted to ſhut the apertures of the barrels or pipes. Sometimes they are made of two round pieces of leather encloſed between two others of braſs; having divers perforations, which are covered with another piece of braſs, moveable upwards and downwards, on a kind of axis, which goes through the middle of them all. Sometimes they are made of braſs, covered over with leather, and furniſhed with a fine ſpring, which gives way upon a force applied againſt it; but upon the ceaſing of that, returns the Valve over the aperture. See PUMP. See alſo Deſaguliers' Exper. Philoſ. vol. 2, p. 156, and p. 180.

VANE, in a ſhip, &c, a thin ſlip of ſome kind of matter, placed on high in the open air, turning eaſily round on an axis or ſpindle, and veered about by the wind, to ſhew its direction or courſe.

VANES, in Mathematical or Philoſophical Inſtruments, are ſights made to ſlide and move upon croſs-ſtaves, fore-ſtaves, quadrants, &c.

VAPOUR, in Meteorology, a watery exhalation raiſed up either by the heat of the ſun, or any other heat, as fire, &c. Vapour is conſidered as a thin veſicle of water, or other humid matter, filled or inflated with air; which, being rarefied to a certain degree by the action of heat, aſcends to ſome height in the atmo-

atmosphere, where it is suspended, till it returns in form of rain, snow, or the like. An assemblage of a number of particles or vesicles of vapour, constitutes what is called a cloud.

Some use the term Vapour indifferently, for all fumes emitted, either from moist bodies, as fluids of any kind; or from dry bodies, as sulphur, &c. But Newton, and other authors, better distinguish between humid and dry fumes, calling the latter *exhalations.*

For the manner in which Vapours are raised, and again precipitated, see CLOUD, DEW, RAIN, BAROMETER, and particularly EVAPORATION.

It may here be added, with respect to the principles of solution adopted to account for evaporation, and largely illustrated under that article, that Dr. Halley, about the beginning of the present century, seems to have been acquainted with the solvent power of air on water; for he says, that supposing the earth to be covered with water, and the sun to move diurnally round it, the air would of itself imbibe a certain quantity of aqueous Vapours, and retain them like salts dissolved in water; and that the air warmed by the sun would sustain a greater proportion of Vapours, as warm water will hold more dissolved salts; which would be discharged in dews, similar to the precipitation of salts on the cooling of liquors. Philos. Trans. Abr. vol. 2, p. 127.

Mr. Eeles, in 1755, endeavoured to account for the ascent of Vapour and exhalation, and their suspension in the atmosphere, by means of the electric fire. The sun, he acknowledges, is the great agent in detaching Vapour and exhalations from their masses, whether he acts immediately by himself, or by his rendering the electric fire more active in its vibrations: but their subsequent ascent he attributes entirely to their being rendered specifically lighter than the lower air, by their conjunction with electrical fire: each particle of Vapour, with the electrical fluid that surrounds it, occupying a greater space than the same weight of air. Mr. Eeles also endeavours to shew, that the ascent and descent of Vapour, attended by this fire, are the cause of all the winds, and that they furnish a satisfactory solution of the general phenomena of the weather and barometer. Philos. Trans. vol. 49, pa. 124.

Dr. Darwin, in 1757, published remarks on the theory of Mr. Eeles, with a view of confuting it; and attempting to account for the ascent of Vapours, by considering the power of expansion which the constituent parts of some bodies acquire by heat, and also that some bodies have a greater affinity to heat, or acquire it sooner, and retain it longer, than others. On these principles, he thinks, it is easily understood how water, whose parts appear from the æolipile to be capable of immeasurable expansion, should by heat alone become specifically lighter than the common atmosphere. A small degree of heat is sufficient to detach or raise the Vapour of water from the mass to which it belongs; and the rays of the sun communicate heat only to those bodies by which they are refracted, reflected, or obstructed, whence, by their impulse, a motion or vibration is caused in the parts of such bodies. Hence he infers, that the sphericles of Vapour will, by refracting the solar rays, acquire a constant heat, though the surrounding atmosphere remain cold. If it be asked, how clouds are supported in the absence of the sun? It must be remembered, that large masses of Vapour must for a considerable time retain much of the heat they have acquired in the day; at the same time reflecting how small a quantity of heat was necessary to raise them, and that doubtless even a less will be sufficient to support them; as from the diminished pressure of the atmosphere at a given height, a less power may be able to continue them in their present state of rarefaction; and lastly, that clouds of particular shapes will be sustained or elevated by the motion they acquire from winds. Philos. Trans. vol. 50, p. 246.

For the Effect of Vapour in the Formation of Springs, &c, see SPRING, and RIVER.

The quantity of Vapour raised from the sea by the warmth of the sun, must be far greater than is commonly imagined. Dr. Halley has attempted to estimate it. For the result of his calculations, see EVAPORATION.

VARIABLE, in Geometry and Analytics, is a term applied by mathematicians, to such quantities as are considered in a Variable or changeable state, either increasing or decreasing. Thus, the abscisses and ordinates of an ellipsis, or other curve line, are Variable quantities; because these vary or change their magnitude together, the one at the same time with the other. But some quantities may be Variable by themselves alone, or while those connected with them are constant: as the abscisses of a parallelogram, whose ordinates may be considered as all equal, and therefore constant. Also the diameter of a circle, and the parameter of a conic section, are *constant*, while their abscisses are *Variable.*

Variable quantities are usually denoted by the last letters of the alphabet, z, y, x, &c; while the constant ones are denoted by the leading letters, a, b, c, &c.

Some authors, instead of *Variable* and *constant* quantities, use the terms *fluent* and *stable* quantities.

The indefinitely small quantity by which a Variable quantity is continually increased or decreased, in very small portions of time, is called the *differential*, or *increment* or *decrement*. And the rate of its increase or decrease at any point, is called its *fluxion*; while the Variable quantity itself is called the *fluent*. And the calculation of these, is the subject of the new *Methodus Differentialis*, or *Doctrine of Fluxions.*

VARENIUS (BERNARD), a learned Dutch geographer and physician, of the last century, who was author of the best mathematical treatise on Geography, intitled, *Geographia Universalis*, in qua affectiones generalis Telluris explicantur. This excellent work has been translated into all languages, and was honoured by an edition, with improvements, by Sir Isaac Newton, for the use of his academical students at Cambridge.

VARIATION, *of Quantities*, in Algebra. See CHANGES, and COMBINATION.

VARIATION, in Astronomy.—*The Variation of the Moon*, called by Bulliald, the *Reflection of her Light*, is the third inequality observed in the moon's motion; by which, when out of the quadratures, her true place differs from her place twice equated. See PLACE, EQUATION, &c.

Newton makes the moon's variation to arise partly from the form of her orbit, which is an ellipsis; and partly

partly from the inequality of the ſpaces, which the moon deſcribes in equal times, by a radius drawn to the earth.

To find the Greateſt Variation. Obſerve the moon's longitude in the octants; and to the time of obſervation compute the moon's place twice equated; then the difference between the computed and obſerved place, is the greateſt Variation.

Tycho makes the greateſt Variation 40′ 30″; and Kepler makes it 51′ 49″.—But Newton makes the greateſt Variation, at a mean diſtance between the ſun and the earth, to be 35′ 10″: at the other diſtances, the greateſt Variation is in a ratio compounded of the duplicate ratio of the times of the moon's ſynodical revolution directly, and the triplicate ratio of the diſtance of the ſun from the earth inverſely. And therefore in the ſun's apogee, the greateſt Variation is 33′ 14″, and in his perigee 37′ 11″; provided that the eccentricity of the ſun be to the tranſverſe ſemidiameter of the orbis magnus, as $16\frac{15}{16}$ to 1000. Or, taking the mean motions of the moon from the ſun, as they are ſtated in Dr. Halley's tables, then the greateſt Variation at the mean diſtance of the earth from the ſun will be 35′ 7″, in the apogee of the ſun 33′ 27″, and in his perigee 36′ 51″. Philoſ. Nat. Princ. pr. 29, lib. 3.

VARIATION, in Geography, Navigation, &c, a term applied to the deviation of the magnetic needle, or compaſs, from the true north point, either towards the eaſt or weſt; called alſo the *declination*. Or the Variation of the compaſs is properly defined, the angle which a magnetic needle, ſuſpended at liberty, makes with the meridian line on an horizontal plane; or an arch of the horizon, comprehended between the true and the magnetic meridians.

In the ſea-language, the Variation is uſually called *north-eaſting*, or *north-weſting*.

All magnetic bodies are found to range themſelves, in ſome ſort, according to the meridian; but they ſeldom agree preciſely with it: in one place they decline, from the north toward the eaſt, in another toward the weſt; and that too differently at different times.

The Variation of the compaſs could not long remain a ſecret, after the invention of the compaſs itſelf: accordingly Ferdinand, the ſon of Columbus, in his life written in Spaniſh, and printed in Italian at Venice in 1571, aſſerts, that his father obſerved it on the 14th of September 1492: though others ſeem to attribute the diſcovery of it to Sebaſtian Cabat, a Venetian, employed in the ſervice of our king Henry VII, about the year 1500.—It now appears however, that this Variation or declination of the needle was known even ſome centuries earlier, though it does not appear that the uſe of the needle itſelf in navigation was then known. For it ſeems there is in the library of the univerſity of Leyden, a ſmall manuſcript tract on the Magnet, in Latin, written by one Peter Adſiger, bearing date the 8th of Auguſt 1269; in which the declination of the needle is particularly mentioned. Mr. Cavallo has printed the chief part of this letter in the Supplement to his Treatiſe on Magnetiſm, with a tranſlation; and I think it is to be wiſhed he had printed the whole of ſo curious a paper. The curioſity of this letter, ſays Mr. Cavallo, conſiſts in its containing almoſt all that is at preſent known of the ſubject, at leaſt the moſt remarkable parts of it, mixed however with a good deal of abſurdity. The laws of magnetic attraction, and of the communication of that power to iron, the directive property of the natural magnet, as well as of the iron that has been touched by it, and even the declination of the magnetic needle, are clearly and unequivocally mentioned in it.

As this Variation differs in different places, Gonzales d'Oviedi found there was none at the Azores; from whence ſome geographers thought fit in their maps to make the firſt meridian paſs through one of theſe iſlands: it not being then known that the Variation altered in time. See MAGNET; alſo Gilbert De Magnete, Lond. 1600, p. 4 and 5; or Purchas's Pilgrims, Lond. 1625, book 2, ſect. 1.

Various are the hypotheſes that have been framed to account for this extraordinary phenomenon: we ſhall only notice ſome of the latter, and more probable: juſt premiſing, that Robert Norman, the inventor of the Dipping-needle, diſputes againſt Cortes's notion, that the Variation was cauſed by a point in the heavens; contending that it ſhould be ſought for in the earth, and propoſes how to diſcover its place.

The firſt is that of Gilbert (De Magnete, lib. 4, p. 151 &c), which is followed by Cabeus, &c. This notion is, that it is the earth, or land, that draws the needle out of its meridian direction: and hence they argue, that the needle varied more or leſs, as it was more or leſs diſtant from any great continent; and conſequently that if it were placed in the middle of an ocean, equally diſtant from equal tracts of land on each ſide, eaſtward and weſtward, it would not decline either to the one or the other, but point exactly north and ſouth. Thus, ſay they, in the Azores iſlands, which are equally diſtant from Africa on the eaſt, and America on the weſt, there is no Variation: but as you ſail from thence towards Africa, the needle begins to decline toward the eaſt, and that ſtill more and more till you reach the ſhore. If you proceed ſtill farther eaſtward, the declination gradually diminiſhes again, by reaſon of the land left behind on the weſt, which continues to draw the needle. The ſame holds till you arrive at a place where the tracts of land on each ſide are equal; and there again the Variation will be nothing. But the misfortune is, the law does not hold univerſally; for multitudes of obſervations of the Variation, in different parts, made and collected by Dr. Halley, overturn the whole theory.

Others therefore have recourſe to the frame and compages of the earth, conſidered as interſperſed with rocks and ſhelves, which being generally found to run towards the polar regions, the needle comes to have a general tendency that way; but it ſeldom happens that their direction is exactly in the meridian, and the needle has conſequently, for the moſt part, ſome Variation.

Others hold that divers parts of the earth have different degrees of the magnetic virtue, as ſome are more intermixed with heterogeneous matters, which prevent the free action or effect of it, than others are.

Others again aſcribe all to magnetic rocks and iron mines, which, affording more of the magnetic matter than other parts, draw the needle more.

 Laſtly,

Laſtly, others imagine that earthquakes, or high tides, have diſturbed and diſlocated ſeveral conſiderable parts of the earth, and ſo changed the magnetic axis of the globe, which was originally the ſame with the axis of the earth itſelf.

But none of theſe theories can be the true one; for ſtill that great phenomenon, the *Variation of the Variation*, i. e. the continual change of the declination, in one and the ſame place, is not accountable for, on any of theſe foundations, nor is it even conſiſtent with them.

Doctor Hook communicated to the Royal Society, in 1674, a theory of the Variation; the ſubſtance of which is, that the magnet has its peculiar pole, diſtant 10 degrees from the pole of the earth, about which it moves, ſo as to make a revolution in 370 years: whence the Variation, he ſays, has altered of late about 10 or 11 minutes every year, and will probably ſo continue to do for ſome time, when it will begin to proceed ſlower and ſlower, till at length it become ſtationary and retrograde, and ſo return back again. Birch's Hiſt. of the Royal Society, vol. 3, p. 131.

Dr. Halley has given a new ſyſtem, the reſult of numerous obſervations, and even of a number of voyages made at the public expence on this account. The light which this author has thrown upon this obſcure part of natural hiſtory, is very great, and of important conſequence in navigation, &c. In this ſyſtem he has reduced the ſeveral Variations in divers places to a preciſe rule, or order, which before appeared all precarious and arbitrary.

His theory will therefore deſerve a more ample detail. The obſervations it is built upon, as laid down in the Philoſ. Tranſ. number 148, or Abr. vol. 2, p. 610, are as follow:

Obſerved Variations of the Needle in divers places, and at divers times.

Places obſerved at.	Longitude from London.	Latitude	Year of Obſervation.	Variation obſerved.
	° ′	° ′		° ′
London - - -	0 0	51 31 n	1580	11 15 e
			1622	6 0 e
			1634	4 5 e
			1672	2 30 w
			1683	4 30 w
Paris - - - -	2 25 e	48 51 n	1640	3 0 e
			1666	0 0
			1681	2 30 w
Uraniburg - -	13 0 e	55 54 n	1672	2 35 w
Copenhagen -	12 53 e	55 41 n	1649	1 53 e
			1672	3 45 w
Dantzick - -	19 0 e	54 23 n	1679	7 0 w
Montpelier - -	4 0 e	43 37 n	1674	1 10 w
Breſt - - -	4 25 w	48 23 n	1680	1 45 w
Rome - - -	13 0 e	41 50 n	1681	5 0 w
Bayonne - -	1 20 w	43 30 n	1680	1 20 w
Hudſon's Bay -	79 40 w	51 0 n	1668	19 15 w
In Hudſon's Straits -	57 0 w	61 0 n	1668	29 30 w
Beffin's Bay, Sir T. Smith's Sound - -	80 0 w	78 0 n	1616	57 0 w
At Sea - - -	57 0 w	38 40 n	1682	7 30 w
At Sea - - -	31 30 w	43 50 n	1682	5 30 w
At Sea - - -	42 0 w	21 0 n	1678	0 40 e
Cape St. Auguſtine - -	35 30 w	28 0 s	1670	5 30 e
Off the mouth of River Plate	53 0 w	39 30 s	1670	20 30 e
Cape Frio - -	41 10 w	22 40 s	1670	12 10 e
Entrance of Magellan's Straits - -	68 0 w	52 30 s	1670	17 0 e
Weſt Entrance of ditto -	75 0 w	53 0 s	1670	14 10 e

Places obſerved at.	Longitude from London.	Latitude	Year of Obſervation.	Variation obſerved.
	° ′	° ′		° ′
Baldivia - -	73 0 w	40 0 s	1670	8 10 e
Cape Aguillas -	16 30 e	34 50 s	1622	2 0 w
			1675	8 0 w
At Sea - - -	1 0 e	34 30 s	1675	0 0
At Sea - - -	20 0 w	34 0 s	1675	10 30 e
At Sea - - -	32 0 w	24 0 s	1675	10 30 e
St. Helena - -	6 30 w	16 0 s	1677	0 40 e
Iſle Aſcenſion -	14 30 w	7 50 s	1678	1 0 e
Johanna - - -	44 0 e	12 15 s	1675	19 30 w
Mombaſa - -	40 0 e	4 0 s	1675	16 0 w
Zocatra - - -	56 0 e	12 30 n	1674	17 0 w
Aden, Mouth of Red Sea	47 30 e	13 0 n	1674	15 0 w
Diego Roiz - -	61 0 e	20 0 s	1676	20 30 w
At Sea - - -	64 30 e	0 0	1676	15 30 w
At Sea - - -	55 0 e	27 0 s	1676	24 0 w
Bombay - - -	72 30 e	19 0 n	1676	12 0 w
Cape Comorin -	76 0 e	8 15 n	1680	8 48 w
Ballaſore - -	87 0 e	21 30 n	1680	8 10 w
Fort St. George	80 0 e	13 15 n	1680	8 10 w
Weſt Point of Java - -	104 0 e	6 40 s	1676	3 10 w
At Sea - - -	58 0 e	39 0 s	1677	27 30 w
I. St. Paul - -	72 0 e	38 0 s	1677	23 30 w
At Van Diemen's	142 0 e	42 25 s	1642	0 0
At New Zealand - -	170 0 e	40 50 s	1642	9 0 e
Three - kings Iſle in ditto	169 30 e	34 35 s	1642	8 40 e
I. Rotterdam in the South Sea	184 0 e	20 15 s	1642	6 20 e
Coaſt of New Guinea - -	149 0 e	4 30 s	1643	8 45 e
Weſt Point of ditto - -	126 0 e	0 26 s	1643	5 30 e

Upon

Upon theſe obſerved Variations Dr. Halley makes ſeveral remarks, as to the Variation in different parts of the world at the time of his writing, eaſtward and weſtward, and the ſituation and direction of the lines or places of no Variation: from the whole he deduces the following theory.

Dr. *Halley's Theory of the Variation of the Needle.* That the whole globe of the earth is one great magnet, having four magnetical poles, or points of attraction; near each pole of the equator two; and that in thoſe parts of the world which lie nearly adjacent to any one of theſe magnetic poles, the needle is governed by it; the neareſt pole being always predominant over the more remote.

The pole which at preſent is neareſt to us, he conjectures to lie in or near the meridian of the Land's-end of England, and not above 7° from the north pole: by this pole, the Variations in all Europe and Tartary, and the North Sea, are chiefly governed; though ſtill with ſome regard to the other northern pole, whoſe ſituation is in the meridian paſſing about the middle of California, and about 15° from the north pole of the world, to which the needle has chiefly reſpect in all North America, and in the two oceans on either ſide of it, from the Azores weſtward to Japan, and farther.

The two ſouthern magnetic poles, he imagines, are rather more diſtant from the ſouth pole of the world; the one being about 16° from it, on a meridian 20° to the weſtward of the Magellanic Streights, or 95° weſt from London: this pole commands the needle in all South America, in the Pacific Ocean, and the greateſt part of the Ethiopic Ocean. The other magnetic pole ſeems to have the greateſt power, and the largeſt dominion of all, as it is the moſt remote from the pole of the world, being little leſs than 20° diſtant from it, in the meridian which paſſes through New Holland, and the iſland Celebes, about 120° eaſt from London: this pole is predominant in the ſouth part of Africa, in Arabia, and the Red Sea, in Perſia, India, and its iſlands, and all over the Indian ſea, from the Cape of Good Hope eaſtward, to the middle of the Great South Sea that divides Aſia from America.

Such, he obſerves, ſeems to be the preſent diſpoſition of the magnetic virtue thoughout the whole globe of the earth. It is then ſhewn how this hypotheſis accounts for all the Variations that have been obſerved of late, and how it anſwers to the ſeveral remarks drawn from the table.

It is there inferred that from the whole it appears, that the direction of the needle, in the temperate and frigid zones, depends chiefly upon the counterpoiſe of the forces of two magnetic poles of the ſame nature: as alſo why, under the ſame meridian, the Variation ſhould be in one place $29\frac{1}{2}$ degrees weſt, and in another $20\frac{1}{2}$ degrees eaſt.

In the torrid zone, and particularly about the equator, reſpect muſt be had to all the four poles, and their poſitions muſt be well conſidered, otherwiſe it will not be eaſy to determine what the Variation ſhould be, the neareſt pole being always ſtrongeſt; yet ſo however as to be ſometimes counterbalanced by the united forces of two more remote ones. Thus, in ſailing from St. Helena, by the iſle of Aſcenſion, to the equator, on the north-weſt courſe, the Variation is very little eaſterly, and unalterable in that whole track; becauſe the South-American pole (which is much the neareſt in the aforeſaid places), requiring a great eaſterly variation, is counterpoiſed by the contrary attraction of the North-American and the Aſiatic ſouth poles; each of which ſingly is, in theſe parts, weaker than the American ſouth pole; and upon the north-weſt courſe the diſtance from this latter is very little varied; and as you recede from the Aſiatic ſouth pole, the balance is ſtill preſerved by an acceſs towards the North-American pole. In this caſe no notice is taken of the European north pole; its meridian being a little removed from thoſe of theſe places, and of itſelf requiring the ſame Variations which are here found.

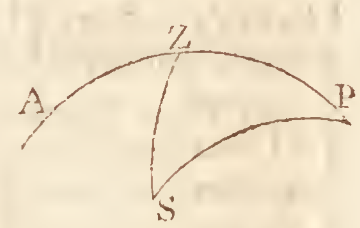
After the ſame manner may the Variations in other places about the equator be accounted for, upon Dr. Halley's hypotheſis.

To obſerve the Variation of the Needle. Draw a meridian line, as directed under Meridian; then a ſtile being erected in the middle of it, place a needle upon it, and draw the right line which it hangs over. Thus will the quantity of the Variation appear.

Or thus: As the former method of finding the Variation cannot be applied at ſea, others have been deviſed, the principal of which are as follow. Suſpend a thread and plummet over the compaſs, till the ſhadow paſs through the centre of the card; obſerve the rhumb, or point of the compaſs which the ſhadow touches when it is the ſhorteſt. For the ſhadow is then a meridian line; and conſequently the Variation is ſhewn.

Or thus: Obſerve the point of the compaſs upon which the ſun, or ſome ſtar, riſes and ſets; biſect the arch intercepted between the riſing and ſetting, and the line of biſection will be the meridian line; conſequently the Variation is had as before. The ſame may alſo be obtained from two equal altitudes of the ſame ſtar, obſerved either by day or night. Or thus: Obſerve the rhumb upon which the ſun or ſtar riſes and ſets; and from the latitude of the place find the eaſtern or weſtern amplitude: for the difference between the amplitude, and the diſtance of the rhumb obſerved, from the eaſtern rhumb of the card, is the Variation ſought.

Or thus: Obſerve the altitude of the ſun, or ſome ſtar S, whoſe declination is known; and note the rhumb in the compaſs to which it then correſponds. Then in the triangle ZPS, are known three ſides, viz, PZ the colatitude, PS the codeclination, and ZS the coaltitude; the angle PZS is thence found by ſpherical trigonometry; the ſupplement to which, viz. AZS, is the azimuth from the ſouth. Then the difference between the azimuth and the obſerved diſtance of the rhumb from the ſouth, is the Variation ſought. See *Azimuth* Compass.



The uſe of the Variation is to correct the courſes a ſhip has ſteered by the compaſs, which muſt always be done before they are worked, or calculated.

Variation *of the Variation*, is a gradual and continual change in the Variation, obſerved in any place, by which the quantity of the Variation is found to be different at different times.

This Variation, according to Henry Bond (in his *Longitude found*, Lond. 1670, pa. 6) "was firſt found to decreaſe by Mr. John Mair; 2dly, by Mr. Edmund Gunter: 3dly, by Mr. Henry Gellibrand; 4thly, by myſelf (Henry Bond) in 1640; and laſtly, by Dr. Robert Hook, and others, in 1665;" which they found out by comparing together obſervations made at the ſame place, at different times. The diſcovery was ſoon known abroad; for Kircher, in his treatiſe intitled Magnes, firſt printed at Rome in 1641, ſays that our countryman Mr. John Greaves had informed him of it, and then he gives a letter of Merſenne's, containing a diſtinct account of it.

This continual change in the Variation, is gradual and univerſal, as appears by numerous obſervations. Thus, the Variation was,

At Paris, according to Orontius Finæus,

in 1550	-	8°	0′E.
in 1640	-	3	0 E.
in 1660	-	0	0
in 1681	-	2	2W.
in 1759	-	18	10W.
in 1760	-	18	20W.

M. De la Lande (Expoſition du Calcul Aſtronomique) obſerves, that the Variation has changed, at Paris, 26° 20′ in the ſpace of 150 years, allowing that in 1610 the Variation was 8° E: and ſince 1740 the needle, which was always uſed by Maraldi, is more than 3° advanced toward the weſt, beyond what it was at that period; which is a change after the rate nearly of 9′½ per year.

At Cape d'Agulhas, in 1600, it had no Variation; (whence the Portugueſe gave it that name);

in 1622 it was		2°W.
in 1675	-	8 W.
in 1692	-	11 W.

which is a change of nearly 8′ per year.

At St. Helena, the Variation,

in 1600 was	8°	0′E.	
in 1623	-	6	0 E.
in 1677	-	0	40 E.
in 1692	.	1	0 W.

which is a change of nearly 5′½ per year.

At Cape Comorin, the Variation,

in 1620 was	14°	20′W.	
in 1680	-	8	44 W.
in 1688	-	7	30 W.

which is a change of nearly 6′½ per year.

At London, the Variation,

in 1580 was	11°	15′ E.	
in 1622	-	6	0 E.
in 1634	-	4	5 E.
in 1657	-	0	0
in 1672	-	2	30 W.
in 1692	-	6	0 W.
in 1723	-	14	17 W.
in 1747	-	17	40 W.
in 1780	-	22	41 W.

which is a change after the rate of 10′ per year, upon a courſe of exactly 200 years. See Philoſ. Tranſ. No. 148 and No. 383, or Abr. vol. 2, p. 615, and vol. 7, p. 290; and Philoſ. Tranſ. vol. 45, p. 280, and vol. 66, p. 393. On the ſubject of the Variation, ſee alſo Norman's New Attractive 1614; Burrows's Diſcovery of the Variation 1581; Bond's Longitude found, 1676; &c.

Mr. Thomas Harding, in the Tranſactions of the Royal Iriſh Academy, vol. 4, has given obſervations on the Variation of the magnetic needle, at Dublin, which are rather extraordinary. He ſays the change in the Variation at that place is *uniform*. That from the year 1657, in which the Variation was nothing (the ſame as at London in that year), it has been going on at the medium rate of 12′ 20″ annually, and was in May 1791, 27° 23′ weſt: exceeding that at London now by 3 or 4 degrees. He brings proof of his aſſertion of the uniformity of the Variation, from different authentic records, and ſtates the operations by which it is calculated. He concludes with recommending accuracy in marking the exiſting Variation when maps are made, as not only conducing to the exact definition of boundaries, but as laying the beſt foundation for a diſcovery of the longitude by ſea or land.

Theory of the Variation of the Variation. According to Dr. Halley's theory, this change in the Variation of the compaſs, is ſuppoſed owing to the difference of velocity in the motions of the internal and external parts of the globe. From the obſervations that have been cited, it ſeems to follow, that all the magnetical poles have a motion weſtward, but yet not exactly round the axis of the earth, for then the Variations would continue the ſame in the ſame parallel of latitude, contrary to experience.

From the diſagreement of ſuch a ſuppoſition with experiments therefore, the learned author of the theory invented the following hypotheſis: The external parts of the globe he conſiders as the ſhell, and the internal as a nucleus, or inner globe; and between the two he conceives a fluid medium. That inner earth having the ſame common centre and axis of diurnal rotation, may revolve with our earth every 24 hours: Only the outer ſphere having its turbinating motion ſomewhat ſwifter or ſlower than the internal ball; and a very minute difference in length of time, by many repetitions, becoming ſenſible; the internal parts will gradually recede from the external, and they will appear to move, either eaſtward or weſtward, by the difference of their motions.

Now, ſuppoſing ſuch an internal ſphere, having ſuch a motion, the two great difficulties in the former hypotheſis are eaſily ſolved; for if this exterior ſhell of earth be a magnet, having its pole at a diſtance from the poles of diurnal rotation; and if the internal nucleus be likewiſe a magnet, having its poles in two other places, diſtant alſo from the axis; and theſe latter, by a ſlow gradual motion, change their place in reſpect of the external, a reaſonable account may then be given of the four magnetical poles before mentioned, and alſo of the changes of the needle's Variation.

The author thinks that two of theſe poles are fixed, and the other two moveable; viz, that the fixed poles are the poles of the external cortex or ſhell of the earth;

earth; and the other the poles of the magnetical nucleus, included and moveable within the former. From the obfervations he infers, that the motion is weftwards, and confequently that the nucleus has not precifely attained the fame velocity with the exterior parts in their diurnal rotation; but fo very nearly equals it, that in 365 revolutions the difference is fcarcely fenfible.

That there is any difference of this kind, arifes from hence, that the impulfe by which the diurnal motion was impreffed on the earth, was given to the external parts, and from thence in time communicated to the internal; but fo as not yet perfectly to equal the velocity of the firft motion impreffed on the fuperficial parts of the globe, and ftill preferved by them.

As to the precife period, obfervations are wanting to determine it, though the author thinks we may reafonably conjecture that the American pole has moved weftward 46° in 90 years, and that its whole period is performed in about 700 years.

Mr. Whifton, in his New Laws of Magnetifm, raifes feveral objections againft this theory. See MAGNETISM.

M. Euler, too, the fon of the celebrated mathematician of that name, has controverted and cenfured Dr. Halley's theory. He thinks, that two magnetic poles, placed on the furface of the earth, will fufficiently account for the Variation: and he then endeavours to fhew, how we may determine the declination of the needle, at any time, and on every part of the globe, from this hypothefis. For the particulars of this reafoning, fee the Hiftoire de l'Academie des Sciences & Belles Lettres of Berlin, for 1757; alfo Mr. Cavallo's Treatife on Magnetifm, p. 117.

Variation of the Needle by Heat and Cold.—There is a fmall Variation of the Variation of the magnetic needle, amounting only to a few minutes of a degree in the fame place, at different hours of the fame day, which is only difcoverable by nice obfervations. Mr. George Graham made feveral obfervations of this kind in the years 1722 and 1723, profeffing himfelf altogether ignorant of the caufe of the phenomena he obferved. Philof. Tranf. No. 383, or Abr. vol. 7, p. 290.

About the year 1750, Mr. Wargentin, fecretary of the Swedifh Academy of Sciences, took notice both of the regular diurnal Variation of the needle, and alfo of its being difturbed at the time of the aurora borealis, as recorded in the Philof. Tranf. vol. 47, p. 126.

About the year 1756, Mr. Canton commenced a feries of obfervations, amounting to near 4000, with an excellent Variation-compafs, of about 9 inches diameter. The number of days on which thefe obfervations were made, was 603, and the Diurnal Variation on 574 of them was regular, fo as that the abfolute Variation of the needle weftward was increafing from about 8 or 9 o'clock in the morning, till about 1 or 2 in the afternoon, when the needle became ftationary for fome time; after that, the abfolute Variation weftward was decreafing, and the needle came back again to its former fituation, or nearly fo, in the night, or by the next morning. The Diurnal Variation is irregular when the needle moves flowly eaftward in the latter part of the morning, or weftward in the latter part of the afternoon; alfo when it moves much either way after night, or fuddenly both ways in a fhort time. Thefe irregularities feldom happen more than once or twice in a month, and are always accompanied, as far as Mr. Canton obferved, with an aurora borealis.

Mr. Canton lays down and evinces, by experiment, the following principle, viz, that the attractive power of the magnet (whether natural or artificial) will decreafe while the magnet is heating, and increafe while it is cooling. He then proceeds to account for both the regular and irregular Variation. It is evident, he fays, that the magnetic parts of the earth in the north, on the eaft fide, and on the weft fide of the magnetic meridian, equally attract the north end of the needle. If then the eaftern magnetic parts be heated fafter by the fun in the morning, than the weftern parts, the needle will move weftward, and the abfolute Variation will increafe: when the attracting parts of the earth on each fide of the magnetic meridian have their heat increafing equally, the needle will be ftationary, and the abfolute Variation will then be greateft; but when the weftern magnetic parts are either heating fafter, or cooling flower, than the eaftern, the needle will move eaftward, or the abfolute Variation will decreafe; and when the eaftern and weftern magnetic parts are cooling equally faft, the needle will again be ftationary, and the abfolute Variation will then be leaft.

By this theory, the Diurnal Variation in the fummer ought to exceed that in winter; and accordingly it is found by obfervation, that the Diurnal Variation in the months of June and July is almoft double of that in December and January.

The irregular Diurnal Variation muft arife from fome other caufe than that of heat communicated by the fun; and here Mr. Canton has recourfe to fubterranean heat, which is generated without any regularity as to time, and which will, when it happens in the north, affect the attractive power of the magnetic parts of the earth on the north end of the needle. That the air neareft the earth will be moft warmed by the heat of it, is obvious; and this has been often noticed in the morning, before day, by means of thermometers at different diftances from the ground. Philof. Tranf. vol. 48, pa. 526.

Mr. Canton has annexed to his paper on this fubject, a complete year's obfervations; from which it appears, that the Diurnal Variation increafes from January to June, and decreafes from June to December. Philof. Tranf. vol. 51, pa. 398.

It has alfo been obferved, that different needles, efpecially if touched with different loadftones, will differ a few minutes in their Variation. See Poleni Epift. Phil. Tranf. num. 421.

Dr. Lorimer (in the Supp. to Cavallo's Magnetifm) adduces fome ingenious obfervations on this fubject. It muft be allowed, fays he, according to the obfervations of feveral ingenious gentlemen, that the collective magnetifm of this earth arifes from the magnetifm of all the ferruginous bodies contained in it, and that the magnetic poles fhould therefore be confidered as the centres of the powers of thofe magnetic fubftances. Thefe poles muft therefore change their places according as the magnetifm of fuch fubftances is affected; and if

with Mr. Canton we allow, that the general caufe of the Diurnal Variation arifes from the fun's heat in the forenoon and afternoon of the fame day, it will naturally occur, that the fame caufe, being continued, may be fufficient to produce the general Variation of the magnetic needle for any number of years. For we muft confider, that ever fince any attentive obfervations have been made on this fubject, the natural direction of the magnetic needle in Europe has been conftantly moving, from weft to eaft, and that in other parts of the world it has continued its motion with equal conftancy.

As we muft therefore admit, fays Dr. Lorimer, that the heat in the different feafons depends chiefly on the fun, and that the months of July and Auguft are commonly the hotteft, while January and February are the coldeft months of the year; and that the temperature of the other months falls into the refpective intermediate degrees; fo we muft confider the influence of heat upon magnetifm to operate in the like manner, viz, that for a fhort time it fcarcely manifefts itfelf; yet in the courfe of a century, the conftancy and regularity of it becomes fufficiently apparent. It would therefore be idle to fuppofe, that fuch an influence could be derived from an uncertain or fortuitous caufe. But if it be allowed to depend upon the conftancy of the fun's motion, and this appears to be a caufe fufficient to explain the phenomena, we fhould (agreeably to Newton's firft law of philofophizing) look no farther.

As we therefore confider, fays he, the magnetic powers of the earth to be concentrated in the magnetic poles, and that there is a diurnal Variation of the magnetic needle, thefe poles muft perform a fmall diurnal revolution proportional to fuch Variation, and return again to the fame point nearly. Suppofe then that the fun in his diurnal revolution paffes along the northern tropic, or along any parallel of latitude between it and the equator, when he comes to that meridian in which the magnetic pole is fituated, he will be much nearer to it, than in any other; and in the oppofite meridian he will of courfe be the fartheft from it. As the influence of the fun's heat will therefore act moft powerfully at the leaft, and lefs forcibly at the greateft diftance, the magnetic pole will confequently defcribe a figure fomething of the elliptical kind; and as it is well known that the greateft heat of the day is fome time after the fun has paffed the meridian, the longeft axis of this elliptical figure will lie north-eafterly in the northern, and fouth-eafterly in the fouthern hemifphere. Again, as the influence of the fun's heat will not from thofe quarters have fo much power, the magnetic poles cannot be moved back to the very fame point, from which they fet out; but to one which will be a little more northerly and eafterly, or more foutherly and eafterly, according to the hemifpheres in which they are fituated. The figures therefore which they defcribe, may more properly be termed elliptoidal fpirals.

In this manner the Variation of the magnetic needle in the northern hemifphere may be accounted for. But with refpect to the fouthern hemifphere we muft recollect, that though the lines of declination in the northern hemifphere have conftantly moved from weft to eaft, yet in the fouthern hemifphere, it is equally certain that they have moved from eaft to weft, ever fince any obfervations have been made on the fubject. Hence then the lines of magnetic declination, or Halleyan curves, as they are now commonly called, appear to have a contrary motion in the fouthern hemifphere, to what they have in the northern; though both the magnetic poles of the earth move in the fame direction, that is from weft to eaft.

In the northern hemifphere there was a line of no Variation, which had eaft Variation on its eaftern fide, and weft Variation on its weftern fide. This line evidently moved from weft to eaft during the two laft centuries; the lines of eaft Variation moving before it, while the lines of weft Variation followed it with a proportional pace. Thefe lines firft paffed the Azores or Weftern Iflands, then the meridian of London, and after a certain number of years ftill later, they paffed the meridian of Paris. But in the fouthern hemifphere there was another line of no Variation, which had eaft Variation on its weftern fide, and weft Variation on its eaftern; the lines of eaft Variation moving before it, while thofe of the weft Variation followed it. This line of no Variation firft paffed the Cape des Aiguilles, and then the Cape of Good Hope; the lines of 5°, 10°, 15°, and 20° weft Variation following it, the fame as was the cafe in the northern hemifphere, but in the contrary direction.

We may juft farther mention the idea of Dr. Gowin Knight, which was, that this earth had originally received its magnetifm, or rather that its magnetical powers had been brought into action, by a fhock, which entered near the fouthern tropic, and paffed out at the northern one. His meaning appears to have been, that this was the courfe of the magnetic fluid, and that the magnetic poles were at firft diametrically oppofite to each other. Though, according to Mr. Canton's doctrine, they would not have long continued fo; for from the intenfe heat of the fun in the torrid zone, according to the principles already explained, the north pole muft have foon retired to the north-eaftward, and the fouth pole to the fouth-eaftward. It is alfo curious to obferve, that on account of the fouthern hemifphere being colder upon the whole than the northern hemifphere, the magnetic poles would have moved with unequal pace: that is, the north magnetic pole would have moved farther in any given time to the north-eaft, than the fouth magnetic pole could have moved to the fouth-eaft. And, according to the opinions of the moft ingenious authors on this fubject, it is generally allowed, that at this time the north magnetic pole is confiderably nearer to the north pole of the earth, than the fouth magnetic pole is to the fouth pole of the earth.

It may farther be added, that feveral ingenious fea officers are of opinion, that in the weftern parts of the Englifh Channel the Variation of the magnetic needle has already begun to decreafe; having in no part of it ever amounted to 25°. There are however other perfons who affert that the Variation is ftill increafing in the Channel, and as far weftward as the 15th degree of longitude and 51° of latitude, at which place they fay that it amounts to about 30°.

Of the Variation Chart. Doctor Halley having collected a multitude of obfervations made on the Variation of the needle in many parts of the world, was hence enabled to draw, on a Mercator's chart, certain lines, fhewing the Variation of the compafs in all thofe places

places over which they passed, in the year 1700, when he published the first chart of this kind, called the *Variation Chart.*

From the construction of this chart it appears, that the longitude of any of those places may be found by it, when the latitude and the Variation in that place are known. Thus, having found the Variation of the compass, draw a parallel of latitude on the chart through the latitude found by observation; and the point where it cuts the curved line, whose Variation is the same with that observed, will be the ship's place. A similar project of thus finding the longitude, from the known latitude and inclination or dip of the needle, was before proposed by Henry Bond, in his treatise intitled, The Longitude Found, printed in 1676.

This method however is attended with two considerable inconveniences: 1st, That wherever the Variation lines run east and west, or nearly so, this way of finding the longitude becomes imperfect, as their intersection with the parallel of latitude must be very indefinite: and among all the trading parts of the world, this imperfection is at present found chiefly on the western coasts of Europe, between the latitudes of 45° and 53°; and on the eastern shores of North America, with some parts of the Western Ocean and Hudson's Bay, lying between the said shores: but for the other parts of the world, a Variation Chart may be attended with considerable benefit. However, the Variation curves, when they run east and west, may sometimes be applied to good purpose in correcting the latitude, when meridian observations cannot be had, as it often happens on the northern coasts of America, in the Western Ocean, and about Newfoundland; for if the Variation can be obtained exactly, then the east and west curve, answering to the Variation in the chart, will shew the latitude.

2dly, As the deviation of the magnetical meridian, from the true one, is subject to continual alteration, therefore a chart to which the Variation lines are fitted for any year, must in time become useless, unless new lines, shewing the state of the Variation at that time, be drawn on the chart: but as the change in the Variation is very slow, therefore new Variation Charts published every 7 or 8 years, will answer the purpose tolerably well. And thus it has happened that Halley's Variation Chart has become useless, for want of encouragement to renew it from time to time.

However, in the year 1744, Mr. William Mountaine and Mr. James Dodson published a new Variation Chart, adapted for that year, which was well received; and several instances of its great utility having been communicated to them, they fitted the Variation lines anew for the year 1756, and in the following year published the 3d Variation Chart, and also presented to the Royal Society a curious paper concerning the Variation of the magnetic needle, with a set of tables annexed, containing the result of upwards of 50 thousand observations, in six periodical reviews, from the year 1700 to 1756 inclusive, and adapted to every 5 degrees of latitude and longitude in the more frequented oceans; which paper and tables were printed in the Transactions for the year 1757.

From these tables of observations, such extraordinary and whimsical irregularities occur in the Variation, that we cannot think it wholly under the direction of one general and uniform law; but rather conclude, with Dr. Gowen, in the 87th prop. of his Treatise upon Attraction and Repulsion, that it is influenced by various and different magnetic attractions, perhaps occasioned by the heterogeneous compositions in the great magnet, the earth.

Many other observations on the Variation of the magnetic needle, are to be found in several volumes of the Philos. Trans. See particularly vol. 48, p. 875; vol. 50, p. 329; vol. 56, p. 220; and vol. 61, p. 422.

VARIATION *Compass*. See COMPASS.

VARIATION *of Curvature*, in Geometry, is used for that inequality or change which takes place in the curvature of all curves except the circle, by which their curvature is more or less in different parts of them. And this Variation constitutes the quality of the curvature of any line.

Newton makes the index of the inequality, or Variation of Curvature, to be the ratio of the fluxion of the radius of curvature to the fluxion of the curve itself: and Maclaurin, to avoid the perplexity that different notions, connected with the same terms, occasion to learners, has adopted the same definition: but he suggests, that this ratio gives rather the Variation of the ray of curvature, and that it might have been proper to have measured the Variation of Curvature rather by the ratio of the fluxion of the curvature itself to the fluxion of the curve; so that, the curvature being inversely as the radius of curvature, and consequently its fluxion as the fluxion of the radius itself directly, and the square of the radius inversely, its Variation would have been directly as the measure of it according to Newton's definition, and inversely as the square of the radius of curvature.

According to this notion, it would have been measured by the angle of contact contained by the curve and circle of curvature, in the same manner as the curvature itself is measured by the angle of contact contained by the curve and tangent. The reason of this remark may appear from this example: The Variation of curvature, according to Newton's explication, is uniform in the logarithmic spiral, the fluxion of the radius of curvature in this figure being always in the same ratio to the fluxion of the curve; and yet, while the spiral is produced, though its curvature decreases, it never vanishes; which must appear a strange paradox to those who do not attend to the import of Newton's definition. Newton's Method of Fluxions and Inf. Series, pa. 76. Maclaurin's Flux. art. 386. Philos. Trans. num. 468, pa. 342.

The Variation of curvature at any point of a conic section, is always as the tangent of the angle contained by the diameter that passes through the point of contact, and the perpendicular to the curve at the same point, or to the angle formed by the diameter of the section, and of the circle of curvature. Hence the Variation of curvature vanishes at the extremities of either axis, and is greatest when the acute angle, contained by the diameter, passing through the point of contact and the tangent, is least.

When the conic section is a parabola, the Variation is

as the tangent of the angle, contained by the right line drawn from the point of contact to the focus, and the perpendicular to the curve. See CURVATURE.

From Newton's definition may be derived practical rules for the Variation of curvature, as follows:

1. Find the radius of curvature, or rather its fluxion; then divide this fluxion by the fluxion of the curve, and the quotient will give the Variation of curvature; exterminating the fluxions when neceſſary, by the equation of the curve, or perhaps by expreſſing their ratio by help of the tangent, or ordinate, or ſubnormal, &c.

2. Since $\frac{\dot{z}^3}{-\dot{x}\ddot{y}}$, or $\frac{\dot{z}^3}{-\ddot{y}}$ (putting $\dot{x} = 1$) denotes the radius of curvature of any curve z, whoſe abſciſs is x, and ordinate y; if the fluxion of this be divided by $\dot{z}$, and $\dot{z}$ and $\ddot{z}$ be exterminated, the general value of the Variation will come out $\frac{-3\dot{y}\ddot{y}^2 + \dddot{y}(1+\dot{y}^2)}{\ddot{y}^2}$; then ſubſtituting the values of $\dot{y}$, $\ddot{y}$, $\dddot{y}$ (found from the equation of the curve) into this quantity, it will give the Variation ſought.

Ex. Let the curve be the parabola, whoſe equation is $ax = y^2$. Here then $2y\dot{y} = a\dot{x} = a$, and $\dot{y} = \frac{a}{2y}$; hence $\ddot{y} = \frac{-a\dot{y}}{2yy} = \frac{-aa}{4y^3}$, and $\dddot{y} = \frac{-3aa\dot{y}}{2y^4} = \frac{3a^3}{8y^5}$. Therefore $\frac{-3\dot{y}\ddot{y}^2 + \dddot{y}(1+\dot{y}^2)}{\ddot{y}^2} = -3\dot{y} + \dddot{y} \times \frac{1+\dot{y}^2}{\ddot{y}^2} = \frac{-3a}{2y} + \frac{3a^3}{8y^5} \times \left(1 + \frac{aa}{4yy}\right) \times \frac{16y^6}{a^4} = \frac{6y}{a}$, the Variation ſought. Emerſon's Flux. pa. 228.

VARIGNON (PETER), a celebrated French mathematician and prieſt, was born at Caen in 1654, and died ſuddenly in 1722, at 68 years of age. He was the ſon of an architect in middling circumſtances, but had a college education, being intended for the church. An accident threw a copy of Euclid's Elements in his way, which gave him a ſtrong turn to that kind of learning. The ſtudy of geometry led him to the works of Des Cartes on the ſame ſcience, and there he was ſtruck with that new light which has, from thence, ſpread over the world.

He abridged himſelf of the neceſſaries of life to purchaſe books of this kind, or rather conſidered them of that number, as indeed they ought to be. What contributed to heighten this paſſion in him was, that he ſtudied in private: for his relations obſerving that the books he ſtudied were not ſuch as were commonly uſed by others, ſtrongly oppoſed his application to them. As there was a neceſſity for his being an eccleſiaſtic, he continued his theological ſtudies, yet not entirely ſacrificing his favourite ſubject to them.

At this time the Abbé St. Pierre, who ſtudied philoſophy in the ſame college, became acquainted with him. A taſte in common for rational ſubjects, whether phyſics or metaphyſics, and continual diſputations, formed the bonds of their friendſhip. They were mutually ſerviceable to each other in their ſtudies. The Abbé, to enjoy Varignon's company with greater eaſe, lodged him with himſelf; thus, growing ſtill more ſenſible of his merit, he reſolved to give him a fortune, that he might fully purſue his genius, and improve his talents; and, out of only 18 hundred livres a year, which he had himſelf, he conferred 300 of them upon Varignon.

The Abbé, perſuaded that he could not do better than go to Paris to ſtudy philoſophy, ſettled there in 1686, with M. Varignon, in the ſuburbs of St. Jacques. There each ſtudied in his own way; the Abbé applying himſelf to the ſtudy of men, manners, and the principles of government; whilſt Varignon was wholly occupied with the mathematics.

I, ſays Fontenelle, who was their countryman, often went to ſee them, ſometimes ſpending two or three days with them. They had alſo room for a couple of viſitors, who came from the ſame province. We joined together with the greateſt pleaſure. We were young, full of the firſt ardour for knowledge, ſtrongly united, and, what we were not then perhaps diſpoſed to think ſo great a happineſs, little known. Varignon, who had a ſtrong conſtitution, at leaſt in his youth, ſpent whole days in ſtudy, without any amuſement or recreation, except walking ſometimes in fine weather. I have heard him ſay, that in ſtudying after ſupper, as he uſually did, he was often ſurpriſed to hear the clock ſtrike two in the morning; and was much pleaſed that four hours reſt were ſufficient to refreſh him. He did not leave his ſtudies with that heavineſs which they uſually create; nor with that wearineſs that a long application might occaſion. He left off gay and lively, filled with pleaſure, and impatient to renew it. In ſpeaking of mathematics, he would laugh ſo freely, that it ſeemed as if he had ſtudied for diverſion. No condition was ſo much to be envied as his; his life was a continual enjoyment, delighting in quietneſs.

In the ſolitary ſuburb of St. Jacques, he formed however a connection with many other learned men; as Du Hamel, Du Verney, De la Hire, &c. Du Verney often aſked his aſſiſtance in thoſe parts of anatomy connected with mechanics: they examined together the poſitions of the muſcles, and their directions; hence Varignon learned a good deal of anatomy from Du Verney, which he repaid by the application of mathematical reaſoning to that ſubject.

At length, in 1687, Varignon made himſelf known to the public by a Treatiſe on New Mechanics, dedicated to the Academy of Sciences. His thoughts on this ſubject were, in effect, quite new. He diſcovered truths, and laid open their ſources. In this work, he demonſtrated the neceſſity of an equilibrium, in ſuch caſes as it happens in, though the cauſe of it is not exactly known. This diſcovery Varignon made by the theory of compound motions, and is what this eſſay turns upon.

This new Treatiſe on Mechanics was greatly admired by the mathematicians, and procured the author two conſiderable places, the one of Geometrician in the Academy of Sciences, the other of Profeſſor of Mathematics in the College of Mazarine, to which he was the firſt perſon raiſed.

Varignon catched eagerly at the Science of Infiniteſimals as ſoon as it appeared in the world, and became one of its moſt early cultivators. When that ſublime and beautiful method was attacked in the Academy itſelf

self (for it could not escape the fate of all innovations) he became one of its most zealous defenders, and in its favour he put a violence upon his natural character, which abhorred all contention. He sometimes lamented, that this dispute had interrupted him in his enquiries into the Integral Calculation so far, that it would be difficult for him to resume his disquisition where he had left it off. He sacrificed Infinitesimals to the interest of Infinitesimals, and gave up the pleasure and glory of making a farther progress in them when called upon by duty to undertake their defence.

All the printed volumes of the Academy bear witness to his application and industry. His works are never detached pieces, but complete theories of the laws of motion, central forces, and the resistance of mediums to motion. In these he makes such use of his rules, that nothing escapes him that has any connection with the subject he treats.

Geometrical certainty is by no means incompatible with obscurity and confusion, and those are sometimes so great, that it is surprising a mathematician should not miss his way in so dark and perplexing a labyrinth. The works of M. Varignon never occasion this disagreeable surprise, he makes it his chief care to place every thing in the clearest light; he does not, as some great men do, consult his ease by declining to take the trouble of being methodical, a trouble much greater than that of composition itself; he does not endeavour to acquire a reputation for profoundness, by leaving a great deal to be guessed by the reader.

He was perfectly acquainted with the history of mathematics. He learned it not merely out of curiosity, but because he was desirous of acquiring knowledge from every quarter. This historical knowledge is doubtless an ornament in a mathematician, but it is an ornament which is by no means without its utility. Indeed it may be laid down as a maxim, the more different ways the mind is occupied in, upon a subject, the more it improves.

Though Varignon's constitution did not seem easy to be impaired, assiduity and constant application brought upon him a severe disease in 1705. Great abilities are generally dangerous to the possessors. He was six months in danger, and three years in a languid state, which proceeded from his spirits being almost entirely exhausted. He said that sometimes when delirious with a fever, he thought himself in the midst of a forest, where all the leaves of the trees were covered with algebraical calculations. Condemned by his physicians, his friends, and himself, to lay aside all study, he could not, when alone in his chamber, avoid taking up a book of mathematics, which he hid as soon as he heard any person coming. He again resumed the attitude and behaviour of a sick man, and seldom had occasion to counterfeit.

In regard to his character, Fontenelle observes, that it was at this time that a writing of his appeared, in which he censured Dr. Wallis for having advanced that there are certain spaces more than infinite, which that great geometrician ascribes to hyperbolas. He maintained, on the contrary, that they were finite. The criticism was softened with all the politeness and respect imaginable; but a criticism it was, though he had written it only for himself. He let M. Carré see it, when he was in a state that rendered him indifferent about things of that kind; and that gentleman, influenced only by the interest of the sciences, caused it to be printed in the memoirs of the Academy of Sciences, unknown to the author, who thus made an attack against his inclination.

He recovered from his disease; but the remembrance of what he had suffered did not make him more prudent for the future. The whole impression of his *Project for a New System of Mechanics*, having been sold off, he formed a design to publish a second edition of it, or rather a work entirely new, though upon the same plan, but more extended. It must be easy to perceive how much learning he must have acquired in the interval; but he often complained, that he wanted time, though he was by no means disposed to lose any. Frequent visits, either of French or of foreigners, some of whom went to see him that they might have it to say that they had seen him; and others to consult him and improve by his conversation: works of mathematics, which the authority of some, or the friendship he had for others, engaged him to examine, and which he thought himself obliged to give the most exact account of; a literary correspondence with all the chief mathematicians of Europe; all these obstructed the book he had undertaken to write. Thus a man acquires reputation by having a great deal of leisure time, and he loses this precious leisure as soon as he has acquired reputation. Add to this, that his best scholars, whether in the College of Mazarine or the Royal College (for he had a professor's chair in both), sometimes requested private lectures of him, which he could not refuse. He sighed for his two or three months of vacation, for that was all the leisure time he had in the year; no sooner were they come but he retired into the country, where his time was entirely his own, and the days seemed always quickly ended.

Notwithstanding his great desire of peace, in the latter part of his life he was involved in a dispute. An Italian monk, well versed in mathematics, attacked him upon the subject of tangents and the angle of contact in curves, such as they are conceived in the arithmetic of infinites; he answered by the last memoir he ever gave to the Academy, and the only one which turned upon a dispute.

In the last two years of his life he was attacked with an asthmatic complaint. This disorder increased every day, and all remedies were ineffectual. He did not however cease from any of his customary business; so that, after having finished his lecture at the College of Mazarine, on the 22d of December 1722, he died suddenly the following night.

His character, says Fontenelle, was as simple as his superior understanding could require. He was not apt to be jealous of the fame of others: indeed he was at the head of the French mathematicians, and one of the best in Europe. It must be owned however, that when a new idea was offered to him, he was too hasty to object. The fire of his genius, the various insights into every subject, made too impetuous an opposition to those that were offered; so that it was not easy to obtain from him a favourable attention.

His works that were published separately, were,

1. Projet d'une Nouvelle Mechanique; 4to, Paris 1687.

2. Des Nouvelles Conjectures fur la Pefanteur.

3. Nouvelle Mechanique ou Statique, 2 tom. 4to, 1725.

As to his memoirs in the volumes of the Academy of Sciences, they are far too numerous to be here particularized; they extend through almoft all the volumes, down to his death in 1722.

VASA *Concordiæ*, in Hydraulics, are two veffels, fo conftructed, as that one of them, though full of wine, will not run a drop, unlefs the other, being full of water, do run alfo. Their ftructure and apparatus may be feen in Wolfius, Element. Mathef. tom. 3, Hydraul.

VAULT, in Architecture, an arched roof, fo contrived, as that the feveral ftones of which it confifts, by their difpofition into the form of a curve, mutually fuftain each other; as the arches of bridges, &c.

Vaults are to be preferred, on many occafions, to foffits, or flat ceilings, as they give a greater rife and elevation, and are alfo more firm and durable.

The Ancients, Salmafius obferves, had only three kinds of vaults: the firft the *fornix*, made cradlewife; the 2d, the *teftudo*, tortoife-wife, or oven-wife; the 3d, the *concha*, made fhell-wife.

But the Moderns fubdivide thefe three forts into a great many more, to which they give different names, according to their figures and ufe: fome are circular, others elliptical, &c.

Again, the fweeps of fome are larger, and others lefs portions of a fphere: all above hemifpheres are called *high*, or *furmounted Vaults*; all that are lefs than hemifpheres, are *low*, or *furbafed Vaults*, &c.

In fome the height is greater than the diameter; in others it is lefs: there are others again quite flat, only made with haunfes; others oven-like, and others growing wider as they lengthen, like a trumpet.

Of Vaults, fome are *fingle*, others *double*, *crofs*, *diagonal*, *horizontal*, *afcending*, *defcending*, *angular*, *oblique*, *pendent*, &c, &c. There are alfo *Gothic* Vaults, with *pendentives*, &c.

Mafter VAULTS, are thofe which cover the principal parts of buildings; in contradiftinction from the *lefs*, or fubordinate Vaults, which only cover fome fmall part; as a paffage, a gate, &c.

Double VAULT, is fuch a one as, being built over another, to make the exterior decoration range with the interior, leaves a fpace between the convexity of the one, and the concavity of the other: as in the dome of St. Paul's at London, and that of St. Peter's at Rome.

VAULTS *with Compartiments*, are fuch whofe fweep, or inner face, is enriched with pannels of fculpture, feparated by platbands. Thefe compartiments, which are of different figures, according to the Vaults, and are ufually gilt on a white ground, are made with ftucco, on brick Vaults; as in the church of St. Peter's at Rome; and with plafter, on timber Vaults.

Theory of VAULTS.—In a femicircular Vault, or arch, being a hollow cylinder cut by a plane through its axis, ftanding on two impofts, and all the ftones that compofe it, being cut and placed in fuch a manner, as that their joints, or beds, being prolonged, do all meet in the centre of the vault; it is evident that all the ftones muft be cut wedge-wife, or wider at top and above, than below; by virtue of which, they fuftain each other, and mutually oppofe the effort of their weight, which determines them to fall.

The ftone in the middle of the Vault, which is perpendicular to the horizon, and is called the *key of the Vault*, is fuftained on each fide by the two contiguous ftones, as by two inclined planes.

The fecond ftone, which is on the right or left of the key-ftone, is fuftained by a third; which, by virtue of the figure of the Vault, is neceffarily more inclined to the fecond, than the fecond is to the firft; and confequently the fecond, in the effort it makes to fall, employs a lefs part of its weight than the firft.

For the fame reafon, all the ftones, reckoning from the keyftone, employ ftill a lefs and lefs part of their weight to the laft; which, refting on the horizontal plane, employs no part of its weight, or makes no effort to fall, as being entirely fupported by the impoft.

Now a great point to be aimed at in Vaults, is, that all the feveral ftones make an equal effort to fall: to effect this, it is evident that as each ftone, reckoning from the key to the impoft, employs a ftill lefs and lefs part of its whole weight; the firft only employing, for example, one-half; the 2d, one-third; the 3d, one-fourth; &c; there is no other way to make thefe different parts equal, but by a proportionable augmentation of the whole; that is, the fecond ftone muft be heavier than the firft, the third heavier than the fecond, and fo on to the laft, which fhould be vaftly heavier.

La Hire demonftrates what that proportion is, in which the weights of the ftones of a femicircular arch muft be increafed, to be in equilibrio, or to tend with equal forces to fall; which gives the firmeft difpofition that a vault can have. Before him, the architects had no certain rule to conduct themfelves by; but did all at random. Reckoning the degrees of the quadrant of the circle, from the keyftone to the impoft; the length or weight of each ftone muft be fo much greater, as it is farther from the key. La Hire's rule is, to augment the weight of each ftone above that of the key ftone, as much as the tangent of the arch to the ftone exceeds the tangent of the arch of half the key. Now the tangent of the laft ftone becomes infinite, and confequently the weight fhould be fo too; but as infinity has no place in practice, the rule amounts to this, that the laft ftone be loaded as much as poffible, and the others in proportion, that they may the better refift the effort which the Vault makes to feparate them; which is called the *fhoot* or *drift* of the Vault.

M. Parent, and other authors, have fince determined the curve, or figure, which the extrados or outfide of a Vault, whofe intrados or infide is fpherical, ought to have, that all the ftones may be in equilibrio.

The above rule of La Hire's has fince been found not accurate. See ARCH, and BRIDGE. See alfo my Treatife on the Principles of Bridges, and Emerfon's Conftruction of Arches.

Key of a VAULT. See KEY, and VOUSSOIR.

Reins or *fillings up of a* VAULT, are the fides which fuftain it.

Pendentive of a VAULT. See PENDENTIVE.

Impoft of a VAULT, is the ftone upon which is laid the firft vouffoir, or arch-ftone of the Vault.

VEADAR,

VEADAR, in Chronology, the 13th month of the Jewiſh eccleſiaſtical year, anſwering commonly to our March; this month is intercalated, to prevent the beginning of Niſan from being removed to the end of February.

VECTIS, in Mechanics, one of the ſimple mechanical powers, more uſually called the LEVER.

VECTOR, or *Radius Vector*, in Aſtronomy, is a line ſuppoſed to be drawn from any planet moving round a centre, or the focus of an ellipſe, to that centre, or focus. It is ſo called, becauſe it is that line by which the planet ſeems to be carried round its centre; and with which it deſcribes areas proportional to the times.

VELOCITY, or *Swiftneſs*, in Mechanics, is that affection of motion, by which a moving body paſſes over a certain ſpace in a certain time. It is alſo called celerity; and it is always proportional to the ſpace moved over in a given time, when the Velocity is uniform, or always the ſame during that time.

Velocity is either *uniform* or *variable*. *Uniform*, or equal *Velocity*, is that with which a body paſſes always over equal ſpaces in equal times. And it is *variable*, or *unequal*, when the ſpaces paſſed over in equal times are unequal; in which caſe it is either *accelerated* or *retarded* Velocity; and this acceleration, or retardation, may alſo be equal or unequal, i. e. uniform or variable, &c. See ACCELERATION, and MOTION.

Velocity is alſo either *abſolute* or *relative*. *Abſolute Velocity* is that we have hitherto been conſidering, in which the Velocity of a body is conſidered ſimply in itſelf, or as paſſing over a certain ſpace in a certain time. But *relative* or *reſpective Velocity*, is that with which bodies approach to, or recede from one another, whether they both move, or one of them be at reſt. Thus, if one body move with the abſolute Velocity of 2 feet per ſecond, and another with that of 6 feet per ſecond; then if they move directly towards each other, the relative velocity with which they approach is that of 8 feet per ſecond; but if they move both the ſame way, ſo that the latter overtake the former, then the relative Velocity with which that overtakes it, is only that of 4 feet per ſecond, or only half of the former; and conſequently it will take double the time of the former before they come in contact together.

VELOCITY *in a Right Line*.—When a body moves with a uniform Velocity, the ſpaces paſſed over by it, in different times, are proportional to the times; alſo the ſpaces deſcribed by two different uniform Velocities, in the ſame time, are proportional to the Velocities; and conſequently, when both times and Velocities are unequal, the ſpaces deſcribed are in the compound ratio of the times and Velocities. That is, $S \propto TV$, and $s \propto tv$; or $S : s :: TV : tv$. Hence alſo, $V : v :: \frac{S}{T} : \frac{s}{t}$, or the Velocity is as the ſpace directly and the time reciprocally.

But in uniformly accelerated motions; the laſt degree of Velocity uniformly gained by a body in beginning from reſt, is proportional to the time; and the ſpace deſcribed from the beginning of the motion, is as the product of the time and Velocity, or as the ſquare of the Velocity, or as the ſquare of the time. That is, in uniformly accelerated motions, $v \propto t$, and $s \propto tv$ or $\propto v^2$ or $\propto t^2$. And, in fluxions, $\dot{s} = vt$.

VELOCITY *of Bodies moving in Curves*.—According to Galileo's ſyſtem of the fall of heavy bodies, which is now univerſally admitted among philoſophers, the Velocities of a body falling vertically are, at each moment of its fall, as the ſquare roots of the heights from whence it has fallen; reckoning from the beginning of the deſcent. And hence he inferred, that if a body deſcend along an inclined plane, the Velocities it has, at the different times, will be in the ſame ratio: for ſince its Velocity is all owing to its fall, and it only falls as much as there is perpendicular height in the inclined plane, the Velocity ſhould be ſtill meaſured by that height, the ſame as if the fall were vertical.

The ſame principle led him alſo to conclude, that if a body fall through ſeveral contiguous inclined planes, making any angles with each other, much like a ſtick when broken, the Velocity would ſtill be regulated after the ſame manner, by the vertical heights of the different planes taken together, conſidering the laſt Velocity as the ſame that the body would acquire by a fall through the ſame perpendicular height.

This concluſion it ſeems continued to be acquieſced in, till the year 1672, when it was demonſtrated to be falſe, by James Gregory, in a ſmall piece of his intitled *Tentamina quædam Geometrica de Motu Penduli & Projectorum*. This piece has been very little known, becauſe it was only added to the end of an obſcure and pſeudonymous piece of his, then publiſhed, to expoſe the errors and vanity of Mr. Sinclair, profeſſor of natural philoſophy at Glaſgow. This little jeu d'eſprit of Gregory is intitled, *The great and new Art of Weighing Vanity: or a diſcovery of the Ignorance and Arrogance of the great and new Artiſt, in his Pſeudo-Philoſophical writings: by M. Patrick Mathers, Arch-Bedal to the Univerſity of S. Andrews*. In the *Tentamina*, Gregory ſhews what the real Velocity is, which a body acquires by deſcending down two contiguous inclined planes, forming an obtuſe angle, and that it is different from the Velocity a body acquires by deſcending perpendicularly through the ſame height; alſo that the Velocity in quitting the firſt plane, is to that with which it enters the ſecond, and in this latter direction, as radius to the coſine of the angle of inclination between the two planes.

This concluſion however, Gregory obſerves, does not apply to the motions of deſcent down any curve lines, becauſe the contiguous parts of curve lines do not form any angle between them, and conſequently no part of the Velocity is loſt by paſſing from one part of the curve to the other; and hence he infers, that the Velocities acquired in deſcending down a continued curve line, are the ſame as by falling perpendicularly through the ſame height. This principle is then applied, by the author, to the motion of pendulums and projectiles.

Varignon too, in the year 1693, followed in the ſame track, ſhewing that the Velocity loſt in paſſing from one right lined direction to another, becomes indefinitely ſmall in the courſe of a curve line; and that therefore the doctrine of Galileo holds good for the deſcent of bodies down a curve line, viz. that the Velocity

 acquired

acquired at any point of the curve, is equal to that which would be acquired by a fall through the fame perpendicular altitude.

The nature of every curve is abundantly determined by the ratio of the ordinates to the correfponding abfciffes; and the effence of curves in general may be conceived as confifting in this ratio, which may be varied in a thoufand different ways. But this fame ratio will be alfo that of two fimple Velocities, by whofe joint effect a body may defcribe the curve in queftion; and confequently the effence of all curves, in general, is the fame thing as the concourfe or combination of all the forces which, taken two by two, may move the fame body. Thus we have a moft fimple and general equation of all poffible curves, and of all poffible Velocities. By means of this equation, as foon as the two fimple Velocities of a body are known, the curve refulting from them is immediately determined.

It may be obferved, in particular, according to this equation, that an uniform Velocity, combined with a Velocity that always varies as the fquare roots of the heights, the two produce the particular curve of a parabola, independent of the angle made by the directions of the two forces that give the Velocities; and confequently a cannon ball, fhot either horizontally or obliquely to the horizon, muft always defcribe a parabola, were it not for the refiftance of the air.

Circular Velocity. See Circular.

Initial Velocity, in Gunnery, denotes the Velocity with which military projectiles iffue from the mouth of the piece by which they are difcharged. This, it is now known, is much more confiderable than was formerly apprehended. For the method of eftimating it, and the refult of a variety of experiments, by Mr. Robins, and myfelf, &c, fee the articles Gun, Gunnery, Projectile, and Resistance.

Mr. Robins had hinted in his New Principles of of Gunnery, at another method of meafuring the Initial Velocities of military projectiles, viz, from the arc of vibration of the gun itfelf, in the act of expulfion, when it is fufpended by an axis like a pendulum. And Mr. Thompfon, in his experiments (Philof. Tranf. vol. 71, p. 229) has purfued the fame idea at confiderable length, in a number of experiments, from whence he deduces a rule for computing the Velocity, which is fomewhat different from that of Mr. Robins, but which agrees very well with his own experiments.

This rule however being drawn only from the experiments with a mufket barrel, and with a fmall charge of powder, and befides being different from that in the theory as propofed by Robins; it was fufpected that it would not hold good when applied to cannon, or other large pieces of ordnance, of different and various lengths, and to larger charges of powder. For this reafon, a great multitude of experiments, as related in my Tracts, vol. 1, were inftituted with cannon of various lengths and charged with many different quantities of powder; and the Initial Velocities of the fhot were computed both from the vibration of a balliftic pendulum, and from the vibration of the gun itfelf; but the confequence was, that thefe two hardly ever agreed together, and in many cafes they differed by almoft 400 feet per fecond in the Velocity. A brief abftract for a comparifon between thefe two methods, is contained in the following tablet, viz.

Comparifon of the Velocities by the Gun and Pendulum.

Gun No.	2 Ounces.			4 Ounces.			8 Ounces.			16 Ounces.		
	Velocity by		Diff.	Velocity by		Diff.	Velocity by		Diff.	Velocity by		Diff.
	Gun	Pend.		Gun	Pend.		Gun	Pend.		Gun	Pend.	
1	830	780	50	1135	1100	35	1445	1430	15	1345	1377	−32
2	863	835	28	1203	1180	23	1521	1580	−59	1485	1656	−171
3	919	920	−1	1294	1300	−6	1631	1790	−159	1680	1998	−318
4	929	970	−41	1317	1370	−53	1669	1940	−271	1730	2106	−376

In this table, the firft column fhews the number of the gun, as they were of different lengths; viz, the length of number 1 was $30\frac{1}{3}$ inches, number 2 was $40\frac{1}{3}$ inches, number 3 was 60 inches, and number 4 was 83 inches, nearly. After the firft column, the reft of the table is divided into four fpaces, for the four charges, 2, 4, 8, 16 ounces of powder: and each of thefe is divided into three columns: in the firft of the three is the Velocity of the ball as determined from the vibration of the gun; in the fecond is the Velocity as determined from the vibration of the pendulum; and in the third is the difference between the two, being fo many feet per fecond, which is marked with the negative fign, or −, when the former Velocity is too little, otherwife it is pofitive.

From the comparifon contained in this table, it appears, in general, that the Velocities, determined by the two different ways, do not agree together; and that therefore the method of determining the Velocity of the ball from the recoil of the gun, is not generally true, although Mr. Robins and Mr. Thompfon had fufpected it to be fo: and confequently that the effect of the inflamed powder on the recoil of the gun, is not exactly the fame when it is fired without a ball, as when it is fired with one. It alfo appears, that this difference is no ways regular, neither in the different

guns

guns with the fame charge of powder, nor in the fame gun with different charges: That with very fmall charges, the Velocity by the gun is greater than that by the pendulum; but that the latter always gains upon the former, as the charge is increafed, and foon becomes equal to it; and afterwards goes on to exceed it more and more: That the particular charge, at which the two Velocities become equal, is different in the different guns; and that this charge is lefs, or the equality fooner takes place, as the gun is longer. And all this, whether we ufe the actual Velocity with which the ball ftrikes the pendulum, or the fame increafed by the Velocity loft by the refiftance of the air, in its flight from the gun to the pendulum.

VENTILATOR, a machine by which the noxious air of any clofe place, as an hofpital, gaol, fhip, chamber, &c, may be difcharged and changed for frefh air.

The noxious qualities of bad air have been long known; and Dr. Hales and others have taken great pains to point out the mifchiefs arifing from foul air, and to prevent or remedy them. That philofopher propofed an eafy and effectual one, by the ufe of his Ventilators; the account of which was read before the Royal Society in May 1741; and a farther account of it may be feen in his Defcription of Ventilators, printed at London in 8vo, 1743; and ftill farther in part 2, p. 32, printed in 1758; where the ufes and applications of them are pointed out for fhips, and prifons, &c. For what is faid of the foul air of fhips may be applied to that of gaols, mines, workhoufes, hofpitals, barracks, &c. In mines, Ventilators may guard againft the fuffocations, and other terrible accidents arifing from damps. The air of gaols has often proved infectious; and we had a fatal proof of this, by the accident that happened fome years fince at the Old Bailey feffions. After that, Ventilators were ufed in the prifon, which were worked by a fmall windmill, placed on the top of Newgate; and the prifon became more healthy.

Dr. Hales farther fuggefts, that Ventilators might be of ufe in making falt; for which purpofe there fhould be a ftream of water to work them; or they might be worked by a windmill, and the brine fhould be in long narrow canals, covered with boards of canvas, about a foot above the furface of the brine, to confine the ftream of air, fo as to make it act upon the furface of the brine, and carry off the water in vapours. Thus it might be reduced to a dry falt, with a faving of fuel, in winter and fummer, or in rainy weather, or any ftate of the air whatever. Ventilators, he apprehends, might alfo ferve for drying linen hung in low, long, narrow galleries, efpecially in damp or rainy weather, and alfo in drying woollen cloths, after they are fulled or dyed; and in this cafe, the Ventilators might be worked by the fulling water-mill. Ventilators might alfo be an ufeful appendage to malt and hop kilns; and the fame author is farther of opinion, that a ventilation of warm dry air from the adjoining ftove, with a cautious hand, might be of fervice to trees and plants in green-houfes; where it is well known that air full of the rancid vapours which perfpire from the plants, is very unkindly to them, as well as the vapours from human bodies are to men: for frefh air is as neceffary to the healthy ftate of vegetables, as of animals.—Ventilators are alfo of excellent ufe for drying corn, hops, and malt.—Gunpowder may be thoroughly dried, by blowing air up through it by means of Ventilators; which is of great advantage to the ftrength of it. Thefe Ventilators, even the fmaller ones, will alfo ferve to purify moft eafily, and effectually, the bad air of a fhip's well, before a perfon is fent down into it, by blowing air through a trunk, reaching near the bottom of it. And in a fimilar manner may ftinking water, and ill-tafted milk, &c, be fweetened, viz, by paffing a current of air through them, from bottom to top, which will carry the offenfive particles along with it.

For thefe and other ufes to which they might be applied, as well as for a particular account of the conftruction and difpofition of Ventilators in fhips, hofpitals, prifons, &c, and the benefits attending them, fee Hales's Treatife on Ventilators, part 2 paffim; and the Philof. Tranf. vol 49, p. 332.

The method of drawing off air from fhips by means of fire-pipes, which fome have preferred to Ventilators, was publifhed by Sir Robert Moray in the Philof. Tranf. for 1665. Thefe are metal pipes, about $2\frac{1}{2}$ inches diameter, one of which reaches from the fire-place to the well of the fhip, and other three branches go to other parts of the fhip; the ftove hole and afh hole being clofed up, the fire is fupplied with air through thefe pipes. The defects of thefe, compared with Ventilators, are particularly examined by Dr. Hales, ubi fupra, p. 113.

In the latter part of the year 1741, M. Triewald, military architect to the king of Sweden, informed the fecretary to the Royal Society, that he had in the preceding fpring invented a machine for the ufe of fhips of war, to draw out the foul air from under their decks, which exhaufted 36172 cubic feet of air in an hour, or at the rate of 21732 tuns in 24 hours. In 1742 he fent one of thefe to France, which was approved of by the Academy of Sciences at Paris, and the navy of France was ordered to be furnifhed with the like Ventilators.

Mr. Erafmus King propofed to have Ventilators worked by the fire engines, in mines. And Mr. Fitzgerald has fuggefted an improved method of doing this, which he has alfo illuftrated by figures. See Philof. Tranf. vol. 50, p, 727.

There are various ways of Ventilation, or changing the air of rooms. Mr. Tidd contrived to admit frefh air into a room, by taking out the middle upper fafh pane of glafs, and fixing in its place a frame box, with a round hole in its middle, about 6 or 7 inches diameter; in which hole are fixed, behind each other, a fet of fails of very thin broad copper-plates, which fpread over and cover the circular hole, fo as to make the air which enters the room, and turning round thefe fails, to fpread round in thin fheets fideways; and fo not to incommode perfons, by blowing directly upon them, as it would do if it were not hindered by the fails.

This method however is very unfeemly and difagreeable in good rooms: and therefore, inftead of it, the late ingenious Mr. John Whitehurft fubftituted another; which was, to open a fmall fquare or rectanglar hole in the party wall of the room, in the upper part near the cieling, at a corner or part diftant from the fire-

fire; and before it he placed a thin piece of metal or pasteboard &c, attached to the wall in its lower part just below the hole, but declining from it upwards, so as to give the air, that enters by the hole, a direction upwards against the cieling, along which it sweeps and disperses itself through the room, without blowing in a current against any person. This method is very useful to cure smoky chimneys, by thus admitting conveniently fresh air. A picture placed before the hole prevents the sight of it from disfiguring the room. This, and many other methods of Ventilating, he meant to have published, and was occupied upon, when death put an end to his useful labours. These have since been published, viz in 1794, 4to, by Dr. Willan.

VENUS, in Astronomy, one of the inferior planets, but the brightest and to appearance the largest of all the planets; and is designed by the mark ♀, supposed to be a rude representation of a female figure, with her trailing robe.

Venus is easily distinguished from all the other planets, by her whiteness and brightness, in which she exceeds all the rest, even Jupiter himself, and which is so considerable, that in a dusky place she causes an object to project a sensible shadow, and she is often visible in the day-time. Her place in the system is the second from the sun, viz, between Mercury and the earth, and in magnitude is about equal to the earth, or rather a little larger according to Dr. Herschel's observations.

As Venus moves round the sun, in a circle beneath that of the earth, she is never seen in opposition to him, nor indeed very far from him; but seems to move backward and forward, passing him from side to side, to the distance of about 47 or 48 degrees, both ways, which is her greatest elongation.

When she appears west of the sun, which is from her inferior conjunction to her superior, she rises before him, or is a morning star, and is called *Phosphorus*, or *Lucifer*, or the *Morning Star*; and when she is eastwards from the sun, which is from her superior conjunction to her inferior, she sets after him, or is an evening star, and is called *Hesperus*, or *Vesper*, or the *Evening star*: being each of those in its turn for 290 days.

The real diameter of Venus is nearly equal to that of the earth, being about 7900 miles; her apparent mean diameter seen from the earth 59″, seen from the sun, or her horizontal parallax, 30″; but as seen from the earth 18″·79 according to Dr. Herschel: her distance from the sun 70 million of miles; her eccentricity $\frac{7}{1000}$ths of the same, or 490,000 miles; the inclination of her orbit to the plane of the ecliptic 3° 23′; the points of their intersection or nodes are 14° of ♊ and ♐; the place of her aphelion ♒ 4° 20′; her axis inclined to her orbit 75° 0′; her periodical course round the sun 224 days 17 hours; the diurnal rotation round her axis very uncertain, being according to Cassini only 23 hours, but according to the observations of Bianchini it is in 24 days 8 hours; though Dr. Herschel thinks it cannot be so much. See also Planets.

Venus, when viewed through a telescope, is rarely seen to shine with a full face, but has phases and changes just like those of the moon, being increasing, decreasing, horned, gibbous, &c: her illuminated part being constantly turned toward the sun, or directed toward the east when she is a morning star, and toward the west when an evening star.

These different phases of Venus were first discovered by Galileo; who thus fulfilled the prediction of Copernicus: for when this excellent astronomer revived the ancient Pythagorean system, asserting that the earth and planets move round the sun, it was objected that in such a case the phases of Venus should resemble those of the moon; to which Copernicus replied, that some time or other that resemblance would be found out. Galileo sent an account of the first discovery of these phases in a letter, written from Florence in 1611, to William de Medici, the duke of Tuscany's ambassador at Prague; desiring him to communicate it to Kepler. The letter is extant in the preface to Kepler's Dioptrics, and a translation of it in Smith's Optics, p. 416. Having recited the observations he had made, he adds, "We have hence the most certain, sensible decision and demonstration of two grand questions, which to this day have been doubtful and disputed among the greatest masters of reason in the world. One is, that the planets in their own nature are opake bodies, attributing to Mercury what we have seen in Venus: and the other is, that Venus necessarily moves round the sun; as also Mercury and the other planets; a thing well believed indeed by Pythagoras, Copernicus, Kepler, and myself, but never yet proved, as now it is, by ocular inspection upon Venus."

Cassini and Campani, in the years 1665 and 1666, discovered spots in the face of Venus: from the appearances of which the former ascertained her motion round her axis; concluding that this revolution was performed in less than a day; or at least that the bright spot which he observed, finished its period either by revolution or libration in about 23 hours. And de la Hire, in 1700, through a telescope of 16 feet, discovered spots in Venus; which he found to be larger than those in the moon.

The next observations of the same kind that occur, are those of signior Binanchini at Rome, in 1726, 1727, 1728, who, with Campani's glasses, discovered several dark spots in the disc of Venus, of which he gave an account and a representation in his book entitled Hesperi et Phosphori Nova Phenomena, published at Rome in 1728. From several successive observations Bianchini concludes, that a rotation of Venus about her axis was not completed in 23 hours, as Cassini imagined, but in $24\frac{1}{3}$ days; that the north pole of this rotation faced the 20th degree of Aquarius, and was elevated 15° above the plane of the ecliptic, and that the axis kept parallel to itself, during the planet's revolution about the sun. Cassini the son, though he admits the accuracy of Bianchini's observations, disputes the conclusion drawn from them, and finally observes, that if we suppose the period of the rotation of Venus to be 23 h. 20 min. it agrees equally well with the observations both of his father and Bianchini; but if she revolve in 24 d. 8 h. then his father's observations must be rejected as of no consequence.

In the Philos. Trans. 1792, are published the results of a course of observations on the planet Venus, begun in the year 1780, by Mr. Schroeter, of Lilienthal, Bremen. From these observations, the author infers, that

that Venus has an atmosphere in some respects similar to that of our earth, but far exceeding that of the moon in density, or power to weaken the rays of the sun: that the diurnal period of this planet is probably much longer than that of other planets: that the moon also has an atmosphere, though less dense and high than that of Venus: and that the mountains of this planet are 5 or 6 times as high as those on the earth.

Dr. Herschel too, between the years 1777 and 1793, has made a long series of observations on this planet, accounts of which are given in the Philos. Trans. for 1793. The results of these observations are: that the planet revolves about its axis, but the time of it is uncertain: that the position of its axis is also very uncertain: that the planet's atmosphere is very considerable: that the planet has probably hills and inequalities on its surface, but he has not been able to see much of them, owing perhaps to the great density of its atmosphere; as to the mountains of Venus, no eye, he says, which is not considerably better than his, or assisted by much better instruments, will ever get a sight of them: and that the apparent diameter of Venus, at the mean distance from the earth, is 18″·79; from whence it may be inferred, that this planet is somewhat larger than the earth, instead of being less, as former astronomers have imagined.

Sometimes Venus is seen in the disc of the sun, in form of a dark round spot. These appearances, called Transits, happen but seldom, viz, when the earth is about her nodes at the time of her inferior conjunction. One of these transits was seen in England in 1639 by Mr. Horrox and Mr. Crabtree; and two in the present century, viz, the one June 6, 1761, and the other in June 1769. There will not happen another of them till the year 1874. See Parallax.

Except such transits as these, Venus exhibits the same appearances to us regularly every 8 years; her conjunctions, elongations, and times of rising and setting, being very nearly the same, on the same days, as before.

In 1672 and 1686, Cassini, with a telescope of 34 feet, thought he saw a satellite move round this planet, at the distance of about $\frac{3}{5}$ of Venus's diameter. It had the same phases as Venus, but without any well defined form; and its diameter scarce exceeded $\frac{1}{4}$ of the diameter of Venus. Dr. Gregory (Astron. lib. 6, prop. 3) thinks it more than probable that this was a satellite; and supposes that the reason why it is not more frequently seen, is the unfitness of its surface to reflect the rays of the sun's light; as is the case of the spots in the moon; for if the whole disc of the moon were composed of such, he thinks she could not be seen so far as to Venus.

Mr. Short, in 1740, with a reflecting telescope of $16\frac{1}{2}$ inches focus, perceived a small star near Venus: with another telescope of the same focus, magnifying 50 or 60 times, and fitted with a micrometer, he found its distance from Venus about 10′; and with a magnifying power of 240, he observed the star assume the same phases with Venus; its diameter seemed to be about $\frac{1}{3}$, or somewhat less, of the diameter of Venus; its light not so bright and vivid, but exceeding sharp and well defined. He viewed it for the space of an hour; but never had the good fortune to see it after the first morning. Philos. Trans. number 459, p. 646, or Abr. vol. 8, p. 208.

M. Montaign, of Limoges in France, preparing for observing the transit of 1761, discovered in the preceding month of May a small star, about the distance of 20′ from Venus, the diameter of it being about $\frac{1}{4}$ of that of the planet. Others have also thought they saw a like appearance. And indeed it must be acknowledged, that Venus may have a satellite, though it is difficult for us to see it. Its enlightened side can never be fully turned towards us, but when Venus is beyond the sun; in which case Venus herself appears little bigger than an ordinary star, and therefore her satellite may be too small to be perceived at such a distance. When she is between us and the sun, her moon has its dark side turned towards us; and when Venus is at her greatest elongation, there is but half the enlightened side of the moon turned toward us, and even then it may be too far distant to be seen by us. But it was presumed, that the two transits of 1761, and 1769, would afford opportunity for determining this point; and yet we do not find, although many observers directed their attention to this object, that any satellite was then seen in the sun's disc; unless we except two persons, viz, an anonymous writer in the London Chronicle of May 18, who says that he saw the satellite of Venus on the sun the day of the transit, at St. Neot's in Huntingdonshire; that it moved in a track parallel to that of Venus, but nearer the ecliptic; that Venus quitted the sun's disc at 31 minutes after 8, and the satellite at 6 minutes after 9; and M. Montaign at Limoges, whose account of his observations is in the Memoirs of the Academy of Paris, from whence the following certificate is extracted:—Certificate. "We having examined, by order of the Academy, the remarks of M. Baudouin on a new observation of the satellite of Venus, made at Limoges the 11th of May by M. Montaign. This fourth observation, of great importance for the theory of the satellite, has shewn that its revolution must be longer than appeared by the first three observations. M. Baudouin believes it may be fixed at 12 days; as to its distance, it appears to him to be 50 semidiameters of Venus; whence he infers that the mass of Venus is equal to that of the earth. This mass of Venus is a very essential element to astronomy, as it enters into many computations, and produces different phenomena: &c.

Signed L' Abbé De La Caille,
De La Lande."

VERBERATION, in Physics, a term used to express the cause of sound, which arises from a Verberation of the air, when struck, in divers manners, by the several parts of the sonorous body first put into a vibratory motion.

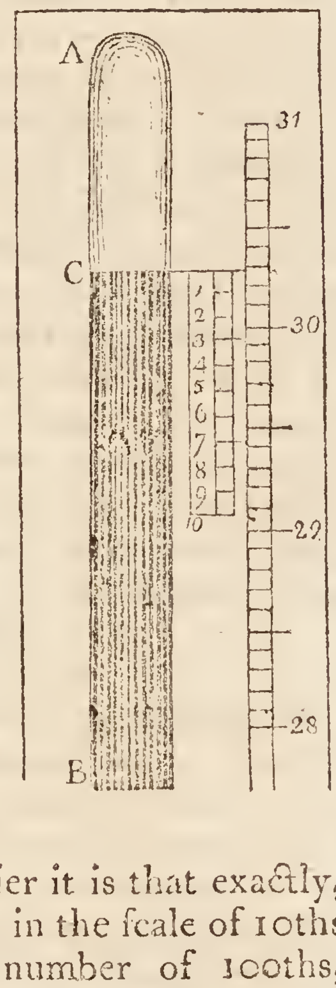
VERNAL, something belonging to the spring season: as vernal signs, vernal equinox, &c.

VERNIER, is a scale, or a division, well adapted for the graduation of mathematical instruments, so called from its inventor Peter Vernier, a gentleman of Franche Comté, who communicated the discovery to the world in a small tract, entitled La Construction, l'Usage, et les Proprietez du Quadrant Nouveau de Mathematique &c, printed at Brussels in 1631. This

was an improvement on the method of diviſion propoſed by Jacobus Curtius, printed by Tycho in Clavius's Aſtrolabe, in 1593. Vernier's method of diviſion, or dividing plate, has been very commonly, though erroneouſly, called by the name of Nonius; the method of Nonius being very different from that of Vernier, and much leſs convenient.

When the relative unit of any line is ſo divided into many ſmall equal parts, thoſe parts may be too numerous to be introduced, or if introduced, they may be too cloſe to one another to be readily counted or eſtimated; for which reaſon there have been various methods contrived for eſtimating the aliquot parts of the ſmall diviſions, into which the relative unit of a line may be commodiouſly divided; and among thoſe methods, Vernier's has been moſt juſtly preferred to all others. For the hiſtory of this, and other inventions of a ſimilar nature, ſee Robins's Math. Tracts, vol. 2, p. 265, &c.

Vernier's ſcale is a ſmall moveable arch, or ſcale, ſliding along the limb of a quadrant, or any other graduated ſcale, and divided into equal parts, that are one leſs in number than the diviſions of the portion of the limb correſponding to it. So, if we want to ſubdivide the graduations on any ſcale into for ex. 10 equal parts; we muſt make the Vernier equal in length to 11 of thoſe graduations of the ſcale, but dividing the ſame length of the Vernier itſelf only into 10 equal parts; for then it is evident that each diviſion on the Vernier will be $\frac{1}{10}$th part longer than the gradations on the inſtrument, or that the diviſion of the former is equal to $\frac{11}{10}$ of the degree on the latter, as that gains 1 in 10 upon this.



Thus let AB be a part of the upper end of a barometer tube, the quickſilver ſtanding at the point C; from 28 to 31 is a part of the ſcale of inches, viz, from 28 inches to 31 inches, divided into 10ths of inches; and the middle piece, from 1 to 10, is the Vernier, that ſlides up and down in a groove, and having 10 of its diviſions equal to 11 tenths of the inches, for the purpoſe of ſubdividing every 10th of the inch into 10 parts, or, the inches into centeſms or 100th parts. In practice, the method of counting is by obſerving (when the Vernier is ſet with its index at top pointing exactly againſt the upper ſurface of the mercury in the tube) which diviſion of the Vernier it is that exactly, or neareſt, coincides with a diviſion in the ſcale of 10ths of inches, for that will ſhew the number of 100ths, over the 10ths of inches next below the index at top. So, in the annexed figure, the top of the Vernier is between 2 and 3 tenths above the 30 inches of the barometer; and becauſe the 8th diviſion of the Vernier is ſeen to coincide with a diviſion of the ſcale, this ſhews that it is 8 centeſms more: ſo that the height of the quickſilver altogether, is 30·28, that is, 30 inches, and 28 hundredths, or 2 tenths and 8 hundredths.

If the ſcale were not inches and 10ths, but degrees of a quadrant, &c, then the 8 would be $\frac{8}{10}$ of a degree, or 48′; or if every diviſion on the ſcale be 10 minutes, then the Vernier will ſubdivide it into ſingle minutes, and the 8 will then be 8 minutes. And ſo for any other caſe.

By altering the number of diviſions, either in the degrees or in the Vernier, or in both, an angle can be obſerved to many different degrees of accuracy. Thus, if a degree on a quadrant be divided into 12 parts, each being 5 minutes, and the length of the Vernier be 21 ſuch parts, or $1°\frac{3}{4}$, and divided into 20 parts, then

$$\frac{1}{12} \times \frac{1}{20} = \frac{1°}{240} = \frac{1'}{4} = 15'',$$

is the ſmalleſt diviſion the Vernier will meaſure to. Or, if the length of the Vernier be $2°\frac{7}{12}$, and divided into 30 parts, then

$$\frac{1}{12} \times \frac{1}{30} = \frac{1°}{360} = \frac{1'}{6} = 10'',$$

is the ſmalleſt part in this caſe: Alſo

$$\frac{1}{12} \times \frac{1}{50} = \frac{1°}{600} = \frac{1'}{10} = 6'',$$

is the ſmalleſt part when the Vernier extends $4°\frac{1}{4}$. See Robertſon's Navigation, book 5, p. 279.

For the method of applying the Vernier to a quadrant, ſee *Hadley's* QUADRANT. And for the application of it to a teleſcope, and the principles of its conſtruction, ſee Smith's Optics, book 3, ſect. 861.

VERSED-*Sine*, of an arch, is the part of the diameter intercepted between the ſine and the commencement of the arc; and it is equal to the difference between the radius and the coſine. See *Verſed*-SINE. And for *coverſed ſine*, ſee COVERSED-*Sine*.

VERTEX *of an Angle*, is the angular point, or the point where the legs or ſides of the angle meet.

VERTEX *of a Figure*, is the uppermoſt point, or the vertex of the angle oppoſite to the baſe.

VERTEX *of a Curve*, is the extremity of the axis or diameter, or it is the point where the diameter meets the curve; which is alſo the vertex of the diameter.

VERTEX *of a Glaſs*, in Optics, the ſame as its pole.

VERTEX is alſo uſed, in Aſtronomy, for the point of the heavens vertically or perpendicularly over our heads, alſo called the zenith.

VERTEX, *Path of the*. See PATH.

VERTICAL, ſomething relating to the vertex or higheſt point. As,

VERTICAL *Point*, in Aſtronomy, is the ſame with vertex, or zenith.—Hence a ſtar is ſaid to be Vertical, when it happens to be in that point which is perpendicularly over any place.

VERTICAL *Circle*, is a great circle of the ſphere, paſſing through the zenith and nadir of a place.—The Vertical circles are alſo called *azimuths*. The meridian of any place is a Vertical circle, viz, that particular one which paſſes through the north or ſouth point of the horizon.—All the Vertical circles interſect one another in the zenith and nadir.

The uſe of the Vertical circles is to eſtimate or meaſure the height of the ſtars &c, and their diſtances from the zenith, which is reckoned on theſe circles; and to find their eaſtern and weſtern amplitude, by obſerving how many degrees the Vertical, in which the ſtar riſes or ſets, is diſtant from the meridian.

Prime VERTICAL, is that Vertical circle, or azimuth, which paſſes through the poles of the meridian; or which is perpendicular to the meridian, and paſſes through the equinoctial points.

Prime VERTICALS, in Dialling. See PRIME *Verticals*.

VERTICAL *of the Sun*, is the Vertical which paſſes through the centre of the ſun at any moment of time. —Its uſe is, in Dialling, to find the declination of the plane on which the dial is to be drawn, which is done by obſerving how many degrees that Vertical is diſtant from the meridian, after marking the point or line of the ſhadow upon the plane at any times.

VERTICAL *Dial*. See *Vertical* DIAL.

VERTICAL *Line*, in Dialling, is a line in any plane perpendicular to the horizon.——This is beſt found and drawn on an erect and reclining plane, by ſteadily holding up a ſtring and plummet, and then marking two points of the ſhadow of the thread on the plane, a good diſtance from one another: and drawing a line through theſe marks.

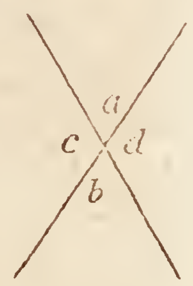
VERTICAL *Line*, in Conics, is a line drawn on the Vertical plane, and through the vertex of the cone.

VERTICAL *Line*, in Perſpective. See *Vertical* LINE.

VERTICAL *Plane*, in Conics, is a plane paſſing through the vertex of a cone, and parallel to any conic ſection.

VERTICAL *Plane*, in Perſpective. See PLANE and PERSPECTIVE.

VERTICAL *Angles*, or *Oppoſite Angles*, in Geometry, are ſuch as have their legs or ſides continuations of each other, and which conſequently have the ſame vertex or angular point. So the angles *a* and *b* are Vertical angles; as alſo the angles *c* and *d*.



VERTICITY, is that property of the magnet or loadſtone, or of a needle &c touched with it, by which it turns or directs itſelf to ſome peculiar point, as to its pole.—The attraction of the magnet was known long before its Verticity.

VERU, a comet, according to ſome writers, reſembling a ſpit, being nearly the ſame as the lonchites, only its head is rounder, and its train longer and ſharper pointed.

VESPER, in Aſtronomy, called alſo *Heſperus*, and the *Evening Star*, is the planet Venus, when ſhe is eaſtward of the ſun, and conſequently ſets after him, and ſhines as an evening ſtar.

VESPERTINE, in Aſtronomy, is when a planet is deſcending to the weſt after ſun-ſet, or ſhines as an evening ſtar.

VIA LACTEA, in Aſtronomy, the milky way, or Galaxy. See GALAXY.

VIA SOLIS, or *ſun's way*, is uſed among aſtronomers, for the ecliptic line, or path in which the ſun ſeems always to move.

VIBRATION, in Mechanics, a regular reciprocal motion of a body, as, for example, a pendulum, which being freely ſuſpended, ſwings or vibrates from ſide to ſide.

Mechanical authors, inſtead of Vibration, often uſe the term *oſcillation*, eſpecially when ſpeaking of a body that thus ſwings by means of its own gravity or weight.

The Vibrations of the ſame pendulum are all iſochronal; that is, they are performed in an equal time, at leaſt in the ſame latitude; for in lower latitudes they are found to be ſlower than in higher ones. See PENDULUM. In our latitude, a pendulum $39\frac{1}{8}$ inches long, vibrates ſeconds, making 60 Vibrations in a minute.

The Vibrations of a longer pendulum take up more time than thoſe of a ſhorter one, and that in the ſubduplicate ratio of the lengths, or the ratio of the ſquare roots of the lengths. Thus, if one pendulum be 40 inches long, and another only 10 inches long, the former will be double the time of the latter in performing a Vibration; for $\sqrt{40} : \sqrt{10} :: \sqrt{4} : \sqrt{1}$, that is as 2 to 1. And becauſe the number of Vibrations, made in any given time, is reciprocally as the duration of one Vibration, therefore the number of ſuch Vibrations is in the reciprocal ſubduplicate ratio of the lengths of the pendulums.

M. Mouton, a prieſt of Lyons, wrote a treatiſe, expreſsly to ſhew, that by means of the number of Vibrations of a given pendulum, in a certain time, may be eſtabliſhed an univerſal meaſure throughout the whole world; and may fix the ſeveral meaſures that are in uſe among us, in ſuch a manner, as that they might be recovered again, if at any time they ſhould chance to be loſt, as is the caſe of moſt of the ancient meaſures, which we now only know by conjecture.

The VIBRATIONS *of a Stretched Chord*, or *String*, ariſe from its elaſticity; which power being in this caſe ſimilar to gravity, as acting uniformly, the Vibrations of a chord follow the ſame laws as thoſe of pendulums. Conſequently the Vibrations of the ſame chord equally ſtretched, though they be of unequal lengths, are iſochronal, or are performed in equal times; and the ſquares of the times of Vibration are to one another inverſely as their tenſions, or powers by which they are ſtretched.

The Vibrations of a ſpring too are proportional to the powers by which it is bent. Theſe follow the ſame laws as thoſe of the chord and pendulum; and conſequently are iſochronal; which is the foundation of ſpring watches.

VIBRATIONS are alſo uſed in *Phyſics*, &c, and for ſeveral other regular alternate motions. Senſation, for inſtance, is ſuppoſed to be performed by means of the vibratory motion of the contents of the nerves, begun by external objects, and propagated to the brain.

This doctrine has been particularly illuſtrated by Dr. Hartley, who has extended it farther than any other writer, in eſtabliſhing a new theory of our mental operations.

The ſame ingenious author alſo applies the doctrine of Vibrations to the explanation of muſcular motion, which he thinks is performed in the ſame general manner as ſenſation and the perception of ideas. For a particular account of his theory, and the arguments by which it is ſupported, ſee his Obſervations on Man, vol. 1.

The feveral forts and rays of light, Newton conceives to make Vibrations of divers magnitudes; which, according to thofe magnitudes, excite fenfations of feveral colours; much after the fame manner as Vibrations of air, according to their feveral magnitudes, excite fenfations of feveral founds. See the article COLOUR.

Heat, according to the fame author, is only an accident of light, occafioned by the rays putting a fine, fubtile, ethereal medium, which pervades all bodies, into a vibrative motion, which gives us that fenfation. See HEAT.

From the Vibrations or pulfes of the fame medium, he accounts for the alternate fits of eafy reflexion and eafy tranfmiffion of the rays.

In the Philofophical Tranfactions it is obferved, that the butterfly, into which the filk-worm is transformed, makes 130 Vibrations or motions of its wings, in one coition.

VIETA (FRANCIS), a very celebrated French mathematician, was born in 1540 at Fontenai, or Fontenaile-Comté, in Lower Poitou, a province of France. He was Mafter of Requefts at Paris, where he died in 1603, being the 63d year of his age. Among other branches of learning in which he excelled, he was one of the moft refpectable mathematicians of the 16th century, or indeed of any age. His writings abound with marks of great originality, and the fineft genius, as well as intenfe application. His application was fuch, that he has fometimes remained in his ftudy for three days together, without eating or fleeping. His inventions and improvements in all parts of the mathematics were very confiderable. He was in a manner the inventor and introducer of Specious Algebra, in which letters are ufed inftead of numbers, as well as of many beautiful theorems in that fcience, a full explanation of which may be feen under the article ALGEBRA. He made alfo confiderable improvements in geometry and trigonometry. His angular fections are a very ingenious and mafterly performance: by thefe he was enabled to refolve the problem of Adrian Roman, propofed to all mathematicians, amounting to an equation of the 45th degree. Romanus was fo ftruck with his fagacity, that he immediately quitted his refidence of Wirtzbourg in Franconia, and came to France to vifit him, and folicit his friendfhip. His Apollonius Gallus, being a reftoration of Apollonius's tract on Tangencies, and many other geometrical pieces to be found in his works, fhew the fineft tafte and genius for true geometrical fpeculations.—He gave fome mafterly tracts on Trigonometry, both plane and fpherical, which may be found in the collection of his works, publifhed at Leyden in 1646, by Schooten, befides another large and feparate volume in folio, publifhed in the author's life-time at Paris in 1579, containing extenfive trigonometrical tables, with the conftruction and ufe of the fame, which are particularly defcribed in the introduction to my Logarithms, pa. 4 &c. To this complete treatife on Trigonometry, plane and fpherical, are fubjoined feveral mifcellaneous problems and obfervations, fuch as, the quadrature of the circle, the duplication of the cube, &c. Computations are here given of the ratio of the diameter of a circle to the circumference, and of the length of the fine of 1 minute, both to a great many places of figures; by which he found that the fine of 1 minute is

between 2908881959
and 2908882056;

alfo the diameter of a circle being 1000 &c, that the perimeter of the infcribed and circumfcribed polygon of 393216 fides, will be as follows, viz, the

perim. of the infcribed polygon 31415926535
perim. of the circumfcribed polygon 31415926537

and that therefore the circumference of the circle lies between thofe two numbers.

Vieta having obferved that there were many faults in the Gregorian Calendar, as it then exifted, he compofed a new form of it, to which he added perpetual canons, and an explication of it, with remarks and objections againft Clavius, whom he accufed of having deformed the true Lelian reformation, by not rightly underftanding it.

Befides thofe, it feems a work greatly efteemed, and the lofs of which cannot be fufficiently deplored, was his *Harmonicon Cœlefte*, which, being communicated to father Merfenne, was, by fome perfidious acquaintance of that honeft-minded perfon, furreptitioufly taken from him, and irrecoverably loft, or fuppreffed, to the great detriment of the learned world. There were alfo, it is faid, other works of an aftronomical kind, that have been buried in the ruins of time.

Vieta was alfo a profound decipherer, an accomplifhment that proved very ufeful to his country. As the different parts of the Spanifh monarchy lay very diftant from one another, when they had occafion to communicate any fecret defigns, they wrote them in ciphers and unknown characters, during the diforders of the league: the cipher was compofed of more than 500 different characters, which yielded their hidden contents to the penetrating genius of Vieta alone. His fkill fo difconcerted the Spanifh councils for two years, that they publifhed it at Rome, and other parts of Europe, that the French king had only difcovered their ciphers by means of magic.

VINCULUM, in Algebra, a mark or character, either drawn over, or including, or fome other way accompanying, a factor, divifor, dividend, &c, when it is compounded of feveral letters, quantities, or terms, to connect them together as one quantity, and fhew that they are to be multiplied, or divided, &c, together.

Vieta, I think, firft ufed the bar or line over the quantities, for a Vinculum, thus $\overline{a+b}$; and Albert Girard the parenthefis thus $(a+b)$; the former way being now chiefly ufed by the Englifh, and the latter by moft other Europeans. Thus $\overline{a+b} \times c$, or $(a+b) \times c$, denotes the product of c and the fum $a+b$ confidered as one quantity. Alfo $\sqrt{a+b}$, or $\surd(a+b)$, denotes the fquare root of the fum $a+b$. Sometimes the mark : is fet before a compound factor, as a Vinculum, efpecially when it is very long, or an infinite feries; thus $3a \times : 1 - 2x + 3x^2 - 4x^3 + 5x^4$ &c.

VINDEMIATRIX, or VINDEMIATOR, a fixed ftar of the third magnitude, in the northern wing of the conftellation Virgo.

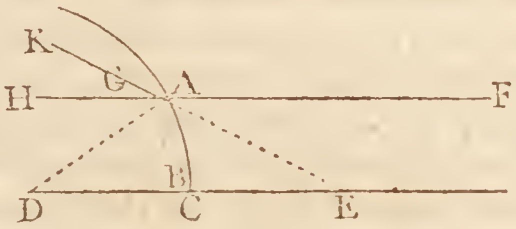
VIRGO,

VIRGO, in Aſtronomy, one of the ſigns or conſtellations of the zodiac, which the ſun enters about the 21ſt or 22d of Auguſt; being one of the 48 old conſtellations, and is mentioned by the aſtronomers of all ages and nations, whoſe works have reached us. Anciently the figure was that of a girl, almoſt naked, with an ear of corn in her hand, evidently to denote the time of harveſt among the people who invented this ſign, whoever they were. But the Greeks much altered the figure, with clothes, wings, &c, and variouſly explained the origin of it by their own fables: thus, they tell us that the virgin, now exalted into the ſkies, was, while on earth, that Juſtitia, the daughter of Aſtræus and Ancora, who lived in the golden age, and taught mankind their duty: but, who, when their crimes increaſed, was obliged to leave the earth, and take her place in the heavens. Again, Heſiod gives the celeſtial maid another origin, and ſays ſhe was the daughter of Jupiter and Themis. There are alſo others who depart from both theſe accounts, and make her to have been Erigone, the daughter of Icarius: while others make her Parthene, the daughter of Apollo, who placed her there; and others, from the ear of corn, make it a repreſentation of Ceres; and others, from the obſcurity of her head, of Fortune.

The ancients, as they gave each of the 12 months of the year to the care of ſome one of the 12 principal deities, ſo they alſo threw into the protection of each of theſe one of the 12 ſigns of the zodiac. Hence Virgo, from the ear of corn in her hand, naturally fell to the lot of Ceres, and we accordingly find it called Signum Cereris.

The ſtars in the conſtellation Virgo, in Ptolomy's catalogue, are 32; in Tycho's 33; in Hevelius's 50; and in the Britannic 110.

VIRTUAL *Focus*, in Optics, is a point in the axis of a glaſs, where the continuation of a refracted ray meets it. Thus, let D be the centre, and DBE



the axis of the glaſs AB; upon which falls the ray FA. Now this ray will not proceed ſtraight forward, as AH, after paſſing the glaſs, but will take the courſe AK, being deflected from the perpendicular AD. If then the refracted ray KA be produced, by AE, to the axis at E, this point E Mr. Molineux calls the *Virtual focus*, or *point of divergence*.

VIS, a Latin word, ſignifying force or power; adopted by writers on phyſics, to expreſs divers kinds of natural powers or faculties.

The term Vis is either active or paſſive: the *Vis activa* is the power of producing motion; the *Vis paſſiva* is that of receiving or loſing it. The *Vis activa* is again ſubdivided into *Vis viva* and *Vis mortua*.

Vis *Abſoluta*, or *abſolute force*, is that kind of centripetal force which is meaſured by the motion that would be generated by it in a given body, at a given diſtance, and depends on the efficacy of the cauſe producing it.

Vis *Acceleratrix*, or *accelerating force*, is that centripetal force which produces an accelerated motion, and is proportional to the velocity which it generates in a given time; or it is as the motive or abſolute force directly, and as the quantity of matter moved inverſely.

Vis *Impreſſa* is defined by Newton to be the action exerciſed on any body to change its ſtate, either of reſt or moving uniformly in a right line.

This force conſiſts altogether in the action; and has no place in the body after the action is ceaſed: for the body perſeveres in every new ſtate by the *Vis inertiæ* alone.

This Vis impreſſa may ariſe from various cauſes; as from percuſſion, preſſion, and centripetal force.

Vis *Inertiæ*, or power of inactivity, is defined by Newton to be a power implanted in all matter, by which it reſiſts any change endeavoured to be made in its ſtate, that is, by which it becomes difficult to alter its ſtate, either of reſt or motion.

This power then agrees with the *Vis reſiſtendi*, or power of reſiſting, by which every body endeavours, as much as it can, to perſevere in its own ſtate, whether of reſt or uniform rectilinear motion; which power is ſtill proportional to the body, or to the quantity of matter in it, the ſame as the weight or gravity of the body; and yet it is quite different from, and even independent of the force of gravity, and would be and act juſt the ſame if the body were devoid of gravity. Thus, a body by this force reſiſts the ſame in all directions, upwards or downwards or obliquely; whereas gravity acts only downwards.

Bodies only exert this power in changes brought on their ſtate by ſome *Vis impreſſa*, force impreſſed on them. And the exerciſe of this power is, in different reſpects, both reſiſtance and impetus; reſiſtance, as the body oppoſes a force impreſſed on it to change its ſtate; and impetus, as the ſame body endeavours to change the ſtate of the reſiſting obſtacle. Phil. Nat. Princ. Math. lib. 1.

The *Vis inertiæ*, the ſame great author elſewhere obſerves, is a paſſive principle, by which bodies perſiſt in their motion or reſt, and receive motion, in proportion to the force impreſſing it, and reſiſt as much as they are reſiſted. See RESISTANCE.

Vis *Inſita*, or *innate force* of matter, is a power of reſiſting, by which every body, as much as in it lies, endeavours to perſevere in its preſent ſtate, whether of reſt or of moving uniformly forward in a right line. This force is always proportional to the quantity of matter in the body, and differs in nothing from the Vis inertiæ, but in our manner of conceiving it.

Vis *Centripeta*. See CENTRIPETAL *Force*.

Vis *Motrix*, or *moving force* of a centripetal body, is the tendency of the whole body towards the centre, reſulting from the tendency of all the parts, and is proportional to the motion which it generates in a given time; ſo that the Vis motrix is to the Vis acceleratrix, as the motion is to the celerity: and as the quantity of motion in a body is eſtimated by the product of the velocity into the quantity of matter, ſo the Vis motrix ariſe

arifes from the Vis acceleratrix multiplied by the quantity of matter.

The followers of Leibnitz ufe the term *Vis motrix* for the force of a body in motion, in the fame fenfe as the Newtonians ufe the term *Vis inertiæ*; this latter they allow to be inherent in a body at reft; but the former, or Vis motrix, a force inherent in the fame body only whilft in motion, which actually carries it from place to place, by acting upon it always with the fame intenfity in every phyfical part of the line which it defcribes.

Vis *Mortua*, and Vis *Viva*, in Mechanics, are terms ufed by Leibnitz and his followers for force, which they diftinguifh into two kinds, *Vis mortua*, and *Vis viva*; underftanding by the former any kind of preffure, or an endeavour to move, not fufficient to produce actual motion, unlefs its action on a body be continued for fome time; and by the latter, that force or power of acting which refides in a body in motion.

VISIBLE, fomething that is an object of vifion or fight, or the property of a thing feen.

The Cartefians fay that light alone is the proper object of vifion. But according to Newton, colour alone is the proper object of fight; colour being that property of light by which the light itfelf is Vifible, and by which the images of opake bodies are painted on the retina.

As to the Situation and Place of Vifible Objects:

Philofophers in general had formerly taken for granted, that the place to which the eye refers any Vifible object, feen by reflection or refraction, is that in which the vifual ray meets a perpendicular from the object upon the reflecting or the refracting plane. That this is the cafe with refpect to plane mirrors is univerfally acknowledged; and fome experiments with mirrors of other forms feem to favour the fame conclufion, and thus afford reafon for extending the analogy to all cafes of vifion. If a right line be held perpendicularly over a convex or concave mirror, its image feems to make one line with it. The fame is the cafe with a right line held perpendicularly within water; for the part which is within the water feems to be a continuation of that which is without. But Dr. Barrow called in queftion this method of judging of the place of an object, and fo opened a new field of inquiry and debate in this branch of fcience. This, with other optical inveftigations, he publifhed in his Optical Lectures, firft printed in 1674. According to him, we refer every point of an object to the place from which the pencils of light iffue, or from which they would have iffued, if no reflecting or refracting fubftance intervened. Purfuing this principle, Dr. Barrow proceeded to inveftigate the place in which the rays iffuing from each of the points of an object, and that reach the eye after one reflection or refraction, meet; and he found that when the refracting furface was plane, and the refraction was made from a denfer medium into a rarer, thofe rays would always meet in a place between the eye and a perpendicular to the point of incidence. If a convex mirror be ufed, the cafe will be the fame; but if the mirror be plane, the rays will meet in the perpendicular, and beyond it if it be concave. He alfo determined, according to thefe principles, what form the image of a right line will take when it is prefented in different manners to a fpherical mirror, or when it is feen through a refracting medium.

Dr. Barrow however notices an objection againft the maxim above mentioned, concerning the fuppofed place of vifible objects, and candidly owns that he was not able to give a fatisfactory folution of it. The objection is this: Let an object be placed beyond the focus of a convex lens, and if the eye be clofe to the lens, it will appear confufed, but very near to its true place. If the eye be a little withdrawn, the confufion will increafe, and the object will feem to come nearer; and when the eye is very near the focus, the confufion will be very great, and the object will feem to be clofe to the eye. But in this experiment the eye receives no rays but thofe that are converging; and the point from which they iffue is fo far from being nearer than the object, that it is beyond it; notwithftanding which the object is conceived to be much nearer than it is, though no very diftinct idea can be formed of its precife diftance.

The firft perfon who took much notice of Dr. Barrow's hypothefis, and the difficulty attending it, was Dr. Berkeley, who (in his Effay on a New Theory of Vifion, p. 30) obferves, that the circle formed upon the retina, by the rays which do not come to a focus, produce the fame confufion in the eye, whether they crofs one another before they reach the retina, or tend to it afterwards; and therefore that the judgment concerning diftance will be the fame in both the cafes, without any regard to the place from which the rays originally iffued; fo that in this cafe, by receding from the lens, as the confufion increafes, which always accompanies the nearnefs of an object, the mind will judge that the object comes nearer. See *Apparent* Distance.

M. Bouguer (in his Traité d'Optique, p. 104) adopts Barrow's general maxim, in fuppofing that we refer objects to the place from which the pencils of rays feemingly converge at their entrance into the pupil. But when rays iffue from below the furface of a veffel of water, or any other refracting medium, he finds that there are always two different places of this feeming convergence: one of them of the rays that iffue from it in the fame vertical circle, and therefore fall with different degrees of obliquity upon the furface of the refracting medium; and another of thofe that fall upon the furface with the fame degree of obliquity, entering the eye laterally with refpect to one another. He fays, fometimes one of thefe images is attended to by the mind, and fometimes the other; and different images may be obferved by different perfons. And he adds, that an object plunged in water affords an example of this duplicity of images.

G. W. Krafft has ably fupported Barrow's opinion, that the place of any point feen by reflection from the furface of any medium, is that in which rays iffuing from it, infinitely near to one another, would meet; and confidering the cafe of a diftant object viewed in a concave mirror, by an eye very near it, when the image, according to Euclid and other writers, would be between the eye and the object, and Barrow's rule cannot be applied, he fays that in this cafe the fpeculum may

be confidered as a plane, the effect being the fame, only that the image is more obfcure. Com. Petrepol. vol. 12, p. 252, 256. See Prieftley's Hift. of Light &c, p. 89, 688.

From the principle above illuftrated feveral remarkable phenomena of vifion may be accounted for: as—That if the diftance between two Vifible objects be an angle that is infenfible, the diftant bodies will appear as if contiguous: whence, a continuous body being the refult of feveral contiguous ones, if the diftances between feveral Vifibles fubtend infenfible angles, they will appear one continuous body; which gives a pretty illuftration of the notion of a continuum.—Hence alfo parallel lines, and long viftas, confifting of parallel rows of trees, feem to converge more and more the farther they are extended from the eye; and the roofs and floors of long extended alleys feen, the former to defcend, and the latter to afcend, and approach each other; becaufe the apparent magnitudes of their perpendicular intervals are perpetually dimiuifhing, while at the fame time we miftake their diftance.

As to the Different Diftances of Vifible Objects:

The mind perceives the diftance of Vifible objects, 1ft, From the different configurations of the eye, and the manner in which the rays ftrike the eye, and in which the image is impreffed upon it. For the eye difpofes itfelf differently, according to the different diftances it is to fee; viz, for remote objects the pupil is dilated, and the cryftalline brought nearer the retina, and the whole eye is made more globous; on the contrary, for near objects, the pupil is contracted, the cryftalline thruft forwards, and the eye lengthened. The mode of performing this however, has greatly divided the opinions of philofophers. See Prieftley's Hift. of Light &c, p. 638—652, where the feveral opinions of Defcartes, Kepler, La Hire, are Le Roi, Porterfield, Jurin, Muffchenbroek, &c, ftated and examined.

Again, the diftance of Vifible objects is judged of by the angle the object makes; from the diftinct or confufed reprefentation of the objects; and from the brifknefs or feeblenefs, or the rarity or denfity of the rays.

To this it is owing, 1ft, That objects which appear obfcure or confufed, are judged to be more remote; a principle which the painters make ufe of to caufe fome of their figures to appear farther diftant than others on the fame plane. 2d, To this it is likewife owing, that rooms whofe walls are whitened, appear the fmaller; that fields covered with fnow, or white flowers, fhew lefs than when clothed with grafs; that mountains covered with fnow, in the night time, appear the nearer, and that opake bodies appear the more remote in the twilight.

The Magnitude of Vifible Objects.

The quantity or magnitude of Vifible objects, is known chiefly by the angle contained between two rays drawn from the two extremes of the object to the centre of the eye. An object appears fo large as is the angle it fubtends; or bodies feen under a greater angle appear greater; and thofe under a lefs angle lefs, &c. Hence the fame thing appears greater or lefs as it is nearer the eye or farther off. And this is called the apparent magnitude.

But to judge of the real magnitude of an object, we muft confider the diftance; for fince a near and a remote object may appear under equal angles, though the magnitudes be different, the diftance muft neceffarily be eftimated, becaufe the magnitude is great or fmall according as the diftance is. So that the real magnitude is in the compound ratio of the diftance and the apparent magnitude; at leaft when the fubtended angle, or apparent magnitude, is very fmall; otherwife, the real magnitude will be in a ratio compounded of the diftance and the fine of the apparent magnitude, nearly, or nearer ftill its tangent.

Hence, objects feen under the fame angle, have their magnitudes in the fame ratio as their diftances. The chord of an arc of a circle appears of equal magnitude from every point in the circumference, though one point be vaftly nearer than another. Or if the eye be fixed in any point in the circumference, and a right line be moved round fo as its extremes be always in the periphery, it will appear of the fame magnitude in every pofition. And the reafon is, becaufe the angle it fubtends is always of the fame magnitude. And hence alfo, the eye being placed in any angle of a regular polygon, the fides of it will all appear of equal magnitude; being all equal chords of a circle defcribed about it.

If the magnitude of an object directly oppofite to the eye be equal to its diftance from the eye, the whole object will be diftinctly feen, or taken in by the eye, but nothing more. And the nearer you approach an object, the lefs part you fee of it.

The leaft angle under which an ordinary object becomes vifible, is about one minute of a degree.

Of the Figure of Vifible Objects.

This is eftimated chiefly from our opinion of the fituation of the feveral parts of the object. This opinion of the fituation, &c, enables the mind to apprehend an external object under this or that figure, more juftly than any fimilitude of the images in the retina, with the object can; the images being often elliptical, oblong, &c, when the objects they exhibit to the mind are circles, or fquares, &c.

The laws of vifion, with regard to the figures of Vifible objects, are,

1. That if the centre of the eye be exactly in the direction of a right line, the line will appear only as a point.

2. If the eye be placed in the direction of a furface, it will appear only as a line.

3. If a body be oppofed directly towards the eye, fo as only one plane of the furface can radiate on it, the body will appear as a furface.

4. A remote arch, viewed by an eye in the fame plane with it, will appear as a right line.

5. A fphere, viewed at a diftance, appears a circle.

6. Angular figures, at a diftance, appear round.

7. If the eye look obliquely on the centre of a regular figure, or a circle, the true figure will not be feen; but the circle will appear oval, &c.

VISIBLE *Horizon, Place, &c.* See the fubftantives.

VISION, is the act of feeing, or of perceiving external objects by the organ of fight.

When an object is fo difpofed, that the rays of light, coming from all parts of it, enter the pupil of the eye, and prefent its image on the retina, that object is then feen. This is proved by experiment; for if the eye of any animal be taken out, and the fkin and fat be carefully ftripped off from the back part of it, till only the thin membrane, which is called the retina, remains to terminate it behind, and any object be placed before the front of the eye, the picture of that object will be feen figured as with a pencil on that membrane. There are thoufands of experiments which prove that this is the mechanical effect of Vifion, or feeing, but none of them all appear fo conveniently as this, which is made with the very eye itfelf of an animal; an eye of an ox newly killed fhews this happily, and with very little trouble. It will indeed appear fingular in this, that the object is inverted, in the picture thus drawn of it, in the eye; and the cafe is the fame in the eye of a living perfon.

Various other opinions however have been held concerning the means of Vifion among philofophers.

The Platonifts and Stoics held Vifion to be effected by the emiffion of rays out of the eyes; conceiving that there was a fort of light thus darted out; which, with the light of the external air, taking hold as it were of the objects, rendered them vifible; and thus returning back again to the eye, altered and new modified by the contact of the object, made an impreffion on the pupil, which gave the fenfation of the object.

Our own countryman, Roger Bacon, diftinguifhed as he was in many refpects, alfo affents to the opinion that vifual rays proceed from the eye; giving this reafon for it, that every thing in nature is qualified to difcharge its proper functions by its own powers, in the fame manner as the fun, and other celeftial bodies. Opus Majus, pa. 289.

The Epicureans held, that Vifion is performed by the emanation of corporeal fpecies or images from objects; or a fort of atomical effluvia continually flying off from the intimate parts of objects, to the eye.

The Peripatetics hold, with Epicurus, that Vifion is performed by the reception of fpecies; but they differ from him in the circumftances; for they will have the fpecies (which they call *intentionales*) to be incorporeal. It is true, Ariftotle's doctrine of Vifion, delivered in his chapter De Afpectu, amounts to no more than this, that objects muft have fome intermediate body, that by this they may move the organ of fight. To which he adds, in another place, that when we perceive bodies, it is their fpecies, not their matter, that we receive; as a feal makes an impreffion on wax, without the wax receiving any thing of the feal.

But this vague and obfcure account the Peripatetics have thought fit to improve. Accordingly, what their mafter calls fpecies, the difciples, underftanding of real proper fpecies, affert, that every vifible object expreffes a perfect image of itfelf in the air contiguous to it; and this image another, fomewhat lefs, in the next air; and the third another; and fo on till the laft image arrives at the cryftalline, which they hold for the chief organ of fight, or that which immediately moves the foul. Thefe images they call *intentional fpecies.*

The modern philofophers, as the Cartefians and Newtonians, give a better account of Vifion. They all agree, that it is performed by rays of light reflected from the feveral points of objects received in at the pupil, refracted and collected in their paffage, through the coats and humours, to the retina; and this ftriking, or making an impreffion, on fo many points of it; which impreffion is conveyed, by the correfpondent capillaments of the optic nerve, to the brain, &c.

Baptifta Porta's experiments with the camera obfcura, about the middle of the 16th century, convinced him that vifion is performed by the intermiffion of fomething into the eye, and not by vifual rays proceeding from the eye, as had been the general opinion before his time; and he was the firft who fully fatisfied himfelf and others upon this fubject; though feveral philofophers ftill adhered to the old opinion.

As for the Peripatetic feries or chain of images, it is a mere chimera; and Ariftotle's meaning is better underftood without than with them. In fact, fetting thefe afide, the Ariftotelian, Cartefian, and Newtonian doctrines of Vifion, are very confiftent with one another; for Newton imagines that Vifion is performed chiefly by the vibrations of a fine medium (which penetrates all bodies) excited in the bottom of the eye by the rays of light, and propagated through the capillaments of the optic nerves, to the fenforium. And Des Cartes maintains, that the fun preffing the materia fubtilis, with which the whole univerfe is every where filled, the vibrations and pulfes of that matter reflected from objects, are communicated to the eye, and thence to the fenfory: fo that the action or vibration of a medium is equally fuppofed in all.

It is generally concluded then, that the images of objects are reprefented on the retina; which is only an expanfion of the fine capillaments of the optic nerve, and from whence the optic nerve is continued into the brain. Now any motion or vibration, impreffed on one extremity of the nerve, will be propagated to the other: hence the impulfe of the feveral rays, fent from the feveral points of the object, will be propagated as they are on the retina (that is, in their proper colours, &c, or in particular vibrations, or modes of preffure, correfponding to them) to the place where thofe capillaments are interwoven into the fubftance of the brain. And thus is Vifion brought to the common cafe of fenfation.

Experience teaches us that the eye is capable of viewing objects at a certain diftance, without any mental exertion. Beyond this diftance, no mental exertion can be of any avail: but, within it, the eye poffeffes a power of adapting itfelf to the various occafions that occur, the exercife of which depends on the volition of the mind. How this is effected, is a problem that has very much engaged the attention of optical writers: but it is doubted whether it has yet been fatisfactorily explained. The firft theory for the folution of this problem is that of Kepler. He fuppofes that the ciliary proceffes contract the diameter of the eye, and lengthen its axis by a mufcular power. But Mr. Thomas Young (in fome ingenious Obfervations on Vifion in the Philof. Tranf. 1793) obferves,

serves, that these processes neither appear to contain any muscular fibres, nor have any attachment by which they can be capable of performing this action.

Des Cartes ascribed this contraction and elongation to a muscularity of the crystalline, of which he supposed the ciliary processes to be the tendons: but he neither demonstrated this muscularity, nor sufficiently considered the connection with the ciliary processes.

De la Hire allows of no change in the eye, except the contraction and dilatation of the pupil: this opinion he founds on an experiment which Dr. Smith has shewn to be fallacious. Haller adopted his hypothesis, notwithstanding its inconsistency with the principles of optics and constant experience.

Dr. Pemberton supposes that the crystalline contains muscular fibres, by which one of its surfaces is flattened, while the other is made convex: but he has not demonstrated the existence of these fibres; and Dr. Jurin has proved that such a change as this is inadequate to the effect.

Dr. Porterfield conceives that the ciliary processes draw the crystalline forward, and make the cornea more convex. But the ciliary processes are incapable of this action; and it appears from Dr. Jurin's calculations, that a sufficient motion of this kind requires a very visible increase in the length of the axis of the eye; an increase which has never yet been observed.

Dr. Jurin maintains that the uvea, at its attachment to the cornea, is muscular; and that the contraction of this ring makes the cornea more convex. But this hypothesis is not sufficiently confirmed by observation.

Musschenbroek conjectures that the relaxation of this ciliary zone, which is nothing but the capsule of the vitreous humour where it receives the impression of the ciliary processes, permits the coats of the eye to push forward the crystalline and cornea. Such a voluntary relaxation however, Mr. Young observes, is wholly without example in the animal economy: besides, if it actually occurred, the coats of the eye could not act as he conceives; nor could they act in this manner without being observed. He adds, that the contraction of the ciliary zone is equally inadequate and unnecessary.

Mr. Young, having examined these theories, and some others of less moment, proceeds to investigate a more probable solution of this optical difficulty.—Adverting to the observation of Dr. Porterfield, that those who have been couched have not the power of accommodating the eye to different distances; and to the reflections of other writers on this subject; he was led to conclude that the rays of light, emitted by objects at a small distance, could only be brought to foci on the retina by a nearer approach of the crystalline to a spherical form; and he imagined that no other power was capable of producing this change, beside a muscularity of part, or of the whole of its capsule:—but, on closely examining first with the naked eye and then with a magnifier, the crystalline of an ox's eye turned out of its capsule, he discovered a structure which seemed to remove the difficulties that have long embarrassed this branch of optics.

"The crystalline of the ox, says he, is composed of various similar coats, each of which consists of six muscles, intermixed with a gelatinous substance, and attached to six membranous tendons. Three of the tendons are anterior, three posterior; their length is about two-thirds of the semidiameter of the coat; their arrangement is that of three equal and equidistant rays, meeting in the axis of the crystalline; one of the anterior is directed towards the outer angle of the eye, and one of the posterior towards the inner angle, so that the posterior are placed opposite to the middle of the interstices of the anterior: and planes passing through each of the six, and through the axis, would mark on either surface six regular equidistant rays. The muscular fibres arise from both sides of each tendon; they diverge till they reach the greatest circumference of the coat; and, having passed it, they again converge, till they are attached respectively to the sides of the nearest tendons of the opposite surface. The anterior or posterior portion of the six, viewed together, exhibits the appearance of three penniform-radiated muscles. The anterior tendons of all the coats are situated in the same planes, and the posterior ones in the continuations of these planes beyond the axis. Such an arrangement of fibres can be accounted for on no other supposition than that of muscularity. This mass is inclosed in a strong membranous capsule, to which it is loosely connected by minute vessels and nerves; and the connection is more observable near its greatest circumference. Between the mass and its capsule is found a considerable quantity of an aqueous fluid, the liquid of the crystalline.

"When the will is exerted to view an object at a small distance, the influence of the mind is conveyed through the lenticular ganglion, formed from branches of the third and fifth pair of nerves by the filaments perforating the sclerotica, to the orbiculus ciliaris, which may be considered as an annular plexus of nerves and vessels; and thence by the ciliary processes to the muscle of the crystalline, which, by the contraction of its fibres, becomes more convex, and collects the diverging rays to a focus on the retina. The disposition of fibres in each coat is admirably adapted to produce this change; for, since the least surface that can contain a given bulk is that of a sphere (Simpson's Fluxions, pa. 486) the contraction of any surface must bring its contents nearer to a spherical form. The liquid of the crystalline seems to serve as a synovia in facilitating the motion, and to admit a sufficient change of the muscular part, with a smaller motion of the capsule.

Mr. Young proceeds to enquire whether these fibres can produce an alteration in the form of the lens sufficiently great to account for the known effects; and he finds, by calculation, that, supposing the crystalline to assume a spherical form, its diameter will be 642 thousandths of an inch, and its focal distance in the eye ·926. Then, disregarding the thickness of the cornea, we find (by Smith, art. 370) that such an eye will collect those rays on the retina, which diverge from a point at the distance of 12 inches and 8 tenths. This is a greater change than is necessary for an ox's eye; for, if it be supposed capable of distinct Vision at a distance somewhat less than 12 inches, yet it is probably far short of being able to collect parallel rays. The human crystalline is susceptible of a much greater change of form. The ciliary zone may admit of as much extension as this diminution of the diameter of the crystalline

fine will require; and its elasticity will assist the cellular texture of the vitreous humour, and perhaps the gelatinous part of the crystalline, in restoring the indolent form.—Mr. Young apprehends that the sole office of the optic nerve is to convey sensation to the brain; and that the retina does not contribute to supply the lens with nerves.—As the human crystalline resembles that of the ox, it may reasonably be presumed that the action of both organs depends on the same general principles.

This theory of Mr. Young's however is strongly opposed by Dr. Hosack, (Philos. Trans. 1794, part 2, pa. 196). He contests the existence of the muscles, which Mr. Young has described, for several reasons. First, from the transparency they must possess; otherwise there would be some irregularity in the refraction of those rays which pass through the several parts, differing both in shape and density. Another circumstance is the number of these muscles. Mr. Young describes 6 in each lamina; and as Leuwenhoek makes 2000 laminæ in all, therefore the number of muscles must amount to 12 thousand, the action of which, Dr. Hosack apprehends, must exceed comprehension. But the existence of these muscles is still more doubtful, if the accuracy of Dr. Hosack's observations be admitted. With the assistance of the best glasses, and with the greatest attention, he could not discover the structure of the crystalline described by Mr. Young, but found it to be perfectly transparent. He first observed the lens in its viscid state, and then exposed different lenses to a moderate degree of heat, so that they became opaque and dry; and it was easy to separate the distinct layers described by Mr. Young. These were so numerous as not to admit of having, each of them, 6 muscles. Another consideration, which seems to prove that these layers possess no distinct muscles, is that, in this opaque state, they are not visible, but consist of an almost infinite number of concentric fibres, not divided into particular bundles, but similar to as many of the finest hairs of equal thickness, arranged in similar order. This regular structure of layers, composed of concentric fibres, Dr. Hosack thinks is much better adapted to the transmission of the rays of light than the irregular structure of muscles. Besides, it ought to be considered that the crystalline lens is not the most essential organ in viewing objects at different distances; and if this be the case, the power of the eye cannot be owing to any changes in this lens. It is a fact, says Dr. Hosack, that we can, in a great degree, do without it; as is the case after couching or extraction, by which operation all its parts must be destroyed. Dr. Porterfield, however, and Mr. Young, on his authority, maintain that patients, after the operation of couching, have not the power of accommodating the eye to different distances of objects. On the whole, Dr. Hosack concludes that no such muscles, as Mr. Young has described, exist, and that he must have been deceived by some other appearances that resembled muscles: neither will he allow the effects ascribed to the ciliary processes in changing the shape or situation of the lens.

Dr. Hosack then proceeds to illustrate the structure and use of the external muscles of the eye; which are 6 in number, 4 called recti or straight, and 2 oblique, and by means of which he thinks the business is effected. The common purposes to which these muscles are subservient are well known: but beside these, Dr. Hosack suggests that it is not inconsistent with the general laws of nature, nor even with the animal œconomy, to imagine that, from their combination, they should have a different action and an additional use. In describing the precise action of these muscles, he supposes an object to be seen distinctly first at the distance of 6 feet; in which case the picture of it falls exactly on the retina. He then directs his attention to another object at the distance of 6 inches, as nearly as possible in the same line. While he is viewing this, he loses sight of the first object, though the rays proceeding from it still fall on the eye; and hence he infers that the eye must have undergone some change; so that the rays meet either before or behind the retina. But, as rays from a more distant object concur sooner than those from a nearer one, the picture of the more remote object must fall before the retina, while the others form a distinct image upon it. But yet the eye continued in the same place; and therefore the retina must, by some means, have been removed to a greater distance from the fore-part of the eye, so as to receive the picture of the nearer object. This object, he contends, could not be seen distinctly, unless the retina were removed to a greater distance, or the refracting power of the media through which the rays passed were augmented:—but as the lens is the chief refracting medium, if we admit that this has no power of changing itself, we are under the necessity of adopting the first of these two suppositions.

The next object of inquiry is, how the external muscles are capable of producing these changes. The recti are strong, broad, and flat, and arise from the back part of the orbit of the eye; and, passing over the ball as over a pulley, they are inserted by broad flat tendons at the anterior part of the eye. The oblique are inserted towards the posterior part by similar tendons. When these different muscles act jointly, the eye being in the horizontal position, and every muscle in action contracting itself, the four recti by their combination must compress the various parts of the eye and lengthen its axis, while the oblique muscles serve to keep the eye in its proper direction and situation. The convexity of the cornea, by means of its great elasticity, is also increased in proportion to the degree of pressure, and thus the rays of light passing through it are necessarily more converged. The elongation of the eye serves also to lengthen the media, in the aqueous, crystalline, and vitreous humours through which the rays pass, so that their powers of refraction are proportionably increased. This is the general effect of the contraction of the external muscles, according to Dr. Hosack's statement of it: to which it may be added, that we possess the same power of relaxing them in proportion to the greater distance of the object, till we arrive at the utmost extent of indolent Vision. Dr. Hosack also illustrates this hypothesis by some experiments.

The misrepresentations of Vision often depend upon the distance of the object. Thus, if an opake globe be placed at a moderate distance from the eye, the picture of it upon the retina will be a circle properly diversified with light and shade, so that it will excite in the mind the sensation of a sphere or globe; but, if the

the globe be placed at a great diſtance from the eye, the diſtance between thoſe lights and ſhades, which form the picture of a globe, will be imperceptible, and the globe will appear no otherwiſe than as a circular plane. In a luminous globe, diſtance is not neceſſary in order to take off the repreſentation of prominent and flat; an iron bullet, heated very red hot, and held but a few yards diſtance from the eye, appears a plane, not a prominent body; it has not the look of a globe, but of a circular plane. It is owing to this miſrepreſentation of Viſion that we ſee the ſun and moon flat by the naked eye; and the planets alſo, through teleſcopes, flat. It is in this light that aſtronomers, when they ſpeak of the ſun, moon, and planets, as they appear to our view, call them the diſcs of the ſun, moon, and planets, which we ſee.

The nearer a globe is to the eye, the ſmaller ſegment of it is viſible, the farther off the greater, and at a due diſtance the half; and, on the ſame principle, the nearer the globe is to the eye, the greater is its apparent diameter, that is, under the greater angle it will appear, the farther off the globe is placed, the leſs is its apparent diameter. This is a propoſition of importance, for, on this principle, we know that the ſame globe, when it appears larger, is nearer to our eye, and, when ſmaller, is farther off from it. Therefore, as we know that the globes of the ſun and moon continue always of the ſame ſize, yet appear ſometimes larger, and ſometimes ſmaller, to us, it is evident, that they are ſometimes nearer, and ſometimes farther off from the place whence we view them. Two globes, of different magnitude, may be made to appear of exactly the ſame diameter, if they be placed at different diſtances, and thoſe diſtances be exactly proportioned to their diameters. To this it is owing, that we ſee the ſun and moon nearly of the ſame diameter; they are, indeed, vaſtly different in real bulk, but, as the moon is placed greatly nearer to our eyes, the apparent magnitude of that little globe is nearly the ſame with that of the greater.

In this inſtance of the ſun and moon (for there cannot be a more ſtriking one) we ſee the miſrepreſentation of Viſion in two or three ſeveral ways. The apparent diameters of theſe globes are ſo nearly equal, that, in their ſeveral changes of place, they do, at times, appear to us abſolutely equal, or mutually greater than one another. This is often to be ſeen, but it is at no time ſo obvious, and ſo perfectly evinced, as in eclipſes of the ſun, which are total. In theſe we ſee the apparent magnitudes of the two globes vary ſo much according to their diſtances, that ſometimes the moon is large enough exactly to cover the diſc of the ſun, ſometimes it is larger, and a part of it every where extends beyond the diſc of the ſun; and, on the contrary, ſometimes it is ſmaller, and, though the eclipſe be abſolutely central, yet it is annular, or a part of the ſun's diſc is ſeen in the middle of the eclipſed part, enlightened, and ſurrounding the opake body of the moon in form of a lucid ring.

When an object, which is ſeen above, without other objects of compariſon, is of a known magnitude, we judge of its diſtance by its apparent magnitude; and cuſtom teaches us to do this with tolerable accuracy. This is a practical uſe of the miſrepreſentation of Viſion, and in the ſame manner, knowing that we ſee things, which are near us, diſtinctly, and thoſe which are diſtant, confuſedly, we judge of the diſtance of an object by the clearneſs, or confuſion, in which we ſee it. We alſo judge yet more eaſily and truly of the diſtance of an object by comparing it to another ſeen at the ſame time, the diſtance of which is better known, and yet more by comparing it with ſeveral others, the diſtances of which are more or leſs known, or more or leſs eaſily judged of. Theſe are the circumſtances which aſſiſt us, even by the miſrepreſentation of Viſion, to judge of diſtance; but, without one or more of theſe, the eye does not, in reality, enable us to judge concerning the diſtance of objects.

This miſrepreſentation, although it ſerves us on ſome occaſions, yet is very limited in its effects. Thus, though it helps us greatly in diſtinguiſhing the diſtance of objects that are about us, both with reſpect to ourſelves and them, and with reſpect to themſelves with one another, yet it can do nothing with the very remote. We ſee that immenſe concave circle, in which we ſuppoſe the fixed ſtars to be placed, at all this vaſt remove from us, and no change of place that we could make to get nearer to it, would be of any avail for determining the diſtance of the ſtars from one another. If we look at three or four churches from a diſtance of as many miles, we ſee them ſtand in a certain poſition with regard to one another. If we advance a great deal nearer to them, we ſee that poſition differ, but, if we move forward only eight or ten feet, the difference is not ſeen.

Vision, in Optics. The laws of Viſion, brought under mathematical demonſtrations, make the ſubject of Optics, taken in the greateſt latitude of that word: for, among mathematical writers, optics is generally taken, in a more reſtricted ſignification, for the doctrine of *direct Viſion*; catoptrics, for the doctrine of *reflected Viſion*; and dioptrics, for that of *refracted Viſion*.

Direct or *Simple* Vision, is that which is performed by means of direct rays; that is, of rays paſſing directly, or in right lines, from the radiant point to the eye. Such is that explained in the preceding article Vision.

Reflected Vision, is that which is performed by rays reflected from ſpeculums, or mirrors. The laws of which, ſee under Reflection, and Mirror.

Refracted Vision, is that which is performed by means of rays refracted, or turned out of their way, by paſſing through mediums of different denſity; chiefly through glaſſes and lenſes. The laws of this, ſee under the article Refraction.

Arch of Vision. See Arch.

Diſtinct Vision, is that by which an object is ſeen diſtinctly. An object is ſaid to be ſeen diſtinctly, when its outlines appear clear and well-defined, and the ſeveral parts of it, if not too ſmall, are plainly diſtinguiſhable, ſo that they can eaſily be compared one with another, in reſpect to their figure, ſize, and colour.

In order to ſuch Diſtinct Viſion, it had commonly been thought that all the rays of a pencil, flowing from a phyſical point of an object, muſt be exactly united in a phyſical, or at leaſt in a ſenſible point of the retina. But Dr. Jurin has made it appear from experiments, that ſuch an exact union of rays is not always neceſſary to

to Diſtinct Viſion. He ſhews that objects may be ſeen with ſufficient diſtinctneſs, though the pencils of rays iſſuing from the points of them do not unite preciſely in the ſame point on the retina; but that ſince, in this caſe, pencils from every point either meet before they reach the retina, or tend to meet beyond it, the light that comes from them muſt cover a circular ſpot upon it, and will therefore paint the image larger than perfect Viſion would repreſent it. Whence it follows, that every object placed either too near or too remote for perfect Viſion, will appear larger than it is by a penumbra of light, cauſed by the circular ſpaces, which are illuminated by pencils of rays proceeding from the extremities of the object.

The ſmalleſt diſtance of perfect Viſion, or that in which the rays of a ſingle pencil are collected into a phyſical point on the retina in the generality of eyes, Dr. Jurin, from a number of obſervations, ſtates at 5, 6, or 7 inches. The greateſt diſtance of diſtinct and perfect Viſion he found was more difficult to determine; but by conſidering the proportion of all the parts of the eye, and the refractive power of each, with the interval that may be diſcerned between two ſtars, the diſtance of which is known, he fixes it, in ſome caſes, at 14 feet 5 inches; though Dr. Porterfield had reſtricted it to 27 inches only, with reſpect to his own eye.

For other obſervations on this ſubject, ſee Jurin's Eſſay on Diſtinct and Indiſtinct Viſion, at the end of Smith's Optics; and Robins's Remarks on the ſame, in his Math. Tracts, vol. 2, pa. 278 &c. See alſo an ingenious paper on Viſion in the Philoſ. Tranſ. 1793, pa. 169, by Mr. Thomas Young.

Field of VISION. See FIELD.

VISUAL, relating to ſight, or ſeeing.

VISUAL *Angle*, is the angle under which an object is ſeen, or which it ſubtends. See ANGLE.

VISUAL *Line*. See LINE.

VISUAL *Point*, in Perſpective, is a point in the horizontal line, where all the ocular rays unite. Thus, a perſon ſtanding in a ſtraight long gallery, and looking forward; the ſides, floor, and cieling ſeem to meet and touch one another in this point, or common centre.

VISUAL *Rays*, are lines of light, conceived to come from an object to the eye.

VITELLIO, or VITELLO, a Poliſh mathematician of the 13th century, as he flouriſhed about 1254. We have of his a large *Treatiſe on Optics*, the beſt edition of which is that of 1572. Vitello was the firſt optical writer of any conſequence among the modern Europeans. He collected all that was given by Euclid, Archimedes, Ptolomy, and Alhazen; though his work is of but little uſe now.

VITREOUS *Humour*, or *Vitreus Humor*, denotes the third or glaſſy humour of the eye; thus called from its reſemblance to melted glaſs. It lies under the cryſtalline; by the impreſſion of which, its forepart is rendered concave. It greatly exceeds in quantity both the aqueous and cryſtalline humours taken together, and conſequently occupies much the greateſt part of the cavity of the globe of the eye. Scheiner ſays, that the refractive power of this humour is a medium between thoſe of the aqueous, which does not differ much from water, and of the cryſtalline, which is nearly the ſame with glaſs. Hawkſbee makes its refractive power the ſame with that of water; and, according to Robertſon, its ſpecific gravity agrees nearly with that of water.

VITRUVIUS (MARCUS VITRUVIUS POLLIO), a celebrated Roman architect, of whom however nothing is known, but what is to be collected from his ten books *De Architectura*, ſtill extant. In the preface to the ſixth book he writes, that he was carefully educated by his parents, and inſtructed in the whole circle of arts and ſciences; a circumſtance which he ſpeaks of with much gratitude, laying it down as certain, that no man can be a complete architect, without ſome knowledge and ſkill in every one of them. And in the preface to the firſt book he informs us, that he was known to Julius Cæſar; that he was afterwards recommended by Octavia to her brother Auguſtus Cæſar; and that he was ſo favoured and provided for by this emperor, as to be out of all fear of poverty as long as he might live.

It is ſuppoſed that Vitruvius was born either at Rome or Verona; but it is not known which. His books of architecture are addreſſed to Auguſtus Cæſar, and not only ſhew conſummate ſkill in that particular ſcience, but alſo very uncommon genius and natural abilities. Cardan, in his 16th book *De Subtilitate*, ranks Vitruvius as one of the 12 perſons, whom he ſuppoſes to have excelled all men in the force of genius and invention; and would not have ſcrupled to have given him the firſt place, if it could be imagined that he had delivered nothing but his own diſcoveries. Thoſe 12 perſons were, Euclid, Archimedes, Apollonius Pergæus, Ariſtotle, Archytas of Tarentum, Vitruvius, Achindus, Mahomet Ibn Moſes the inventor or improver of Algebra, Duns Scotus, John Suiſſet ſurnamed the Calculator, Galen, and Heber of Spain.

The architecture of Vitruvius has been often printed; but the beſt edition is that of Amſterdam in 1649. Perrault alſo, the noted French architect, gave an excellent French tranſlation of the ſame, and added notes and figures: the firſt edition of which was publiſhed at Paris in 1673, and the ſecond much improved, in 1684. —Mr. William Newton too, an ingenious architect, and late Surveyor to the works at Greenwich Hoſpital, publiſhed in 1780 &c, curious commentaries on Vitruvius, illuſtrated with figures; to which is added a deſcription, with figures, of the Military Machines uſed by the Ancients.

VIVIANI (VINCENTIO), a celebrated Italian mathematician, was born at Florence in 1621, ſome ſay 1622. He was a diſciple of the illuſtrious Galileo, and lived with him from the 17th to the 20th year of his age. After the death of his great maſter, he paſſed two or three years more in proſecuting geometrical ſtudies without interruption, and in this time it was that he formed the deſign of his Reſtoration of Ariſteus. This ancient geometrician, who was contemporary with Euclid, had compoſed five books of problems *De Locis Solidis*, the bare propoſitions of which were collected by Pappus, but the books are entirely loſt; which Viviani undertook to reſtore by the force of his genius.

He broke this work off before it was finiſhed, in order to apply himſelf to another of the ſame kind, and that

that was, to restore the 5th book of Apollonius's Conic Sections. While he was engaged in this, the famous Borelli found, in the library of the Grand Duke of Tuscany, an Arabic manuscript, with a Latin inscription, which imported, that it contained the eight books of Apollonius's Conic Sections: of which the 8th however was not found to be there. He carried this manuscript to Rome, in order to translate it, with the assistance of a professor of the Oriental languages. Viviani, very unwilling to lose the fruits of his labours, procured a certificate that he did not understand the Arabic language, and knew nothing of that manuscript: he was so jealous on this head, that he would not even suffer Borelli to send him an account of any thing relating to it. At length he finished his book, and published it, 1659, in folio, with this title, *De Maximis & Minimis Geometrica Divinatio in quintum Conicorum Apollonii Pergæi*. It was found that he had more than divined; as he seemed superior to Apollonius himself.

After this he was obliged to interrupt his studies for the service of his prince, in an affair of great importance, which was, to prevent the inundations of the Tiber, in which Cassini and he were employed for some time, though nothing was entirely executed.

In 1664 he had the honour of a pension from Louis the 14th, a prince to whom he was not subject, nor could be useful. In consequence he resolved to finish his Divination upon Aristeus, with a view to dedicate it to that prince; but he was interrupted in this task again by public works, and some negotiations which his master entrusted to him.—In 1666 he was honoured by the Grand Duke with the title of his first mathematician.—He resolved three problems, which had been proposed to all the mathematicians of Europe, and dedicated the work to the memory of Mr. Chapelain, under the title of *Enodatio Problematum* &c.—He proposed the problem of the quadrable arc, of which Leibnitz and l'Hospital gave solutions by the Calculus Differentialis.—In 1669, he was chosen to fill, in the Royal Academy of Sciences, a place among the eight foreign associates. This new favour reanimated his zeal; and he published three books of his Divination upon Aristeus, at Florence in 1701, which he dedicated to the King of France. It is a thin folio, intitled, *De Locis Solidis secunda Divinatio Geometrica*, &c. This was a second edition enlarged; the first having been printed at Florence in 1673.—Viviani laid out the fortune, which he had raised by the bounties of his prince, in building a magnificent house at Florence; in which he placed a bust of Galileo, with several inscriptions in honour of that great man; and died in 1703, at 81 years of age.

Viviani had, says Fontenelle, that innocence and simplicity of manners which persons commonly preserve, who have less commerce with men than with books; without that roughness and a certain savage fierceness which those often acquire who have only to deal with books, not with men. He was affable, modest, a fast and faithful friend, and, what includes many virtues in one, he was grateful in the highest degree for favours.

ULLAGE, *of a Cask*, in Gauging, is so much as it wants of being full.

ULTERIOR, in Geography, is applied to some part of a country or province, which, with regard to the rest of that country, is situate on the farther side of a river, or mountain, or other boundary, which divides the country into two parts.

ULTRAMUNDANE, beyond the world, is that part of the universe supposed to be without or beyond the limits of our world or system.

UMBILICUS, and UMBILICAL *Point*, in Geometry, the same with focus.

UMBRA, a Shadow. See LIGHT, SHADOW, PENUMBRA, &c.

UNCIA, a term generally used for the 12th part of a thing. In which sense it occurs in Latin writers, both for a weight, called by us an *ounce*, and a measure called an *inch*.

UNCIÆ, in Algebra, first used by Vieta, are the numbers prefixed to the letters in the terms of any power of a binomial; now more usually, and generally, called *coefficients*. Thus, in the 4th power of $a + b$, viz,

$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4,$$

the Unciæ are 1, 4, 6, 4, 1.

Briggs first shewed how to find these Unciæ, one from another, in any power, independent of the foregoing powers. They are now usually found by what is called Newton's binomial theorem, which is the same rule as Briggs's in another form. See BINOMIAL.

UNDECAGON, is a polygon of eleven sides.

If the side of a regular Undecagon be 1, its area will be $9{\cdot}3656399 = \frac{11}{4} \times$ tang. of $73\frac{7}{11}$ degrees; and therefore if this number be multiplied by the square of the side of any other regular Undecagon, the product will be the area of that Undecagon. See my Mensuration, pa. 114 &c, 2d edit.

UNDETERMINED, is sometimes used for INDETERMINATE.

UNDULATORY *Motion*, is applied to a motion in the air, by which its parts are agitated like the waves of the sea; as is supposed to be the case when the string of a musical instrument is struck. This Undulatory motion of the air is supposed the matter or cause of sound.—Instead of the Undulatory, some authors choose to call this a *vibratory* motion.

UNEVEN *Number*, the same as odd number, or such as cannot be divided by 2 without leaving 1 remaining. The series of Uneven Numbers are 1, 3, 5, 7, 9, &c. See NUMBER, and ODD *Number*.

UNGULA, in Geometry, is a part cut off a cylinder, cone, &c, by a plane passing obliquely through the base, and part of the curve surface; so called from its resemblance to the (ungula) hoof of a horse &c. For the contents &c of such Ungulas, see my Mensuration, pa. 218—246, 2d edition.

UNICORN, in Astronomy. See MONOCEROS.

UNIFORM, or *Equable Motion*, is that by which a body passes always with the same celerity, or over equal spaces in equal times. See MOTION.

In Uniform motions, the spaces described or passed over, are in the compound ratio of the times and velocities; but the spaces are simply as the times, when the velocity is given; and as the velocities, when the time is given.

Uniform *Matter*, in Natural Philoſophy, is that which is all of the ſame kind and texture.

UNISON, in Muſic, is when two ſounds are exactly alike, or the ſame note, or tone.

What conſtitutes a Uniſon, is the equality of the number of vibrations, made in the ſame time, by the two ſonorous bodies.

It is a noted phenomenon in muſic, that an intenſe ſound being raiſed, either with the voice, or a ſonorous body, another ſonorous body near it, whoſe tone is either Uniſon, or octave to that tone, will ſound its proper note Uniſon, or octave, to the given note. The experiment is eaſily tried with the ſtrings of two inſtruments; or with a voice and harpſichord; or a bell, or even a drinking glaſs.

This phenomenon is thus accounted for: one ſtring being ſtruck, and the air put into a vibratory motion by it; every other ſtring, within the reach of that motion, will receive ſome impreſſion from it: but each ſtring can only move with a determinate velocity of recourſes or vibrations; and all Uniſons proceed from equal vibrations; and other concords from other
p 276 proportions of vibration. The Uniſon ſtring then, keeping equal pace with the ſounding ſtring, as having the ſame meaſure of vibrations, muſt have its motion continued, and ſtill improved, till at length its motion become ſenſible, and it give a diſtinct ſound. Other concording ſtrings have their motions propagated in different degrees, according to the frequency of the coincidence of their vibrations with thoſe of the ſounded ſtring: the octave therefore moſt ſenſibly; then the 5th; after which, the croſſing of the motions prevents any effect.

This is illuſtrated, as Galileo firſt ſuggeſted, by the pendulum, which being ſet a-moving, the motion may be continued and augmented, by making frequent, light, coincident impulſes; as blowing on it when the vibration is juſt finiſhed: but if it be touched by any croſs or oppoſite motion, and that frequently too, the motion will be interrupted, and ceaſe altogether. So, of two Uniſon ſtrings, if the one be forcibly ſtruck, it communicates motion, by the air, to the other; and both performing their vibrations together, the motion of that other will be improved and heightened by the frequent impulſes received from the vibrations of the firſt, becauſe given preciſely when the other has finiſhed its vibration, and is ready to return: but if the vibrations of the chords be unequal in duration, there will be a croſſing of motions, more or leſs, according to the proportion of the inequality; by which the motion of the untouched ſtring will be ſo checked, as never to be ſenſible. And this we find to be the caſe in all conſonances, except Uniſon, octave, and the fifth.

UNIT, Unite, or Unity, in Arithmetic, the number one, or one ſingle individual part of diſcrete quantity. See Number.—The place of units, is the firſt place on the right hand in integer numbers.

According to Euclid, Unity is not a number, for he defines number to be a multitude of Units.

UNITY, the abſtract or quality which conſtitutes or denominates a thing *one*.

UNIVERSE, a collective name, ſignifying the aſſemblage of heaven and earth, with all things in them.

The Ancients, and after them the Carteſians, imagine the Univerſe to be infinite; and the reaſon they give is, that it implies a contradiction to ſuppoſe it finite or bounded; ſince it is impoſſible not to conceive ſpace beyond any limits that can be aſſigned it; which ſpace, according to the Carteſians, is body, and conſequently part of the Univerſe.

UNLIKE *Quantities*, in Algebra, are ſuch as are expreſſed by different letters, or by different powers of the ſame letter. Thus, a, and b, and a^2, and ab are all Unlike quantities.

Unlike *Signs*, are the different ſigns $+$ and $-$.

UNLIMITED or *Indeterminate Problem*, is ſuch a one as admits of many, or even of infinite anſwers. As, to divide a given triangle into two equal parts; or to deſcribe a circle through two given points. See Diophantine, and Indeterminate.

VOID *Space*, in Phyſics. See Vacuum.

VOLUTE, in Architecture, a kind of ſpiral ſcroll, and uſed in the Ionic and Compoſite capitals; of which it makes the principal characteriſtic and ornament.

VORTEX, or *Whirlwind*, in Meteorology, a ſudden, rapid, violent motion of the air, in circular whirling directions.

Vortex is alſo uſed for an eddy or whirlpool, or a body of water, in certain ſeas and rivers, which runs rapidly round, forming a ſort of cavity in the middle.

Vortex, in the Carteſian Philoſophy, is a ſyſtem or collection of particles of matter moving the ſame way, and round the ſame axis.

Such Vortices are the grand machines by which theſe philoſophers ſolve moſt of the motions and other phenomena of the heavenly bodies. And accordingly, the doctrine of theſe Vortices makes a great part of the Carteſian philoſophy.

The matter of the world they hold to have been divided at the beginning into innumerable little equal particles, each endowed with an equal degree of motion, both about its own centre, and ſeparately, ſo as to conſtitute a fluid.

Several ſyſtems, or collections of this matter, they farther hold to have been endowed with a common motion about certain points, as common centres, placed at equal diſtances, and that the matters, moving round theſe, compoſed ſo many Vortices.

Then, the primitive particles of the matter they ſuppoſe, by theſe inteſtine motions, to become, as it were, ground into ſpherical figures, and ſo to compoſe globules of divers magnitudes; which they call the matter of the ſecond element: and the particles rubbed, or ground off them, to bring them to that form, they call the matter of the firſt element.

And ſince there would be more of the firſt element than would ſuffice to fill all the vacuities between the globules of the ſecond, they ſuppoſe the remaining part to be driven towards the centre of the Vortex, by the circular motion of the globules; and that being there amaſſed into a ſphere, it would produce a body like the ſun.

This ſun being thus formed, and moving about its own axis with the common matter of the Vortex, would neceſſarily throw out ſome parts of its matter, through the vacuities of the globules of the ſecond element conſtituting the Vortex; and this eſpecially at ſuch places as are fartheſt from its poles; receiving, at the ſame time, in,

in, by these poles, as much as it loses in its equatorial parts. And, by this means, it would be able to carry round with it those globules that are nearest, with the greater velocity; and the remoter, with less. And by this means, those globules, which are nearest the centre of the sun, must be smallest; because, were they greater, or equal, they would, by reason of their velocity, have a greater centrifugal force, and recede from the centre. If it should happen, that any of these sun-like bodies, in the centres of the several Vortices, should be so incrustated, and weakened, as to be carried about in the Vortex of the true sun; if it were of less solidity, or had less motion, than the globules towards the extremity of the solar Vortex, it would descend towards the sun, till it met with globules of the same solidity, and susceptible of the same degree of motion with itself; and thus, being fixed there, it would be for ever after carried about by the motion of the Vortex, without either approaching any nearer to the sun, or receding from it; and so would become a planet.

Supposing then all this; we are next to imagine, that our system was at first divided into several Vortices, in the centre of each of which was a lucid spherical body; and that some of these, being gradually incrustated, were swallowed up by others which were larger, and more powerful, till at length they were all destroyed, and swallowed up by the largest solar Vortex; except some few which were thrown off in right lines from one Vortex to another, and so become comets.

But this doctrine of Vortices is, at best, merely hypothetical. It does not pretend to shew by what laws and means the celestial motions are effected, so much as by what means they possibly might, in case it should have so pleased the Creator. But we have another principle which accounts for the same phenomena as well, nay, better than that of Vortices; and which we plainly find has an actual existence in the nature of things: and this is gravity, or the weight of bodies.

The Vortices, then, should be thrown out of philosophy, were it only that two different adequate causes of the same phenomena are inconsistent.

But there are other objections against them. For, 1°, if the bodies of the planets and comets be carried round the sun in Vortices, the bodies with the parts of the Vortex immediately investing them, must move with the same velocity, and in the same direction; and besides, they must have the same density, or the same vis inertiæ. But it is evident, that the planets and comets move in the very same parts of the heavens with different velocity, and in different directions. It follows, therefore, that those parts of the Vortex must revolve at the same time, in different directions, and with different velocities; since one velocity, and direction, will be required for the passage of the planets, and another for that of the comets.

2°, If it were granted, that several Vortices are contained in the same space, and do penetrate each other, and revolve with divers motions; since these motions must be conformable to those of the bodies, which are perfectly regular, and performed in conic sections; it may be asked, How they should have been preserved entire so many ages, and not disturbed and confounded by the adverse actions and shocks of so much matter as they must meet withal?

3°, The number of comets is very great, and their motions are perfectly regular, observing the same laws with the planets, and moving in orbits, that are exceedingly eccentric. Accordingly, they move every way, and towards all parts of the heavens, freely pervading the planetary regions, and going frequently contrary to the order of the signs; which would be impossible unless these Vortices were away.

4°, If the planets move round the sun in Vortices, those parts of the Vortices next the planets, we have already observed, would be equally dense with the planets themselves: consequently the vortical matter, contiguous to the perimeter of the earth's orbit, would be as dense as the earth itself: and that between the orbits of the earth and Saturn, must be as dense, or denser. For a Vortex cannot maintain itself, unless the more dense parts be in the centre, and the less dense towards the circumference: and since the periodical times of the planets are in sesquialterate ratio of their distances from the sun, the parts of the Vortex must be in the same ratio. Whence it follows, that the centrifugal forces of the parts will be reciprocally as the squares of the distances. Such, therefore, as are at a greater distance from the centre, will endeavour to recede from it with the less force. Accordingly, if they be less dense, they must give way to the greater force, by which the parts nearer the centre endeavour to rise. Thus, the more dense will rise, and the less dense descend; and thus there will be a change of places, till the whole fluid matter of the Vortex be so adjusted as that it may rest in equilibrio.

Thus will the greatest part of the Vortex without the earth's orbit, have a degree of density and inactivity, not less than that of the earth itself. Whence the comets must meet with a very great resistance, which is contrary to all appearances. Cotes, Præf. ad Newt. Princip. The doctrine of Vortices, Newton observes, labours under many difficulties: for a planet to describe areas proportional to the times, the periodical times of a Vortex should be in a duplicate ratio of their distances from the sun; and for the periodical time of the planets, to be in a sesquiplicate proportion of their distances from the sun, the periodical times of the parts of the Vortex should be in the same proportion of their distances: and, lastly, for the less Vortices about Jupiter, Saturn, and the other planets, to be preserved, and swim securely in the sun's Vortex, the periodical times of the sun's Vortex should be equal. None of which proportions are found to obtain in the revolutions of the sun and planets round their axes. Phil. Nat. Princ. Math. apud Schol. Gen. in Calce.

Besides, the planets, according to this hypothesis, being carried about the sun in ellipses, and having the sun in the focus of each figure, by lines drawn from themselves to the sun, they always describe areas proportionable to the times of their revolutions, which that author shews the parts of no Vortex can do. Schol. prop. ult. lib. ii. Princip.

Again, Dr. Keill proves, in his Examination of Burnet's Theory, that if the earth were carried in a Vortex, it would move faster in the proportion of three to two,

two, when it is in Virgo than when it is in Piſces; which all experience proves to be falſe.

There is, in the Philoſophical Tranſactions, a Phyſico-mathematical demonſtration of the impoſſibility and inſufficiency of Vortices to account for the Celeſtial Phenomena; by Monſ. de Sigorne. See Num. 457. Sect. vi. pa. 409 et ſeq.

This author endeavours to ſhew, that the mechanical generation of a Vortex is impoſſible; and that it has only an axifugal force, and not a centrifugal and centripetal one; that it is not ſufficient for explaining gravity and its properties; that it deſtroys Kepler's aſtronomical laws; and therefore he concludes, with Newton, that the hypotheſis of Vortices is fitter to diſturb than explain the celeſtial motions. We muſt refer to the diſſertation itſelf for the proof of theſe aſſertions. See Cartesian Philosophy.

VOSSIUS (Gerard John), one of the moſt learned and laborious writers of the 17th century, was of a conſiderable family in the Netherlands: and was born in 1577, in the Palatinate near Heidelberg, at a place where his father, John Voſſius, was miniſter. He firſt learned Latin, Greek, and Philoſophy at Dort, where his father had ſettled, and died. In 1595 he went to Leyden, where he farther purſued theſe ſtudies, joining mathematics to them, in which ſcience he made a conſiderable progreſs. He became Maſter of Arts and Doctor in Philoſophy in 1598; and ſoon after, Director of the College at Dort; then, in 1614, Director of the Theological College juſt founded at Leyden; and, in 1618, Profeſſor of Eloquence and Chronology in the Academy there, the ſame year in which appeared his Hiſtory of the Pelagian Controverſy. This hiſtory procured him much odium and diſgrace on the continent, but an ample reward in England, where archbiſhop Laud obtained leave of king Charles the 1ſt for Voſſius to hold a prebendary in the church of Canterbury, while he reſided at Leyden: this was in 1629, when he came over to be inſtalled, took a Doctor of Laws degree at Oxford, and then returned.—In 1633 he was called to Amſterdam to fill the chair of a Profeſſor of Hiſtory; where he died in 1649, at 72 years of age; after having written and publiſhed as many works as, when they came to be collected and printed at Amſterdam in 1695 &c, made 6 volumes folio, works which will long continue to be read with pleaſure and profit. The principal of theſe are, —1. *Etymologicon Linguæ Latinæ.*—2. *De Origine & Progreſſu Idololatriæ.*—3. *De Hiſtoricis Græcis.*—4. *De Hiſtoricis Latinis.*—5. *De Arte Grammatica.*—6. *De Vitiis Sermonis & Gloſſematis Latino-Barbaris.*—7. *Inſtitutiones Oratoriæ.*—8. *Inſtitutiones Poeticæ.*—9. *Ars Hiſtorica.*—10. *De quatuor Artibus popularibus, Grammatice, Gymnaſtice, Muſice, & Graphice.*—11. *De Philologia.*—12. *De Univerſa Matheſeos Natura & Conſtitutione.*—13. *De Philoſophia.*—14. *De Philoſophorum Sectis.*—15. *De Veterum Poetarum Temporibus.*

Vossius (*Denis*), ſon of the foregoing Gerard John, died at 22 years of age, a prodigy of learning, whoſe inceſſant ſtudies brought on him ſo immature a death. There are of his, among other ſmaller pieces, Notes upon Cæſar's Commentaries, and upon Maimonides on Idolatry.

Vossius (*Francis*), brother of Denis and ſon of Gerard John, died in 1645, after having publiſhed a Latin poem in 1640, on a naval victory gained by the celebrated Van Tromp.

Vossius (*Gerard*), brother of Denis and Francis, and ſon of Gerard John, wrote Notes upon Paterculus, which were printed in 1639. He was one of the moſt learned critics of the 17th century; but died in 1640, like his two brothers, at a very early age, and before their father.

Vossius (*Iſaac*), was the youngeſt ſon of Gerard John, and the only one that ſurvived him. He was born at Leyden in 1618, and was a man of great talents and learning. His father was his only preceptor, and his whole time was ſpent in ſtudying His merit recommended him to a correſpondence with queen Chriſtina of Sweden, who employed him in ſome literary commiſſions. At her requeſt, he made ſeveral journeys into Sweden, where he had the honour to teach her the Greek language; though ſhe afterwards diſcarded him on hearing that he intended to write againſt Salmaſius, for whom ſhe had a particular regard. In 1663 he received a handſome preſent of money from Louis the 14th of France, accompanied with a complimentary letter from the miniſter Colbert.—In 1670 he came over to England, when he was created Doctor of Laws at Oxford, and king Charles the 2d made him Canon of Windſor; though he knew his character well enough to ſay, there was nothing that Voſſius refuſed to believe, excepting the Bible. He appears indeed, by his publications, which are neither ſo numerous nor ſo uſeful as his father's, to have been a moſt credulous man, while he afforded many circumſtances to bring his religious faith in queſtion. He died at his lodgings in Windſor Caſtle, in 1688; leaving behind him the beſt private library, as it was then ſuppoſed, in the world; which, to the ſhame and reproach of England, was ſuffered to be purchaſed and carried away by the univerſity of Leyden. His publications chiefly were:—1. *Periplus Scylacis Caryandenſis, &c*, 1639.—2. Juſtin, with Notes, 1640.—3. *Ignatii Epiſtolæ, & Barnabæ Epiſtola*, 1646.—4. *Pomponius Mela de Situ Orbis*, 1648.—5. *Diſſertatio de vera Ætate Mundi, &c*, 1659.—6. *De Septuaginta Interpretibus, &c*, 1661.—7. *De Luce*, 1662.—8. *De Motu Marium & Ventorum.*—9. *De Nili & aliorum Fluminum Origine.*—10. *De Poematum Cantu & Viribus Rythmi*, 1673.—11. *De Sybillinis aliiſque, quæ Chriſti natalem præceſſere*, 1679.—12. *Catullus, & in eum Iſaaci Voſſii Obſervationes*, 1684.—13. *Variarum Obſervationum liber*, 1685, in which are contained the following pieces: viz, *De Antiquæ Romæ & aliarum quarundam Urbium Magnitudine; De Artibus & Scientiis Sinarum; De Origine & Progreſſu Pulveris Bellici apud Europæos; De Triremium & Liburnicarum Conſtructione; De Emendatione Longitudinum; De patefacienda per Septentrionem ad Japonenſes & Indos Navigatione; De apparentibus in Luna circulis; Diurna Telluris converſione omnia gravia ad medium tendere.*

VOUSSOIRS, vault-ſtones, are the ſtones which immediately form the arch of a bridge, &c, being cut ſomewhat in the manner of a truncated pyramid, their under ſides conſtituting the intrados, to which their joints

joints or ends ſhould be every where in a perpendicular direction.

The length of the middle Vouſſoir, or key-ſtone, and which is the leaſt of all, ſhould be about $\frac{1}{15}$th or $\frac{1}{18}$th of the ſpan of the arch; from hence theſe ſtones ſhould be made larger and larger, all the way down to the impoſt; that they may the better ſuſtain the great weight which reſts upon them, without being cruſhed or broken, and that they may alſo bind the firmer together.

To find the juſt length of the Vouſſoirs, or the figure of the extrados, when that of the intrados is given; ſee my Principles of Bridges, or Emerſon's Conſtruction of Arches, in his volume of Miſcellanies.

URANIBURGH, or celeſtial town, the name of a celebrated obſervatory, in a caſtle in the little iſland Weenen, in the Sound; built by the celebrated Daniſh aſtronomer, Tycho Brahe, who furniſhed it with inſtruments for obſerving the courſe and motions of the heavenly bodies.

This obſervatory, which was finiſhed about the year 1580, had not ſubſiſted above 17 years when Tycho, who little thought to have erected an edifice of ſo ſhort a duration, and who had even publiſhed the figure and poſition of the heavens, which he had choſen for the moment to lay the firſt ſtone in, was obliged to abandon his country.

Soon after this, the perſons to whom the property of the iſland was given, demoliſhed the building: part of the ruins was diſperſed into divers places: the reſt ſerved to build Tycho a handſome ſeat upon his ancient eſtate, which to this day bears the name of Uraniburgh; and it was here that Tycho compoſed his catalogue of the ſtars. Its latitude is 55° 54′ north, and longitude 12° 47′ eaſt of Greenwich.

M. Picart, making a voyage to Uraniburgh, found that Tycho's meridian line, there drawn, deviated from the meridian of the world; which ſeems to confirm the conjecture of ſome perſons, that the poſition of the meridian line may vary.

URSA, in Aſtronomy, the Bear, a name common to two conſtellations of the northern hemiſphere, near the pole, diſtinguiſhed by *Major* and *Minor*.

Ursa *Major*, or the *Great Bear*, one of the 48 old conſtellations, and perhaps more ancient than many of the others; being familiarly known and alluded to by the oldeſt writers, and is mentioned by Homer as obſerved by navigators. It is ſuppoſed that this conſtellation is that mentioned in the book of Job, under the name of *Cheſil*, which our tranſlation has rendered Orion, where it is ſaid, "Canſt thou looſe the bands of Cheſil (Orion)?" It is farther ſaid that the Ancients repreſented each of theſe two conſtellations under the form of a waggon drawn by a team of horſes, and the Greeks originally called them waggons and two bears; they are to this day popularly called the wains, or waggons, and the greater of them Charles's Wain. Hence is remarked the propriety of the expreſſion, "looſe the bands &c," the binding and looſing being terms very applicable to a harneſs, &c.

Perhaps the Egyptians, or whoever elſe were the people that invented the conſtellations, placed thoſe it s, which are near the pole, in the figure of a bear, as being an animal inhabiting towards the north pole, and making neither long journeys, nor ſwift motions. But the Greeks, in their uſual way, have adapted ſome of their fables to it. They ſay this bear was Calliſto, daughter of Lycaon, king of Arcadia; that being debauched by Jupiter, he afterwards placed her in the heavens, as well as her ſon Arcturus.

The Greeks called this conſtellation Arctos and Helice, from its turning round the pole. The Latins from the name of the nymph, as variouſly written, Calliſto, Megiſto, and Flemiſto, and from the Arabians, ſometimes Feretrum Majus, the Great Bier. And the Urſa Minor, they called Feretrum Minus, the Little Bier. The Italians have followed the ſame cuſtom, and call them Cataletto. They ſpoke alſo of the Phenicians being guided by the Leſſer Bear, but the Greeks by the Greater.

There are two remarkable ſtars in this conſtellation, viz, thoſe in the middle of his body, conſidered as the two hindermoſt of the wain, and called the pointers, becauſe they always point nearly in a direction towards the north pole ſtar, and ſo are uſeful in finding this ſtar out.

The ſtars in Urſa Major, are, according to Ptolomy's catalogue, 35; in Tycho's 56; in Hevelius's 73; but in the Britannic catalogue 87.

Ursa *Minor*, the *Little Bear*, called alſo *Arctos Minor*, *Phœnice*, and *Cynoſura*, one of the 48 old conſtellations, and near the north pole, the large ſtar in the tip of its tale being very near to it, and thence called the pole-ſtar.

The Phenicians guided their navigations by this conſtellation, for which reaſon it was called Phenice, or the Phenician conſtellation. It was alſo called Cynoſura by the Greeks, becauſe, according to ſome, that was one of the dogs of the huntreſs Calliſto, or the Great Bear; but according to others Cynoſura was one of the Idæan nymphs that nurſed the infant Jupiter; and ſome ſay that Calliſto was another of them, and that, for their care, they were taken up together to the ſkies.

Ptolomy places in this conſtellation 8 ſtars, Tycho 7, Hevelius 12, and Flamſteed 24.

URSUS (Nicholas Raimarus), a very extraordinary perſon, and diſtinguiſhed in the ſcience of aſtronomy, was born at Henſtedt in Dithmarſen, in the duchy of Holſtein, about the year 1550 He was a ſwineherd in his youth, and did not begin to read till he was 18 years of age; but then he employed all the hours he could ſpare from his daily labour, in learning to read and write. He afterwards applied himſelf to learn the languages; and, having a ſtrong genius, made a rapid progreſs in Greek and Latin. He quickly learned alſo the French language, the mathematics, aſtronomy, and philoſophy; and moſt of them without the aſſiſtance of a maſter.

Having left his native country, he gained a maintenance by teaching; which he did in Denmark in 1584, and on the frontiers of Pomerania and Poland in 1585. It was in this place that he invented a new ſyſtem of aſtronomy, very little different from that of Tycho Brahe. This he communicated, in 1586, to the landgrave of Heſſe, which gave riſe to a terrible diſpute between him and Tycho. This celebrated aſtronomer charged him with being a plagiary: who, as he related, happening to come with his maſter into his ſtudy, ſaw there, drawn on a piece of paper, the figure of his

his system; and afterwards insolently boasted that he himself was the inventor of it. Ursus, upon this accusation, wrote furiously against Tycho, called the honour of his invention in question, ascribing the system to Apollonius Pergæus; and in short abused him in so brutal a manner, that he was going to be prosecuted for it.

Ursus was afterwards invited by the emperor to teach the mathematics in Prague; from which city, to avoid the presence of Tycho, he withdrew silently in 1589, and died soon after.

He made some improvements in trigonometry, and wrote several books, which discover the marks of his hasty studies; his erudition being indigested, and his style incorrect, as is almost always to be observed of persons that are late-learned.

VULPECULA *et* ANSER, the *Fox and Goose*, in Astronomy, one of the new constellations of the northern hemisphere, made out of the unformed stars by Hevelius, in which he reckons 27 stars; but Flamsteed counts 35.

W.

WAD, or WADDING, in Gunnery, a stopple of paper, hay, straw, old rope-yarn, or tow, rolled firmly up like a ball, or a short cylinder, and forced into a gun upon the powder, to keep it close in the chamber; or put up close to the shot, to keep it from rolling out, as well as, according to some, to prevent the inflamed powder from dilating round the sides of the ball, by its windage, as it passes along the chace, which it was thought would much diminish the effort of the powder. But, from the accurate experiments lately made at Woolwich, it has not been found to have any such effect.

WADHOOK, or WORM, a long pole with a screw at the end, to draw out the wad, or the charge, or paper &c from a gun.

WAGGONER, in Astronomy, is the constellation Ursa Major, or the Great Bear, called also vulgarly Charles's Wain.

WAGGONER is also used for a routier, or book of charts, describing the seas, their coasts, &c.

WALLIS (Dr. JOHN), an eminent English mathematician, was the son of a clergyman, and born at Ashford in Kent, Nov. 23. 1616. After being instructed, at different schools, in grammar learning, in Latin, Greek, and Hebrew, with the rudiments of logic, music, and the French language, he was placed in Emanuel college, Cambridge. About 1640 he entered into orders, and was chosen fellow of Queen's college. He kept his fellowship till it was vacated by his marriage, but quitted his college to be chaplain to Sir Richard Darley; after a year spent in this situation, he spent two more as chaplain to lady Vere. While he lived in this family, he cultivated the art of deciphering, which proved very useful to him on several occasions: he met with rewards and preferment from the government at home for deciphering letters for them; and it is said, that the elector of Brandenburg sent him a gold chain and medal, for explaining for him some letters written in ciphers.

In 1643 he published *Truth Tryed*, or Animadversions on lord Brooke's treatise, called The Nature of Truth &c; styling himself "a minister in London," probably of St. Gabriel Fenchurch, the sequestration of which had been granted to him.—In 1644 he was chosen one of the scribes or secretaries to the assembly of divines at Westminster.

Academical studies being much interrupted by the civil wars in both the universities, many learned men from them resorted to London, and formed assemblies there. Wallis belonged to one of these, the members of which met once a week, to discourse on philosophical matters; and this society was the rise and beginning of that which was afterwards incorporated by the name of the Royal Society, of which Wallis was one of the most early members.

The Savilian professor of geometry at Oxford being ejected by the parliamentary visitors, in 1649, Wallis was appointed to succeed him, and he opened his lectures there the same year. In 1650 he published some Animadversions on a book of Mr. Baxter's, intitled, "Aphorisms of Justification and the Covenant." And in 1653, in Latin, a Grammar of the English tongue, for the use of foreigners; to which was added, a tract *De Loquela seu Sonorum formatione, &c*, in which he considers philosophically the formation of all sounds used in articulate speech, and shews how the organs being put into certain positions, and the breath pushed out from the lungs, the person will thus be made to speak, whether he hear himself or not. Pursuing these reflections, he was led to think it possible, that a deaf person might be taught to speak, by being directed so to apply the organs of speech, as the sound of each letter required, which children learn by imitation and frequent attempts, rather than by art. He made a trial or two with success; and particularly upon one Popham, which involved him in a dispute with Dr. Holder, of which some account has already been given in the life of that gentleman.

In 1654 he took the degree of Doctor in Divinity; and the year after became engaged in a long controversy with Mr. Hobbes. This philosopher having, in 1655, printed his treatise *De Corpore Philosophico*, Dr. Wallis the same year wrote a confutation of it in Latin, under the title of *Elenchus Geometriæ Hobbianæ*; which so provoked Hobbes, that in 1656 he published it in English, with the addition of what he called, "Six Lessons to the Professors of Mathematics in Oxford." Upon this Dr. Wallis wrote an answer in English, intitled, "Due Correction for Mr. Hobbes; or School discipline for not saying his Lessons right," 1656: to which Mr. Hobbes replied in a pamphlet called "ΣΤΙΓΜΑΙ, &c, or Marks of the absurd Geometry, Rural Language, Scottish Church-politics, and Barbarisms, of John Wallis, 1657." This was immediately rejoined to by Dr. Wallis, in *Hobbiani Puncti Dispunctio*, 1657. And here this controversy seems to have ended, at this time: but in 1661 Mr. Hobbes printed *Examinatio & Emendatio Mathematicorum Hodiernorum in sex Dialogis*; which occasioned Dr. Wallis to publish the next year, *Hobbius Heautontimorumenos*, addressed to Mr. Boyle.

In 1657 he collected and published his mathematical works, in two parts, entitled, *Mathesis Universalis*, in 4to; and in 1658, *Commercium Epistolicum de Quæstionibus quibusdam Mathematicis nuper habitum*, in 4to; which was a collection of letters written by many learned men, as Lord Brounker, Sir Kenelm Digby, Fermat, Schooten, Wallis, and others.

He was this year chosen *Custos Archivorum* of the university. Upon this occasion Mr. Stubbe, who, on account of his friend Mr. Hobbes, had before waged war against Wallis, published a pamphlet, intitled, "The Savilian Professor's Case Stated," 1658. Dr. Wallis replied to this; and Mr. Stubbe republished his case, with enlargements, and a vindication against the exceptions of Dr. Wallis.

Upon the Restoration he met with great respect; the king thinking favourably of him on account of some services he had done both to himself and his father Charles the first. He was therefore confirmed in his places, also admitted one of the king's chaplains in ordinary, and appointed one of the divines empowered to revise the book of Common Prayer. He complied with the terms of the act of uniformity, and continued a steady conformist till his death. He was a very useful member of the Royal Society; and kept up a literary correspondence with many learned men. In 1670 he published his *Mechanica; sive de Motu*, 4to. In 1676 he gave an edition of *Archimedis Syracusani Arenarius & Dimensio Circuli*; and in 1682 he published from the manuscripts, *Claudii Ptolomæi Opus Harmonicum*, in Greek, with a Latin version and notes; to which he afterwards added, *Appendix de veterum Harmonica ad hodiernam comparata, &c.* In 1685 he published some theological pieces; and, about 1690, was engaged in a dispute with the Unitarians; also, in 1692, in another dispute about the Sabbath. Indeed his books upon subjects of divinity are very numerous, but nothing near so important as his mathematical works.

In 1685 he published his History and Practice of Algebra, in folio; a work that is full of learned and useful matter. Besides the works above mentioned, he published many others, particularly his *Arithmetic of Infinites*, a book of genius and good invention, and perhaps almost his only work that is so, for he was much more distinguished for his industry and judgment, than for his genius. Also a multitude of papers in the Philos. Trans. in almost every volume, from the 1st to the 25th volume. In 1697, the curators of the University press at Oxford thought it for the honour of the university to collect the doctor's mathematical works, which had been printed separately, some in Latin, some in English, and published them all together in the Latin tongue, in 3 vols folio, 1699.

Dr. Wallis died at Oxford the 28th of October 1703, in the 88th year of his age, leaving behind him one son and two daughters. We are told that he was of a vigorous constitution, and of a mind which was strong, calm, serene, and not easily ruffled or discomposed. He speaks of himself, in his letter to Mr. Smith, in a strain which shews him to have been a very cautious and prudent man, whatever his secret opinions and attachments might be: he concludes, "It hath been my endeavour all along to act by moderate principles, being willing, whatever side was uppermost, to promote any good design, for the true interest of religion, of learning, and of the public good."

WARD (Dr. Seth), an English prelate, chiefly famous for his knowledge in mathematics and astronomy, was the son of an attorney, and born at Buntingford, Hertfordshire, in 1617 or 1618. From hence he was removed and placed a student in Sidney college, Cambridge, in 1632. Here he applied with great vigour to his studies, particularly to the mathematics, and was chosen fellow of his college. In 1640 he was pitched upon by the Vice-chancellor to be Prævaricator, which at Oxford is called Terræ-filius; whose office it was to make a witty speech, and to laugh at any thing or any body: a privilege which he exercised so freely, that the Vice-chancellor actually suspended him from his degree; though he reversed the censure the day following.

The civil war breaking out, Ward was involved not a little in the consequences of it. He was ejected from his fellowship for refusing the Covenant; against which he soon after joined with several others, in drawing up that noted treatise, which was afterwards printed. Being now obliged to leave Cambridge, he resided for some time with certain friends about London, and at other times at Aldbury in Surry, with the noted mathematician Oughtred, where he prosecuted his mathematical studies. He afterwards lived for the most part, till 1649, with Mr. Ralph Freeman at Aspenden in Hertfordshire, whose sons he instructed as their preceptor; after which he resided some months with lord Wenman, of Thame Park, in Oxfordshire.

He had not been long in this family before the visitation of the university of Oxford began; the effect of which was, that many learned and eminent persons were turned out, and among them Mr. Greaves, the Savilian professor of Astronomy: this gentleman laboured to procure Ward for his successor, whose abilities in his way were universally known and acknowledged; and effected it; Dr. Wallis succeeding to the Geometry professorship at the same time. Mr. Ward then entered himself of Wadham college, for the sake of

Dr. Wilkins, who was the warden; and he presently applied himself to bring the astronomy lectures, which had long been neglected and disused, into repute again; and for this purpose he read them very constantly, never missing one reading day, all the while he held the lecture.

In 1654, both the Savilian professors did their exercises, in order to proceed doctors in divinity; and when they were to be presented, Wallis claimed precedency. This occasioned a dispute; which being decided in favour of Ward, who was really the senior, Wallis went out grand compounder, and so obtained the precedency. In 1659, Ward was chosen president of Trinity college; but was obliged at the Restoration to resign that place. He had amends made him, however, by being presented in 1660 to the rectory of St. Laurence Jewry. The same year he was also installed precentor of the church of Exeter. In 1661 he became fellow of the Royal Society, and dean of Exeter; and the year following he was advanced to the bishopric of the same church. In 1667 he was translated to the see of Salisbury; and in 1671 was made chancellor of the order of the garter; an honour which he procured to be permanently annexed to the see of Salisbury, after it had been held by laymen for above 150 years.

Dr. Ward was one of those unhappy persons who have the misfortune to survive their senses, which happened in consequence of a fever ill cured: he lived till the Revolution, but without knowing any thing of the matter; and died in January 1689, about 71 years of age. He was the author of several Latin works in astronomy and different parts of the mathematics, which were thought excellent in their day; but their use has been superseded by later improvements and the Newtonian philosophy. Some of these were,

1. A Philosophical Essay towards an Eviction of the Being and Attributes of God, &c. 1652.
2. De Cometis, &c; 4to, 1653.
3. In Ismaelis Bullialdi Astronomia Inquisitio; 4to, 1653.
4. Idea Trigonometriæ demonstratæ; 4to, 1654.
5. Astronomia Geometrica; 8vo, 1656. In this work, a method is proposed, by which the astronomy of the planets is geometrically resolved, either upon the Elliptical or Circular motion; it being in the third or last part of this work that he proposes and explains what is called Ward's Circular Hypothesis.
6. Exercitatio epistolica in Thomæ Hobbii Philosophiam, ad D. Joannem Wilkins; 1656, 8vo.

But that by which he hath chiefly signalized himself, as to astronomical invention, is his celebrated approximation to the true place of a planet, from a given mean anomaly, founded upon an hypothesis, that the motion of a planet, though it be really performed in an elliptic orbit, may yet be considered as equable as to angular velocity, or with an uniform circular motion round the upper focus of the ellipse, or that next the aphelion, as a centre. By this means he rendered the praxis of calculation much easier than any that could be used in resolving what has been commonly called Kepler's problem, in which the coequate anomaly was to be immediately investigated from the mean elliptic one. His hypothesis agrees very well with those orbits which are elliptical but in a very small degree, as that of the Earth and Venus: but in others, that are more elliptical, as those of Mercury, Mars, &c, this approximation stood in need of a correction, which was made by Bulliald. Both the method, and the correction, are very well explained and demonstrated, by Keill, in his Astronomy, lecture 24.

WARGENTIN (Peter), an ingenious Swedish mathematician and astronomer, was born Sept. 22, 1717, and died Dec. 13, 1783. He became secretary to the Academy at Stockholm in 1749, when he was only 32 years of age; and he became successively a member of most of the literary academies in Europe, as London, Paris, Petersburg, Gottingen, Upsal, Copenhagen, Drontheim, &c. In this country he is probably most known on account of his tables for computing the eclipses of Jupiter's satellites, which are annexed to the Nautical Almanac of 1779. I know not that he has published any separate work; but his communications were very numerous to several of those Academies of which he was a member; as the Academy of Stockholm, in which are 52 of his memoirs; in the Philosophical Transactions, the Upsal Acts, the Paris Memoirs, &c.

WATCH, a small portable machine, or movement, for measuring time; having its motion commonly regulated by a spiral spring. Perhaps, strictly speaking, watches are all such movements as *shew* the parts of time; as clocks are such as *publish* them, by striking on a bell, &c. But commonly, the term Watch is appropriated to such as are carried in the pocket; and clock to the large movements, whether they strike the hour or not.

Spring or *Pendulum* Watches stand pretty much on the same principle with pendulum clocks. For if a pendulum, describing small circular arcs, make vibrations of unequal lengths, in equal times, it is because it describes the greater arc with a greater velocity; so a spring put in motion, and making greater and less vibrations, as it is more or less stiff, and as it has a greater or less degree of motion given it, performs them nearly in equal times. Hence, as the vibrations of the pendulum had been applied to large clocks, to rectify the inequality of their motions; so, to correct the unequal motions of the balance in Watches, a spring is added, by the isochronism of whose vibrations the correction is to be affected. The spring is usually wound into a spiral; that, in the little compass allotted it, it may be as long as possible; and may have strength enough not to be mastered, and dragged about, by the inequalities of the balance it is to regulate. The vibrations of the two parts, viz, the spring and the balance, should be of the same length; but so adjusted, as that the spring, being more regular in the length of its vibrations than the balance, may occasionally communicate its regularity to the latter.

The Invention of Spring or Pocket Watches, is due to the last age. It is true, it is said, in the history of Charles the 5th, that a Watch was presented to that prince: but this was probably no more than a kind of clock to be set on a table: some resemblance of which we have still remaining in the ancient pieces made before the year 1670. Some accounts also say, the first Watches were made at Nuremberg in 1500, by Peter Hell,

ſhell, and were called Nuremberg eggs, on account of their oval form. And farther, that the ſame year George Purbach, a mathematician of Vienna, employed a watch that pointed to ſeconds, for aſtronomical obſervations, which was probably a kind of clock. In effect, it is between Hook and Huygens that the glory of this excellent invention lies: but to which of them it properly belongs, has been greatly diſputed; the Engliſh aſcribing it to the former, and the French, Dutch, &c, to the latter. Derham, in his Artificial Clockmaker, ſays roundly, that Dr. Hook was the inventor; and adds, that he contrived various ways of regulation: one way was with a loadſtone: another with a tender ſtraight ſpring, one end of which played backward and forward with the balance; ſo that the balance was to the ſpring as the ball of a pendulum, and the ſpring as the rod of the ſame: a third method was with two balances, of which there were divers ſorts; ſome having a ſpiral ſpring to the balance for a regulalator, and others without. But the way that prevailed, and which ſtill continues in mode, was with one balance, and one ſpring running round the upper part of the verge of it: though this has a diſadvantage, which thoſe with two ſprings &c were free from; in that, a ſudden jerk, or confuſed ſhake will alter its vibrations, and flurry it very much.

The time of theſe inventions was about the year 1658; as appears, among other evidences, from an inſcription on one of the double-balance Watches preſented to king Charles the ſecond, viz, Rob. Hook inven. 1658. T. Tompion fecit, 1675. The invention ſoon came into repute both at home and abroad; and two of the machines were ſent for by the Dauphin of France. Soon after this, M. Huygens's Watch with a ſpiral ſpring got abroad, and made a great noiſe in England, as if the longitude could be found by it. It is certain however, that this invention was later than the year 1673, when his book De Horol. Oſcillat. was publiſhed; in which there is no mention of this, though he ſpeaks of ſeveral other contrivances in the ſame way.

One of theſe the lord Brounker ſent for out of France, where M. Huygens had got a patent for them. This Watch agreed with Dr. Hook's, in the application of the ſpring to the balance; only that of Huygens had a longer ſpiral ſpring, and its pulſes and beats were much ſlower; alſo the balance, inſtead of turning quite round, as Dr. Hook's, turned ſeveral times every vibration. Huygens alſo invented divers other kinds of Watches, ſome of them without any ſtring or chain at all, which he called pendulum Watches.

Mr. Derham ſuggeſts that he ſuſpects Huygens's fancy was firſt ſet to work by ſome intelligence he might have of Hook's invention from Mr. Oldenburg, or ſome other of his correſpondents in England; though Mr. Oldenburg vindicates himſelf againſt that charge, in the Philoſ. Tranſ. numbers 118 and 129.

Watches, ſince their firſt invention, have gone on in a continued courſe of improvement, and they have lately been brought to great perfection, both in England and in France, but more eſpecially the former, particularly owing to the great encouragement that has been given to them by the Board of Longitude. Some of the chief writers and improvers of Watches, are, Le Roy, Cummins, Harriſon, Mudge, Emery, and Arnold, whoſe Watches are now in very high repute, and in frequent uſe in the navy and India ſhips, for keeping the longitude. See Derham's Artificial Clockmaker; Cummins's Principles of Clock and Watch work; Mudge's Thoughts on the Means of improving Watches, &c.

Striking WATCHES, are ſuch as, beſides the proper Watch part, for meaſuring time, have a clock part, for ſtriking the hours, &c. Theſe are real clocks; only moved by a ſpring inſtead of a weight; and are properly called pocket-clocks.

Repeating WATCHES, are ſuch as, by pulling a ſtring, &c, repeat the hour, quarter, or minute, at any time of the day or night.—This repetition was the invention of Mr. Barlow, being firſt put in practice by him in larger movements or clocks, about the year 1676. The contrivance immediately ſet the other artiſts to work, who ſoon contrived divers ways of effecting the ſame. But its application to pocket Watches was not known before K. James the ſecond's reign; when the ingenious inventor above mentioned was ſoliciting a patent for it. The talk of a patent engaged Mr. Quare to reſume the thoughts of a like contrivance, which he had in view ſome years before: he now effected it; and being preſſed to endeavour to prevent Mr. Barlow's patent, a Watch of each kind was produced before the king and council; upon trial of which, the preference was given to Mr. Quare's. The difference between them was, that Barlow's was made to repeat by puſhing in two pieces on each ſide the Watch-box; one of which repeated the hour, and the other the quarter: whereas Quare's was made to repeat by a pin that ſtuck out near the pendant, which being thruſt in (as now is done by thruſting in the pendant itſelf) repeated both the hour and quarter with the ſame thruſt.

Of the Mechaniſm of a WATCH.

Watches, as well as clocks, are compoſed of wheels and pinions, with a regulator to direct the quickneſs or ſlowneſs of the wheels, and of a ſpring which communicates motion to the whole machine. But the regulator and ſpring of a Watch are vaſtly inferior to the weight and pendulum of a clock, neither of which can be employed in Watches. Inſtead of a pendulum, therefore, they are obliged to uſe a balance (Pl. 34, fig. 4) to regulate the motion of a Watch; and of a ſpring (fig. 6), which ſerves inſtead of a weight, to give motion to the wheels and balance.

The wheels of a Watch, like thoſe of a clock, are placed in a frame, formed of two plates and four pillars. Fig. 3 repreſents the inſide of a Watch, after the plate (Fig. 5) is taken off. A is the barrel which contains the ſpring (fig. 6); the chain is rolled about the barrel, with one end of it fixed to the barrel A, and the other to the fuſee B.

When a Watch is wound up, the chain which was upon the barrel winds about the fuſee, and by this means the ſpring is ſtretched; for the interior end of the ſpring is fixed by a ſpring to the immoveable axis, about which the barrel revolves; the exterior end of the ſpring is fixed to the inſide of the barrel, which turns upon an axis. It is there eaſy to perceive how the ſpring extends itſelf, and how its elaſticity forces the

the barrel to turn round, and confequently obliges the chain which is upon the fufee to unfold and turn the fufee; the motion of the fufee is communicated to the wheel CC; then by means of the teeth, to the pinion *c*, which carries the wheel D; then to the pinion *d*, which carries the wheel E; then to the pinion *e*, which carries the wheel F; then to the pinion *f*, upon which is the balance-wheel G, whofe pivot runs in the piece A, called the potance, and B called a follower, which are fixed on the plate fig. 5. This plate, of which only a part is reprefented, is applied to that of fig. 3, in fuch a manner, that the pivots of the wheels enter into holes made in the plate fig. 3. Thus the impreffed force of the fpring is communicated to the wheels: and the pinion *f* being then connected to the wheel F, obliges it to turn (fig. 7). This wheel acts upon the pallats of the verge 1, 2. (fig. 4) the axis of which carries the balance HH (fig. 4). The pivot I, in the end of the verge, enters into the hole G in the potance A (fig. 5). In this figure the pallats are reprefented; but the balance is on the other fide of the plate, as may be feen in fig. 11. The pivot 3 of the balance enters into a hole of the cock BC (fig. 10), a perfpective view of which is reprefented in fig. 12. Thus the balance, turns between the cock and the potance *c* (fig. 5), as in a kind of cage. The action of the balance-wheel upon the pallats 1, 2, (fig. 4) is the fame with that of the fame wheel in the clock; i. e. in a Watch the balance-wheel obliges the balance to vibrate backwards and forwards like a pendulum.

At each vibration of the balance a pallat allows a tooth of the balance-wheel to efcape; fo that the quicknefs of the motion of the wheels is entirely determined by the quicknefs of the vibrations of the balance, and thefe vibrations of the balance and motion of the wheels are produced by the action of the fpring.

But the quicknefs or flownefs of the vibrations of the balance depends not folely upon the action of the great fpring, but chiefly upon the action of the fpring *abc*, called the fpiral fpring (fig. 13) fituated under the balance H, and reprefented in perfpective (fig. 11); the exterior end of the fpiral is fixed to the pin *a* (fig. 13). This pin is applied near the plate in *a* (fig. 11); the interior end of the fpiral is fixed by a peg to the centre of the balance. Hence if the balance be turned upon itfelf, the plates remaining immoveable, the fpring will extend itfelf, and make the balance perform one revolution. Now, after the fpiral is thus extended, if the balance be left to itfelf, the elafticity of the fpiral will bring back the balance, and in this manner the alternate vibrations of the balance are produced.

In fig. 7 all the wheels above defcribed are reprefented in fuch a manner, that we may eafily perceive at firft fight how the motion is communicated from the barrel to the balance.

In fig. 8 are reprefented the wheels under the dial-plate, by which the hands are moved. The pinion *a* is adjufted to the force of the prolonged pivot of the wheel D (fig. 7), and is called a cannon pinion. This wheel revolves in an hour. The end of the axis of the pinion *a*, upon which the minute hand is fixed, is fquare; the pinion (fig. 8) is indented into the wheel *b*, which is carried by the pinion *a*. Fig. 9 is a wheel fixed upon a barrel, into the cavity of which the pinion *a* enters, and upon which it turns freely. This wheel *d* revolves in 12 hours, and carries along with it the hour-hand.

WATER, in Phyfiology, a clear, infipid, and colourlefs fluid, coagulable into a tranfparent folid fubftance, called ice, when placed in a temperature of 32° of Fahrenheit's thermometer, or lower, but volatile and fluid in every degree of heat above that; and when pure, or freed from heterogeneous particles, is reckoned one of the four elements.

By fome late experiments of Meffrs. Lavoifier, Watt, Cavendifh, Prieftley, Kirwan, &c, it appears, that Water confifts of dephlogifticated air, and inflammable air or phlogifton intimately united; or, as Mr. Watt conceives, of thofe two principles deprived of part of their latent heat. And in fome inftances it appears that air and Water are mutually convertible into each other. Thus, Mr. Cavendifh (Philof. Tranf. vol. 74, p. 128) recites feveral experiments, in which he changed common air into pure Water, by decompofing it in conjunction with inflammable air. Dr. Prieftley likewife, having decompofed dephlogifticated and inflammable air, by firing them together by the electric explofion, found a manifeft decompofition of Water, which, as nearly as he could judge, was equal in weight to that of the decompofed air. He alfo made a number of other curious experiments, which feemed to favour the idea of a converfion of Water into air, without abfolutely proving it. The difficulty which M. De Luc and others have found in expelling all air from Water, is beft accounted for on the fuppofition of the generation of air from Water; and admitting that the converfion of Water into air is effected by the intimate union of what is called the principle of heat with the Water, it appears fufficiently analogous to other changes, or rather combinations, of fubftances. Is not, fays Dr. Prieftley, the acid of nitre, and alfo that of vitriol, a thing as unlike to air as Water is, their properties being as remarkably different? And yet it is demonftrable that the acid of nitre is convertible into the pureft refpirable air, and probably by the union of the fame principle of heat. Philof. Tranf. vol. 73, p. 414 &c.

Indeed there feems to be Water in all bodies, and particles of almoft all kinds of matter in Water; fo that it is hardly ever fufficiently pure to be confidered as an element. Water, if it could be had alone, and pure, Boerhaave argues, would have all the requifites of an element, and be as fimple as fire; but there is no expedient hitherto difcovered for procuring it fo pure. Rain Water, which feems the pureft of all thofe we know of, is replete with infinite exhalations of all kinds, which it imbibes from the air: fo that if filtered and diftilled a thoufand times, there ftill remain fæces. Befides this, and the numberlefs impurities it acquires after it is raifed, by mixing with all forts of effluvia in the atmofphere, and by falling upon and running over the earth, houfes, and other places. There is alfo fire contained in all Water; as appears from its fluidity, which is owing to fire alone. Nor can any kinds of filtering through fand, ftone, &c, free it entirely from falts &c. Nor have all the experiments that have been invented by the philofophers, ever been able to derive Water perfectly pure. Hence Boerhaave fays, that he is convinced nobody ever faw a drop of pure Water; that

that the utmoſt of its purity known, only amounts to its being free from this or that ſort of matter; and that it can never, for inſtance, be quite deprived of ſalt; ſince air will always accompany Water, and air always contains ſalt.

Water ſeems to be diffuſed everywhere, and to be preſent in all ſpace wherever there is matter. There are hardly any bodies in nature but what will yield Water: it is even aſſerted that fire itſelf is not without it. A ſingle grain of the fiery ſalt, which in a moment's time will penetrate through a man's hand, readily imbibes half its weight of Water, and melts even in the drieſt air imaginable. Among innumerable inſtances, hartſhorn, kept 40 years, and turned as hard and dry as any metal, ſo that it will yield ſparks of fire when ſtruck againſt a flint, yet being put into a glaſs veſſel, and diſtilled, will afford $\frac{1}{8}$th part of its quantity of Water. Bones dead and dried 25 years, and thus become almoſt as hard as iron, yet by diſtillation have yielded half their weight of Water. And the hardeſt ſtones, ground and diſtilled, always diſcover a portion of it. But hitherto no experiment ſhews, that Water enters as a principle into the combination of metallic matters, or even into that of vitreſcible ſtones.

From ſuch conſiderations, philoſophers have been led to hold the opinion, that all things were made of Water. Baſil Valentine, Paracelſus, Van Helmont, and others have maintained, that Water is the elemental matter or ſtamen of all things, and ſuffices alone for the production of all the viſible creation. Thus too Newton: "All birds, beaſts, and fiſhes, inſects, trees, and vegetables, with their ſeveral parts, do grow out of Water, and watery tinctures, and ſalts; and by putrefaction they all return again to watery ſubſtances." And the ſame doctrine is held, and confirmed by experiments, by Van Helmont, Boyle, and others.

But Dr. Woodward endeavours to ſhew that the whole is a miſtake.—Water containing extraneous corpuſcles, ſome of which, according to him, are the proper matter of nutrition; the Water being ſtill found to afford ſo much the leſs nouriſhment, the more it is purified by diſtillation. So that Water, as ſuch, does not ſeem to be the proper nutriment of vegetables; but only the vehicle which contains the nutritious particles, and carries them along with it, through all the parts of the plant.

Helmont however carries his ſyſtem ſtill farther, and imagines that all bodies may be reconverted into Water. His alkaheſt, he affirms, adequately reſolves plants, animals, and minerals, into one liquor, or more, according to their ſeveral internal differences of parts; and the alkaheſt, being abſtracted again from theſe liquors, in the ſame weight, and with the ſame virtues, as when it diſſolved them, the liquors may, by frequent cohobations from chalk, or ſome other proper matter, be totally deprived of their ſeminal endowments, and at laſt return to their firſt matter; which is inſipid Water.

Spirit of wine, of all other ſpirits, ſeems freeſt from Water: yet Helmont affirms, it may be ſo united with Water, as to become Water itſelf. He adds, that it is material Water, only under a ſulphureous diſguiſe. And the ſame thing he obſerves of all ſalts, and of oils, which may be almoſt wholly changed into Water.

No ſtandard for the Weight and Purity of WATER.—Water ſcarce ever continues two moments exactly of the ſame weight; by reaſon of the air and fire contained in it. The expanſion of Water in boiling ſhews what effect the different degrees of fire have on the gravity of Water. This makes it difficult to fix the ſpecific gravity of Water, in order to ſettle its degree of purity. However, the pureſt Water we can obtain, according to the experiments of Mr. Hawſkbee, is 850 times heavier than air: or according to the experiments of Mr. Cavendiſh, the thermometer being at 50° and the barometer at $29\frac{3}{4}$, about 800 times as heavy as air: and according to the experiments of Sir Geo. Shuckburgh, when the barometer is at 29·27 and the thermometer at 53°, Water is 836 times heavier than air; whence alſo may be deducted this general proportion, which may be accounted a ſtandard, viz, that, when the barometer is at 30° and the thermometer at 55°, then Water is 820 times heavier than air; alſo that in ſuch a ſtate the cubic foot of Water weighs 1000 ounces avoirdupois, and that of air 1·222, or $1\frac{2}{9}$ nearly, alſo that of mercury 13600 ounces; and for other ſtates of the thermometer and barometer, the allowance is after this rate, viz, that the column of mercury in the barometer varies its length by the 10 thouſandth part of itſelf for a change of each ſingle degree of temperature, and Water changes by $\frac{1}{2\,000}$ part of its height or magnitude by each degree of the ſame. However, we have not any very exact ſtandard in air; for Water being ſo much heavier than air, the more Water there is contained in the air, the heavier of courſe muſt the air be; as indeed a conſiderable part of the weight of the atmoſphere ſeems to ariſe from the Water that is in it.

Properties and Effects of WATER.——Water is a very volatile body. It is entirely reduced into vapours and diſſipated, when expoſed to the fire and unconfined.

Water heated in an open veſſel, acquires no more than a certain determinate degree of heat, whatever be the intenſity of the fire to which it is expoſed; which greateſt degree of heat is when it boils violently.

It has been found that the degree of heat neceſſary to make Water boil, is variable, according to the purity of the Water and the weight of the atmoſphere. The following table ſhews the degree of heat at which Water boils, at various heights of the barometer, being a medium between thoſe reſulting from the experiments of Sir Geo. Shuckburgh and M. De Luc:

Height of the Barometer.	Heat of Boiling Water.
Inches.	°
26	205
$26\frac{1}{2}$	206
27	206·9
$27\frac{1}{2}$	207·7
28	208·5
$28\frac{1}{2}$	209·4
29	210·3
$29\frac{1}{2}$	211·2
30	212·0
$30\frac{1}{2}$	212·8
31	213·6

Water

Water is found the moſt penetrative of all bodies, after fire, and the moſt difficult to confine; paſſing through leather, bladders, &c, which will confine air; making its way gradually through woods; and is only retainable in glaſs and metals; nay it was found by experiment at Florence, that when ſhut up in a ſpherical veſſel of gold, which was preſſed with a great force, it made its way through the pores even of the gold itſelf.

Water, by this penetrative quality alone, may be inferred to enter the compoſition of all bodies, both vegetable, animal, foſſil, and even mineral; with this particular circumſtance, that it is eaſily, and with a gentle heat, ſeparable again from bodies it had united with.

And yet the ſame Water, as little coheſive as it is, and as eaſily ſeparated from moſt bodies, will cohere firmly with ſome others, and bind them together in the moſt ſolid maſſes; as in the tempering of earth, or aſhes, clay, or powdered bones, &c, with Water, and then dried and burnt, when the maſſes become hard as ſtones, though without the Water they would be mere duſt or powder. Indeed it appears wonderful that Water, which is otherwiſe an almoſt univerſal diſſolvent, ſhould nevertheleſs be a great coagulator.

Some have imagined that Water is incompreſſible, and therefore nonelaſtic; founding their opinion on the celebrated Florentine experiment above mentioned, with the globe of gold; when the Water being, as they ſay, incapable of condenſation, rather than yield, tranſuded through the pores of the metal, ſo that the ball was found wet all over the outſide; till at length making a cleft in the gold, it ſpun out with great vehemence. But the truth of the concluſions drawn from this Florentine experiment has been very juſtly queſtioned; Mr. Canton having proved by accurate experiments, that Water is actually compreſſed even by the weight of the atmoſphere. See Compression.

Beſides, the diminution of ſize which Water ſuffers when it paſſes to a leſs degree of heat, ſufficiently ſhews that the particles of this fluid are, like thoſe of all other known ſubſtances, capable of approaching nearer together.

Ditch Water, is often uſed as an object for the microſcope, and ſeldom fails to afford a great variety of animalcules; often appearing of a greeniſh, reddiſh, or yellowiſh colour, from the great multitudes of them. And to the ſame cauſe is to be aſcribed the green ſkim on the ſurface of ſuch Water. Dunghill Water is alſo full of an immenſe crowd of animalcules.

Freſh Water, is ſaid of that which is inſipid, or without ſalt, and inodorous; being the natural and pure ſtate of the element.

Hard Water, or *Crude* Water, is that in which ſoap does not diſſolve completely or uniformly, but is curdled. The diſſolving power of hard Water is leſs than that of ſoft; and hence its unfitneſs for waſhing, bleaching, dyeing, boiling kitchen vegetables, &c.

The hardneſs of Water may ariſe either from ſalts, or from gas. That which ariſes from ſalts, may be diſcovered and remedied by adding ſome drops of a ſolution of fixed alkali; but the latter by boiling, or expoſure to the open air.

Spring Waters are often hard; but river Water ſoft. Hard Waters are remarkably indiſpoſed to corrupt; they even preſerve putreſcible ſubſtances for a conſiderable length of time: hence they ſeem to be beſt fitted for keeping at ſea, eſpecially as they are ſo eaſily ſoftened by a little alkaline ſalt.

Putrid Water, is that which has acquired an offenſive ſmell and taſte by the putreſcence of animal or vegetable ſubſtances contained in it. This kind of Water is in the higheſt degree pernicious to the human frame, and capable of bringing on mortal diſeaſes even by its ſmell. Quicklime put into water is uſeful to preſerve it longer ſweet; or even expoſure to the air in broad ſhallow veſſels. And putrid Water may be in a great meaſure ſweetened, by paſſing a current of freſh air through it, from bottom to top.

Rain Water may be conſidered as the pureſt diſtilled Water, but impregnated during its paſſage through the air with a conſiderable quantity of phlogiſtic and putreſcent matter; whence it is ſuperior to any other in fertilizing the earth. Hence alſo it is inferior for domeſtic purpoſes to ſpring or river-Water, even if it could be readily procured: but ſuch as is gotten from ſpouts placed below the roofs of houſes, the common way of procuring it in this country, is evidently very impure, and becomes putrid in a ſhort time.

River or *Running* Water, is next in purity to ſnow or diſtilled water; and for domeſtic purpoſes ſuperior to both, in having leſs putreſcent matter, and more fixed air. That however is much the pureſt that runs over a clean rocky or ſtony bottom.

River Waters generally putrefy ſooner than thoſe of ſprings. During the putrefaction, they throw off a part of their heterogeneous matter, and at length become ſweet again, and purer than at firſt; after which they commonly preſerve a long time: this is remarkably the caſe with the Thames Water, taken up about London; which is commonly uſed by ſeamen, in their voyages.

Salt Water, ſuch as has much ſalt in it, ſo as to be ſenſible to the taſte.

Sea Water, or Water of the ſea, is an aſſemblage of bodies, in which Water can ſcarce be ſaid to have the principal part: it is an univerſal colluvies of all the bodies in nature, ſuſtained and kept ſwimming in Water as a vehicle: being a ſolution of common ſalt, ſal catharticus amarus, a ſelenitic ſubſtance, and a compound of muriatic acid with magneſia, mixed together in various proportions. It may be freſhened by ſimple diſtillation without any addition, and thus it has ſometimes been uſeful in long voyages at ſea. Sea Water by itſelf has a purgative quality, owing to the ſalts it contains; and has been greatly recommended in ſcrophulous diſorders.

Sea Water is about 3 parts in 100 heavier than common Water; and its temperature at great depths is from 34 to 40 degrees; but near the ſurface it follows more nearly the temperature of the air.

Snow Water, is the pureſt of all the common Waters, when the ſnow has been collected pure. Kept in a warm place, in clean glaſs veſſels, not cloſely ſtopped, but covered from duſt, &c, ſnow-water becomes in time putrid; though in well-ſtopped bottles it remains unaltered for ſeveral years. But diſtilled Water ſuffers no alteration in either circumſtance.

Spring Water is commonly impregnated with a

ſmall portion of imperfect neutral ſalt, extracted from the different ſtrata through which it percolates. Some contain a vaſt quantity of ſtony matter, which they depoſit as they run along, and thus form maſſes of ſtone; ſometimes incruſtating various animal and vegetable matters, which they are therefore ſaid to petrify. Spring-Water is much uſed for domeſtic purpoſes, and on account of its coolneſs is an agreeable drink; but on account of its being uſually ſomewhat hard, is inferior to that which has run for a conſiderable way in a channel.

Spring-water ariſes from the rain, and from the miſts and moiſture in the atmoſphere. Theſe falling upon hills and other parts of the earth, ſoak into the ground, and paſs along till they find a vent out again, in the form of a ſpring.

WATER-*Bellows*, in Mechanics, are bellows, for blowing air into furnaces, that are worked by the force of water.

WATER-*Clock*. See CLEPSYDRA.

WATER-*Engine*, an engine for extinguiſhing fires; or any engine to raiſe water; or any engine moved by the force of Water. See ENGINE, and STEAM-*Engine*.

WATER-*Gage*, an inſtrument for meaſuring the depth or quantity of any water. See GAGE.

WATER-*Level*, is the true level which the ſurface of ſtill Water takes, and is the trueſt of any.

WATER-*Logged*, in Sea-Language, denotes the ſtate of a ſhip when, by receiving a great quantity of Water into her hold, by leaking, &c, ſhe has become heavy and inactive upon the ſea, ſo as to yield without reſiſtance to the effort of every wave ruſhing over her deck.

WATER-*Machine*. See MACHINE.

WATER-*Meaſure*. Salt, ſea-coal, &c, while on board veſſels in the pool, or river, are meaſured with the corn-buſhel heaped up; or elſe 5 ſtriked pecks are allowed to the buſhel. This is called Water-meaſure; and it exceeds Wincheſter-meaſure by about 3 gallons in the buſhel.

WATER-*Microſcope*. See MICROSCOPE.

WATER-*Mill*. See MILL.

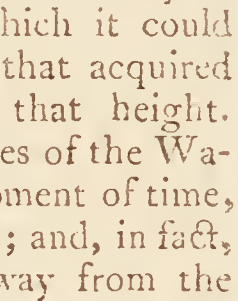
Motion of WATER, in Hydraulics. The theory of the motion of running Water is one of the principal objects of hydraulics, and to which many eminent mathematicians have paid their attention. But it were to be wiſhed that their theories were more conſiſtent with each other, and with experience. The inquiſitive reader may conſult Newton's Principia, lib. 2, pr. 36, with the comment. Dan. Bernoulli's Hydrodynamica. J. Bernoulli, Hydraulica, Oper. tom. 4, pa. 389. Dr. Jurin, in the Philoſ. Tranſ. num. 452, or Abridg. vol. 8, pa. 282. Graveſande, Phyſic. Elem. Mathem. lib. 3, par. 2. Maclaurin's Flux. art. 537. Poleni de Caſtellis, Ximenes, D'Alembert, Boſſu, Buat, and many others.

But notwithſtanding the labours of all theſe eminent authors, this intricate ſubject ſtill remains in a great meaſure obſcure and uncertain. Even the ſimple caſe of the motion of running water, when it iſſues from a hole in the bottom of a veſſel, has never yet been determined, ſo as to give univerſal ſatisfaction to the learned. On this head, it is now pretty generally allowed, that the velocity of the iſſuing ſtream, is equal to that which a heavy body acquires by falling through the height of the fluid above the hole, as may be demonſtrated by theory: but in practice, the quantity of the effluent Water is much leſs than what is given by this theory; owing to the obſtruction to the motion in the hole, partly from the ſides of it, and partly from the different directions of the parts of the Water in entering it, which thence obſtruct each other's motion. And this obſtruction, and the diminution in the quantity of Water run out, is ſtill the more in proportion as the hole is the ſmaller; in ſuch ſort, that when the hole is very ſmall, the quantity is diminiſhed in the ratio of $\sqrt{2}$ to 1 very nearly, which is the ratio of the greateſt diminution; and for larger holes, the diminution is always leſs and leſs. This fact is aſcertained, or admitted by Newton, and all the other philoſophers abovementioned, with ſome ſmall variations.

That the velocity of the Water in the hole, or at leaſt ſome part of it, as that for example in the middle of the ſtream, is equal to that abovementioned, is even evinced by experiment, by directing the ſtream either ſideways, or upwards: for in the former caſe, it is found to range upon an horizontal plane, a diſtance that juſt anſwers to that velocity, by the nature of projectiles; and in the latter caſe, the jet riſes nearly to the height of the Water in the veſſel; which it could not do, if its velocity were not equal to that acquired by the free deſcent of a body through that height. Hence it is evident then, that the particles of the Water, which are in the hole at the ſame moment of time, do not all burſt out with the ſame velocity; and, in fact, the velocity is found to decreaſe all the way from the middle of the hole, where it is greateſt, towards the ſide or edge, where it is the leaſt.

At a ſmall diſtance from the hole, the diameter of the vein of Water is much leſs than that of the hole. Thus, if the diameter of the hole be 1, the diameter of the vein of Water juſt without it, will be $\frac{21}{25}$, or 0·84, according to Newton's meaſure, who firſt obſerved this phenomenon; and according to Poleni's meaſure 0·78 nearly.

By the experiments of Buat (Principes d'Hydraulique), the quantity by theory is to that by experiment, for a ſmall hole made in the thin ſide of a reſervoir, as 8 to 5. When a ſhort pipe is added to the hole outwards, of the length of two or three times its diameter, that ratio is as 16 to 13. And when the ſhort pipe is all within ſide the veſſel, as in the margin, the ſame ratio becomes that of 3 to 2. Poleni alſo found that the quantity of Water flowing through a pipe or tube, was much greater than that through a hole of the ſame diameter in the thin ſide or bottom of the veſſel, the height of the head of Water above each being the ſame. See alſo many other curious circumſtances in Buat's Principes above mentioned.



Some authors give this rule for finding the height due to the velocity in a flat orifice, or a medium among all the parts of it, ſuch that this medium velocity being drawn into the area of the hole, ſhall give the quantity per ſecond that runs through; viz, let *A* denote the area

area of the ſurface of the Water in the veſſel, a the area of the orifice by which the Water iſſues, and H the height of the Water above the orifice; then, as $2A - a : A :: H : h$, the height due to the medium velocity, or the height from which a body muſt freely deſcend, by the force of gravity, to acquire that mean velocity.

Authors are not yet agreed as to the force with which a vein of Water, ſpouting from a round hole in the ſide of a veſſel, preſſes upon a plane directly oppoſed to the motion of the vein. Moſt authors agree, that the preſſure of this vein, flowing uniformly, ought to be equal to the weight of a cylinder of Water, whoſe baſe is equal to the hole through which the Water flows, and its height equal to the height of the Water in the veſſel above the hole. The experiments made by Mariotte, and others, ſeem to countenance this opinion. But Dan. Bernoulli rejects it, and eſtimates this preſſure by the weight of a column of the fluid, whoſe diameter is equal to the contracted vein (according to Newton's obſervation abovementioned), and the height of which is equal to double the altitude due to the real velocity of the ſpouting Water; and this preſſure is alſo equal to the force of repulſion, ariſing from the reaction of the ſpouting Water upon the veſſel. The ingenious author remarks that he ſpeaks only of ſingle veins of Water, the whole of which are received by the planes upon which they preſs; for as to the preſſures exerted by fluids ſurrounding the bodies they preſs upon, as the wind, or a river, the caſe is different, though confounded with the former by writers on this ſubject. Hydrodynamica, pa. 289.

Another rule however had been adopted by the Academicians of Paris, who made a number of experiments to confirm or eſtabliſh it. Hiſt. Acad. Paris, ann. 1679, ſect. 3, cap. 5.

D. Bernoulli, on the other hand, thinks his own theory ſufficiently eſtabliſhed by the experiments he relates; for the particulars of which ſee the Acta Petropolitana, vol. 8, pa. 122.

This ingenious author is of opinion that his theory of the quantity of the force of repulſion, exerted by a vein of ſpouting Water, might be uſefully applied to move ſhips by pumping; and he thinks the motion produced by this repulſive force would fall little, if at all, ſhort of that produced by rowing. He has given his reaſons and computations at length in his Hydrodynamica, pa. 293 &c.

This ſcience of the preſſures exerted by Water or other fluids in motion, is what Bernoulli calls *Hydraulico-ſtatica*. This ſcience differs from hydroſtatics, which conſiders only the preſſure of Water and other fluids at reſt; whereas hydraulico-ſtatics conſiders the preſſure of Water in motion. Thus the preſſure exerted by Water moving through pipes, upon the ſides of thoſe pipes, is an hydraulico-ſtatical conſideration, and has been erroneouſly determined by many, who have given no other rules in theſe caſes, but ſuch as are applicable only to the preſſure of fluids at reſt. See Hydrodynam. pa. 256 &c.

WATER-*Poiſe*. See HYDROMETER, and AREOMETER.

Dr. Hook contrived a Water-poiſe, which may be of good ſervice in examining the purity &c of Water. It conſiſts of a round glaſs ball, like a bolt head, about 3 inches diameter, with a narrow ſtem or neck, the 24th of an inch in diameter; which being poiſed with red lead, ſo as to make it but little heavier than pure ſweet Water, and thus fitted to one end of a fine balance, with a counterpoiſe at the other end; upon the leaſt addition of even the 2000th part of ſalt to a quantity of Water, half an inch of the neck will emerge above the water. Philoſ. Tranſ. num. 197.

Raiſing of WATER, in Hydraulics. The great uſe of raiſing Water by engines for the various purpoſes of life, is well known. Machines have in all ages been contrived with this view; a detail of the beſt of which, with the theory of their conſtruction, would be very curious and inſtructive. M. Belidor has executed this in part in his Architecture Hydraulique. Dr. Deſaguliers has alſo given a deſcription of ſeveral engines to raiſe Water, in his Courſe of Experimental Philoſophy, vol. 2; and there are ſeveral other ſmaller works of the ſame kind.

Engines for raiſing Water are either ſuch as throw it up with a great velocity, as in jets; or ſuch as raiſe it from one place to another by a gentle motion. For the general theory of theſe engines, ſee Bernoulli's Hydrodynamica.

Deſaguliers has ſettled the maximum of engines for raiſing water, thus: a man with the beſt Water engine cannot raiſe above one hogſhead of Water in a minute, 10 feet high, to hold it all day; but he can do almoſt twice as much for a minute or two.

WATER-*Spout*. See SPOUT.

WATER-*Wheel*, an engine for raiſing Water in great quantity out of a deep well, &c. See PERSIAN-*Wheel*.

WATER-*Works*. See *Raiſing of* WATER.

WAVE, in Phyſics, a volume of water elevated by the action of the wind &c, upon its ſurface, into a ſtate of fluctuation, and accompanied by a cavity. The extent from the bottom or loweſt point of one cavity, and acroſs the elevation, to the bottom of the next cavity, is the breadth of the Wave.

Waves are conſidered as of two kinds, which may be diſtinguiſhed from one another by the names of natural and accidental Waves. The natural Waves are thoſe which are regularly proportioned in ſize to the ſtrength of the wind which produces them. The accidental Waves are thoſe occaſioned by the wind's reacting upon itſelf by repercuſſion from hills or high ſhores, and by the daſhing of the Waves themſelves, otherwiſe of the natural kind, againſt rocks and ſhoals; by which means theſe Waves acquire an elevation much above what they can have in their natural ſtate.

Mr. Boyle proved, by numerous experiments, that the moſt violent wind never penetrates deeper than 6 feet into the water; and it ſeems a natural conſequence of this, that the water moved by it can only be elevated to the ſame height of 6 feet from the level of the ſurface in a calm; and theſe 6 feet of elevation being added to the 6 of excavation, in the part from whence that water ſo elevated was raiſed, ſhould give 12 feet for the utmoſt elevation of a Wave. This is a calculation that does great honour to its author; as many experiments and obſervations

obſervations have proved that it is very nearly true in deep ſeas, where the Waves are purely natural, and have no accidental cauſes to render them larger than their juſt proportion.

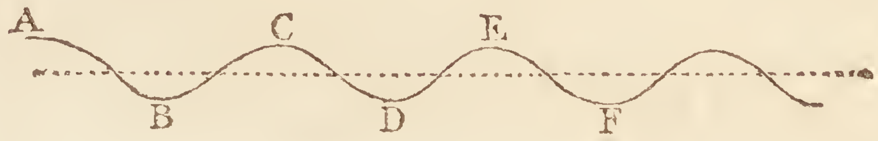
It is not to be underſtood however, that no Wave of the ſea can riſe more than 6 feet above its natural level in open and deep water; for Waves vaſtly higher than theſe are formed in violent tempeſts in the great ſeas. Theſe however are not to be accounted Waves in their natural ſtate, but as compound Waves formed by the union of many others; for in theſe wide plains of water, when one Wave is raiſed by the wind, and would elevate itſelf up to the exact height of 6 feet, and no more, the motion of the water is ſo great, and the ſucceſſion of Waves ſo quick, that while this is riſing, it receives into it ſeveral other Waves, each of which would have been at the ſame height with itſelf; theſe run into the firſt Wave one after another, as it is riſing; by which means its riſe is continued much longer than it naturally would have been, and it becomes accumulated to an enormous ſize. A number of theſe complicated Waves riſing together, and being continued in a long ſucceſſion by the continuation of the ſtorm, make the Waves ſo dangerous to ſhips, which the ſailors in their phraſe call mountains high.

Different Waves do not diſturb one another when they move in different directions. The reaſon is, that whatever figure the ſurface of the water has acquired by the motion of the Waves, there may in that be an elevation and depreſſion; as alſo ſuch a motion as is required in the motion of a Wave.

Waves are often produced by the motion of a tremulous body, which alſo expand themſelves circularly, though the body goes and returns in a right line; for the water which is raiſed by the agitation, deſcending, forms a cavity, which is every where ſurrounded with a riſing.

The Motion of the WAVES, makes an article in the Newtonian philoſophy; that author having explained their motions, and calculated their velocity from mathematical principles, ſimilar to the motion of a pendulum, and to the reciprocation of water in the two legs of a bent and inverted ſyphon or tube.

His propoſition concerning ſuch canal or tube is the 44th of the 2d book of his Principia, and is this: "If water aſcend and deſcend alternately in the erected legs of a canal or pipe; and a pendulum be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation, is equal to half the length of the water in the canal; then the water will aſcend and deſcend in the ſame times in which the pendulum oſcillates." The author hence infers, in prop. 45, that the velocity of Waves is in the ſubduplicate ratio of their breadths; and in prop. 46, he proceeds "To find the velocity of Waves," as follows: "Let a pendulum be conſtructed, whoſe length between the point of ſuſpenſion and the centre of oſcillation is equal to the breadth of the Waves; and in the time that the pendulum will perform one ſingle oſcillation, the Waves will advance forward nearly a ſpace equal to their breadth. That which I call the breadth of the Waves, is the tranſverſe meaſure lying between the deepeſt part of the hollows, or between the tops of the ridges. Let ABCDEF repreſent the ſurface of ſtagnant water aſcending and deſcending in ſucceſſive Waves; alſo let



A, C, E, &c, be the tops of the Waves; and B, D, F, &c, the intermediate hollows. Becauſe the motion of the Waves is carried on by the ſucceſſive aſcent and deſcent of the water, ſo that the parts of it, as A, C, E, &c, which are higheſt at one time, become loweſt immediately after; and becauſe the motive force, by which the higheſt parts deſcend and the loweſt aſcend, is the weight of the elevated water, that alternate aſcent and deſcent will be analogous to the reciprocal motion of the water in the canal, and obſerve the ſame laws as to the times of its aſcent and deſcent; and therefore (by prob. 44, above mentioned) if the diſtances between the higheſt places of the Waves A, C, E, and the loweſt B, D, F, be equal to twice the length of any pendulum, the higheſt parts A, C, E, will become the loweſt in the time of one oſcillation, and in the time of another oſcillation will aſcend again. Therefore between the paſſage of each Wave, the time of two oſcillations will intervene; that is, the Wave will deſcribe its breadth in the time that the pendulum will oſcillate twice; but a pendulum of 4 times that length, and which therefore is equal to the breadth of the Waves, will juſt oſcillate once in that time. *Q. E. I.*

"*Corol.* 1. Therefore Waves, whoſe breadth is equal to $39\frac{1}{8}$ inches, or $3\frac{25}{96}$ feet, will advance through a ſpace equal to their breadth in one ſecond of time; and therefore in one minute they will go over a ſpace of $195\frac{5}{8}$ feet; and in an hour a ſpace of 11737 feet, nearly, or 2 miles and almoſt a quarter.

"*Corol.* 2. And the velocity of greater or leſs Waves, will be augmented or diminiſhed in the ſubduplicate ratio of their breadth.

"Theſe things (Newton adds) are true upon the ſuppoſition, that the parts of water aſcend or deſcend in a right line; but in fact, that aſcent and deſcent is rather performed in a circle; and therefore I propoſe the time defined by this propoſition as only near the truth."

Stilling WAVES *by means of Oil.* This wonderful property, though well known to the Ancients, as appears from the writings of Pliny, was for many ages either quite unnoticed, or treated as fabulous by ſucceeding philoſophers. Of late it has, by means of Dr. Franklin, again attracted the attention of the learned; though it appears, from ſome anecdotes, that ſeafaring people have always been acquainted with it. In Martin's deſcription of the Weſtern Iſlands of Scotland, we have the following paſſage: "The ſteward of Kilda, who lives in Pabbay, is accuſtomed, in time of a ſtorm, to tie a bundle of puddings, made of the fat of ſea-fowl, to the end of his cable, and lets it fall into the ſea behind his rudder. This, he ſays, hinders the Waves from breaking, and calms the ſea." Mr. Pennant, in his Britiſh Zoology, vol. iv, under the article Seal,

Seal, takes notice, that when these animals are devouring a very oily fish, which they always do under water, the Waves above are remarkably smooth; and by this mark the fishermen know where to find them. Sir Gilbert Lawson, who served long in the army at Gibraltar, assured Dr. Franklin, that the fishermen in that place are accustomed to pour a little oil on the sea, in order to still its motion, that they may be enabled to see the oysters lying at its bottom, which are there very large, and which they take up with a proper instrument. A similar practice obtains among fishermen in various other parts, and Dr. Franklin was informed by an old sea-captain, that the fishermen of Lisbon, when about to return into the river, if they saw too great a surf upon the bar, would empty a bottle or two of oil into the sea, which would suppress the breakers, and allow them to pass freely.

The Doctor having revolved in his mind all these pieces of information, became impatient to try the experiment himself. At last having an opportunity of observing a large pond very rough with the wind, he dropped a small quantity of oil upon it. But having at first applied it on the lee-side, the oil was driven back again upon the shore. He then went to the windward side, and poured on about a tea-spoon full of oil; this produced an instant calm over a space several yards square, which spread amazingly, and extended itself gradually till it came to the lee-side; making all that quarter of the pond, perhaps half an acre, as smooth as glass. This experiment was often repeated in different places, and always with success. Our author accounts for it in the following manner:

"There seems to be no natural repulsion between water and air, to keep them from coming into contact with each other. Hence we find a quantity of air in water; and if we extract it by means of the air pump, the same water again exposed to the air will soon imbibe an equal quantity.—Therefore air in motion, which is wind, in passing over the smooth surface of water, may rub as it were upon that surface, and raise it into wrinkles; which, if the wind continues, are the elements of future Waves. The smallest Wave once raised does not immediately subside and leave the neighbouring water quiet; but in subsiding raises nearly as much of the water next to it, the friction of the parts making little difference. Thus a stone dropped into a pool raises first a single Wave round itself, and leaves it, by sinking to the bottom; but that first Wave subsiding raises a second, the second a third, and so on in circles to a great extent.

"A small power continually operating, will produce a great action. A finger applied to a weighty suspended bell, can at first move it but little; if repeatedly applied, though with no greater strength, the motion increases till the bell swings to its utmost height, and with a force that cannot be resisted by the whole strength of the arm and body. Thus the small first raised Waves being continually acted upon by the wind, are, though the wind does not increase in strength, continually increased in magnitude, rising higher and extending their bases, so as to include a vast mass of water in each Wave, which in its motion acts with great violence. But if there be a mutual repulsion between the particles of oil, and no attraction between oil and water, oil dropped on water will not be held together by adhesion to the spot whereon it falls; it will not be imbibed by the water; it will be at liberty to expand itself; and it will spread on a surface that, besides being smooth to the most perfect degree of polish, prevents, perhaps by repelling the oil, all immediate contact, keeping it at a minute distance from itself; and the expansion will continue, till the mutual repulsion between the particles of the oil is weakened and reduced to nothing by their distance.

"Now I imagine that the wind blowing over water thus covered with a film of oil cannot easily catch upon it, so as to raise the first wrinkles, but slides over it, and leaves it smooth as it finds it. It moves the oil a little indeed, which being between it and the water, serves it to slide with, and prevents friction, as oil does between those parts of a machine that would otherwise rub hard together. Hence the oil dropped on the windward side of a pond proceeds gradually to leeward, as may be seen by the smoothness it carries with it quite to the opposite side. For the wind being thus prevented from raising the first wrinkles that I call the elements of Waves, cannot produce Waves, which are to be made by continually acting upon and enlarging those elements; and thus the whole pond is calmed.

"Totally therefore we might suppress the Waves in any required place, if we could come at the windward place where they take their rise. This in the ocean can seldom if ever be done. But perhaps something may be done on particular occasions to moderate the violence of the Waves when we are in the midst of them, and prevent their breaking when that would be inconvenient. For when the wind blows fresh, there are continually rising on the back of every great Wave a number of small ones, which roughen its surface, and give the wind hold, as it were, to push it with greater force. This hold is diminished by preventing the generation of those small ones. And possibly too, when a Wave's surface is oiled, the wind, in passing over it, may rather in some degree press it down, and contribute to prevent its rising again, instead of promoting it.

"This, as mere conjecture, would have little weight, if the apparent effects of pouring oil into the midst of Waves were not considerable, and as yet not otherwise accounted for.

"When the wind blows so fresh, as that the Waves are not sufficiently quick in obeying its impulse, their tops being thinner and lighter, are pushed forward, broken, and turned over in a white foam. Common Waves lift a vessel without entering it; but these, when large, sometimes break above and pour over it, doing great damage.

"That this effect might in any degree be prevented, or the height and violence of Waves in the sea moderated, we had no certain account; Pliny's authority for the practice of seamen in his time being slighted. But discoursing lately on this subject with his excellency Count Bentinck of Holland, his son the honourable Captain Bentinck, and the learned professor Allemand (to all whom I showed the experiment of smoothing in a windy day the large piece of water at the head of the

green park), a letter was mentioned which had been received by the Count from Batavia, relative to the saving of a Dutch ship in a storm by pouring oil into the sea."

WAY *of a Ship*, is sometimes used for her wake or track. But more commonly the term is understood of the course or progress which she makes on the water under sail: thus, when she begins her motion, she is said to be *under Way*; when that motion increases, she is said to have *fresh Way* through the water; when she goes apace, they say *she has a good Way*; and the account of her rate of sailing by the log, they call, *keeping an account of her Way*. And because most ships are apt to fall a little to the leeward of their true course; it is customary, in casting up the log-board, to allow something for her *leeward Way*, or *leeway*. Hence also a ship is said to have *head-Way*, and *stern-Way*.

WAYWISER, an instrument for measuring the road, or distance travelled; called also PERAMBULATOR, and PEDOMETER. See these two articles.

Mr. Lovell Edgworth communicated to the Society of Arts, &c, an account of a Way-wiser of his invention; for which he obtained a silver medal. This machine consists of a nave, formed of two round flat pieces of wood, 1 inch thick and 8 inches in diameter. In each of the pieces there are cut eleven grooves, $\frac{5}{8}$ of an inch wide, and $\frac{3}{8}$ deep; and when the two pieces are screwed together, they enclose eleven spokes, forming a wheel of spokes, without a rim: the circumference of the wheel is exactly one pole; and the instrument may be easily taken to pieces, and put up in a small compass. On each of the spokes there is driven a ferril, to prevent them from wearing out; and in the centre of the nave, there is a square hole to receive an axle. Into this hole is inserted an iron or brass rod, which has the thread of a very fine screw worked upon it from one end to the other; upon this screw hangs a nut which, as the rod turns round with the wheel, advances towards the nave of the wheel or recedes from it. The nut does this, because it is prevented from turning round with the axle, by having its centre of gravity placed at some distance below the rod, so as always to hang perpendicularly like a plummet. Two sides of this screw are filed away flat, and have figures engraved upon them, to shew by the progressive motion of the nut, how many circumvolutions of the wheel and its axle have been made: on one side the divisions of miles, furlongs, and poles are in a direct order, and on the other side the same divisions are placed in a retrograde order.

If the person who uses this machine places it at his right hand side, holding the axle loosely in his hands, and walks forward, the wheel will revolve, and the nut advance from the extremity of the rod towards the nave of the wheel. When two miles have been measured, it will have come close to the wheel. But to continue this measurement, nothing more is necessary than to place the wheel at the left hand of the operator; and the nut will, as he continues the course, recede from the axletree, till another space of two miles is measured.

It appears from the construction of this machine, that it operates like circular compasses; and does not, like the common wheel Way-wiser, measure the surface of every stone and molehill, &c, but passes over most of the obstacles it meets with, and measures the chords only, instead of the arcs of any curved surfaces upon which it rolls.

WEATHER, denotes the state or disposition of the atmosphere, with regard to heat and cold, drought and moisture, fair or foul, wind, rain, hail, frost, snow, fog, &c. See ATMOSPHERE, HAIL, HEAT, FROST, RAIN, &c.

There does not seem in all philosophy any thing of more immediate concernment to us, than the state of the Weather; as it is in, and by means of the atmosphere, that all plants are nourished, and all animals live and breathe; and as any alterations in the density, heat, purity, &c, of that, must necessarily be attended with proportionable ones in the state of these.

The great, but regular alterations, a little change of Weather makes in many parts of inanimate matter, every person knows, in the common instance of barometers, thermometers, hygrometers, &c; and it is owing partly to our inattention, and partly to our unequal and intemperate course of life, that we also, like many other animals, do not feel as great and as regular ones in the tubes, chords, and fibres of our own bodies.

To establish a proper theory of the Weather, it would be necessary to have registers carefully kept in divers parts of the globe, for a long series of years; from whence we might be enabled to determine the directions, breadth, and bounds of the winds, and of the weather they bring with them; with the correspondence between the Weather of divers places, and the difference between one sort and another at the same place. We might thus in time learn to foretell many great emergencies; as, extraordinary heats, rains, frosts, droughts, dearths, and even plagues, and other epidemical diseases, &c.

It is however but very few, and partial registers or accounts of the Weather, that have been kept. The Royal Society, the French Academy, and a few particular philosophers, have at times kept such registers as their fancies have dictated, but at no time a regular and correspondent series in many different places, at the same time, followed with particular comparisons and deductions from the whole, &c. The most of what has been done in this way, is as follows: The volumes of the Philosophical Transactions from year to year; the same, for instructions and examples pertaining to the subject, vol. 65, part 2, art. 16; Eras. Bartholin has observations of the Weather for every day in the year 1671: Mr. W. Merle made the like at Oxford, for 7 years: Dr. Plot did the same at the same place, for the year 1684: Mr. Hillier, at Cape Corse, for the years 1686 and 1687: Mr. Hunt and others at Gresham College, for the years 1695 and 1696: Dr. Derham at Upminster in Essex, for the years 1691, 1692, 1697, 1698, 1699, 1703, 1704, 1705: Mr. Townley, in Lancashire, in 1697, 1698: Mr. Cunningham, at Emin in China, for the years 1698, 1699, 1700, 1701: Mr. Locke, at Oats in Essex, 1692; Dr. Scheuchzer, at Zurich, 1708; and Dr. Tilly, at Pisa, the same year: Professor Toaldo, at Padua, for many years: Mr. T. Barker, at Lyndon, in Rutland, for many years in the Philos. Trans.: Mr. Dalton for Kendal, and Mr. Crosthwaite for Keswick, in the years

1788,

1788, 1789, 1790, 1791, 1792, &c; and several others. The register now kept, for many years, in the Philos. Trans. contains an account, two times every day, of the thermometer, barometer, hygrometer, quantity of rain, direction and strength of the wind, and appearance of the atmosphere, as to fair, cloudy, foggy, rainy, &c. And if similar registers were kept in many other parts of the globe, and printed in such-like public Transactions, they might readily be consulted, and a proper use made of them, for establishing this science on the true basis of experiment.

From many experiments, some general observations have been made, as follow: That barometers generally rise and fall together, even at very distant places, and a consequent conformity and similarity of Weather; but this is the more uniformly so, as the places are nearer together, as might be expected. That the variations of the barometer are greater, as the places are nearer the pole; thus, for instance, the mercury at London has a greater range by 2 or 3 lines than at Paris; and at Paris, a greater than at Zurich; and at some places near the equator, there is scarce any variation at all. That the rain in Switzerland and Italy is much greater in quantity, for the whole year, than in Essex; and yet the rains are more frequent, or there are more rainy days, in Essex, than at either of those places. That cold contributes greatly to rain; and this apparently by condensing the suspended vapours, and so making them descend: thus, very cold months, or seasons, are commonly followed immediately by very rainy ones; and cold summers are always wet ones. That high ridges of mountains, as the Alps, and the snows with which they are covered, not only affect the neighbouring places by the colds, rains, vapours, &c, which they produce; but even distant countries, as England, often partake of their effects. See a collection of ingenious and meteorological observations and conjectures, by Dr. Franklin, in his Experiments, &c, pa. 182, &c. Also a Meteorological Register kept at Mansfield Woodhouse, from 1784 to 1794, Nottingham 1795, 8vo; and Kirwin's ingenious papers on this subject in the Transactions of the Irish Academy, vol. 5. See also the articles EVAPORATION, RAIN, and WIND.

Other Prognostics and Observations, are as follow:

That a thick dark sky, lasting for some time, without either sun or rain, always becomes first fair, and then foul, i. e. it changes to a fair clear sky, before it turns to rain. And the reason is obvious: the atmosphere is replete with vapours which, though sufficient to reflect and intercept the sun's rays from us, yet want density to descend; and while the vapours continue in the same state, the Weather will do so too: accordingly, such Weather is commonly attended with moderate warmth, and with little or no wind to disturb the vapours, and a heavy atmosphere to sustain them; the barometer being commonly high: but when the cold approaches, and by condensing the vapours drives them into clouds or drops, then way is made for the sun beams; till the same vapours, by farther condensation, be formed into rain, and fall down in drops.

That a change in the warmth of the Weather is followed by a change in the wind. Thus, the northerly and southerly winds, though commonly accounted the *causes* of cold and warm Weather, are really the *effects* of the cold or warmth of the atmosphere; of which Dr. Derham assures us he had so many confirmations, that he makes no doubt of it. Thus, it is common to see a warm southerly wind suddenly changed to the north, by the fall of snow or hail; or to see the wind, in a cold frosty morning, north, when the sun has well warmed the air, wheel towards the south; and again turn northerly or easterly in the cold evening.

That most vegetables expand their flowers and down in sunshiny Weather: and towards the evening, and against rain, close them again; especially at the beginning of their flowering, when their seeds are tender and sensible. This is visible enough in the down of Dandelion, and other downs; and eminently so in the flowers of pimpernel; the opening and shutting of which make what is called the countryman's *Weather-wiser*, by which he foretels the Weather of the following day. The rule is, when the flowers are close shut up, it betokens rain, and foul Weather; but when they are spread abroad, fair Weather.

The stalk of trefoil, lord Bacon observes, swells against rain, and grows more upright: and the like may be observed, though less sensibly, in the stalks of most other plants. He adds, that in the stubble fields there is found a small red flower, called by the country people pimpernel, which opening in a morning, is a sure indication of a fine day.

It is very conceivable that vegetables should be affected by the same causes as the Weather, as they may be considered as so many hygrometers and thermometers, consisting of an infinite number of tracheæ, or air-vessels; by which they have an immediate communication with the air, and partake of its moisture, heat, &c.

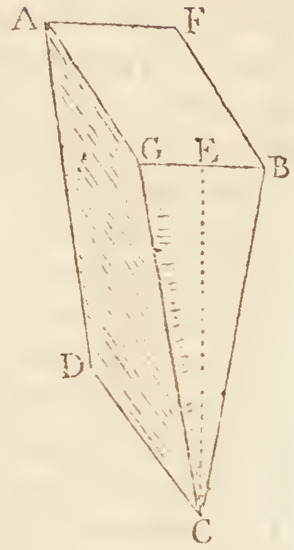
Hence it is, that all wood, even the hardest and most solid, swells in moist Weather; the vapours easily insinuating into the pores, especially of the lighter and drier kinds. And hence is derived a very extraordinary use of wood, viz, for breaking rocks or millstones. The method at the quarries is this: Having cut a rock into the form of a cylinder, the workmen divide it into several thinner cylinders, of horizontal courses, by making holes at proper distances round the great one; into these holes they drive pieces of sallow wood, dried in an oven; these in moist Weather, imbibing the humidity from the air, swell, and acting like wedges they break or cleave the rock into several flat stones. And, in like manner, to separate large blocks of stone in the quarry, they wedge such pieces of wood into holes, forming the block into the intended shape, and then pour water upon the wedges, to produce the effect more immediately.

WEATHER-*Glasses*, are instruments contrived to shew the state of the atmosphere, as to heat, cold, moisture, weight, &c; and so to measure the changes that take place in those respects; by which means we are enabled to predict the alteration of Weather, as to rain, wind, frost, &c.

Under the class of Weather-glasses, are comprehended barometers, thermometers, hygrometers, manometers, and anemometers.

WEDGE,

WEDGE, in Geometry, is a ſolid having a rectangular baſe, and two of its oppoſite ſides ending in an acies or edge. Thus, AB is the rectangular baſe; and DC the edge; a perpendicular CE, from the edge to the baſe, is the height of the Wedge. When the length of the edge DC is equal to the length of the baſe BF, which is the moſt common form of it, the Wedge is equal to half a rectangular priſm of the ſame baſe AB and height EC; or it is then a whole triangular priſm, having the triangle BCG for its baſe, and AG or DC for its height. If the edge be more or leſs than AG, its ſolid content will be more or leſs. But, in all caſes of the Wedge, the following is a general rule for finding the content of it, viz,



To twice the length of the baſe add the length of the edge, multiply the ſum by the breadth of the baſe, and the product by the height of the Wedge; then $\frac{1}{6}$ of the laſt product will be the ſolid content.

That is, $\overline{2AG + DC} \times AF \times \frac{1}{6} EC =$ the content. See this rule demonſtrated, and illuſtrated with examples, in my Menſuration, p. 191, 2d edition.

WEDGE, in Mechanics, one of the five mechanical powers, or ſimple engines; being a geometrical Wedge, or very acute triangular priſm, applied to the ſplitting of wood, or rocks, or raiſing great weights.

The Wedge is made of iron, or ſome other hard matter, and applied to the raiſing of vaſt weights, or ſeparating large or very firm blocks of wood or ſtone, by introducing the thin edge of the Wedge, and driving it in by blows ſtruck upon the back by hammers or mallets.

The Wedge is the moſt powerful of all the ſimple machines, having an almoſt unlimited and double advantage over all the other ſimple mechanical powers; both as it may be made vaſtly thin, in proportion to its height; in which conſiſts its own natural power; and as it is urged by the force of percuſſion, or of ſmart blows, which is a force incomparably greater than any mere dead weight or preſſure, ſuch as is employed upon other machines. And accordingly we find it produces effects vaſtly ſuperior to thoſe of any other power whatever; ſuch as the ſplitting and raiſing the largeſt and hardeſt rocks; or even the raiſing and lifting the largeſt ſhip, by driving a Wedge below it; which a man can do by the blow of a mallet: and thus the ſmall blow of a hammer, on the back of a Wedge, appears to be incomparably greater than any mere preſſure, and will overcome it.

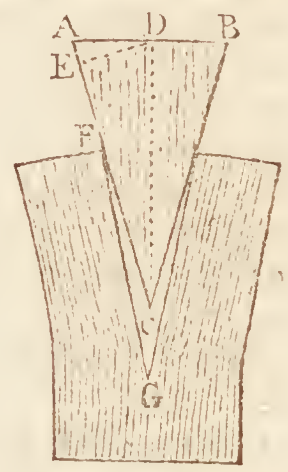
To the Wedge may be referred all edge-tools, and tools that have a ſharp point, in order to cut, cleave, ſlit, ſplit, chop, pierce, bore, or the like; as knives, hatchets, ſwords, bodkins, &c.

In the Wedge, the friction againſt the ſides is very great, at leaſt equal to the force to be overcome; becauſe the Wedge retains any poſition to which it is driven; and therefore the reſiſtance is at leaſt doubled by the friction.

Authors have been of various opinions concerning the principle from whence the Wedge derives its power. Ariſtotle conſiders it as two levers of the firſt kind, inclined towards each other, and acting oppoſite ways. Guido Ubaldi, Merſenne, &c, will have them to be levers of the ſecond kind. But De Lanis ſhews, that the Wedge cannot be reduced to any lever at all. Others refer the Wedge to the inclined plane. And others again, with De Stair, will hardly allow the Wedge to have any force at all in itſelf; aſcribing much the greateſt part to the mallet which drives it.

The doctrine of the force of the Wedge, according to ſome writers, is contained in this propoſition: "If a power directly applied to the head of a Wedge, be to the reſiſtance to be overcome, as the breadth of the back GB, is to the height EC; then the power will be equal to the reſiſtance; and if increaſed, it will overcome it."

But Deſaguliers has proved that, when the reſiſtance acts perpendicularly againſt the ſides of the Wedge, the power is to the whole reſiſtance, as the thickneſs of the back is to the length of both the ſides taken together. And the ſame proportion is adopted by Wallis (Op. Math. vol. 1, p. 1016), Keill (Intr. ad Ver. Phyſ.), Graveſande (Elem. Math. Lib. 1, cap. 14), and by almoſt all the modern mathematicians. Graveſande indeed diſtinguiſhes the mode in which the Wedge acts, into two caſes, one in which the parts of a block of wood, &c, are ſeparated farther than the edge has penetrated to, and the other in which they have not ſeparated farther: In his Scholium de Ligno findendo (ubi ſupra), he obſerves, that when the parts of the wood are ſeparated before the Wedge, the equilibrium will be when the force by which it is puſhed in, is to the reſiſtance of the wood, as the line DE drawn from the middle of the baſe to the ſide of the Wedge but perpendicular to the ſeparated ſide of the wood continued FG, is to the height of the Wedge DC; but when the parts of the wood are ſeparated no farther than the Wedge is driven in, the equilibrium will be, when the power is to the reſiſtance, as the half baſe AD, is to its ſide AC.



Mr. Ferguſon, in eſtimating the proportion of equilibrium in the two caſes laſt mentioned by Graveſande, agrees with this author, and other modern philoſophers, in the latter caſe; but in the former he contends, that when the wood cleaves to any diſtance before the Wedge, as it generally does, then the power impelling the Wedge, will be to the reſiſtance of the wood, as half its thickneſs, is to the length of either ſide of the cleft, eſtimated from the top or acting part of the Wedge: for, ſuppoſing the Wedge to be lengthened down to the bottom of the cleft, the power will be to the reſiſtance, as half the thickneſs of the Wedge is to the length of either of its ſides. See Ferguſon's Lect. p. 40, &c, 4to. See alſo Deſagu. Exp. Phil. vol. 1, p. 107; and Ludlam's Eſſay on the Power of the Wedge, printed in 1770; &c.

The generally acknowledged property of the Wedge, and the ſimpleſt way of demonſtrating it, ſeem to be the following: When a Wedge is kept in equilibrio, the power acting againſt the back, is to the force acting prepen-

perpendicularly against either side, as the breadth of the back AB, is to the length of the side AC or BC.—*Demonstra.* For any three forces which sustain one another in equilibrio, are as the corresponding sides of a triangle that are drawn perpendicular to the directions in which the forces act. But AB is perpendicular to the force acting on the back, to drive the Wedge forward; and the sides AC and BC are perpendicular to the forces acting upon them; therefore the three forces are as the said lines AB, AC, BC.

Hence, the thinner a Wedge is, the greater is its effect, in splitting any body, or in overcoming any resistance against the side of the Wedge.

WEDNESDAY, the 4th day of the week, formerly consecrated by the inhabitants of the northern nations to Woden or Oden; who, being reputed the author of magic and inventor of all the arts, was thought to answer to the Mercury of the Greeks and Romans, in honour of whom the same day was by them called *dies Mercurii*; and hence it is denoted by astronomers by the character of Mercury ☿.

WEEK, a division of time that comprises seven days.

The origin of this division of Weeks, or of computing time by sevenths, is much controverted. It has often been thought to have taken its rise from the four quarters or intervals of the moon, between her changes of phases, which, being about 7 days distant, gave occasion to the division: but others more probably from the seven planets.

Be this however as it may, the division is certainly very ancient. The Syrians, Egyptians, and most of the oriental nations, appear to have used it from the earliest ages: though it did not get footing in the west till brought in by christianity. The Romans reckoned their days not by sevenths, but by ninths; and the ancient Greeks by decads, or tenths; in imitation of which the new French calendar seems to have been framed.

The Jews divided their time by Weeks, of 7 days each, as prescribed by the law of Moses; in which they were appointed to work 6 days, and to rest the 7th, in commemoration of the creation, which being effected in 6 days, God rested on the 7th.

Some authors will even have the use of Weeks, among the other eastern nations, to have proceeded from the Jews; but with little appearance of probability. It is with better reason that others suppose the use of Weeks, among the eastern nations, to be a remnant of the tradition of the creation, which they had still retained with divers others; or else from the number of the planets.

The Jews denominated the days of the Week, the first, second, third, fourth, and fifth; and the sixth day they named the preparation of the sabbath, or 7th day, which answered to our Saturday. And the like method is still kept up by the christian Arabs, Persians, Ethiopians, &c.

The ancient heathens denominated the days of the Week from the seven planets; which names are still mostly retained among the christians of the west; thus, the first day was called *dies solis*, *sun-day*; the 2d *dies lunæ*, *moon-day*; &c; a practice the more natural on Dion's principle, that the Egyptians took the division of the Week itself from the seven planets.

In fact, the true reason for these denominations seems to be founded in astrology. For the astrologers distributing the government and direction of all the hours in the Week among the seven planets, ♄ ♃ ♂ ☉ ♀ ☿ ☽, so as that the government of the first hour of the first day fell to Saturn, that of the second day to Jupiter, &c, they gave each day the name of the planet which, according to their doctrine, presided over the first hour of it, and that according to the order above stated. So that the order of the planets in the Week, bears little relation to that in which they follow in the heavens; the former being founded on an imaginary power each planet has, in its turn, on the first hour of each day.

Dion Cassius gives another reason for the denomination, drawn from the celestial harmony. For it being observed, that the harmony of the diatessaron, which consists in the ratio of 4 to 3, is of great force and effect in music; it was judged meet to proceed directly from Saturn to the Sun; because, according to the old system, there are three planets between Saturn and the Sun, and 4 from the Sun to the Moon.

Our Saxon ancestors, before their conversion to Christianity, named the seven days of the Week from the Sun and Moon and some of their deified heroes, to whom they were peculiarly consecrated, and representing the ancient gods or planets; which names we received and still retain: Thus, Sunday was devoted to the Sun; Monday to the Moon; Tuesday to Tuisco; Wednesday to Woden; Thursday to Thor, the thunderer; Friday to Friga or Friya or Fræa, the wife of Thor; and Saturday to Seater. And nearly according to this order, the modern astronomers express the days of the Week by the seven planets as below:

☉ Sunday
☾ Monday
♂ Tuesday
☿ Wednesday
♃ Thursday
♀ Friday
♄ Saturday.

In the same order and number also do these obtain in the Hindoo days of the Week. See Kindersley's Specimens of Hindoo Literature, just published, 8vo.

WEIGH, WAY, or WEY, a weight of cheese, wool, &c, containing 256 pounds avoirdupois. Of corn, the Weigh contains 40 bushels; of barley or malt, 6 quarters.

WEIGHT, or *Gravity*, in Physics, a quality in natural bodies, by which they tend downwards toward the centre of the earth. See GRAVITY.

Weight, like gravity, may be distinguished into *absolute*, *specific*, and *relative*.

Newton demonstrates, 1. That the Weights of all bodies, at equal distances from the centre of the earth, are directly proportional to the quantities of matter that each contains: Whence it follows, that the Weights of bodies have no dependence on their shapes or textures; and that all spaces are not equally full of matter.

2. On different parts of the earth's surface, the Weight of the same body is different; owing to the spheroidal figure of the earth, which causes the body on the surface to be nearer the centre in going from the equator toward the poles: and the increase in the Weight is nearly

nearly in proportion to the versed sine of double the latitude; or, which is the same thing, to the square of the right sine of the latitude: the Weight at the equator to that at the pole, being as 229 to 230; or the whole increase of Weight from the equator to the pole, is the 229th part of the former.

3. That the Weights of the same body, at different distances above the earth, are inversely as the squares of the distances from the centre. So that, a body at the distance of the moon, which is 60 semidiameters from the earth's centre, would weigh only the 3600th part of what it weighs at the earth's surface.

4. That at different distances within the earth, or below the surface, the weights of the same body are directly as the distances from the earth's centre: so that, at half way toward the centre, a body would weigh but half as much, and at the very centre it would be no Weight at all.

5. A body immersed in a fluid, which is specifically lighter than itself, loses so much of its Weight, as is equal to the Weight of a quantity of the fluid of the same bulk with itself. Hence, a body loses more of its weight in a heavier fluid than in a lighter one; and therefore it weighs more in a lighter fluid than in a heavier one.

The Weight of a cubic foot of pure water, is 1000 ounces, or $62\frac{1}{2}$ pounds, avoirdupois. And the Weights of the cubic foot of other bodies, are as set down under the article *Specific* GRAVITY.

In the Philos. Trans. (number 458, p. 457 &c) is contained some account of the analogy between English Weights and measures, by Mr. Barlow. He states, that anciently the cubic foot of water was assumed as a general standard for liquids. This cubic foot, of $62\frac{1}{2}$ lb, multiplied by 32, gives 2000, the weight of a ton: and hence 8 cubic feet of water made a hogshead, and 4 hogsheads a tun, or ton, in capacity and denomination, as well as Weight.

Dry measures were raised on the same model. A bushel of wheat, assumed as a general standard for all sorts of grain, also weighed $62\frac{1}{2}$lb. Eight of these bushels make a quarter, and 4 quarters, or 32 bushels, a ton Weight. Coals were sold by the chaldron, supposed to weigh a ton, or 2000 pounds; though in reality it weighs perhaps upwards of 3000 pounds.

Hence a ton in Weight is the common standard for liquids, wheat, and coals. Had this analogy been adhered to, the confusion now complained of would have been avoided.—It may reasonably be supposed that corn and other commodities, both dry and liquid, were first sold by Weight; and that measures, for convenience, were afterwards introduced, as bearing some analogy to the Weights before used.

WEIGHT, *Pondus*, in Mechanics, denotes any thing to be raised, sustained, or moved by a machine; or any thing that in any manner resists the motion to be produced.

In all machines, there is a natural and fixed ratio between the Weight and the moving power: and if they be such as to balance each other in equilibrio, and then the machine be put in motion by any other force; the Weight and power will always be reciprocally as the velocities of them, or of their centres of gravity; or their momentums will be equal, that is, the product of the Weight multiplied by its velocity, will be equal to the product of the power multiplied by its velocity.

WEIGHT, in Commerce, denotes a body of a known Weight, appointed to be put into a balance against other bodies, whose Weight is required to be known. These Weights are usually of lead, iron, or brass; though in several parts of the East Indies common flints are used; and in some places a sort of little beans.

The diversity of Weights, in all nations, and at all times, makes one of the most perplexing circumstances in commerce, &c. And it would be a very great convenience if all nations could agree upon a universal standard, and system, both of Weights and measures.

Weights may be distinguished into *ancient* and *modern*, *foreign* and *domestic*.

Modern WEIGHTS, *used in the several parts of Europe, and the Levant.*

English WEIGHTS. By the 27th chapter of Magna Charta, the Weights are to be the same all over England: but for different commodities there are two different sorts, viz, *troy Weight*, and *averdupois Weight*.

The origin from which both of these are raised, is the grain of wheat, gathered in the middle of the ear;

32 of these, well dried, made one pennyweight,
20 pennyweights - - - - - one ounce, and
12 ounces - - - - - - - - one pound troy;

by Stat. 51 Hen. III; 31 Edw. I; 12 Henry VII.

A learned writer has shewn that, by the laws of assize, from William the Conqueror to the reign of Henry VII, the legal pound Weight contained a pound of 12 ounces, raised from 32 grains of wheat; and the legal gallon measure contained 8 of those pounds of wheat, 8 gallons making the bushel, and 8 bushels the quarter.

Henry VII. altered the old English Weight, and introduced the troy pound in its stead, being 3 quarters of an ounce only heavier than the old Saxon pound, or 1-16th heavier. The first statute that directs the use of the averdupois Weight, is that of 24 Henry VIII; and the particular use to which this Weight is thus directed, is simply for weighing butcher's meat in the market; though it is now used for weighing all sorts of coarse and large articles. This pound contains 7000 troy grains; while the troy pound itself contains only 5760 grains, and the old Saxon pound Weight but 5400 grains. Philos. Trans. vol. 65, art. 3.

Hence there are now in common use in England, two different Weights, viz, troy Weight, and averdupois Weight, the former being employed in weighing such fine articles as jewels, gold, silver, silk, liquors, &c; and the latter for coarse and heavy articles, as bread, corn, flesh, butter, cheese, tallow, pitch, tar, iron, copper, tin, &c. and all grocery wares. And Mr. Ward supposes that it was brought into use from this circumstance, viz, as it was customary to allow larger Weight, of such coarse articles, than the law had expressly enjoined, and this he observes happened to be a 6th part more. Apothecaries buy their drugs by averdupois Weight, but they compound them by troy Weight, though under some little variation of name and divisions.

The troy or trone pound Weight in Scotland, which by ſtatute is to be the ſame as the French pound, is commonly ſuppoſed equal to $15\frac{3}{4}$ Engliſh troy ounces, or 7560 grains; but by a mean of the ſtandards kept by the dean of gild of Edinburgh, it weighs $7599\frac{1}{16}$ or 7600 grains nearly.

The following tables ſhew the diviſions of the troy and averdupois Weights.

Table of Troy Weight, as uſed,

1. By the Goldſmiths, &c.

Grains	Pennywt.	Ounce	Pound
24 =	1 dwt.		
480 =	20 =	1 oz.	
5760 =	240 =	12 =	1 lb.

2. By the Apothecaries.

Grains	Scruples	Drams	Ounces	Pound
20 =	1 ℈			
60 =	3 =	1 ʒ		
480 =	24 =	8 =	1 ℥	
5760 =	288 =	96 =	12 =	1 lb.

Table of Averdupois Weight.

Drams	Ounces	Pounds	Quarters	Hund. wt.	Ton
16 =	1				
256 =	16 =	1			
7168 =	448 =	28 =	1		
28672 =	1792 =	112 =	4 =	1	
573440 =	35840 =	2240 =	80 =	20 =	1

Mr. Ferguſon (Lect. on Mech. p. 100, 4to) gives the following compariſon between troy and averdupois Weight.

175 troy pounds are equal to 144 averdup. pounds.
175 troy ounces are equal to 192 averdup. ounces.
1 troy pound contains 5760 grains.
1 averdupois pound contains 7000 grains.
1 averdupois ounce contains $437\frac{1}{2}$ grains.
1 averdupois dram contains 27·34375 grains.
1 troy pound contains 13 oz. 2·651428576 drams averdupois
1 averdup. lb. contains 1 lb 2 oz 11 dwts 16 gr troy

The moneyers, jewellers, &c, have a particular claſs of Weights, for gold and precious ſtones, viz, *carat* and *grain*; and for ſilver, the *pennyweight* and *grain*. The moneyers have alſo a peculiar ſubdiviſion of the troy grain: thus, dividing

the grain into 20 mites
the mite into 24 droits
the droit into 20 periots
the periot into 24 blanks.

The dealers in wool have likewiſe a particular ſet of Weights; viz, the *ſack*, *weigh*, *tod*, *ſtone*, and *clove*, the proportions of which are as below: viz,

the ſack containing	2	weighs
the weigh - - - -	$6\frac{1}{2}$	tods
the tod - - - - - -	2	ſtones
the ſtone - - - - -	2	cloves
the clove - - - - -	7	pounds.

Alſo 12 ſacks make a laſt or 4368 pounds.

Farther,
56 lb of old hay, or 60 lb new hay, make a truſs.
40 lb of ſtraw make a truſs.
36 truſſes make a load, of hay or ſtraw.
14 lb make a ſtone.
5 lb of glaſs a ſtone.

French WEIGHTS. The common or Paris pound Weight, is to the Engliſh troy pound, as 21 to 16, and to the averdupois pound as 27 to 25; it therefore contains 7560 troy grains; and it is divided into 16 ounces like the pound averdupois, but more particularly thus: the pound into 2 *marcs*; the marc into 8 *ounces*; the ounce into 8 *gros*, or *drams*; the groſs or dram into 3 deniers, Paris ſcruples or pennyweights; and the pennyweight into 24 *grains*; the grain being an equivalent to a grain of wheat. So that the Paris ounce contains $472\frac{1}{2}$ troy grains, and therefore it is to the Engliſh troy ounce as 63 to 64. But in ſeveral of the French provinces, the pound is of other different Weights. A *quintal* is equal to 100 pounds.

The Weights above enumerated under the two articles of Engliſh and French Weights, are the ſame as are uſed throughout the greateſt part of Europe; only under ſomewhat different names, diviſions, and proportions. And beſides, particular nations have alſo certain Weights peculiar to themſelves, of too little conſequence here to be enumerated. But to ſhew the proportion of theſe ſeveral Weights to one another, there may be here added a reduction of the divers pounds in uſe throughout Europe, by which the other Weights are eſtimated, to one ſtandard pound, viz, the pound of Amſterdam, Paris, and Bourdeaux; as they were accurately calculated by M. Ricard, and publiſhed in the new edition of his Traité de Commerce, in 1722.

Proportion of the WEIGHTS *of the chief Cities in Europe, to that of Amſterdam.*

100 pounds of Amſterdam are equal to

lbs	of	lbs	of
108	Alicant	100	Bilboa
105	Antwerp	105	Bois le Duc
120	Archangel, or 3 poedes	151	Bologna
		100	Bourdeaux
105	Arſchot	104	Bourg en Breſſe
120	Avignon	103	Bremen
98	Baſil	125	Breſlaw
100	Bayonne	105	Bruges
166	Bergamo	105	Bruſſels
97	Berg. op Zoom	105	Cadiz
$95\frac{1}{4}$	Bergen, Norw.	105	Cologne
111	Bern	$107\frac{1}{2}$	Copenhagen
100	Beſançon	87	Conſtantinople

WEIGHTS *continued.*

100 pounds of Amsterdam are equal to

113½ lbs	of Dantzic	154 lbs	of Messina
100	Dort	168	Milan
97	Dublin	120	Moutpelier
97	Edinburgh	125	Muscovy
143	Florence	100	Nantes
98	Franckfort, sur Maine	100	Nancy
		169	Naples
105	Gaunt	98	Nuremberg
89	Geneva	100	Paris
163	Genoa	112½	Revel
102	Hamburgh	109	Riga
125	Koningsberg	100	Rochel
105	Leipsic	146	Rome
106	Leyden	100	Rotterdam
143	Leghorn	96	Rouen
105½	Liege	100	S. Malo
106	Lisbon	100	S. Sebastian
114	Lisle	158½	Saragosa
109	London, averdupois	100	Seville
		114	Smyrna
105	Louvain	110	Stetin
105	Lubeck	81	Stockholm
141½	Lucca	118	Tholouse
116	Lyons	151	Turin
114	Madrid	158½	Valencia
105	Malines	182	Venice.
123½	Marseilles		

Ancient WEIGHTS.

1. The Weights of the ancient Jews, reduced to the English troy Weights, will stand as below:

	lb	oz	dwt	gr
Shekel - - - -	0	0	9	2 $\frac{4}{7}$
Manch - - - -	2	3	6	10 $\frac{2}{7}$
Talent - - - -	113	10	1	10 $\frac{2}{7}$

2. Grecian and Roman Weights, reduced to English troy Weight, are as in the following table:

	lb	oz	dwt	gr
Lentes - - - -	0	0	0	0 $\frac{85}{112}$
Siliquæ - - - -	0	0	0	3 $\frac{1}{28}$
Obolus - - - -	0	0	0	9 $\frac{3}{28}$
Scriptulum - -	0	0	0	18 $\frac{3}{14}$
Drachma - - -	0	0	2	6 $\frac{5}{14}$
Sextula - - - -	0	0	3	0 $\frac{6}{7}$
Sicilicus - - - -	0	0	4	13 $\frac{2}{7}$
Duella - - - - -	0	0	6	1 $\frac{5}{7}$
Uncia - - - - -	0	0	18	5 $\frac{1}{7}$
Libra - - - - -	0	10	18	13 $\frac{5}{7}$

The Roman ounce is the English averdupois ounce, which they divide into 7 denarii, as well as 8 drachms: and as they reckoned their denarius equal to the Attic drachm, this will make the Attic Weights one-eighth heavier than the correspondent Roman Weights. Arbuth.

Regulation of WEIGHTS *and Measures.* This is a branch of the king's prerogative. For the public convenience, these ought to be universally the same throughout the nation, the better to reduce the prices of articles to equivalent values. But as Weight and measure are things in their nature arbitrary and uncertain, it is necessary that they be reduced to some fixed rule or standard. It is however impossible to fix such a standard by any written law or oral proclamation; as no person can, by words only, give to another an adequate idea of a pound Weight, or foot-rule. It is therefore expedient to have recourse to some visible, palpable, material standard; by forming a comparison with which, all Weights and measures may be reduced to one uniform size. Such a standard was anciently kept at Winchester: and we find in the laws of king Edgar, near a century before the conquest, an injunction that that measure should be observed throughout the realm.

Most nations have regulated the standard of measures of length from some parts of the human body; as the palm, the hand, the span, the foot, the cubit, the ell (*ulna* or arm), the pace, and the fathom. But as these are of different dimensions in men of different proportions, ancient historians inform us, that a new standard of length was fixed by our king Henry the first; who commanded that the *ulna* or ancient ell, which answers to the modern yard, should be made of the exact length of his own arm.

A standard of long measure being once gained, all others are easily derived from it; those of greater length by multiplying that original standard, those of less by dividing it. Thus, by the statute called *compositio ulnarum et perticarum*, 5½ yards make a perch; and the yard is subdivided into 3 feet, and each foot into 12 inches; which inches will be each of the length of 3 barley corns. But some, on the contrary, derive all measures, by composition, from the barley corn.

Superficial measures are derived by squaring those of length; and measures of capacity by cubing them.

The standard of Weights was originally taken from grains or corns of wheat, whence our lowest denomination of Weights is still called a *grain*; 32 of which are directed, by the statute called *compositio mensurarum*, to compose a pennyweight, 20 of which make an ounce, and 12 ounces a pound, &c.

Under king Richard the first it was ordained, that there should be only one Weight and one measure throughout the nation, and that the custody of the assize or standard of Weights and measures, should be committed to certain persons in every city and borough; from whence the ancient office of the king's ulnager seems to have been derived. These original standards were called *pondus regis*, and *mensura domini regis*, and are directed by a variety of subsequent statutes to be kept in the exchequer chamber, by an officer called the *clerk of the market*, except the wine gallon, which is committed to the city of London, and kept in Guildhall.

The Scottish standards are distributed among the oldest boroughs. The elwand is kept at Edinburgh, the pint at Stirling, the pound at Lanark, and the firlot at Linlithgow.

The two principal Weights established in Great Britain, are troy Weight, and avoirdupois Weight, as

as before mentioned. Under the head of the former it may farther be added, that

A carat is a Weight of 4 grains; but when the term is applied to gold, it denotes the degree of fineness. Any quantity of gold is supposed divided into 24 parts. If the whole mass be pure gold, it is said to be 24 carats fine; if there be 23 parts of pure gold, and one part of alloy or base metal, it is said to be 23 carats fine, and so on.

Pure gold is too soft to be used for coin. The standard coin of this kingdom is 22 carats fine. A pound of standard gold is coined into $44\frac{1}{2}$ guineas, and therefore every guinea should weigh 5 dwts $9\frac{4}{89}$ grains.

A pound of silver for coin contains 11 oz 2 dwts pure silver, and 18 dwts alloy: and standard silver-plate, 11 ounces pure silver, with 1 ounce alloy. A pound of standard silver is coined into 62 shillings; and therefore the Weight of a shilling should be 3 dwts $20\frac{2}{3}\frac{8}{1}$ grains.

Universal Standard for Weights *and Measures.*

Philosophers, from their habits of generalizing, have often made speculations for forming a general standard for Weights and measures through the whole world. These have been devised chiefly of a philosophical nature, as best adapted to universality. After the invention of pendulum clocks, it first occurred that the length of a pendulum which should vibrate seconds, would be proper to be made a universal standard for lengths; whether it should be called a yard, or any thing else. But it was found, that it would be difficult in practice, to measure and determine the true length of such a pendulum, that is the distance between the point of suspension and the point of oscillation. Another cause of inaccuracy was afterwards discovered, when it was found that the seconds pendulum was of different lengths in all the different latitudes, owing to the spheroidal figure of the earth, which causes that all places in different latitudes are at different distances from the centre, and consequently the pendulums are acted upon by different forces of gravity, and therefore require to be of different lengths. In the latitude of London this is found to be $39\frac{1}{8}$ inches.

The Society of Arts in London, among their many laudable and patriotic endeavours, offered a handsome premium for the discovery of a proper standard for Weights and measures. This brought them many frivolous expedients, as well as one which was an improvement on the method of the pendulum, by one Hatton. This consisted in measuring the difference of the lengths of two pendulums of different times of vibration; which could be performed more easily and accurately than that of the length of one single pendulum. This method was put in practice, and fully explained and illustrated, by the late Mr. Whitehurst, in his attempt to ascertain an Universal Standard of Weights and Measures. But still the same kind of inaccuracy of measurement &c, obtains in this way, as in the single pendulum, though in a smaller degree.

Another method that has been proposed for this purpose, is the space that a heavy body falls freely through in 1 second of time. But this is an experiment more difficult than the former to be made with accuracy; on which account, different persons will all make the space fallen to be of different quantities, which would give as many different standards of length. Add to this, that the spheroidal form of the earth here again introduces a diversity in the space, owing to the different distances from the centre, and the consequent diversity in the force of gravity by which the body falls. This space has been found to be 193 inches, or $16\frac{1}{12}$ feet, in the latitude of London; but it will be a different quantity in other latitudes.

Many other inferior expedients have also been proposed for the purpose of universal measures, and Weights; but there is another which now has the best prospect of success, and is at present under particular experiments, by the philosophers both of this and the French nation. This method is by the measure of the degrees of latitude; which would give a large quantity, and admit of more accurate measures, by subdivision, than what could be obtained by beginning from a small quantity, or measure, and thence to proceed increasing, by multiples. This measure might be taken either from the extent of the whole compass of the earth, or of all the 360 degrees, or a medium degree among them all, or from the measure of a degree in the medium latitude of 45 degrees. It will also be most convenient to make the subdivisions of this measure, when found, to proceed decimally, or continually by 10ths.

The universal standard for lengths being once established, those of Weights, &c, would easily follow. For instance, a vessel, of certain dimensions, being filled with distilled water, or some other homogeneous matter, the Weight of that may be considered as a standard for Weights.

Weight of the Air, Water, &c. See those articles severally. See also Specific Gravity.

WERST, a Russian measure of length, equal to 3500 English feet.

WEST, one of the cardinal points of the horizon, or of the compass, diametrically opposite to the east, or lying on the left hand when we face the north. Or West is strictly the intersection of the prime vertical with the horizon, on that side where the sun sets.

West *Wind*, is also called *Zephyrus*, and *Favonius*.

West *Dial*. See Dial.

WESTERN *Amplitude*, *Horizon*, *Ocean*. See the several articles.

WESTING, in Navigation, is the quantity of departure made good to the westward from the meridian.

WEY. See Weigh.

WHALE, in Astronomy, one of the constellations. See Cetus.

WHEEL, in Mechanics, a simple machine, consisting of a circular piece of wood, metal, or other matter, that revolves on an axis. This is otherwise called *Wheel and Axle*, or Axis *in Peritrochio*, as a mechanical power, being one of the most frequent and useful of any. In this capacity of it, the Wheel is a kind of perpetual lever, and the axis another lesser one; or the radius of the Wheel and that of its axis may be considered as the longer and shorter arms of a lever, the centre of the Wheel being the fulcrum or point of

suspen-

fuspension. Whence it is, that the power of this machine is estimated by this rule, as the radius of the axis is to the radius of the Wheel or of the circumference, so is any given power, to the weight it will sustain.

Wheels, as well as their axes, are frequently dented, or cut into teeth, and are then of use upon innumerable occasions; as in jacks, clocks, mill-work, &c; by which means they are capable of moving and acting on one another, and of being combined together to any extent; the teeth either of the axis or circumference working in those of other Wheels or axles; and thus, by multiplying the power to any extent, an amazing great effect is produced.

To compute the power of a combination of Wheels; the teeth of the axis of every Wheel acting on those in the circumference of the next following. Multiply continually together the radii of all the axes, as also the radii of all the Wheels; then it will be, as the former product is to the latter product, so is a given power applied to the circumference, to the weight it can sustain. Thus, for example, in a combination of five Wheels and axles, to find the weight a man can sustain, or raise, whose force is equal to 150 pounds, the radii of the Wheels being 30 inches, and those of the axes 3 inches. Here $3 \times 3 \times 3 \times 3 \times 3 = 243$,
and $30 \times 30 \times 30 \times 30 \times 30 = 24300000$,
therefore as $243 : 24300000 :: 150 : 15000000$ lb, the weight he can sustain, which is more than 6696 tons weight. So prodigious is the increase of power in a combination of Wheels!

But it is to be observed, that in this, as well as every other mechanical engine, whatever is gained in power, is lost in time; that is, the weight will move as much slower than the power, as the force is increased or multiplied, which in the example above is 100000 times slower.

Hence, having given any power, and the weight to be raised, with the proportion between the Wheels and axles necessary to that effect; to find the number of the Wheels and axles. Or, having the number of the Wheels and axles given, to find the ratio of the radii of the Wheels and axles. Here, putting

p = the power acting on the last wheel,
w = the weight to be raised,
r = the radius of the axles,
R = the radius of the wheels,
n = the number of the wheels and axles;

then, by the general proportion, as $r^n : R^n :: p : w$; therefore $pR^n = wr^n$ is a general theorem, from whence may be found any one of these five letters or quantities, when the other four are given. Thus, to find n the number of Wheels: we have first

$$\frac{R^n}{r^n} = \frac{w}{p}, \text{ then } n = \frac{\log. w - \log. p}{\log. R - \log. r}.$$

And to find $\frac{R}{r}$, the ratio of the Wheel to the axle; it is

$$\frac{R}{r} = \sqrt[n]{\frac{w}{p}}.$$

Wheels *of a Clock*, &c, are, the crown wheel, contrat wheel, great wheel, second wheel, third wheel, striking wheel, detent wheel, &c.

Wheels *of Coaches, Carts, Waggons*, &c. With respect to Wheels of carriages, the following particulars are collected from the experiments and observations of Desaguliers, Beighton, Camus, Ferguson, Jacob, &c.

1. The use of Wheels, in carriages, is twofold; viz, that of diminishing or more easily overcoming the resistance or friction from the carriage; and that of more easily overcoming obstacles in the road. In the first case the friction on the ground is transferred in some degree from the outer surface of the Wheel to its nave and axle; and in the latter, they serve easily to raise the carriage over obstacles and asperities met with on the roads. In both these cases, the height of the Wheel is of material consideration, as the spokes act as levers, the top of an obstacle being the fulcrum, their length enables the carriage more easily to surmount them; and the greater proportion of the Wheel to the axle serves more easily to diminish or to overcome the friction of the axle. See Jacob's Observations on Wheel Carriages, p. 23 &c.

2. The Wheels should be exactly round; and the fellies at right angles to the naves, according to the inclination of the spokes.

3. It is the most general opinion, that the spokes be somewhat inclined to the naves, so that the Wheels may be dishing or concave. Indeed if the Wheels were always to roll upon smooth and level ground, it would be best to make the spokes perpendicular to the naves, or to the axles; because they would then bear the weight of the load perpendicularly. But because the ground is commonly uneven, one Wheel often falls into a cavity or rut, when the other does not, and then it bears much more of the weight than the other does; in which case it is best for the Wheels to be dished, because the spokes become perpendicular in the rut, and therefore have the greatest strength when the obliquity of the road throws most of the weight upon them; whilst those on the high ground have less weight to bear, and therefore need not be at their full strength.

4. The axles of the Wheels should be quite straight, and perpendicular to the shafts, or to the pole. When the axles are straight, the rims of the Wheels will be parallel to each other, in which case they will move the easiest, because they will be at liberty to proceed straight forwards. But in the usual way of practice, the ends of the axles are bent downwards; which always keeps the sides of the Wheels that are next the ground nearer to one another than their upper sides are; and this not only makes the Wheels drag sideways as they go along, and gives the load a much greater power of crushing them than when they are parallel to each other, but also endangers the overturning the carriage when a Wheel falls into a hole or rut, or when the carriage goes on a road that has one side lower than the other, as along the side of a hill. Mr. Beighton however has offered several reasons to prove that the axles of Wheels ought not to be straight; for which see Desaguliers's Exp. Phil. vol. 2, Appendix.

5. Large Wheels are found more advantageous for rolling than small ones, both with regard to their power as a longer lever, and to the degree of friction, and to the advantage in getting over holes, rubs, and stones,

ftones, &c. If we confider Wheels with regard to the friction upon their axles, it is evident that fmall Wheels, by turning oftener round, and fwifter about the axles, than large ones, muft have much more friction. Again, if we confider Wheels as they fink into holes or foft earth, the large Wheels, by finking lefs, muft be much eafier drawn out of them, as well as more eafily over ftones and obftacles, from their greater length of lever or fpokes. Defaguliers has brought this matter to a mathematical calculation, in his Experim. Philof. vol. 1, p. 171, &c. See alfo Jacob's Obferv. p. 63.

From hence it appears then, that Wheels are the more advantageous as they are larger, provided they are not more than 5 or 6 feet diameter; for when they exceed thefe dimenfions, they become too heavy; or if they are made light, their ftrength is proportionably diminifhed, and the length of the fpokes renders them more liable to break: befides, horfes applied to fuch Wheels would not be capable of exerting their utmoft ftrength, by having the axles higher than their breafts, fo that they would draw downwards; which is even a greater difadvantage than fmall Wheels have in occafioning the horfes to draw upwards.

6. Carriages with 4 Wheels, as waggons or coaches, are much more advantageous than carriages with 2 Wheels, as carts and chaifes; for with 2 wheels it is plain the tiller horfe carries part of the weight, in one way or other: in going down hill, the weight bears upon the horfe; and in going up hill, the weight falls the other way, and lifts the horfe, which is ftill worfe. Befides, as the Wheels fink into the holes in the roads, fometimes on one fide, fometimes on the other, the fhafts ftrike againft the tiller's fides, which deftroys many horfes: moreover, when one of the Wheels finks into a hole or rut, half the weight falls that way, which endangers the overturning of the carriage.

7. It would be much more advantageous to make the 4 Wheels of a coach or waggon large, and nearly of a height, than to make the fore Wheels of only half the diameter of the hind Wheels, as is ufual in many places. The fore Wheels have commonly been made of a lefs fize than the hind ones, both on account of turning fhort, and to avoid cutting the braces. Crane-necks have alfo been invented for turning yet fhorter, and the fore Wheels have been lowered, fo as to go quite under the bend of the crane-neck.

It is held, that it is a great difadvantage in fmall Wheels, that as their axle is below the bow of the horfes breafts, the horfes not only have the loaded carriage to draw along, but alfo part of its weight to bear, which tires them foon, and makes them grow much ftiffer in their hams, than they would be if they drew on a level with the fore axle.

But Mr. Beighton difputes the propriety of fixing the line of traction on a level with the breaft of a horfe, and fays it is contrary to reafon and experience. Horfes, he fays, have little or no power to draw but what they derive from their weight; without which they could not take hold of the ground, and then they muft flip, and draw nothing. Common experience alfo teaches, that a horfe muft have a certain weight on his back or fhoulders, that he may draw the better. And when a horfe draws hard, it is obferved that he bends forward, and brings his breaft near the ground; and then if the Wheels are high, he is pulling the carriage againft the ground. A horfe tackled in a waggon will draw two or three ton, becaufe the point or line of traction is below his breaft, by the lownefs of the Wheels. It is alfo common to fee, when one horfe is drawing a heavy load, efpecially up hill, his fore feet will rife from the ground; in which cafe it is ufual to add a weight on his back, to keep his fore part down, by a perfon mounting on his back or fhoulders, which will enable him to draw that load, which he could not move before. The greateft ftrefs, or main bufinefs of drawing, fays this ingenious writer, is to overcome obftacles; for on level plains the drawing is but little, and then the horfe's back need be preffed but with a fmall weight.

8. The utility of broad Wheels, in amending and preferving the roads, has been fo long and generally acknowledged, as to have occafioned the legiflature to enforce their ufe. At the fame time, the proprietors and drivers of carriages feem to be convinced by experience, that a narrow-wheeled carriage is more eafily and fpeedily drawn by the fame number of horfes, than a broad-wheeled one of the fame burthen: probably becaufe they are much lighter, and have lefs friction on the axle.

On the fubject of this article, fee Jacob's Obferv. &c. on Wheel-Carriages, 1773, p. 81. Defagul. Exper. Phil. vol. 1, p. 201. Fergufon's Lect. 4to, p. 56. Martin's Phil. Brit. vol. 1, p. 229.

Blowing WHEEL, is a machine contrived by Defaguliers, for drawing the foul air out of any place, or for forcing in frefh, or doing both fucceffively, without opening doors or windows. See Philof. Tranf. number 437. The intention of this machine is the fame as that of Hales's ventilator, but not fo effectual, nor fo convenient. See Defag. Exper. Philof. vol. 2, p. 563, 568.—This Wheel is alfo called a *centrifugal Wheel*, becaufe it drives the air with a centrifugal force.

Water WHEEL, of a Mill, that which receives the impulfe of the ftream by means of ladle-boards or float-boards. M. Parent, of the Academy of Sciences, has determined that the greateft effect of an underfhot Wheel, is when its velocity is equal to the 3d part of the velocity of the water that drives it; but it ought to be the half of that velocity, as is fully fhewn in the article Mill, pa. 111. In fixing an underfhot Wheel, it ought to be confidered whether the water can run clear off, fo as to caufe no back-water to ftop its motion. Concerning this article, fee Defagul. Exp. Philof. vol. 2, p. 422. Alfo a variety of experiments and obfervations relating to underfhot and overfhot Wheels, by Mr. Smeaton, in the Philof. Tranf. vol. 51, p. 100.

Ariftotle's WHEEL. See ROTA *Ariftotelica*.

Meafuring WHEEL. See PERAMBULATOR.

Orffyreus's WHEEL. See ORFFYREUS.

Perfian WHEEL. See PERSIAN.

WHEEL-*Barometer*. See BAROMETER.

WHIRL-POOL, an eddy, vortex, or gulph, where the water is continually turning round.

WHIRLING-TABLE, a machine contrived for repre-

representing several phenomena in philosophy, and nature; as, the principal laws of gravitation, and of the planetary motions in curvilinear orbits.

The figure of this instrument is exhibited fig. 1, pl. 35: where AA is a strong frame of wood; B a winch fixed on the axis C of the wheel D, round which is the catgut string F, which also goes round the small wheels G and K, crossing between them and the great wheel D. On the upper end of the axis of the wheel G, above the frame, is fixed the round board *d*, to which may be occasionally fixed the bearer MSX. On the axis of the wheel H is fixed the bearer NTZ, and when the winch B is turned, the wheels and bearers are put into a Whirling motion. Each bearer has two wires W, X, and Y, Z, fixed and screwed tight into them at the ends by nuts on the outside; and when the nuts are unscrewed, the wires may be drawn out in order to change the balls U, V, which slide upon the wires by means of brass loops fixed into the balls, and preventing their touching the wood below them. Through each ball there passes a silk line, which is fixed to it at any length from the centre of the bearer to its end, by a nut-screw at the top of the ball; the shank of the screw going into the centre of the ball, and pressing the line against the under side of the whole which it goes through. The line goes from the ball, and under a small pulley fixed in the middle of the bearer; then up through a socket in the round plate (S and T) in the middle of each bearer; then through a slit in the middle of the square top (O and P) of each tower, and going over a small pulley on the top comes down again the same way, and is at last fastened to the upper end of the socket fixed in the middle of the round plate above mentioned. Each of these plates S and T has four round holes near their edges, by which they slide up and down upon the wires which make the corner of each tower. The balls and plates being thus connected, each by its particular line, it is plain that if the balls be drawn outward, or towards the end M and N of their respective bearers, the round plates S and T will be drawn up to the top of their respective towers O and P.

There are several brass weights, some of two, some of three, and others of four ounces, to be occasionally put within the towers O and P, upon the round plates S and T: each weight having a round hole in the middle of it, for going upon the sockets or axes of the plates, and being slit from the edge to the hole, that it may slip over the line which comes from each ball to its respective plate.

For a specimen of the experiments which may be made with this machine, may be subjoined the following.

1. Removing the bearer MX, put the loop of the line *b* to which the ivory ball *a* is fastened over a pin in the centre of the board *d*, and turn the winch B; and the ball will not immediately begin to move with the board, but, on account of its inactivity, endeavour to remain in its state of rest. But when the ball has acquired the same velocity with the board, it will remain upon the same part of the board, having no relative motion upon it. However, if the board be suddenly stopped, the ball will continue to revolve upon it, until the friction thereof stops its motion: so that matter resists every change of state, from that of rest to that of motion, and *vice versa*.

2. Put a longer cord to this ball; let it down through the hollow axis of the bearer MX and wheel G, and fix a weight to the end of the cord below the machine; and this weight, if left at liberty, will draw the ball from the edge of the Whirling board to its centre. Draw off the ball a little from the centre, and turn the winch; then the ball will go round and round with the board, and gradually fly farther from the centre, raising up the weight below the machine. And thus it appears that all bodies, revolving in circles, have a tendency to fly off from those circles, and must be retained in them by some power proceeding from or tending to the centre of motion. Stop the machine, and the ball will continue to revolve for some time upon the board; but as the friction gradually stops its motion, the weight acting upon it will bring it nearer and nearer to the centre in every revolution, till it brings it quite thither. Hence it appears, that if the planets met with any resistance in going round the sun, its attractive power would bring them nearer and nearer to it in every revolution, till they would fall into it.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, and it will revolve upon the board. Then, observing the velocity of its motion, pull the cord below the machine, and thus bring the ball nearer the centre of the board, and the ball will be seen to revolve with an increasing velocity, as it approaches the centre: and thus the planets which are nearest the sun perform quicker revolutions than those which are more remote, and move with greater velocity in every part of their respective circles.

4. Remove the ball *a*, and apply the bearer MX, whose centre of motion is in its middle at *w*, directly over the centre of the Whirling board *d*. Then put two balls (V and U) of equal weight upon their bearing wires, and having fixed them at equal distances from their respective centres of motion *w* and *x* upon their silk cords, by the screw nuts, put equal weights in the towers O and P. Lastly, put the catgut strings E and F upon the grooves G and H of the small wheels, which, being of equal diameters, will give equal velocities to the bearers above, when the winch B is turned; and the balls U and V will fly off toward M and N, and raise the weights in the towers at the same instant. This shews, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away these equal balls, and put a ball of 6 ounces into the bearer MX, at a 6th part of the distance *wz* from the centre, and put a ball of one ounce into the opposite bearer, at the whole distance $xy = wz$; and fix the balls at these distances on their cords, by the screw nuts at the top: then the ball U, which is 6 times as heavy as the ball V, will be at only a 6th part of the distance from its centre of motion; and consequently will revolve in a circle of only a 6th part of the circumference of the circle in which V revolves. Let equal weights be put into the towers, and the winch be turned; which (as the catgut string

ts on equal wheels below, will cauſe the balls to revolve in equal times: but V will move 6 times as faſt as U, becauſe it revolves in a circle of 6 times its radius, and both the weights in the towers will riſe at once. Hence it appears, that the centrifugal forces of revolving bodies are in direct proportion to their quantities of matter multiplied into their reſpective velocities, or into their diſtance from the centres of their reſpective circles.

If theſe two balls be fixed at equal diſtances from their reſpective centres of motion, they will move with equal velocities; and if the tower O has 6 times as much weight put into it as the tower P has, the balls will raiſe their weights exactly at the ſame moment: i. e. the ball U, being 6 times as heavy as the ball V, has 6 times as much centrifugal force in deſcribing an equal circle with an equal velocity.

6. Let two balls, U and V, of equal weights, be fixed on their cords at equal diſtances from their reſpective centres of motion w and x; and let the catgut ſtring E be put round the wheel K (whoſe circumference is only half that of the wheel H or G) and over the pulley s to keep it tight, and let 4 times as much weight be put into the tower P as in the tower O. Then turn the winch B, and the ball V will revolve twice as faſt as the ball U in a circle of the ſame diameter, becauſe they are equidiſtant from the centres of the circles in which they revolve; and the weights in the towers will both riſe at the ſame inſtant; which ſhews that a double velocity in the ſame circle will exactly balance a quadruple power of attraction in the centre of the circle: for the weights in the towers may be conſidered as the attractive forces in the centres, acting upon the revolving balls; which moving in equal circles, are as if they both moved in the ſame circle. Whence it appears that, if bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the ſquares of the velocities.

7. The catgut ſtring remaining as before, let the diſtance of the ball V from the centre x be equal to 2 of the diviſions on its bearer; and the diſtance of the ball U from the centre w be 3 and a 6th part; the balls themſelves being equally heavy, and V making two revolutions by turning the winch, whilſt U makes one; ſo that if we ſuppoſe the ball V to revolve in one moment, the ball U will revolve in 2 moments, the ſquares of which are 1 and 4: therefore, the ſquare of the period of V is contained 4 times in the ſquare of the period of U. But the diſtance of V is 2, the cube of which is 8, and the diſtance of U is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained 4 times: and therefore, the ſquares of the periods V and U are to one another as the cubes of their diſtances from x and w, the centres of their reſpective circles. And if the weight in the tower O be 4 ounces, or equal to the ſquare of 2, which is the diſtance of V from the centre x; and the weight in the tower P be 10 ounces, nearly equal to the ſquare of $3\frac{1}{6}$, the diſtance of U from w; it will be found upon turning the machine by the winch, that the balls U and V will raiſe their reſpective weights at very nearly the ſame inſtant of time. This experiment confirms the famous propoſition of Kepler, viz. that the ſquares of the periodical times of the planets round the ſun are in proportion as the cubes of their diſtances from him; and that the ſun's attraction is inverſely as the ſquare of the diſtance from his centre.

8. Take off the ſtring E from the wheels D and H, and let the ſtring F remain upon the wheels D and G; take away alſo the bearer MX from the Whirling-board d, and inſtead of it put on the machine AB (fig. 2), fixing it to the centre of the board by the pins c and d, ſo that the end ef may riſe above the board to an angle of 30 or 40 degrees. On the upper part of this machine, there are two glaſs tubes a and b, cloſe ſtopped at both ends, each tube being about three quarters full of water. In the tube a is a little quickſilver, which naturally falls down to the end a in the water; and in the tube b is a ſmall cork, floating on the top of the water, and ſmall enough to riſe or fall in the tube. While the board b with this machine upon it continues at reſt, the quickſilver lies at the bottom of the tube a, and the cork floats on the water near the top of the tube b. But, upon turning the winch and moving the machine, the contents of each tube fly off towards the uppermoſt ends, which are fartheſt from the centre of motion; the heavieſt with the greateſt force. Conſequently, the quickſilver in the tube a will fly off quite to the end f, occupying its bulk of ſpace, and excluding the water, which is lighter than itſelf: but the water in the tube b, flying off to its higher end e, will exclude the cork from that place, and cauſe it to deſcend toward the loweſt end of the tube; for the heavier body, having the greater centrifugal force, will poſſeſs the upper part of the tube, and the lighter body will keep between the heavier and the lower part.

This experiment demonſtrates the abſurdity of the Carteſian doctrine of vortices; for, if a planet be more denſe or heavy than its bulk of the vortex, it will fly off in it farther and farther from the ſun; if leſs denſe, it will come down to the loweſt part of the vortex, at the ſun: and the whole vortex itſelf, unleſs prevented by ſome obſtacle, would fly quite off, together with the planets.

9. If a body be ſo placed upon the Whirling-board of the machine (fig. 1.) that the centre of gravity of the body be directly over the centre of the board, and the board be moved ever ſo rapidly by the winch B, the body will turn round with the board, without removing from its middle; for, as all parts of the body are in equilibrio round its centre of gravity, and the centre of gravity is at reſt in the centre of motion, the centrifugal force of all parts of the body will be equal at equal diſtances from its centre of motion, and therefore the body will remain in its place. But if the centre of gravity be placed ever ſo little out of the centre of motion, and the machine be turned ſwiftly round, the body will fly off towards that ſide of the board on which its centre of gravity lies. Then if the wire C (fig. 3) with its little ball B be taken away from the ſemi-globe A, and the flat ſide f of the ſemiglobe be laid upon the Whirling-board, ſo that their centres may coincide; if then the board be turned ever ſo quickly by the winch, the ſemi-globe will remain where it was placed: but if the wire C be ſcrewed into the ſemi-globe at d, the whole becomes one body, whoſe centre of gravity is at or near d. Fix the pin c in

in the centre of the Whirling-board, and let the deep groove *b* cut in the flat fide of the femi-globe be put upon the pin, fo that the pin may be in the centre of A (fee fig. 4) where the groove is to be reprefented at *b*, and let the board be turned by the winch, which will carry the little ball B (fig. 3) with its wire C, and the femi-globe A, round the centre-pin *c i*; and then, the centrifugal force of the little ball B, weighing one ounce, will be fo great as to draw off the femi-globe A, weighing two pounds, until the end of the groove at *c* ftrikes againft the pin *c*, and fo prevents A from going any farther: otherwife, the centrifugal force of B would have been great enough to have carried A quite off the whirling-board. Hence we fee that, if the fun were placed in the centre of the orbits of the planets, it could not poffibly remain there; for the centrifugal forces of the planets would carry them quite off, and the fun with them; efpecially when feveral of them happened to be in one quarter of the heavens. For the fun and planets are as much connected by the mutual attraction fubfifting between them, as the bodies A and B are by the wire C fixed into them both. And even if there were but one planet in the whole heavens to go round ever fo large a fun in the centre of its orbit, its centrifugal force would foon carry off both itfelf and the fun: for the greateft body placed in any part of free fpace could be eafily moved; becaufe, if there were no other body to attract it, it would have no weight or gravity of itfelf, and confequently, though it could have no tendency of itfelf to remove from that part of fpace, yet it might be very eafily moved by any other fubftance.

10. As the centrifugal force of the light body B will not allow the heavy body A to remain in the centre of motion, even though it be 24 times as heavy as B; let the ball A (fig. 5) weighing 6 ounces be connected by the wire C with the ball B, weighing one ounce, and let the fork E be fixed into the centre of the Whirling-board; then, hang the balls upon the fork by the wire C in fuch a manner that they may exactly balance each other, which will be when the centre of gravity between them, in the wire at *d*, is fupported by the fork. And this centre of gravity is as much nearer to the centre of the ball A than to the centre B, as A is heavier than B; allowing for the weight of the wire on each fide of the fork. Then, let the machine be moved, and the balls A and B will go round their common centre of gravity *d*, keeping their balance, becaufe either will not allow the other to fly off with it. For, fuppofing the ball B to be only one ounce in weight, and the ball A to be fix ounces; then, if the wire C were equally heavy on each fide of the fork, the centre of gravity *d* would be 6 times as far from the centre of B as from the centre of A, and confequently B will revolve with a velocity 6 times as great as A does; which will give B 6 times as much centrifugal force as any fingle ounce of A has; but then as B is only one ounce, and A fix ounces, the whole centrifugal force of A will exactly balance that of B; and therefore, each body will detain the other, fo as to make it keep in its circle.

Hence it appears, that the fun and planets muft all move round the common centre of gravity of the whole fyftem, in order to preferve that juft balance which takes place among them.

11. Take away the forks and balls from the Whirling-board, and place the trough AB (fig. 6) thereon, fixing its centre to that of the board by the pin H. In this trough are two balls D and E of unequal weights, connected by a wire *f*, and made to flide eafily upon the wire ftretched from end to end of the trough, and made faft by nut fcrews on the outfide of the ends. Place thefe balls on the wire *c*, fo that their common centre of gravity *g*, may be directly over the centre of the Whirling-board. Then turn the machine by the winch ever fo fwiftly, and the trough and balls will go round their centre of gravity, fo as neither of them will fly off; becaufe, on account of the equilibrium, each ball detains the other with an equal force acting againft it. But if the ball E be drawn a little more towards the end of the trough at A, it will remove the centre of gravity towards that end from the centre of motion; and then, upon turning the machine, the little ball E will fly off, and ftrike with a confiderable force againft the end A, and draw the great ball B into the middle of the trough. Or, if the great ball D be drawn towards the end B of the trough, fo that the centre of gravity may be a little towards that end from the centre of motion; and the machine be turned by the winch, the great ball D will fly off, and ftrike violently againft the end B of the trough, and will bring the little ball E into the middle of it. If the trough be not made very ftrong, the ball D will break through it.

12. Mr. Fergufon has explained the reafon why the tides rife at the fame time on oppofite fides of the earth, and confequently in oppofite directions, by the following new experiment on the Whirling-table. For this purpofe, let *a b c d* (fig. 7) reprefent the earth, with its fide *c* turned toward the moon, which will then attract the water fo as to raife them from *c* to *g*: and in order to fhew that they will rife as high at the fame time on the oppofite fide from *a* to *e*; let a plate AB (fig. 8) be fixed upon one end of the flat bar DC, with fuch a circle drawn upon it as *a b c d* (fig. 7) to reprefent the round figure of the earth and fea; and an ellipfe as *e f g h* to reprefent the fwelling of the tide at *e* and *g*, occafioned by the influence of the moon. Over this plate AB fufpend the three ivory balls *e*, *f*, *g*, by the filk lines *h*, *i*, *k*, faftened to the tops of the wires H, I, K, fo that the ball at *e* may hang freely over the fide of the circle *e*, which is fartheft from the moon M at the other end of the bar; the ball at *f* over the centre, and the ball at *g* over the fide of the circle *g*, which is neareft the moon. The ball *f* may reprefent the centre of the earth, the ball *g* water on the fide next the moon, and the ball *e* water on the oppofite fide. On the back of the moon M is fixed a fhort bar N parallel to the horizon, and there are three holes in it above the little weights *p*, *q*, *r*. A filken thread *o* is tied to the line *k* clofe above the ball *g*, and paffing by one fide of the moon M goes through a hole in the bar N, and has the weight *p* hung to it. Such another thread *m* is tied to the line *i*, clofe above the ball *f*, and, paffing through the centre of the moon M and middle of the bar N, has the weight *q* hung to it which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, clofe above

above the ball *e*, and, paffing by the other fide, of the moon M through the bar N, has the weight *r* hung to it, which is lighter than the weight *q*. The ufe of thefe three unequal weights is to reprefent the moon's unequal attraction at different diftances from her; fo that if they are left at liberty, they will draw all the three balls towards the moon with different degrees of force, and caufe them to appear as in fig. 9, in which cafe they are evidently farther from each other than if they hung freely by the perpendicular lines *h*, *i*, *k*. Hence it appears, that as the moon attracts the fide of the earth which is neareft her with a greater degree of force than fhe does the centre of the earth, fhe will draw the water on that fide more than the centre, and caufe it to rife on that fide: and as fhe draws the centre more than the oppofite fide, the centre will recede farther from the furface of the water on that oppofite fide, and leave it as high there as fhe raifed it on the fide next her. For, as the centre will be in the middle between the tops of the oppofite elevations, they muft of courfe be equally high on both fides at the fame time.

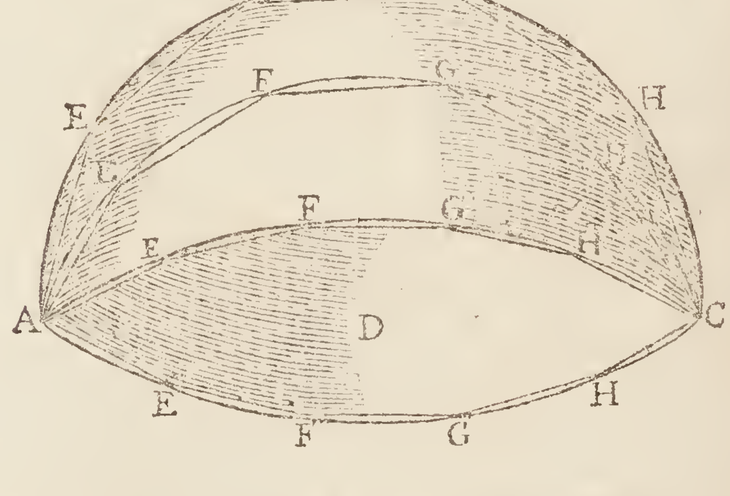
However, upon this fuppofition, the earth and moon would foon come together; and this would be the cafe if they had not a motion round their common centre of gravity, to produce a degree of centrifugal force, fufficient to balance their mutual attraction. Such motion they have; for as the moon revolves in her orbit every month, at the diftance of 240000 miles from the earth's centre, and of 234000 miles from the centre of gravity of the earth and moon, the earth alfo goes round the fame centre of gravity every month at the diftance of 6000 miles from it, i. e. from it to the centre of the earth. But the diameter of the earth being, in round numbers, 8000 miles, its fide next the moon is only 2000 miles from the common centre of gravity of the earth and moon, its centre 6000 miles from it, and its fartheft fide from the moon 10000 miles. Confequently the centrifugal forces of thefe parts are as 2000, 6000, and 10000; i. e. the centrifugal force of any fide of the earth, when it is turned from the moon, is five times as great as when it is turned toward the moon. And as the moon's attraction, expreffed by the number 6000 at the earth's centre, keeps the earth from flying out of this monthly circle, it muft be greater than the centrifugal force of the waters on the fide next her; and confequently, her greater degree of attraction on that fide is fufficient to raife them; but as her attraction on the oppofite fide is lefs than the centrifugal force of the water there, the excefs of this force is fufficient to raife the water juft as high on the oppofite fide.

To prove this experimentally, let the bar DC with its furniture be fixed on the Whirling-board of the machine (fig. 1.) by pufhing the pin P into the centre of the board; which pin is in the centre of gravity of the whole bar with its three balls, *e*, *f*, *g*, and moon M. Now if the Whirling-board and bar be turned flowly round by the winch, till the ball *f* hangs over the centre of the circle, as in fig. 10, the ball *g* will be kept towards the moon by the heavieft weight *p* (fig. 8), and the ball *e*, on account of its greater centrifugal force, and the lefs weight *r*, will fly off as far to the other fide, as in fig. 10. And thus, whilft the machine is kept turning, the balls *e* and *g* will hang over the ends of the ellipfe *l f k*. So that the centrifugal force of the ball *e* will exceed the moon's attraction juft as much as her attraction exceeds the centrifugal force of the ball *g*, whilft her attraction juft balances the centrifugal force of the ball *f*, and makes it keep in its circle. Hence it is evident, that the tides muft rife to equal heights at the fame time on oppofite fides of the earth. See Fergufon's Lectures on Mechanics, lect. 2, and Defag. Ex. Phil. vol. 1, lect. 5.

WHIRLWIND, a wind that rifes fuddenly, is exceedingly rapid and impetuous, in a Whirling direction, and often progreffively alfo; but it is commonly foon fpent.

Dr. Franklin, in his Phyfical and Meteorological Obfervations, read to the Royal Society in 1756, fuppofes a Whirlwind and a waterfpout to proceed from the fame caufe: their only difference being, that the latter paffes over the water, and the former over the land. This opinion is corroborated by the obfervations of M. de la Pryme, and many others, who have remarked the appearances and effects of both to be the fame. They have both a progreffive as well as a circular motion; they ufually rife after calms and great heats, and moftly happen in the warmer latitudes: the wind blows every way from a large furrounding fpace, both to the waterfpout and whirlwind; and a waterfpout has, by its progreffive motion, paffed from the fea to the land, and produced all the phenomena and effects of a Whirlwind: fo that there is no reafon to doubt that they are meteors arifing from the fame general caufe, and explicable upon the fame principles, furnifhed by electrical experiments and difcoveries. See HURRICANE, and WATERSPOUT. For Dr. Franklin's ingenious method of accounting for both thefe phenomena, fee his Letters and Papers, &c, vol. 1, p. 191, 216, &c.

WHISPERING-*Places*, are places where a Whifper, or other fmall noife, may be heard from one part to another, to a great diftance. They depend on a principle, that the voice, &c, being applied to one end of an arch, eafily paffes by repeated reflections to the other. Thus,



let ABC reprefent the fegment of a fphere; and fuppofe a low voice uttered at A, the vibrations extending themfelves every way, fome of them will impinge upon the points E, E, &c; and thence be reflected to the points F, F, &c; thence to G, G, &c; till at laft they meet in C; where by their union they caufe a much ftronger found than in any part of the fegment whatever,

whatever, even louder than at the point from whence they set out. Accordingly, all the contrivance in a Whispering-place is, that near the person who Whispers, there be a smooth wall, arched either cylindrically, or elliptically, &c. A circular arch will do, but not so well.

Some of the most remarkable places for Whispering, are the following: viz, The prison of Dionysius at Syracuse, which increased a soft Whisper to a loud noise; or a clap of the hand to the report of a cannon, &c. The aqueducts of Claudius, which carried a voice 16 miles: beside divers others mentioned by Kircher in his Phonurgia. In England, the most considerable Whispering places are, the dome of St. Paul's church, London, where the ticking of a watch may be heard from side to side, and a very soft Whisper may be sent all round the dome: this Dr. Derham found to hold not only in the gallery below, but above upon the scaffold, where a Whisper would be carried over a person's head round the top of the arch, though there be a large opening in the middle of it into the upper part of the dome. And the celebrated Whispering-place in Gloucester cathedral, which is only a gallery above the east end of the choir, leading from one side of it to the other. See Birch's Hist. of the Royal Soc. vol. 1, pa. 120.

WHISTON (William), an English divine, philosopher, and mathematician, of uncommon parts, learning, and extraordinary character, was born the 9th of December 1667, at Norton in the county of Leicester, where his father was rector. He was educated under his father till he was 17 years of age, when he was sent to Tamworth school, and two years after admitted of Clare-hall, Cambridge, where he pursued his studies, and particularly the mathematics, with great diligence. During this time he became afflicted with a great weakness of sight, owing to close study in a whitened room; which was in a good measure relieved by a little relaxation from study, and taking off the strong glare of light by hanging the place opposite his seat with green.

In 1693 he was made master of arts and fellow of the college, and soon after commenced one of the tutors; but his ill state of health soon after obliged him to relinquish this profession. Having entered into orders, in 1694 he became chaplain to Dr. More, bishop of Norwich; and while in this station he published his first work, intitled, *A New Theory of the Earth &c*; in which he undertook to prove that the Mosaic doctrine of the earth was perfectly agreeable to reason and philosophy: which work, having much ingenuity, though it was written against by Mr. John Keill, brought considerable reputation to the author.

In the year 1698, bishop More gave him the living of Lowestoff in Suffolk, where he immediately went to reside, and devoted himself with great diligence to the discharge of that trust.—In the beginning of this century he was made Sir Isaac Newton's deputy, and afterwards his successor in the Lucasian professorship of mathematics; when he resigned his living at Lowestoff, and went to reside at Cambridge. From this time his publications became very frequent, both in theology and mathematics. Thus, in 1702 he published, A Short View of the Chronology of the Old Testament, and of the Harmony of the four Evangelists.—In 1707, *Prælectiones Astronomicæ*; beside eight Sermons on the Accomplishment of the Scripture Prophecies, preached at Boyle's lecture; and Newton's Arithmetica Universalis.—In 1708, Tacquet's Euclid, with select Theorems of Archimedes; the former of which had accidentally been his first introduction to the study of the mathematics.—In the same year he drew up an Essay upon the Apostolical Constitutions, which the Vice chancellor refused his licence for printing. The author tells us, he had read over the two first centuries of the church, and found that the Eusebian or Arian doctrine was chiefly the doctrine of those ages, which, though deemed heterodox, he thought it his duty to discover.—In 1709, he published a volume of Sermons and Essays on various subjects.—In 1710, Prælectiones Physico-Mathematicæ, which with the Prælectiones Astronomicæ, were translated and published in English. And it may be said, with no small honour to the memory of Mr. Whiston, that he was one of the first who explained the Newtonian philosophy in a popular way, so as to be intelligible to the generality of readers.—Among other things also, he translated the Apostolical Constitutions into English, which favoured the doctrine of the supremacy of the father and subordination of the son, vulgarly called the Arian heresy: Upon which his friends began to be alarmed for him; and the consequence shewed it was not groundless; for, Oct. 30, 1710, he was deprived of his professorship, and expelled the university of Cambridge, after he had been formally convened and interrogated for some days together.—At the conclusion of this year, he wrote his Historical preface, afterwards prefixed to his Primitive Christianity Revived, containing the reasons for his dissent from the commonly received notions of the Trinity, which work he published the next year, in 4 volumes 8vo, for which the Convocation fell upon him most vehemently.

In 1713, he and Mr. Ditton composed their scheme for finding the longitude, which they published the year following, a method which consisted in measuring distances by means of the velocity of sound; some more particulars of which are related in the life of Mr. Ditton.—In 1719, he published an ironical Letter of Thanks to doctor Robinson, bishop of London, for his late Letter to his clergy against the use of New Forms of Doxology. And, the same year, a Letter to the earl of Nottingham, Concerning the Eternity of the Son of God, and his Holy Spirit.—In 1720, he was proposed by Sir Hans Sloane and Dr. Halley to the Royal Society as a member; but was refused admittance by Sir Isaac Newton the president.

On Mr. Whiston's expulsion from Cambridge, he went to London, where he conferred with Doctors Clarke, Hoadly, and other learned men, who endeavoured to moderate his zeal, which however he would not suffer to be tainted or corrupted, and many were not much satisfied with the authority of these constitutions, but approved his integrity. Mr. Whiston now settled in London with his family; where, without suffering his zeal to be intimidated, he continued to write, and to propagate his Primitive Christianity with as much ardour as if he had been in the most flourishing circumstances; which however were so bad, that, in

 1721,

1721, a subscription was made for the support of his family, which amounted to 470l. For though he drew some profits from reading astronomical and philosophical lectures, and also from his publications, which were very numerous, yet these of themselves were very insufficient: nor, when joined with the benevolence and charity of those who loved and esteemed him for his learning, integrity, and piety, did they prevent his being frequently in great distress.—In 1722 he published an Essay towards restoring the true text of the Old Testament.—In 1724, The Literal Accomplishment of Scripture Prophecies.—Also, The Calculation of Solar Eclipses without Parallaxes.—In 1726, Of the Thundering Legion &c.—In 1727, A Collection of Authentic Records belonging to the Old and New Testament.—In 1730, Memoirs of the Life of Dr. Samuel Clarke.—In 1732, A Vindication of the Testimony of Phlegon, or an Account of the Great Darkness and Earthquake at our Saviour's Passion, described by Phlegon.—In 1736, Athanasian Forgeries, &c. And the Primitive Eucharist revived.—In 1737, The Astronomical Year, particularly of the Comet foretold by Sir Isaac Newton.—Also the Genuine Works of Flavius Josephus.—In 1739, Mr. Whiston put in his claim to the mathematical professorship at Cambridge, then vacant by the death of Dr. Saunderson, in a letter to Dr. Ashton, the master of Jesus-college; but no regard was paid to it.—In 1745, he published his Primitive New Testament in English.—In 1748, his Sacred History of the Old and New Testament. Also, Memoirs of his own Life and Writings, which are very curious.

Whiston continued many years a member of the established church; but at length forsook it, on account of the reading of the Athanasian Creed, and went over to the Baptists; which happened while he was at the house of Samuel Barker, Esq. at Lindon in Rutlandshire, who had married his daughter; where he died, after a week's illness, the 22d of August 1752, at upwards of 84 years of age.—We have mentioned the principal of his writings in the foregoing memoir; to which may be added, Chronological Tables, published in 1750.

The character of this conscientious and worthy man has been attempted by two very able personages, who were well acquainted with him, namely, bishop Hare and Mr. Collins, who unite in giving him the highest applauses, for his integrity, piety, &c.—Mr. Whiston left some children behind him; among them, Mr. John Whiston, who was for many years a very considerable bookseller in London.

WHITE, one of the colours of bodies. Though White cannot properly be said to be one colour, but rather a composition of all the colours together: for Newton has demonstrated that bodies only appear White by reflecting all the kinds of coloured rays alike; and that even the light of the sun is only White, because it consists of all colours mixed together.

This may be shewn mechanically in the following manner: Take seven parcels of coloured fine powders, the same as the primary colours of the rainbow, taking such quantities of these as shall be proportional to the respective breadths of these colours in the rainbow, which are of red 45 parts, orange 27, yellow 48, green 60, blue 60, indigo 40, and of violet 80; then mix intimately together these seven parcels of powders, and the mixture will be a pretty White colour: and this is only similar to the uniting the prismatic colours together again, to form a White ray or pencil of light of the whole of them. The same thing is done conveniently thus: Let the flat upper surface of a top be divided into 360 equal parts, all around its edge; then divide the same surface into seven sectors in the proportion of the numbers above, by seven radii or lines drawn from the centre; next let the respective colours be painted in a lively manner on these spaces, but so as the edge of each colour may be made nearly like the colour next adjoining, that the separation may not be well distinguished by the eye; then if the top be made to spin, the colours will thus seem to be mixed all together, and the whole surface will appear of a uniform whiteness: and if a large round black spot be painted in the middle, so as there may be only a broad flat ring of colours around it, the experiment will succeed the better. See Newton's Optics, prop. 6, book 1; and Ferguson's Tracts, pa. 296.

White bodies are found to take heat slower than black ones; because the latter absorb or imbibe rays of all kinds and colours, and the former reflect them. Hence it is that black paper is sooner put in flame, by a burning-glass, than White; and hence also black clothes, hung up in the sun by the dyers, dry sooner than white ones.

WHITEHURST (John), an ingenious English philosopher, was born at Congleton in the county of Cheshire, the 10th of April 1713, being the son of a clock and watch-maker there. Of the early part of his life but little is known; he who dies at an advanced age, leaving few behind him to communicate anecdotes of his youth. On his quitting school, where it seems the education he received was very defective, he was bred by his father to his own profession, in which he soon gave hopes of his future eminence.

It was very early in life that, from his vicinity to the many stupendous phenomena in Derbyshire, which were constantly presented to his observation, his attention was excited to enquire into the various causes of them.

At about the age of 21, his eagerness after new ideas carried him to Dublin, having heard of an ingenious piece of mechanism in that city, being a clock with certain curious appendages, which he was very desirous of seeing, and no less so of conversing with the maker. On his arrival however, he could neither procure a sight of the former, nor draw the least hint from the latter concerning it. Thus disappointed, he fell upon an expedient for accomplishing his design; and accordingly took up his residence in the house of the mechanic, paying the more liberally for his board, as he had hopes from thence of more readily obtaining the indulgence wished for. He was accommodated with a room directly over that in which the favourite piece was kept carefully locked up: and he had not long to wait for his gratification: for the artist, while one day employed in examining his machine, was suddenly called down stairs; which the young enquirer happening to overhear, softly slipped into the room, inspected the machine, and, presently satisfying himself as to the secret, escaped undiscovered

covered to his own apartment. His end thus compaffed, he fhortly after bid the artift farewell, and returned to his father in England.

About two or three years after his return from Ireland, he left Congleton, and entered into bufinefs for himfelf at Derby, where he foon got into great employment, and diftinguifhed himfelf very much by feveral ingenious pieces of mechanifm, both in his own regular line of bufinefs, and in various other refpects, as in the conftruction of curious thermometers, barometers, and other philofophical inftruments, as well as in ingenious contrivances for water-works, and the erection of various larger machines: being confulted in almoft all the undertakings in Derbyfhire, and in the neighbouring counties, where the aid of fuperior fkill, in mechanics, pneumatics, and hydraulics, was requifite.

In this manner his time was fully and ufefully employed in the country, till, in 1775, when the act paffed for the better regulation of the gold coin, he was appointed ftamper of the money-weights; an office conferred upon him, altogether unexpectedly, and without folicitation. Upon this occafion he removed to London, where he fpent the remainder of his days, in the conftant habits of cultivating fome ufeful parts of philofophy and mechanifm. And here too his houfe became the conftant refort of the ingenious and fcientific at large, of whatever nation or rank, and this to fuch a degree, as very often to impede him in the regular profecution of his own fpeculations.

In 1778, Mr. Whitehurft publifhed his Inquiry into the Original State and Formation of the Earth; of which a fecond edition appeared in 1786, confiderably enlarged and improved; and a third in 1792. This was the labour of many years; and the numerous inveftigations neceffary to its completion, were in themfelves alfo of fo untoward a nature, as at times, though he was naturally of a ftrong conftitution, not a little to prejudice his health. When he firft entered upon this fpecies of refearch, it was not altogether with a view to inveftigate the formation of the earth, but in part to obtain fuch a competent knowledge of fubterraneous geography as might become fubfervient to the purpofes of human life, by leading mankind to the difcovery of many valuable fubftances which lie concealed in the lower regions of the earth.

May the 13th, 1779, he was elected and admitted a Fellow of the Royal Society. He was alfo a member of fome other philofophical focieties, which admitted him of their refpective bodies, without his previous knowledge; but fo remote was he from any thing that might favour of oftentation, that this circumftance was known only to a very few of his moft confidential friends. Before he was admitted a member of the Royal Society, three feveral papers of his had been inferted in the Philofophical Tranfactions, viz, Thermometrical Obfervations at Derby, in vol. 57; An Account of a Machine for raifing Water, at Oulton, in Chefhire, in vol. 65; and Experiments on Ignited Subftances, vol. 66: which three papers were printed afterwards in the collection of his works in 1792.

In 1783 he made a fecond vifit to Ireland, with a view to examine the Giant's Caufeway, and other northern parts of that ifland, which he found to be chiefly compofed of volcanic matter: an account and reprefentations of which are inferted in the latter editions of his Inquiry. During this excurfion, he erected an engine, for raifing water from a well, to the fummit of a hill, in a bleaching ground, at Tullidoi, in the county of Tyrone: it is worked by a current of water, and for its utility is perhaps unequalled in any country.

In 1787 he publifhed, An Attempt toward obtaining Invariable Meafures of Length, Capacity, and Weight, from the Menfuration of Time. His plan is, to obtain a meafure of the greateft length that conveniency will permit, from two pendulums whofe vibrations are in the ratio of 2 to 1, and whofe lengths coincide nearly with the Englifh ftandard in whole numbers. The numbers which he has chofen fhew much ingenuity. On a fuppofition that the length of a feconds pendulum, in the latitude of London, is $39\frac{1}{5}$ inches, the length of one vibrating 42 times in a minute, muft be 80 inches; and of another vibrating 84 times in a minute muft be 20 inches; and their difference, 60 inches, or 5 feet, is his ftandard meafure. By the experiments however, the difference between the lengths of the two pendulum rods, was found to be only 59·892 inches, inftead of 60, owing to the error in the affumed length of the feconds pendulum, $39\frac{1}{5}$ inches being greater than the truth, which ought to be $39\frac{1}{8}$ very nearly. By this experiment, Mr. Whitehurft obtained a fact, as accurately as may be in a thing of this nature, viz, the difference between the lengths of two pendulum rods whofe vibrations are known: a datum from whence may be obtained, by calculation, the true lengths of pendulums, the fpaces through which heavy bodies fall in a given time, and many other particulars relating to the doctrine of gravitation, the figure of the earth, &c, &c.

Mr. Whitehurft had been at times fubject to flight attacks of the gout, and he had for feveral years felt himfelf gradually declining. By an attack of that difeafe in his ftomach, after a ftruggle of two or three months, it put an end to his laborious and ufeful life, on the 18th of February 1788, in the 75th year of his age, at his houfe in Bolt-court, Fleet-ftreet, being the fame houfe where another eminent felf-taught philofopher, Mr. James Ferguson, had immediately before him lived and died.

For feveral years before his death, Mr. Whitehurft had been at times occupied in arranging and completing fome papers, for a treatife on Chimneys, Ventilation, and Garden-ftoves; which have fince been collected and given to the public, by Dr. Willan, in 1794.

However refpectable Mr. Whitehurft may have been in mechanics, and thofe parts of natural fcience which he more immediately cultivated, he was of ftill higher account with his acquaintance and friends on the fcore of his moral qualities. To fay nothing of the uprightnefs and punctuality of his dealings in all tranfactions relative to bufinefs; few men have been known to poffefs more benevolent affections than he, or, being poffeffed of fuch, to direct them more judicioufly to their proper ends. As to his perfon, he was above the middle ftature, rather thin than otherwife, and of a countenance expreffive at once of penetration and mildnefs. His fine gray locks, unpolluted by art, gave a venerable air to his whole appearance. In drefs he was plain, in diet temperate, in his general intercourfe with mankind.

mankind easy and obliging. In company he was cheerful or grave alike, according to the dictate of the occasion; with now and then a peculiar species of humour about him, delivered with such gravity of manner and utterance, that those who knew him but slightly were apt to understand him as serious, when he was merely playful. But where any desire of information on subjects in which he was conversant was expressed, he omitted no opportunity of imparting it.

WHITSUNDAY, the 50th day or seventh sunday from Easter.—The season properly called Pentecost, is popularly called *Whitsuntide;* because, it is said, in the primitive church, the newly baptized persons came to church between Easter and Pentecost in *white* garments.

WILKINS (Dr. John), a very ingenious and learned English bishop and mathematician, was the son of a goldsmith at Oxford, and born in 1614. After being educated in Greek and Latin, in which he made a very quick progress, he was entered a student of New-Inn in that university, when he was but 13 years of age; but after a short stay there, he was removed to Magdalen Hall; where he took his degrees. Having entered into holy orders, he first became chaplain to William Lord Say, and afterwards to Charles Count Palatine of the Rhine, with whom he continued some time. Adhering to the Parliament during the civil wars, they made him warden of Wadham college about the year 1648. In 1656 he married the sister of Oliver Cromwell, then lord protector of England, who granted him a dispensation to hold his wardenship, notwithstanding his marriage. In 1659, he was by Richard Cromwell made master of Trinity college in Cambridge; but ejected the year following, upon the restoration. He was then chosen preacher to the society of Gray's Inn, and rector of St. Lawrence Jewry, London, upon the promotion of Dr. Seth Ward to the bishoprick of Exeter. About this time he became a member of the Royal Society, was chosen of their council, and proved one of their most eminent members. He was afterwards made dean of Rippon, and in 1668 bishop of Chester; but died of the stone in 1672, at 58 years of age.

Bishop Wilkins was a man who thought it prudent to submit to the powers in being; he therefore subscribed to the solemn league and covenant, while it was enforced; and was equally ready to swear allegiance to king Charles when he was restored: this, with his moderate spirit towards dissenters, rendered him not very agreeable to the churchmen; and yet several of them could not but give him one of the best of characters. Burnet writes, that "he was a man of as great a mind, as true a judgment, as eminent virtues, and of as good a soul, as any he ever knew: that though he married Cromwell's sister, yet he made no other use of that alliance, but to do good offices, and to cover the university of Oxford from the sourness of Owen and Goodwin. At Cambridge, he joined with those who studied to propagate better thoughts, to take men off from being in parties, or from narrow notions, from superstitious conceits, and fierceness about opinions. He was also a great observer and promoter of experimental philosophy, which was then a new thing, and much looked after. He was naturally ambitious, but was the wisest clergyman I ever knew. He was a lover of mankind, and had a delight in doing good." The same historian mentions afterwards another quality which Wilkins possessed in a supreme degree, and which it was well for him he did, since he had great occasion for the use of it; and that was, says he, "a courage, which could stand against a current, and against all the reproaches with which ill-natured clergymen studied to load him."

Of his publications, which are all of them very ingenious and learned, and many of them particularly curious and entertaining, the first was in 1638, when he was only 24 years of age, viz, The Discovery of a New World; or, A Discourse to prove, that it is probable there may be another Habitable World in the Moon; with a Discourse concerning the Possibility of a Passage thither.—In 1640, A Discourse concerning a New Planet, tending to prove that it is probable our earth is one of the Planets.—In 1641, Mercury; or, the Secret and Swift Messenger; shewing, how a man may with Privacy and Speed communicate his Thoughts to a Friend at any Distance, 8vo.—In 1648, Mathematical Magic; or, the Wonders that may be performed by Mathematical Geometry, 8vo. All these pieces were published entire in one volume 8vo, in 1708, under the title of, The Mathematical and Philosophical Works of the right rev. John Wilkins, &c; with a print of the author and general title page handsomely engraven, and an account of his life and writings. To this collection is also subjoined an abstract of a larger work, printed in 1668, folio, intitled, An Essay towards a Real Character and a Philosophical Language. These were all his mathematical and philosophical works; beside which, he wrote several tracts in theology, natural religion, and civil polity, which were much esteemed for their piety and moderation, and went through several editions.

WINCH, a popular term for a windlass. Also the bent handle for turning round wheels, grind-stones, &c.

WIND, a current, or stream of air, especially when it is moved by some natural cause.

Winds are denominated from the point of the compass or horizon they blow from; as the east Wind, north Wind, south Wind, &c.

Winds are also divided into several kinds; as *general, particular, perennial, stated, variable, &c.*

Constant or *Perennial* WINDS, are those that always blow the same way; such as the remarkable one between the two tropics, blowing constantly from east to west, called also the *general trade-Wind.*

Stated or *Periodical* WINDS, are those that constantly return at certain times. Such are the sea and land breezes, blowing from land to sea in the morning, and from sea to land in the evening. Such also are the shifting or particular trade Winds, which blow one way during certain months of the year, and the contrary way the rest of the year.

Variable or *Erratic* WINDS, are such as blow without any regularity either as to time, place, or direction. Such as the Winds that blow in the interior parts of England, &c: though some of these claim their certain times of the day; as, the north-Wind is most frequent in the morning, the west-Wind about noon, and the south-Wind in the night.

General

General WIND, is such as blows at the same time the same way, over a very large tract of ground, most part of the year; as the general trade-Wind.

Particular WINDS, include all others, excepting the general trade Winds.

Those peculiar to one little canton or province, are called *topical* or *provincial Winds*. The Winds are also divided, with respect to the points of the compass or of the horizon, into *cardinal* and *collateral*.

Cardinal WINDS, are those blowing from the four cardinal points, east, west, north, and south.

Collateral WINDS, are the intermediate Winds between any two cardinal Winds, and take their names from the point of the compass or horizon they blow from.

In Navigation, when the Wind blows gently, it is called a *breeze*; when it blows harder, it is called a *gale*, or a *stiff gale*; and when it blows very hard, a *storm*.

For a particular account of the trade-Winds, monsoons, &c, see Philos. Trans. number 183, or Abridg. vol. 2, p. 133. Also Robertson's Navigation book 5, sect. 6.

A Wind blowing from the sea, is always moist; as bringing with it the copious evaporation and exhalations from the waters: also, in summer, it is cool; and in winter warm. On the contrary, a Wind from the continent, is always dry; warm in summer, and cold in winter. Our northerly and southerly Winds however, which are usually accounted the causes of cold and warm weather, Dr. Derham observes, are really rather the effect of the cold or warmth of the atmosphere. Hence it is that we often find a warm southerly Wind suddenly change to the north, by the fall of snow or hail; and in a cold frosty morning, we find the Wind north, which afterward wheels about to the southerly quarter, when the sun has well warmed the air; and again in the cold evening, turns northerly, or easterly.

Physical Cause of WINDS. Some philosophers, as Descartes, Rohault, &c, account for the general Wind, from the diurnal rotation of the earth; and from this general Wind they derive all the particular ones. Thus, as the earth turns eastward, the particles of the air near the equator, being very light, are left behind; so that, in respect of the earth's surface, they move westwards, and become a constant easterly wind, as they are found between the tropics, in those parallels of latitude where the diurnal motion is swiftest. But yet, against this hypothesis, it is urged, that the air, being kept close to the earth by the principle of gravity, would in time acquire the same degree of velocity that the earth's surface moves with, as well in respect of the diurnal rotation, as of the annual revolution about the sun, which is about 30 times swifter.

Dr. Halley therefore substitutes another cause, capable of producing a like constant effect, not liable to the same objections, but more agreeable to the known properties of the elements of air and water, and the laws of the motion of fluid bodies. And that is the action of the sun's beams, as he passes every day over the air, earth, and water, combined with the situation of the adjoining continents. Thus, the air which is less rarefied or expanded by heat, must have a motion towards those parts which are more rarefied, and less ponderous, to bring the whole to an equilibrium; and as the sun keeps continually shifting to the westward, the tendency of the whole body of the lower air is that way. Thus a general easterly Wind is formed, which being impressed upon the air of a vast ocean, the parts impel one another, and so keep moving till the next return of the sun, by which so much of the motion as was lost, is again restored; and thus the easterly Wind is made perpetual. But as the air towards the north and south is less rarefied than in the middle, it follows that from both sides it ought to tend towards the equator.

This motion, compounded with the former easterly Wind, accounts for all the phenomena of the general trade-Winds, which, if the whole surface of the globe were sea, would blow quite round the world, as they are found to do in the Atlantic and the Ethiopic oceans. But the large continents of land in this middle tract, being excessively heated, communicate their heat to the air above them, by which it is exceedingly rarefied, which makes it necessary that the cooler and denser air should rush in towards it, to restore the equilibrium. This is supposed to be the cause why, near the coast of Guinea, the wind always sets in upon the land, blowing westerly instead of easterly.

From the same cause it happens, that there are such constant calms in that part of the ocean called the *rains*; for this tract being placed in the middle, between the westerly Winds blowing on the coast of Guinea, and the easterly trade-Winds blowing to the westward of it; the tendency of the air here is indifferent to either, and so stands in equilibrio between both; and the weight of the incumbent atmosphere being diminished by the continual contrary Winds blowing from hence, is the reason that the air here retains not the copious vapour it receives, but lets it fall in so frequent rains.

It is also to be considered, that to the northward of the Indian ocean there is every where land, within the usual limits of the latitude of 30°, viz, Arabia, Persia, India, &c, which are subject to excessive heats when the sun is to the north, passing nearly vertical; but which are temperate enough when the sun is removed towards the other tropic, because of a ridge of mountains at some distance within the land, said to be often in winter covered with snow, over which the air as it passes must needs be much chilled. Hence it happens that the air coming, according to the general rule, out of the north-east, to the Indian sea, is sometimes hotter, sometimes colder, than that which, by a circulation of one current over another, is returned out of the south-west; and consequently sometimes the under current, or Wind, is from the north-east, sometimes from the south-west.

That this has no other cause, appears from the times when these Winds set, viz, in April; when the sun begins to warm these countries to the north, the south-west monsoons begin, and blow during the heats till October, when the sun being retired, and all things growing cooler northward, but the heat increasing to the south, the north-east Winds enter, and blow all the winter, till April again. And it is doubtless from the same principle, that to the southward of the equator, in part of the Indian ocean, the north-west Winds succeed the south-east, when the sun draws near the tropic

tropic of Capricorn. Philos. Trans. num. 183; or Abridg. vol. 2, pa. 193.

But some philosophers, not satisfied with Dr. Halley's theory above recited, or thinking it not sufficient for explaining the various phenomena of the Wind, have had recourse to another cause, viz, the gravitation of the earth and its atmosphere towards the sun and moon, to which the tides are confessedly owing. They allege that, though we cannot discover aërial tides, of ebb or flow, by means of the barometer, because columns of air of unequal height, but different density, may have the same pressure or weight; yet the protuberance in the atmosphere, which is continually following the moon, must, say they, occasion a motion in all parts, and so produce a Wind more or less to every place, which conspiring with, or being counteracted by the Winds arising from other causes, makes them greater or less. Several dissertations to this purpose were published, on occasion of the subject proposed by the Academy of Sciences at Berlin, for the year 1746. But Musschenbroek will not allow that the attraction of the moon is the cause of the general Wind; because the east Wind does not follow the motion of the moon about the earth; for in that case there would be more than 24 changes, to which it would be subject in the course of a year, instead of two. Introd. ad Phil. Nat. vol. 2, pa. 1102.

And Mr. Henry Eeles, conceiving that the rarefaction of the air by the sun cannot simply be the cause of all the regular and irregular motions which we find in the atmosphere, ascribes them to another cause, viz, the ascent and descent of vapour and exhalation, attended by the electrical fire or fluid; and on this principle he has endeavoured to explain at large the general phenomena of the weather and barometer. Philos. Trans. vol. 49, pa. 124.

Laws of the Production of Wind.

The chief laws concerning the production of Wind, may be collected under the following heads.

1. If the spring of the air be weakened in any place more than in the adjoining places, a Wind will blow through the place where the diminution is; because the less elastic or forcible will give way to that which is more so, and thence induce a current of air into that place, or a Wind. Hence, because the spring of the air increases, as the compressing weight increases, and compressed air is denser than that which is less compressed; all Winds blow into rarer air, out of a place filled with a denser.

2. Therefore, because a denser air is specifically heavier than a rarer; an extraordinary lightness of the air in any place must be attended with extraordinary Winds, or storms. Now, an extraordinary fall of the mercury in the barometer shewing an extraordinary lightness of the atmosphere, it is no wonder if that foretels storms of Wind and rain.

3. If the air be suddenly condensed in any place, its spring will be suddenly diminished: and hence, if this diminution be great enough to affect the barometer, a Wind will blow through the condensed air. But since the air cannot be suddenly condensed, unless it has before been much rarefied, a Wind will blow through the air, as it cools, after having been violently heated.

4. In like manner, if air be suddenly rarefied, its spring is suddenly increased; and it will therefore flow through the air not acted on by the rarefying force. Hence a Wind will blow out of a place, in which the air is suddenly rarefied; and on this principle probably it is, that the sun, by rarefying the air, must have a great influence on the production of Winds.

5. Most caves are found to emit Wind, either more or less. Musschenbroek has enumerated a variety of causes that produce Winds, existing in the bowels of the earth, on its surface, in the atmosphere, and above it. See Introd. ad Phil. Nat. vol. 2, pa. 1116.

6. The rising and changing of the Winds are determined by weathercocks, placed on the tops of high buildings, &c. But these only indicate what passes about their own height, or near the surface of the earth. And Wolfius assures us, from observations of several years, that the higher Winds, which drive the clouds, are different from the lower ones, which move the weathercocks. Indeed it is no uncommon thing to see one tier of clouds driven one way by a Wind, and another tier just over the former driven the contrary way, by another current of air, and that often with very different velocities. And the late experiments with air balloons have proved the frequent existence of counter Winds, or currents of air, even when it was not otherwise visible, nor at all expected; by which they have been found to take very different and unexpected courses, as they have ascended higher and higher in the atmosphere.

Laws of the Force and Velocity of the Wind.

Wind being only air in motion, and the motion of a fluid against a body at rest, creating the same resistance as when the body moves with the same velocity through the fluid at rest; it follows, that the force of the Wind, and the laws of its action upon bodies, may be referred to those of their resistance when moved through it; and as these circumstances have been treated pretty fully under the article Resistance *of the Air*, there is no occasion here to make a repetition of them. We there laid down both the quantity and laws of such a force, upon bodies of different shapes and sizes, moving with all degrees of velocity up to 2000 feet per second, and also for planes set at all degrees of obliquity, or inclination to the direction of motion; all which circumstances having, for the first time, been determined by real experiments.

As to the Velocity of the Wind: philosophers have made use of various methods for determining it. The method employed by Dr. Derham, was by letting light downy feathers fly in the air, and nicely observing the distance to which they were carried in any number of half seconds. He says that he thus measured the velocity of the Wind in the great storm of August 1705, which he found moved at the rate of 33 feet in half a second, or 45 miles per hour: whence he concludes, that the most vehement Wind does not fly at the rate of above 50 or 60 miles an hour; and that at a medium the velocity of Wind is at the rate of 12 or 15 miles per hour. Philos. Trans. number 313, or Abridg. vol. 4, p. 411.

Mr. Brice observes however, that experiments with feathers are liable to much uncertainty; as they hardly ever

ever go forward in a ſtraight direction, but ſpirally, or elſe irregularly from ſide to ſide, or up and down.

He therefore conſiders the motion of a cloud, by means of its ſhadow over the ſurface of the earth, as a much more accurate meaſure of the velocity of the Wind. In this way he found that the Wind, in a conſiderable ſtorm, moved at the rate of near 63 miles an hour; and when it blew a freſh gale, at the rate of 21 miles per hour; and in a ſmall breeze it was near 10 miles an hour. Philoſ. Tranſ. vol. 56, p. 226.

The velocity and force of the Wind are alſo determined experimentally by various machines, called *anemometers*, *wind-meaſurers*, or *wind-gages*; the deſcription of which ſee under theſe articles.

In the Philoſ. Tranſ. for 1759, p. 165, Mr. Smeaton has given a table, communicated to him by a Mr. Rouſe, for ſhewing the force of the Wind, with ſeveral different velocities, which I ſhall inſert below, as I find the numbers nearly agree with my own experiments made on the reſiſtance of the air, when the reſiſting ſurfaces are reduced to the ſame ſize, by a due proportion for the reſiſtance, which is in a higher degree than that of the ſurfaces.

N. B. The table of my reſults is printed in pa. 111, vol. 1, under the article Anemometer; where it is to be noted, that the numbers in the third column of that table, for the velocity of the Wind per hour, are all erroneouſly printed, only the 4th part of what each of them ought to be; ſo that thoſe numbers muſt be all multiplied by 4.

A Table of the different Velocities and Forces of the Wind, according to their common appellations.

Velocity of the Wind		Perpendicular force on one ſq. foot, in averdupois pounds.	Common appellations of the Winds.
Miles in one hour.	= feet in one ſecond.		
1	1·47	·005	Hardly perceptible.
2	2·93	·020	Juſt perceptible.
3	4·40	·044	
4	5·87	·079	Gentle pleaſant wind.
5	7·33	·123	
10	14·67	·492	Pleaſant briſk gale.
15	22·00	1·107	
20	29·34	1·968	Very briſk.
25	36·67	3·075	
30	44·01	4·429	High Winds.
35	51·34	6·027	
40	58·68	7·873	Very high.
45	66·01	9·963	
50	73·35	12·300	A ſtorm or tempeſt.
60	88·02	17·715	A great ſtorm.
80	117·36	31·490	A hurricane.
100	146·70	49·200	A hurricane that tears up trees, and carries buildings &c before it.

The force of the Wind is nearly as the ſquare of the velocity, or but little above it, in theſe velocities. But the force is much more than in the ſimple ratio of the ſurfaces, with the ſame velocity, and this increaſe of the ratio is the more, as the velocity is the more. By accurate experiments with two planes, the one of $17\frac{1}{4}$ ſquare inches, the other of 32, which are nearly in the ratio of 5 to 9, I found their reſiſtances, with a velocity of 20 feet per ſecond, to be, the one 1·196 ounces, and the other 2·542 ounces; which are in the ratio of 8 to 17, being an increaſe of between $\frac{1}{5}$ and $\frac{1}{6}$ part more than the ratio of the ſurfaces.

Wind-*Gage*, in Pneumatics, an inſtrument ſerving to determine the velocity and force of the Wind. See Anemometer, Anemoscope, and the article juſt above concerning the Force and Velocity of the Wind.

Dr. Hales had various contrivances for this purpoſe. He found (Statical Eſſays, vol. 2, p. 326) that the air ruſhed out of a ſmith's bellows, at the rate of $68\frac{3}{4}$ feet in a ſecond of time, when compreſſed with a force of half a pound upon every ſquare inch lying on the whole upper ſurface of the bellows. The velocity of the air, as it paſſed out of the trunk of his ventilators, was found to be at the rate of 3000 feet in a minute, which is at the rate of 34 miles an hour. The ſame author ſays, that the velocity with which impelled air paſſes out at any orifice, may be determined by hanging a light valve over the noſe of a bellows, by pliant leathern hinges, which will be much agitated and lifted up from a perpendicular to a more than horizontal poſition by the force of the ruſhing air. There is alſo another more accurate way, he ſays, of eſtimating the velocity of air, viz, by holding the orifice of an inverted glaſs ſiphon full of water, oppoſite to the ſtream of air, by which the water will be depreſſed in one leg, and raiſed in the other, in proportion to the force with which the water is impelled by the air. Deſcrip. of Ventilators, 1743, p. 12. And this perhaps gave Dr. Lind the idea of his Wind-gage, deſcribed below.

M. Bouguer contrived a ſimple inſtrument, by which may be immediately diſcovered the force which the Wind exerts on a given ſurface. This is a hollow tube AABB (fig. 14, pl. 30), in which a ſpiral ſpring CD is fixed, that may be more or leſs compreſſed by a rod FSD, paſſing through a hole within the tube at AA. Then having obſerved to what degree different forces or given weights are capable of compreſſing the ſpiral, mark diviſions on the rod in ſuch a manner, that the mark at S may indicate the weight requiſite to force the ſpring into the ſituation CD: afterwards join at right angles to this rod at F, a plane ſurface CFE of any given area at pleaſure; then let this inſtrument be oppoſed to the Wind, ſo that it may ſtrike the ſurface perpendicularly, or parallel to the rod; then will the mark at S ſhew the weight to which the force of the Wind is equivalent.

Dr. Lind has alſo contrived a ſimple and eaſy apparatus of this kind, nearly upon the laſt idea of Dr. Hales mentioned above. This inſtrument is fully explained at the article Anemometer, vol. 1, pa. 111, and a figure of it given, pl. 3, fig. 4.

Mr. Benjamin Martin, from a hint firſt ſuggeſted by Dr. Burton, contrived an anemoſcope, or Wind-gage, of a conſtruction like a Wind-mill, with four ſails; but the axis which the ſails turn, is not cylindrical, but conical, like the fuſee of a watch; about this fuſee winds a cord, having a weight at the end, which is

wound always, by the force of the Wind, upon the fails, till the weight juft balances that force, which will be at a thicker part of the fufee when the Wind is ftrong, and at a fmaller part of it when it is weaker. But although this inftrument fhews when a Wind is ftronger or weaker, it will neither fhew what is the actual velocity of the Wind, nor yet its force upon a fquare foot of direct furface; becaufe the fails are fet at an uncertain oblique angle to the Wind, and this acts at different diftances from the axis or centre of motion. Martin's Phil. Brit. vol. 2, p. 211. See the fig. 5, plate 3, vol. 1.

Wind-*Gun*, the fame as Air-*Gun*; which fee.

Wind-*Mill*, a kind of mill which receives its motion from the impulfe of the Wind.

The internal ftructure of the Windmill is much the fame with that of watermills: the difference between them lying chiefly in an external apparatus, for the application of the power. This apparatus confifts of an axis EF (fig. 11, pl. 36), through which pafs perpendicular to it, and to each other, two arms or yards, AB and CD, ufually about 32 feet long: on thefe yards are formed a kind of fails, vanes, or flights, in a trapezoid form, with parallel ends; the greater of which HI is about 6 feet, and the lefs FG are determined by radii drawn from the centre E, to I and H.

Thefe fails are to be capable of being always turned to the wind, to receive its impulfe: for which purpofe there are two different contrivances, which conftitute the two different kinds of Windmills in common ufe.

In the one, the whole machine is fupported upon a moveable arbor, or axis, fixed upright on a ftand or foot; and turned round occafionally to fuit the wind, by means of a lever.

In the other, only the cover or roof of the machine, with the axis and fails, in like manner turns round with a parallel or horizontal motion. For this purpofe, the cover is built turret-wife, and encompaffed with a wooden ring, having a groove, at the bottom of which are placed, at certain diftances, a number of brafs truckles; and within the groove is another ring, upon which the whole turret ftands. To the moveable ring are connected beams *ab* and *fe*; and to the beam *ab* is faftened a rope at *b*, having its other end fitted to a windlafs, or axis-in-peritrochio: this rope being drawn through the iron hook G, and the windlafs turned, the fails are moved round, and fet fronting the wind, or with the axis pointing ftraight againft the wind.

The internal mechanifm of a Windmill is exhibited in fig. 12; where AHO is the upper room, and H*o*Z the lower one; AB the axle-tree paffing through the mill; STVW the fails covered with canvas, fet obliquely to the wind, and turning round in the order of the letters; CD the cogwheel, having about 48 cogs or teeth, *a*, *a*, *a*, &c, which carry round the lantern EF, having 8 or 9 trundles or rounds *c*, *c*, *c*, &c, together with its upright axis GN; IK is the upper millftone, and LM the lower one; QR is the bridge, fupporting the axis or fpindle GN; this bridge is fupported by the beams *cd*, XY, wedged up at *c*, *d* and X; ZY is the lifting tree, which ftands upright; *ab* and *ef* are levers, whofe centres of motion are Z and *e*; *fghi* is a cord, with a ftone *i*, going about the pins *g* and *h*, and ferving as a balance or counterpoife. The fpindle *t*N is fixed to the upper millftone IK, by a piece of iron called the rynd, and fixed in the lower fide of the ftone, which is the only one that turns about, and its whole weight refts upon a hard ftone, fixed in the bridge QR at N. The trundle EF, and its axis G*t*, may be taken away; for it refts by its lower part at *t* by a fquare focket, and the top runs in the edge of the beam *w*. By bearing down the end *f* of the lever *fe*, *b* is raifed, which raifes ZY, and this raifes YX, which lifts up the bridge QR, with the axis NG, and the upper ftone IK; and thus the ftones are fet at any diftance. The lower or immoveable ftone is fixed upon ftrong beams, and is broader than the upper one: the flour is conveyed through the tunnel *no* into a cheft; P is the hopper, into which is put the corn, which runs through the fpout *r* into the hole *t*, and fo falls between the ftones, where it is ground to meal. The axis G*t* is fquare, which fhaking the fpout *r*, as it goes round, makes the corn run out; *rs* is a ftring going about the pin *s*, and ferving to move the fpout nearer to the axis or farther from it, fo as to make the corn run fafter or flower, according to the velocity and force of the wind. And when the wind is ftrong, the fails are only covered in part, or on one fide, or perhaps only one half of two oppofite fails. Toward the end B of the axletree is placed another cogwheel, trundle, and millftones, with an apparatus like that juft defcribed; fo that the fame axis moves two ftones at once; and when only one pair is to grind, one of the trundles and its fpindle are taken out: *xyl* is a girth of pliable wood, fixed at the end *x*; the other end *l* being tied to the lever *km*, moveable about *k*; and the end *m* being put down, draws the girth *xyl* clofe to the cogwheel, which gently and gradually ftops the motion of the mill, when required: *pq* is a ladder for afcending to the higher part of the mill; and the corn is drawn up by means of a rope, rolled about the axis AB, when the mill is at work. See Mill.

Theory of the Windmill, *Pofition of the Sails, &c.*

Were the fails fet fquare upon their arms or yards, and perpendicular to the axletree, or to the wind, no motion would enfue, becaufe the direct wind would keep them in an exact balance. But by fetting them obliquely to the common axis, like the fails of a fmoke-jack, or inclined like the rudder of a fhip, the wind, by ftriking the furface of them obliquely, turns them about. Now this angle which the fails are to make with their common axis, or the degree of *weathering*, as the mill-wrights call it, fo as that the wind may have the greateft effect, is a matter of nice enquiry, and has much occupied the thoughts of the mathematician and the artift.

In examining the compound motions of the rudder of a fhip, we find that the more it approaches to the direction of the keel, or to the courfe of the water, the more weakly this ftrikes it; but, on the other hand, the greater is the power of the lever to turn the veffel about. The obliquity of the rudder therefore has, at the fame time, both an advantage and a difadvantage. It has been a point of inquiry therefore to find the pofition of the rudder when the ratio of the advantage over the difadvantage is the greateft. And M. Renau, in his

his theory of the working of ſhips, has found, that the beſt ſituation of the rudder is when it makes an angle of about 55 degrees with the keel.

The obliquity of the ſails, with regard to their axis, has preciſely the ſame advantage, and diſadvantage, with the obliquity of the rudder to the keel. And M. Parent, ſeeking by the new analyſis the moſt advantageous ſituation of the ſails on the axis, finds it the ſame angle of about 55 degrees. This obliquity has been determined by many other mathematicians, and found to be more accurately 54° 44′. See Maclaurin's Fluxions, p. 733; Simpſon's Fluxions, prob. 17, p. 521; Martin's Philoſ. Britan. vol. 1, p. 220, vol. 2, p. 212; &c.

This angle, however, is only that which gives the wind the greateſt force to put the ſail in motion, but not the angle which gives the force of the wind a maximum upon the ſail when in motion: for when the ſail has a certain degree of velocity, it yields to the wind; and then that angle muſt be increaſed, to give the wind its full effect. Maclaurin, in his Fluxions, p. 734, has ſhewn how to determine this angle.

It may be obſerved, that the increaſe of this angle ſhould be different according to the different velocities from the axletree to the further extremity of the ſail. At the beginning, or axis, it ſhould be 54° 44′; and thence continually increaſing, giving the vane a twiſt, and ſo cauſing all the ribs of the vane to lie in different planes.

It is farther obſerved, that the ribs of the vane or ſail ought to decreaſe in length from the axis to the extremity, giving the vane a curvilinear form; ſo that no part of the force of any one rib be ſpent upon the reſt, but all move on independent of each other. The twiſt above mentioned, and the diminution of the ribs, are exemplified in the wings of birds.

As the ends of the ſail neareſt the axis cannot move with the ſame velocity which the tips or fartheſt ends have, although the wind acts equally ſtrong upon them both, Mr. Ferguſon (Lect. on Mech. pa. 52) ſuggeſts, that perhaps a better poſition than that of ſtretching them along the arms directly from the centre of motion, might be, to have them ſet perpendicularly acroſs the farther ends of the arms, and there adjuſted lengthwiſe to the proper angle: for in that caſe both ends of the ſails would move with the ſame velocity; and being farther from the centre of motion they would have ſo much the more power, and then there would be no occaſion for having them ſo large as they are generally made; which would render them lighter, and conſequently there would be ſo much the leſs friction on the thick neck of the axle, when it turns in the wall.

Mr. Smeaton (Philoſ. Tranſ. 1759), from his experiments with Windmill ſails, deduces ſeveral practical maxims: as,

1. That when the wind falls upon a concave ſurface, it is an advantage to the power of the whole, though every part, taken ſeparately, ſhould not be diſpoſed to the beſt advantage. By ſeveral trials he has found that the curved form and poſition of the ſails will be beſt regulated by the numbers in the following table.

6th Parts of the radius or ſail.	Angle with the axis.	Angle with the plane of motion.
1	72°	18°
2	71	19
3	72	18 middle.
4	74	16
5	$77\frac{1}{2}$	$12\frac{1}{2}$
6	83	7 end.

2. That a broader ſail requires a greater angle; and that when the ſail is broader at the extremity, than near the centre, this ſhape is more advantageous than that of a parallelogram.

3. When the ſails, made like ſectors of circles, joining at the centre or axis, filled up about 7-8ths of the whole circular ſpace, the effect was the greateſt.

4. The velocity of Windmill ſails, whether unloaded, or loaded ſo as to produce a maximum of effect, is nearly as the velocity of the Wind; their ſhape and poſition being the ſame.

5. The load at the maximum is nearly, but ſomewhat leſs than, as the ſquare of the velocity of the wind.

6. The effects of the ſame ſails at a maximum, are nearly, but ſomewhat leſs than, as the cubes of the velocity of the wind.

7. In ſails of a ſimilar figure and poſition, the number of turns in a given time, are reciprocally as the radius or length of the ſail.

8. The effects of ſails of ſimilar figure and poſition, are as the ſquare of their length.

9. The velocity of the extremities of Dutch mills, as well as of the enlarged ſails, in all their uſual poſitions, is conſiderably greater than the velocity of the wind.

M. Parent, in conſidering what figure the ſails of a Windmill ſhould have, to receive the greateſt impulſe from the wind, finds it to be a ſector of an ellipſis, whoſe centre is that of the axletree of the mill; and the leſs ſemiaxis the height of 32 feet; as for the greater, it follows neceſſarily from the rule that directs the ſail to be inclined to the axis in the angle of 55 degrees.

On this foundation he aſſumes four ſuch ſails, each being a quarter of an ellipſe; which he ſhews will receive all the wind, and loſe none, as the common ones do. Theſe 4 ſurfaces, multiplied by the lever, with which the wind acts on one of them, expreſs the whole power the wind has to move the machine, or the whole power the machine has when in motion.

A Windmill with 6 elliptical ſails, he ſhews, would ſtill have more power than one with only four. It would only have the ſame ſurface with the four; ſince the 4 contain the whole ſpace of the ellipſis, as well as the 6. But the force of the 6 would be greater than that of the 4, in the ratio of 245 to 231. If it were deſired to have only two ſails, each being a ſemiellipſis, the ſurface would be ſtill the ſame; but the power would be diminiſhed by near 1-3d of that with 6 ſails; becauſe the greatneſs of the ſectors would much ſhorten the lever with which the wind acts.

The ſame author has alſo conſidered which form, among the rectangular ſails, will be moſt advantageous; i. e.

i. e. that which ſhall have the product of the ſurface by the lever of the wind, the greateſt. The reſult of this enquiry is, that the width of the rectangular ſail ſhould be nearly double its length ; whereas uſually the length is made almoſt 5 times the width.

The power of the mill, with four of theſe new rectangular ſails, M. Parent ſhews, will be to the power of four elliptic ſails, nearly as 12 to 23 ; which leaves a conſiderable advantage on the ſide of the elliptic ones; and yet the force of the new rectangular ſails will ſtill be conſiderably greater than that of the common ones.

M. Parent alſo conſiders what number of the new ſails will be moſt advantageous ; and finds that the fewer the ſails, the more ſurface there will be, but the power the leſs. Farther, the power of a Windmill with 6 ſails is denoted by 14, that of another with 4 will be as 13, and another with 2 ſails will be denoted by 9. That as to the common Windmill, its power ſtill diminiſhes as the breadth of the ſails is ſmaller, in proportion to the length : and therefore the uſual proportion of 5 to 1 is exceedingly diſadvantageous.

WINDWARD, in Sea Language, denotes any thing towards that point from whence the wind blows, in reſpect of a ſhip.

Sailing to WINDWARD. See SAILING.

WINDWARD *Tide*, denotes a tide that runs againſt the wind.

WINDAGE *of a Gun*, is the difference between the diameter of the bore of the gun and the diameter of the ball.

Heretofore the Windage appointed in the Engliſh ſervice, viz, 1-20th of the diameter of the ball, which has been uſed almoſt from the beginning, has been far too much, owing perhaps to the firſt want of roundneſs in the ball, or to ruſt, foulneſs, or irregularities in the bore of the gun. But lately a beginning has been made to diminiſh the Windage, which cannot fail to be of very great advantage ; as the ſhot will both go much truer, and have leſs room to bounce about from ſide to ſide, to the great damage of the gun ; and beſides much leſs powder will ſerve for the ſame effect, as in ſome caſes $\frac{1}{3}$ or $\frac{1}{2}$ the inflamed powder eſcapes by the Windage. The French allowance of Windage is 1-25th of the diameter of the ball.

WINDLASS, or WINDLACE, a particular machine uſed for raiſing heavy weights, as guns, ſtones, anchors, &c.

This is a very ſimple machine, conſiſting only of an axis or roller, ſupported horizontally at the two ends by two pieces of wood and a pulley : the two pieces of wood meet at top, being placed diagonally ſo as to prop each other ; and the axis or roller goes through the two pieces, and turns in them. The pulley is faſtened at top, where the pieces join. Laſtly, there are two ſtaves or hand ſpikes which go through the roller, to turn it by ; and the rope, which comes over the pulley, is wound off and on the ſame.

WINDLASS, in a Ship, is an inſtrument in ſmall ſhips, placed upon the deck, juſt abaft the foremaſt. It is made of a long and thick piece of timber, either cylindrical, or octagonal, &c, in form of an axletree, placed horizontally acroſs the ſhip, a foot or more above the deck ; and it is turned about by the help of handſpikes put into holes made for that purpoſe.

This machine will purchaſe or raiſe much more than a capſtan, and that without any danger to thoſe that heave ; for if in heaving the Windlaſs about, any of the handſpikes ſhould happen to ſlip or break, the Windlaſs will ſtop of itſelf, as it does at the end of every pull or heave of the men, being prevented from returning by means of a catch that falls into notches. See fig. 15, pl. 35.

WINDOW, q. d. *wind-door*, an aperture or opening in the wall of a houſe, to admit the air and light.

Before the uſe of glaſs became general, which was not till towards the end of the 12th century, the Windows in England ſeem generally to have been compoſed of paper, oiled, both to defend it againſt the weather, and to make it more tranſparent ; as now is ſometimes uſed in workſhops and unfiniſhed buildings. Some of the better ſort were furniſhed with lattices of wood or ſheets of linen. Theſe it ſeems were fixed in frames, called *capſamenta*, and hence our *caſements* ſtill ſo common in ſome of the counties.

The chief rules with regard to Windows are, 1. That they be as few in number, and as moderate in dimenſions, as may be conſiſtent with other reſpects; inaſmuch as all openings are weakenings.

2. That they be placed at a convenient diſtance from the angles or corners of the buildings : both for ſtrength and beauty.

3. That they be made all equal one with another, in their rank and order ; ſo that thoſe on the right hand may anſwer to thoſe on the left, and thoſe above be right over thoſe below : both for ſtrength and beauty.

As to their dimenſions, care is to be taken, to give them neither more nor leſs than is needful ; regard being had to the ſize of the rooms, and of the building. The apertures of Windows in middle-ſized houſes, may be from 4 to 5 feet ; in the ſmaller ones leſs ; and in large buildings more. And the height may be double their width at the leaſt : but in lofty rooms, or large buildings, the height may be a 4th, or 3d, or half their breadth more than the double.

Such are the proportions for Windows of the firſt ſtory ; and the breadth muſt be the ſame in the upper ſtories ; but as to the height, the ſecond ſtory may be a 3d part lower than the firſt, and the third ſtory a 4th part lower than the ſecond.

WINTER, one of the four ſeaſons or quarters of the year.

Winter properly commences on the day when the ſun's diſtance from the zenith of the place is the greateſt, or when his declination is the greateſt on the contrary ſide of the equator ; and it ends on the day when that diſtance is a mean between the greateſt and leaſt, or when he next croſſes the equinoctial.

At and near the equator, the Winter, as well as the other ſeaſons, return twice every year ; but all other places have only one Winter in the year ; which in the northern hemiſphere begins when the ſun is in the tropic of Capricorn, and in the ſouthern hemiſphere when he is in the tropic of Cancer : ſo that all places in the ſame hemiſphere have their Winter at the ſame time.

Notwithſtanding the coldneſs of this ſeaſon it is proved in aſtronomy, that the ſun is really nearer to the earth in our Winter than in ſummer : the reaſon of the defect

defect of heat being owing to the lownefs of the fun, or to the obliquity of his rays.

WOLFF, Wolfius, (Christian), baron of the Roman empire, privy counfellor to the king of Pruffia, and chancellor to the univerfity of Halle in Saxony, as well as member of many of the literary academies in Europe, was born at Breflau in 1679. After ftudying philofophy and mathematics at Breflau and Jena, he obtained permiffion to give lectures at Leipfic; which, in 1703, he opened with a differtation called *Philofophia Practica Univerfalis, Methodo Mathematica confcripta*, which ferved greatly to enhance the reputation of his talents. He publifhed two other differtations the fame year; the firft *De Rotis Dentatis*, the other *De Algorithmo Infinitefimali Differentiali*; which obtained him the honourable appellation of Affiftant to the Faculty of Philofophy at Leipfic.

He now accepted the profefforfhip of mathematics at Halle, and was elected into the fociety at Leipfic, at that time engaged in publifhing the *Acta Eruditorum*. After having inferted in this work many important pieces relating to mathematics and phyfics, he undertook, in 1709, to teach all the various branches of philofophy, beginning with a fmall Logical treatife in Latin, being Thoughts on the Powers of the Human Underftanding. He carried himfelf through thefe great purfuits with amazing affiduity and ardour: the king of Pruffia rewarded him with the office of counfellor to the court in 1721, and augmented the profits of that poft by very confiderable appointments: he was alfo chofen a member of the Royal Society of London and of Pruffia.

In the midft of all this profperity however, Wolff raifed an ecclefiaftical ftorm againft himfelf, by a Latin oration he delivered in praife of the Chinefe philofophy: every pulpit immediately refounded againft his tenets; and the faculty of theology, who entered into a ftrict examination of his productions, refolving that the doctrine he taught was dangerous to the laft degree, an order was obtained in 1723 for difplacing him, and commanding him to leave Halle in 24 hours.

Wolff now retired to Caffel, where he obtained the profefforfhip of mathematics and philofophy in the univerfity of Marbourg, with the title of Counfellor to the Landgrave of Heffe; to which a profitable penfion was annexed. Here he renewed his labours with redoubled ardour; and it was in this retreat that he publifhed the greateft part of his numerous works.

In 1725, he was declared an honorary profeffor of the academy of fciences at Peterfburg, and in 1733 was admitted into that of Paris. The king of Sweden alfo declared him one of the council of regency; but the pleafing fituation of his new abode, and the multitude of honours which he had received, were too alluring to permit him to accept of many advantageous offers; among which was the office of prefident of the academy at Peterfburg.

The king of Pruffia too, who was now recovered from the prejudices he had been made to conceive againft Wolff, wanted to re-eftablifh him in the univerfity of Halle in 1733, and made another attempt to effect it in 1739; which Wolff for a time thought fit to decline, but at laft fubmitted: he returned therefore in 1741, invefted with the characters of privy counfellor, vice chancellor, and profeffor of the law of nature and of nations. The king afterwards, upon a vacancy, raifed him to the dignity of chancellor of the univerfity; and the elector of Bavaria created him a baron of the empire. He died at Halle in Saxony, of the gout in his ftomach, in 1754, in the 76th year of his age, after a life filled up with a train of actions as wife and fyftematical as his writings, of which he compofed in Latin and German more than 60 diftinct pieces. The chief of his mathematical compofitions, is his *Elementa Mathefeos Univerfæ*, the beft edition of which is that of 1732 at Geneva, in 5 vols 4to; which does not however comprife his Mathematical Dictionary in the German language, in 1 vol. 8vo, nor many other diftinct works on different branches of the mathematics, nor his Syftem of Philofophy, in 23 vols. in 4to.

WORKING *to Windward*, in Sea Language, is the operation by which a fhip endeavours to make a progrefs againft the wind.

WREN (Sir Christopher), a great philofopher and mathematician, and one of the moft learned and eminent architects of his age, was the fon of the rev. Chriftopher Wren, dean of Windfor, and was born at Knoyle in Wiltfhire in 1632. He ftudied at Wadham college, Oxford; where he took the degree of mafter of arts in 1653, and was chofen fellow of Allfouls college there. Soon after, he became one of that ingenious and learned fociety, who then met at Oxford for the improvement of natural and experimental philofophy, and which at length produced the Royal Society.

When very young, he difcovered a furprifing genius for the mathematics, in which fcience he made great advances before he was 16 years of age.—In 1657 he was made profeffor of aftronomy in Grefham college, London; and his lectures, which were much frequented, tended greatly to the promotion of real knowledge: in his inaugural oration, among other things, he propofed feveral methods by which to account for the fhadows returning backward 10 degrees on the dial of king Ahaz, by the laws of nature. One fubject of his lectures was upon telefcopes, to the improvement of which he had greatly contributed: another was on certain properties of the air, and the barometer. In the year 1658 he read a defcription of the body and different phafes of the planet Saturn; which fubject he propofed to inveftigate while his colleague, Mr. Rooke, then profeffor of geometry, was carrying on his obfervations upon the fatellites of Jupiter. The fame year he communicated fome demonftrations concerning cycloids to Dr. Wallis, which were afterwards publifhed by the doctor at the end of his treatife upon that fubject. About that time alfo, he refolved the problem propofed by Pafcal, under the feigned name of John de Montford, to all the Englifh mathematicians; and returned another to the mathematicians in France, formerly propofed by Kepler, and then refolved likewife by himfelf, to which they never gave any folution.—In 1660, he invented a method for the conftruction of folar eclipfes: and in the latter part of the fame year, he with ten other gentlemen formed themfelves into a fociety, to meet weekly, for the improvement of natural and experimental philofophy; being the foundation of the Royal Society.—In the beginning of 1661, he was chofen Savilian profeffor of aftronomy at Oxford,

in the room of Dr. Seth Ward; where he was the fame year created Doctor of Laws.

Among his other accomplifhments, Dr. Wren had gained fo confiderable a fkill in architecture, that he was fent for the fame year, from Oxford, by order of king Charles the 2d, to affift Sir John Denham, furveyor general of the works.—In 1663, he was chofen fellow of the Royal Society; being one of thofe who were firft appointed by the Council after the grant of their charter. Not long after, it being expected that the king would make the fociety a vifit, the lord Brounker, then prefident, by a letter requefted the advice of Dr. Wren, concerning the experiments which might be moft proper on that occafion: to whom the doctor recommended principally the Torricellian experiment, and the weather needle, as being not mere amufements, but ufeful, and alfo neat in their operation. Indeed upon many occafions Dr. Wren did great honour to that illuftrious body, by many curious and ufeful difcoveries, in aftronomy, natural philofophy, and other fciences, related in the Hiftory of the Royal Society, where Dr. Sprat has inferted them from the regifters and other books of the fociety to 1665. Among others of his productions there enumerated, is a lunar globe; reprefenting the fpots and various degrees of whitenefs upon the moon's furface, with the hills, eminences and cavities: the whole contrived fo, that by turning it round to the light, it fhews all the lunar phafes, with the various appearances that happen from the fhadows of the mountains and valleys, &c: this lunar model was placed in the king's cabinet. Another of thefe productions, is a tract on the Doctrine of Motion that arifes from the impact between two bodies, illuftrated by experiments. And a third is, The Hiftory of the Seafons, as to the temperature, weather, productions, difeafes, &c, &c. For which purpofe he contrived many curious machines, feveral of which kept their own regifters, tracing out the lines of variations, fo that a perfon might know what changes the weather had undergone in his abfence: as windgages, thermometers, barometers, hygrometers, raingages, &c.—He made alfo great additions to the new difcoveries on pendulums; and among other things fhewed, that there may be produced a natural ftandard for meafure from the pendulum for common ufe.—He invented many ways to make aftronomical obfervations more eafy and accurate: He fitted and hung quadrants, fextants, and radii more commodioufly than formerly: he made two telefcopes to open with a joint like a fector, by which obfervers may infallibly take a diftance to half minutes, &c. He made many forts of retes, fcrews, and other devices, for improving telefcopes to take fmall diftances, and apparent diameters, to feconds: He made apertures for taking in more or lefs light, as the obferver pleafes, by opening and fhutting, the better to fit glaffes for crepufculine obfervations.—He added much to the theory of dioptrics; much to the manufacture of grinding good glaffes: He attempted, and not without fuccefs, the making of glaffes of other forms than fpherical. He exactly meafured and delineated the fpheres of the humours of the eye, the proportions of which to one another were only gueffed at before: a difcuffion fhewing the reafons why we fee objects erect, and that reflection conduces as much to vifion as refraction. He difplayed a natural and eafy theory of refractions, which exactly anfwered every experiment. He fully demonftrated all dioptrics in a few propofitions, fhewing not only, as in Kepler's Dioptrics, the common properties of glaffes, but the proportions by which the individual rays cut the axis, and each other, upon which the charges of the telefcopes, or the proportion of the eye-glaffes and apertures, are demonftrably difcovered.—He made conftant obfervations on Saturn, and a true theory of that planet, before the printed difcourfe by Huygens, on that fubject, appeared.—He made maps of the Pleiades and other telefcopic ftars: and propofed methods to determine the great queftion as to the earth's motion or reft, by the fmall ftars about the pole to be feen in large telefcopes—In navigation he made many improvements. He framed a magnetical terella, which he placed in the midft of a plane board with a hole, into which the terella is half immerfed, till it be like a globe with the poles in the horizon: the plane is then dufted over with fteel filings from a fieve: the duft, by the magnetical virtue, becomes immediately figured into furrows that bend like a fort of helix, proceeding as it were out at one pole, and returning in by the other; the whole plane becoming figured like the circles of a planifphere.—It being a queftion in his time among the problems of navigation, to what mechanical powers failing againft the wind was reducible; he fhewed it to be a wedge: and he demonftrated, how a tranfient force upon an oblique plane would caufe the motion of the plane againft the firft mover: and he made an inftrument mechanically producing the fame effect, and fhewed the reafon of failing on all winds. The geometrical mechanifm of rowing, he fhewed to be a lever on a moving or cedent fulcrum: for this end, he made inftruments and experiments, to find the refiftance to motion in a liquid medium; with other things that are the neceffary elements for laying down the geometry of failing, fwimming, rowing, flying, and conftructing of fhips—He invented a very fpeedy and curious way of etching. He ftarted many things towards the emendation of water-works. He likewife made fome inftruments for refpiration, and for ftraining the breath from fuliginous vapours, to try whether the fame breath, fo purified, will ferve again.—He was the firft inventor of drawing pictures by microfcopical glaffes. He found out perpetual, or at leaft longlived lamps, for keeping a perpetual regular heat, in order to various ufes, as hatching of eggs and infects, production of plants, chemical preparations, imitating nature in producing foffils and minerals, keeping the motion of watches equal, for the longitude and aftronomical ufes.—He was the firft author of the anatomical experiment of injecting liquor into the veins of animals. By this operation, divers creatures were immediately purged, vomited, intoxicated, killed, or revived, according to the quality of the liquor injected. Hence arofe many other new experiments, particularly that of transfufing blood, which has been profecuted in fundry curious inftances. This is a fhort account of the principal difcoveries which Dr. Wren prefented, or fuggefted, to the Royal Society, or were improved by him.

As to his architectural works: It has before been

obferved

obſerved that he had been ſent for to aſſiſt Sir John Denham. In 1665 he travelled into France, to examine the moſt beautiful edifices and curious mechanical works there, when he made many uſeful obſervations. Upon his return home, he was appointed architect, and one of the commiſſioners for repairing St. Paul's cathedral. Within a few days after the fire of London, 1666, he drew a plan for a new city, and preſented it to the king; but it was not approved of by the parliament. In this model, the chief ſtreets were to croſs each other at right angles, with leſſer ſtreets between them; the churches, public buildings, &c, ſo diſpoſed as not to interfere with the ſtreets, and four piazzas placed at proper diſtances.—Upon the death of Sir John Denham, in 1668, he ſucceeded him in the office of ſurveyor-general of the king's works; and from this time he had the direction of a great many public edifices, by which he acquired the higheſt reputation. He built the magnificent theatre at Oxford, St. Paul's cathedral, the Monument, the modern part of Hampton Court, Chelſea-college, one of the wings of Greenwich hoſpital, the churches of St. Stephen Walbrook, and St. Mary-le-bow, with upwards of 60 other churches and public works, which that dreadful fire made neceſſary. In the management of which buſineſs, he was aſſiſted in the meaſurements, and laying out of private property, by the ingenious Dr. Robert Hook. The variety of buſineſs in which he was by this means engaged, requiring his conſtant attendance and concern, he reſigned his Savilian profeſſorſhip at Oxford in 1673; and the year following he received from the king the honour of knighthood.—He was one of the commiſſioners who, on the motion of Sir Jonas Moore, ſurveyor-general of the ordnance, had been appointed to find out a proper place for erecting an obſervatory; and he propoſed Greenwich, which was approved of; the foundation ſtone of which was laid the 10th of Auguſt 1675, and the building was preſently finiſhed under the direction of Sir Jonas, with the advice and aſſiſtance of Sir Chriſtopher.

In 1680 he was choſen preſident of the Royal Society; afterwards appointed architect and commiſſioner of Chelſea-college; and in 1684, principal officer or comptroller of the works in Windſor-caſtle. Sir Chriſtopher ſat twice in Parliament, as a repreſentative for two different boroughs. While he continued ſurveyor-general, his reſidence was in Scotland-yard; but after his removal from that office, in 1718, he lived in St. James's-ſtreet, Weſtminſter. He died the 25th of February 1723, at 91 years of age; and he was interred with great ſolemnity in St. Paul's cathedral, in the vault under the ſouth wing of the choir, near the eaſt end.

As to his perſon, Sir Chriſtopher Wren was of a low ſtature, and thin frame of body; but by temperance and ſkilful management he enjoyed a good ſtate of health, to a very unuſual length of life. He was modeſt, devout, ſtrictly virtuous, and very communicative of his knowledge. Beſides his peculiar eminence as an architect, his learning and knowledge were very extenſive in all the arts and ſciences, and eſpecially in the mathematics.

Sir Chriſtopher never printed any thing himſelf, but ſeveral of his works have been publiſhed by others; ſome in the Philoſophical Tranſactions, and ſome by Dr. Wallis and other friends.—His poſthumous works and draughts were publiſhed by his ſon.

WRIGHT (Edward), a noted Engliſh mathematician, who flouriſhed in the latter part of the 16th century, and beginning of the 17th; dying in the year 1615. He was contemporary with Mr. Briggs, and much concerned with him in the buſineſs of the logarithms, the ſhort time they were publiſhed before his death. He alſo contributed greatly to the improvement of navigation and aſtronomy. The following memoirs of him are tranſlated from a Latin paper in the annals of Gonvile and Caius college in Cambridge, viz, " This year (1615) died at London, Edward Wright of Garveſton in Norfolk, formerly a fellow of this college; a man reſpected by all for the integrity and ſimplicity of his manners, and alſo famous for his ſkill in the mathematical ſciences: ſo that he was not undeſervedly ſtyled a moſt excellent mathematician by Richard Hackluyt, the author of an original treatiſe of our Engliſh navigations. What knowledge he had acquired in the ſcience of mechanics, and how uſefully he employed that knowledge to the public as well as private advantage, abundantly appear both from the writings he publiſhed, and from the many mechanical operations ſtill extant, which are ſtanding monuments of his great induſtry and ingenuity. He was the firſt undertaker of that difficult but uſeful work, by which a little river is brought from the town of Ware in a new canal, to ſupply the city of London with water; but by the tricks of others he was hindered from completing the work he had begun. He was excellent both in contrivance and execution, nor was he inferior to the moſt ingenious mechanic in the making of inſtruments, either of braſs or any other matter. To his invention is owing whatever advantage Hondius's geographical charts have above others; for it was Wright who taught Jodocus Hondius the method of conſtructing them, which was till then unknown; but the ungrateful Hondius concealed the name of the true author, and arrogated the glory of the invention to himſelf. Of this fraudulent practice the good man could not help complaining, and juſtly enough, in the preface to his treatiſe of the Correction of Errors in the Art of Navigation; which he compoſed with excellent judgment, and after long experience, to the great advancement of naval affairs. For the improvement of this art he was appointed mathematical lecturer by the Eaſt-India Company, and read lectures in the houſe of that worthy knight Sir Thomas Smith, for which he had a yearly ſalary of 50 pounds. This office he diſcharged with great reputation, and much to the ſatisfaction of his hearers. He publiſhed in Engliſh a book on the doctrine of the ſphere, and another concerning the conſtruction of ſun-dials. He alſo prefixed an ingenious preface to the learned Gilbert's book on the loadſtone. By theſe and other his writings, he has tranſmitted his fame to lateſt poſterity. While he was yet a fellow of this college, he could not be concealed in his private ſtudy, but was called forth to the public buſineſs of the nation, by the queen, about the year 1593. [Other accounts ſay 1589.] He was ordered to attend the earl of Cumberland in ſome

ſome maritime expeditions. One of theſe he has given a faithful account of, in the manner of a journal or ephemeris, to which he has prefixed an elegant hydrographical chart of his own contrivance. A little before his death he employed himſelf about an Engliſh tranſlation of the book of logarithms, then lately diſcovered by lord Napier, a Scotchman, who had a great affection for him. This poſthumous work of his was publiſhed ſoon after, by his only ſon Samuel Wright, who was alſo a ſcholar of this college. He had formed many other uſeful deſigns, but was hindered by death from bringing them to perfection. Of him it may truly be ſaid, that he ſtudied more to ſerve the public than himſelf; and though he was rich in fame, and in the promiſes of the great, yet he died poor, to the ſcandal of an ungrateful age." So far the memoir; other particulars concerning him, are as follow.

Mr. Wright firſt diſcovered the true way of dividing the meridian line, according to which the Mercator's charts are conſtructed, and upon which Mercator's failing is founded. An account of this he ſent from Caius college, Cambridge, where he was then a fellow, to his friend Mr. Blondeville, containing a ſhort table for that purpoſe, with a ſpecimen of a chart ſo divided, together with the manner of dividing it. All which Blondeville publiſhed, in 1594, among his Exerciſes. And, in 1597, the reverend Mr. William Barlowe, in his Navigator's Supply, gave a demonſtration of this diviſion as communicated by a friend.

At length, in 1599, Mr. Wright himſelf printed his celebrated treatiſe, intitled, *The Correction of certain Errors in Navigation*, which had been written many years before; where he ſhews the reaſon of this diviſion of the meridian, the manner of conſtructing his table, and its uſes in navigation, with other improvements. In 1610 a ſecond edition of Mr. Wright's book was publiſhed, and dedicated to his royal pupil, prince Henry; in which the author inſerted farther improvements; particularly he propoſed an excellent way of determining the magnitude of the earth; at the ſame time recommending very judiciouſly, the making our common meaſures in ſome certain proportion to that of a degree on its ſurface, that they might not depend on the uncertain length of a barley-corn. Some of his other improvements were; The Table of Latitudes for dividing the meridian, computed as far as to minutes: An inſtrument, he calls the Sea-rings, by which the variation of the compaſs, the altitude of the ſun, and the time of the day, may be readily determined at once in any place, provided the latitude be known: The correcting of the errors ariſing from the eccentricity of the eye in obſerving by the croſs-ſtaff: A total amendment in the Tables of the declinations and places of the ſun and ſtars, from his own obſervations, made with a ſix-foot quadrant, in the years 1594, 95, 96, 97: A ſea-quadrant, to take altitudes by a forward or backward obſervation; having alſo a contrivance for the ready finding the latitude by the height of the pole-ſtar, when not upon the meridian. And that this book might be the better underſtood by beginners, to this edition is ſubjoined a tranſlation of Zamorano's Compendium; and added a large table of the variation of the compaſs as obſerved in very different parts of the world, to ſhew it is not occaſioned by any magnetical pole. The work has gone through ſeveral other editions ſince. And, beſide the books above mentioned, he wrote another on navigation, intitled, *The Haven-finding Art*. Other accounts of him ſay alſo, that it was in the year 1589 that he firſt began to attend the earl of Cumberland in his voyages. It is alſo ſaid that he made, for his pupil, prince Henry, a large ſphere with curious movements, which, by the help of ſpring-work, not only repreſented the motions of the whole celeſtial ſphere, but ſhewed likewiſe the particular ſyſtems of the ſun and moon, and their circular motions, together with their places and poſſibilities of eclipſing each other: there is in it a work for a motion of 17100 years, if it ſhould not be ſtopt, or the materials fail. This ſphere, though thus made at a great expence of money and ingenious induſtry, was afterwards in the time of the civil wars caſt aſide, among duſt and rubbiſh, where it was found, in the year 1646, by Sir Jonas Moore, who at his own expence reſtored it to its firſt ſtate of perfection, and depoſited it at his own houſe in the Tower, among his other mathematical inſtruments and curioſities.

X.

XENOCRATES, an eminent philoſopher among the ancient Greeks, was born at Chalcedon, and died 314 years before Chriſt, at about 90 years of age. He became early a diſciple of Plato, ſtudying under this great maſter at the ſame time with Ariſtotle, though he was not poſſeſſed of equal talents; the former wanting a ſpur, and the latter a bridle. He was fond of the mathematics; and permitted none of his ſcholars to be ignorant of them. There was ſomething ſlovenly in the behaviour of Xenocrates; for which reaſon Plato frequently exhorted him to ſacrifice to the graces. Serioufneſs and ſeverity were always ſeen in his deport-

deportment: yet notwithſtanding this ſevere caſt of mind, he was very compaſſionate. There was ſomething extraordinary in the rectitude of his morals: he was abſolute maſter of his paſſions; and was not fond of pleaſure, riches, or applauſe. Indeed, ſo great was his reputation for ſincerity and probity, that he was the only perſon whom the magiſtrates of Athens diſpenſed from confirming his teſtimony with an oath. And yet he was ſo ill treated by them, as to be ſold becauſe he could not pay the poll-tax laid upon foreigners. Demetrius Phalereus bought Xenocrates, paid the debt to the Athenians, and immediately gave him his liberty. At Alexander's requeſt, he compoſed a treatiſe on the Art of Reigning; 6 books on Nature; 6 books on Philoſophy; one on Riches, &c; but none of them have come down to theſe times:—His theology it ſeems was but poor ſtuff: Cicero refutes him in the firſt book of the Nature of the Gods.

XENOPHANES, a Greek philoſopher, born in Colophon, was, according to ſome authors, the diſciple of Archelaus; in which caſe he muſt have been contemporary with Socrates. Others relate, that he taught himſelf all he knew, and that he lived at the ſame time with Anaximander: according to which account he muſt have flouriſhed before Socrates, and about the 60th Olympiad, as Diogenes Laertius affirms. He founded the Eleatic ſect; and wrote ſeveral poems on philoſophical ſubjects; as alſo a great many on the foundation of Colophon, and on that of the colony of Elea. He wrote alſo againſt Homer and Heſiod. He was baniſhed from his country, withdrew to Sicily, and lived in Zanche and Catana. His opinion with regard to the nature of God differs not much from that of Spinoza.—When he ſaw the Egyptians pour forth lamentations during their feſtivals, he thus adviſed them: "If the objects of your worſhip are Gods, do not weep: if they are men, offer not ſacrifices to them." The anſwer he made to a man with whom he refuſed to play at dice, is highly worthy of a philoſopher: This man calling him a coward, "Yes, replied he, I am exceſſively ſo with regard to all ſhameful actions."

XENOPHON, a celebrated Greek general, philoſopher, and hiſtorian, was born at Athens, and became early a diſciple of Socrates, who, ſays Strabo, ſaved his life in battle. About the 50th year of his age he engaged in the expedition of Cyrus, and accompliſhed his immortal retreat in the ſpace of 15 months. The jealouſy of the Athenians baniſhed him from his native city, for engaging in the ſervice of Sparta and Cyrus. On his return therefore he retired to Scillus, a town of Elis, where he built a temple to Diana, which he mentions in his epiſtles, and devoted his leiſure to philoſophy and rural ſports. But commotions ariſing in that country, he removed to Corinth, where it ſeems he wrote his Grecian Hiſtory, and died at the age of 90, in the year 360 before Chriſt.

By his wife Phileſia he had two ſons, Diodorus and Gryllus. The latter rendered himſelf immortal by killing Epaminondas in the famous battle of Mantinea, but periſhed in that exploit, which his father lived to record.

The beſt editions of his works are thoſe of Franckfort in 1674, and of Oxford, in Greek and Latin, in 1703, 5 vols. 8vo. Separately have been publiſhed his *Cyropædia*, Oxon. 1727, 4to, and 1736, 8vo. *Cyri Anabaſis*, Oxon. 1735, 4to, and 1747, 8vo. *Memorabilia Socratis*, Oxon. 1741, 8vo.—His *Cyropædia* has been admirably tranſlated into Engliſh by Spelman.

XIPHIAS, in Aſtronomy, is the Dorado or Swordfiſh, a conſtellation of the ſouthern hemiſphere; being one of the new conſtellations added by modern aſtronomers; and conſiſting of 6 ſtars only. See Dorado.

Y.

YARD, a lineal meaſure, or meaſure of length, uſed in England and Spain chiefly to meaſure cloth, ſtuffs, &c. The Yard was ſettled by Henry the 1ſt, from the length of his own arm.

The Engliſh Yard contains 3 feet; and it is equal
to 4-5ths of the Engliſh ell,
to 7-9ths of the Paris ell,
to 4-3ds of the Flemiſh ell,
to 56-51ſts of the Spaniſh vara or Yard.

Yard, or *Golden* Yard, is alſo a popular name given to the 3 ſtars which compoſe the belt of Orion.

YEAR, in the full extent of the word, is a ſyſtem or cycle of ſeveral months, uſually 12. Others define Year, in the general, a period or ſpace of time, meaſured out by the revolution of ſome celeſtial body in its orbit. Thus, the time in which the fixed ſtars make a revolution, is called the *great Year*; and the times in which Jupiter, Saturn, the Sun, Moon, &c, complete their courſes, and return to the ſame point of the zodiac, are reſpectively called the Years of Jupiter, and Saturn, and the Solar, and Lunar Years, &c.

As Year denoted originally a revolution, and was not limited to that of the ſun; accordingly we find by the oldeſt accounts, that people have, at different times, expreſſed other revolutions by it, particularly that of the moon: and conſequently that the Years of ſome accounts, are to be reckoned only months, and ſometimes periods of 2, or 3, or 4 months. This will help us greatly in underſtanding the accounts that certain nations give of their own antiquity, and per-

 haps

haps of the age of men. We read exprefsly, in feveral of the old Greek writers, that the Egyptian Year, at one period, was only a month; and we are farther told that at other periods it was 3 months, or 4 months: and it is probable that the children of Ifrael followed the Egyptian account of their Years. The Egyptians talked, almoft 2000 years ago, of having accounts of events 48 thoufand Years diftance. A great deal muft be allowed to fallacy, on the above account; but befide this, the Egyptians had, in the time of the Greeks, the fame ambition which the Chinefe have at prefent, and wanted to pafs themfelves upon that people, as thefe others do upon us, for the oldeft inhabitants of the earth. They had recourfe alfo to the fame means, and both the prefent and the early impoftors have pretended to ancient obfervations of the heavenly bodies, and recounted eclipfes in particular, to vouch for the truth of their accounts. Since the time in which the folar Year, or period of the earth's revolution round the fun, has been received, we may account with certainty; but for thofe remote ages, in which we do not know of a certainty what is meant by the term Year, it is impoffible to form any conjecture of the duration of time in the accounts. The Babylonians pretend to an antiquity of the fame romantic kind; they talk of 47 thoufand Years in which they had kept obfervations; but we may judge of thefe as of the others, and of the obfervations as of the Years. The Egyptians fpeak of the ftars having four times altered their courfes in that period which they claim for their hiftory, and that the fun fet twice in the eaft. They were not fuch perfect aftronomers, but, after a round-about voyage, they might perhaps miftake the eaft for the weft when they came in again.

YEAR, or SOLAR YEAR, properly, and by way of eminence fo called, is the fpace of time in which the fun moves through the 12 figns of the ecliptic. This, by the obfervations of the beft modern aftronomers, contains 365 days, 5 hours, 48 min. 48 feconds: the quantity affumed by the authors of the Gregorian calendar is 365 days, 5 hours, 49 min. But in the civil or popular account, this Year only contains 365 days; except every 4th Year, which contains 366.

The viciffitude of feafons feems to have given occafion to the firft inftitution of the Year. Man, naturally curious to know the caufe of that diverfity, foon found it was the proximity and diftance of the fun; and therefore gave the name Year to the fpace of time in which that luminary performed his whole courfe, by returning to the fame point of his orbit. According to the accuracy in their obfervations, the Year of fome nations was more perfect than that of others, but none of them quite exact, nor whofe parts did not fhift with regard to the parts of the fun's courfe.

According to Herodotus, it was the Egyptians who firft formed the Year, making it to contain 360 days, which they fubdivided into 12 months, of 30 days each. Mercury Trifmegiftus added 5 days more to the account. And on this footing it is faid that Thales inftituted the Year among the Greeks; though that form of the Year did not hold throughout all Greece. Alfo, the Jewifh, Syrian, Roman, Perfian, Ethiopic, Arabic, &c Years, were all different. In fact, confidering the imperfect ftate of aftronomy in thofe ages, it is no wonder that different people fhould difagree in the calculation of the fun's courfe. We are even affured by Diod. Siculus, lib. 1. Plutarch, in Numa, and Pliny, lib. 7, cap. 48, that the Egyptian Year itfelf was at firft very different from that now reprefented.

The folar Year is either *aftronomical* or *civil*.

The *Aftronomical Solar* YEAR, is that which is determined precifely by aftronomical obfervations; and is of two kinds, *tropical*, and *fidereal* or *aftral*.

Tropical, or *Natural* YEAR, is the time the fun takes in paffing through the zodiac; which, as before obferved, is 365 d. 5 h. 48 m. 48 fec.; or 365 d. 5h. 49 min. This is the only proper or natural Year, becaufe it always keeps the fame feafons to the fame months.

Sidereal or *Aftral* YEAR, is the fpace of time the fun takes in paffing from any fixed ftar, till his return to it again. This confifts of 365 d. 6 h. 9 m. 17 fec.; being 20 m. 29 fec. longer than the true folar year.

Lunar YEAR, is the fpace of 12 lunar months. Hence, from the two kinds of fynodical lunar months, there arife two kinds of lunar Years; the one *aftronomical*, the other *civil*.

Lunar Aftronomical YEAR, confifts of 12 lunar fynodical months; and therefore contains 354d. 8h. 48m. 38 fec. and is therefore 10d. 21h. 0m. 10f. fhorter than the folar Year. A difference which is the foundation of the Epact.

Lunar Civil YEAR, is either common or embolifmic.

The *Common Lunar* YEAR confifts of 12 lunar civil months; and therefore contains 354 days. And

The *Embolifmic* or *Intercalary Lunar* YEAR, confifts of 13 lunar civil months, and therefore contains 384 days.

Thus far we have confidered Years and months, with regard to aftronomical principles, upon which the divifion is founded. By this, the various forms of civil Years that have formerly obtained, or that do ftill obtain, in divers nations, are to be examined.

Civil YEAR, is that form of Year which every nation has contrived or adopted, for computing their time by. Or the civil is the tropical Year, confidered as only confifting of a certain number of whole days: the odd hours and minutes being fet afide, to render the computation of time, in the common occafions of life, more eafy. As the tropical Year is 365 d. 5 h. 49 m. or almoft 365 d. 6 h. which is 365 days and a quarter; therefore if the civil Year be made 365 days, every 4th year it muft be 366 days, to keep nearly to the courfe of the fun. And hence the civil Year is either *common* or *biffextile*. The

Common Civil YEAR, is that confifting of 365 days; having feven months of 31 days each, four of 30 days, and one of 28 days; as indicated by the following well known memorial verfes:

Thirty days hath September,
April, June, and November;
February twenty-eight alone,
And all the reft have thirty-one.

Biffextile or *Leap* YEAR, confifts of 366 days; having one day extraordinary; called the intercalary, or biffextile day; and takes place every 4th Year. This additional day to every 4th Year, was firft introduced by

by Julius Cæsar; who, to make the civil Years keep pace with the tropical ones, contrived that the 6 hours which the latter exceeded the former, should make one day in 4 years, and be added between the 24th and 23d of February, which was their 6th of the calends of March; and as they then counted this day twice over, or had *bis sexto calendas*, hence the Year itself came to be called *bis sextus*, and *bissextile*.

However, among us, the intercalary day is not introduced by counting the 23d of February twice over, but by adding a day at the end of that month, which therefore in that Year contains 29 days.

A farther reformation was made in this year by Pope Gregory. See *Gregorian* YEAR, CALENDAR, BISSEXTILE, and LEAP-*Year*.

The Civil or Legal Year, in England, formerly commenced on the day of the Annunciation, or 25th of March; though the historical Year began on the day of the Circumcision, or 1st of January; on which day the German and Italian Year also begins. The part of the Year between these two terms was usually expressed both ways: as 1745-6, or 174$\frac{5}{6}$. But by the act for altering the stile, the civil Year now commences with the 1st of January.

Ancient Roman YEAR. This was the lunar Year, which, as first settled by Romulus, contained only ten months, of unequal numbers of days in the following order: viz,

March 31; April 30; May 31; June 30; Quintilis 31; Sextilis 30; September 30; October 31; November 30; December 30; in all 304 days; which came short of the true lunar Year by 50 days; and of the solar by 61 days. Hence, the beginning of Romulus's Year was vague, and unfixed to any precise season; to remove which inconvenience, that prince ordered so many days to be added yearly as would make the state of the heavens correspond to the first month, without calling them by the name of any month.

Numa Pompilius corrected this irregular constitution of the Year, composing two new months, January and February, of the days that were used to be added to the former Year. Thus Numa's year consisted of 12 months, of different days, as follow; viz,

January - 29;	February 28;	March - - 31;
April - - 29;	May - - 31;	June - - - 29;
Quintilis 31;	Sextilis 29;	September 29;
October - 31;	November 29;	December 29;

in all 355 days; therefore exceeding the quantity of a lunar civil Year by one day; that of a lunar astronomical Year by $15^h\ 11^m\ 22^s$; but falling short of the common solar Year by 10 days; so that its beginning was still vague and unfixed.

Numa, however, desiring to have it begin at the winter solstice, ordered 22 days to be intercalated in February every 2d Year, 23 every 4th, 22 every 6th, and 23 every 8th Year.

But this rule failing to keep matters even, recourse was had to a new way of intercalating; and instead of 22 days every 8th Year, only 15 were to be added. The care of the whole was committed to the pontifex maximus; who however, neglecting the trust, let things run to great confusion. And thus the Roman Year stood till Julius Cæsar reformed it. See CALENDAR. And for the manner of reckoning the days of the Roman months, see CALENDS, NONES, and IDES.

Julian YEAR. This is in effect a solar Year, commonly containing 365 days; though every 4th Year, called Bissextile, it contains 366. The months of the Julian Year, with the number of their days, stood thus:

January - 31;	February - 28;	March - 31;
April - - 30;	May - - 31;	June - - 30;
July - - 31;	August - - 31;	September 30;
October - 31;	November 30;	December 31.

But every Bissextile Year had a day added in February, making it then to contain 29 days.

The mean quantity therefore of the Julian Year is 365¼ days, or $365^d\ 6^h$; exceeding the true solar Year by somewhat more than 11 minutes; an excess which amounts to a whole day in almost 131 years. Hence the times of the equinoxes go backward, and fall earlier by one day in about 130 or 131 Years. And thus the Roman Year stood, till it was farther corrected by pope Gregory.

For settling this Year, Julius Cæsar brought over from Egypt, Sosigenes, a celebrated mathematician; who, to supply the defect of 67 days, which had been lost through the neglect of the priests, and to bring the beginning of the Year to the winter solstice, made one Year to consist of 15 months, or 445 days; on which account that Year was used to be called *annus confusionis*, the *Year of confusion*. See *Julian* CALENDAR.

Gregorian YEAR. This is the Julian Year corrected by this rule, viz, that instead of every secular or 100th Year being a bissextile, as it would be in the former way, in the new way three of them are common Years, and only the 4th is bissextile.

The error of 11 minutes in the Julian Year, by continual repetition, had accumulated to an error of 13 days from the time when Cæsar made his correction; by which means the equinoxes were greatly disturbed. In the Year 1582, the equinoxes were fallen back 10 days, and the full moons 4 days, more backward than they were in the time of the Nicene council, which was in the Year 325; viz, the former from the 20th of March to the 10th, and the latter from the 5th to the 1st of April. To remedy this increasing irregularity, pope Gregory the 13th, in the year 1582, called together the chief astronomers of his time, and concerted this correction, throwing out the 10 days above mentioned. He exchanged the lunar cycle for that of the epacts, and made the 4th of October of that Year to be the 15th; by that means restoring the vernal equinox to the 21st of March. It was also provided, by the omission of 3 intercalary days in 400 Years, to make the civil Year keep pace nearly with the solar Year, for the time to come. See CALENDAR.

In the Year 1700, the error of 10 days was grown to 11; upon which, the protestant states of Germany, to prevent farther confusion, adopted the Gregorian correction. And the same was accepted also in England in the year 1752, when 11 days were thrown out after the 2d of September that Year, by accounting the 3d to be the 14th day of the month: calling this the new stile, and the former the old stile. And the Gregorian, or new

new ſtile, is now in like manner uſed in moſt countries of Europe.

Yet this laſt correction is ſtill not quite perfect; for as it has been ſhewn that in 4 centuries, the Julian Year gains $3^d\ 2^h\ 40^m$; and as it is only the 3 days that are kept out in the Gregorian Year; there is ſtill an exceſs of $2^h\ 40^m$ in 4 centuries, which amounts to a whole day in 36 centuries, or in 3600 Years. See CALENDAR, *New* or *Gregorian* STILE, &c.

Egyptian YEAR, called alſo the *Year of Nabonaſſar*, on account of the epoch of Nabonaſſar, is the ſolar Year of 365 days, divided into 12 months, of 30 days each, beſide 5 intercalary days, added at the end. The order and names of theſe months are as follow:

1. Thoth; - -	2. Paophi; - -	3. Athyr;
4. Chojac; - -	5. Tybi; - -	6. Mecheir;
7. Phamenoth;	8. Pharmuthi;	9. Pachon;
10. Pauni; - -	11. Epiphi; -	12. Meſori.

As the Egyptian Year, by neglecting the 6 hours, in every 4 Years loſes a whole day of the Julian Year, its beginning runs through every part of the Julian Year in the ſpace of 1460 Years; after which, they meet again; for which reaſon it is called the *erratic* Year. And becauſe this return to the ſame day of the Julian Year, is performed in the ſpace of 1460 Julian Years, this circle is called the Sothic period.

This Year was applied by the Egyptians to civil uſes, till Anthony and Cleopatra were defeated; but the mathematicians and aſtronomers uſed it till the time of Ptolomy, who made uſe of it in his Almageſt; ſo that the knowledge of it is of great uſe in aſtronomy, for comparing the ancient obſervations with the modern.

The ancient Egyptians, we are told by Diodorus Siculus, (Plutarch, lib. 1, in the life of Numa, and Pliny, lib. 7, cap. 48) meaſured their Years by the courſe of the moon. At firſt they were only one month, then 3, then 4, like that of the Arcadians; and then 6, like that of the people of Acarnania. Thoſe authors add, that it is on this account that they reckon ſuch a vaſt number of Years from the beginning of the world; and that in the hiſtory of their kings, we meet with ſome who lived 1000, or 1200 Years. The ſame thing is maintained by Kircher; Oedip. Egypt. tom. 2, pa. 252. And a late author obſerves, that Varro has affirmed the ſame of all nations, that has been quoted of the Egyptians. By which means many account for the great ages of the more ancient patriarchs; expounding the gradual decreaſe in their ages, by the ſucceſſive increaſe of the number of months in their years.

Upon the Egyptians being ſubdued by the Romans, they received the Julian Year, though with ſome alteration; for they ſtill retained their ancient months, with the five additional days, and every 4th Year they intercalated another day, for the 6 hours, at the end of the Year, or between the 28th and 29th of Auguſt. Alſo, the beginning of their Year, or the firſt day of the month Thoth, anſwered to the 29th of Auguſt of the Julian Year, or to the 30th if it happened to be leap Year.

The Ancient Greek YEAR.—This was a lunar Year, conſiſting of 12 months, which at firſt had each 30 days, then alternately 29 and 30 days, computed from the firſt appearance of the new moon; with the addition of an emboliſmic month of 30 days, every 3d, 5th, 8th, 11th, 14th, 16th, and 19th Year of a cycle of 19 Years; in order to keep the new and full moons to the ſame terms or ſeaſons of the Year.

Their Year commenced with that new moon which was neareſt to the ſummer ſolſtice. And the order of the months, with the number of their days, were as follow: 1. Εκατομβαιων, of 29 days; 2. Μηταγειτνιων 30; 3. Βοηδρομιων 29; 4. Μαιμακτηριων 30; 5. Πυανεψιων 29; 6. Ποσειδεων 30; 7. Γαμηλιων 29; 8. Ανθεστηριων 30; 9. Ελαφηβολιων 29; 10. Μουνυχιων 30; 11. Θαργηλιων 29; 12. Σκιροφοριων 30.—But many of the Greek nations had other names for their months.

The Ancient Jewiſh YEAR.—This is a lunar Year, uſually conſiſting of 11 months, containing alternately 30 and 29 days. And it was made to agree with the ſolar Year, by adding 11, and ſometimes 12 days, at the end of the Year, or by an emboliſmic month. The order and quantities of the months were as follow: 1. Niſan or Abib 30 days; 2. Jiar or Zius 29; 3. Siban or Sievan 30; 4. Thamuz or Tamuz 29; 5. Ab 30; 6. Elul 29; 7. Tiſri or Ethanim 30; 8. Marcheſvam or Bul 29; 9. Ciſleu 30; 10. Tebeth 29; 11. Sabat or Schebeth 30; 12. Adar 30 in the emboliſmic year, but 29 in the common year.—Note, in the defective Year, Ciſleu was only 29 days; and in the redundant Year, Marcheſvam was 30.

The Modern Jewiſh YEAR is likewiſe lunar, conſiſting of 12 months in common Years, but of 13 in emboliſmic Years; which, in a cycle of 19 Years, are the 3d, 6th, 8th, 11th, 14th, 17th, and 19th. Its beginning is fixed to the new moon next after the autumnal equinox. The names and order of the months, with the number of the days, are as follow: 1. Tiſri 30 days; 2. Marcheſvan 29; 3. Ciſleu 30; 4. Tebeth 29; 5. Schebeth 30; 6. Adar 29; 7. Veadar, in the emboliſmic year, 30; 8. Niſan 30; 9. Ilar 29; 10. Sivan 30; 11. Thamuz 29; 12. Ab 30; 13. Elul 29.

The Syrian YEAR, is a ſolar one, having its beginning fixed to the beginning of October in the Julian Year; from which it only differs in the names of the months, the quantities being the ſame; as follow: 1. Tiſhrin, anſwering to our October, and containing 31 days; 2. Latter Tiſhrin, containing, like November, 30 days; 3. Canun 31; 4. Latter Canun 31; 5. Shabat 28, or 29 in a leap-year; 6. Adar 31; 7. Niſan 30; 8. Aiyar 31; 9. Haziram 30; 10. Thamuz 31; 11. Ab 31; 12. Elul 30.

The Perſian YEAR, is a ſolar one, of 365 days, conſiſting of 12 months of 30 days each, with 5 intercalary days added at the end. The months are as follow: 1. Afrudia meh; 2. Ardihaſcht meh; 3. Cardi meh; 4. Thir meh; 5. Merded meh; 6. Schabarir meh; 7. Mehar meh; 8. Aben meh; 9. Adar meh; 10. Di meh; 11. Behen meh; 12. Affirer meh. This Year is the ſame as the Egyptian Nabonaſſarean, and is called the *yezdegerdic Year*, to diſtinguiſh it from the fixed ſolar Year, called the Gelalean Year, which the Perſians began to uſe in the Year 1079, and which was

formed

formed by an intercalation, made ſix or ſeven times in four Years, and then once every 5th Year.

The Arabic, Mahometan, and Turkiſh YEAR, called alſo the Year of the *Hegira*, is a lunar Year, equal to $354^d\ 8^h\ 48^m$, and conſiſts of 12 months, containing alternately 30 and 29 days. Though ſometimes it contains 13 months; the names &c being as follow: 1. Muharram of 30 days; 2. Saphar 29; 3. Rabia 30; 4. Latter Rabia 29; 5. Jomada 30; 6. Latter Jomada 29; 7. Rajab 30; 8. Shaaban 29; 9. Ramadan 30; 10. Shawal 29; 11. Dulkaadah 30; 12. Dulheggia 29, but in the emboliſmic year 30. An intercalary day is added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21ſt, 24th, 26th, 29th, in a cycle of 29 Years. The months commence with the firſt appearance of the new moons after the conjunctions.

Ethiopic YEAR, is a ſolar Year perfectly agreeing with the Actiac, except in the names of the months, which are; 1. Maſcaram; 2. Tykympt; 3. Hydar; 4. Tyſhas; 5. Tyr; 6. Jacatil; 7. Magabit; 8. Mijazia; 9. Ginbat; 10. Syne; 11. Hamel; 12. Hahaſe. Intercalary days 5. It commences with the Egyptian Year, on the 29th of Auguſt of the Julian Year.

YESDEGERDIC YEAR. See *Perſian* YEAR.

Z.

ZENITH, in Aſtronomy, the vertical point, or point in the heavens directly overhead. Or, the Zenith is a point in the ſurface of the ſphere, from which a right line drawn through the place of any ſpectator, paſſes through the centre of the earth.

The Zenith of any place, is alſo the pole of the horizon, being 90 degrees diſtant from every point of it. And through the Zenith paſs all the azimuths, or vertical circles.

The point diametrically oppoſite to the Zenith, is called the *nadir*, being the point in the ſphere directly under our feet: and it is the Zenith to our antipodes, as our Zenith is their nadir.

ZENITH-*Diſtance*, is the diſtance of the ſun or ſtar from our Zenith; and is the complement of the altitude, or what it wants of 90 degrees.

ZENO, ELEATES, or of *Elea*, one of the greateſt philoſophers among the Ancients, flouriſhed about 500 years before the Chriſtian æra. He was the diſciple of Parmenides, and even, according to ſome writers, his adopted ſon. Ariſtotle aſſerts that he was the inventor of logic: but his logic ſeems to have been calculated and employed to perplex all things, and not to clear up any thing. For Zeno employed it only to diſpute againſt all comers, and to ſilence his opponents, whether they argued right or wrong. Among many other ſubtleties and embarraſſing arguments, he propoſed ſome with regard to motion, denying that ther was any ſuch thing in nature; and Ariſtotle, in the 6th book of his phyſics, has preſerved ſome of them, which are extremely ſubtile, eſpecially the famous argument named Achilles; which was to prove this propoſition, that the ſwifteſt animal could never overtake the ſloweſt, as a greyhound a tortoiſe, if the latter ſet out a little fore the former: for ſuppoſe the tortoiſe to be 100 yards before the dog, and that this runs 100 times as faſt as the other; then while the dog runs the firſt 100 yards, the tortoiſe runs 1, and is therefore 1 yard before the dog; again, while the dog runs over this yard, the tortoiſe will run the 100th part of a yard, and will be ſo much before the dog; and again, while the dog runs over this 100th part of a yard, the tortoiſe will have got the 100th part of that 100th part before him; and ſo on continually, ſays he, the dog will always be ſome ſmall part behind the tortoiſe. But the fallacy will ſoon be detected, by conſidering where the tortoiſe will be when the dog has run over 200 yards; for as the former can have run only two yards in the ſame time, and therefore muſt then be 98 yards behind the dog, he conſequently muſt have overtaken and paſſed the tortoiſe. It has been ſaid that, to prove to him, or ſome diſciple of his, that there is ſuch a thing as motion, Diogenes the Cynic roſe up and walked over the floor.—Zeno ſhewed great courage in ſuffering pain; for having joined with others to endeavour to reſtore liberty to his country, which groaned under the oppreſſion of a tyrant, and the enterprize being diſcovered, he ſupported with extraordinary firmneſs the ſharpeſt tortures. It is even ſaid that he had the courage to bite off his tongue, and ſpit it in the tyrant's face, for fear of being forced, by the violence of his torments, to diſcover his accomplices. Some ſay that he was pounded to death in a mortar.

ZENO, a celebrated Greek philoſopher, was born at Citium, in the Iſle of Cyprus, and was the founder of the Stoics; a ſect which had its name from that of a portico at Athens, where this philoſopher choſe to hold his diſcourſes. He was caſt upon that coaſt by ſhipwreck; and he ever after regarded this as a great happineſs, praiſing the winds for having ſo happily driven him into the port of Piræum.—Zeno was the diſciple of Crates, and had a great number of followers. He made the ſovereign good to conſiſt in dying in conformity to nature, guided by the dictates of right reaſon. He acknowledged but one God; and admitted an inevitable deſtiny over all events. His

servant taking advantage of this last opinion, cried, while he was beating him for dishonesty, "I was destined to steal;" to which Zeno replied, "Yes, and to be beaten too." This philosopher used to say, "That if a wise man ought not to be in love, as some pretended; none would be more miserable than beautiful and virtuous women, since they would have none for their admirers but fools." He also said, "That a part of knowledge consists in being ignorant of such things as ought not to be known: that a friend is another self: that a little matter gives perfection to a work, though perfection is not a little matter." He compared those who spoke well and lived ill, to the money of Alexandria, which was beautiful, but composed of bad metal.—It is said that being hurt by a fall, he took that as a sign he was then to quit this life, and laid violent hands on himself, about 264 years before Christ.

Cleanthes, Crysippus, and the other successors of Zeno maintained, that with virtue we might be happy in the midst even of disgrace and the most dreadful torments. They admitted the existence of only one God, the soul of the world, which they considered as his body, and both together forming a perfect being. It is remarked that, of all the sects of the ancient philosophers, this was one of those which produced the greatest men.

We ought not to confound the two Zenos above mentioned, with

ZENO, a celebrated Epicurean philosopher, born at Sidon, who had Cicero and Pomponius Atticus for his disciples, and who wrote a book against the mathematics, which, as well as that of Possidonius's refutation of it, is lost; nor with several other Zenos mentioned in history.

ZENSUS, or ZENZUS, in Arithmetic and Algebra, a name used by some of the older authors, especially in Germany, for a square number, or the 2d power: being a corruption from the Italic *censi*, of Pacioli, Tartalea, &c, or the Latin *census*, which signified the same thing.

ZETETICE, or ZETETIC *Method*, in Mathematics, was the method made use of to investigate, or find out the solution of a problem; and was much the same thing as analytics, or the analytic method.

Vieta has an ingenious work of this kind in 5 books; *Zeteticorum libri quinque.*

ZOCCO, ZOCCOLO, ZOCLE, or SOCLE, in Architecture, a square body, less in height than breadth, placed under the bases of pedestals, statues, vases, &c. See SOCLE and PLINTH.

ZODIAC, in Astronomy, an imaginary ring or broad circle, in the heavens, in form of a belt or girdle, within which the planets all make their excursions. In the very middle of it runs the ecliptic, or path of the sun in his annual course; and its breadth, comprehending the deviations or latitudes of the planets, is by some authors accounted 16°, some 18, and others 20 degrees.

The Zodiac, cutting the equator obliquely, makes with it the same angle as the ecliptic, which is its middle line, which angle, continually varying, is now nearly equal to 23° 28′; which is called the obliquity of the Zodiac or ecliptic, and is also the sun's greatest declination.

The Zodiac is divided into 12 equal parts, of 30 degrees each, called the signs of the Zodiac, being so named from the constellations which anciently passed them. But, the stars having a motion from west to east, those constellations do not now correspond to their proper signs; from whence arises what is called the *precession of the equinoxes.* And therefore when a star is said to be in such a sign of the Zodiac, it is not to be understood of that constellation, but only of that dodecatemory or 12th part of it.

Cassini has also observed a tract in the heavens, within whose bounds most of the comets, though not all of them, are observed to keep, and which he therefore calls the *Zodiac of the comets.* This he makes as broad as the other Zodiac, and marks it with signs or constellations, like that; as Antinous, Pegasus, Andromeda, Taurus, Orion, the Lesser Dog, Hydra, the Centaur, Scorpion, and Sagittary.

ZODIACAL *Light*, a brightness sometimes observed in the zodiac, resembling that of the galaxy or milky way. It appears at certain seasons, viz, towards the end of winter and in spring, after sunset, or before his rising, in autumn and beginning of winter, resembling the form of a pyramid, lying lengthways with its axis along the zodiac, its base being placed obliquely with respect to the horizon. This phenomenon was first described and named by the elder Cassini, in 1683. It was afterwards observed by Fatio, in 1684, 1685, and 1686; also by Kirch and Eimmart, in 1688, 1689, 1691, 1693, and 1694. See Mairan, Suite des Mem. de l'Acad. Royale des Sciences 1731, pa. 3.

The Zodiacal light, according to Mairan, is the solar atmosphere, a rare and subtile fluid, either luminous by itself, or made so by the rays of the sun surrounding its globe; but in a greater quantity, and more extensively, about his equator, than any other part.

Mairan says, it may be proved from many observations, that the sun's atmosphere sometimes reaches as far as the earth's orbit, and there meeting with our atmosphere, produces the appearance of an Aurora borealis.

The length of the Zodiacal light varies sometimes in reality, and sometimes in appearance only, from various causes.

Cassini often mentions the great resemblance between the Zodiacal light and the tails of comets. The same observation has been made by Fatio: and Euler endeavoured to prove that they were owing to similar causes. See Decouverte de la Lumiere Celeste que paroit dans le Zodiaque, art. 41. Lettre à M. Cassini, printed at Amsterdam in 1686. Euler, in Mem. de l'Acad. de Berlin, tom. 2.

This light seems to have no other motion than that of the sun itself: and its extent from the sun to its point, is seldom less than 50 or 60 degrees in length, and more than 20 degrees in breadth: but it has been known to extend to 100 or 103°, and from 8 to 9° broad.

It is now generally acknowledged, that the electric fluid is the cause of the aurora borealis, ascribed by Mairan

Mairan to the solar atmosphere, which produces the Zodiacal light; and which is thrown off chiefly and to the greatest distance from the equatorial parts of the sun, by means of the rotation on his axis, and extending visibly as far as the orbit of the earth, where it falls into the upper regions of our atmosphere, and is collected chiefly towards the polar parts of the earth, in consequence of the diurnal revolution, where it forms the aurora borealis. And hence it has been suggested, as a probable conjecture, that the sun may be the fountain of the electrical fluid, and that the Zodiacal light, and the tails of comets, as well as the aurora borealis, the lightning, and artificial electricity, are its various and not very dissimilar modifications.

ZONE, in Geography and Astronomy, a division of the earth's surface, by means of parallel circles, chiefly with respect to the degree of heat in the different parts of that surface.

The ancient astronomers used the term Zone, to explain the different appearances of the sun and other heavenly bodies, with the length of the days and nights; and the geographers, as they used the climates, to mark the situation of places; using the term climate when they were able to be more exact, and the term Zone when less so.

The Zones were commonly accounted five in number; one a broad belt round the middle of the earth, having the equator in the very middle of it, and bounded, towards the north and south, by parallel circles passing through the tropics of Cancer and Capricorn. This they called the *torrid Zone*, which they supposed not habitable, on account of its extreme heat. Though sometimes they divided this into two equal torrid Zones, by the equator, one to the north, and the other south; and then the whole number of Zones was accounted 6.

Next, from the tropics of Cancer and Capricorn, to the two polar circles, were two other spaces called *temperate Zones*, as being moderately warm; and these they supposed to be the only habitable parts of the earth.

Lastly, the two spaces beyond the temperate Zones, about either pole, bounded within the polar circles, and having the poles in the middle of them, are the two *frigid* or *frozen Zones*, and which they supposed not habitable, on account of the extreme cold there.

Hence, the breadth of the torrid Zone, is equal to twice the greatest declination of the sun, or obliquity of the ecliptic, equal to 46° 56′, or twice 23° 28′. Each frigid Zone is also of the same breadth, the distance from the pole to the polar circle being equal to the same obliquity 23° 28′. And the breadth of each temperate Zone is equal to 43° 4′, the complement of twice the same obliquity. See these Zones exhibited in plate 35, fig. 16.

The difference of Zones is attended with a great diversity of phenomena. 1. In the torrid Zone, the sun passes through the zenith of every place in it twice a year; making as it were two summers in the year; and the inhabitants of this Zone are called *amphiscians*, because they have their noon-day shadows projected different ways in different times of the year, northward at one season, and southward at the other.

2. In the temperate and frigid Zones, the sun rises and sets every natural day of 24 hours. Yet every where, but under the equator, the artificial days are of unequal lengths, and the inequality is the greater, as the place is farther from the equator. The inhabitants of the temperate Zones are called *heteroscians*, because their noon-day shadow is cast the same way all the year round, viz, those in the north Zone toward the north pole, and those in the south Zone toward the south pole.

3. Within the frigid Zones, the inhabitants have their artificial days and nights extended out to a great length; the sun sometimes skirting round a little above the horizon for many days together; and at another season never rising above the horizon at all, but making continual night for a considerable space of time. The inhabitants of these Zones are called *periscians*, because sometimes they have their shadows going quite round them in the space of 24 hours.

ADDENDA ET CORRIGENDA.

A.

A C H

ACCELERATED *Motion*, pa. 18, col. 1, line 17 from the bottom, *after* ſecond inſtant, *add*, or ſmall part of time.—l. 6 from the bottom, *for* in every inſtant, *read* at every moment.—l. 2 from bottom, *for* $16\frac{1}{12}$, *read* $32\frac{1}{6}$.—col. 2, l. 1 and 2, *for* $32\frac{1}{6}$, $48\frac{1}{4}$, $64\frac{1}{3}$, *read* $64\frac{1}{3}$, $96\frac{1}{2}$, $128\frac{2}{3}$.

ACCELERATING *Force*, pa. 21, col. 2, l. 27, *for* requires, *read* acquires.

Pa. 22, col. 2, l. 16 from the bottom, for $t = v\dot{t}$ read $\dot{s} = \dot{v}t$.—next line, for t and s, read $\dot{t}$ and $\dot{s}$.

ACHROMATIC, pa. 25, col. 2, l. 14, *for* fractions, *read* refractions.

Pa. 26, col. 1, l. 12, *for* Veritus, *read* Veritas.

After l. 9, *add*, Since this article was printed, I obſerve, in the 3d volume of the Edinburgh Philoſophical Tranſactions, an account of a curious ſet of experiments, on the unequal refrangibility of light, with obſervations on Achromatic teleſcopes, by Dr. Robert Blair. This ingenious gentleman ſets out with obſerving, "If the theory of the Achromatic teleſcope is ſo complete as it has been repreſented, may it not reaſonably be demanded, whence it proceeds, that Hugenius and others could execute teleſcopes with ſingle object glaſſes 8 inches and upwards in diameter, while a compound object glaſs of half theſe dimenſions, is hardly to be met with? or how it can ariſe from any defect in the execution, that reflectors can be made ſo much ſhorter than Achromatic refractors of equal apertures, when it is well known that the latter are much leſs affected by any imperfections in the execution of the lenſes compoſing the object glaſs, than reflectors are by equal defects in the figure of the great ſpeculum?—The general anſwer made by artiſts to enquiries of this kind, is, that the fault lies in the imperfection of glaſs, and particularly in that kind of glaſs of which the concave lens of the compound object glaſs is formed, called flint glaſs.—It was in order to ſatisfy myſelf concerning the reality of this difficulty, and to attempt to remove it, that I engaged in the following courſe of experiments."

Dr. Blair deſcribes the apparatus and manner of making the experiments. He employed various priſms of different kinds of glaſs; alſo lenſes of glaſs, and of a great variety of fluid mediums, having different degrees of refraction. Having detailed the whole at conſiderable length, for which a reference muſt be made to the work itſelf, and it is very deſerving of attentive peruſal, he concludes with the following recapitulation of the contents and ſcope of the whole diſcourſe.

"The unequal refrangibility of light, as diſcovered and fully explained by Sir Iſaac Newton, ſo far ſtands its ground uncontroverted, that when the refraction is made in the confine of any medium whatever, and a vacuum, the rays of different colours are unequally refracted, the red-making rays being the leaſt refrangible, and the violet-making rays the moſt refrangible.

"The diſcovery of what has been called a different diſperſive power in different refractive mediums, proves thoſe theorems of Sir Iſaac Newton not to be univerſal, in which he concludes that the difference of refraction of the moſt and leaſt refrangible rays, is always in a given proportion to the refraction of the mean refrangible ray. There can be no doubt that this poſition is true with reſpect to the mediums on which he made his experiments; but there are many exceptions to it.

"For the experiments of Mr. Dollond prove, that the difference of refraction between the red and violet rays, in proportion to the refraction of the whole pencil, is greater in ſome kinds of glaſs than in water, and greater in flint-glaſs than in crown-glaſs.

"The firſt ſet of experiments above recited, prove, that the quality of diſperſing the rays in a greater degree than crown-glaſs, is not confined to a few mediums, but is poſſeſſed by a great variety of fluids, and by ſome of theſe in a moſt extraordinary degree. Solutions of metals, eſſential oils, and mineral acids, with the exception of the vitriolic, are moſt remarkable in this reſpect.

"Some conſequences of the combinations of mediums of different diſperſive powers, which have not been ſufficiently attended to, are then explained. Although the greater refrangibility of the violet rays than of the red rays, when light paſſes from any medium whatever into a vacuum, may be conſidered as a law of nature, yet in the paſſage of light from one medium into another, it depends entirely on the qualities of the mediums, which of theſe rays ſhall be the moſt refrangible, or whether there ſhall be any difference in their refrangibility.

"The application of the demonstrations of Hugenius to the correction of the aberration from the spherical figures of lenses, whether solid or fluid, is then taken notice of, as being the next step towards perfecting the theory of telescopes.

"Next it appears from trials made with object-glasses of very large apertures, in which both aberrations are corrected as far as the principles will admit, that the correction of colour which is obtained by the common combination of two mediums which differ in dispersive power, is not complete. The homogeneal green rays emerge most refracted, next to these the united blue and yellow, then the indigo and orange united, and lastly the united violet and red, which are least refracted.

"If this production of colour were constant, and the length of the secondary spectrum were the same in all combinations of mediums when the whole refraction of the pencil is equal, the perfect correction of the aberration from difference of refrangibility would be impossible, and would remain an insurmountable obstacle to the improvement of dioptrical instruments.

"The object of the next experiment is, therefore, to search, whether nature affords mediums which differ in the degree in which they disperse the rays composing the prismatic spectrum, and at the same time separate the several orders of rays in the same proportion. For if such could be found, the above-mentioned secondary spectrum would vanish, and the aberration from difference of refrangibility might be removed. The result of this investigation was unsuccessful with respect to its principal object. In every combination that was tried, the same kind of uncorrected colour was observed, and it was thence concluded, that there was no direct method of removing the aberration.

"But it appeared in the course of the experiments, that the breadth of the secondary spectrum was less in some combinations than in others, and thence an indirect way opened, leading to the correction sought after; namely by forming a compound concave lens of the materials which produce most colour, and combining it with a compound convex lens formed of the materials which produce least colour; and it was observed in what manner this might be effected by means of three mediums, though apparently four are required.

"In searching for mediums best adapted for the above purpose, a very singular and important quality was detected in the muriatic acid. In all the dispersive mediums hitherto examined, the green rays, which are the mean refrangible in crown-glass, were found among the less refrangible, and thence occasion the uncorrected colour which has been described. In the muriatic acid, on the contrary, these same rays make a part of the more refrangible; and in consequence of this, the order of the colours in the secondary spectrum, formed by a combination of crown glass with this fluid, is inverted, the homogeneal green being now the least refrangible, and the united red and violet the most refrangible.

"This remarkable quality found in the marine acid led to complete success in removing the great defect of optical instruments, that dissipation or aberration of the rays, arising from their unequal refrangibility, which has rendered it impossible hitherto to converge all of them to one point either by single or opposite refractions. A fluid in which the particles of marine acid and metalline particles hold a due proportion, at the same time that it separates the extreme rays of the spectrum much more than crown-glass, refracts all the orders of rays exactly in the same proportion as the glass does; and hence rays of all colours, made to diverge by the refraction of the glass, may either be rendered parallel by a subsequent refraction made in the confine of the glass and this fluid, or by weakening the refractive density of the fluid, the refraction which takes place in the confine of it and glass, may be rendered as regular as reflexion, while the errors arising from unavoidable imperfections of workmanship, are far less hurtful than in reflexion, and the quantity of light transmitted by equal apertures of the telescopes much greater.

"Such are the advantages which the theory presents. In reducing this theory to practice, difficulties must be expected in the first attempts. Many of these it was necessary to surmount before the experiments could be completed. For the delicacy of the observations is such as to require a considerable degree of perfection in the execution of the object-glasses, in order to admit of the phenomena being rendered more apparent by means of high magnifying powers. Great pains seem to have been taken by mathematicians to little purpose, in calculating the radii of the spheres requisite for Achromatic telescopes, from their not considering that the object-glass itself is a much nicer test of the optical properties of refracting mediums than the gross experiments made by prisms, and that the results of their demonstrations cannot exceed the accuracy of the data, however much they may fall short of it.

"I shall conclude this paper, which has now greatly exceeded its intended bounds, by enumerating the several cases of unequal refrangibility of light, that their varieties may at once be clearly apprehended.

"In the refraction which takes place in the confine of every known medium and a vacuum, rays of different colours are unequally refrangible, and the red-making rays are least refrangible, and the violet-making rays are most refrangible.

"This difference of refrangibility of the red and violet rays is not the same in all mediums. Those mediums in which the difference is greatest, and which, by consequence, separate or disperse the rays of different colours most, have been distinguished by the term dispersive, and those mediums which separate the rays least have been called indispersive. Dispersive mediums differ from indispersive, and still more from each other, in another very essential circumstance.

"It appears from the experiments which have been made on indispersive mediums, that the mean refrangible light is always the same, and of a green colour.

"Now, in by far the largest class of dispersive mediums, including flint glass, metallic solutions, essential oils, the green light is not the mean refrangible order, but forms one of the less refrangible orders of light, being found in the prismatic spectrum nearer to the deep red than the extreme violet.

"In another class of dispersive mediums, which includes the muriatic and nitrous acids, this same green light becomes one of the more refrangible orders, being now found nearer to the extreme violet than the deep red.

"These

" These are the varieties in the refrangibility of light, when the refraction takes place in the confine of a vacuum; and the phenomena will scarce differ sensibly in refractions made in the confine of dense mediums and air.

" But when light passes from one dense medium into another, the cases of unequal refrangibility are more complicated.

" In refractions made in the confine of mediums which differ only in strength, not in quality, as in the confine of water and crown-glass, or in the confine of the different kinds of dispersive fluids more or less diluted, the difference of refrangibility will be the same as above stated in the confine of dense mediums and air, only the whole refraction will be less.

" In the confine of an indispersive medium, and a rarer medium belonging to either class of the dispersive, the red and violet rays may be rendered equally refrangible. If the dispersive power of the rare medium be then increased, the violet rays will become the least refrangible, and the red rays the most refrangible. If the mean refractive density of the two mediums be rendered equal, the red and violet rays will be refracted in opposite directions, the one towards, the other from the perpendicular.

" Thus it happens to the red and violet rays, whichsoever class of dispersive mediums be employed. But the refrangibility of the intermediate orders of rays, and especially of the green rays, will be different when the class of dispersive mediums is changed.

" Thus, in the first case, where the red and violet rays are rendered equally refrangible, the green rays will emerge most refrangible if the first class of dispersive mediums is used, and least refrangible if the second class is used. And in the other two cases, where the violet becomes least refrangible, and the red most refrangible, and where these two kinds of rays are refracted in opposite directions, the green rays will join the red if the first class of dispersive mediums be employed, and will arrange themselves with the violet if the second class be made use of.

"Only one case more of unequal refrangibility remains to be stated; and that is, when light is refracted in the confine of mediums belonging to the two different classes of dispersive fluids. In its transition, for example, from an essential oil, or a metallic solution, into the muriatic acid, the refractive density of these fluids may be so adjusted, that the red and violet rays shall suffer no refraction in passing from the one into the other, how oblique soever their incidence be. But the green rays will then suffer a considerable refraction, and this refraction will be from the perpendicular, when light passes from the muriatic acid into the essential oil, and towards the perpendicular, when it passes from the essential oil into the muriatic acid. The other orders of rays will suffer similar refractions, which will be greatest in those adjoining the green, and will diminish as they approach the deep red on the one hand, and the extreme violet on the other, where the refraction ceases entirely.

" The manner of the production of these effects, by the attraction of the several mediums, may be thus explained. We shall suppose the attractive forces, which produce the refractions of the red, green and violet light, to be represented by the numbers, 8, 12, and 16, in glass; 6, 9, 14, in the metallic solution; 6, 11, 14, in the muriatic acid; and 6, 10, 14, in a mixture of these two fluids. The excess of attraction of glass for the red and violet light is equal to 2, whichsoever of the three fluids be employed. The refraction of these two orders of rays will therefore be the same in all the three cases. But the excess of attraction for the green light is equal to 3, when the metallic solution is used, and therefore the green light will be more refracted than the red and violet, in this case. When the muriatic acid is used, the excess of attraction of glass for the green light is only 1, and therefore the green light will now be less refracted than the red and violet.

" We shall next suppose the metallic solution and the acid to adjoin each other. The attractions of both these mediums, for the red light being 6, and for the violet light 14, these two orders of rays will suffer no refraction in the confine of the two fluids, the difference of their attractions being equal to nothing.

" But the attractive force of the metallic solution for the green ray being only 9, and that of the muriatic acid for the same ray being 11, the green light will be attracted towards the muriatic acid with the force 2; and therefore the difference between the refraction of the green light and the unrefracted red and violet light, which takes place in the confine of these fluids, will greatly exceed the difference of refraction of the green light, and equally refracted red and violet light, which is produced in the confine of glass and either of the fluids.

" Lastly, in a mixture of the two kinds of fluids, the attraction for the red, green and violet rays, being 6, 10 and 14, and that of the glass, 8, 12 and 16, the excess of the attraction of the glass for the green rays, is the same which it is for the red and violet rays. These three orders of rays will therefore suffer an equal refraction, being each of them attracted towards the glass with the force 2; and when this is the case, it appears, from the observations, that the indefinite variety of rays of intermediate colours and shades of colours, which altogether compose solar light, will also be regularly bent from their rectilinear course, constituting what has been termed a planatic refraction."

In short, Dr. Blair says, that he "uses more transparent mediums than the common ones; avoids or greatly diminishes the reflections at the surfaces of the mediums; applies fluid mediums more homogeneous than thick flint or crown glass, which at the same time disperse the different coloured rays of light in the same proportion, by which means an image is produced perfectly Achromatic, which is but imperfectly so in Dollond's object glasses made of flint and crown glass combined.

ACOUSTICS, at the end, *add*, But this statute was repealed by the 15th of Geo. the 3d, cap. 32.

AEROSTATION, pa. 45, col. 2, l. 40, *for* 800, *read* 680.—l. 46, *for* $28\frac{1}{3}$ *read* 26.—l. 48, *for* balloon *read* parachute.—l. 51 and 52, *for* $28\frac{1}{3}$ *read* 26, and *for* 13 *read* 12.—l. 55, *read* 2 feet 3 inches.

Pa. 46,

Pa. 46, col. 2, at the end of the article on *Aeroſtation*, *add*, See an ingenious and learned treatiſe on the mathematical and phyſical principles of Air-balloons, by the late Dr. Damen, profeſſor of philoſophy and mathematics in the Univerſity of Leyden, entitled, Phyſical and Mathematical Contemplations on Aeroſtatic Balloons, &c; in 8vo, at Utrecht, 1784.

Pa. 70, col. 1, l. 4. *dele* $-\sqrt{3 - 1 = 2}$.—l. 5, at the end *add* $-\sqrt{3} - 1 = 2$.

Pa. 71. col. 1, l. 9, *for* $y^2 + 2y - 7$ *read* $y^2 + 2y - 7$.

AFFECTED *Equations*, *add* (from Francis Maſeres, Eſq.)—"This expreſſion of *Affected Equations* ſeems to require ſome further explanation. It was introduced by the celebrated Vieta, the great father and reſtorer of Algebra. He has many expreſſions peculiar to himſelf, and which have not been adopted by ſubſequent Algebraiſts. Amongſt theſe are the following ones. He calls a ſet of quantities in continual geometrical proportion, (ſuch as the quantities $1, x, x^2, x^3, x^4, x^5, x^6\ x^7$, &c,) a ſet of *ſcalar* quantities, or *magnitudines ſcalares*; and, when there are ſeveral of theſe *ſcalar* quantities mentioned together, (as in the compound quantity $x^5 + ax^4 - b^2x^3$,) he calls the higheſt quantity, or that which is fartheſt in the ſcale of quantities $1, x, x^2, x^3, x^4, x^5, x^6, x^7$, &c. (to wit, the quantity x^5 in the ſaid compound quantity $x^5 + ax^4 - b^2x^3$,) *the power* of the fundamental quantity x, or of the ſecond term in the ſaid ſcale; and he calls the lower ſcalar quantities which are involved in the ſecond and third terms of the ſaid compound quantity $x^5 + ax^4 - b^2x^3$, to wit, the quantities x^4 and x^3, (or, in our preſent language, the inferior powers of x,) ſcalar quantities of a *parodic* degree to x^5, or the power of the fundamental quantity x. This word *parodic* I take to be derived (though Vieta does not tell us ſo) from the Greek words παρὰ and ὁδὸς, which ſignify *near* and *a way* or *road*, becauſe theſe inferior ſcalar quantities x^3 and x^4 lie *in the way* as you paſs along in the ſcale of the aforeſaid quantities $1, x, x^2, x^3, x^4, x^5, x^6\ x^7$, &c, from 1 to x^5, which he calls the power of x in the ſaid compound quantity $x^5 + ax^4 - b^2x^3$. Theſe inferiour ſcalar quantities x^3 and x^4 are therefore *parodic*, or *ſituated in the way to*, or are *leading to*, the higher ſcalar quantity x^5. He then proceeds to define *a pure power* and *an affected power*, and tells us that *a pure power* is a ſcalar quantity that is not affected with any *parodic*, or *inferiour* ſcalar quantity, and that *an affected power* is a ſcalar quantity that is connected by addition, or ſubtraction with one, or more, *inferiour*, or *parodic*, ſcalar quantities, combined with co-efficients that raiſe them to the ſame dimenſion as the power itſelf, or make them *homogeneous* to it, and conſequently capable of being added to it, or ſubtracted from it. Thus x^5 alone is a *pure power* of x, namely, its fifth power; and $x^5 + ax^4 - b^2x^3$ is *an affected power* of x, namely, its fifth power *affected by*, or *connected with*, the two *parodic*, or *inferiour*, ſcalar quantities x^3 and x^4, which are multiplied into bb and a, in order to make them *homogeneous* to, or *of the ſame dimenſion with*, x^5 itſelf, and capable of being added to it or ſubtracted from it. See Schooten's Edition of Vieta's works, publiſhed at Leyden in Holland in the year 1646, pages 3 and 4.

"This, then, being the meaning of the expreſſion, *a pure power* and *an affected power*, the meaning of the correſponding expreſſions of *a pure equation* and *an affected equation* follows from it of courſe: *a pure equation* ſignifying an equation in which a pure power of an unknown quantity is declared to be equal to ſome known quantity; ſuch as the equation $x^5 = 79$; and *an affected equation* ſignifying an equation in which a power of an unknown quantity affected by, or connected, either by addition or ſubtraction, with, ſome inferiour powers of the ſame unknown quantity, (multiplied into proper co-efficients in order to make them *homogeneous* to the ſaid higheſt power of the ſaid unknown quantity,) is declared to be equal to ſome known quantity; ſuch as the equation $x^5 + ax^4 - b^2x^3 = 79$. This I take to be the original meaning of the expreſſion *an affected equation*. But, as the language of *Vieta* has not been adopted by ſubſequent writers of Algebra, I ſhould think it would be more convenient to call them by ſome other name. And, perhaps thoſe of *binomial*, *trinomial*, *quadrinomial*, *quinquinomial*, and, in general, that of *multinomial* equations, would be as convenient as any. Thus, $xx + ax = rr$, and $x^3 + ax^2 = r^3$, and $x^3 + a^2x = r^3$, and $x^4 + a^3x = r^4$, and $x^4 + ax^3 = r^4$, might all be called *binomial* equations, becauſe they would be equations in which a *binomial* quantity, or quantity conſiſting of two terms that involved the unknown quantity x, is declared to be equal to a known quantity; and, for a like reaſon, the equations $x^3 + ax^2 + b^2x = r^3$, and $x^4 - ax^3 + b^2x^2 = r^4$, and $x^4 - ax^3 + b^3x = r^4$, and $x^5 + ax^4 + b^2x^3 = r^5$, and $x^5 + ax^4 - b^2x^3 = r^5$, and $x^5 + b^2x^3 + c^4x = r^5$, might be called *trinomial* equations. And the like names might be given to equations of a greater number of terms. Dr. Hutton, I obſerve, in his excellent new Mathematical and Philoſophical Dictionary, juſt now publiſhed, (Feb. 2. 1795,) calls them *compound* equations; which is likewiſe a very proper name for them, and leſs obſcure than that of *affected* equations."

Pa. 76, col. 1, l. 25, *for* $\sqrt{3}+1-\sqrt{3-1}$, *read* $\sqrt{3+1}-\sqrt{3-1}$.

Pa. 94, col. 2, l. 34, *for* Spaniard, *read* Portugueſe.

Pa. 95, col. 2, after l. 21, or the end of the paragraph relating to Dr. Barrow, add as follows:—Of theſe lectures, the 13th deſerves the moſt ſpecial notice, being entirely employed upon Equations, delivered in a very curious way. He there treats of the nature and number of their roots, and the limits of their magnitudes, from the deſcription of lines accommodated to each, viz, treating the ſubject as a branch of the doctrine of maxima and minima, which, in the opinion of ſome perſons, is the right way of conſidering them, and far preferable to the ſo much boaſted invention of the generation of Equations from each other diſcovered by Harriot and Deſcartes.

Pa. 97, col. 2, after l. 3, *add*—Dr. Waring and the Rev. M. Vince, of Cambridge, have both given many improve-

improvements and diſcoveries in ſeries and in other branches of analyſis. Thoſe of Mr. Vince are chiefly contained in the latter volumes of the Philoſophical Tranſactions; where alſo are ſeveral of Dr. Waring's; but the bulk of this gentleman's improvements are contained in his ſeparate publications, particularly the *Meditationes Algebraicæ*, publiſhed in 1770; the *Proprietates Algebraicarum Curvarum*, 1772; and the *Meditationes Analyticæ*, 1776; an account of the chief contents of which, a friend has favoured me with, as follows.

Of Dr. Waring's Meditationes Algebraicæ.

The firſt chapter treats of the transformation of algebraical equations into others, of which the roots have given algebraical relation to the roots of the given equations.

The general reſolution of this problem requires the finding the aggregates of each of the values of algebraical functions of the roots of the given equation: for this purpoſe the author begins with finding the ſum of the m^{th} power of each of the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c = 0$ by a ſeries proceeding according to the dimenſions of p the ſum of the roots: this ſeries (when continued in infinitum and converges) finds alſo the ſum of any root of the above-mentioned quantities. From this ſeries is deduced the law of the reverſion of the ſeries $y = ax + bx^2 + cx^3 + \&c$, which finds x in terms of y; and alſo the law of a ſeries, which expreſſes the greateſt or leaſt roots, and their powers or roots of a given algebraical equation, and which may be applied whether that root is poſſible or impoſſible, if the root be much greater or leſs than each of the remaining ones. All the powers and roots of this ſeries, when continued in infinitum, obſerve the ſame law.

On this ſubject are further added ſome elegant theorems; of which, one finds the ſum of all quantities of this kind $\alpha^a\beta^b\gamma^c$, &c; where α, β, γ, &c, denote the roots of the given equation. This has been ſince publiſhed by the celebrated mathematician Mr. le Grange in the Academy of Sciences at Paris.

There is alſo added a method of conſiderable utility in theſe matters; viz, the aſſuming equations whoſe roots are known, and thence deducing the coefficients of the equations ſought: and alſo from the terms of an inferior equation deducing the terms of a ſuperior.

The ſecond chapter principally treats of the limits and number of impoſſible and affirmative and negative roots of algebraical equations.

Some new properties are added, of the limiting equations reſulting from multiplying the ſucceſſive terms of the given equation into an arithmetical ſeries; and a method of finding limits between each of the roots of a given equation, ſince publiſhed in the Berlin Acts, and alſo ſome new methods of finding equations whoſe roots are limits between the roots of other equations. In theor. 4 and 5 are contained quantities which are always greater than certain others, when they are all poſſible; from whence may be deduced Newton's and ſeveral other rules for finding the number of impoſſible roots: theſe rules may be rendered ſomewhat more general by multiplying the given equations into others, whoſe roots are all poſſible, and finding whether impoſſible roots may be deduced by the rule in the reſulting equation, which cannot from it be diſcovered in the given one. A rule is given, deduced from each ſucceſſive four terms of the given equation, and conſequently much more general than rules deduced from each ſucceſſive three terms. The former always diſcovers the true number of impoſſible roots contained in quadratic and cubic equations, the latter in quadratic only. There is alſo a rule given for finding the number of impoſſible roots from an equation, of which the roots are the ſquares, &c, of the roots of a given equation; and a ſecond from an equation of which the roots are the ſquares of the differences of the roots of a given equation; and a third rule for finding an equation, of which the root is $z = nx^{n-1} - \overline{n-1}\, px^{n-2} + \&c$; if $x^n - px^{n-1} + qx^{n-2} - \&c = 0$ be the given equation, &c, theſe latter reſolutions always diſcover the true number of impoſſible roots contained in cubic, biquadratic and ſurſolid equations; and alſo whether or not any impoſſible roots are contained in any given equation; and alſo from the laſt term whether the number of impoſſible roots contained be 2, 6, 10, &c, or 0, 4, 8, &c. The principle of a 4th rule is given by finding when two roots once, twice, thrice, &c, or four, &c, roots become equal. From a method given of finding the number of impoſſible roots contained in an equation involving only one unknown quantity, is deduced a method of diſcovering limits between which are contained any number of impoſſible roots in an equation involving two or more unknown quantities. From the number of impoſſible, affirmative and negative roots contained in a given equation, is delivered a method of finding the number of impoſſible, &c roots contained in an equation of which the roots have a given algebraical relation to the roots of the given equation.

The principles are ſubjoined of finding the number of affirmative and negative roots contained in an algebraical equation: but this neceſſarily ſuppoſes a method of finding the number of its impoſſible roots known. It is demonſtrated, that if the equation $x^n - px^{n-1} + qx^{n-2} - \&c = 0$ be multiplied by $x - a$, then every change of ſigns in the given, will have one, or three, or five, &c in the reſulting equation; and if it be multiplied by $x + a$, then every continuation from $+$ to $+$ or $-$ to $-$, will produce one, or three, or five, &c ſuch continuations in the reſulting, whence every equation $x^n - px^{n-1} + \&c = 0$ will contain at leaſt ſo many changes of ſigns in its ſucceſſive terms as there are affirmative roots, and ſo many continued progreſſes from $+$ to $+$ and $-$ to $-$, as there are negative. In a biquadratic $x^4 + px^3 + qx^2 + rx + s = 0$, of which two roots are impoſſible, and s an affirmative quantity, then it is demonſtrated that the two poſſible ones will be both negative or both affirmative, according as $p^3 - 4pq + 8r$ is an affirmative or negative quantity, if the ſigns of the coefficients, p, q, r, s are neither all affirmative, nor alternately $-$ and $+$. The number of impoſſible and affirmative and negative roots contained in the equation $x^n + Ax^m + B = 0$ is likewiſe given, &c. If $lx^m - px^{m-1} + qx^{m-2} - \&c = 0$ and $bx^n - ax^{n-1} + bx^{n-2} - \&c = v$, and further $bx^n - ax^{n-1} + bx^{n-2}$ &c $= 0$ and $lx^m - px^{m-1} + \&c = w$, then the content of all the values

of

of the quantity w will be to the content of all the values of the quantity v : : $\pm l^{n}$: b^{m}, from whence are deduced some properties of parabolic curves. *Ex. gr.* Let the equation expressing the relation between the abscifs x and ordinate y be $y = ax^{n} + bx^{n-1} + cx^{n-2} + \&c.$ Then will the content under the $(n-1)$ greatest ordinates be to the square of the content of all the distances between any two points in which the abscifs cuts the curve : : $a^{n-1} : n^{n-2}$. The quotient of the content of all the sines divided by the content of all the cosines to the points in which the abscifs cuts the curve, will be to the content of all the abovementioned greatest ordinates : : $n^{n}a$: 1. Similar propositions are deduced concerning the ordinates to the points of contrary flexure, &c.

The third chapter is versant, concerning, 1st finding the roots of equations or irrational quantities, which have given relations to each other: this is performed by substitution or division and finding the common divisors of the quantities resulting; and 2d concerning more (n) equations containing a less number (m) of supposed unknown quantities, which consequently require n-m equations, since named equations of condition; these are likewise deduced from the method of finding common divisors. 3dly, Concerning the resolution of equations; in this case is given, 1. The reduction or resolution of some recurring equations. 2. Some properties of the roots of the equation $x^{n} \pm 1 = 0$. 3. Resolution of a biquadratic $x^4 + px^3 + qx^2 + rx + s = 0$, by reducing it to an equation $z^4 + az^2 + b = 0$. 4. A resolution of the biquadratic $x^4 + 2px^3 = qx^2 + rx + s$ by adding $(p^2 + 2n) x^2 + 2pnx + n^2$ to both sides of the equation, so as to complete the square; and the deducing that the values of n are $\frac{\alpha\beta + \gamma\delta}{2}$, $\frac{\alpha\gamma + \beta\delta}{2}$, $\frac{\alpha\delta + \beta\gamma}{2}$; the values of $\sqrt{(q + p^2 + 2n)}$ are $\frac{\alpha + \beta - \gamma - \delta}{2}$, $\frac{\alpha + \gamma - \beta - \delta}{2}$, &c, and the values of $\sqrt{(s + n^2)}$ are $\frac{\alpha\beta - \gamma\delta}{2}$, $\frac{\alpha\gamma - \beta\delta}{2}$, &c; if $\alpha, \beta, \gamma, \delta$, are the roots of the given equation. 5. A resolution of equations as general as any yet discovered, viz, the assuming $x = a\sqrt[n]{p} + b\sqrt[n]{p^2} + c\sqrt[n]{p^3} + \&c$; and exterminating the irrational quantities, viz, from assuming $x = a\sqrt[3]{p} + b\sqrt[3]{p^2}$ are deduced different resolutions of cubic; from $x = a\sqrt[4]{p} + b\sqrt[4]{p^2} + c\sqrt[4]{p^3}$ different resolutions of biquadric; from the equations $x = a\sqrt[n]{p} + b\sqrt[n]{p^2}$, $x = a\sqrt[m]{p} + b\sqrt[m]{p^{n-1}}$; $x = a\sqrt[n]{p} + b\sqrt[n]{p^3}$, $x = a\sqrt[n]{p} + \sqrt[n]{p^{n-2}}$, &c, are deduced De Moivre's equation, and several others of new formula not before delivered. 6. The resolution $x = \sqrt[n]{\alpha} + \sqrt[n]{\beta} + \sqrt[n]{\gamma} + \&c$, first given by Euler, shewn to be a very particular; but this is rendered here much more general by assuming a more general resolution. 7. The resolution and reduction of equations from exterminating irrational quantities. 8. Reduction of some equations, when they are deduced from others by reducing them to the original equations. 9. The finding a quantity, which multiplied into a given irrational will produce a rational quantity, and thence deducing from a given equation involving irrational quantities the dimensions to which the equation freed from them will ascend. 10. Let P = a series either ascending or descending according to the dimensions of x, from thence is deduced the sum of a series consisting of its alternate terms, or terms at (n) distance from each other. 11. It is proved, that Cardan's resolution of a cubic, is a resolution of an equation of 9 dimensions or three different cubics: similar principles are applied to some other equations. 12. General principles are given for the deducing the function of the roots of the given, which constitute the coefficients or roots of the transformed equation. E. g. Let a cubic equation $x^3 + qx - r = 0$ and $z - \frac{q}{3z} = x$, thence is shewn the function of the roots of x, which constitute z, and further the cases of the cubic, which are resolvable by the transformed equation, whose root is z: the same principles are applied to biquadratics. 13. The correspondent impossible roots of a given irrational quantity are deduced; and also the different roots of a given resolution. 14. The biquadratic of the formula $x^2 - 2(a + b\sqrt{-1})x - c - d\sqrt{-1} = 0$ is distinguished into two quadratic equations involving only possible quantities, and thence every algebraic equation is proved to consist of simple and quadratic divisors involving only possible quantities. 15. A method is delivered of transforming irrational quantities into others; but it is cautioned, that in reduction and transformation correspondent roots should be used, otherwise it is probable that we shall fall into errors, of which examples are given. 16. The convergency of a root found by the common method of approximations is given; and it is discovered that the convergency principally depends on the quantity assumed for the root being much more near to one root than to any other; and independent of it, not on how near it is to a root.

The fourth chapter is principally conversant concerning more algebraical equations and their reductions to one. 1. It gives the law of the resolution of any number of simple equations; and the reduction of n simple equations to $n - 1$ by means of others. 2. The method of reducing more (n) equations into one so as to exterminate $n - 1$ unknown quantities by the method of common divisors, and further delivers the principles of investigating the roots or values of the unknown quantities, which result from this, or, which is much the same, from the common method of Erasmus Bartholinus, and which are not contained in the given equations. 3. If two algebraical equations of n and m dimensions of the unknown quantities x and y are reduced to one so as to exterminate one of the unknown quantities, the principles are given of finding the dimensions to which the other will ascend: if it ascends to $n \times m$ dimensions; then the sum of the roots depends on the terms of n and $n - 1$ dimensions in the one, and m and $m - 1$ in the other, and similarly of the products of every two; &c. From this principle are deduced several properties of algebraical curves. The

The same principles are applied to more equations involving more unknown quantities. 4. Some two equations of given formulæ are reduced to one so as to exterminate one unknown quantity. 5. Two equations are likewise reduced to one so as to exterminate unknown quantities by means of infinite series. 6. A method of finding whether some equations contain the same roots of the unknown quantities as others. 7. From the correspondent roots of the unknown quantities in given equations are found the constitution of their coefficients; and from thence the aggregates of the functions of the roots of two or more equations. 8. Some things are given concerning the transformations of more equations than one, of their impossible roots, of their roots which have a given relation to each other. 9. Some reductions and resolutions of more equations involving more unknown quantities. 10. If two equations similarly involve two unknown quantities x and y; then the equation of which the root is x or y is demonstrated to have twice the dimensions of the equation whose root is any rational function of $x + y$ or $x^2 + y^2$ or any rational recurring function of x and y; and if for y be substituted $1 - y$; then in the equation whose root is the resulting quantity the dimensions will be the same as in the equations whose root is x or y, but its formula will be of half the number of dimensions. The same principles are applied to more equations similarly involving more unknown quantities. 11. If there are two equations involving two unknown quantities, one deduced from the other, by some substitutions investigated from equations similarly involving two unknown quantities; then the equation whose root is one of the unknown quantities will be recurring. 12. Let A and B be functions of x and y, a method is given of finding, whether A is a function of B. 13. Methods of approximations to the roots of equations when they are unequal, or two or more nearly equal, possible or impossible; and also some remarks on the increments or decrements of the roots, in passing from one equation to others of the same number of dimensions are given.

The fifth chapter treats of rational and integral values of the unknown quantities of given equations.

1. It finds the rational and integral simple, quadratic, &c divisors (by a method different to Waessaner's) of a given equation, which involves one or more unknown quantities. 2. If two equations involve two unknown quantities x and y; the same irrationality, which is contained in x will likewise be contained in its correspondent value of y, unless two or more values of the quantity (x or y) are equal, &c. 3. A method is given of finding integral correspondent values of the unknown quantities of two or more equations involving as many unknown quantities. 4. A method is also delivered of deducing when a given equation can be resolved by means of square, cube, &c roots; and when by similar methods it can be reduced to equations of $\frac{1}{2}$, $\frac{1}{3}$, &c, its dimensions. 5. A method is given of finding a quantity or number, in which are contained all the divisors of any given rational or integral quantities. 6. A method different from Schooten's, Newton's, and Euler's, of extracting the root of a binomial surd $a + \sqrt{b}$ is given, and the principle demonstrated on which all the rules are founded given by Schooten, viz, the multiplying the binomial surd so that the n^{th} root of $A^2 - B$ can be extracted, where $A + \sqrt{B}$ is the resulting surd; and it is further proved that multiplying the given surd $a + \sqrt{b}$ into 2^n will render Newton's resolution as general as the others; and lastly the extraction of the (m^{th}) root of the quantity $A + B\sqrt[n]{p} + C\sqrt[n]{p^2} + \dots + \sqrt[n]{p^{n-1}}$ is given. 7. The law of Dr. Wallis's approximations in terms of the successive quotients, as also of continual fractions is deduced. 8. A method of deducing the integral values of each of the unknown quantities x, y, z, v, &c, contained in the equation $ax + by + cz + dv \pm$ &c $+ f = 0$ in terms of quantities, for which may be assumed any whole numbers. 9. Two or more equations are reduced to one, so as to exterminate unknown quantities; and if the unknown quantities of the resulting equations be integral or fractional, then the unknown quantities of the given equations will also be integral or fractional. 10. Principles are delivered of deducing equations of which the unknown quantities admit of correspondent and known integral or rational values. 11. Correspondent integral or rational values of the unknown quantities in several equations are given, and from some values of the abovementioned kind given, are deduced others. 12. A method of denoting any numbers either by fours, fives, sixes, &c, and their powers; and similar properties deduced as in decimal arithmetic. 13. It is demonstrated that the sum of the divisors of the number $1.2.3\dots x = N$ has to N a greater ratio than the sum of the divisors of any number L less than N has to L; and some other similar properties. 14. In the Philosophical Transactions are given properties similar to Mr. Euler's of the sum of divisors of the natural numbers, and some others. 15. Let $N = a^2 + rb^2$, where a, b, r, p and q are whole numbers, then N^{2m+1} and N^{2m+2} can be compounded by $(m + 1)$ different ways of the quantities $p^2 + rq^2$; the different ways were first given in the Medit. 16. Every number consists of 1, 2, 3 or 4 squares, and of 1, 2, 3, 4, .. 9 cubes, and therefore if a number N is equal to 3 squares or 8 cubes, the problem may not be possible. 17. Let x and z be any whole numbers, and a and b numbers prime to each other, then $ax + bz$ can constitute any number, which exceeds $a \times b - a - b$. 18. Let r the greatest common divisor of m and $n - 1$, where n is a prime number; the number of remainders from the division of the number 1^m, 2^m, 3^m, &c, in infinitum by n will be $\frac{n-1}{r} + 1$: from which are deduced several propositions. 19. Sir John Wilson's property delivered and demonstrated, viz, $1.2.3\dots n-1 + 1$ will be divisible by n, if n be a prime number. 20. The sum of the powers $1^r + 2^r + 3^r + \dots x^r$ are found divisible by $x.\ x+1$, if r be a whole number; from whence is deduced an elegant property of all parabolas correspondent to the property of Archimedes of the inscribed triangles in a conical parabola. 21. Some properties of exponential equations; several other new properties of algebraical quantities and equations are given in these Meditations. They were sent to the Royal Society in 1757, and since published in the years 1760, 62, and 69.

Properties

Properties of Algebraical Curves.

The equation expreffing the relation between the abfcifs and its correfpondent ordinates of a curve is transformed into another which expreffes the relation between different abfciffæ and their ordinates, from which is deduced, that there may be n and not more different diameters in a curve of $n-1$ order, which cuts its ordinates in a given angle; and likewife that a diameter can have no more than $n-1$ different inclinations of its ordinates, unlefs the diameter be a general one. 2. The formula of the equations to curves, all whofe diameters are parallel, or cut each other in a given point, or which have a general diameter to which the lines any how inclined are ordinates. 3. It is proved that there cannot be more than $\frac{n}{m}$ different inclinations of parallel ordinates, which cut the curve in $n-m$ points only, poffible or impoffible. 4. Something is added concerning diameters, which cut their ordinates on both fides into equal parts. 5. It is demonftrated that there are curves of any number of odd orders, that cut a right line in 2, 4, 6, &c, points only; and of any number of even orders that cut a right line in 3, 5, 7, &c points; and confequently that the order of the curve cannot be denounced from the number of points, in which it cuts a right line. 6. The principles are delivered of finding the afymptotes, parabolical legs, ovals, points, &c, of a curve, of which the equation marking the relation between the abfcifs and its ordinates is given; and alfo given the number of afymptotes, parabolical legs of different kinds, ovals, points of different kinds, the leaft order of a curve, which receives them, is deduced. 7. An equation expreffing the relation between an abfcifs and its ordinates, is transformed into an equation expreffing the relation between the diftances from two or more points, the latter may be varied an infinite number of ways; and thence are deduced fome properties. Many refolutions of this kind are only refolutions of a particular cafe contained in it; and confequently can never be deduced from any general reafoning; they are often deduced from fome particular cafes, which are known to anfwer feveral conditions of the problem. Transformations of a given curve into others by fubftitutions, and properties of the loci of fome points are deduced, from which Mr. Cotes's property of algebraical curves, and others of a fimilar and fomewhat different nature are derived. 8. Let a curve of n dimenfions have n afymptotes, then the content of the n abfciffæ will be to the content of the n ordinates, in the fame ratio in the curve and afymptotes, the fum of their (n) fubnormals to ordinates perpendicular to their abfciffæ will be equal to the curve and the afymptotes; and they will have the fame central and diametrial curves. 9. Some propofitions are added concerning the conftruction of equations, and fome equations are conftructed from the principles of Slufius.—If two curves of n and m dimenfions have a common afymptote; or the terms of the equations to the curves of the greateft dimenfions have a common divifor, then the curves cannot interfect each other in $n \times m$ points, poffible or impoffible. If the two curves have a common general centre, and interfect each other in $n \times m$ points, then the fum of the affirmative abfciffæ &c to thofe points will be equal to the fum of the negative; and the fum of the n fubnormals to a curve which has a general centre will be proportional to the diftance from that centre. 10. Something is added on the defcription of curves. 11. No curve which has an hyperbolical leg of the conical kind can in general be fquared. 12. It is demonftrated that no oval figure, which does not interfect itfelf in a given point, can in general be expreffed in finite algebraical terms. 13. Given an algebraical equation, and fimilarly equations expreffing a relation between x and y, &c; and alfo a fluxional quantity which is an algebraical function (z) of x and y and their fluxions; a method is given of deducing an equation whofe root is z; and thence fome properties of curves. 14. Properties fimilar to the fubfequent of conic fections, are extended to curves of fuperior orders, viz, if lines be drawn from given points in them in given angles to four lines infcribed in the conic fection, then will the rectangle under two of thofe lines be to the rectangle under the other two in a given ratio. Several properties are added, which follow from the application of algebraical propofitions invented in the Medit. Algebr. to curve lines.

The fecond chapter treats of curvoids and epicurvoids, or curves generated by the rotation of given curves on right lines or curves, and gives a method of rectifying and fquaring them; and from the radii of curvature of the generating curves being given, it deduces the length and radius of curvature of the curve generated at the correfpondent point; it alfo afferts that from them may be deduced the conftruction of the fluxional equations of the different orders.

The third chapter treats of algebraical folids. 1. It deduces the equation to every fection of a folid generated by the rotation of a curve round its axis; and from thence the different fections generated by the rotation of conic fections round their axis. 2. The equation to folids contains the relation between the two abfciffæ and their ordinates, and the order of the folid may be diftinguifhed according to the dimenfions of the equation; or the folid may be defined by two equations expreffing the relation between the three abovementioned quantities, and a fourth which may be the axis of the fection: there is further given a method of deducing the equation to any fection of thefe folids, and from it the equation to the curve projected on a plane by a given curve. 3. A method of deducing the projection of a curve or folid on each other. 4. If the equation be $x - a = 0$, (x being the diftance from a given point) then it may denote the periphery of a circle if one plane, or the furface of a globe if it refers to a folid. 5. Let x and y denote the diftances from two refpective points, then an equation expreffing the relation between x and y defigns the periphery of a curve, if contained in the fame plane, or the furface of a folid generated by the rotation of a curve round its axis, paffing through the two given points, if a folid. 6. An equation expreffing the relation between lines drawn from three or more points may denote an equation to a folid. 7. If x, z and y denote the two abfciffes and correfpondent ordinates to a folid, and the terms of x and y, or x and z, or y and z; or x, z and y be fimilarly involved; then may the folid be divided into two or

or fix fimilar and equal parts; and if no unequal power of x or y or z; or x and y, &c; or x, y and z be contained in the equation, then the curve may further be divided in general into twice, four or fix times the preceding number of equal parts. 7. Curves of double curvature are defigned by two equations expreffing the relation between two abfciffæ and correfpondent ordinates, or between lines drawn from three or more points; fimilar properties may be deduced from thefe as from the equations to curves.

Chapter the 4th treats of the maxims and minims of polygons infcribed and circumfcribed about curves, and thence deduces certain quantities equal to each other, when maxims and minims are contained at every point of the curve: it further contains feveral properties of conic fections. 1. If any rectilinear figure circumfcribes an ellipfe, the content under the alternate fegments of the line made by the points in which the line touches the ellipfe will be equal. 2. If a right line cuts a conic fection, and the parts of the line without the conic fection on both fides are equal; and any rectilinear figure, which begins and ends at the bounds of the abovementioned line, be defcribed round the conic fection, then the contents under the alternate fegments of the circumfcribing lines as divided in the points of contact will be equal. 3. If two polygons be circumfcribed about an ellipfe, and the fides are cut by the points of contacts in the fame ratios in the one as in the other; then will the areas of the two polygons be equal. 4. If two lines cut a conic fection proportionally, i. e. they are divided by the conic fection in the fame ratio in the one as in the other, and if polygons be defcribed round the conic fection, terminated at the ends of thofe lines, of which the fides are divided by the points of contact in the fame ratio in the one as in the other, then will the area of the two polygons be equal, as likewife the curvilinear area. 5. If all the fides of two polygons infcribed in an ellipfe make the two angles at the fame point equal, and two polygons of this kind be infcribed in the curve, then will the fum of the fides of the one polygon be equal to the fum of the fides of the other. Several other fimilar properties are added, as alfo properties of folids generated by the rotation of a conic fection round its axis; to which I fhall mention the three or four following. 1. The diagonals of a parallelogram circumfcribing an ellipfe or hyperbola will be conjugate diameters. 2. The fections of a folid generated by the rotation of a conic fection round its axis, which pafs through its focus, will have that point for the focus of all the fections. 3. If 4 perpendiculars be drawn from any point in an hyperbola to its periphery; and two lines from the fame point to the afymptotes and the ordinates from the 4 points of the curve and the 2 of the afymptotes be drawn to the abfcifs; then will the fum of the refulting abfciffæ to the former be double to the fum of the abfciffæ to the latter. 4. If an arc of the periphery of a circle be divided into n equal parts, a, $2a$, $3a$, &c, and $p =$ chord of the arc $180 - na$, and α and β be the roots of the quadratic $x^2 - px + 1 = 0$ and radius 1: then will $\alpha^n + \beta^n =$ chord of the arc $180 - na$, from whence may be deduced the divifors of the quantity $x^{2n} - Ax^n + 1$; and alfo the equation whofe roots are the diftances of a point in the circle from thofe points of equal divifion, and further may be deduced the fum of all the values of any algebraical function of thofe lines.

Moft of the properties of circles given by Archimedes are extended to conic fections, and fome of the algebraical and geometrical properties of Pappus are rendered more general; and the principles invented applied to many other cafes. In the firft edition of this book publifhed in 1762 were nearly enumerated the lines of the fourth order on the fame principles as Newton's enumeration of lines of the third order; but this has fince been rejected by the author as not fufficiently diftinguifhing the curve, and as being of no great utility.

Meditationes Analyticæ.

The firft *chapter* treats of finding the fluxion of a fluent, when the quantity or fluent is confidered as generated by motion; or the parts from the whole when the whole or quantity is confidered as confifting of innumerable parts. It further gives the law of a feries, which expreffes the fluxion of an exponential of any order.

Chapter 2, is verfant about the fluents of fluxions. 1. It finds the general fluent of a fluxion $P\dot{x}$, when P is any algebraical function of x however irrational but not exponential; for which intent it inveftigates the common divifors of any two quantities contained under the different vincula; and thence the common divifors of the refulting divifors, and fo on; and likewife all the equal divifors contained in any of the abovementioned quantities; whence it fo reduces the quantity P, that no equal nor common divifors may be contained in any of the refulting quantities under the different vincula; and from the common method deduces the terms of a feries to the number, which the feries is fhewn to confift of, when it does not proceed in infinitum. 2. It demonftrates, that if the dimenfions of x in the denominator of P exceed its dimenfions in the numerator by 1, then the fluent cannot be expreffed in finite terms; and alfo if one factor of P be $(A \pm (A^2 + a)^{\frac{1}{2}})^{\lambda}$, where a is an invariable quantity, and in fome other cafes the fubftitution required muft be fomewhat different. 3. The fluents of fome fluential and exponential fluxions, or fluxions involving fluents and exponential quantities, are given. 4. A general method of difcovering whether the fluent of any fluxion of any order involving one, two or more variable quantities, and their fluxions, can be expreffed in terms of the variable quantities and their fluxions. 5. The correction of fluents of all orders, and thence the fluent contained between any values of the variable quantities and their fluxions, is given; in thefe corrections the fame roots of the irrational quantities are to be ufed in the correction as in the fluent. 6. From the transformation of equations and the principles before delivered, are deduced fluents equal to each other. 7. Some exponential quantities given which continually change from poffibility to impoffibility, and from impoffibility to poffibility. 8. Is a method of finding whether the fluent of any fluxion contained between any limits are finite or not. 9. The fum of the fluents of a fluxion which is an algebraical function of the letter x multiplied into $\dot{x}$ can always be expreffed by finite terms, circular arcs and logarithms, the extraction of the roots of equations being granted.

10. Some

10. Some fluxions involving irrational quantities are reduced to others, in which no irrationality is contained. 11. The general principles of deducing whether the fluent of a given fluxion can generally be expreſſed by finite algebraical terms, their circular arcs and logarithms. 12. Some equal correſpondent fluents are found by ſubſtitutions deduced from equations in which two variable quantities are ſimilarly involved. 13. Some neceſſary corrections are given of finding the fluents of all the fluxions of the formula

$$x^{pn \pm \sigma n - 1}\dot{x} \times R^{m \pm \lambda} \times S^{o \pm \mu} \times T^{t \times \nu} \times \&c,$$

(where σ, λ, μ, ν, &c denote any whole numbers,

$$\text{and } R = e + fx^{n} + gx^{2n} + \ldots x^{\alpha n},$$

$$S = h + kx^{n} + lx^{2n} + \ldots x^{\beta n},$$

$$T = q + rx^{n} + sx^{2n} + \ldots x^{\gamma n}, \&c)$$

from $\alpha + \beta + \gamma +$ &c, independent fluents; but perhaps not from $\alpha + \beta + \gamma +$ &c fluents, which have different values of the quantities, σ, λ, μ, ν, &c. 14. The number of independent fluents of the formulæ $x^{\theta + \alpha n + \beta m} \times (a + bx^{n} + cx^{m})^{\lambda + \pi} \times \dot{x}$, where α, β and π denote whole affirmative numbers, &c; and the number of independent fluents of the formulæ $\dot{X} f Y\dot{x}$, where $\dot{X}$ is a fluxion of which the fluent can be found, from which can be deduced all of the ſame formula, is immediately known from the number of independent fluents of the formula $Y\dot{x}$ and $XY\dot{x}$ which determine all of thoſe formulæ. 15. Let $a + bx^{n} + cx^{2n} + \ldots kx^{\mu n} = p$, and from ſome fluents of the fluxions of the formulæ $p \times x^{\mu n - 1}\dot{x}$, where μ is a whole affirmative number, are determined the remaining ones of the ſame formula. 16. Something is added concerning finding the value of a fraction, when both the numerator and denominator vaniſh; and laſtly from the fluents of ſome fluxions being given, the method of deducing the fluents of others.

Chapter 3, principally treats of algebraical and fluxional equations. 1. It gives the method of transforming two or more fluxional equations into one ſo as to exterminate one or more variable quantities and their fluxions, and finds the order of the reſulting equation. 2. It reduces ſome fluxional equations into more. 3. A method of reducing fluxional equations involving fluents ſo as to exterminate the fluents. 3. Some caſes are given, in which the two variable quantities contained in a given equation are expreſſed in terms of a third. 4. Given an algebraical equation expreſſing the relation between x and y; a method is given of finding the fluent of $y\dot{x}^{n}$ or other fluxions in finite terms of x and y, if they can be expreſſed by ſuch; or elſe by infinite ſeries; this was firſt taught in the Philoſophical Tranſactions in the year 1764. 5. Something is added concerning the correction of fluxional equations. 6. A method of inveſtigating, whether a given equation is the general fluent of a given fluxional equation. 7. The method of deducing, whether a given equation is a particular or general fluent of a given fluxional equation. In both by ſubſtituting for the fluxions their values deduced from the fluential equation their values &c in the fluxional, the fluxional muſt reſult $= 0$; and in the general fluent there muſt be contained ſo many invariable quantities to be aſſumed at will independently as is the order of the fluent; and in both all the variable quantities muſt neceſſarily be variable, and no function of them vaniſh out of the fluxional equation from the ſubſtitution; for then all the conditions of the fluxional equation are anſwered by the fluential. 8. An inveſtigation, when fluxional equations are integrable. 9. From ſome fluents are deduced others, *e. g.* if the area between any two ordinates to one abſciſſa can in general be found, then the area between any two ordinates of any other abſciſſa can be found &c. 10. From given fluxional equations and the fluents of ſome fluxions are deduced the fluents of many others. 11. The fluent of the firſt order of a fluxional equation of the nth order will have (n) different values and n different multipliers; and the fluent of the ſecond order $n \cdot \frac{n-1}{2}$ different values, &c. 12. Let $\alpha = 0$, $\beta = 0$, $\gamma = 0$, &c, (n) general fluents of the fluxional equation, $\lambda = 0$, then will any function of the fluents α, β, γ, &c be a fluent of the ſame fluxional equation $\lambda = 0$. 13. From aſſuming equations, which contain only ſimple powers of the invariable quantities to be aſſumed at will, may eaſily be deduced fluxional equations, of which the general reſolutions are known: 2. From aſſuming the values of any variable quantities and ſubſtituting then their fluxions for the variable quantities, &c. in any functions π, ϱ, &c of the variables aſſumed, let the quantities reſulting be A, B, &c; then generally will $\pi = A$, $\varrho = B$, &c. be fluxional equations, of which the particular fluentials are known. It may be obſerved in this place as before, that from no general reaſoning can particular fluents be deduced. 14. In the reſolution of fluxional equations it is obſerved, that from the logarithmic and exponential quantities contained in the fluxional, may be deduced by chapter 1 the exponentials &c contained in the fluential: 2, and in a ſimilar manner from the irrational quantities and denominators contained in it, the correſpondent irrational quantities and denominators contained in the fluential: 3, the greateſt dimenſions of y multiplied into $\dot{x}$ muſt be greater than thoſe of y into $\dot{y}$ by unity; when there are two of this kind &c, $\alpha y\dot{x} + \beta x\dot{y} = x^{m}y^{n}(\delta\dot{x} + \epsilon\dot{y})$ the reſolution is given; and ſo of more. 15. In the given equation, if the fluxion of the greateſt order does not aſcend to one dimenſion only; then by extraction &c ſo reduce the equation, that it may aſcend to one dimenſion only; and thence find the fluent of any fluxion $P^{n}\dot{y} + Q^{n-1}\dot{y} +$ &c, $+ R^{\prime n}\dot{z} +$ &c. 16. Let a fluxional equation be given involving x and y, in which x flows uniformly, a method is given of finding whether it admits of a multiplier, which is a function of x ∴ and ſimilarly of multipliers of other formulæ. 17. The method of deducing the multipliers of fluxional equations by infinite ſeries. 18. Some fluxional equations are reduced by ſubſtitutions, which ſubſtitutions are commonly eaſily deducible from the fluxional equation given. 19. Somewhat concerning the reduction of ſome fluxional equations to homogeneous, and concerning homogeneous equations of different orders; and of reducing an homogeneous fluxional equation of n order to a fluxional equation of $n - 1$ order: and alſo

alſo of reducing m fluxional equations of n order to one of $mn-1$ orders, and ſo of all others to one degree leſs than the order generally occurring if they had not been homogeneous. 20. The ſubſtitution of an exponential for a variable quantity in equations which contain no exponential quantity; for ſometimes u has been ſubſtituted for a quantity which flows uniformly, and then w ſuppoſed to flow uniformly, which leads to a falſe reſolution. 21. A caution is given not to ſubſtitute homogeneous functions of no dimenſions for variable quantities; and in the general reſolution to obſerve, that there is contained an invariable quantity to be aſſumed at will, which is not contained in the fluxional equation. 22. Something more added concerning the fluents of $p^n\dot{y} + q^{-1}\dot{x}\dot{y} + r^{n-3}\dot{y}\dot{x}^2 +$ &c, $= 0$, where p, q, r, &c. are functions of x, and ſo of ſome other fluxional equations. 23. Fluxional equations are deduced, of which the variable quantities cannot be expreſſed in terms of each other, but both may be expreſſed in terms of a third. 24. Every fluxion or fluent which is a function x, y, z, and x, y &c. is expreſſed in terms of partial differences. 25. The reſolution of ſome equations expreſſing the relation between partial differences &c is given. 26. Some obſervations on finding the fluents of fluxions, when the variable quantities become infinite.

The ſecond book treats of increments and their integrals. 1. Some new laws of the increments are given. 2. The fluxion of the increment of P will be equal to the increment of the fluxion; where P is any function of x, if only the fluxion of the increment of x be equal to the increment of the fluxion. 3. Increments are reduced to others of given formulæ *e.g.* $\alpha + \frac{\beta}{x} + \frac{\gamma}{x(x+\dot{x})} +$, &c, and it is obſerved that if β be not $= 0$, then the integral cannot be found in finite terms of the variable quantity, &c. It may be obſerved, that Taylor, Monmort, &c, firſt found the integral of the two increments

$$x.\overline{x-\dot{x}}.\overline{x-2\dot{x}}\ldots\overline{x-\overline{n-1}\,\dot{x}} \text{ and } \frac{1}{x.\overline{x-\dot{x}}\ldots\overline{x-\overline{n-1}\,\dot{x}}}$$

but did not proceed much further (correſpondent to the finding the fluxion of the fluent x^n); the increments of fluents have been ſince deduced, &c. In this book are diſcovered propoſitions correſpondent to moſt of the inventions in fluxions, *e.g.* a method of finding the integral of any increment expreſſed in algebraical or exponential terms of the variable quantity or quantities, and when the fluent cannot be expreſſed: it is obſerved that they cannot be expreſſed in finite terms of the variable x, &c, if the dimenſions of x, &c, in the denominator exceed its dimenſions in the numerator by 1; or if any factor in the denominator of the fraction reduced to its lower terms have not another contained likewiſe in the denominator, diſtant by a whole number, multiplied into the increment of x. —The increments of ſome integrals are deduced from the integrals of other increments; the integrals of ſome incremental equations from different methods; their general integrals, and particular corrections, &c, &c; but here it is to be obſerved, that the general problem of increments cannot be extended beyond the particular of fluxions, but ſomewhat more may be added, when both are joined together. The third book is verſant concerning infinite ſeries. 1. It gives the ratio of the apparent and real convergency. 2. A method of finding limits between which the ſum of the ſeries conſiſts; and alſo whether the ſum of the ſeries is finite or not from the terms being given or equation between the terms. 3. The convergency of the whole ſeries is judged from the ratio of convergency of the terms at an infinite diſtance. 4. The ſeries from the fluent converges, if the ſeries from the fluxion does, there are ſeveral propoſitions on infinite ſeries deducible from the common algebra. 5. Let an equation $0 = a - bx + cx^2 - dx^3 +$ &c; and $\frac{b}{a}$ much greater than $\frac{c}{b}$, $\frac{c}{b}$ than $\frac{d}{c}$; &c. then will all the roots be poſſible, and $\frac{a}{b}$ an approximation to the leaſt root, $\frac{b}{c}$ to the next, &c: if an equation $y^n + ay^{n-1} + \ldots + fy^{n-m} + gy^{n-m-1} + = 0$, and if one root be much leſs than any m root, but much greater than the remaining; or if the equation be $x^n - px^{n-1} + qx^{n-2} \ldots \pm gx^{n-m+1} \mp hx^{n-m} \pm ix^{n-m-1} \mp kx^{n-m-2} \pm$ &c. $= 0$, then will the approximation to the above root be $\frac{i}{h} - (\frac{k}{i} - \frac{gi^2}{h^3}) +$ &c. 6. Somewhat on the approximations when the approximation given is much more near to one, two, or more roots than to any other, and on the degree of convergency of the ſubſequent approximations deduced; and their ultimate approximations. 7. Given approximations to m roots of a given equation are deduced more near approximations to them. 8. The incremental equation given and applied to approximations. 9. From given approximations to two or more unknown quantities contained in two or more equations are deduced more near approximations to them, either when the approximations given are more near to one, or to two, or more roots of one or more of the unknown quantities than to any others, and ſo of infinite equations. 10. New ſeries are given for the fluents of different fluxions. 1. Log. $\overline{x \pm e} =$ log. $x \pm \frac{e}{x} - \frac{e^2}{2x^2} \pm$ &c.; the number whoſe log. is $v \pm e$ (if N be log. of v) $= N \pm Ne \pm \frac{Ne^2}{2} \pm$ &c; the log. of $\frac{a+x+e}{a-x-e} =$ log. $\frac{a+x}{a-x} + \frac{e}{a^2-x^2} -$ &c. The ſine of the arc $A \pm e$ is $S \pm Ce - \frac{1}{2}Se^2$, &c, and coſine of the ſame arc $= C \pm Se - \frac{1}{2.3}Ce^2 \pm$ &c. S and C being the ſine and coſine of A, the fluent of the fluxion of an elliptical arc $\frac{\sqrt{(1-cx^2)}\dot{x}}{\sqrt{(1-x^3)}}$ which differs little from the arc of a circle when e is a very ſmall quantity $= A - \frac{c}{2} \times \frac{1.A - xP}{2} -$ &c, where $A = \int\frac{\dot{x}}{P}$, $B = \frac{1A - xP}{2}$, C =

$C = \frac{3B - x^2 P}{3}$, &c. and $P = \sqrt{1 - x^2}$, and A = arc of a circle of which the fine is x.

A fimilar feries may be applied from the arc of an hyperbola or ellipfe, to find a correfpondent arc of an hyperbola or ellipfe not much different from the preceding. In this method the feries proceeds according to the dimenfions of fome fmall quantities, and the firft term of the feries is generally a near value of the quantity fought. Thefe feries properly inftituted will generally converge the fwifteft. 11. Something new is added concerning the fluent of the fluxional equation $\ddot{y} = y\dot{z}^2$ viz $-y = E \times$ fin. arc: (z) + F + cof. (ar. (z)); E and F being any quantities to be affumed at will; and of correfpondent equations to logarithms, and finding their values when z is increafed by e. 12. A feries for the increafe of the arc from a fmall increafe of the tangent, fine, &c. 12. When the terms a and x of the binomial $a \pm x$ are equal, the cafes are given in which the feries $a^m \pm ma^{m-1} x$ + &c, = $\overline{a \pm x}^m$ or the feries $a^m x \pm \frac{m}{2} a^{m-1} x$ + &c, &c. will ultimately converge. 13. If any algebraical quantity V a function of x be reduced into a feries proceeding according to the dimenfions of x, a general method of finding what are the limits between which it converges; or the feries from $\int V\dot{x}$, &c; and the method of interpolations fo as to render them converging. 14. The convergency of different feries are compared together. *e. g.* is given $\int \frac{\dot{x}}{1+x} = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 -$, &c. $= \frac{x}{1+x} + \frac{x^2}{2(1+x)^2} + \frac{x^3}{3(1+x)^3}$ + &c. or $\int \frac{\dot{x}}{1 \pm x} = \frac{x}{1 \pm x} \pm \frac{x^2}{2(1+x)^2}$ + &c: there is an erratum contained in this example, for $a -$ is fometimes printed inftead of $a +$: this feries is eafily deduced from Bernouilli's method of deducing infinite feries, and has been fince printed in the Philofophical Tranfactions. 15. Given algebraical or fluxional equations, and a fluxional quantity, a method is given of finding a feries, which expreffes the fluent of the fluxional quantity, from which principles are deduced new feries for the area of a fegment of a circle, the periphery of the ellipfe, hyperbola, &c. 16. It is fhewn, that feriefes proceeding according to the dimenfions of a quantity x always diverge, when feriefes for the fame purpofe proceeding according to the reciprocal of its dimenfions converge; unlefs fometimes in the cafe when they both become the fame. 17. As feries proceeding in infinitum according to the dimenfions of the quantity x were firft invented or ufed for the finding the fluents of fluxions, it being reduced into terms, whofe fluents were known: fo in finding integrals of increments it may be neceffary to reduce the quantity into an infinite feries of terms, whofe integrals are known, and which converges. Examples of formulæ of feriefes of this kind are given. 18. Methods are given of finding the value of one unknown quantity contained in one or more equations involving more unknown quantities, and the law of their convergencies and the interpolations neceffary to render feriefes for finding fluents converging, fimilar principles may be applied to incremental and fluxional equations. 19. It is obferved, that in finding the value of any variable quantity in a feries proceeding according to the dimenfions of another, there will occur in a fluxional or incremental equation of (n) order in the feries n invariable quantities to be affumed at will; and alfo the fluxional equations, &c. from whence they will arife. 20. The finding the integral of $\frac{\dot{z}}{z}$, &c. 21. From the correfpondent relation between the fums of two feries refulting, which are functions of a variable quantity y, when the relation between x and z two values of y are given, is given a method of finding the coefficients of the feries. 22. The rule generally called the reductio ad abfurdum extended to more fubftitutions.

The fourth book treats of the fummation of feries, a method of correfpondent values and feveral other problems. 1. Of finding the fum of a feries expreffed by a rational function of z into x^{nz}; where z denotes fucceffively the numbers 1, 2, 3, &c, in infinitum. 2. Given an equation expreffing the relation between the fucceffive fums, the relation between the fucceffive terms is known, and the *vice verfa*, &c. 3. It is found from an equation expreffing the relation between the fucceffive fums, terms and z the diftance from the firft term of the feries, whether the fum of the feries is finite or not. 4. The difference between z^{-o} and $\overline{z+1}^{-o}$, where z denotes the diftance from the firft term of the feries, will be $-o \times z^{-o-1}$, which is greater than the fimple ratio let o be as fmall as poffible, and confequently the fum of the feries finite. 5. If a feries $a + bx + cx^2 + x^3$, of which at an infinite diftance the preceding coefficients have to the fubfequent the ratio of $r : 1$, be multiplied into a function = 0, when $x = \delta$, then if α be greater than r the feries will diverge; if lefs converge. 6. From adding feveral terms of one or more feries together may be formed a feries, of which the fum from the fums of the preceding feries is known. 6. Seriefes are formed, of which the fums are known from varying the divifors, &c. 7. From given feries are deduced others, of which the fums are known, and the fum of many feries are deduced from finding the fluxions of fluents and fluents of fluxions. 8. From the relation between the different terms given is deduced the correfpondent fluxional equation. 9. The finding the terms of any feries, which can be deduced from given feries; and thence deducing many feries of which the fums can be found from the fum of the given feries. 10. Series are given of which the fums can be found from finite terms, circular arcs, logarithms, elliptical and hyperbolical arcs. 11. From a general expreffion, when algebraical, fluxional, incremental, &c, for the fum of a feries can be deduced a fimilar expreffion for the fum of every fecond, third, &c, terms. 12. An infinite feries may be a particular refolution of infinite fluxional equations. 13. The terms of fome feries may be infinite and their fums known. 14. The general fluent of $\dot{y}^n = y\dot{x}^n$ is given by a feries of the fame kind, and the fame of fome other fluxional equations. 15. A quantity is found which multiplied into a feries more

more ſwiftly converging gives a given ſeries. 16. The firſt differences of the terms of ſome ſeries are given; if the terms are in geometrical ratio to each other the abovementioned differences will alſo be in geometrical ratio to each other: whence it appears, that the ſeries from this method of differences will converge leaſt when the given ſeries converges ſwifteſt, &c, but not always the contrary. Several other propoſitions are added concerning the method of differences applied to ſeries. 17. A parabolico-hyperbolical curve is drawn through any number of points, as alſo an algebraical ſolid —. 18. Something is given concerning the convergency &c. of ſeries deduced from the differences of the numerators of a given ſeries, of which the denominators conſtitute a geometrical progreſſion. 19. A rule is given for rendering ſeries converging, in which it is obſerved that the ſum of ſo many terms ſhould be found that z the diſtance from the firſt term of the ſeries may exceed the greateſt root of the equation reſulting from the quantity which expreſſes the term made $= 0$. 20. An equation expreſſing the relation between the ſums and terms is reduced to an infinite fluxional equation expreſſing the relation between the ſum or term, its fluxions, and z the diſtance from the firſt term of the ſeries. 21. From a method being known of finding the ſum of a ſeries, which involves one variable only, is given a method of finding the ſum of ſeries which involve more variable quantities: and from aſſuming ſums of ſerieſes of this kind are deduced their terms. 22. The ſums of ſeries are found conſiſting of irrational terms. 23. The principle of the convergency of the approximations found in drawing parabolical curves through given points. 24. Something new is given concerning the interpolations of quantities.

25. $\frac{e^{\alpha x} + e^{\beta x} + e^{\gamma x} + \&c}{n} = 1 + \frac{x^n}{1.2..n} + \frac{x^{2n}}{1.2..2n} +$ &c. if α, β, γ, &c, are the roots of $x^n - 1 = 0$, &c. 26. Something is added concerning ſeries from $\int\frac{\dot{x}}{x}\int\frac{\dot{x}}{x}\int\frac{\dot{x}}{x}$, &c, $\times \int\frac{\dot{x}}{1 + ax^n}$. 27. Nandens's Problems are ſomewhat extended. 28. Something is added on changing continual fractions into others. 29. A method of transforming ſeries into continual factors. 30. A rule for finding the ſine and coſine of $\frac{n}{m}$ the arc; and transforming an algebraical equation into an equation expreſſed in terms of ſines and coſines, and thence from an approximation to the ſine is found one more near; the ſame might have been performed by tangents, cotangents, ſecants, coſecants, &c. 31. From ſome fluents given have been found others, and conſequently by reducing the fluents to infinite ſeries from ſome infinite ſeries given may others be deduced. 32. The fluent of $\frac{x^{\alpha}\dot{x}}{1 \pm x^n}$ is found by approximation, where α is an irrational quantity, which method of finding approximations to the indices may be applied to other caſes. 33. The ſum of the fractions are found when the denominators $= 0$, and conſequently each particular infinite. 34. It is aſſerted, that the ſum of certain fractions given become $= 0$, when the terms are expreſſed by a fraction of which the denominator is a rational function of the diſtance from the firſt term of the ſeries. 35. $\int x^{\alpha - \beta - 1}\dot{x}\int x^{\beta - \gamma - 1}\dot{x}\int x^{\gamma - \delta - 1}\dot{x} \times P$, where $P = Ax^n + Bx^{n+m} + Cx^{n+2m} + Dx^{n+3m} +$, &c, will be to $\int x^{\beta-\alpha-1}\dot{x}\int x^{\alpha-\gamma-1}\dot{x}\int x^{\gamma-\delta-1}\dot{x}\int$ &c. $\times P :: x^{\alpha} : x^{\beta}$ if the fluents are contained between the ſame values of x. 36. Are given ſome ſeries conſiſting of two, of which the one converges, when the other diverges, and conſequently the ſum of both diverges; &c. 37. From the law of a ſeries being given, the law of the ſeries which expreſſes the ſquare, or ſome function of the given ſeries, is found.

1. A method of differences, which deduces from the ſums given any ſucceſſive ſums, e. g. Let S^1, S^2, S^3, S^4, be the logarithms of the ratios $r : r + p$, $r : r + 2p$, $r : r + 3p$, $r : r + 4p$, then will the logarithm of $r : r + 5p$ be $5 \times (S^4 - S^1) + 10\,(S^2 - S^3)$ nearly: then rules are given in general, and likewiſe their errors from the true values.

2. A method of correſpondent values is given, e. g. Let a, b, c, d, &c, be values of x; and S^a, S^b, S^c, S^d, &c, correſpondent values of y; then may

$$y = \frac{(x-b)\ (x-c)\ (x-d)\ \&c}{(a-b)\ (a-c)\ (a-d)\ \&c} \times S^a + \frac{(x-a)\ (x-c)\ (x-d)\ \&c}{(b-a)\ (b-c)\ (b-d)\ \&c} \times S^b + \&c.$$

3. If the formula of the ſeries be $A + Bx + Cx^2 +$ &c $= y$; or $y = \frac{x}{a} \times \frac{(x-b)\ (x-c)\ \&c}{(a-b)\ (a-c)\ \&c} \times S^a + \frac{x}{b} \times \frac{(x-a)\ (x-c)\ \&c}{(b-a)\ (b-c)\ \&c} \times S^b +$ &c; if the formula of the ſeries be $Ax + Bx^2 +$ &c $= y$, which anſwers to Briggs's or Newton's method of interpolations; or the ſeries will be

$$\frac{x^h}{a^h} \times \frac{(x^k - b^k)\ (x^k - c^k)\ (x^k - d^k)\ \&c}{(a^k - b^k)\ (a^k - c^k)\ (a^k - d^k)\ \&c} \times S^a + \frac{x^h}{b^h} \times \frac{(x^k - a^k)\ (x^k - c^k)\ (x^k - d^k)\ \&c}{(b^k - a^k)\ (b^k - c^k)\ (b^k - d^k)\ \&c} \times S^b +$$

&c; if the formula of the ſeries be $Ax^h + Bx^{h+k} + Cx^{h+2k} +$ &c, $= y$; a general formula, which includes the preceding.

5. The ſeries is given for deducing others when the number of correſpondent values given are either even or odd, and the values of x are equidiſtant from each other. 6. And alſo from correſpondent values of x and y to a number of equidiſtant values of x is deduced the value of y to the next ſucceſſive or any ſucceſſive value of x. 7. Some arithmetical theorems are deduced from the preceding propoſitions. 8. Another method is given of reſolving the preceding problem. 9. A method of correcting the ſolution from a ſolution given

given which finds (n) values of y to (n) given values of x true, and m falfe to (m) other values. 10. A fimilar refolution is added from correfpondent values of x, y, z, &c given; and more general refolutions. 11. Given the refolution of fome cafes, and formula in which the general is contained, a method is given in fome cafes of deducing it. 12. The principles of a method of deductions and reductions are added.

In a Pamphlet publifhed at Cambridge, algebraical quantities are tranflated into probable relations, and fome theorems on probabilities thence deduced; to which are adjoined,

1. The theorem $\overline{a+b}\cdot\overline{a+b\pm l}\cdot\overline{a+b\pm 2l}$

$$\overline{a+b\pm 2l}\ldots\overline{a+b\pm\overline{n-1}l} = a\cdot\overline{a\pm l}\cdot\overline{a\pm 2l}\ldots$$

$$\overline{a\pm\overline{n-1}l} + n\times a\cdot\overline{a\pm l}\cdot\overline{a\pm 2l}\ldots\overline{a\pm\overline{n-2}l}\times$$

$$b + n\cdot\frac{n-1}{2}a\cdot a^{\pm l}\cdot\overline{a\pm 2l}\ldots\overline{a\pm\overline{n-3}l}\times$$

$$b\cdot\overline{b\pm l} + n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}\times a\cdot\overline{a\pm l}\cdot\overline{a\pm 2l}\ldots$$

$\overline{a\pm\overline{n-4}l}\times b\cdot\overline{b\pm l}\cdot\overline{b\pm 2l}$ + &c; this becomes the binomial theorem when $l=0$; and it will afford anfwers to fimilar cafes when the whole number of chances are increafed or diminifhed conftantly by l, as the binomial does when they remain the fame, a fimilar multinomial theorem is given. In the fame pamphlet are further added fome new propofitions on chances, on the values of lives, furvivorfhips, &c. In thefe books are alfo contained the inventions of others on fimilar fubjects, which in the prefaces are afcribed to their refpective authors.

In the Philofophical Tranfactions are given fome properties of numbers, &c, of which fome have been publifhed in the books above mentioned; to which may be fubjoined fomething in mixed mathematics, viz, a paper on central forces, which extends not only to central forces, but alfo to forces applied in any other direction, as in the direction of the tangent, and confequently includes refiftances, &c. It gives a rule for finding the forces tending to two or more given points when the curve defcribed and velocity of the body in every point of it is given, *e.g.* Let the curve be an ellipfe, and the velocity the fame at every point, and the two centres of force be the foci of the ellipfe; then will the forces tending to the two foci be equal, and vary as the fquare of the fine of the angle contained between the diftance from the centre of force to the point in which the body is fituated, and the tangent to the curve at that point.

The method of deducing the fluxional equations which exprefs the curve defcribed by a body acted on by any forces tending to given points, or applied in any given directions: fome other propofitions are contained on fimilar fubjects. 2. A paper on the fluxions of the attractions of lines, furfaces, and folids, and from the different methods of deducing them are found different fluents equal to each other: a third paper gives a folution of Kepler's problem of cutting the area of a circle defcribed round a point by approximations, which alfo is applied to other cafes; this likewife contains fome other problems. Many of thefe difcoveries have fince been publifhed, fome in the London, and other foreign tranfactions.

Let $e^l = N$, then will l denote the log. of N to the modulus c. If e the modulus $= 10$, then will the fyftem be the common or Briggs's fyftem of logarithms. Logarithms, and the fums of fome other feriefes, of the formulæ $ax^h + bx^{h+k}$ + &c may be deduced in a manner fimilar to that which was ufed by the Ancients for finding the fines of the arcs of circles.

To particularife the numerous propofitions contained in thefe works, would exceed the limits of our defign. Befides thofe already mentioned, others are interfperfed through the whole works.

ANEMOMETER, p. 111, col. 2, l. 1, *after* 12 ounces, *add* or $\frac{3}{4}$ of a pound. Owing to an overfight in the fucceeding lines, of confidering this 12 ounces as 12 pounds, in the calculations, feveral errors have been incurred, and the 3d column of the table of numbers, in that page, or the column for the velocity, has the numbers only $\frac{1}{4}$ of what they ought to be, or they require to be all multiplied by 4, the fquare-root of 16, the number of ounces in a pound. Hence, in line 6, *for* $\sqrt{12}$ *r.* $\sqrt{\frac{3}{4}}$; l. 7 and 8, *for* $22\frac{4}{5}$ *r.* $91\frac{1}{5}$; l. 8, *for* $15\frac{1}{2}$ *r.* 62. And the whole fucceeding table corrected will be as follows:

Table of the correfponding Height of Water, Force on a Square Foot, and Velocity of Wind,

Height of Water.	Force of Wind.	Velocity of Wind per Hour
Inches.	Pounds.	Miles.
$0\frac{1}{4}$	1·3	18·0
$0\frac{1}{2}$	2·6	25·6
1	5·2	36·0
2	10·4	50·8
3	15·6	62·0
4	20·8	76·0
5	26·0	80·4
6	31·25	88·0
7	36·5	95·2
8	41·7	101·6
9	46·9	108·0
10	52·1	113·6
11	57·3	119·2
12	62·5	124·0

In one inftance Dr. Lind found that the force of the wind was fuch as to be equal $34\frac{6}{10}$ pounds, on a fquare foot; and this by proportion, in the foregoing table, will be found to anfwer to a velocity of 93 miles per hour.

ARCH, p. 137, col. 1, l. 29, *for* fuch cafes as they, *read* fuch cafes they. Line 30, *after* hanches, *add*, See Bridge.

ARCHIMEDES, p. 139, col. 1, l. 52 and 53, *for* preface, a commentary, *read* preface. We find here alfo Eutocius's commentary. Pa. 59, *after* college, *add*, who had the fole care of this edition.

ASSU-

ASSURANCE *on Lives.* Pa. 150, col. 2, in the 3d paragraph, for want of sufficient information concerning the London and Royal Exchange Assurance Offices, that paragraph gives an imperfect and, in some respect, erroneous account of them: it refers to their state 30 years ago, but the Companies have since that, altered their method of proceeding. Instead of that paragraph therefore, take the following account of their present constitution; viz,

The London ASSURANCE, is a corporation established by a charter of king George the 1st, viz, in 1720; under power of which, Assurances are made from the risk of sea-voyages, and from the danger of fire to houses and goods; the prices of which are regulated by the apparent risk to be assured. They also make Assurances on lives; the prices of which are formed on an estimation of the probable duration of life at different ages, on the consideration of the apparent health of the persons to be assured, and of their avocations in life.

This corporation, and the Royal Exchange corporation, gave each the sum of 150,000 pounds to government, for an *exclusive right* of making Assurances as *corporate bodies.* They are known to possess a large and undeniable fund to answer losses. And the prudent management of these corporations has enabled them, of late years, to increase gradually their dividends to the proprietors of their stock. This *exclusive privilege* to make Assurances as corporate bodies, is of great advantage and convenience to the public; and as they act under a common seal, the assured may have a speedy and easy mode of recovering losses, and cannot be subject to any calls or deductions whatever. When their charters were granted to them, it was enacted, that if a proprietor of the stock of one corporation should at the same time, directly or indirectly, be a proprietor of stock in the other corporation, the respective stock so held is to be forfeited, one moiety to the king, the other to the informer. This was evidently settled, to prevent their interest from becoming a joint one; so that they should be made to act in competition to each other, for the greater benefit of the public.

The Royal Exchange ASSURANCE, is a corporation established by charter, as above, under the power of which, Assurances are made from the risk of sea-voyages, and from the danger of fire to houses and goods; the prices of which are regulated by the greater or less risk supposed to be assured. They also make Assurances on lives, the prices of which are formed on estimation of the probable duration of life at different ages, and under different circumstances. The present rates of Assurances on lives are as in the table below. And though a duty on these Assurances should take place on the plan lately proposed to the House of Commons, there is no great probability that these prices will be increased.

This corporation has also, like the former, been empowered to grant life annuities by an act of parliament, which requires that the prices of the annuities should be expressed in tables, hung up in some conspicuous place in their offices, for public inspection; and no agreement for any price is valid, but such as shall be expressed in the tables last made and published by the corporation.

From

From the Office of the CORPORATION *of the* ROYAL EXCHANGE ASSURANCE, *on the* ROYAL EXCHANGE, LONDON.

RATES OF ASSURANCES ON LIVES.

SINGLE LIVES.

Age.	Premium per cent. for an assurance for one year. £. s. d.	Premium per cent. per annum, for an assurance for seven years. £. s. d.	Premium per ct. per ann. for an assurance for the whole continuance of life. £. s. d.
8 to 14	1 2 3	1 6 9	2 7 0
15	1 2 6	1 8 9	2 8 3
16	1 4 0	1 10 9	2 9 9
17	1 6 6	1 12 9	2 11 0
18	1 9 0	1 14 3	2 12 3
19	1 11 3	1 15 9	2 13 6
20	1 14 0	1 16 9	2 14 6
21	1 16 0	1 17 9	2 15 9
22	1 16 6	1 18 3	2 17 9
23	1 17 3	1 18 9	2 19 9
24	1 17 9	1 19 6	2 19 0
25	1 18 3	2 0 3	3 0 3
26	1 19 0	2 0 9	3 1 3
27	1 19 6	2 1 6	3 2 6
28	2 0 3	2 2 3	3 4 0
29	2 1 0	2 3 0	3 5 3
30	2 1 6	2 3 9	3 6 9
31	2 2 3	2 4 6	3 8 3
32	2 3 0	2 5 3	3 9 9
33	2 3 9	2 6 0	3 11 6
34	2 4 9	2 7 3	3 13 0
35	2 5 6	2 8 6	3 14 9
36	2 6 3	2 9 6	3 16 9
37	2 7 3	2 11 0	3 18 6
38	2 8 3	2 12 3	4 0 9
39	2 9 0	2 13 9	4 2 9
40	2 11 0	2 15 3	4 4 0
41	2 12 6	2 16 9	4 7 3
42	2 14 6	2 18 3	4 9 9
43	2 15 9	2 19 9	4 12 3
44	2 17 0	3 1 6	4 14 9
45	2 18 6	3 3 6	4 17 6
46	2 19 9	3 5 9	5 0 3
47	3 1 3	3 8 0	5 3 3
48	3 2 9	3 10 6	5 6 6
49	3 5 3	3 13 3	5 9 9
50	3 9 0	3 16 0	5 13 6
51	3 11 9	3 18 6	5 17 0
52	3 14 0	4 1 0	6 0 6
53	3 16 3	4 3 9	6 4 6
54	3 18 9	4 6 9	6 8 6
55	4 1 3	4 10 0	6 13 0
56	4 4 0	4 13 6	6 17 9
57	4 7 3	4 17 0	7 2 6
58	4 10 3	5 0 9	7 7 9
59	4 14 0	5 4 9	7 13 6
60	4 17 9	5 9 0	7 19 3
61	5 1 9	5 13 9	8 5 6
62	5 5 0	5 18 9	8 12 3
63	5 9 9	6 4 9	8 19 6
64	5 13 6	6 11 0	9 7 3
65	5 19 0	6 18 6	9 16 0
66	6 5 3	7 7 0	10 5 3
67	6 12 0	7 16 6	10 15 3

JOINT LIVES.

For the Assurance of a Gross Sum, payable when One of Two Joint Lives that shall be named shall drop.

Age of the life to be assured.	Age of the life against which the assurance is to be made.	Premium per cent. per ann. £. s. d.
10	10	1 15 9
	20	1 16 6
	30	1 15 6
	40	1 14 9
	50	1 13 9
	60	1 12 6
	70	1 11 3
	80	1 9 3
20	10	2 5 9
	20	2 6 3
	30	2 4 9
	40	2 3 6
	50	2 2 0
	60	2 0 3
	70	1 18 3
	80	1 15 3
30	10	2 16 9
	20	2 17 6
	30	2 15 9
	40	2 13 6
	50	2 11 3
	60	2 8 6
	70	2 5 9
	80	2 2 3
40	10	3 14 0
	20	3 14 9
	30	3 12 9
	40	3 10 0
	50	3 6 0
	60	3 1 9
	70	2 17 6
	80	2 12 3
50	10	5 1 3
	20	5 2 3
	30	5 0 3
	40	4 17 3
	50	4 12 3
	60	4 4 6
	70	3 17 0
	80	3 8 9
60	10	7 6 0
	20	7 7 9
	30	7 5 3
	40	7 2 6
	50	6 18 3
	60	6 8 0
	70	5 12 3
	80	4 17 6
67	10	10 1 3
	20	10 3 6
	30	10 1 0
	40	9 18 3
	50	9 14 6
	60	9 6 0
	70	8 3 6
	80	6 16 0

For the Assurance of a Gross Sum, payable when either of Two Joint Lives shall drop.

Age.	Age.	Premium per ct. per annum. £. s. d.
10	10	3 11 6
	15	3 16 6
	20	4 2 0
	25	4 6 0
	30	4 12 3
	35	4 19 6
	40	5 8 6
	45	5 19 0
	50	6 14 9
	55	7 13 6
	60	8 18 6
	67	11 12 9
15	15	4 1 3
	20	4 7 0
	25	4 11 6
	30	4 17 0
	35	5 4 0
	40	5 13 0
	45	6 4 3
	50	6 19 0
	55	7 17 9
	60	9 2 6
	67	11 16 9
20	20	4 12 6
	25	4 16 9
	30	5 2 3
	35	5 9 0
	40	5 18 3
	45	6 9 6
	50	7 4 3
	55	8 2 9
	60	9 7 9
	67	12 2 3
25	25	5 1 0
	30	5 6 3
	35	5 12 9
	40	6 1 9
	45	6 12 9
	50	7 7 3
	55	8 5 9
	60	9 10 6
	67	12 4 9
30	30	5 11 3
	35	5 17 9
	40	6 6 3
	45	6 17 0
	50	7 11 3
	55	8 9 3
	60	9 13 9
	67	12 7 9

Age.	Age.	Premium per cent. per ann. £. s. d.
35	35	6 3 9
	40	6 12 0
	45	7 2 3
	50	7 16 3
	55	8 14 0
	60	9 18 3
	67	12 11 6
40	40	6 19 9
	45	7 9 9
	50	8 3 6
	55	9 0 6
	60	10 4 3
	67	12 17 0
45	45	7 19 3
	50	8 12 3
	55	9 8 9
	60	10 12 0
	67	13 4 0
50	50	9 4 9
	55	10 0 3
	60	11 2 9
	67	13 13 6
55	55	10 15 3
	60	11 16 3
	67	14 5 6
60	60	12 16 0
	67	15 2 9
67	67	17 4 9

By whom the Affurance is made.	Name, age, and defcription of the life to be Affured.	Time for which the Affurance is made.	*Conditions of Affurance made by Perfons on their own Lives.* The Affurance to be void if the perfon whofe life is Affured fhall depart beyond the limits of Europe, fhall die upon the feas, or enter into or engage in any military or naval fervice whatever, without the previous confent of the company; or fhall come by death by fuicide, duelling, or the hand of Juftice; or fhall not be, at the time the Affurance is made, in good health.	Sum affured.	Rate *per* cent. *per* annum.

By whom the Affurance is made.	Name, age and defcription of the life to be Affured.	Time for which the Affurance is made.	*Conditions of Affurance made by Perfons on the Lives of others.* The Affurance to be void if the perfon whofe life is Affured fhall depart beyond the limits of Europe, fhall die upon the feas, or enter into or engage in any military or naval fervice whatever, without the previous confent of the company; or fhall not be at the time the Affurance is made in good health.	Sum affured.	Rate *per* cent. *per* annum.

Place and date of birth.
If had the fmall-pox.
Whether in the army or navy.
The life Affured to appear at the office, or pay
10*s. per cent.* on Affurances for one year.
15*s. per cent.* for more than one year, and not exceeding feven years.
20*s. per cent.* if for the whole continuance of life.
} In the firft payment only.

Reference to be made to two perfons of repute to afcertain his or her identity.

*** Attendance daily from ten to half paft two o'clock and from five to feven, Saturday in the afternoon excepted.

☞ The lives of perfons engaged in the army or navy may be Affured by fpecial agreement.

N. B. THE CORPORATION ALSO GRANT ANNUITIES ON LIVES.

AUTOMATON. To the end of this article, in pa. 176, col. 2, may be added the following curious particulars, extracted from a letter of an ingenious gentleman since that article was published, viz, Thomas Collinson, Esq. nephew of the late ingenious Peter Collinson, Esq. F. R. S. "Turning over the leaves of your late valuable publication (says my worthy correspondent), part 1. of the Mathematical and Philosophical Dictionary, I observed under the article *Automaton*, the following:" 'But all these seem to be inferior to M. Kempell's chess-player, which may truly be considered as the greatest master-piece in mechanics that ever appeared in the world;' (upon which Mr. Collinson observes) "So it certainly would have been, had its scientific movements depended merely on mechanism. Being slightly acquainted with M. Kempell when he exhibited his chess-playing figure in London, I called on him about five years since at his house at Vienna; another gentleman and myself being then on a tour on the continent. The baron (for I think he is such) shewed me some working models which he had lately made—among them, an improvement on Arkwright's cotton-mill, and also one which he thought an improvement on Boulton and Watt's last steam-engine. I asked him after a piece of speaking mechanism, which he had shewn me when in London. It spoke as before, and I gave the same word as I gave when I first saw it, *Exploitation*, which it distinctly pronounced with the French accent. But I particularly noticed, that not a word passed about the chess-player; and of course I did not ask to see it.—In the progress of the tour I came to Dresden, where becoming acquainted with Mr. Eden, our envoy there, by means of a letter given me by his brother lord Auckland, who was ambassador when I was at Madrid, he obligingly accompanied me in seeing several things worthy of attention. And he introduced my companion and myself to a gentleman of rank and talents, named Joseph Friedrick Freyhere, who seems completely to have discovered the *Vitality* and soul of the chess-playing figure. This gentleman courteously presented me with the treatise he had published, dated at Dresden, Sept. 30, 1789, explaining its principles, accompanied with curious plates neatly coloured. This treatise is in the German language; and I hope soon to get a translation of it. A well-taught boy, very thin and small of his age (sufficiently so that he could be concealed in a drawer almost immediately under the chess-board), agitated the whole. Even after this abatement of its being strictly an automaton, much ingenuity remains to the contriver.—This discovery at Dresden accounts for the silence about it at Vienna; for I understand, by Mr. Eden, that Mr. Freyhere had sent a copy to baron Kempell: though he seems unwilling to acknowledge that Mr. F. has completely analysed the whole.

"I know that long and uninteresting letters are formidable things to men who know the value of time and science: but as this happens to be upon the subject, forgive me for adding one very admirable piece of mechanism to those you have touched upon. When at Geneva, I called upon Droz, son of the original Droz of la Chaux de Fonds (where I also was). He shewed me an oval gold snuff box, about (if I recollect right) 4 inches and a half long, by 3 inches broad, and about an inch and a half thick. It was double, having an horizontal partition; so that it may be considered as one box placed on another, with a lid of course to each box—One contained snuff—In the other, as soon as the lid was opened, there rose up a very small bird, of green enamelled gold, sitting on a gold stand. Immediately this minute curiosity wagged its rail, shook its wings, opened its bill of white enamelled gold, and poured forth, minute as it was (being only three quarters of an inch from the beak to the extremity of the tail) such a clear melodious song, as would have filled a room of 20 or 30 feet square with its harmony.—Droz agreed to meet me at Florence; and we visited the Abbé Fontana together. He afterwards joined me at Rome, and exhibited his bird to the pope and the cardinals in the Vatican palace, to the admiration, I may say to the astonishment of all who saw and heard it."

Another extract from a second letter upon the same subject, by Mr. Collinson, is as follows: "Permit me to speak of another Automaton of Droz's, which several years since he exhibited in England; and which, from my personal acquaintance, I had a commodious opportunity of particularly examining. It was a figure of a man, I think the size of life. It held in its hand a metal style; a card of Dutch vellum being laid under it. A spring was touched, which released the internal clockwork from its stop, when the figure immediately began to draw. Mr. Droz happening once to be sent for in a great hurry to wait upon some considerable personage at the west end of the town, left me in possession of the keys, which opened the recesses of all his machinery. He opened the drawing-master himself; wound it up; explained its leading parts; and taught me how to make it obey my requirings, as it had obeyed his own. Mr. Droz then went away. After the first card was finished, the figure rested. I put a second; and so on, to five separate cards, all different subjects: but five or six was the extent of its delineating powers. The first card contained, I may truly say, elegant portraits and likenesses of the king and queen, facing each other: and it was curious to observe with what precision the figure lifted up his pencil, in the transition of it from one point of the draft to another, without making the least slur whatever: for instance, in passing from the forehead to the eye, nose, and chin; or from the waving curls of the hair to the ear, &c. I have the cards now by me, &c, &c."

Pa. 177, col. 1, l. 2, *for* August *read* September.

Page

B.

PAGE 195, col. 1, at the end of the article on Barometrical Meaſurements of Altitudes, *add*, See a learned paper in vol. 1. of the Tranſactions of the R. Soc. of Edinburgh, " On the Cauſes which affect the Accuracy of Barometrical Meaſurements; by John Playfair, A. M. F. R. S. Edin. and Profeſſor of Mathematics in the Univerſity of Edinburgh." Alſo another by Dr. Damen, late Profeſſor of Mathematics and Philoſophy in the Univerſity of Leyden, intitled, " Diſſertatio Phyſica & Mathematica de Montium Altitudine Barometro Metienda: Accedit Refractionis Aſtronomicæ Theoria; in 8vo, at the Hague, 1783.

Pa. 205, col. 1, after the life of Dan. Bernoulli, *add* the following life of James.

BERNOULLI (James), another mathematical branch of the foregoing celebrated family. He was born at Baſil in October 1759; being the ſon of John Bernoulli, and grandſon of the firſt John Bernoulli, before mentioned, and the nephew of Daniel Bernoulli laſt noticed above. Our author's elder brother John, who ſtill lives at Berlin, is alſo well known in the republic of ſcience, particularly for his aſtronomical labours.

The gentleman to whom this article relates, was educated, as moſt of his relations had been, for the profeſſion of law: but his genius led him very early into the ſtudy of mathematics; and at 20 years of age he read public lectures on experimental philoſophy in the univerſity of Baſil, for his uncle Daniel Bernoulli, whom he hoped to have ſucceeded as profeſſor. Being diſappointed in this view, he reſolved to leave his native place, and to ſeek his fortune elſewhere; hence he accepted the office of ſecretary to Count Breuner, the emperor's envoy to the republic of Venice; and in this city he remained till the year 1786, when, on the recommendation of his countryman, M. Fuſs, he was invited to Peterſburgh to ſucceed M. Lexell in the academy there, where he continued till his death, which happened the 3d of July 1789, at not quite 30 years of age, and when he had been married only two months, to the youngeſt daughter of John Albert Euler, the ſon of the ſo celebrated Leonard Euler.

Impoſſible or Imaginary BINOMIAL. After this article, in pa. 208, the middle of col. 1, *add* what here follows.

In the foregoing article are given ſeveral rules for the roots of Binomials. Dr. Maſkelyne, the Aſtronomer Royal, has alſo given a method of finding any power of an Impoſſible Binomial, by another like Binomial. This rule is given in his Introduction prefixed to Taylor's Tables of Logarithms, pa. 56; and is as follows.

The logarithms of a and b being given, it is required to find the power of the Impoſſible Binomial $a \pm \sqrt{-b^2}$ whoſe index is $\frac{m}{n}$, that is, to find $(a \pm \sqrt{-b^2})^{\frac{m}{n}}$ by another Impoſſible Binomial; and thence the value of $(a+\sqrt{-b^2})^{\frac{m}{n}} + (a-\sqrt{-b^2})^{\frac{m}{n}}$, which is always poſſible, whether a or b be the greater of the two.

Solution. Put $\frac{b}{a}$ = tang. z. Then

$$(a\pm\sqrt{-b^2})^{\frac{m}{n}} = (a^2+b^2)^{\frac{m}{2n}} \times (\text{coſ.}\tfrac{m}{n}z \pm \sqrt{-\text{ſin.}^2\tfrac{m}{n}z}).$$

Hence $(a+\sqrt{-b^2})^{\frac{m}{n}} + (a-\sqrt{-b^2})^{\frac{m}{n}} = (a^2+b^2)^{\frac{m}{2n}} \times 2\,\text{coſ.}\frac{m}{n}z = (a \times \text{ſec.}\,z)^{\frac{m}{n}} \times 2\,\text{coſin.}\frac{m}{n}z = (b \times \text{coſec}\,z.)^{\frac{m}{n}} \times 2\,\text{coſin.}\frac{m}{n}z$, where the firſt or ſecond of theſe two laſt expreſſions is to be uſed, according as z is an extreme or mean arc; or rather, becauſe $\frac{b}{a}$ is not only the tangent of z, but alſo of $z + 360°$, $z + 720°$, &c; therefore the factor in the anſwer will have ſeveral values, viz,

$2\,\text{coſ.}\frac{m}{n}z$; $2\,\text{coſ.}\frac{m}{n}(z+360°)$; $2\,\text{coſ.}\frac{m}{n}(z+720°)$; &c; the number of which, if m and n be whole numbers, and the fraction $\frac{m}{n}$ be in its leaſt terms, will be equal to the denominator n; otherwiſe infinite.

By Logarithms. Put log. b + 10 − log. a = log. tan. z.

Then log. $\left((a+\sqrt{-b^2})^{\frac{m}{n}} + (a-\sqrt{-b^2})^{\frac{m}{n}}\right) =$

$$= \frac{m}{n} \times (\text{l.}\,a + 10 - \text{l. coſ.}\,z) + \text{l.}\,2 + \text{l. coſ.}\tfrac{m}{n}z - 10$$

$$= \frac{m}{n} \times (\text{l.}\,b + 10 - \text{l. ſin.}\,z) + \text{l.}\,2 + \text{l. coſ.}\tfrac{m}{n}z - 10;$$

where the firſt or ſecond expreſſion is to be uſed, according as z is an extreme or mean arc. Moreover by taking ſucceſſively, l. coſ. $\frac{m}{n}z$; l. coſ. $\frac{m}{n}(z+360°)$; l. coſ. $\frac{m}{n}(z+720°)$; &c, there will ariſe ſeveral diſtinct anſwers to the queſtion, agreeably to the remark above.

BINOMIAL *Theorem.* Francis Maſeres, Eſq. (Curſitor Baron of the Exchequer) has communicated the

the following obfervations on the Binomial theorem, and its demonftration; viz, About the year 1666 the celebrated Sir Ifaac Newton difcovered that, if m were put for any whole number whatfoever, the coefficients of the terms of the mth power of $1 + x$ would be

$$1, \frac{m}{1}, \frac{m}{1}\cdot\frac{m-1}{2}, \frac{m}{1}\cdot\frac{m-1}{2}\cdot\frac{m-2}{3}, \&c,$$

till we come to the term $\frac{m-(m-1)}{m}$, which will be the laft term. But how he difcovered this propofition, he has not told us, nor has he even attempted to give a demonftration of it. Dr. John Wallis, of Oxford, informs us (in his Algebra, chap. 85, pa. 319) that he had endeavoured to find this manner of generating thefe coefficients one from another, but without fuccefs; and he was greatly delighted with the difcovery, when he found that Mr. Newton had made it. But he likewife has omitted to give a demonftration of it, as well as Sir Ifaac Newton; and probably he did not know how to demonftrate it.

Sir Ifaac Newton, after he had difcovered this rule for generating the coefficients of the powers of $1 + x$ when the indexes of thofe powers were whole numbers, conjectured that it might poffibly be true likewife when they were fractions. He therefore refolved to try whether it was or not, by applying it to fuch indexes in a few eafy inftances, and particularly to the indexes $\frac{1}{2}$ and $\frac{1}{3}$, which, if the rule held good in the cafe of fractional indexes, would enable him to find feriefes equal to the values of $\overline{1 + x)}^{\frac{1}{2}}$ and $\overline{1 + x)}^{\frac{1}{3}}$, or the fquare-root and the cube-root of the Binomial quantity $1 + x$. And, when he had in this manner obtained a feries for $\overline{1 + x)}^{\frac{1}{2}}$, which he fufpected to be equal to $\overline{1 + x)}^{\frac{1}{2}}$, or the fquare root of $1 + x$, he multiplied the faid feries into itfelf, and found that the product was $1 + x$; and when he had obtained a feries for $\overline{1 + x)}^{\frac{1}{3}}$ he multiplied the faid feries twice into itfelf, and found that the product was $1 + x$; and thence he concluded that the former feries was really equal to the fquare-root of $1 + x$, and that the latter feries was really equal to its cube-root. And from thefe and a few more fuch trials, in which he found the rule to anfwer, he concluded univerfally that the rule was always true, whether the index m ftood for a whole number or a fraction of any kind, as $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{2}, \frac{5}{7}, \frac{9}{5}$, or, in general $\frac{p}{q}$.

After the difcovery of this rule by Sir Ifaac Newton, and the publication of it by Dr. Wallis, in his Algebra, chap. 85, in the year 1685, (which I believe was the firft time it was publifhed to the world at large, though it was inferted in Sir Ifaac Newton's firft letter to Mr. Oldenburgh, the fecretary to the Royal Society, dated June 13, 1676, and the faid letter was fhewn to Mr. Leibnitz, and probably to fome other of the learned mathematicians of that time it remained for fome years without a demonftration, either in the cafe of integral powers or of roots. At laft however it was demonftrated in the cafe of integral powers by means of the properties of the figurate numbers, by that learned, fagacious, and accurate mathematician Mr. James Bernoulli, in the 3d chapter of the 2d part of his excellent treatife *De Arte Conjectandi*, or, *On the Art of forming reafonable Conjectures concerning Events that depend on Chance*; which appears to me to be by much the beft written treatife on the doctrine of Chances that has yet been publifhed, though Mr. Demoivre's book on the fame fubject may have carried the doctrine fomething further. This treatife of Mr. James Bernoulli's was not publifhed till the year 1713, which was fome years after his death, which happened in Auguft 1705; but there is reafon to think that it was compofed in the latter years of the preceding century, about the years 1696, 1697, 1698, 1699, and 1700, and even that fome parts of it, or fome of the propofitions inferted in it, had been found out by the author in the years 1689, 1690, 1691, and 1692. For the firft part of his very curious tract, intitled, *Pofitiones Arithmeticæ de Seriebus Infinitis* was publifhed at Bafil or Bafle in Switzerland (which was his native place, and in which he was at that time profeffor of mathematics) in the year 1689; and the fecond part of the faid *Pofitiones* (in the 19th Pofition of which thofe properties of the figurate numbers from which the Binomial theorem may be deduced, are fet down) was publifhed at the fame place in the year 1692. But the demonftrations of thofe properties of the figurate numbers, and of the Binomial theorem, which depends upon them, were never as I believe communicated to the public till the year 1713, when the author's pofthumous treatife *De Arte Conjectandi* made its appearance. Thefe demonftrations are founded on clear and fimple principles, and afford as much fatiffaction as can well be expected on the fubject. But the full difplay and explanation of thefe principles, and the deduction of the faid properties of the figurate numbers, and ultimately of the Binomial theorem, from them, is a matter of confiderable length. It will not therefore be amifs to give a fhorter proof of the truth of this important theorem, that fhall not require a previous knowledge of the properties of the figurate numbers, but yet fhall be equally conclufive with that which is derived from thofe properties. Now this may be done in the manner following.

Let us fuppofe that the coefficients of the terms of the firft fix powers of the Binomial quantity $1 + x$ have been found, upon trial, to be fuch as would be produced by the general expreffions

$$1, \frac{m}{1}, \frac{m}{1}\cdot\frac{m-1}{2}, \frac{m}{1}\cdot\frac{m-1}{2}\cdot\frac{m-2}{3}, \&c,$$

by fubftituting in them firft 1, then 2, then 3, then 4, then 5, and laftly 6, inftead of m. This may eafily be tried by raifing the faid firft fix powers of $1 + x$ by repeated multiplications by $1 + x$ in the common way, and afterwards finding the terms of the fame powers by means of the faid general expreffions above; which will be found to produce the very fame terms as arofe from the multiplications. After thefe trials we fhall be fure that thofe general expreffions are the true values of the coefficients of the powers of $1 + x$ at leaft in the faid firft fix powers. And it will therefore only remain

remain to be proved that, ſince the rule is true in the ſaid firſt ſix powers, it will alſo be true in the next following, or the 7th power, and conſequently in the 8th, 9th and 10th powers, and in all higher powers whatſoever.

Now, if the coefficients of the 1ſt, 2d, 3d, 4th, and other following terms of $\overline{1+x}|^{m}$ be denoted by the letters a, b, c, d, &c, reſpectively, it is evident from the nature of multiplication, that the coefficients of the 1ſt, 2d, 3d, 4th, and other following terms of the next higher power of $1+x$, to wit, $\overline{1+x}|^{m+1}$ will be equal to a, $a+b$, $b+c$, $c+d$, &c, reſpectively, or to the ſums of every two contiguous coefficients of the terms of the preceding ſeries which is $=\overline{1+x}|^{m}$. This will appear from the operation of multiplication, which is as follows.

$$\begin{array}{l} a + bx + cx^2 + dx^3 + ex^4 + \&c \\ 1 + x \\ \hline a + bx + cx^2 + dx^3 + ex^4 + \&c \\ \quad + ax + bx^2 + cx^3 + dx^4 + \&c. \\ \hline \end{array}$$

Therefore, if $\overline{1+x}|^{m}$ is equal to the ſeries

$$a + bx + cx^2 + dx^3 + ex^4 + \&c,$$

then $\overline{1+x}|^{m+1}$ will be equal to the ſeries

$$a + \overline{a+b}\,.\,x + \overline{b+c}\,.\,x^2 + \overline{c+d}\,.\,x^3 + \&c.$$

Now let n be $= m+1$. We ſhall then have to prove that, if the coefficients a, b, c, d, &c, be reſpectively equal to

$$1,\ \frac{m}{1},\ \frac{m}{1}.\frac{m-1}{2},\ \frac{m}{1}.\frac{m-1}{2}.\frac{m-2}{3},\ \&c,$$

the coefficients a, $a+b$, $b+c$, &c, will be reſpectively equal to

$$1,\ \frac{n}{1},\ \frac{n}{1}.\frac{n-1}{2},\ \frac{n}{1}.\frac{n-1}{2}.\frac{n-2}{3},\ \&c.$$

In order to prove this, there is nothing more to do than to collect together every two terms of the former of theſe two ſeries, and then ſubſtitute into theſe ſums, n inſtead of $m+1$, when there will immediately come out the terms of the latter ſeries as above, viz,

$$\overline{1+x}|^{n} = 1 + \frac{n}{1}x + \frac{n}{1}.\frac{n-1}{2}x^2 + \&c. \qquad \text{Q. E. D.}$$

BINOMIAL *Theorem, Improvement of*. Mr. Bonnycaſtle, of the Royal Mil. Acad. has lately diſcovered the following ingenious improvement of this theorem, which is now publiſhed for the firſt time.

This celebrated theorem has been given under various forms, ſince the time of its firſt invention; but the following property of it is conceived to be new, and capable of an application of which the original ſeries is not ſuſceptible.

The Newtonian theorem, in one of its moſt commodious forms, is

$$\overline{1+p}|^{n} = 1 + np + \frac{n.n-1}{2}p^2 + \frac{n.n-1.n-2}{2.3}p^3 + \frac{n.n-1.n-2.n-3}{2.3.4}p^4$$

&c; and the new theorem here alluded to, is

$$\overline{1+p}|^{n} = 1 + sn + \frac{1}{2}s^2n^2 + \frac{1}{2.3}s^3n^3 + \frac{1}{2.3.4}s^4n^4\ \&c;$$

where $s = p - \frac{1}{2}p^2 + \frac{1}{3}p^3 - \frac{1}{4}p^4 + \frac{1}{5}p^5$ &c.

Of which the inveſtigation is as follows:

$$\overline{1+p}|^{n} = 1 + np + \frac{n.n-1}{2}p^2 + \frac{n.n-1.n-2}{2.3}p^3 + \frac{n.n-1.n-2.n-3}{2.3.4}p^4\ \&c$$

$$= 1 + np + (n^2-1)\frac{p^2}{2} + (n^3-3n^2+2n)\frac{p^3}{2.3} + (n^4-6n^3+11n^2-6n)\frac{p^4}{2.3.4}$$

$$+ (n^5 - 10n^4 + 35n^3 - 50n^2 + 24n)\frac{p^5}{2.3.4.5}$$

$$+ (n^6 - 15n^5 + 85n^4 - 225n^3 + 274n^2 - 120n)\frac{p^6}{2.3.4.5.6}\ \&c.$$

Then by connecting the ſeveral powers of p with all the like powers of n, the latter ſeries will become

$$\begin{aligned} 1 &+ \Big(p - \frac{p^2}{2} + \frac{2p^3}{2.3} - \frac{6p^4}{2.3.4} + \frac{24p^5}{2.3.4.5} - \frac{120p^6}{2.3.4.5.6}\ \&c\Big)n \\ &+ \Big(\frac{p^2}{2} - \frac{3p^3}{2.3} + \frac{11p^4}{2.3.4} - \frac{50p^5}{2.3.4.5} + \frac{274p^6}{2.3.4.5.6}\ \&c\Big)n^2 \\ &+ \Big(\frac{p^3}{2.3} - \frac{6p^4}{2.3.4} + \frac{35p^5}{2.3.4.5} - \frac{225p^6}{2.3.4.5.6}\ \&c\Big)n^3 \\ &+ \Big(\frac{p^4}{2.3.4} - \frac{10p^5}{2.3.4.5} + \frac{85p^6}{2.3.4.5.6}\ \&c\Big)n^4 \\ &+ \Big(\frac{p^5}{2.3.4.5} - \frac{15p^6}{2.3.4.5.6}\ \&c\Big)n^5 \\ &+ \Big(\frac{p^6}{2.3.4.5.6}\ \&c\Big)n^6 \\ &\qquad \&c; \end{aligned}$$

which by abbreviation, &c, becomes

$$\begin{aligned} 1 &+ \Big(p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} + \frac{p^5}{5} - \frac{p^6}{6}\ \&c\Big)n \\ &+ \tfrac{1}{2}\Big(p^2 - \frac{3p^3}{3} + \frac{11p^4}{3.4} - \frac{50p^5}{3.4.5} + \frac{274p^6}{3.4.5.6}\ \&c\Big)n^2 \\ &+ \frac{1}{2.3}\Big(p^3 - \frac{6p^4}{4} + \frac{35p^5}{4.5} - \frac{225p^6}{4.5.6}\ \&c\Big)n^3 \\ &+ \frac{1}{2.3.4}\Big(p^4 - \frac{10p^5}{5} + \frac{85p^6}{5.6}\ \&c\Big)n^4 \\ &+ \frac{1}{2.3.4.5}\Big(p^5 - \frac{15p^6}{6}\ \&c\Big)n^5 \\ &+ \frac{1}{2.3.4.5.6}\Big(p^6\ \&c\Big)n^6 \\ &\qquad \&c. \end{aligned}$$

In which laſt ſeries, the literal parts of the coefficients of the 3d, 4th, 5th, &c terms, are the ſquare, cube, biquadrate, &c, of the coefficient of the 2d term, as will appear either from the actual involution of $p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4}$ &c, or by comparing its ſeveral powers with the multinomial theorem of Demoivre.

From hence it follows that,

$\overline{1+p}|^{n}$

$$\overline{1+p}|^{n} = 1 + (p - \frac{p^2}{2} + \frac{p^3}{3} \&c)n + \frac{1}{2}(p - \frac{p^2}{2} + \frac{p^3}{3} \&c)n^2$$

$$+ \frac{1}{2.3}(p - \frac{p^2}{2} + \frac{p^3}{3} \&c)n^3 + \frac{1}{2.3.4}(p - \frac{p^2}{2} + \frac{p^3}{3} \&c)n^4$$

$$\&c.$$

And if $p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4}$ &c be put $= s$, we ſhall have

$$\overline{1+p}|^{n} = 1 + sn + \frac{1}{2}s^2n^2 + \frac{1}{2.3}s^3n^3 + \frac{1}{2.3.4}s^4n^4 \ \&c,$$

as was to be ſhewn.

By a ſimilar mode of deduction, it may alſo be proved that

$$\overline{1-p}|^{n} = 1 - \int n + \tfrac{1}{2}\int^2 n^2 - \frac{1}{2.3}\int^3 n^3 + \frac{1}{2.3.4}\int^4 n^4 \&c;$$

where in this caſe $\int = p + \frac{p^2}{2} + \frac{p^3}{3} + \frac{p^4}{4}$ &c.

In each of which formulæ, the index n, may be conſidered either as a whole number, a fraction, a ſurd, a given or an unknown quantity, as the circumſtance may require.

For the application of theſe theorems, ſee LOGARITHMS, and EXPONENTIAL *Equations*, following.

C.

CANAL, in general, denotes a long, round, hollow inſtrument, through which a fluid matter may be conveyed. In which ſenſe, it amounts to the ſame as what is otherwiſe called a pipe, tube, channel, &c. Thus the Canal of an aqueduct, is the part through which the water paſſes; which, in the ancient works of this kind, is lined with a coat of maſtic of a peculiar compoſition.

CANAL more particularly denotes a kind of artificial river, often furniſhed with locks and ſluices, and ſuſtained by banks or mounds. They are contrived for divers purpoſes; ſome for forming a communication between one place and another; as the Canals between Bruges and Ghent, or between Bruſſels and Antwerp: Others for the decoration of a garden, or houſe of pleaſure; as the Canals of Verſailles, Fontainbleau, St. James's Park, &c: And others are made for draining wet and marſhy lands; which laſt however are more properly called water-gangs, drains, ditches, &c.

It is needleſs to enumerate the many advantages ariſing from Canals and artificial navigations. Their utility is now ſo apparent, that moſt nations in Europe give the higheſt encouragement to undertakings of this kind wherever they are practicable. Nor did their advantages eſcape the obſervation of the Ancients. From the earlieſt accounts of ſociety we read of attempts to cut through large iſthmuſes, to make communications by water, either between one ſea and another, or between different nations, or diſtant parts of the ſame nation, where land-carriage was long and expenſive.

Egypt is full of Canals, dug to receive and diſtribute the waters of the Nile, at the time of its inundation. They are dry the reſt of the year, except the Canal of Joſeph, and four or five others, which may be ranked as conſiderable rivers. There were alſo ſubterraneous Canals, or tunnels, dug by an ancient king of Egypt, by which thoſe lakes, formed by the inundations of the Nile, were conveyed into the Mediterranean ſea.

Herodotus relates, that the Cnidians, a people of Coria, in Aſia Minor, deſigned to cut through the iſthmus which joins that peninſula to the continent; but were ſuperſtitious enough to give up the undertaking, becauſe it was interdicted by an oracle.

Several kings of Egypt attempted to join the Red-Sea to the Mediterranean; a project which Cleopatra was very fond of. This Canal was begun, according to Herodotus, by Necus ſon of Pſammeticus, who deſiſted from the attempt on an anſwer from the oracle, after having loſt 120 thouſand men in the enterpriſe. It was reſumed and completed by Darius ſon of Hyſtaſpes, or, according to Diodorus and Strabo, by Ptolomy Philadelphus; who relate that Darius relinquiſhed the work on a repreſentation made to him by unſkilful engineers, that the Red-Sea, being higher than the land of Egypt, would overflow and drown the whole country. It was wide enough for two galleys to paſs abreaſt, and its length was four days ſailing. Diodorus adds, that it was alſo called Ptolomy's river; that this prince built a city at its mouth on the Red-Sea, which he called Arſinoë, from the name of his favourite ſiſter; and that the Canal might be either opened or ſhut, as occaſion required. Diod. Sic. lib. 1; Strabo, Geog. lib. 17; Herod. lib. 2. Soliman the 2d, emperor of the Turks, employed 50 thouſand men in this great work; which was completed under the caliphate of Omar, about the year 635; but was afterward allowed to fall into neglect and diſrepair; ſo that it is now difficult to diſcover any traces of it. Hiſt. Acad. Scienc. ann. 1703, pa. 110.

Both the Greeks and Romans intended to make a Canal acroſs the Iſthmus of Corinth, which joins the Morea and Achaia, for a navigable paſſage by the Ionian ſea into the Archipelago. Demetrius, Julius Cæſar, Caligula, and Nero, made ſeveral unſucceſsful efforts to open this paſſage. But as the Ancients were entirely ignorant of the uſe of water-locks, their whole attention

attention was employed in making level cuts, which is probably the chief reason why they so often failed in their attempts. Charlemagne formed a design of joining the Rhine and the Danube, to make a communication between the Ocean and the Black-Sea, by a Canal from the river Almutz which discharges itself into the Danube, to the Reditz, which falls into the Maine, which last falls into the Rhine near Mayence or Mentz: for this purpose he employed a prodigious number of workmen; but he met with so many obstacles from different quarters, that he was obliged to give up the attempt.

A new Canal for conveying the waters of the Nile from Ethiopia into the Red-Sea without passing into Egypt, was projected by Albuquerque, viceroy of India for the Portuguese, to render Egypt barren and unprofitable to the Turks.—M. Gaildereau attributes the frequency of the plague in Egypt, of late days, to the decay, or stopping up of these Canals; which happened upon the Turks becoming masters of the country.

In China, there is scarce a town or village without the advantage either of an arm of the sea, a navigable river, or a Canal, by which means navigation is rendered so common, that there are almost as many people on the water as the land. The great Canal of China, is one of the wonders of art, extending from north to south quite across the empire, from Pekin to Canton, a distance of 825 miles, and was made upwards of 800 years ago. Its breadth and depth are sufficient to carry barks of considerable burden, which are managed by sails and masts, as well as rowed by hand. On this Canal it seems the emperor employs near ten thousand ships. It passes through, or by, 41 large cities; there are in it 75 vast locks and sluices, to keep up the water, and pass the ships where the ground will not admit of sufficient depth of channel, beside several thousand draw and other bridges. Indeed, F. Magailliane assures us, there are passages from one end of China to the other, the space of 600 French leagues, either by Canals or rivers, except a single day's journey by land, necessary to cross a mountain.

The French at present have many fine Canals. That of Briere, otherwise called the Canal of Burgundy, was begun under Henry IV, and finished under the direction of cardinal Richelieu in the reign of Louis XIII. This Canal makes a communication between the Loire and the Seine, and so to Paris. It extends 11 French great leagues from Briere to Montargis, and has 42 locks upon it.

The Canal of Orleans was begun in 1675, for establishing a communication also between the Seine and the Loire. It is considerably shorter than that of Briere, and has only 20 sluices.

The Canal of Bourbon was but lately undertaken: its design is to make a communication from the river Oise to Paris.

But the greatest and most useful work of this kind, is the junction of the Ocean with the Mediterranean by the Canal of Languedoc, called also the Canal of the two seas. It was proposed in the reigns of Francis I and Henry IV, and was begun and finished under Louis XIV; having been planned by Francis Riquet in the year 1666, and finished before his death, which happened in 1680. It begins with a large reservoir 4000 paces in circumference, and 24 feet deep, which receives many springs from the mountain Noire. The Canal is about 200 miles in length, extending from Narbonne to Tholouse, being supplied by a number of rivulets in the way, and furnished with 104 locks or sluices, of about 8 feet rise each. In some places it is carried over bridges and aqueducts of vast height, which give passage underneath to other rivers; and in some places it is cut through solid rocks for a mile together.

The new Canal of the lake Ladoga, cut from Volhowa to the Neva, by which a communication is made between the Baltic, or rather Ocean, and the Caspian sea, was begun by the czar Peter the 1st in 1719: by means of which the English and Dutch merchandize is easily conveyed into Persia, without being obliged to double the Cape of Good Hope.—There was a former Canal of communication between the Ladoga lake and the river Wolga, by which timber and other goods had been brought from Persia to Petersburg; but the navigation of it was so dangerous, that a new one was undertaken.

The Spaniards have several times had in view the digging a Canal through the Isthmus of Darien, between North and South America, from Panama to Nombre de Dios, to make a ready communication between the Atlantic and the South Sea, and thus afford a straight passage to China and the East Indies.

In the Dutch, Austrian, and French Netherlands, there is a great number of Canals: that from Bruges to Ostend carries vessels of 200 tons. But it would be an endless task to describe the numberless Canals in Holland, Germany, Russia, &c. We may therefore only take a view of those in our own country.

In England, that ancient Canal from the river Nyne, a little below Peterborough, to the river Witham, three miles below Lincoln; called by the modern inhabitants Caerdike; may be ranked among the monuments of the Roman grandeur, though it is now most of it filled up. Morton will have it made under the emperor Domitian. Urns and medals have been discovered on the banks of this Canal, which seem to confirm that opinion. Yet some authors take it to be a Danish work. It was 40 miles in length; and, so far as appears from the ruins, must have been very broad and deep. Notwithstanding that early beginning, it is not long since Canals have been revived in this country. They are now however become very numerous, particularly in the counties of York, Lincoln, and Cheshire. Most of the counties between the mouth of the Thames and the Bristol channel are connected together either by natural or artificial navigations; those upon the Thames and Isis reaching within about 20 miles of those upon the Severn.

The Canal for supplying London with water by means of the New River, was projected and begun by Mr. Edward Wright, author of the celebrated treatise on Navigation, about the year 1608; but finished by Mr. (afterwards Sir Hugh) Middleton, five years after. This Canal commences near Ware, in Hertfordshire, and takes a course of 60 miles before it reaches the cistern at Islington, which supplies the several water-pipes that convey it to the city and parts adjacent. In some places

places it is 30 feet deep, and in others it is conveyed over a valley between two hills, by means of a trough supported on wooden arches, and rising above 23 feet in height.

The Duke of Bridgwater's Canal, projected and executed under the direction of Mr. Brindley, was begun about the year 1759. It was first designed only for conveying coals to Manchester, from a mine in the duke's estate; but has since been applied to many other useful purposes of inland navigation. This Canal begins at a place called Worsley-mill, about 7 miles from Manchester, where a bason is made capable of holding all the boats, and a great body of water which serves as a reservoir or head to the navigation. The Canal runs through a hill by a subterraneous passage, large enough for admitting long flat-bottomed boats, which are towed by a rail on each hand, near three quarters of a mile, to the coal-works. There the passage divides into two channels, one of which goes off 300 yards to the right, and the other as many to the left; and both may be continued at pleasure. The passage is in some places cut through the solid rock, and in others arched over with brick; and air-funnels, some of which are near 37 yards perpendicular, are cut, at certain distances, through the rock to the top of the hill. The arch at its entrance is about 6 feet wide, and about 5 feet high from the surface of the water; but widens within, so that in some places the boats may pass one another, and at the pits it is 10 feet wide. When the boats are loaded and brought out of the bason, five or six of them are linked together, and drawn along the Canal by a single horse, and thus reaching Manchester in a course of nine miles. It is broad enough for two barges to pass or go abreast; and on one side there is a good road for the passage of the people, and the horses or mules employed in the work. The Canal is raised over public roads by means of arches; and it passes over the navigable river Irwell near 50 feet above it; so that large vessels in full sail pass under the Canal, while the duke's barges are at the same time passing over them. This Canal joins that which passes from the river Mersey towards the Trent, taking in the whole a course of 34 miles.

The Lancaster Canal begins near Kendal, and terminates near Eccleston, comprehending the distance of $72\frac{1}{2}$ miles.

The Canal from Liverpool to Leeds is $108\frac{1}{2}$ miles: that from Leeds to Selby, $23\frac{1}{4}$ miles; from Chichester to Middlewich, $26\frac{3}{4}$ miles; from the Trent to the Mersey, 88 miles; from the Trent to the Severn, $46\frac{1}{2}$ miles. The Birmingham Canal joins this near Wolverhampton, and is $24\frac{1}{4}$ miles: the Droitwich Canal is $5\frac{1}{2}$ miles: the Coventry Canal, commencing near Lichfield, and joining that of the Trent, is $36\frac{1}{4}$ miles: the Oxford Canal breaks off from this, and is 82 miles: the Chesterfield Canal joins the Trent near Gainsborough, and is 44 miles.

A communication is now formed, by means of this inland navigation, between Kendal and London, by way of Oxford; between Liverpool and Hull, by the way of Leeds; and between the Bristol channel and the Humber, by the junction formed between the Trent and the Severn. Other schemes have been projected, which the present spirit of improvement will probably soon carry into execution, of opening a communication between the German and Irish seas, so as to reduce a hazardous navigation of more than 800 miles by sea, into a little more than 150 miles by land, or inland navigation; and also of joining the Isis with the Severn.

In Scotland, a navigable Canal between the Forth and Clyde, which divides that country into two parts, was thought of more than a century since, for transports and small ships of war. It was again projected in the year 1722, and a survey made; but nothing more was done till 1761, when the then lord Napier, at his own expence, had a survey, plan, and estimate made on a small scale. In 1764, the trustees for fisheries, &c, in Scotland, procured another survey, plan, and estimate of a Canal 5 feet deep, which was to cost 79,000 pounds. In 1766, a subscription was obtained by a number of the most respectable merchants in Glasgow, for making a Canal 4 feet deep and 24 feet in breadth; but when the bill was nearly obtained in parliament, it was given up on account of the smallness of the scale, and a new subscription set on foot for a Canal 7 feet deep, estimated at 150,000 pounds. This obtained the sanction of parliament; and the work was begun in 1768, by Mr. Smeaton the engineer. The extreme length of the Canal from the Forth to the Clyde is 35 miles, beginning at the mouth of the Carron, and ending at Dalmure Burnfoot on the Clyde, 6 miles below Glasgow, rising and falling 160 feet by means of 39 locks, 20 on the east side of the summit, and 19 on the west, as the tide does not ebb so low in the Clyde as in the Forth by 9 feet; and it was deepened to upwards of 8 feet. This Canal was finished a few years since, after having experienced some interruptions and delays, for want of resources, and is esteemed the greatest work of the kind in this island. Vessels drawing 8 feet water, with 19 feet in the beam and 73 feet in length, pass with ease; and the whole enterprise displays the art of man in a high degree. To supply the Canal with water was of itself a very great work. There is one reservoir of 50 acres 24 feet deep, and another of 70 acres 22 feet deep, in which many rivers and springs terminate, which it is expected will afford sufficient supply of water at all times.

The Practice of Canal Digging and Inland Navigations.

The particular operations necessary for making artificial navigations, depend upon a number of circumstances. The situation of the ground; the vicinity or connection with rivers; the ease or difficulty with which a proper quantity of water can be obtained: these and many other circumstances necessarily produce great variety in the structure of artificial navigations, and augment or diminish the labour and expence of executing them. When the ground is naturally level, and unconnected with rivers, the execution is easy, and the navigation is not liable to be disturbed by floods: but when the ground rises and falls, and cannot be reduced to a level, artificial methods of raising and lowering vessels must be employed; which likewise vary according to circumstances.

Sometimes a kind of temporary sluices are employed, to raise boats over falls or shoals in rivers, by a very simple operation. Two pillars of mason-work, with grooves, are fixed, one on each bank of the river, at some

fome diftance below the fhoal. The boat having paffed thefe pillars, ftrong planks are let down acrofs the river by pulleys into the grooves; by which means the water is dammed up to a proper height for allowing the boat to pafs up the river over the fhoal.

The Dutch and Flemings at this day fometimes, when obftructed by cafcades, form an inclined plane or rolling-bridge upon dry land, along which their veffels are drawn from the river below the cafcade, into the river above it. This it is faid was the only method employed by the Ancients, and ftill fometimes ufed by the Chinefe. Thefe rolling-bridges confift of a number of cylindrical rollers which turn eafily on pivots. And a mill is commonly built near; fo that the fame machinery may ferve the double purpofe of working the mill and drawing up veffels.

But in the prefent improved ftate of inland navigation, thefe falls and fhoals are commonly furmounted by means of what are called locks or fluices. A lock is a bafon placed lengthwife in a river or Canal, lined with walls of mafonry on each fide, and terminated by two gates placed acrofs the Canal, where there is a cafcade or natural fall of the country; and fo conftructed, that the bafon being filled with water by an upper fluice to the level of the waters above, a veffel may afcend through the upper gate; or the water in the lock being reduced to the level of the water at the bottom of the cafcade, the veffel may defcend through the lower gate: for when the waters are brought to a level on either fide, the gate on that fide may be eafily opened.

But as the lower gate is ftrained in proportion to the depth of water it fupports, when the perpendicular height of the water exceeds 12 or 13 feet, it becomes neceffary to have more locks than one. Thus, if the fall be 16 feet, two locks are required, each of 8 feet fall; and if the fall be 25 feet, three locks are neceffary, each having 8 feet 4 inches fall.—It is evident that the fide-walls of locks fhould be made very ftrong: and where the natural foundation is bad, they fhould be founded on piles and platforms of wood. They fhould likewife flope outwards, in order to refift the preffure of the earth from behind.

To illuftrate this by reprefentations: Plate 37, fig. 1, is a perfpective view of part of a Canal, with feveral locks &c; the veffel L being within the lock AC.—Fig. 2 is an elevation or upright fection along the Canal; the veffel L about to enter.—Fig. 3, a like fection of a lock full of water; the veffel L being raifed to a level with the water in the fuperior Canal.—And fig. 4 is the plan or ground fection of a lock: where L is a veffel in the inferior Canal; C, the under gate; A, the upper gate; GH, a fubterraneous paffage for letting water from the fuperior Canal run into the lock; and KF, a fubterraneous paffage for water from the lock to the inferior Canal.

X and Y (fig. 1) are the two flood-gates, each of which confifts of two leaves, refting upon one another, fo as to form an obtufe angle, the better to refift the preffure of the water. The firft (X) prevents the water of the fuperior Canal from falling into the lock; and the fecond (Y) dams up and fuftains the water in the lock. Thefe flood-gates ought to be very ftrong, and to turn freely upon their hinges. They fhould alfo be made very tight and clofe, that as little water as poffible may be loft. And, to make them open and fhut with eafe, each leaf is furnifhed with a long lever A*b*, A*b*; C*b*. C*b*.

By the fubterraneous paffage GH (fig. 2, 3, 4) which defcends obliquely, by opening the fluice G, the water is let down from the fuperior Canal D into the lock, where it is ftopped and retained by the gate C when fhut, till the water in the lock comes to be on a level with the water in the fuperior Canal D; as reprefented in fig. 3. When, on the other hand, the water contained by the lock is to be let out, the paffage GH muft be fhut, by letting down the fluice G; the gate A muft alfo be fhut, and the paffage KF opened by raifing the fluice K. A free paffage being thus given to the water, it defcends through KF, into the inferior Canal, until the water in the lock be on a level with the water in the inferior Canal B; as reprefented in fig. 2.

Now fuppofe it be required to raife the veffel L (fig. 2) from the inferior Canal B to the fuperior one D. If the lock be full of water, the fluice G muft be fhut, as alfo the gate A, and the fluice K opened, fo that the water in the lock may run out till it become to a level with the water in the inferior Canal B. When the water in the lock comes to be on a level with the water at B, the leaves of the gate C are opened by the levers C*b*, which is eafily performed, the water on each fide of the gate being in equilibrio; the veffel then fails into the lock. After this, the gate C and the fluice K are fhut, and the fluice G opened, in order to fill the lock, till the water in the lock, and confequently the veffel, be upon a level with the water in the fuperior Canal D; as is reprefented in fig. 3. The gate A is then opened, and the veffel paffes into the Canal D.

Again let it be required to make a veffel defcend from the Canal D into the inferior Canal B. If the lock be empty, as in fig. 2, the gate C and fluice K muft be fhut, and the upper fluice G opened, fo that the water in the lock may rife to a level with the water in the upper Canal D. Then, opening the gate A, the veffel will pafs through into the lock. This done, fhut the gate A and the fluice G; then open the fluice K, till the water in the lock be on a level with the water in the inferior Canal; this done, the gate C is opened, and the veffel paffes along into the Canal B, as was required.

CATENARY. Line 4, *for* ACB *read* BAC.—l. 6, *for* A and B *read* C and B. *After which add*, It is otherwife called the *Elaftic Curve*.

CHALDRON. Line 4, *for* 2000 pounds, *read* 28 cwt. or 3136 pounds. *At the end add*, By act of parliament, a Newcaftle Chaldron is to weigh $52\frac{1}{2}$ cwt, or 3 waggons of $17\frac{1}{2}$ cwt, or 6 carts of $8\frac{3}{4}$ cwt each, making $52\frac{1}{2}$ cwt to the Chaldron. The ftatute London Chaldron is to confift of 36 bufhels heaped up, each bufhel to contain a Winchefter bufhel and one quart, and to be $19\frac{1}{2}$ inches diameter externally. Now it has been found by repeated trials, that 15 London Chaldrons are equal to 8 Newcaftle Chaldrons, which, reckoning $52\frac{1}{2}$ cwt to the latter, gives 28 cwt to the former, or 3136 lbs to the London Chaldron.

This I find nearly confirmed by experiment. I weighed

weighed one peck of coals, which amounted to $21\frac{3}{4}$ lb. Then 4 times this gives 87 lb for the weight of the bushel; and 36 times the bushel gives 3132 lb for the Chaldron; to which if the weight of the odd quart be added, or 3 lb nearly, it gives 3135 lb for the weight of the Chaldron, which is only one pound less than by statute.

Pa. 287, col. 2, l. 20, *for* $YX - a - x$, *read* $YX = a - x$.

CIRCLE *of Curvature.* To what is said of this article in the Dictionary, may be added what follows.

A circular arc is the only curve line that is equally curved in every point. In all other curve lines, such as the arc of an ellipse, or a parabola, or an hyperbola, or a cycloid, the curvature is different in different points, and the degree of curvature in any point is estimated by the curvature of a Circle which is said to have the same curvature as the proposed curve line in that point; by which is understood the Circle which, having the tangent of the proposed curve in the said point for its tangent, approaches so nearly to the proposed curve that no other Circle whatever can be drawn between it and that curve.

This Circle is also said to *osculate* the curve in the said point, and is therefore often called the *osculating Circle*, as well as the *Circle of equal curvature* with the curve in the said point. And the radius of this Circle is called the *radius of curvature* of the proposed curve in the said point; also its centre is called the *centre of curvature.*

Now there are some curve lines so very highly curved in some particular points, that every Circle, of how small a radius soever, having the tangent to the curve in one of those points for its tangent, will pass without the curve, or between the curve and its tangent. This, for example, is the case with the curve of a cycloid in the two points contiguous to its base, as also with the cissoid at its vertex. And in such points the curvature of these curves is said to be *infinite*, because it is greater than the curvature of any Circle, how small soever. Also the radius of the Circle of curvature in such points is nothing; the length of that radius being always inversely or reciprocally as the degree of curvature at any point.

The theory of these Circles of equal curvature with curves in particular points was first cultivated by Apollonius in his Conic Sections: and it has since been carried much farther by several great mathematicians of modern times; particularly by Mr. Huygens in his doctrine of Evolute Curves and Curves of Evolution, and by the great Sir Isaac Newton. See CURVATURE.

CLARKE (Dr. SAMUEL), a celebrated English divine, philosopher, and metaphysician, was the son of Edward Clarke, Esq. alderman of Norwich, and for several years one of its representatives in parliament; and was born there the 11th of October 1675. He was instructed in classical learning at the free-school of that town; and in 1691 removed thence to Caius college in Cambridge; where his uncommon abilities soon began to display themselves. Though the philosophy of Des Cartes was at that time the established philosophy of the university, yet Clarke easily mastered the new system of Newton; and in order to his first degree of arts, performed a public exercise in the schools upon a question taken from it. He greatly contributed to the establishment of the Newtonian philosophy by an excellent translation of Rohault's Physics, with notes, which he finished before he was 22 years of age: a book which had been for some time the system used in the university, and founded upon Cartesian principles. This was first published in the year 1697, and it soon after went through several other editions, all with improvements.

Mr. Whiston relates that, in that year, 1697, while he was chaplain to Dr. Moore bishop of Norwich, he met with young Clarke, then wholly unknown to him, at a coffee-house in that city; where they entered into a conversation about the Cartesian philosophy, particularly Rohault's Physics, which Clarke's tutor, as he tells us, had put him upon translating. "The result of this conversation was, says Whiston, that I was greatly surprised that so young man as Clarke then was, should know so much of those sublime discoveries, which were then almost a secret to all, but to a few particular mathematicians. Nor did I remember (continues he) above one or two at the most, whom I had then met with, that seemed to know so much of that philosophy as Mr. Clarke."

He afterwards turned his thoughts to divinity; and having taken holy orders, in 1698 he succeeded Mr. Whiston as chaplain to Dr. Moore bishop of Norwich, who was ever after his constant friend and patron. In 1699 he published two treatises: the one on Baptism, Confirmation, and Repentance; the other, Reflections on that part of a book called Amyntor, or a Defence of Milton's Life, which relates to the Writings of the Primitive Fathers, and the Canon of the New Testament. In 1701 he published A Paraphrase upon the Gospel of St. Matthew; which was followed in 1702 by the Paraphrases upon the Gospels of St. Mark and St. Luke, and soon after by a third volume upon St. John.

Mean while bishop Moore gave him the rectory of Drayton near Norwich, with a lectureship in that city. In 1704 he was appointed to preach Boyle's lecture; and the subject he chose was, The Being and Attributes of God. He succeeded so well in this, and gave so much satisfaction, that he was appointed to preach the same lecture the next year, when he chose for his subject, The Evidences of Natural and Revealed Religion. These sermons were first printed in two volumes, in 1705 and 1706; and contained some remarks on such objections as had been made by Hobbes and Spinoza, and other opposers of natural and revealed religion. In the 6th edition was added, A Discourse concerning the Connection of the Prophecies of the Old Testament, and the application of them to Christ.

About this time, Mr. Whiston informs us, he discovered that Mr. Clarke (having read much of the primitive writers) began to suspect that the Athanasian doctrine of the Trinity was not the doctrine of those early ages; and it was particularly remarked of him, that he never read the Athanasian Creed at his parish church.

In 1706 he published A Letter to Mr. Dodwell; answering all the arguments in his epistolary discourse against the immortality of the soul. Bishop Hoadley observes,

observes, that in this letter he answered Mr. Dodwell in so excellent a manner, both with regard to the philosophical part, and to the opinions of some of the primitive writers, upon whom these doctrines were fixed, that it gave universal satisfaction. But this controversy did not stop here; for the celebrated Mr. Collins, coming in as a second to Dodwell, went much farther into the philosophy of the dispute, and indeed seemed to produce all that could be said against the immateriality of the soul, as well as the liberty of human actions. This enlarged the scene of the dispute; into which our author entered, and wrote with such a spirit of clearness and demonstration, as at once shewed him greatly superior to his adversaries in metaphysical and physical knowledge; making every intelligent reader rejoice that such an incident had happened to provoke and extort from him such excellent reasoning and perspicuity of expression.

In the midst of these labours, Mr. Clarke found time to shew his regard to mathematical and philosophical studies, with his exact knowledge and skill in them. And his natural affection and capacity for these studies were not a little improved by the friendship of Sir Isaac Newton; at whose request he translated his Optics into Latin in 1706. With this version Sir Isaac was so highly pleased, that he presented him with the sum of 500l. or 100l. to each of his five children.

The same year also, bishop Moore procured for him the rectory of St. Bennett's, Paul's Wharf, in London; and soon after carried him to court, and recommended him to the favour of queen Anne. She appointed him one of her chaplains in ordinary; and also presented him to the rectory of St. James's, Westminster, when it became vacant in 1709. Upon this occasion he took the degree of D. D. when the public exercise which he performed for it at Cambridge was highly admired.

The same year 1709, Dr. Clarke revised and corrected Whiston's translation of the Apostolical Constitutions into English, at his earnest request. In 1712 he published a most beautiful and pompous edition of Cæsar's Commentaries. And the same year, his celebrated book called, The Scripture Doctrine of the Trinity. Whiston informs us, that some time before the publication of this book, there was a message sent to the author by lord Godolphin, and others of queen Anne's ministers, importing, "That the affairs of the public were with difficulty then kept in the hands of those that were for liberty; that it was therefore an unseasonable time for the publication of a book that would make a great noise and disturbance; and that therefore they desired him to forbear till a fitter opportunity should offer itself:" which message, says he, the doctor paid no regard to, but went on according to the dictates of his own conscience with the publication of his book. The ministers however were very right in their conjectures; for the work made noise and disturbance enough, and occasioned a great many books and pamphlets, written by himself and others. Nor were these the whole that his work occasioned: it rendered the author obnoxious to the ecclesiastical power, and his book was complained of by the lower house of convention. The doctor drew up a preface, and afterwards gave in several explanations, which seemed to satisfy the upper house; at least the affair was not brought to any issue, the members appearing desirous to prevent dissensions and divisions.

In 1715 and 1716 he had a dispute with the celebrated Leibnitz, concerning the principles of natural philosophy and religion; and a collection of the papers which passed between them, was published in 1717. This work was addressed to queen Caroline, then princess of Wales, who was pleased to have the controversy pass through her hands. It related chiefly to the subjects of liberty and necessity.

About the year 1718 he was presented by the lord Lechmere, to the mastership of Wigston's hospital in Leicestershire. In 1724 and 1725 he published 18 sermons, preached on several occasions. In 1727, on the death of Sir Isaac Newton, he had the offer of succeeding him as Master of the Mint, a place worth from 12 to 15 hundred a year: but to this secular preferment he could not reconcile himself; and therefore absolutely refused it.—In 1728 was published, a Letter from Dr. Clarke to Mr. Benjamin Hoadley, occasioned by the Controversy relating to the Proportion of Velocity and Force of Bodies in Motion; and printed in the Philosophical Transactions, num. 401.—In the beginning of 1729 he published the first 12 books of Homer's Iliad: a work which bishop Hoadley calls an accurate performance; and his notes, a treasury of grammatical and critical knowledge. And the same year came out, his Exposition of the Church Catechism, and 10 volumes of Sermons: books so well known and so generally approved, that they need no recommendation. But the same year, on Sunday the 11th of May, going to preach before the Judges at Serjeant's Inn, he was seized with a pain in his side, which made it impossible for him to perform his office. He was carried home and continued under his disorder till the 17th of the same month, when he died, in the 54th year of his age, after long enjoying a vigorous state of health, having scarce ever known sickness.

Three years after the doctor's death, appeared the other 12 books of the Iliad, published in 4to by his son, Mr. Samuel Clarke, who says in the preface, that his father had finished the annotations to the first three of those books, and as far as the 359th verse of the 4th; and had revised the text and version as far as verse 510 of the same book.

Dr. Clarke married Catherine, the only daughter of the Rev. Mr. Lockwood, rector of Little Missingham in the county of Norfolk, by whom he had seven children, four of whom survived him.

Queen Caroline took great pleasure in the doctor's conversation and friendship, seldom missing a week in which she did not receive some proof of the greatness of his genius, and the force of his understanding.

As to the character of Dr. Clarke, he is represented as possessing one of the best dispositions in the world, remarkably humane and tender, free and easy in his conversation, cheerful and even playful in his manner. Bishop Hare says of him, "He was a man who had all the good qualities that could meet together to recommend him. He was possessed of all the parts of learning that are valuable in a clergyman, in a degree that

few poſſeſs any ſingle one. He has joined to a good ſkill in the three learned languages, a great compaſs of the beſt philoſophy and mathematics, as appears by his Latin works; and his Engliſh ones are ſuch a proof of his own piety, and of his knowledge in divinity, and have done ſo much ſervice to religion, as would make any other man, that was not under a ſuſpicion of hereſy, ſecure of the friendſhip of all good churchmen, eſpecially the clergy. And to all this piety and learning was joined, a temper happy beyond expreſſion; a ſweet, eaſy, modeſt, obliging behaviour adorned all his actions; and neither paſſion, vanity, inſolence, or oſtentation appeared either in what he ſaid or wrote. This is the learning, this the temper of the man, whoſe ſtudy of the Scriptures has betrayed him into a ſuſpicion of ſome heretical opinions. Biſhop Hoadley too having remarked how great the doctor was in all branches of learning, adds, If in any one of theſe he had excelled only ſo much as he did in all, he would have been juſtly entitled to the character of a great man: but there is ſomething ſo very extraordinary, that the ſame perſon ſhould excel not only in thoſe parts of knowledge which require the ſtrongeſt judgment, but in thoſe which require the greateſt memory too. So that, in a very high degree, divinity and mathematics, experimental philoſophy and claſſical learning, metaphyſics and critical ſkill, were united in Dr. Clarke.—Much more may be ſeen, ſaid in his praiſe by biſhop Hoadley, Dr. Sykes, and Mr. Whiſton, in their Memoirs of his life.

CLEF, or Cliff, in Muſic, a mark at the beginning of the lines of a ſong, which ſhews the tone or key in which the piece is to begin. Or, it is a letter marked on any line, which explains and gives the name to all the reſt.

Anciently, every line had a letter marked for a Clef; but now a letter on one line ſuffices; ſince by this all the reſt are known; reckoning up and down, in the order of the letters.

It is called the Clef, or key, becauſe that by it are known the names of all the other lines and ſpaces; and conſequently the quantity of every degree, or interval. But becauſe every note in the octave is called a key, though in another ſenſe, this letter marked is called peculiarly the *ſigned Clef*; becauſe, being written on any line, it not only ſigns and marks that one, but it alſo explains all the reſt. By Clef, therefore, for diſtinction ſake, is meant that letter, ſigned on a line, which explains the reſt; and by key, the principal note of a ſong, in which the melody cloſes.

There are three of theſe ſigned Clefs, *c*, *f*, *g*. The Clef of the higheſt part in a ſong, called *treble*, or *alt*, is *g*, ſet on the ſecond line counting upwards. The Clef of the baſs, or the loweſt part, is *f* on the 4th line upwards. For all the other mean parts, the Clef is *c*, ſometimes on one, ſometimes on another line. Indeed, ſome that are really mean parts, are ſometimes ſet with the *g* clef. It muſt however be obſerved, that the ordinary ſignatures of Clefs bear little reſemblance to thoſe letters. Mr. Malcolm thinks it would be well if the letters themſelves were uſed. Kepler takes great pains to ſhew, that the common ſignatures are only corruptions of the letters they repreſent. The figures of theſe now are as follow:

 Character of the treble Clef.

 The mean Clef.

 The baſs Clef.

The Clefs are always taken fifths to one another. So the Clef *f* being loweſt, *c* is a fifth above it, and *g* a fifth above *c*.

When the place of the Clef is changed, which is not frequent in the mean Clef, it is with a deſign to make the ſyſtem comprehend as many notes of the ſong as poſſible, and ſo to have the fewer notes above or below it. So that, if there be many lines above the Clef, and few below it, this purpoſe is anſwered by placing the Clef in the firſt or ſecond line: but if there be many notes below the Clef, it is placed lower in the ſyſtem. In effect, according to the relation of the other notes to the Clef note, the particular ſyſtem is taken differently in the ſcale, the Clef line making one in all the variety.

But ſtill, in whatever line of the particular ſyſtem any Clef is found, it muſt be underſtood to belong to the ſame of the general ſyſtem, and to be the ſame individual note or ſound in the ſcale. By this conſtant relation of Clefs, we learn how to compare the ſeveral particular ſyſtems of the ſeveral parts, and to know how they communicate in the ſcale, that is, which lines are uniſon, and which not: for it is not to be ſuppoſed, that each part has certain bounds, within which another muſt never come. Some notes of the treble, for example, may be lower than ſome of the mean parts, or even of the baſs. Therefore to put together into one ſyſtem all the parts of a compoſition written ſeparately, the notes of each part muſt be placed at the ſame diſtances above and below the proper Clef, as they ſtand in the ſeparate ſyſtem: and becauſe all the notes that are conſonant, or heard together, muſt ſtand directly over each other, that the notes belonging to each part may be diſtinctly known, they may be made with ſuch differences as ſhall not confound, or alter their ſignifications with reſpect to time, but only ſhew that they belong to this or that part. Thus we ſhall ſee how the parts change and paſs through one another; and which, in every note, is higheſt, loweſt, or uniſon.

It muſt here be obſerved, that for the performance of any ſingle piece, the Clef only ſerves for explaining the intervals in the lines and ſpaces: ſo that it need not be regarded what part of any greater ſyſtem it is; but the firſt note may be taken as high or low as we pleaſe. For as the proper uſe of the ſcale is not to limit the abſolute degree of tone; ſo the proper uſe of the *ſigned* Clef is not to limit the pitch, at which the firſt note of any part is to be taken; but to determine the tune of the reſt, with reſpect to the firſt; and conſidering all the parts together, to determine the relation of their ſeveral notes by the relations of their Clefs in the ſcale: thus, their pitch of tune being determined in a certain note of one part, the other notes of that part are determined by the conſtant relations of the letters

letters of the scale, and the notes of the other parts by the relations of their Clefs.

In effect, for performing any single part, the Clef note may be taken in an octave, that is, at any note of the same name; provided we do not go too high, or too low, for finding the rest of the notes of a song. But in a concert of several parts, all the Clefs must be taken, not only in the relations, but also in the places of the system abovementioned; that every part may be comprehended in it.

The natural and artificial note expressed by the same letter, as *c* and *c*※, are both set on the same line or space. When there is no character of flat or sharp, at the beginning with the Clef, all the notes are natural: and if in any particular place the artificial note be required, it is denoted by the sign of a flat or sharp, set on the line a space before that note.

If a sharp or flat be set at the beginning in any line or space with the Clef, all the notes on that line or space are artificial ones; that is, are to be taken a semitone higher or lower than they would be without such sign. And the same affects all their octaves above and below, though they be not marked so. In the course of the song, if the natural note be sometimes required, it is signified by the character ♮.

COMPASS. Pa. 314, col. 1, after l. 6 from the bottom, add, See also a new one in the Supplement to Cavallo's Treatise on Magnetism.

CONDORCET (John-Anthony-Nicholas *de* Caritat, *Marquis of)*, member of the Institute of Bologna, of the Academies of Turin, Berlin, Stockholm, Upsal, Philadelphia, Petersbourg, Padua, &c, and secretary of the Paris Academy of Sciences, was born at Ribemont in Picardie, the 17th of September 1743. His early attachment to the sciences, and progress in them, soon rendered him a conspicuous character in the commonwealth of letters. He was received as a member of the Academy of Sciences at 25 years of age, namely, in March 1769, as Adjunct-Mecanician; afterwards, he became Associate in 1710, Adjunct-Secretary in 1773, and sole Secretary soon after, which he enjoyed till his death, or till the dissolution of the Academy by the Convention.

Condorcet soon became an author, and that in the most sublime branches of science. He published his *Essais d'Analyse* in several parts; the first part in 1765 (at 22 years of age); the second, in 1767; and the third, in 1768. These works are chiefly on the Integral Calculus, or the finding of Fluents, and make one volume in 4to.

He published the Eloges of the Academicians or members of the Academy of Sciences, from the year 1666 till 1700, in several volumes. He wrote also similar Eloges of the Academicians who died during the time that he discharged the important office of Secretary to the Academy; as well as the very useful histories of the different branches of science commonly prefixed to the volumes of Memoirs, till the volume for the year 1783, when it is to be lamented that so useful a part of the plan of the Academy was discontinued.

His other memoirs contained in the volumes of the Academy, are the following.

1. Tract on the Integral Calculus; 1765.
2. On the problem of Three Bodies; 1767.
3. Observations on the Integral Calculus; 1767.
4. On the Nature of Infinite Series; on the Extent of the Solutions which they give; and on a new method of Approximation for Differential Equations of all Orders; 1769.
5. On Equations for Finite Differences; 1770.
6. On Equations for Partial Differences; 1770.
7. On Differential Equations; 1770.
8. Additions to the foregoing Tracts; 1770.
9. On the Determination of Arbitrary Functions which enter the Integrals of Equations to Partial Differences; 1771.
10. Reflexions on the Methods of Approximation hitherto known for Differential Equations; 1771.
11. Theorem concerning Quadratures; 1771.
12. Inquiry concerning the Integral Calculus; 1772.
13. On the Calculation of Probabilities, part 1 and 2; 1781.
14. Continuation of the same, part 3; 1782.
15. Ditto, part 4; 1783.
16. Ditto, part 5; 1784.

Condorcet had the character of being a very worthy honest man, and a respectable author, though perhaps not a first-rate one, and produced an excellent set of Eloges of the deceased Academicians, during the time of his secretaryship. A late French political writer has observed of him, that he laboured to succeed to the literary throne of d'Alembert, but that he cannot be ranked among illustrious authors; that his works have neither animation nor depth, and that his style is dull and dry; that some bold attacks on religion and declamations against despotism have chiefly given a degree of fame to his writings.

On the breaking out of the troubles in France, Condorcet took a decided part on the side of the people, and steadily maintained the cause he had espoused amid all the shocks and intrigues of contending parties; till, under the tyranny of Robespierre, he was driven from the convention, being one of those members proscribed on the 31st of May 1793, and he died about April 1794. The manner of his death is thus described by the public prints of that time. He was obliged to conceal himself with the greatest care for the purpose of avoiding the fate of Brissot and the other deputies who where executed. He did not, however, attempt to quit Paris, but concealed himself in the house of a female, who, though she knew him only by name, did not hesitate to risk her own life for the purpose of preserving that of Condorcet. In her house he remained till the month of April 1794, when it was rumoured that a domiciliary visit was to be made, which obliged him to leave Paris. Although he had neither passport nor civic card, he escaped through the Barrier, and arrived at the Plain of Mont rouge, where he expected to find an asylum in the country-house of an intimate friend. Unfortunately this friend had set out for Paris, where he was to remain for three days.—During all this period, Condorcet wandered about the fields and in the woods, not

not daring to enter an inn on account of not having a civic card. Half dead with hunger, fatigue, and fear, and fcarcely able to walk on account of a wound in his foot, he paffed the night under a tree.

At length his friend returned, and received him with great cordiality; but as it was deemed imprudent that he fhould enter the houfe in the day-time, he returned to the woods till night. In this fhort interval between morning and night his caution forfook him, and he refolved to go to an inn for the purpofe of procuring food. He went to an inn at Clamars, and ordered an omlette. His torn clothes, his dirty cap, his meagre and pale countenance, and the greedinefs with which he devoured the omlette, fixed the attention of the perfons in the inn, among whom was a member of the Revolutionary Committee of Clamars. This man conceiving him to be Condorcet, who had effected his efcape from the Bicetre, afked him whence he came, whither he was going, and whether he had a paffport? The confufed manner in which he replied to thefe queftions, induced the member to order him to be conveyed before the Committee, who, after an examination, fent him to the diftrict of Boury la Reine. He was there interrogated again, and the unfatisfactory anfwers which he gave, determined the directors of the diftrict to fend him to prifon on the fucceeding day.—During the night he was confined in a kind of dungeon. On the next morning, when his keeper entered with fome bread and water for him, he found him ftretched on the ground without any figns of life.

On infpecting the body, the immediate caufe of his death could not be difcovered, but it was conjectured that he had poifoned himfelf. Condorcet indeed always carried a dofe of poifon in his pocket, and he faid to the friend who was to have received him into his houfe, that he had been often tempted to make ufe of it, but that the idea of a wife and daughter, whom he loved tenderly, reftrained him. During the time that he was concealed at Paris, he wrote a hiftory of the Progrefs of the Human Mind, in two volumes.

CUBICS. The method of refolving all the cafes of Cubic equations by the tables of fines, tangents and fecants, are thus given by Dr. Mafkelyne, p. 57, Taylor's Logarithms.

"The following method is adapted to a Cubic equation, wanting the fecond term; therefore, if the equation has the fecond term, it muft be firft taken away in the ufual manner. There are four forms of Cubic equations wanting the fecond term, whofe roots, according to known rules equivalent to Cardan's, are as follow:

1ft. $x^3 + px - q = 0$ - - - - - - $x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$.

2d. $x^3 + px + p = 0$ - - - - - - $x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$

3d. $x^3 - px - q = 0$ - - - - - - $x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}}$

4th. $x^3 - px + q = 0$ - - - - - - $x = -\sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}} - \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} - \frac{p^3}{27}}}$

The roots of the firft and fecond forms are negatives of each other; and thofe of the third and fourth are alfo negatives of each other. The firft and fecond forms have only one root each. The third and fourth forms have alfo only one root each, when the quadratic furd $\sqrt{\frac{q^2}{4} - \frac{p^3}{27}}$ is poffible; but have three roots each when that furd is impoffible.

The roots of all the four forms may, in all cafes, be eafily computed as follows:

Forms 1ft and 2d. Put $\left(\frac{q}{2} \times \frac{3}{p}\right)^{\frac{3}{2}}$ = tang. z; and $\sqrt[3]{\text{tan. } 45^\circ - \frac{1}{2}z}$ = tan. u. Then $x = \pm \sqrt{\frac{4p}{3}} \times$ cot. $2u$; where the upper fign belongs to the firft form, and the lower fign to the fecond form.

Forms 3d and 4th. Put $\left(\frac{2}{q} \times \frac{p}{3}\right)^{\frac{3}{2}}$ if lefs than unity, elfe its reciprocal $\left(\frac{q}{2} \times \frac{3}{4}\right)^{\frac{3}{2}}$ = cof. z. Then,

Cafe 1ft. $\left(\frac{2}{q} \times \frac{p}{3}\right)^{\frac{3}{2}}$ < unity. Put $\sqrt[3]{\text{tan. } 45^\circ - \frac{1}{2}z}$ = tan. u. Then $x = \pm \sqrt{\frac{4p}{3}} \times$ cofec. $2u$; where the upper fign belongs to the third form, and the lower fign to the fourth form.

Cafe 2d. $\left(\frac{2}{q} \times \frac{p}{3}\right)^{\frac{3}{2}}$ > unity. Then x has three values in each form, viz, $x = \pm \sqrt{\frac{4p}{3}} \times$ cof. $\frac{z}{3} = \mp \sqrt{\frac{4p}{3}} \times$ cof. $\overline{60^\circ - \frac{z}{3}} = \mp \sqrt{\frac{4p}{3}} \times$ cof. $\overline{60^\circ + \frac{z}{3}}$; where the upper figns belong to the third form, and the lower figns to the fourth form.

By

By Logarithms.

Forms 1st and 2d. $\text{Log.}\frac{q}{2} + 10 - \frac{3}{2} \times \text{log.}\frac{p}{3} =$ log. tan. z, and $\frac{\text{log. tan.}\ \overline{45^\circ - \frac{1}{2}z} + 20}{3} = \text{log. tan.}\ u.$

Then $\text{log.}\ x = \frac{1}{2}\ \text{log.}\ \frac{4p}{3} + \text{log. cot.}\ 2u - 10$; and x will be affirmative in the first form, and negative in the second form.

Forms 3d and 4th. $\frac{3}{2} \times \text{log.}\frac{p}{3} + 10 - \text{log.}\ \frac{q}{2}$ being less than 10 (which is case first) or $\text{log.}\ \frac{q}{2} + 10 - \frac{3}{2} \times \text{log.}\frac{p}{3}$ being less than 10 (which is case 2d) $=$ log. cos. z.

Case 1st. $\frac{\text{Log. tan.}\ \overline{45^\circ - \frac{1}{2}z} + 20}{3} = \text{log. tan.}\ u.$

Then $\text{log.}\ x = \frac{1}{2}\ \text{log.}\ \frac{4p}{3} + 10 - \text{log. sin.}\ 2u$; and x will be affirmative in the third form, and negative in the 4th form.

Case 2d. Here x has three values.

1st. $\text{Log.} \pm x = \frac{1}{2}\text{log.}\ \frac{4p}{3} + \text{log. cos.}\ \frac{z}{3} - 10.$

2d. $\text{Log.} \mp x = \frac{1}{2}\ \text{log.}\ \frac{4p}{3} + \text{log. cos.}\ \overline{60^\circ - \frac{z}{3}},$

3d. $\text{Log.} \mp x = \frac{1}{2}\ \text{log.}\ \frac{4p}{3} + \text{log. cos.}\ 60^\circ + \frac{z}{3} - 10;$

where the upper signs belong to the third form, and the lower signs to the fourth form; that is, the first value of x in the third form is positive, and its second and third values negative; and the first value of x in the fourth form is negative, and its second and third values affirmative."

See also IRREDUCIBLE *Case*.

CURVE. Pa. 350, col. 2, l. 35, for $dx + x^2$ r. $dx + x^2$.

D.

DIPPING *Needle*. Pa. 383, col. 2, after line 38 *add*, See a new Dipping-needle by Dr. Lorimer, in the Philos. Trans. 1775, also in the Supplement to Cavallo's Treatise on Magnetism.

DOME. In plate 33 is represented the plan and elevation of a Dome constructed without centring, by Mr. S. Bunce; viz, Fig. 1 the plan, and Fig. 2 the elevation. The first course consists of the stones marked 1, 1, 1, &c, of different sizes, the large ones exactly twice the height of the small ones, placed alternately, and forming intervals to receive the stones marked 2, 2, 2. The other courses are continued in the same manner, according to the order of the figures to the top.

It is evident, from the converging or wedgelike form of the intervals, that the stones they receive can only be inserted from the outside, and cannot fall through: therefore the whole Dome may be built without centring or temporary support. To break the upright joints, the stones may be cut of the form marked in Fig. 3; and those marked 16, 17, &c, near the keystones, may be enlarged as at Fig. 4.

Pa. 399, col. 2, line 10 from the bottom, *for* DYMANICS *read* DYNAMICS.

E.

EUTOCIUS, a respectable Greek mathematician, lived at Ascalon in Palestine about the year of Christ 550. He was one of the most considerable mathematicians that flourished about the decline of the sciences among the Greeks, and had for his preceptor Isidorus the principal architect of the church of St. Sophia at Constantinople. He is chiefly known however by his commentaries on the works of the two ancient authors, Archimedes and Apollonius. Those two commentaries are both excellent compositions, to which we owe many useful circumstances in the history of the mathematics.

His commentaries on Apollonius are published in Halley's edition of the works of that author; and those on Archimedes, first in the Basle edition, in Greek and Latin, in 1543, and since in some others, as the late Oxford edition. Of these commentaries, those rank the highest, which illustrate Archimedes's work on the Sphere and Cylinder; in one of which we have a recital of the various methods practised by the ancients in the solution of the Delian problem, or that of doubling the cube. The others are of less value; though it cannot but be regretted that Eutocius did not pursue his plan of commenting on all the works of Archimedes, with the same attention and diligence which he employed in his remarks on the sphere and cylinder.

Pa. 507, line 5 from the bottom, *for* $3 + {}^{1}$ *read* $3 + \frac{1}{7}$.

Pa. 551, line 22 from the bottom, *for* $\frac{7}{50}$ *read* $\frac{7}{30}$.

G.

GROIN, with Builders, is the angle made by the intersection of two arches. It is of two kinds, regular and irregular; viz, Regular when both the arches have the same diameter, but an Irregular Groin when one arch is a semicircle and the other a semiellipsis. Groins are chiefly used in forming arched roofs, where one hollow arched vault intersects with another; as in the roofs of most churches, and some cellars in large houses.

I.

IMPOSSIBLE *Binomial*. See BINOMIAL.

IRREDUCIBLE *Case*, in Algebra. Mr. Bonnycastle has communicated the following additional observations on this case, and, an improved solution by a table of sines. The

IRREDUCIBLE *Case, in Algebra*, is a cubic equation of the form $x^3 - ax = \pm b$, having $\frac{1}{27}a^3$ greater than $\frac{1}{4}b^2$, or $4a^3$ greater than $27b^2$; in which case, it is well known, that the solution cannot be generally obtained, either by Cardan's rule, or any other which has yet been devised.

One of the most convenient methods of determining the roots of equations of this kind, is by means of a Table of Natural Sines, &c, for which purpose the following formulæ will be found extremely commodious, the arc, in each case, being always less than a quadrant, and therefore attended with no ambiguity.

If

If the equation be $x^3 - ax = b$; let A be put = arc whoſe coſ. is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$ to rad. 1, then the three roots, or values of x, will be as follows:

$$x = 2\sqrt{\frac{a}{3}} \times \text{coſine}\ \frac{A}{3}$$

$$x = -2\sqrt{\frac{a}{3}} \times \text{ſine}\ \frac{90^\circ + A}{3}$$

$$x = -2\sqrt{\frac{a}{3}} + \text{ſine}\ \frac{90^\circ - A}{3}$$

And, if the equation be $x^3 - ax = -b$; let A be put = arc whoſe ſine is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$ to rad. 1; then the three roots, or values of x, will be as follows.

$$x = 2\sqrt{\frac{a}{3}} \times \text{ſine}\ \frac{A}{3}$$

$$x = 2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{90^\circ + A}{3}$$

$$x = -2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{90^\circ + A}{3}$$

Ex. 1. Let $x^3 - 3x = 1$, to find the 3 roots of the equation.

Here $\frac{3b}{2a}\sqrt{\frac{3}{a}} = \frac{3}{6}\sqrt{\frac{3}{3}} = \frac{1}{2} = {\cdot}5 = \text{coſ. } 60^\circ = A.$

$$\text{Hence}\begin{cases} x = 2\,\text{coſ.}\frac{60^\circ}{3} = 2\,\text{coſ.}\,20^\circ = 1{\cdot}8793852 \\ x = -2\,\text{ſine}\frac{150^\circ}{3} = -2\,\text{ſine}\,50^\circ = -1{\cdot}5320888 \\ x = -2\,\text{ſine}\,\frac{30^\circ}{3} = -2\,\text{ſine}\,10^\circ = -{\cdot}3472964 \end{cases}$$

Ex. 2d. Let $x^3 - 3x = -1$, to find the 3 roots of the equation.

Here $\frac{3b}{2a}\sqrt{\frac{3}{a}} = \frac{3}{6}\sqrt{\frac{3}{3}} = \frac{1}{2} = {\cdot}5 = \text{ſine } 30^\circ = A,$

$$\text{Hence}\begin{cases} x = 2\,\text{ſine}\,\frac{30^\circ}{3} = 2\,\text{ſine}\,10^\circ = {\cdot}3472964 \\ x = 2\,\text{coſ.}\frac{120^\circ}{3} = 2\,\text{coſ.}\,40^\circ = 1{\cdot}5320888 \\ x = -2\,\text{coſ.}\,\frac{60^\circ}{3} = -2\,\text{coſ.}\,20^\circ = -1{\cdot}8793852 \end{cases}$$

The inveſtigation of this method is as follows:

It is ſhewn, by the writers on Trigonometry, that if c be the coſine of any arc to rad. 1, $4c^3 - 3c$ will be the coſine of 3 times that arc; and conſequently c is the coſine of $\frac{1}{3}$ of the arc whoſe coſine is $4c^3 - 3c$, or any other equal quantity.

In order, therefore, to reduce the equation $x^3 - ax = b$ to this form, let $x = \frac{y}{z}$; then $\frac{y^3}{z^3} - a \times \frac{y}{z} = b$, or $y^3 - az^2y = bz^3$, or $4y^3 - 4az^2y = 4bz^3$; whence if $4az^2$ be put = 3, we ſhall have $z = \sqrt{\frac{3}{4a}} = \frac{1}{2}\sqrt{\frac{3}{a}}$, and conſequently $4y^3 - 3y = \frac{3b}{2a}\sqrt{\frac{3}{a}}$.

From which laſt equation, it appears that y = coſ. $\frac{1}{3}$ arc whoſe coſ. is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$; and therefore $x = \frac{y}{z} = \frac{y}{\frac{1}{2}\sqrt{\frac{3}{a}}}$

$= 2\sqrt{\frac{a}{3}} \times y = 2\sqrt{\frac{a}{3}} \times$ (coſ. $\frac{1}{3}$ arc whoſe coſ. is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$).

or, if A be put = arc whoſe coſ. is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$, x is $= 2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{A}{3}$.

But the arc of which $\frac{3b}{2a}\sqrt{\frac{3}{a}}$ is the coſine, is either A, A + 360° or A + 720°; whence $x = 2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{A}{3}$ or $2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{A + 360^\circ}{3}$, or $2\sqrt{\frac{a}{3}} \times \text{coſ.}\ \frac{A + 720^\circ}{3}$; the two latter of which being converted into ſines, will give the ſame formulæ as in the rule.

In like manner, if s be the ſine of any arc to rad. 1, $3s - 4s^3$ is well known to be the ſine of 3 times that arc; and conſequently s is the ſine of $\frac{1}{3}$ of the arc whoſe ſine is $3s - 4s^3$. Whence, to reduce the equation $x^3 - ax = -b$, to this form, let $x = \frac{y}{z}$, as before;

then $\frac{y^3}{z^3} - a \times \frac{y}{z} = -b$, or $y^3 - az^2y = -bz^3$, or $az^2y - y^3 = bz^3$, or $4az^2y - 4y^3 = 4bz^3$; where, if $4az^2$ be put = 3, we ſhall have $z = \frac{1}{2}\sqrt{\frac{3}{a}}$, and conſequently $3y - 4y^3 = \frac{3b}{2a}\sqrt{\frac{3}{a}}$.

From which laſt equation it appears that y = ſine $\frac{1}{3}$ *arc* whoſe ſine is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$, and therefore $x = \frac{y}{z} = 2\sqrt{\frac{a}{3}} \times$ (ſine $\frac{1}{3}$ *arc* whoſe ſine is $\frac{3b}{2a}\sqrt{\frac{3}{a}}$), which is the ſame as the rule, the other two roots being found as in the former caſe.

L.

LOCK, for Canals, in Inland Navigations. See Canal.

LOGARITHMS. Mr. Bonnycaſtle has communicated the following new method of making theſe uſeful numbers:

Logarithms. The ſeries now chiefly uſed in the computation of Logarithms were originally derived from the hyperbola, by means of which, and the logiſtic curve, the nature and properties of theſe numbers are clearly and elegantly explained.

The doctrine, however, being purely arithmetical, this mode of demonſtrating it, by the intervention of certain curves, was conſidered, by Dr. Halley, as not conformable to the nature of the ſubject.

He

He has, accordingly, inveſtigated the ſame ſeries from the abſtract principles of numbers; but his method, which is a kind of diſguiſed fluxions, is, in many places, ſo extremely abſtruſe and obſcure, that few have been able to comprehend his reaſoning.

An eaſy and perſpicuous demonſtration, of this kind, was therefore ſtill wanting; which may be obtained from the pure principles of Algebra, independently of the doctrine of Curves, as follows:

The Logarithm of any number, is the index of that power of ſome other number, which is equal to the given number.

Thus, if $r^x = a$, the logarithm of a is x, which may be either poſitive or negative, and r any number whatever, according to the different ſyſtems of Logarithms.

When $a = 1$, it is plain that x muſt be $= 0$, whatever be the value of r; and conſequently the Logarithm of 1 is always 0 in every ſyſtem.

If $x = 1$, it is alſo plain that a muſt be $= r$; and therefore r is always the number in every ſyſtem, whoſe Logarithm in that ſyſtem is 1.

To find the Logarithm of any number, in any ſyſtem, it is only neceſſary, from the equation $r^x = a$, to find the value of x in terms of r and a.

This may be ſtrictly effected, by means of a new property of the binomial theorem of Newton; which is given under its proper article in this Appendix. The general Logarithmic equation being $r^x = a$, let

$$a = 1 + p, \text{ and } \frac{1}{x} = z; \text{ then } r = a^{\frac{1}{x}} = \overline{1+p}^{\frac{1}{x}} = \overline{1+p}^{z} =$$

$$1 + (p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} \&c)z + \tfrac{1}{2}(p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} \&c)^2 z^2$$

$$+ \frac{1}{2 \cdot 3}(p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} \&c)^3 z^3 + \frac{1}{2\cdot3\cdot4}(p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} \&c)^4 z^4$$

$$+ \frac{1}{2\cdot3\cdot4\cdot5}(p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} \&c)^5 z^5, \&c.$$ See *Binomial* THEOREM, Appendix.

And if $p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} + \frac{p^5}{5}$ &c be put $= s$, we ſhall have

$$1 + sz + \tfrac{1}{2}s^2z^2 + \frac{1}{2\cdot3}s^3z^3 + \frac{1}{2\cdot3\cdot4}s^4z^4 + \frac{1}{2\cdot3\cdot4\cdot5}s^5z^5 \&c = r,$$

or $sz + \tfrac{1}{2}s^2z^2 + \frac{1}{2\cdot3}s^3z^3 + \frac{1}{2\cdot3\cdot4}s^4z^4 + \frac{1}{2\cdot3\cdot4\cdot5}s^5z^5 \&c = r - 1$,

which let be put $= q$; then, by reverting the ſeries z or $\frac{1}{x}$ will be found

$$= \frac{q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5 \&c}{s} = \frac{q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5 \&c}{p - \frac{1}{2}p^2 + \frac{1}{3}p^3 - \frac{1}{4}p^4 + \frac{1}{5}p^5 \&c}$$

and conſequently $x = \dfrac{p - \frac{1}{2}p^2 + \frac{1}{3}p^3 - \frac{1}{4}p^4 + \frac{1}{5}p^5 \&c}{q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5 \&c}$.

The Logarithm of a, or $1 + p$, is therefore $= \dfrac{p - \frac{1}{2}p^2 + \frac{1}{3}p^3 - \frac{1}{4}p^4 + \frac{1}{5}p^5 \&c}{q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5 \&c}$; or, ſince $p = a - 1$, and $q = r - 1$, the Logarithm of a is

$$= \frac{(a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \frac{1}{4}(a-1)^4 + \frac{1}{5}(a-1)^5}{(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \frac{1}{4}(r-1)^4 + \frac{1}{5}(r-1)^5} \&c;$$

Which is a general expreſſion for the Logarithm of any number, in any ſyſtem of Logarithms, the radix r being taken of any value, greater or leſs than 1.

But as r in every ſyſtem, is a conſtant quantity, being always the number whoſe Logarithm in the ſyſtem to which it belongs is 1, the above expreſſion may be ſimplified, either by aſſuming $r =$ to ſome particular number, and from thence finding the value of the ſeries conſtituting the denominator; or by aſſuming this whole ſeries $=$ to ſome particular number, and from thence finding the value which muſt be given to the radix r.

By the latter of theſe methods, the denominator may be made to vaniſh, by aſſuming the value of the ſeries of which it conſiſts $= 1$, in which caſe, the Logarithm of $1 + p$ becomes $= p - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} + \frac{p^5}{5}$ &c, or the Logarithm of

$a = (a-1) - \frac{1}{2}(a-1)^2 + \frac{1}{3}(a-1)^3 - \frac{1}{4}(a-1)^4 + \frac{1}{5}(a-1)^5$ &c, and r, by reverſion of ſeries is found $= 2.7182818$ &c.

The ſyſtem ariſing from this mode of determining the value of the radix r, is that which furniſhes what have been uſually called hyperbolic Logarithms; and appears to be the ſimpleſt form the general expreſſion admits of.

If, on the contrary, the radix r be aſſumed $=$ to ſome particular number, as for inſtance 10, the value of the ſeries $q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5$ &c, or its equal $(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \frac{1}{4}(r-1)^4 + \frac{1}{5}(r-1)^5$ &c will become $= 2.30258509$ &c, and the

$$\text{Log. of } 1+p = \frac{1}{2.30258509} \times (p - \tfrac{1}{2}p^2 + \tfrac{1}{3}p^3 - \tfrac{1}{4}p^4 + \tfrac{1}{5}p^5 \&c)$$

or the Log. of a

$$= \frac{1}{2.30258509} \times (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \tfrac{1}{4}(a-1)^4 + \tfrac{1}{5}(a-1)^5$$

&c, which gives the ſyſtem that furniſhes Briggs's or the common Logarithms.

And, in like manner, by aſſuming any particular value for r, and thence determining the value of the ſeries $q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5$ &c, or its equal $(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \frac{1}{4}(r-1)^4 + \frac{1}{5}(r-1)^5$ &c; or by aſſuming the ſame ſeries of ſome particular value, and thence determining the value of r, any ſyſtem of Logarithms may be derived.

The ſeries $q - \frac{1}{2}q^2 + \frac{1}{3}q^3 - \frac{1}{4}q^4 + \frac{1}{5}q^5$ &c, or its equal $(r-1) - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \frac{1}{4}(r-1)^4 + \frac{1}{5}(r-1)^5$ &c, which forms the denominator of the above compound expreſſion, exhibiting the Logarithms of numbers according to any ſyſtem, is what was firſt called, by Cotes, the Modulus of the ſyſtem, being always a conſtant quantity, depending only on the aſſumed value of r.

And, as the form of this ſeries is exactly the ſame as that which conſtitutes the numerator, and which has been ſhewn to be the hyperbolic Logarithm of a, it follows that the Modulus of any ſyſtem of Logarithms is equal to the hyperbolic Logarithm of the radix of that ſyſtem,

ſyſtem, or of the number whoſe proper Logarithm in the ſyſtem to which it belongs is 1.

The form of the ſeries here obtained for the hyperbolic Logarithm of a, is the ſame as that which was firſt diſcovered by Mercator; and if the ſeries of Wallis be required, it may be inveſtigated in a ſimilar manner as follows:

The general Logarithmic equation being $r^x = a$, as before, let $a = \frac{1}{1-p}$ and $z = \frac{1}{x}$; then $r = a^{\frac{1}{x}} =$

$$\overline{\frac{1}{1-p}}\Big|^{z} = \frac{1}{\overline{1-p}|^{z}}, \text{ and } \frac{1}{r} = \overline{1-p}|^{z} = 1-(p+\frac{p^2}{2}+\frac{p^3}{3}+\frac{p^4}{4}\&c)$$

$$+\tfrac{1}{2}(p+\frac{p^2}{2}+\frac{p^3}{3}+\frac{p^4}{4}\&c)^2z^2 - \frac{1}{2.3}(p+\frac{p^2}{2}+\frac{p^3}{3}+\frac{p^4}{4}\&c)^3z^3$$

$$+\frac{1}{2.3.4}(p+\frac{p^2}{2}+\frac{p^3}{3}+\frac{p^4}{4}\&c)^4z^4 - \frac{1}{2.3.4.5}(p+\frac{p^2}{2}+\frac{p^3}{3}+\frac{p^4}{4}\&c)^5z^5$$

&c.

And if $p + \frac{p^2}{2} + \frac{p^3}{3} + \frac{p^4}{4} + \frac{p^5}{5}$ &c be put $= s$, we ſhall have $1 - sz + \frac{1}{2}s^2z^2 - \frac{1}{2.3}s^3z^3 + \frac{1}{2.3.4}s^4z^4 \&c = \frac{1}{r}$,

or $sz - \frac{1}{2}s^2z^2 + \frac{1}{2.3}s^3z^3 - \frac{1}{2.3.4}s^4z^4 \&c = 1 - \frac{1}{r}$, which let be put $= q$; then, by converſion of ſeries, z or $\frac{1}{x}$ will be found

$$= \frac{q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}{s} = \frac{q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5}{p+\frac{1}{2}p^2+\frac{1}{3}p^3+\frac{1}{4}p^4+\frac{1}{5}p^5}$$

and conſequently $x = \frac{p+\frac{1}{2}p^2+\frac{1}{3}p^3+\frac{1}{4}p^4+\frac{1}{5}p^5 \&c}{q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}$.

The Logarithm of a or $\frac{1}{1-p}$ is, therefore,

$= \frac{p+\frac{1}{2}p^2+\frac{1}{3}p^3+\frac{1}{4}p^4+\frac{1}{5}p^5 \&c}{q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}$; or ſince

$$p = 1 - \frac{1}{a} = \frac{a-1}{a} \text{ and } q = 1 - \frac{1}{r} = \frac{r-1}{r},$$

the Logarithm of a is $=$

$$\frac{\frac{a-1}{a}+\frac{1}{2}\left(\frac{a-1}{a}\right)^2+\frac{1}{3}\left(\frac{a-1}{a}\right)^3+\frac{1}{4}\left(\frac{a-1}{a}\right)^4+\frac{1}{5}\left(\frac{a-1}{a}\right)^5 \&c}{\frac{r-1}{r}+\frac{1}{2}\left(\frac{r-1}{r}\right)^2+\frac{1}{3}\left(\frac{r-1}{r}\right)^3+\frac{1}{4}\left(\frac{r-1}{r}\right)^4+\frac{1}{5}\left(\frac{r-1}{r}\right)^5 \&c}$$

Which is another general expreſſion for the Logarithm of any number a, in any ſyſtem of Logarithms, that may be ſimplified in the ſame manner as the former, the denominator being ſtill equal to the hyperbolic Logarithm of the radix r; or, which is the ſame thing, to the Modulus of the ſyſtem.

For if the ſeries $q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5$ &c, or its equal

$$\frac{r-1}{r}+\frac{1}{2}\left(\frac{r-1}{r}\right)^2+\frac{1}{3}\left(\frac{r-1}{r}\right)^3+\frac{1}{4}\left(\frac{r-1}{r}\right)^4+\frac{1}{5}\left(\frac{r-1}{r}\right)^5 \&c,$$

be aſſumed $= 1$, the hyperbolic Logarithm of $\frac{1}{1-p}$ will be $= p + \frac{1}{2}p^2 + \frac{1}{3}p^3 + \frac{1}{4}p^4 + \frac{1}{5}p^5$ &c, or the hyperbolic Logarithm of a

$$= \frac{a-1}{a}+\tfrac{1}{2}\left(\frac{a-1}{a}\right)^2+\tfrac{1}{3}\left(\frac{a-1}{a}\right)^3+\tfrac{1}{4}\left(\frac{a-1}{a}\right)^4+\tfrac{1}{5}\left(\frac{a-1}{a}\right)^5 \&c;$$

and r, by reverſion of ſeries will be found $= 2.7182818$, as before. And if, on the contrary, the radix r be aſſumed $= 10$, the value of the ſeries $q+\frac{1}{2}q^2+\frac{1}{3}q^3+\frac{1}{4}q^4+\frac{1}{5}q^5$ &c, or its equal $\frac{r-1}{r}+\frac{1}{2}\left(\frac{r-1}{r}\right)^2+\frac{1}{3}\left(\frac{r-1}{r}\right)^3+\frac{1}{4}\left(\frac{r-1}{r}\right)+\frac{1}{5}\left(\frac{r-1}{r}\right)^5$ &c, will become $= 2.30258509$, as before; and the common Logarithm of

$$\frac{1}{1-p} = \frac{1}{2.30258509} \times (p+\tfrac{1}{2}p^2+\tfrac{1}{3}p^3+\tfrac{1}{4}p^4+\tfrac{1}{5}p^5 \&c),$$

or the common Logarithm of $a = \frac{1}{2.30258509}$

$$\times : \frac{a-1}{a}+\tfrac{1}{2}\left(\frac{a-1}{a}\right)^2+\tfrac{1}{3}\left(\frac{a-1}{a}\right)^3+\tfrac{1}{4}\left(\frac{a-1}{a}\right)^4+\tfrac{1}{5}\left(\frac{a-1}{a}\right)^5 \&c.$$

Or the latter formula, for the Logarithm of $\frac{1}{1-p}$, or its equal a, may be more conciſely derived from the firſt, as follows:

The Logarithm of $1+p$ has been ſhewn to be $= \frac{p-\frac{1}{2}p^2+\frac{1}{3}p^3-\frac{1}{4}p^4+\frac{1}{5}p^5 \&c}{q-\frac{1}{2}q^2+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}$, and if $-p$ be ſubſtituted in the place of $+p$, the logarithm of $1-p$ will become $= \frac{-p-\frac{1}{2}p^2-\frac{1}{3}p^3-\frac{1}{4}p^4-\frac{1}{5}p^5 \&c}{q-\frac{1}{2}q^2+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}$, whence the Logarithm of $\frac{1}{1-p} = \text{Log. } 1 - \text{Log. } (1-p) = 0 -$

$$\left(\frac{-p-\frac{1}{2}p^2-\frac{1}{3}p^3-\frac{1}{4}p^4-\frac{1}{5}p^5 \&c}{q-\frac{1}{2}q^2+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5 \&c} = \frac{p+\frac{1}{2}p^2+\frac{1}{3}p^3+\frac{1}{4}p^4+\frac{1}{5}p^5 \&c}{q-\frac{1}{2}q^2+\frac{1}{3}q^3-\frac{1}{4}q^4+\frac{1}{5}q^5 \&c}\right)$$

$$\text{or Log. } a = \frac{\frac{a-1}{a}+\frac{1}{2}\left(\frac{a-1}{a}\right)^2+\frac{1}{3}\left(\frac{a-1}{a}\right)^3+\frac{1}{4}\left(\frac{a-1}{a}\right)^4 \&c}{(r-1)-\frac{1}{2}(r-1)^2+\frac{1}{3}(r-1)^3-\frac{1}{4}(r-1)^4 \&c};$$

where the denominator is the ſame as in the firſt formula, q being here $= r-1$.

If the denominator, in either of theſe general formulæ, be put $= m$, the Logarithm of $1+p$ will be denoted by $\frac{1}{m} \times (p-\frac{1}{2}p^2+\frac{1}{3}p^3-\frac{1}{4}p^4+\frac{1}{5}p^5$ &c, or the Logarithm of a by

$$\frac{1}{m} \times : (a-1)-\tfrac{1}{2}(a-1)^2+\tfrac{1}{3}(a-1)^3-\tfrac{1}{4}(a\ 1)^4+\tfrac{1}{5}(a-1)^5 \&c.$$

And the Logarithm of $\frac{1}{1-p}$ will be denoted by

$$\frac{1}{m} \times (p+\tfrac{1}{2}p^2+\tfrac{1}{3}p^3+\tfrac{1}{4}p^4+\tfrac{1}{5}p^5 \&c,$$

or the Logarithm of a by

$$\frac{1}{m} \times : \frac{a-1}{a}+\tfrac{1}{2}\left(\frac{a-1}{a}\right)^2+\tfrac{1}{3}\left(\frac{a-1}{a}\right)^3+\tfrac{1}{4}\left(\frac{a-1}{a}\right)^4+\tfrac{1}{5}\left(\frac{a-1}{a}\right)^5 \&c.$$

And ſince the ſum of the Logarithms of any two numbers is equal to the Logarithm of their product, the Logarithm of $\frac{1+p}{1-p}$ will become

$= \frac{2}{m}$

$$= \frac{2}{m} \times (p + \tfrac{1}{3}p^3 + \tfrac{1}{5}p^5 + \tfrac{1}{7}p^7 \text{ \&c}),$$

or the Logarithm of a

$$= \frac{2}{m} \times : \frac{a-1}{a+1} + \tfrac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \tfrac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \tfrac{1}{7}\left(\frac{a-1}{a+1}\right)^7 \text{ \&c.}$$

Which is a third general formula, that converges faſter than either of the former.

The Logarithm of any number may, therefore, be exhibited univerſally, or according to any ſyſtem of Logarithms, in the three following forms:

$$\text{Log.}\,(1+p) = \frac{1}{m} \times : p - \tfrac{1}{2}p^2 + \tfrac{1}{3}p^3 - \tfrac{1}{4}p^4 + \tfrac{1}{5}p^5 \text{ \&c.}$$

$$\text{Log.}\,\frac{1}{1-p} = \frac{1}{m} \times : p + \tfrac{1}{2}p^2 + \tfrac{1}{3}p^3 + \tfrac{1}{4}p^4 + \tfrac{1}{5}p^5 \text{ \&c.}$$

$$\text{Log.}\,\frac{1+p}{1-p} = \frac{2}{m} \times : p + \tfrac{1}{3}p^3 + \tfrac{1}{5}p^5 + \tfrac{1}{7}p^7 + \tfrac{1}{9}p^9 \text{ \&c.}$$

Or

$$\text{Log.}\,a = \frac{1}{m} \times : (a-1) - \tfrac{1}{2}(a-1)^2 + \tfrac{1}{3}(a-1)^3 - \tfrac{1}{4}(a-1)^4 \text{ \&c.}$$

$$\text{Log.}\,a = \frac{1}{m} \times : \frac{a-1}{a} + \tfrac{1}{2}\left(\frac{a-1}{a}\right)^2 + \tfrac{1}{3}\left(\frac{a-1}{a}\right)^3 + \tfrac{1}{4}\left(\frac{a-1}{a}\right)^4 \text{ \&c.}$$

$$\text{Log.}\,a = \frac{2}{m} \times : \frac{a-1}{a+1} + \tfrac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \tfrac{1}{5}\left(\frac{a-1}{a+1}\right)^5 + \tfrac{1}{7}\left(\frac{a-1}{a+1}\right)^7 \text{ \&c.}$$

And if $a+b$ be put $= s$, and $a \backsim b = d$, theſe general formulæ may be eaſily converted into the following:

$$\text{Log.}\,\frac{a}{b} = \frac{1}{m} \times : \frac{d}{b} - \frac{d^2}{2b^2} + \frac{d^3}{3b^3} - \frac{d^4}{4b^4} + \frac{d^5}{5b^5} \text{ \&c.}$$

$$\text{Log.}\,\frac{a}{b} = \frac{1}{m} \times : \frac{d}{a} + \frac{d^2}{2a^2} + \frac{d^3}{3a^3} + \frac{d^4}{4a^4} + \frac{d^5}{5a^5} \text{ \&c.}$$

$$\text{Log.}\,\frac{a}{b} = \frac{2}{m} \times : \frac{d}{s} + \frac{d^3}{3s^3} + \frac{d^5}{5s^5} + \frac{d^7}{7s^7} + \frac{d^9}{9s^9} \text{ \&c.}$$

From which laſt expreſſions, if d or its equal $a \backsim b$ be put $= 1$, we ſhall have, by proper ſubſtitution, and the nature of Logarithms:

$$\text{Log.}\,a = \text{Log.}\,(a-1) + \frac{1}{m} \times : \frac{1}{a} + \frac{1}{2a^2} + \frac{1}{3a^3} + \frac{1}{4a^4} \text{ \&c.}$$

$$\text{Log.}\,a = \text{Log.}\,(a-1) + \frac{1}{m} \times : \frac{1}{a-1} - \frac{1}{2(a-1)^2} + \frac{1}{3(a-1)^3} - \frac{1}{4(a-1)^4} \text{ \&c.}$$

$$\text{Log.}\,a = \text{Log.}\,(a-2) + \frac{1}{m} \times : \frac{1}{a-1} + \frac{1}{3(a-1)^3} + \frac{1}{5(a-1)^5} + \frac{1}{7(a-1)^7} \text{ \&c.}$$

And from the addition and ſubtraction of theſe ſeries, ſeveral others may be derived; but in the actual computation of Logarithms, they will be found to poſſeſs little or no advantage above thoſe here given. The ſame general formula may be derived from the original Logarithmic equation $r^x = a$ in a different way, thus:

Let $r = 1 + q$, then $r^x = \overline{1+q}\,^x = 1 + (q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4 \text{ \&c})x + \tfrac{1}{2}(q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4 \text{ \&c})^2x^2 + \frac{1}{2.3}(q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4 \text{ \&c})^3x^3 + \frac{1}{2.3.4}(q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4 \text{ \&c})^4x^4 \text{ \&c} = a$; or if r be put $= \frac{1}{1-q}$, we ſhall have $\overline{1-q}\,^x = 1 - (q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4 \text{ \&c})x + \tfrac{1}{2}(q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4 \text{ \&c})^2x^2 - \frac{1}{2.3}(q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4 \text{ \&c})^3x^3 + \frac{1}{2.3.4}(q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4 \text{ \&c})^4x^4 \text{ \&c} = \frac{1}{a}$.

And by denoting $q - \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 - \tfrac{1}{4}q^4$ &c in the firſt caſe, or its equal $q + \tfrac{1}{2}q^2 + \tfrac{1}{3}q^3 + \tfrac{1}{4}q^4$, in the latter caſe, by m, theſe expreſſions will become

$$1 + mx + \tfrac{1}{2}m^2x^2 + \frac{1}{2.3}m^3x^3 + \frac{1}{2.3.4}m^4x^4 + \text{\&c} = a;$$

$$\text{and } 1 - mx + \tfrac{1}{2}m^2x^2 - \frac{1}{2.3}m^3x^3 + \frac{1}{2.3.4}m^4x^4 \text{ \&c} = \frac{1}{a};$$

which are the two anti-Logarithmic ſeries of Halley: from whence, by reverſion of ſeries, may be found the Logarithm of any number a, as before.

M.

MICROSCOPE. The following directions are given for uſing the New Univerſal Pocket Microſcope, made and ſold by W. and S. Jones, opticians, No. 135, Holborn, London. See fig. 4, pl. 33.

" This Microſcope is adapted to the viewing of all ſorts of objects, whether *tranſparent*, or *opake;* and for *inſects*, *flowers*, *animalcules*, and the infinite variety of the *minutiæ* of Nature and Art, will be found the moſt complete and portable for the price, of any hitherto contrived.

Place the ſquare pillar of the Microſcope in the ſquare ſocket at the foot D, and faſten it by the pin, as ſhewn in the figure. Place alſo in the foot, the reflecting mirror C. There are three lenſes at the top ſhewn at A, which ſerve to magnify the objects. By uſing theſe lenſes ſeparately or combined, you make ſeven different powers. When tranſparent objects, ſuch as are in the ivory ſliders, number 4, are to be viewed, you place the ſliders over the ſpring, at the underſide of the ſtage B; then looking through the lens or magnifier, at A, at the ſame time reflect up the light, by moving the mirrour C below, and move gently upwards or downwards as may be neceſſary, the ſtage B, upon its ſquare pillar, till you ſee the object illuminated and diſtinctly magnified; and in this manner for the other objects.

For animalcules, you unſcrew the braſs box that is fitted at the ſtage B, containing two glaſſes, and leave the undermoſt glaſs upon the ſtage, to receive the fluids. If you wiſh to view thereon any moving inſect, &c, it may be confined by ſcrewing on the cover: of the two glaſſes, the concave is beſt for fluids. Should the objects be opake, ſuch as ſeeds, &c; they are to be placed upon the black and white ivory round piece, number 3, which is fitted alſo to the ſtage B. If the objects are of a dark colour, you place them contraſtedly on the white ſide of the ivory. If they are of a white, or a light colour, upon the blackened ſide. Some objects will

will be more conveniently viewed, by ſticking them on the point of number 2; or between the nippers at the other end, which open by preſſing the two little braſs pins. This apparatus is alſo fitted to a ſmall hole in the ſtage, made to receive the ſupport of the wire.

The braſs forceps, number 1, ſerve to take up any ſmall object by, in order to place them on the ſtage for view. The inſtrument may be readily converted into an hand Microſcope, to view objects againſt the common light; and which, for ſome tranſparent ones, is better ſo. It is done by only taking out the pillar from its foot in D, turning it half round, and fixing it in again; the foot then becomes a uſeful handle, and the reflector C is laid aſide.

The whole apparatus packs into a fiſh-ſkin caſe, $4\frac{1}{4}$ inches long, $2\frac{1}{4}$ inches broad, and $1\frac{1}{2}$ inches deep.

For perſons more curious and nice in theſe ſort of inſtruments, there is contrived a uſeful adjuſting ſcrew to the ſtage, repreſented at *e*. It is firſt moved up and down like the other, to the focus nearly, and made faſt by the ſmall ſcrew. The utmoſt diſtinctneſs of the object is then obtained, by gently turning the long fine threaded ſcrew, at the ſame time you are looking through the magnifiers A. In this caſe, there may be alſo added an extraordinary deep magnifier, and a concave ſilver ſpeculum, with a magnifier to ſcrew on at A, which will ſerve for viewing the very ſmall, and opake objects, in the completeſt manner, and render the inſtrument as comprehenſive in its uſes and powers, as thoſe formerly ſold under the name of *Wilſon's Microſcope*."

MODULUS, and MODULAR *Ratio*. See p. 49 at the bottom.

N.

NUTATION, in Aſtronomy, a kind of libratory motion of the earth's axis; by which its inclination to the plane of the ecliptic is continually varying, by a certain number of ſeconds, backwards and forwards. The whole extent of this change in the inclination of the earth's axis, or, which is the ſame thing, in the apparent declination of the ſtars, is about 19″, and the period of that change is little more than 9 years, or the ſpace of time from its ſetting out from any point and returning to the ſame point again, about 18 years and 7 months, being the ſame as the period of the moon's motions, upon which it chiefly depends; being indeed the joint effect of the inequalities of the action of the ſun and moon upon the ſpheroidal figure of the earth, by which its axis is made to revolve with a conical motion, ſo that the extremity of it deſcribes a ſmall circle, or rather an ellipſe, of 19·1 ſeconds diameter, and 14″·2 conjugate, each revolution being made in the ſpace of 18 years 7 months, according to the revolution of the moon's nodes.

This is a natural conſequence of the Newtonian ſyſtem of univerſal attraction; the firſt principle of which is, that all bodies mutually attract each other in the direct ratio of their maſſes, and in the inverſe ratio of the ſquares of their diſtances. From this mutual attraction, combined with motion in a right line, Newton deduces the figure of the orbits of the planets, and particularly that of the earth. If this orbit were a circle, and if the earth's form were that of a perfect ſphere, the attraction of the ſun would have no other effect than to keep the earth in its orbit, without cauſing any irregularity in the poſition of its axis. But neither is the earth's orbit a circle, nor its body a ſphere; for the earth is ſenſibly protuberant towards the equator, and its orbit is an ellipſis, which has the ſun in its focus. Now when the poſition of the earth is ſuch, that the plane of the equator paſſes through the centre of the ſun, the attractive power of the ſun acts only ſo as to draw the earth towards it, ſtill parallel to itſelf, and without changing the poſition of its axis; a circumſtance which happens only at the time of the equinoxes. In proportion as the earth recedes from thoſe points, the ſun alſo goes out of the plane of the equator, and approaches that of the one or other of the tropics; the ſemidiameter of the earth, then expoſed to the ſun, being unequal to what it was in the former caſe, the equator is more powerfully attracted than the reſt of the globe, which cauſes ſome alteration in its poſition, and its inclination to the plane of the ecliptic: and as that part of the orbit, which is compriſed between the autumnal and vernal equinox, is leſs than that which is compriſed between the vernal and autumnal, it follows, that the irregularity cauſed by the ſun, during his paſſage through the northern ſigns, is not entirely compenſated by that which he cauſes during his paſſage through the ſouthern ſigns; and that the paralleliſm of the terreſtrial axis, and its inclination to the ecliptic, is thence a little altered.

The like effect which the ſun produces upon the earth, by his attraction, is alſo produced by the moon, which acts with greater force, in proportion as ſhe is more diſtant from the equator. Now, at the time when her nodes agree with the equinoxial points, her greateſt latitude is added to the greateſt obliquity of the ecliptic. At this time therefore, the power which cauſes the irregularity in the poſition of the terreſtrial axis, acts with the greateſt force; and the revolution of the nodes of the moon being performed in 18 years 7 months, hence it happens that in this time the nodes will twice agree with the equinoxial points; and conſequently, twice in that period, or once every 9 years, the earth's axis will be more influenced than at any other time.

That the moon has alſo a like motion, is ſhewn by Newton, in the firſt book of the Principia; but he obſerves indeed that this motion muſt be very ſmall, and ſcarcely ſenſible.

As to the hiſtory of the Nutation, it ſeems there have been hints and ſuſpicions of the exiſtence of ſuch a circumſtance, ever ſince Newton's diſcovery of the ſyſtem of the univerſal and mutual attraction of matter; ſome traces of which are found in his Principia, as above mentioned.

We find too, that Flamſteed had hoped, about the year 1690, by means of the ſtars near his zenith, to determine the quantity of the Nutation which ought to follow from the theory of Newton; but he gave up that project, becauſe, ſays he, if this effect exiſts, it muſt remain inſenſible till we have inſtruments much longer than 7 feet, and more ſolid and better fixed than mine. Hiſt. Cæleſt. vol. 3, pa. 113.

And Horrebow gives the following paſſage, extracted from the manuſcripts of his maſter Roemer, who died in 1710, whoſe obſervations he publiſhed in 1753, under

der the title of *Basis Astronomiæ*. By this paragraph it appears that Roemer suspected also a Nutation in the earth's axis, and had some hopes to give the theory of it: it runs thus; "Sed de altitudinibus non perinde certus reddebar, tàm ob refractionum varietatem quàm ob aliam nondum liquido perspectam causam; scilicet per hos duos annos, quemadmodum & alias, expertus sum esse quandam in declinationibus varietatem, quæ nec refractionibus nec parallaxibus tribui potest, sine dubio ad vacillationem aliquam poli terrestris referendam, cujus me verisimilem dare posse theoriam, observationibus munitam, spero." Basis Astronomiæ, 1735, pa. 66.

These ideas of a Nutation would naturally present themselves to those who might perceive certain changes in the declinations of the stars; and we have seen that the first suspicions of Bradley in 1727, were that there was some Nutation of the earth's axis which caused the star γ Draconis to appear at times more or less near the pole; but farther observations obliged him to search another cause for the annual variations (art. ABERRATION): it was not till some years after that he discovered the second motion which we now treat of, properly called the Nutation. See the art. STAR, pa. 500 &c, where Bradley's discovery of it is given at length; to which may be farther added the following summary.

For the better explaining the discovery of the Nutation by Bradley, we must recur to the time when he observed the stars in discovering the aberration. He perceived in 1728, that the annual change of declination in the stars near the equinoxial colure, was greater than what ought to result from the annual precession of the equinoxes being supposed 50″, and calculated in the usual way; the star η Ursæ Majoris was in the month of September 1728, 20″ more south than the preceding year, which ought to have been only 18″; from whence it would follow that the precession of the equinoxes should be 55″½ instead of 50″, without ascribing the difference between the 18 and 20″ to the instrument, because the stars about the solstitial colure did not give a like difference. Philos. Trans. vol. 35, pa. 659.

In general, the stars situated near the equinoctial colure had changed their declination about 2″ more than they ought by the mean precession of the equinoxes, the quantity of which is very well known, and the stars near the solstitial colure the same quantity less than they ought; but, Bradley adds, whether these small variations arise from some regular cause, or are occasioned by some change in the sector, I am not yet able to determine. Bradley therefore ardently continued his observations for determining the period and the law of these variations; for which purpose he resided almost continually at Wansted till 1732, when he was obliged to repair to Oxford to succeed Dr. Halley; he still continued to observe with the same exactness all the circumstances of the changes of declination in a great number of stars. Each year he saw the periods of the aberration confirmed according to the rules he had lately discovered; but from year to year he found also other differences; the stars situated between the vernal equinox and the winter solstice approached nearer to the north pole, while the opposite ones receded farther from it: he began therefore to suspect that the action of the moon upon the elevated equatorial parts of the earth might cause a variation or libration in the earth's axis: his sector having been left fixed at Wansted, he often went there to make observations for many years, till the year 1747, when he was fully satisfied of the cause and effects, an account of which he then communicated to the world. Philos. Trans. vol. 45, an. 1748.

"On account of the inclination of the moon's orbit to the ecliptic, says Dr. Maskelyne (Astronomical Observations 1776, pa. 2), and the revolution of the nodes in antecedentia, which is performed in 18 years and 7 months, the part of the precession of the equinoxes, owing to her action, is not uniform: but subject to an equation, whose maximum is 18″: and the obliquity of the ecliptic is also subject to a periodical equation of 9″·55; being greater by 19·1″ when the moon's ascending node is in Aries, than when it is in Libra. Both these effects are represented together, by supposing the pole of the earth to describe the periphery of an ellipsis, in a retrograde manner, during each period of the moon's nodes, the greater axis, lying in the solstitial colure, being 19·1″, and the lesser axis, lying in the equinoctial colure, 14·2″; being to the greater, as the cosine of double the obliquity of the ecliptic to the cosine of the obliquity itself. This motion of the pole of the earth is called the Nutation of the earth's axis, and was discovered by Dr. Bradley, by a series of observations of several stars made in the course of 20 years, from 1727 to 1747, being a continuation of those by which he had discovered the aberration of light. But the exact law of the motion of the earth's axis has been settled by the learned mathematicians d'Alembert, Euler, and Simpson, from the principles of gravity. The equation hence arising in the place of a fixed star, whether in longitude, right-ascension, or declination (for the latitudes are not affected by it) has been sometimes called Nutation, and sometimes Deviation." And again (says the Doctor, pa. 8), the above "quantity 19·1′, of the greatest Nutation of the earth's axis in the solstitial colure, is what I found from a scrupulous calculation of all Dr. Bradley's observations of γ Draconis, which he was pleased to communicate to me for that purpose. From a like examination of his observation of η Ursæ majoris, I found the lesser axis of the ellipsis of Nutation to be 14·1″, or only $\frac{1}{10}$th of a second less than what it should be from the observations of γ Draconis. But the result from the observations of γ Draconis is most to be depended upon."

Mr. Machin, secretary of the Royal Society, to whom Bradley communicated his conjectures, soon perceived that it would be sufficient to explain, both the Nutation and the change of the precession, to suppose that the pole of the earth described a small circle. He stated the diameter of this circle at 18″, and he supposed that it was described by the pole in the space of one revolution of the moon's nodes. But later calculations and theory, have shewn that the pole describes a small ellipsis, whose axes are 19·1″ and 14·2″, as above mentioned.

To shew the agreement between the theory and observations, Bradley gives a great multitude of observations of a number of stars, taken in different positions; and out of more than 300 observations which he made, he found but 11 which were different from the mean by

se

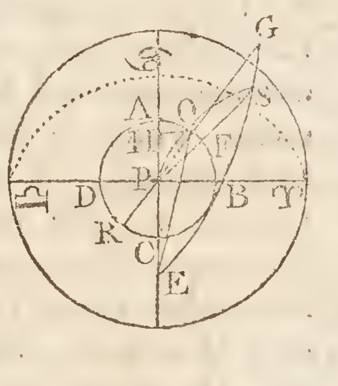
ſo much as 2″. And by the ſuppoſition of the elliptic rotation, the agreement of the theory with obſervation comes out ſtill nearer.

By the obſervations of 1740 and 1741, the ſtar η Urſæ majoris appeared to be 3″ farther from the pole than it ought to be according to the obſervations of other years. Bradley thought this difference aroſe from ſome particular cauſe; which however was chiefly the fault of the circular hypotheſis. He ſuſpected alſo that the ſituation of the apogee of the moon might have ſome influence on the Nutation. He invited therefore the mathematicians to calculate all theſe effects of attraction, which has been ably done by d'Alembert, Euler, Walmeſley, Simpſon, and others; and the aſtronomers to continue to obſerve the poſitions of the ſmalleſt ſtars, as well as the largeſt, to diſcover the phyſical derangements which they may ſuffer, and which had been obſerved in ſome of them.

Several effects ariſe from the Nutation. The firſt of theſe, and that which is the moſt eaſily perceived, is the change in the obliquity of the ecliptic; the quantity of which ought to be varied from that cauſe by 18″ in about 9 years. Accordingly, the obliquity of the ecliptic was obſerved in 1764 to be 23° 28′ 15″, and in 1755 only 23° 28′ 5″: not only therefore had it not diminiſhed by 8″, as it ought to have done according to the regular mean diminution of that obliquity; but it had even augmented by 10″; making together 18″, for the effect of the Nutation in the 9 years.

The Nutation changes equally the longitudes, the right-aſcenſions, and the declinations of the ſtars, as before obſerved; it is the latitudes only which it does not affect, becauſe the ecliptic is immoveable in the theory of the Nutation.

Dr. Bradley illuſtrates the foregoing theory of Nutation in the following manner. Let P repreſent the mean place of the pole of the equator, about which point, as a centre, ſuppoſe the true pole to move in the ſmall circle ABCD, whoſe diameter is 18″. Let E be the pole of the ecliptic, and EP be equal to the mean diſtance between the poles of the equator and ecliptic; and ſuppoſe the true pole of the equator to be at A, when the moon's aſcending node is in the beginning of Aries; and at B,



when the node gets back to Capricorn; and at C, when the ſame node is in Libra: at which time the north pole of the equator being nearer the north pole of the ecliptic, by the whole diameter of the little circle AC, equal to 18″; the obliquity of the ecliptic will then be ſo much leſs than it was, when the moon's aſcending node was in Aries. The point P is ſuppoſed to move round E, with an equal retrograde motion, anſwerable to the mean preceſſion ariſing from the joint actions of the ſun and moon: while the true pole of the equator moves round P, in the circumference ABCD, with a retrograde motion likewiſe, in a period of the moon's nodes, or of 18 years and 7 months. By this means, when the moon's aſcending node is in Aries, and the true pole of the equator, at A, is moving from A towards B; it will approach the ſtars that come to the meridian with the ſun about the vernal equinox, and recede from thoſe that come with the ſun near the autumnal equinox, faſter than the mean pole P does. So that, while the moon's node goes back from Aries to Capricorn, the apparent preceſſion will ſeem ſo much greater than the mean, as to cauſe the ſtars that lie in the equinoctial colure to have altered their declination 9″, in about 4 years and 8 months, more than the mean preceſſion would do; and in the ſame time, the north pole of the equator will ſeem to have approached the ſtars that come to the meridian with the ſun of our winter ſolſtice about 9″, and to have receded as much from thoſe that come with the ſun at the ſummer ſolſtice.

Thus the phenomena before recited are in general conformable to this hypotheſis. But to be more particular; let S be the place of a ſtar, PS the circle of declination paſſing through it, repreſenting its diſtance from the mean pole, and ♈PS its mean right-aſcenſion. Thus if O and R be the points where the circle of declination cuts the little circle ABCD, the true pole will be neareſt that ſtar at O, and fartheſt from it at R; the whole difference amounting to 18″, or to the diameter of the little circle. As the true pole of the equator is ſuppoſed to be at A, when the moon's aſcending node is in Aries; and at B, when that node gets back to Capricorn; and the angular motion of the true pole about P, is likewiſe ſuppoſed equal to that of the moon's node about E, or the pole of the ecliptic; ſince in theſe caſes the true pole of the equator is 90 degrees before the moon's aſcending node, it muſt be ſo in all others.

When the true pole is at A, it will be at the ſame diſtance from the ſtars that lie in the equinoctial colure, as the mean pole P is; and as the true pole recedes back from A towards B, it will approach the ſtars which lie in that part of the colure repreſented by P♈; and recede from thoſe that lie in P♎; not indeed with an equable motion, but in the ratio of the ſine of the diſtance of the moon's node from the beginning of Aries. For if the node be ſuppoſed to have gone backwards from Aries 30°, or to the beginning of Piſces, the point which repreſents the place of the true pole will, in the mean time, have moved in the little circle through an arc, as AO, of 30° likewiſe; and would therefore in effect have approached the ſtars that lie in the equinoctial colure P♈, and have receded from thoſe that lie in P♎ by $4\frac{1}{2}$ ſeconds, which is the ſine of 30° to the radius AP. For if a perpendicular fall from O upon AP, it may be conceived as part of a great circle, paſſing through the true pole and any ſtar lying in the equinoctial colure. Now the ſame proportion that holds in theſe ſtars, will obtain likewiſe in all others; and from hence we may collect a general rule for finding how much nearer, or farther, any ſtar is to, or from, the mean pole, in any given poſition of the moon's node.

For, If *from the right-aſcenſion of the ſtar, we ſubtract the diſtance of the moon's aſcending node from Aries; then radius will be to the ſine of the remainder, as 9″ is to the number of ſeconds that the ſtar is nearer to, or farther from, the true, than the mean pole.*

This motion of the true pole, about the mean at P, will alſo produce a change in the right-aſcenſion of the ſtars,

ftars, and in the places of the equinoctial points, as well as in the obliquity of the ecliptic; and the quantity of the equations, in either of thefe cafes, may be eafily computed for any given pofition of the moon's nodes.

Dr. Bradley then proceeds to find the exact quantity of the mean preceffion of the equinoctial points, by comparing his own obfervations made at Greenwich, with thofe of Tycho Brahe and others; the mean of all which he ftates at 1 degree in $71\frac{1}{2}$ years, or $50\frac{1}{3}''$ per year; in order to fhew the agreement of the foregoing hypothefis with the phenomena themfelves, of the alterations in the polar diftances of the ftars; the conclufions from which approach as near to a coincidence as could be expected on the foregoing circular hypothefis, the diameter of which is $18''$; inftead of the more accurate quantity $19{\cdot}1''$, as deduced by Dr. Mafkelyne, and the elliptic theory as determined by the mathematicians, in which the greater axis ($19{\cdot}1''$) is to the lefs axis ($14{\cdot}2''$), as the cofine of the greateft declination is to the cofine of double the fame.

To give an idea now of the Nutation of the ftars, in longitude, right-afcenfion, and declination; fuppofe the pole of the equator to be at any time in the point O, alfo S the place of any ftar, and OH perpendicular to AE: then, like as AE is the folftitial colure when the pole of the equator was at A, and the longitude of the ftar S equal to the angle AES; fo OE is the folftitial colure when that pole is at O, and the longitude is then only the angle OES; lefs than before by the angle AEO, which therefore is the Nutation in longitude: counting the longitudes from the folftitial inftead of the equinoctial colure, from which they differ equally by 90 degrees, and therefore have the fame difference AEO. Now the angle AEO will be as the line HO = fin. AO to radius PB = fin. AO × PB = fin. AO × 9''; therefore as EO : HO : : radius 1 : $\frac{HO}{EO} = \frac{\text{fin. } AO \times 9''}{\text{fin. } 23^\circ\ 28'} = \frac{\text{fin. node} \times 9''}{\text{fin. } 23^\circ\ 28'}$, fince AO is equal to longitude of the moon's node. This expreffion therefore gives the Nutation in longitude, fuppofing the maximum of Nutation, with Bradley, to be $18''$; and it is negative, or muft be fubtracted from the mean longitude of the ftars, when the moon's node is in the firft 6 figns of its longitude, but additive in the latter 6, to give the true apparent longitude.

This equation of the Nutation in longitude is the fame for all the ftars; but that for the declination and right-afcenfion is various for the different ftars. In the foregoing figure, PS is the mean polar diftance, or mean codeclination, of the ftar S, when the true place of the pole is O; and SO the apparent codeclination; alfo, the angle SPE is the mean right-afcenfion, and SOE the apparent one, counted from the folftitial colure; confequently OPS or OPF the difference between the right-afcenfion of the ftar and that of the pole, which is equal to the longitude of the node increafed by 3 figns or 90 degrees; fuppofing OF to be a fmall arc perpendicular to the circle of declination PFS; then is SF = SO, and PF the Nutation in declination, or the quantity the declination of the ftar has increafed; but radius 1 : $9''$: : cofin. OPF : PF = $9'' \times$ cof. OPF; fo that the equation of declination will be found by multiplying $9''$ by the fine of the ftar's right-afcenfion diminifhed by the longitude of the node; for that angle is the complement of the angle SPO. This Nutation in declination is to be added to the mean declination to give the apparent, when its argument does not exceed 6 figns; and to be fubtracted in the latter 6 figns. But the contrary for the ftars having fouth declination.

To calculate the Nutation in right-afcenfion, we muft find the difference between the angle SOE the apparent, and SPE the mean right-afcenfion, counted from the folftitial colure EO. Now the true right-afcenfion SOE is equal to the difference between the two variable angles GOE and GOS; the former of which arifes from the change of one of the variable circles EO, and depends only on the fituation of the node or of that of the pole O; the latter GOS depends on the angle GPS which is the difference between the right-afcenfion of the ftar and the place of the pole O. Now in the fpherical triangle GPE, which changes into GOE, the fide GE and the angle G remain conftant, and the other parts are variable; hence therefore the fmall variation PO of the fide next the conftant angle G, is to the fmall variation of the angle oppofite to the conftant fide GE, as the tangent of the fide PE oppofite to the conftant angle, is to the fine of the angle GPE oppofite to the conftant fide; that is, as

$$\text{tang. } 23^\circ\ 28' : \text{fin. OPE} : : 9'' : x = \frac{9'' \times \text{fin OPE}}{\text{tang. } 23^\circ\ 28'},$$

the difference between the angles GOE and GPE. This is the change which the Nutation PO produces in the angle GPE, being the firft part of the Nutation fought, and is common to all the ftars and planets. It is to be fubtracted from the mean right-afcenfion in the firft 6 figns of the longitude of the node, and added in the other fix.

In like manner is found the change which the Nutation produces in the other part of the right-afcenfion SPE, that is, in the angle SPG, which becomes SOG by the effect of the Nutation. This fmall variation will be calculated from the fame analogy, by means of the triangle SOG, in which the angle G is conftant, as well as the fide SG, whilft SP changes into SO. Hence therefore, tang. SP : fin. SPG : : $9''$: variation of SPG, that is, the cotangent of the declination is to the cofine of the diftance between the ftar and the node, as $9''$ are to the quantity the angle SPG varies in becoming the angle SOG, being the fecond part of the Nutation in right-afcenfion; and if there be taken for the argument, the right-afcenfion of the ftar minus the longitude of the node, the equation will be fubtractive in the firft and laft quadrant of the argument, and additive in the 2d and 3d, or from 3 to 9 figns. But the contrary for ftars having fouth declination.

This fecond part of the Nutation in right-afcenfion affects the return of the fun to the meridian, and therefore it muft be taken into the account in computing the equation of time. But the former part of the Nutation does not enter into that computation; becaufe it only changes the place of the equinox, without changing the point of the equator to which a ftar correfponds, and confequently without altering the duration of the returns to the meridian.

All these calculations of the Nutation, above explained, are upon Machin's hypothesis, that the pole describes a circle; however Bradley himself remarked that some of his observations differed too much from that theory, and that such observations were found to agree better with theory, by supposing that the pole, instead of the circle, describes an ellipse, having its less axis DB = 16″ in the equinoctial colure, and the greater axis AC = 18″, lying in the solstitial colure. But as even this correction was not sufficient to cause all the inequalities to disappear entirely, Dr. Bradley referred the determination of the point to theoretical and physical investigation. Accordingly several mathematicians undertook the task, and particularly d'Alembert, in his Recherches sur la précession des equinoxes, where he determines that the pole really describes an ellipse, and that narrower than the one assumed above by Bradley, the greater axis being to the less, as the cosine of 23° 28′ to the cosine of double the same. And as Dr. Maskelyne found, from a more accurate reduction of Bradley's observations, that the maximum of the Nutation gives 19·1″ for the greater axis, therefore the above proportion gives 14·2″ for the less axis of it; and according to these data, the theory and observations are now found to agree very near together.

See La Lande's Astron. vol. 3, art. 2874 &c, where he makes the corrections for the ellipse. He observes however that by the circular hypothesis alone, the computations may be performed as accurately as the observations can be made; and he concludes with some corrections and rules for computing the Nutation in the elliptic theory.

The following set of general tables very readily give the effect of Nutation on the elliptical hypothesis; they were calculated by the late M. Lambert, and are taken from the Connoissance des Temps for the year 1788.

General Tables for Nutation in the Ellipse.

Table 1.

Degrees	0·6 + −	1·7 + −	2·8 + −	
	″	″	″	
0	0·00	3·93	6·80	30
1	0·14	4·04	6·86	29
2	0·27	4·16	6·93	28
3	0·41	4·28	6·99	27
4	0·55	4·39	7·06	26
5	0·68	4·50	7·11	25
6	0·82	4·61	7·17	24
7	0·95	4·72	7·23	23
8	1·11	4·83	7·28	22
9	1·23	4·94	7·33	21
10	1·36	5·05	7·38	20
11	1·50	5·15	7·42	19
12	1·63	5·25	7·47	18
13	1·77	5·35	7·51	17
14	1·90	5·45	7·55	16
15	2·03	5·55	7·58	15
16	2·16	5·65	7·62	14
17	2·30	5·74	7·65	13
18	2·43	5·83	7·68	12
19	2·56	5·92	7·71	11
20	2·68	6·01	7·73	10
21	2·81	6·10	7·75	9
22	2·94	6·19	7·76	8
23	3·07	6·27	7·77	7
24	3·19	6·35	7·79	6
25	3·32	6·43	7·80	5
26	3·44	6·51	7·82	4
27	3·56	6·58	7·83	3
28	3·69	6·66	7·84	2
29	3·81	6·73	7·85	1
30	3·93	6·80	7·85	0
	+ − 5·11	+ − 4·10	+ − 3·9	Degrees

Table 2.

Degrees	0·6 + −	1·7 + −	2·8 + −	
	″	″	″	
0	0·00	0·58	1·00	30
1	0·02	0·59	1·01	29
2	0·04	0·61	1·02	28
3	0·06	0·63	1·02	27
4	0·08	0·64	1·03	26
5	0·10	0·66	1·04	25
6	0·12	0·68	1·05	24
7	0·14	0·69	1·06	23
8	0·16	0·71	1·07	22
9	0·18	0·72	1·07	21
10	0·20	0·74	1·08	20
11	0·22	0·75	1·09	19
12	0·24	0·77	1·09	18
13	0·26	0·78	1·10	17
14	0·28	0·80	1·11	16
15	0·30	0·81	1·11	15
16	0·32	0·83	1·12	14
17	0·34	0·84	1·12	13
18	0·35	0·85	1·13	12
19	0·37	0·87	1·13	11
20	0·39	0·88	1·13	10
21	0·41	0·89	1·14	9
22	0·43	0·91	1·14	8
23	0·45	0·92	1·14	7
24	0·47	0·93	1·14	6
25	0·49	0·94	1·15	5
26	0·50	0·95	1·15	4
27	0·52	0·96	1·15	3
28	0·54	0·97	1·15	2
29	0·56	0·99	1·15	1
30	0·58	1·00	1·15	0
	+ − 5·11	+ − 4·10	+ − 3·9	Degrees

Table 3.

Degrees	0·6 − +	1·7 − +	2·8 − +	
	″	″	″	
0	0·00	7·71	13·36	30
1	0·27	7·95	13·50	29
2	0·54	8·18	13·62	28
3	0·81	8·40	13·75	27
4	1·08	8·63	13·87	26
5	1·35	8·85	13·98	25
6	1·61	9·07	14·10	24
7	1·88	9·29	14·20	23
8	2·15	9·50	14·31	22
9	2·41	9·71	14·41	21
10	2·68	9·92	14·50	20
11	2·94	10.12	14·59	19
12	3·21	10·32	14·67	18
13	3·47	10·52	14·76	17
14	3·73	10·72	14·83	16
15	3·99	10·91	14·90	15
16	4·25	11·10	14·97	14
17	4·51	11·28	15·03	13
18	4·77	11·47	15·09	12
19	5·02	11·65	15·15	11
20	5·28	11·82	15·20	10
21	5·53	11·99	15·24	9
22	5·78	12·16	15·28	8
23	6·03	12·32	15 32	7
24	6·28	12·48	15·35	6
25	6·52	12·64	15·37	5
26	6·76	12·79	15·39	4
27	7·01	12·94	15·41	3
28	7·25	13·09	15·42	2
29	7·48	13·23	15·43	1
30	7·71	13·36	15·43	0
	− + 5·11	− + 4·10	− + 3·9	Degrees

The

The Uſe of the Tables.

The right-aſcenſion of a ſtar minus the moon's mean longitude, gives the argument of the firſt of theſe three tables. The ſum of the ſame two quantities gives the argument of the 2d table. Then the ſum or the difference of the quantities found with theſe two arguments, will give the correction to be applied to the mean declination of the ſtar, if it is north declination; but if it is ſouthern, the ſigns + or − are to be changed into − and +.

From each of thoſe two arguments for the declination ſubtracting 3 ſigns, or 90°, gives the arguments for correcting the right-aſcenſion; the ſum or difference of the quantities found, with theſe two arguments, in tables 1 and 2, is to be multiplied by the tangent of the ſtar's declination, and to the product is to be added the quantity taken out of table 3, the argument of which is the mean longitude of the moon's aſcending node: when the declination of the ſtar is ſouth, the tangent will be negative.

Example. To find the Nutation in right-aſcenſion and declination for the ſtar α Aquilæ, the 1ſt of July 1788.

	s	°	′	″
Right-aſcenſion of the ſtar	9	25	7	
Long. of the moon's node	8	15	40	
Diff. being argument 1,	1	9	27	+ 4·99
Sum, argument 2, - -	6	10	47	− 0·22
Correction of the declination - - -				+ 4·77

The above two arguments being each diminiſhed by 3 ſigns, give,

	s	°	′	″
Argument 1 - - - - - - -	10	9	27	− 6·06
Argument 2 - - - - - - -	3	10	47	+ 1·13
				− 4·93
Declin. of ſtar north, its tangent - - -				0·146
The product is - - - - - -				− 0·72
Long. of the ☾'s node, argum. 3 -				+ 14·94
Correction of right-aſcenſion - - -				+ 14·22

In general, let ☊ denote the longitude of the moon's aſcending node; r the right-aſcenſion of a ſtar or planet; d its declination; the Nutation in declination and right-aſcenſion will be expreſſed by the two following formulæ; viz, the Nutation in declination

$$= 7''\cdot85 \times \text{ſin.}\ (r - ☊) + 1''\cdot15 \times \text{ſin.}\ (r + ☊);$$

and the Nutation in right-aſcenſion

$$= [7''\cdot85 \times \text{ſin.}\ (r - ☊ - 90°) + 1''\cdot15 \times \text{ſin.}\ (r + ☊ - 90°)] \times \text{tang.}\ d - 15''\cdot43 \times \text{ſin.}\ ☊.$$

For the mathematical inveſtigation of the effects of univerſal attraction, in producing the Nutation, &c, ſee d'Alembert's Recherches ſur la Preceſſion des Equinoxes; Silvabelle's Treatiſe on the Preceſſion of the Equinoxes &c, in the Philoſ. Tranſ. an. 1754, p. 385; Walmeſley's treatiſe De Præceſſione Equinoctiorum et Axis Terræ Nutatione, in the Philoſ. Tranſ. an. 1756, pa. 700; Simpſon's Miſcellaneous Tracts, pa. 1; and other authors.

S

STEAM. The obſervations on the different degrees of temperature acquired by water in boiling, under different preſſures of the atmoſphere, and the formation of the vapour from water under the receiver of an air-pump, when, with the common temperatures, the preſſure is diminiſhed to a certain degree, have taught us that the expanſive force of vapour or Steam is different in the different temperatures, and that in general it increaſes in a variable ratio as the temperature is raiſed.

But there was wanting, on this important ſubject, a ſeries of exact and direct experiments, by means of which, having given the degree of temperature in boiling water, we may know the expanſive force of the Steam riſing from it; and vice verſa. There was wanting alſo an analytical theorem, expreſſing the relation between the temperature of boiling water, and the preſſure with which the force of its Steam is in equilibrium. Theſe circumſtances then have lately been accompliſhed by M. Betancourt, an ingenious Spaniſh philoſopher, the particulars of which are deſcribed in a memoir communicated to the French Academy of Sciences in 1790, and ordered to be printed in their collection of the Works of Strangers.

The apparatus which M. Betancourt makes uſe of, is a copper veſſel or boiler, with its cover firmly ſoldered on. The cover has three holes, which cloſe up with ſcrews: the firſt is to put the water in and out; through the ſecond paſſes the ſtem of a thermometer, which has the whole of its ſcale or graduations above the veſſel, and its ball within, where it is immerſed either in the water or the Steam according to the different circumſtances; through the third hole paſſes a tube making a communication between the cavity of the boiler and one branch of an inverted ſyphon, which, containing mercury, acts as a barometer for meaſuring the preſſure of the elaſtic vapour within the boiler. There is a fourth hole, in the ſide of the veſſel, into which is inſerted a tube, with a turn-cock, making a communication with the receiver of an air-pump, for extracting the air from the boiler, and to prevent its return.

The apparatus being prepared in good order, and diſtilled water introduced into the boiler by the firſt hole, and then ſtopped, as well as the end of the inverted ſyphon or barometer, M. Betancourt ſurrounded the boiler with ice, to lower the temperature of the water to the freezing point, and then extracting all the air from the boiler by means of the air-pump, the difference between the columns of mercury in the two branches of the barometer is the meaſure of the ſpring of the vapour ariſing from the water in that temperature. Then, lighting the fire below the boiler, he raiſed gradually the temperature of the water from 0 to 110 degrees of Reaumur's thermometer; being the ſame as from 32 to 212 degrees of Fahrenheit's; and for each degree of elevation in the temperature, he obſerved the height of the column of mercury which meaſured the elaſticity or preſſure of the vapour.

The reſults of M. Betancourt's experiments are contained

tained in a table of four columns, which are but little different, according to the different quantities of water in the veſſel. It is here obſervable, that the increaſe in the expanſive force of the vapour, is at firſt very ſlow; but gradually increaſing faſter and faſter, till at laſt it becomes very rapid. Thus, the ſtrength of the vapour, at 80 degrees, is only equal to 28 French inches of mercury; but at 110 degrees it is equal to no leſs than 98 inches, that is 3 times and a half more for the increaſe of only 30 degrees of heat.

To expreſs analytically the relation between the degrees of temperature of the vapour, and its expanſive force, this author employs a method deviſed by M. Prony. This method conſiſts in conceiving the heights of the columns of mercury, meaſuring the expanſive force, to repreſent the ordinates of a curve, and the degrees of heat as the abſciſſes of the ſame; making the ordinates equal to the ſum of ſeveral logarithmic ones, which contain two indeterminates, and determining theſe quantities ſo that the curve may agree with a good number of obſervations taken throughout the whole extent of them. Then conſtructing the curve which reſults immediately from the experiments, and that given by the formula, theſe two curves are found to coincide almoſt perfectly together; the ſmall differences being doubtleſs owing to the little irregularities in the experiments and in dividing the ſcale; ſo that the phenomena may be conſidered as truly repreſented by the formula.

M. Betancourt made alſo experiments with the vapour from ſpirit of wine, ſimilar to thoſe made with water; conſtructing the curve, and giving the formula proper to the ſame. From which is derived this remarkable reſult, that, for any one and the ſame degree of heat, the ſtrength of the vapour of ſpirit of wine, is to that of water, always in the ſame conſtant ratio, viz, that of 7 to 3 very nearly; the ſtrength of the former being always $2\frac{1}{3}$ times the ſtrength of the latter, with the ſame degree of heat in the liquid.

Of the Formula, or Equation to the Curve.

The equation to the curve of temperature and preſſure, denoting the relation between the abſciſſes and ordinates, or between the temperature of the vapour and its ſtrength, is, for water,

$$y = b^{a+cx} - b^{a+c'x} - b^{e+c''x} + b^{e+c'''x}.$$

Where x denotes the abſciſſes of the curve, or the degrees of Reaumur's thermometer; and y the correſponding ordinates, or the heights of the column of mercury in Paris inches, repreſenting the ſtrength or elaſticity of the vapour anſwering to the number x of degrees of the thermometer. Then, by comparing this formula with a proper number of the experiments, the values of the conſtant quantities come out as below:

$b = 10·$
$a = 0·068831$
$c = 0·019438$
$c' = 0·013490$
$e = -\ 4·689760$
$c'' = 0·058622$
$e' = -\ 3·937600$
$c''' = 0·049220$

Hence it is evident by inſpection, that the terms of the equation are very eaſy to calculate. For, b being the radix or root of the common ſyſtem of logarithms, and all the terms on the ſecond ſide of the equation being the powers of b, theſe terms are conſequently the tabular natural numbers having the variable exponents for their logarithms. Now as x riſes only to the firſt power, and is multiplied by a conſtant number, and another conſtant number being added to the product, gives the variable exponent, or logarithm; to which then is immediately found the correſponding natural number in the table of logarithms.

In the above formula, the two laſt terms may be entirely omitted, as very ſmall, as far as to the 90th degree of the thermometer; and even above that temperature thoſe two terms make but a ſmall part of the whole formula.

And for the ſpirit of wine the formula is

$$y = b^{a+cx} + b^{a'+c'x} - b^{e+c''x} + b^{e'+c'''x} - A.$$

Where x and y, as before, denote the abſciſs and ordinate of the curve, or the temperature and expanſive force of the vapour from the ſpirit of wine; alſo the values of the conſtant quantities are as below:

$b = 10·$
$a = -\ 0·04853$
$c = 0·02393$
$a' = -\ 0·63414$
$c' = -\ 0·096532$
$e = -\ 2·509542$
$c'' = 0·046473$
$e' = -\ 1·790192$
$c''' = 0·029448$
$A = 1·12647$

This formula is of the ſame nature as the former, having alſo the like eaſe and convenience of calculation; and perhaps more ſo; as the ſecond term $b^{a'+c'x}$, having its exponent wholly negative, ſoon diminiſhes to no value, ſo as to be omitted from the 10th degree of temperature; alſo the difference between the laſt two terms $-\ b^{e+c''x} + b^{e'+c'''x}$ may be omitted till the 70th degree, for the ſame reaſon. So that, to the 10th degree of temperature the theorem is only $y = b^{a+cx} + b^{a'+c'x} - A$; and from the 10th to the 70th degree it is barely $y = b^{a+cx} - A$; after which, for the laſt 15 or 20 degrees, for great accuracy, the laſt two terms may be taken in.

A compendium of the table of the experiments here follows, for the vapour of both water and ſpirit of wine, the temperature by Reaumur's thermometer, and the barometer in French inches.

Table

Table of the Temperature and Strength of the Vapour of Water and Spirit of Wine, by Reaumur's Thermometer, and French Inches.

Degr. of Reau. Ther.	Height of the Barometer for Vapour of Water.	Height of the Barometer for Vapour of Spirit of Wine.	Deg. of Reau. Ther.	Height of the Barometer for Vapour of Water.	Height of the Barometer for Vapour of Spirit of Wine.
1	0·0176	0·0043	56	7·6948	18·4420
2	0·0346	0·0208	57	8·1412	19·5081
3	0·0538	0·0478	58	8·6221	20·6286
4	0·0747	0·0837	59	9·1071	21·6071
5	0·1038	0·1279	60	9·6280	23·0544
6	0·1211	0·1794	61	10·1767	24·3451
7	0·1508	0·2377	62	10·7098	25·6107
8	0·1741	0·3024	63	11·3602	27·1444
9	0·2073	0·3733	64	11·9976	28·6483
10	0·2304	0·4502	65	12·6687	30·2262
11	0·2681	0·5130	66	13·3743	31·8795
12	0·3039	0·6058	67	14·1161	33·6114
13	0·3419	0·7040	68	14·8958	35·4258
14	0·3877	0·8077	69	15·7153	37·3232
15	0·4258	0·9172	70	16·577	39·3076
16	0·4778	1·0330	71	17·482	41·3807
17	0·5208	1·1553	72	18·433	43·5465
18	0·5730	1·2846	73	19·433	45·8042
19	0·6283	1·4212	74	20·485	48·1589
20	0·6872	1·5655	75	21·587	50·6096
21	0·7497	1·7180	76	22·746	53·1593
22	0·8159	1·8791	77	23·965	55·8095
23	0·8863	2·0494	78	25·260	58·3968
24	0·9610	2·2293	79	26·588	61·3057
25	1·0402	2·4194	80	28·006	64·3524
26	1·1239	2·6202	81	29·455	67·4095
27	1·2127	2·8325	82	30·980	70·4967
28	1·3068	3·0568	83	32·575	73·7647
29	1·4065	3·2937	84	34·251	77·0764
30	1·5019	3·5441	85	35·984	80·4708
31	1·6333	3·8087	86	37·800	83·9351
32	1·7413	4·0883	87	39·697	87·4625
33	1·8671	4·3837	88	41·642	91·1366
34	1·9980	4·6958	89	43·730	94·6580
35	2·1374	5·0256	90	45·870	98·2764
36	2·2846	5·3741	91	48·092	
37	2·4401	5·6423	92	50·408	
38	2·6045	6·1315	93	52·785	
39	2·7780	6·5426	94	55·253	
40	2·9711	6·9770	95	57·801	
41	3·1544	7·4360	96	60·423	
42	3·3583	7·9211	97	63·108	
43	3·5735	8·4336	98	65·877	
44	3·8005	8·9751	99	68·692	
45	4·0399	9·5476	100	71·552	
46	4·2922	10·1516	101	74·444	
47	4·5582	10·7906	102	77·359	
48	4·8386	11·4606	103	80·268	
49	5·1346	12·1800	104	83·259	
50	5·4453	12·9340	105	85·992	
51	5·7706	13·7300	106	88·735	
52	6·1194	14·5720	107	91·367	
53	6·4834	15·4610	108	93·815	
54	6·8667	16·4000	109	96·039	
55	7·2798	17·3930	110	98·356	

M. Betancourt deduces ſeveral uſeful and ingenious conſequences and applications from this courſe of experiments. He ſhews, for inſtance, that the effect of Steam engines muſt, in general, be greater in winter than in ſummer; owing to the different degrees of temperature in the water of injection. And from the very ſuperior ſtrength of the vapour of ſpirit of wine, over that of water, he argues that, by trying other fluids, ſome may be found, not very expenſive, whoſe vapour may be ſo much ſtronger than that of water, with the ſame degree of heat, that it may be ſubſtituted inſtead of water in the boilers of Steam-engines, to the great ſaving in the very heavy expence of fuel: nay, he even declares, that ſpirit of wine itſelf might thus be employed in a machine of a particular conſtruction, which, with the ſame quantity of fuel, and without any increaſe of expence in other things, ſhall produce an effect greatly ſuperior to what is obtained from the ſteam of water. He makes ſeveral other obſervations on the working and improvement of Steam-engines.

Another uſe of theſe experiments, deduced by M. Betancourt, is, to meaſure the height of mountains, by means of a thermometer, immerſed in boiling water, which he thinks may be done with a preciſion equal, if not ſuperior, to that of the barometer. As ſoon as I had obtained exact reſults of my experiments, ſays he, and was convinced that the degree of heat received by water depends abſolutely on the preſſure upon its ſurface, I endeavoured to compare my obſervations with ſuch as have been made on mountains of different heights, to know what is the degree of heat which water can receive when the barometer ſtands at a determinate height; but from ſo few obſervations having been made of this kind, and the different ways employed in graduating inſtruments, it is difficult to draw any certain conſequences from them.

The firſt obſervation which M. Betancourt compared with his experiments, is one mentioned in the Memoirs of the Academy of Sciences, anno 1740, page 92. It is there ſaid, that M. Monnier having made water boil upon the mountain of Canigou, where the barometer ſtood at 20·18 inches, the thermometer immerſed in this water ſtood at a point anſwering to 71 degrees of Reaumur: whereas in M. Betancourt's table of experiments, at an equal preſſure upon the ſurface of the water, the thermometer ſtood at 73·7 degrees. This difference he thinks is owing partly to the want of preciſion in the obſervation, and partly to the different method of graduating the thermometer, and the neglect of purging the barometer tube of air.

M. Betancourt next compared his experiments with ſome obſervations made by M. De Luc on the tops of ſeveral mountains; in which, after reducing the ſcales of this gentleman to the ſame meaſures as his own, he finds a very near degree of coincidence indeed. The following table contains a ſpecimen of theſe compariſons, the inſtances being taken at random from De Luc's treatiſe on the Modifications of the Atmoſphere.

Degrees of Heat in Boiling Water upon the Tops of Mountains, obſerved by De Luc.				Heat of the Water in M. Betancourt's Experim.
Places of Obſervation.	Heat of the air.	Height of the Bar.	Heat of the Wa. by Th.	
Beaucaire -	$14\frac{1}{4}$	28·248	80·37	80·29
Geneva - -	$12\frac{1}{2}$	27·056	79·33	79·33
Grange Town	$16\frac{1}{4}$	24·510	77·11	77·42
Lans le Bourg		24·145	77·18	77·14
Grange le F.	15	24·089	76·76	77·09
Grenairon	$10\frac{1}{4}$	20·427	73·26	73·89
Glaciere de B.	$6\frac{1}{2}$	19·677	72·56	73·24

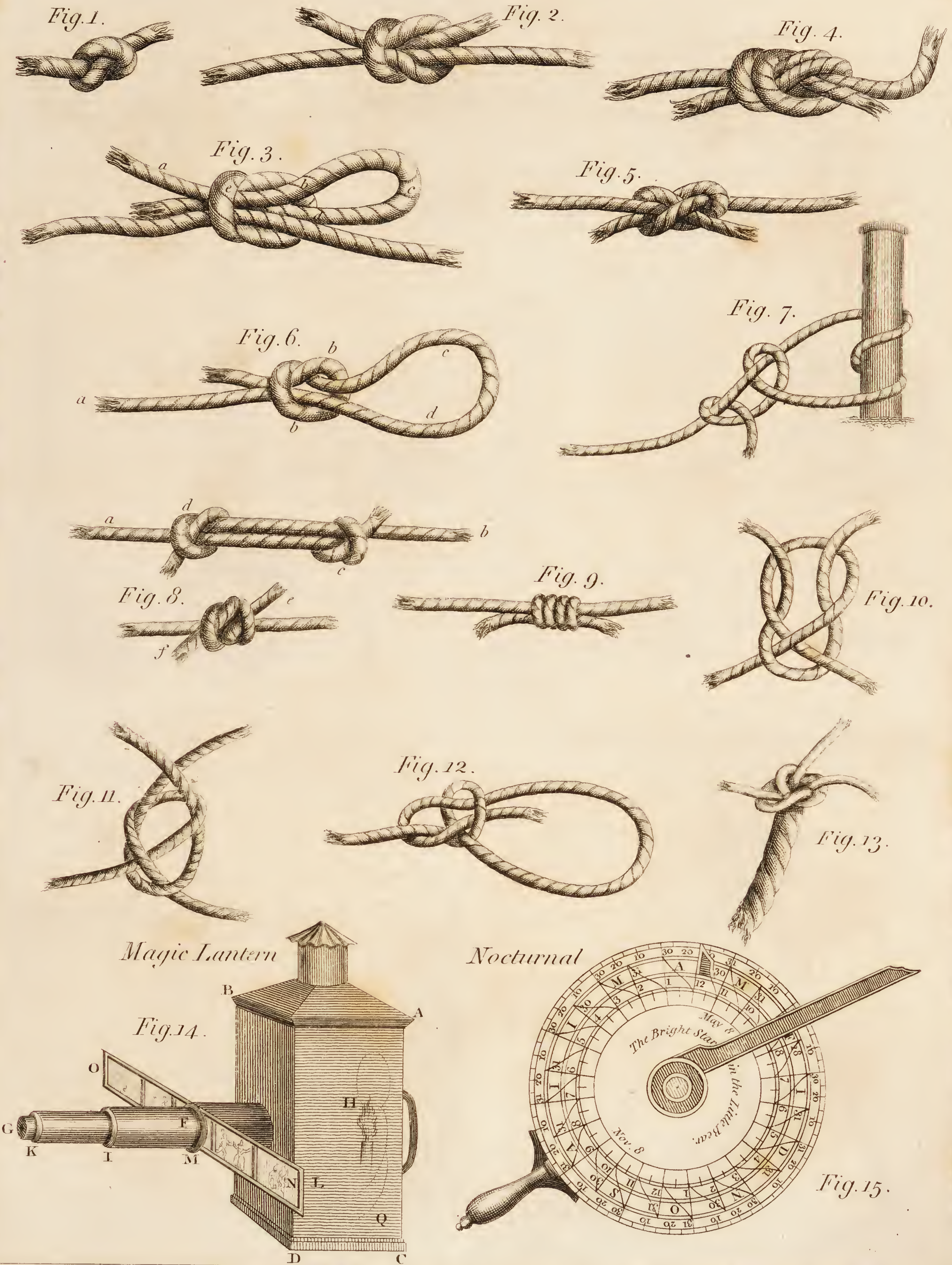
Where it is remarkable, that the difference between the two is of no conſequence in ſuch matters.

Many other advantages might be deduced from the exact knowledge of the effect which the preſſure of the atmoſphere has upon the heat which water can receive: one of which, M. Betancourt obſerves, is of too great importance in phyſics not to be mentioned. As ſoon as the thermometer became known to philoſophers, almoſt every one endeavoured to find out two fixed points to direct them in dividing the ſcale of the inſtrument; having found that thoſe of the freezing and boiling of water were nearly conſtant in different places, they gave theſe the preference over all others: but having diſcovered that water is capable of receiving a greater or leſs quantity of heat, according to the preſſure of the atmoſphere upon its ſurface, they felt the neceſſity of fixing a certain conſtant value to that preſſure, which it was almoſt generally agreed ſhould be equal to a column of 28 French inches of mercury. This agreement however did not remove all the difficulties. For inſtance, if it were required to conſtruct at Madrid a thermometer that might be comparable with another made at Paris, the thing would be found impoſſible by the means hitherto known, becauſe the barometer never riſes ſo high as 27 inches at Madrid; and it was not certainly known how much the ſcale of the thermometer ought to be increaſed to have the point of boiling water in a place where the barometer is at 28 inches. But by making uſe of the foregoing obſervations, the thing appears very eaſy, and it is to be hoped that by the general knowledge of them, thermometers may be brought to great perfection, the accurate uſe of which is of the greateſt importance in phyſics.

Beſides, without being confined to the height of the barometer in the open air, in a given place, we may regulate a thermometer according to any one aſſigned heat of water, by means of ſuch an apparatus as M. Betancourt's. For, in order to graduate a thermometer, having a barometer ready divided; it is evident that by knowing, from the foregoing table of experiments, the degree of heat anſwering to any one expanſive force, we can thence aſſign the degree of the thermometer correſponding to a certain height of the barometer. A determination admitting of great preciſion, eſpecially in the higher temperatures, where the motion of the barometer is ſo conſiderable in reſpect to that of the thermometer.

F I N I S.

KNOTS of different kinds.



LEVELS.

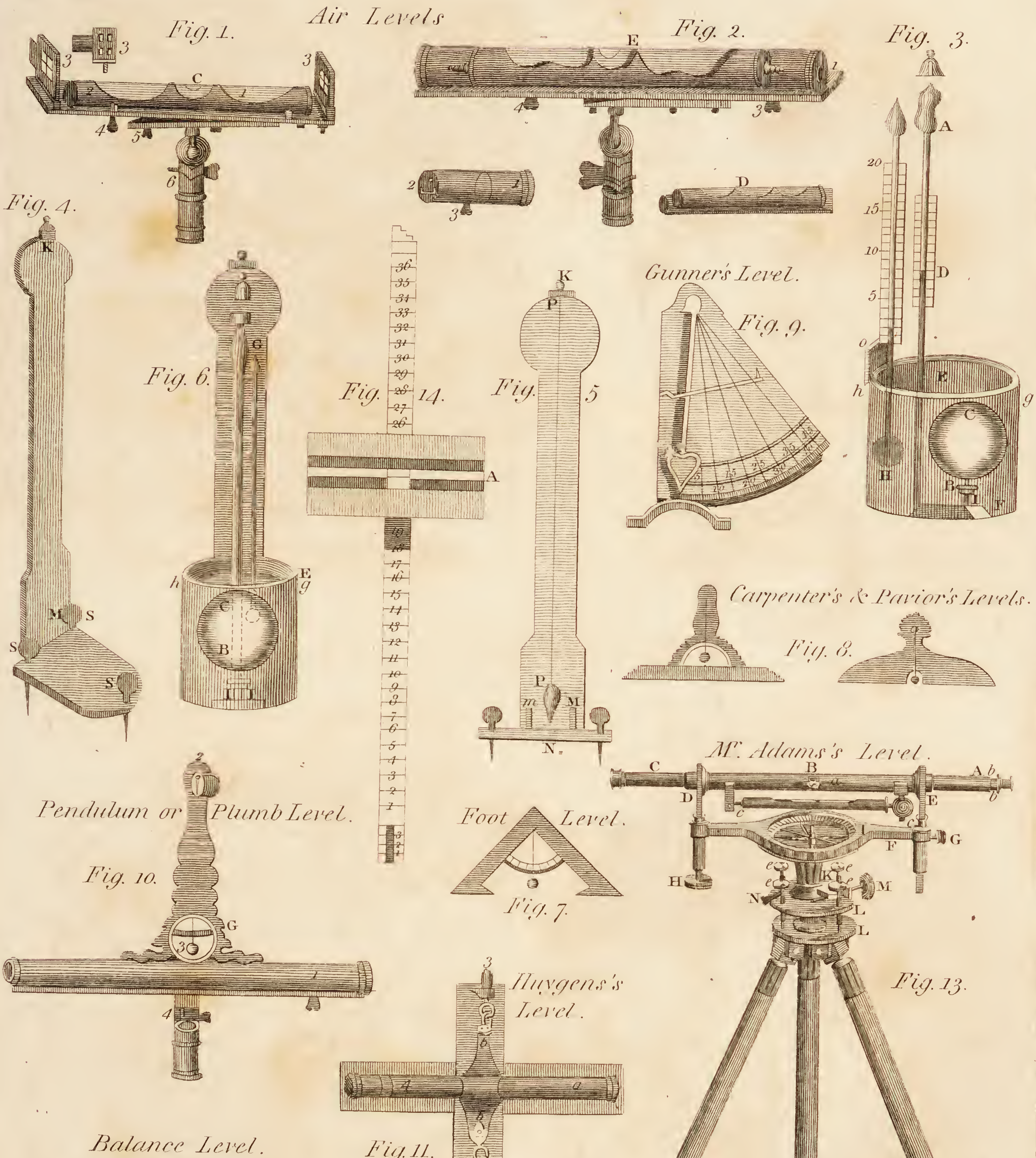
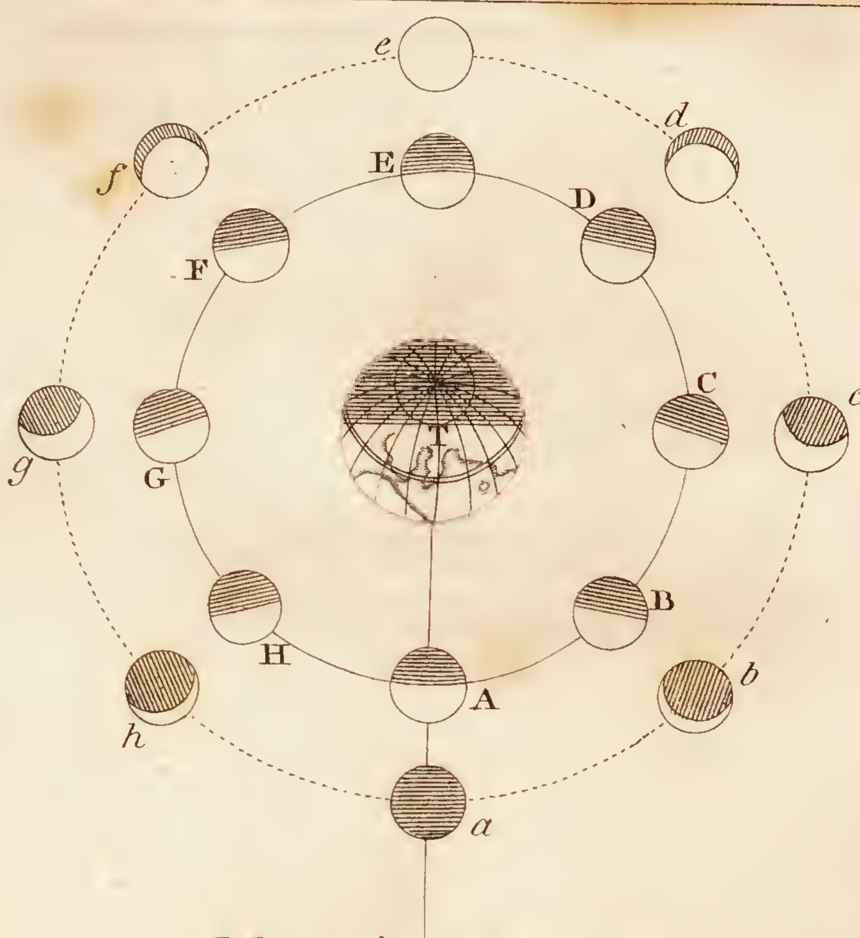

Air Levels
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 6.
Fig. 14.
Fig. 5
Gunner's Level.
Fig. 9.
Carpenter's & Pavior's Levels.
Fig. 8.
Mr. Adams's Level.
Pendulum or Plumb Level.
Foot Level.
Fig. 7.
Fig. 10.
Huygens's Level.
Fig. 13.
Balance Level.
Fig. 11.
Fig. 12.

Fig. 1.

MAGIC Square of Squares.

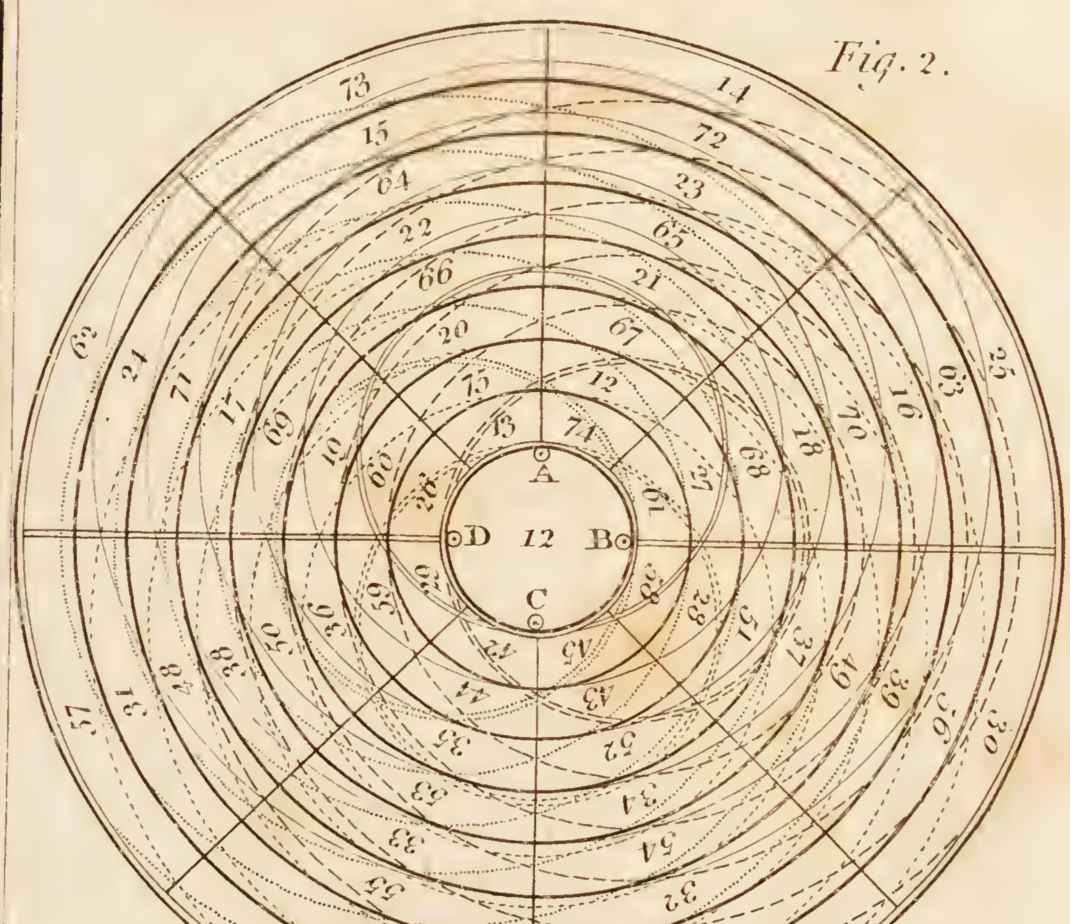
200	217	232	249	8	25	40	57	72	89	104	121	136	153	168	181
58	39	26	7	250	231	218	199	186	167	154	135	122	103	90	71
198	219	230	251	6	27	38	59	70	91	102	123	134	155	166	187
60	37	28	5	252	229	220	197	188	165	156	133	124	101	92	69
201	216	233	248	9	24	41	56	73	88	105	120	137	152	169	184
55	42	23	10	247	234	215	202	183	170	151	138	119	106	87	74
203	214	235	246	11	22	43	54	75	86	107	118	139	150	171	182
53	44	21	12	245	236	213	204	181	172	149	140	117	108	85	76
205	212	237	244	13	20	45	52	77	84	109	116	141	148	173	180
51	46	19	14	243	238	211	206	179	174	147	142	115	110	83	78
207	210	239	242	15	18	47	50	79	82	111	114	143	146	175	178
49	48	17	16	241	240	209	208	177	176	145	144	113	112	81	80
196	221	228	253	4	29	36	61	68	93	100	125	132	157	164	189
62	35	30	3	254	227	222	195	190	163	158	131	126	99	94	67
194	223	226	255	2	31	34	63	66	95	98	127	130	159	162	191
64	33	32	1	256	225	224	193	192	161	160	129	128	97	96	65



MOON'S Phases.
Fig. 3.

MAGIC Circle of Circles.

Fig. 2.



Face of the MOON.

Fig. 4.

North

South

Artificial Magnets.

Fig. 1.

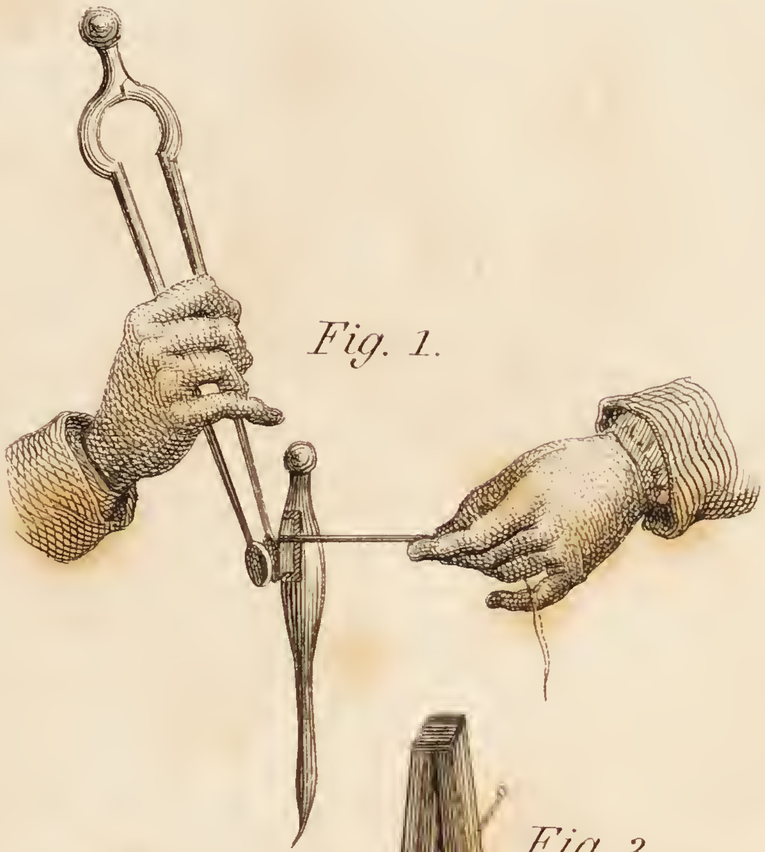


Fig. 2.

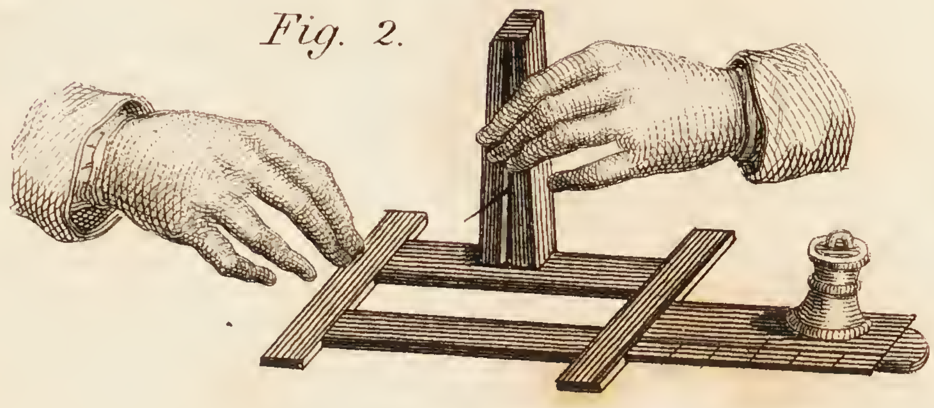


Fig. 3.

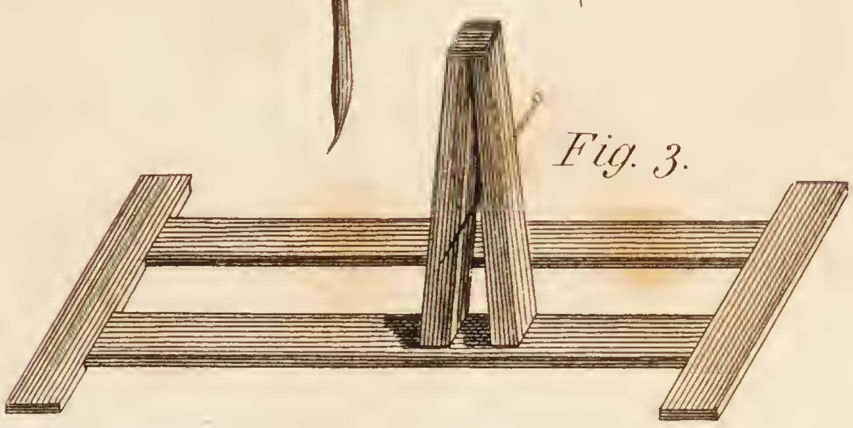


Fig. 4.

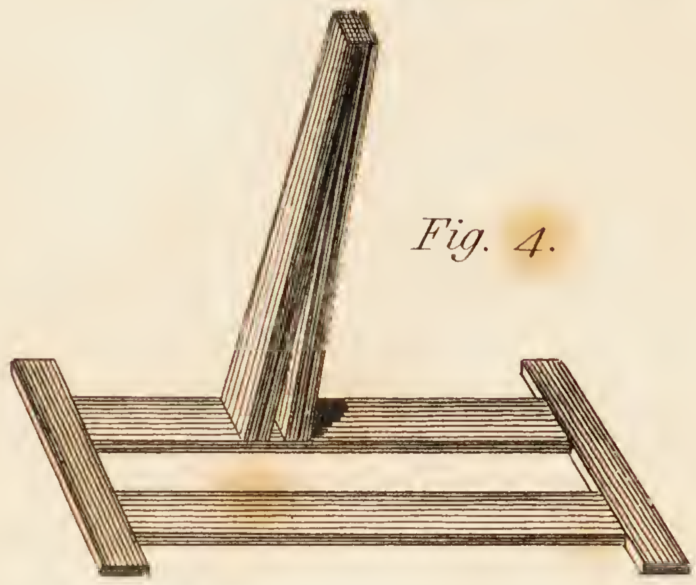


Fig. 5.

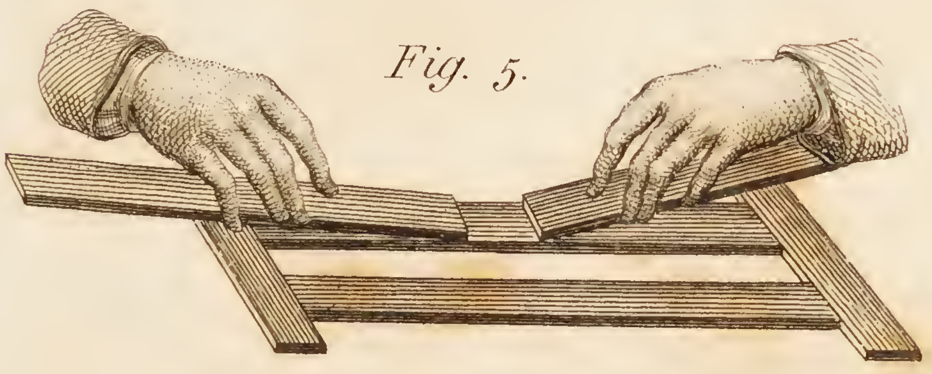
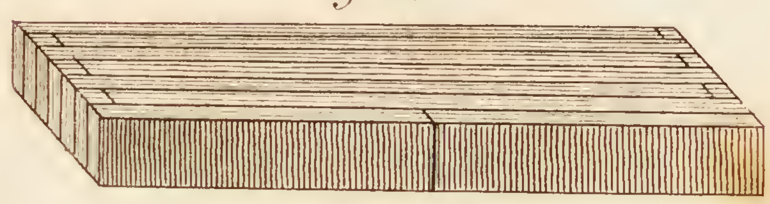


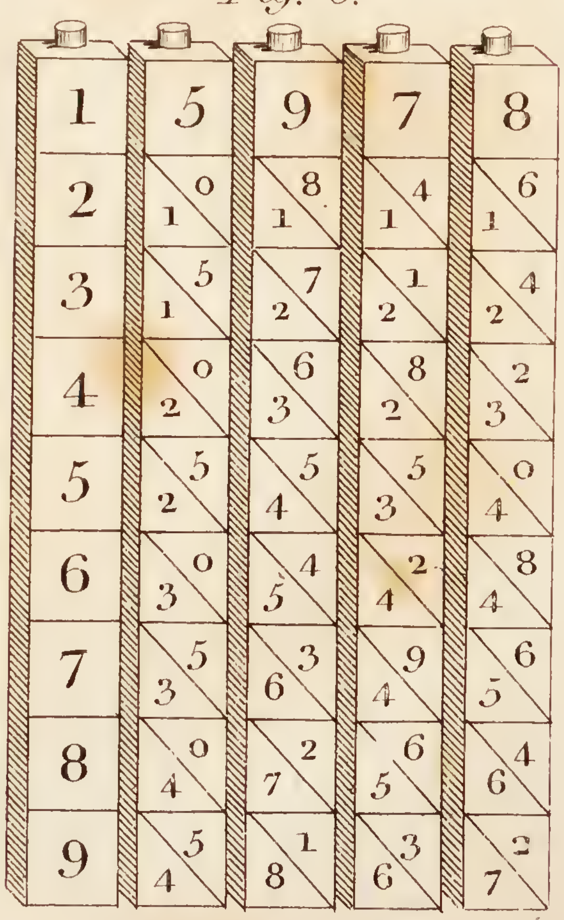
Fig. 6.



Neper's Bones.

Fig. 7.

Fig. 8.



GEOGRAPHICAL MAPS.

Fig. 1.

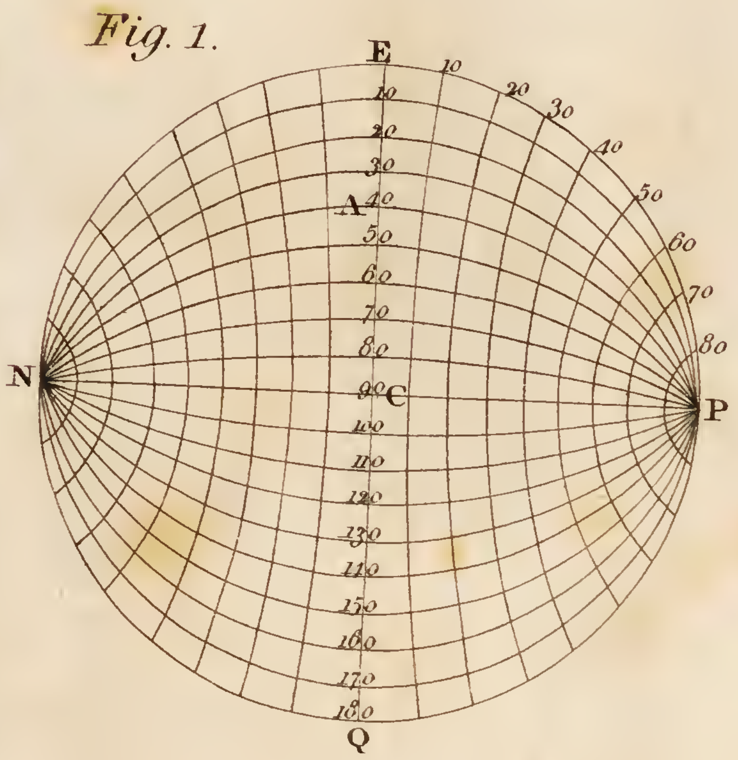


Fig. 2.

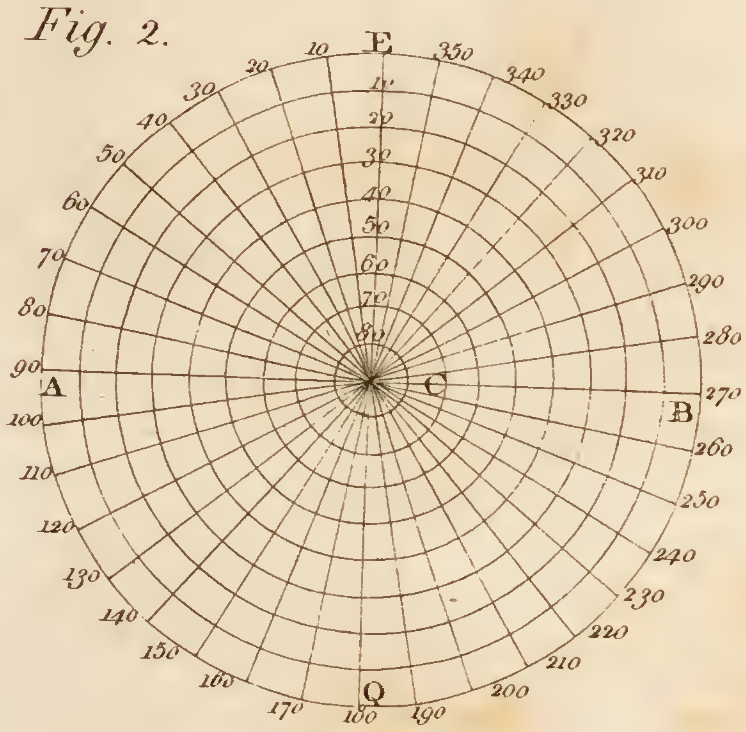


Fig. 3.

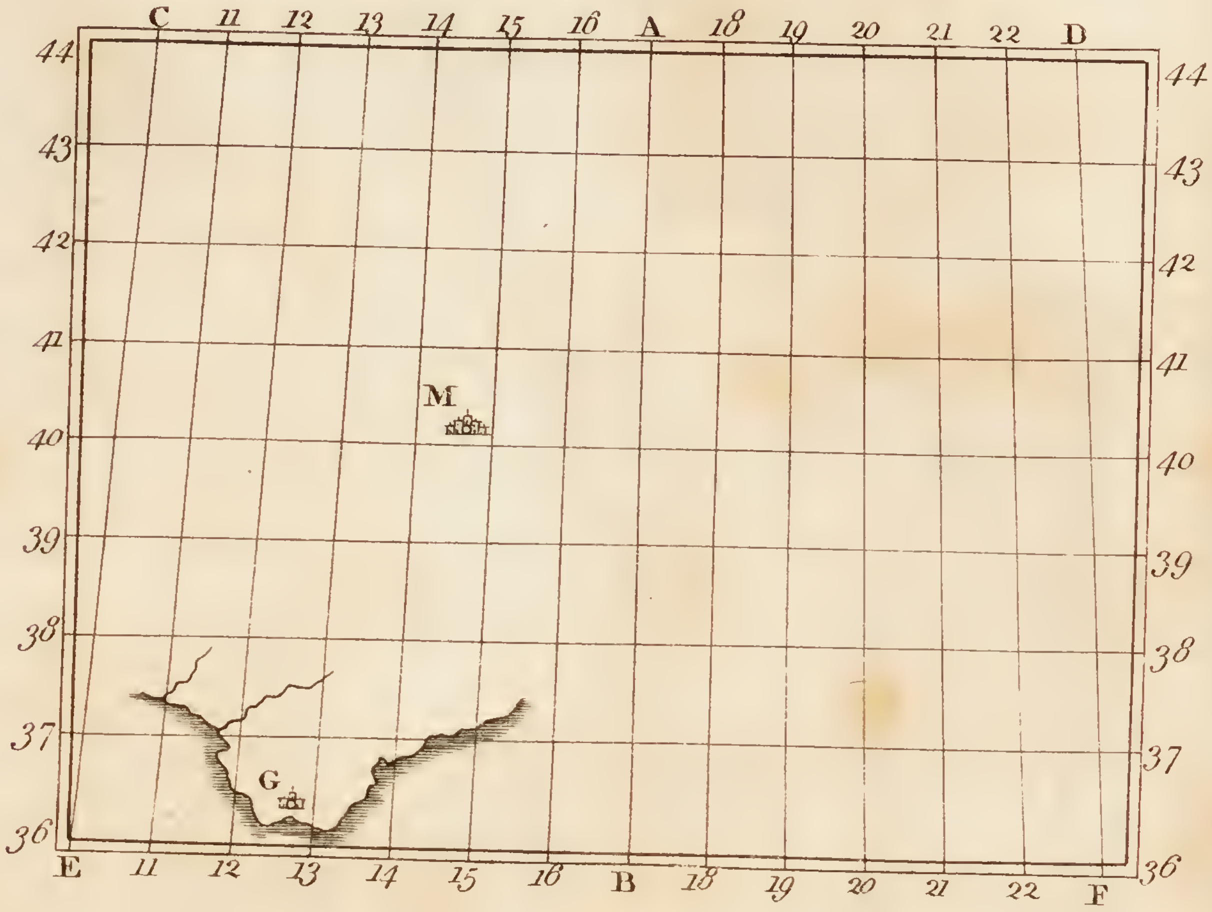
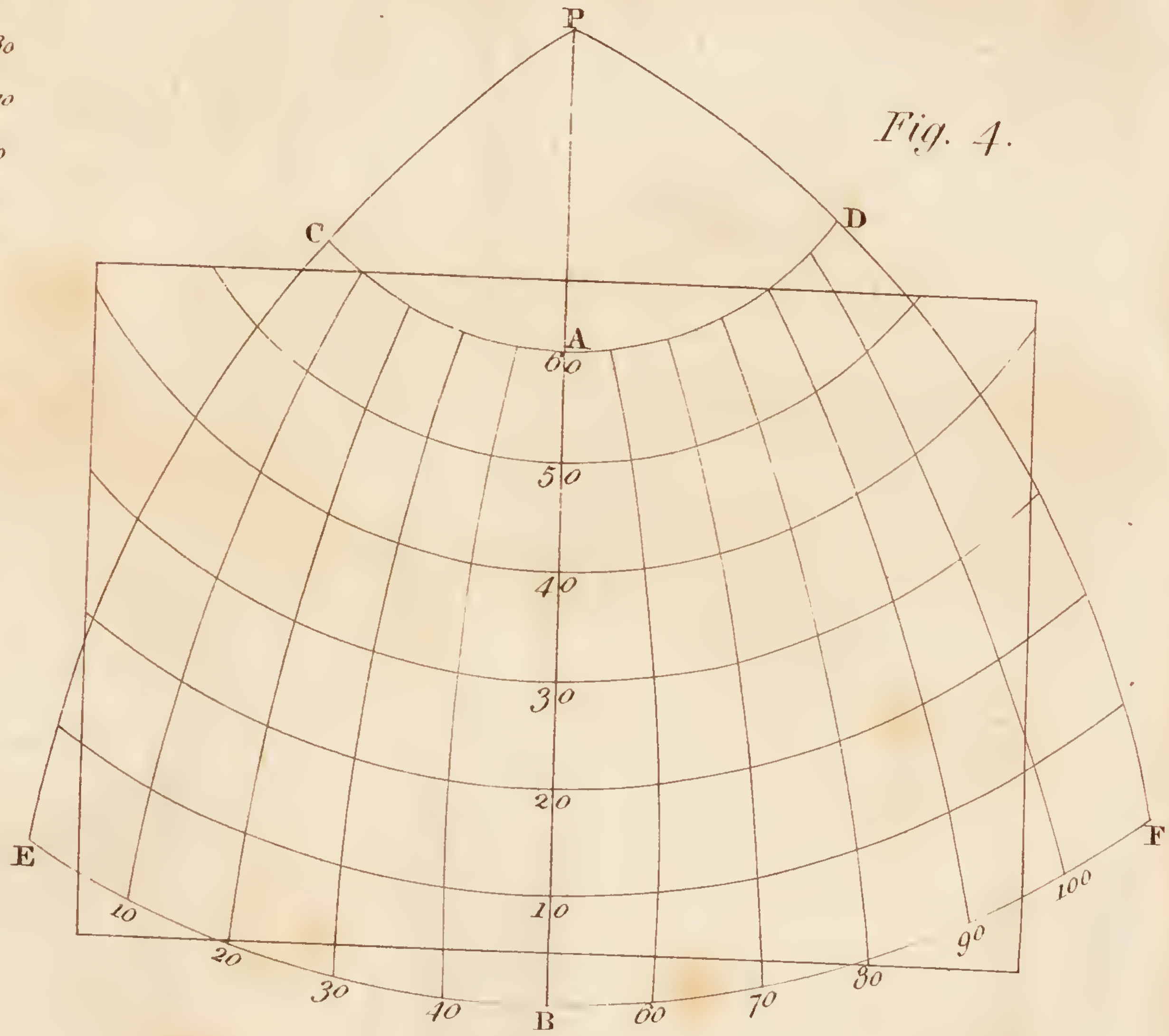
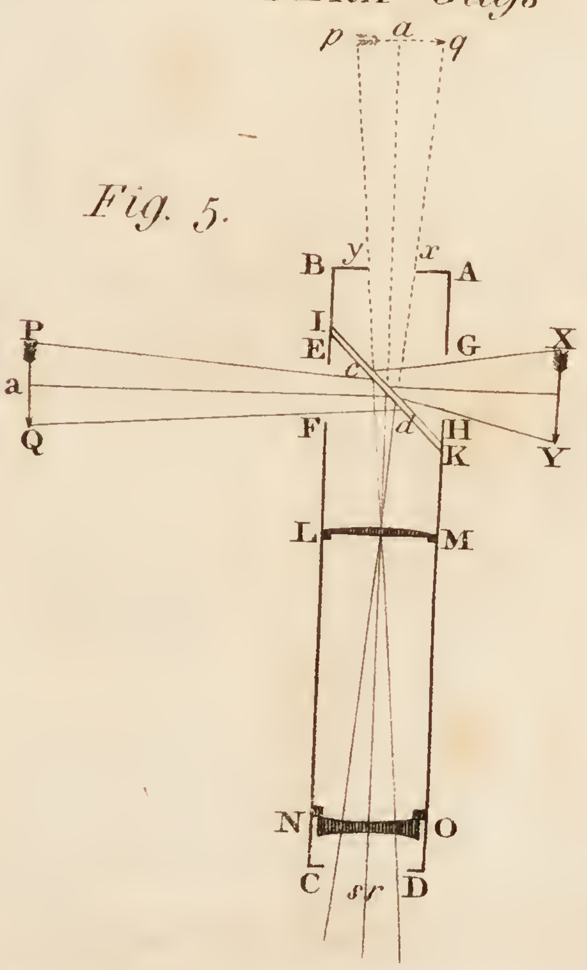


Fig. 4.



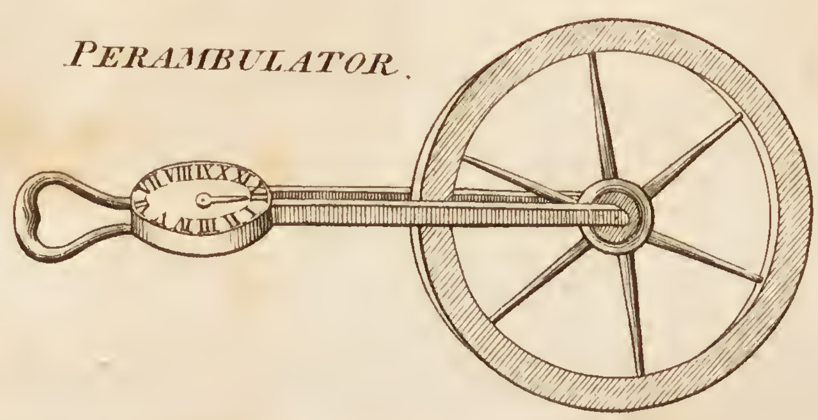
OPERA Glass

Fig. 5.

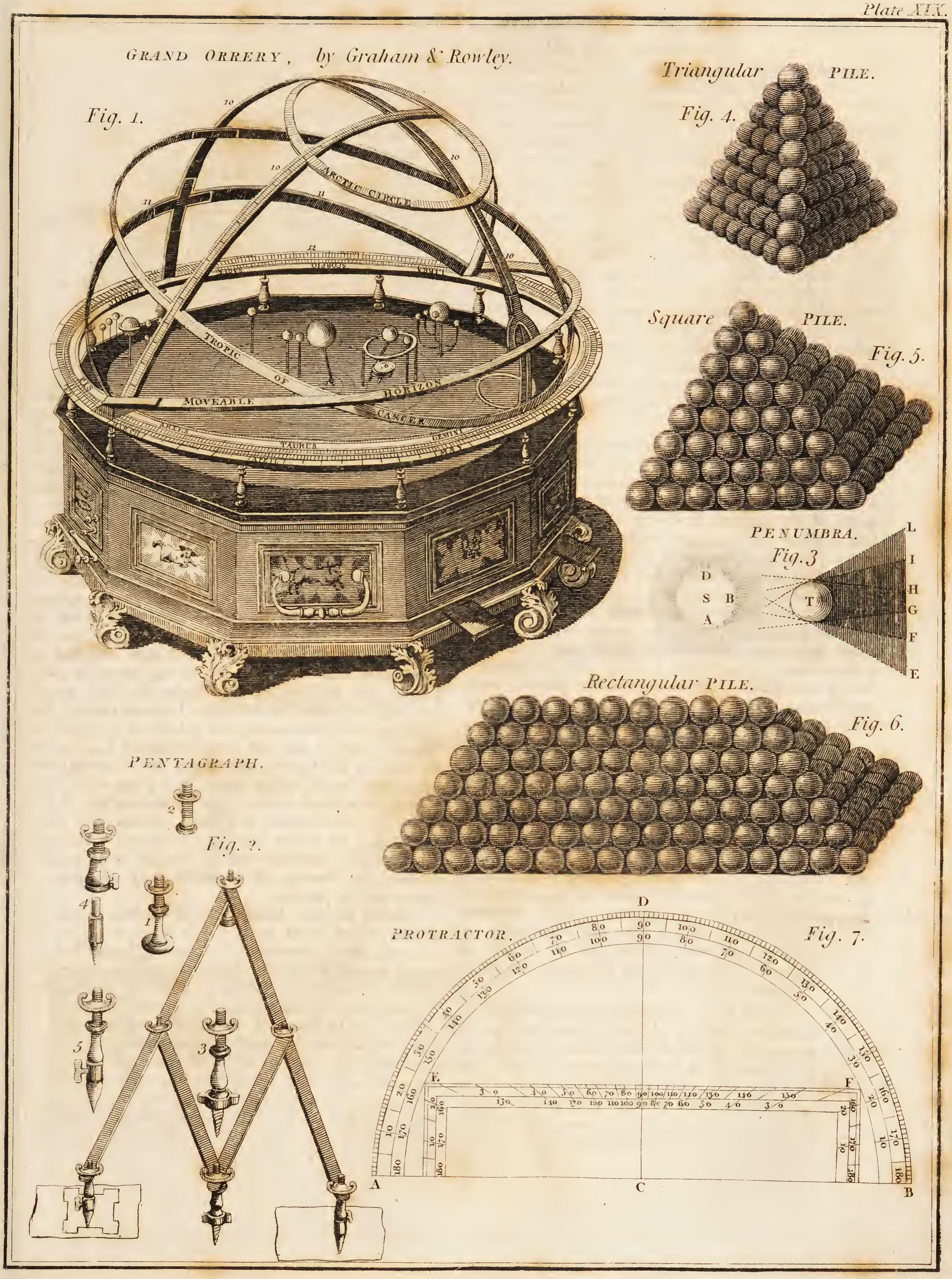


PERAMBULATOR.

Fig. 6.



MICROSCOPES.


GRAND ORRERY, by Graham & Rowley.
Fig. 1.
ARCTIC CIRCLE
TROPIC OF CANCER
MOVEABLE HORIZON
ARIES
TAURUS
GEMINI
Triangular PILE.
Fig. 4.
Square PILE.
Fig. 5.
PENUMBRA.
Fig. 3
D S B A T
L I H G F E
Rectangular PILE.
Fig. 6.
PENTAGRAPH.
Fig. 2.
1 2 3 4 5
PROTRACTOR.
Fig. 7.
D
A C B
E F

PERSIAN Wheel.

Fig. 1.

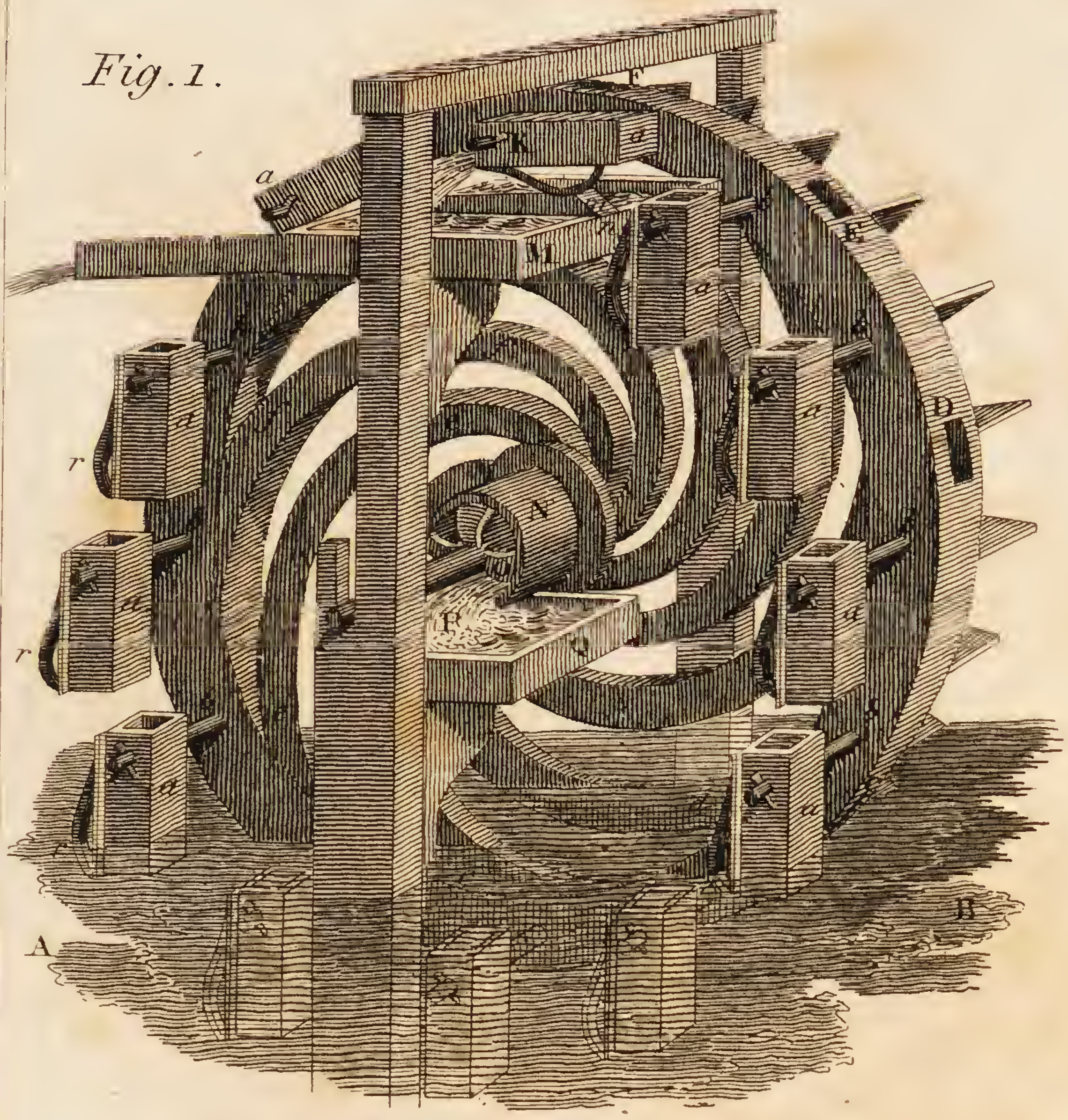
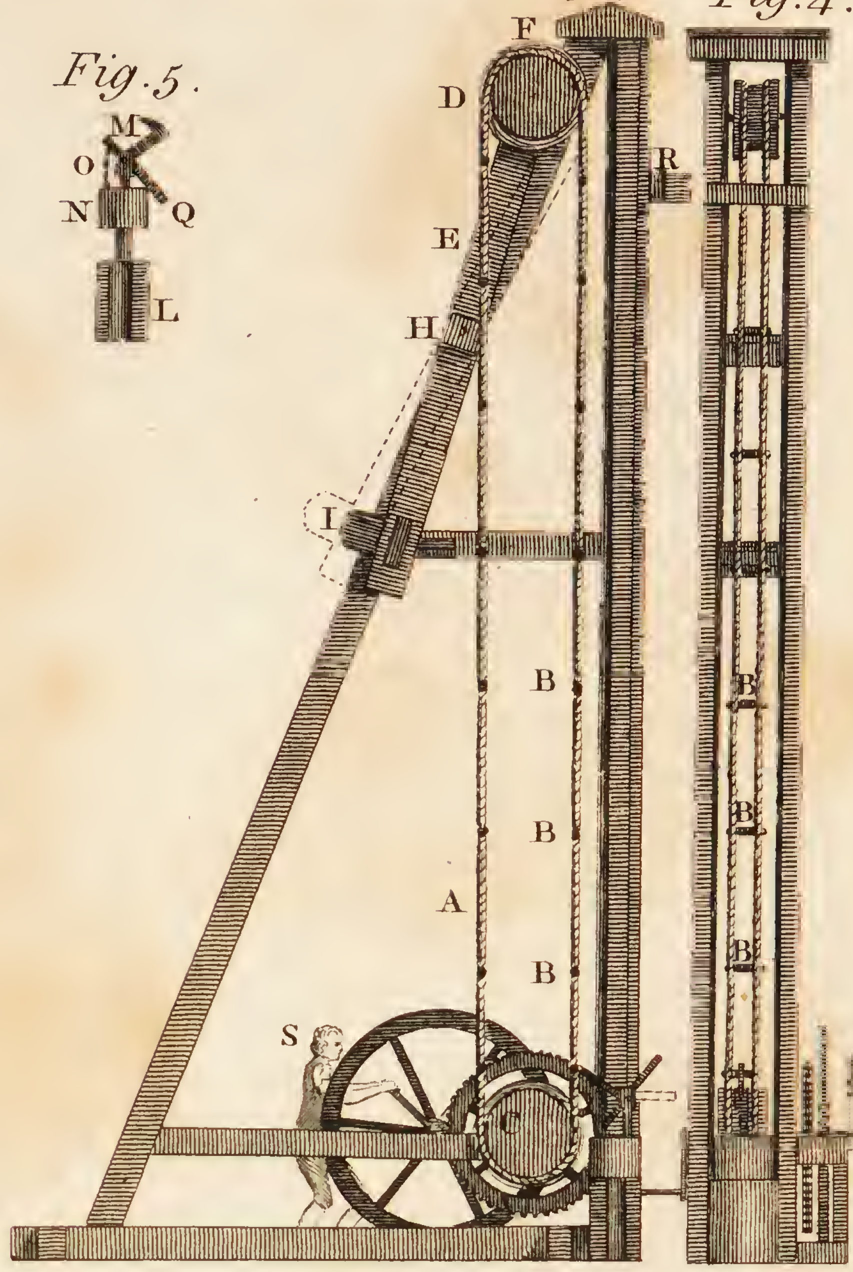


Fig. 5.

Bunce's PILE Engine.

Fig. 3.
Fig. 4.



Vauloue's PILE Engine.

Fig. 2.

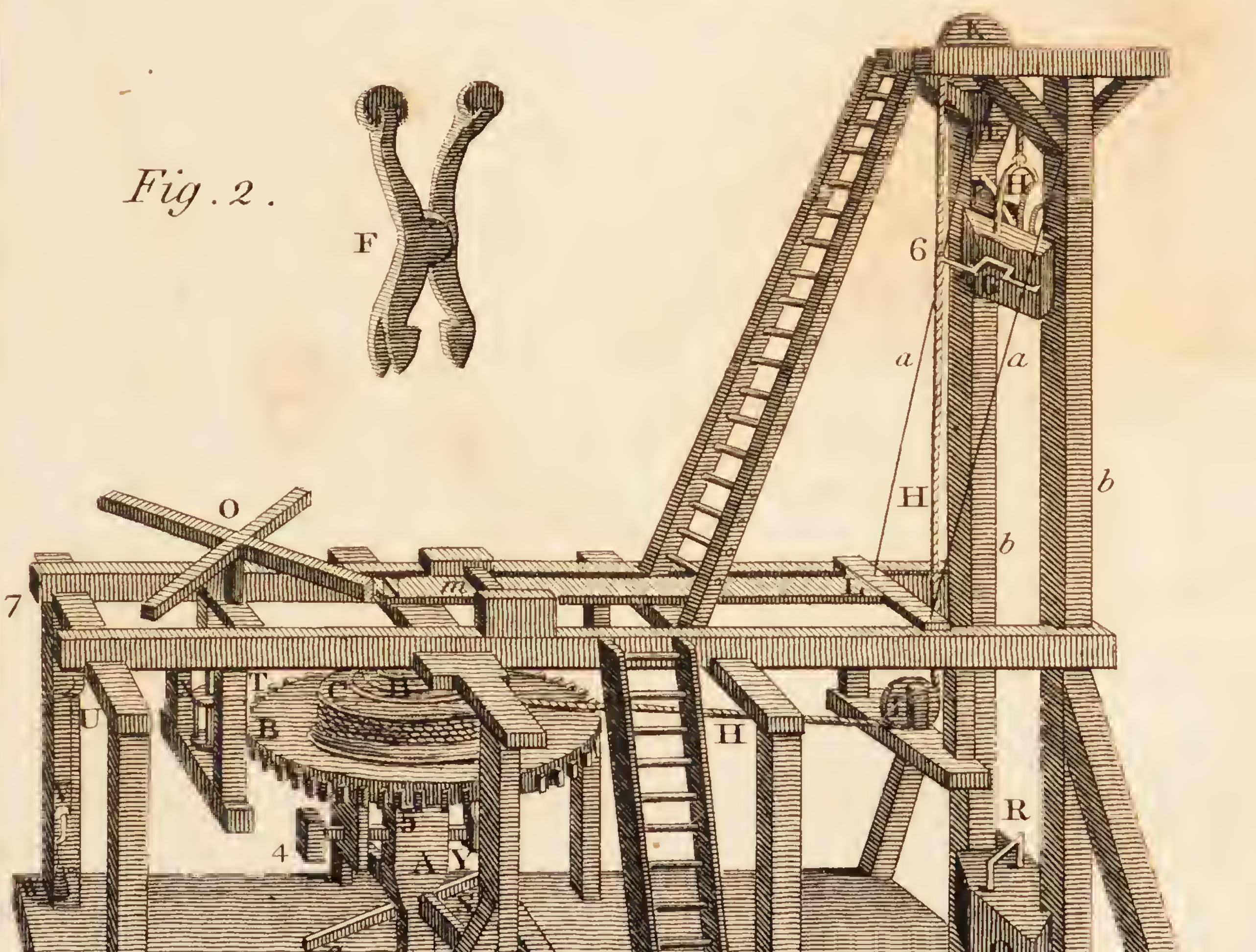
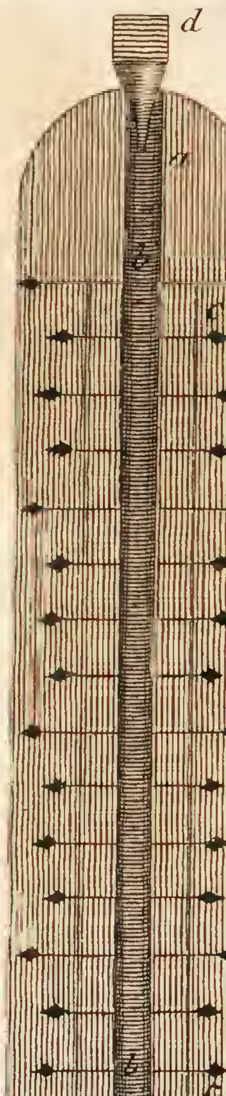


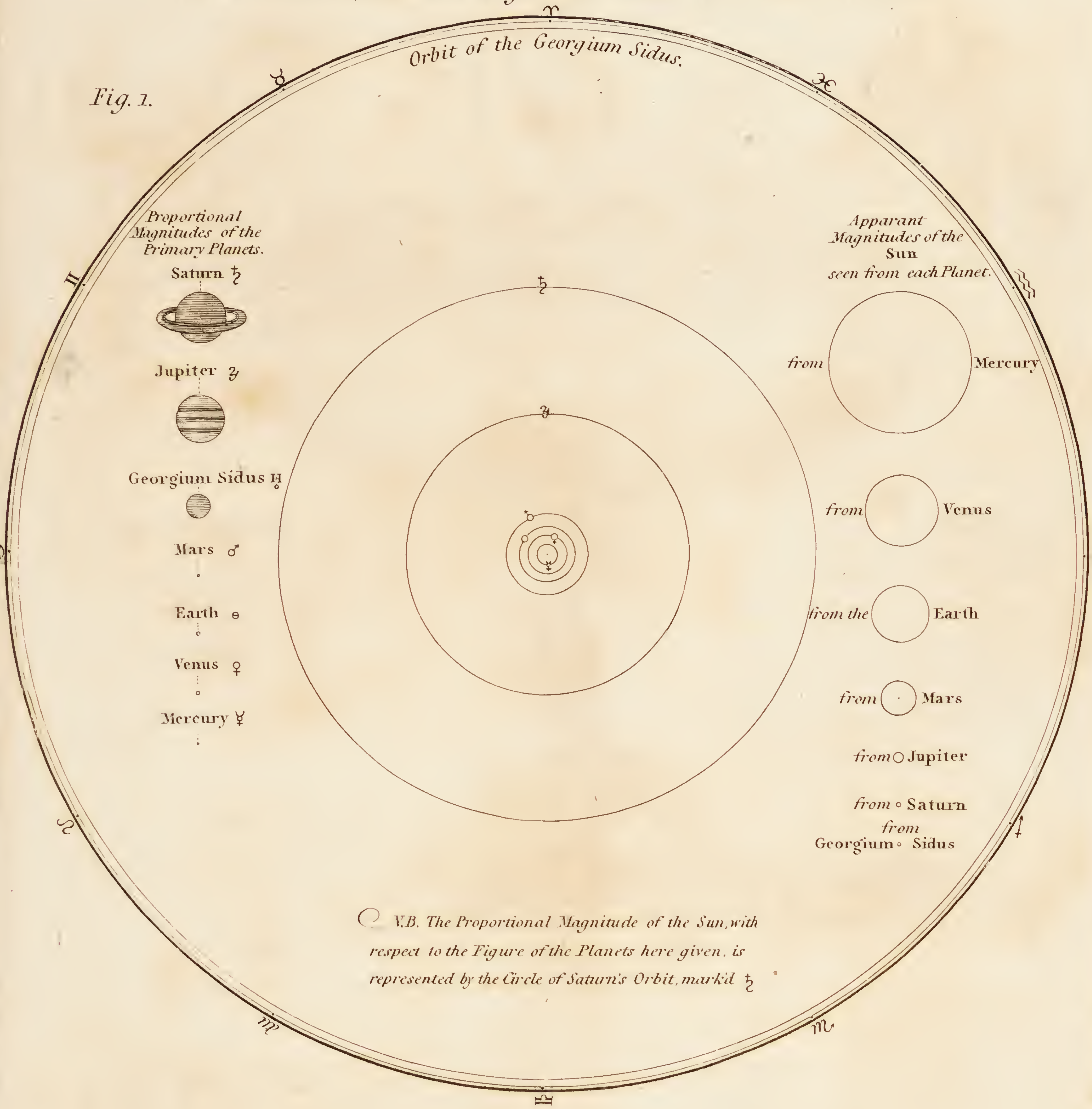
Fig. 6.



PLUVIAMETER

View of the proportional Magnitudes of the PLANETARY ORBITS.

Fig. 1.



PLANE SCALES. Fig. 2.

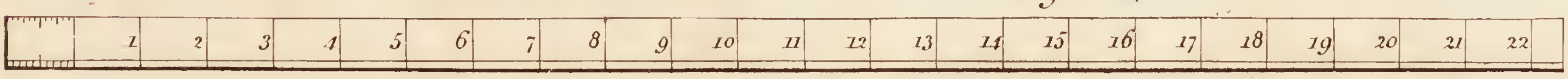
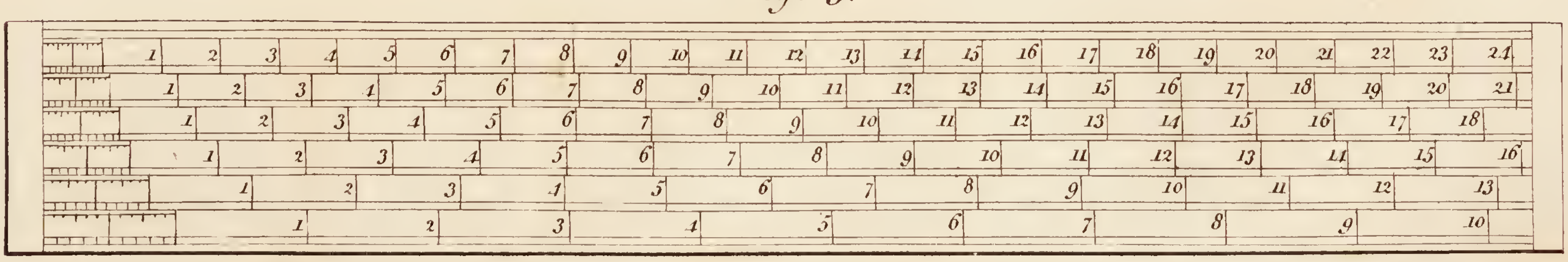
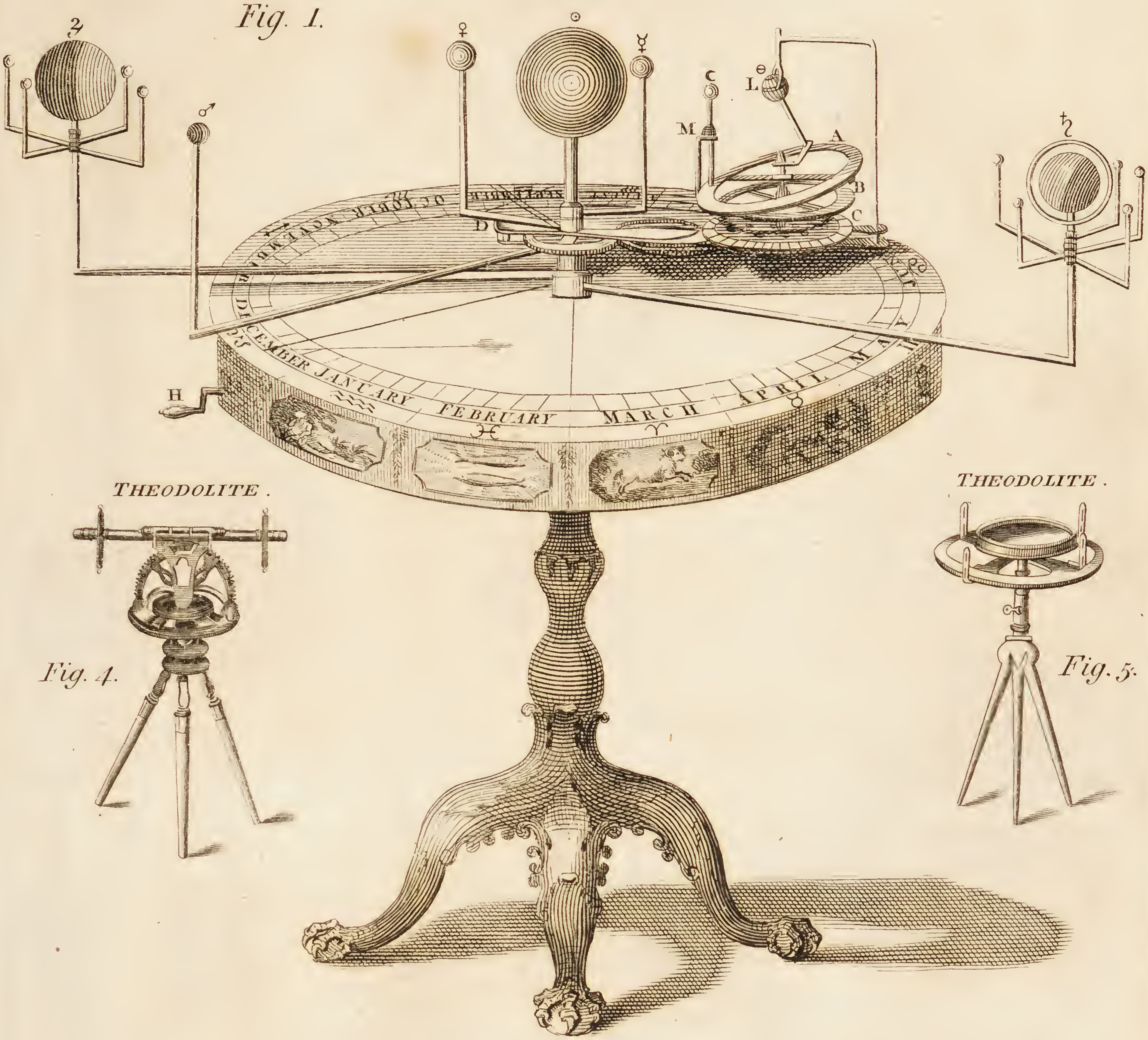


Fig. 3.



PLANETARIUM *by Jones.*

Fig. 1.



THEODOLITE.

Fig. 4.

THEODOLITE.

Fig. 5.

Fig. 2. PLANE DIAGONAL SCALES.

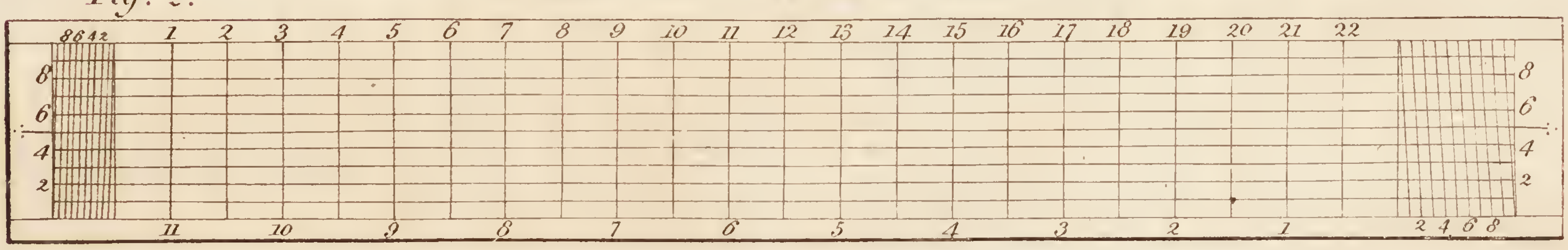
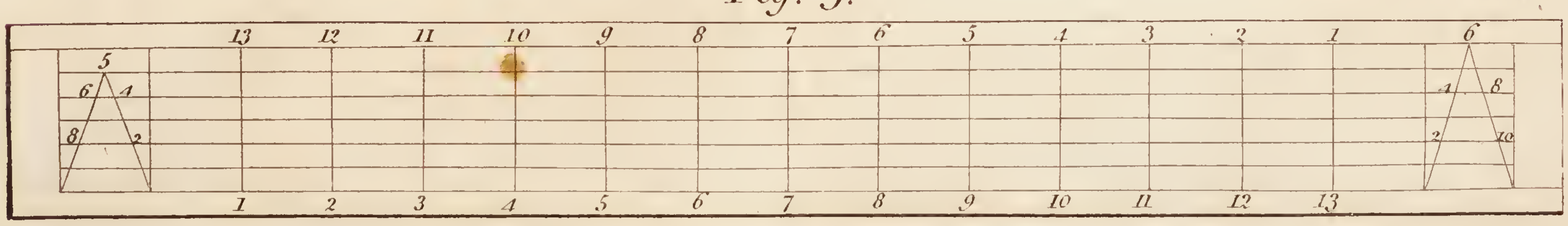
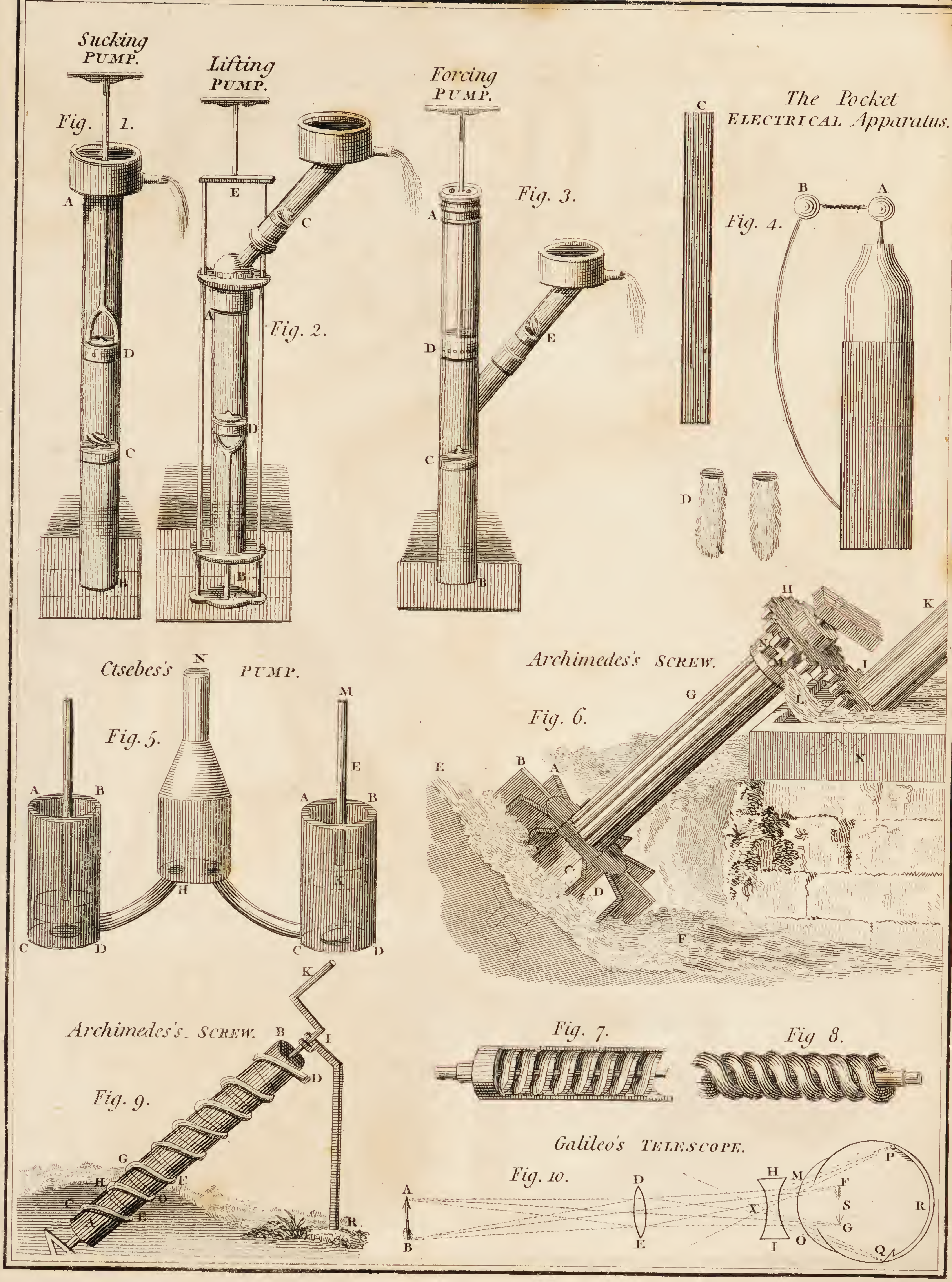
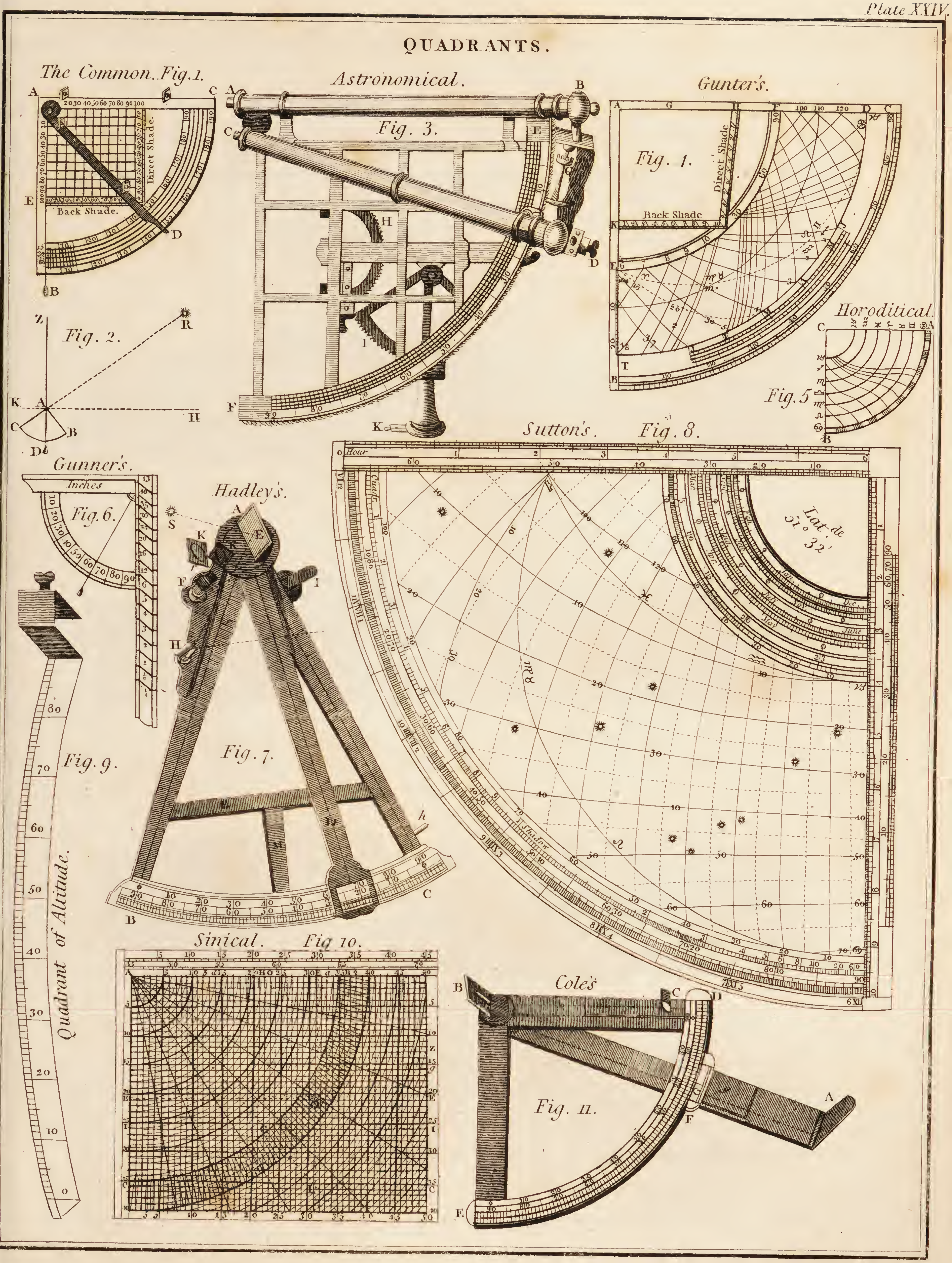


Fig. 3.




Sucking PUMP.
Fig. 1.
Lifting PUMP.
Fig. 2.
Forcing PUMP.
Fig. 3.
The Pocket ELECTRICAL Apparatus.
Fig. 4.
Ctsebes's PUMP.
Fig. 5.
Archimedes's SCREW.
Fig. 6.
Fig. 7.
Fig 8.
Archimedes's SCREW.
Fig. 9.
Galileo's TELESCOPE.
Fig. 10.


QUADRANTS.
The Common. Fig. 1.
Back Shade.
Direct Shade.
Fig. 2.
Astronomical.
Fig. 3.
Gunter's.
Fig. 1.
Back Shade
Direct Shade
Horoditical.
Fig. 5
Sutton's. Fig. 8.
Hour
Lat. de 51° 32′
Shadow
Gunner's.
Inches
Fig. 6.
Hadley's.
Fig. 7.
Fig. 9.
Quadrant of Altitude.
Sinical. Fig 10.
Cole's
Fig. 11.

Telescopic Appearances, of the Western Horizon, in three different States of the Atmosphere, taken from the Laurel Mount, at Traine, in Modbury Devonshire.

THE SCALE — Minutes of a Degree. 0 10 20 30 40 50 60

Ordinary or common Appearance.

Appearance somewhat elevated by Refraction.

Appearance when more considerably elevated.

a. Maker tower, about 12 3/4 miles distant, in a straight line. b. Gate place (at present appearing like an Arch) on a Hill about 3 3/4 miles. cc. Ground about 9 3/4 miles. d. A wood in Mount Edgcumbe park, about 12 1/4 miles. ee. A hill about 3 miles. f. Trees in the Park. g. A mow, seen by the Refraction on the Ground. cc. h. Another set of Trees in the Park.

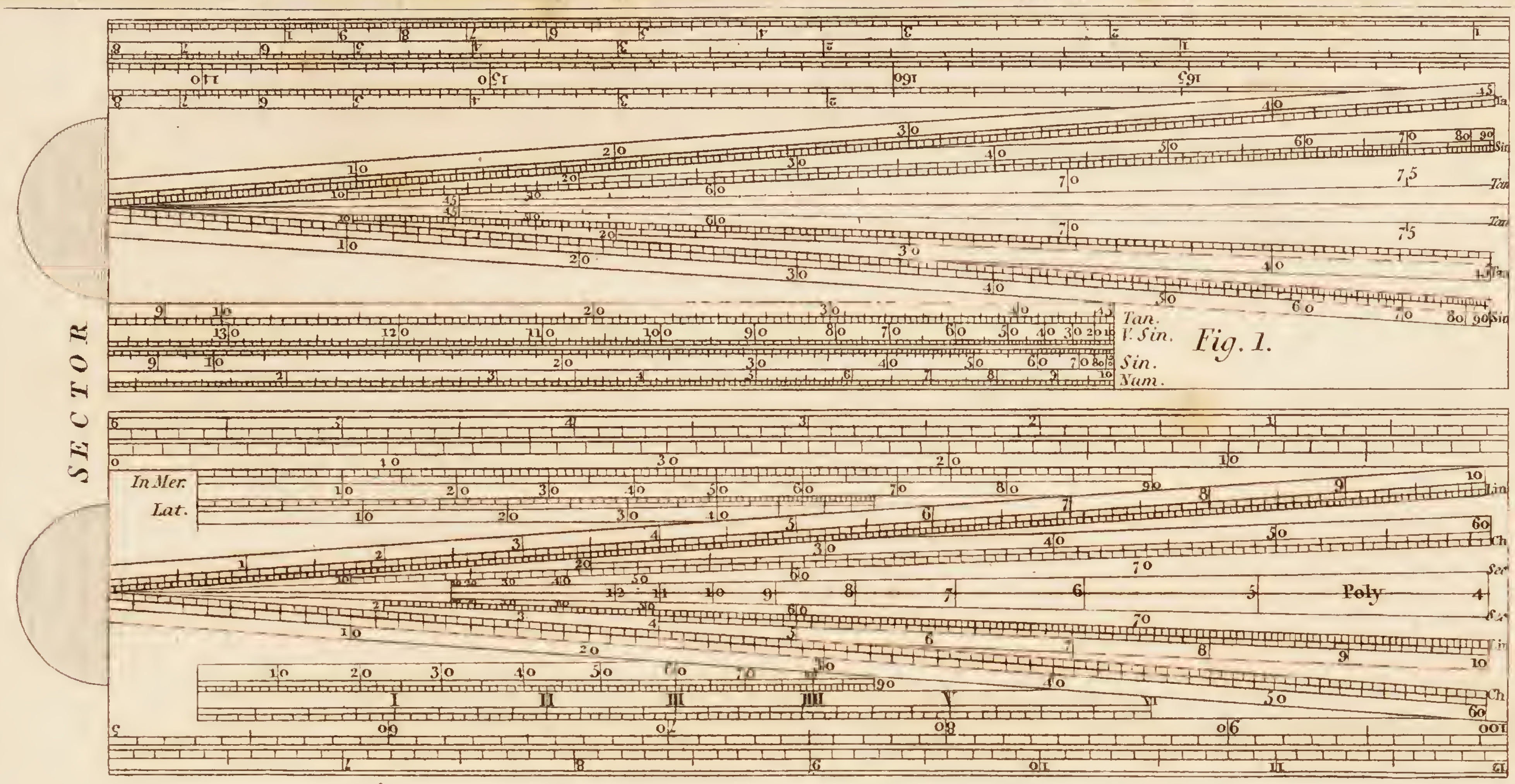


Fig. 1.

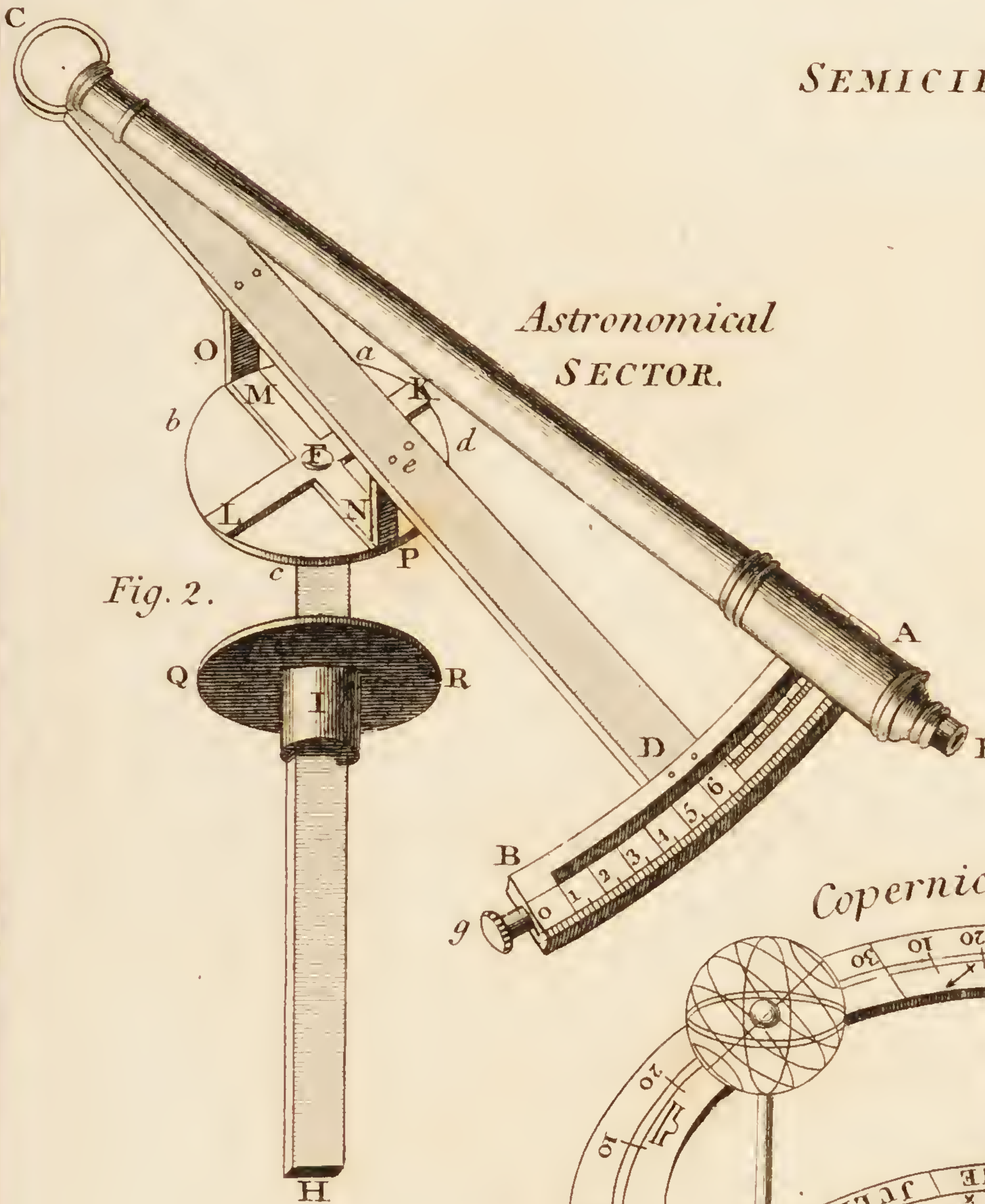


Fig. 2.

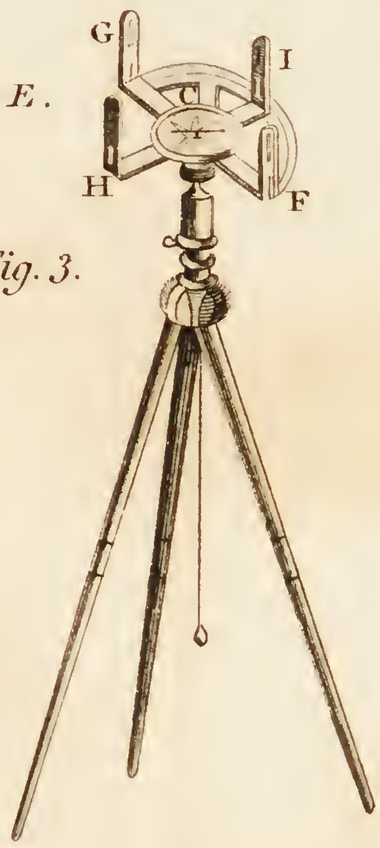


Fig. 3.

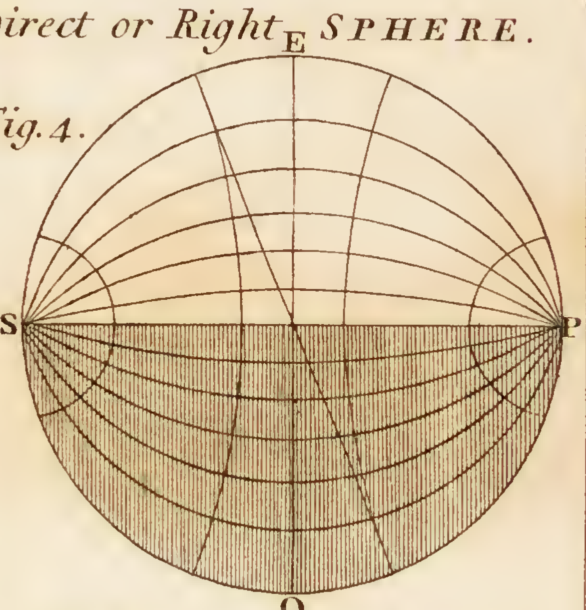


Fig. 4.

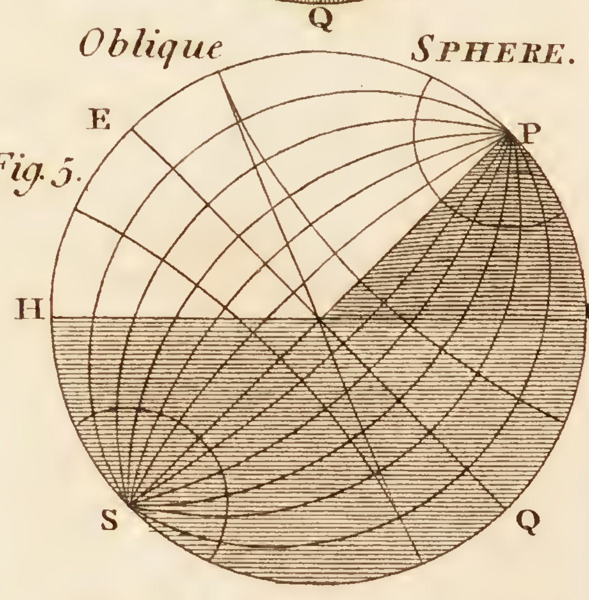


Fig. 5.

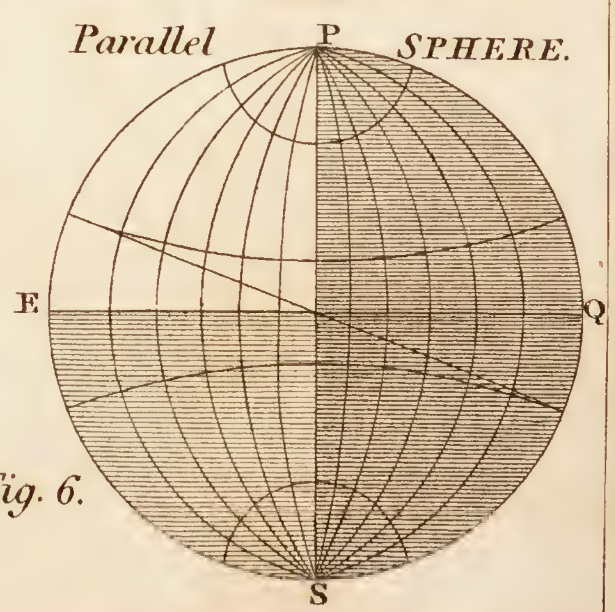


Fig. 6.

Fig. 7.

Water SPOUT

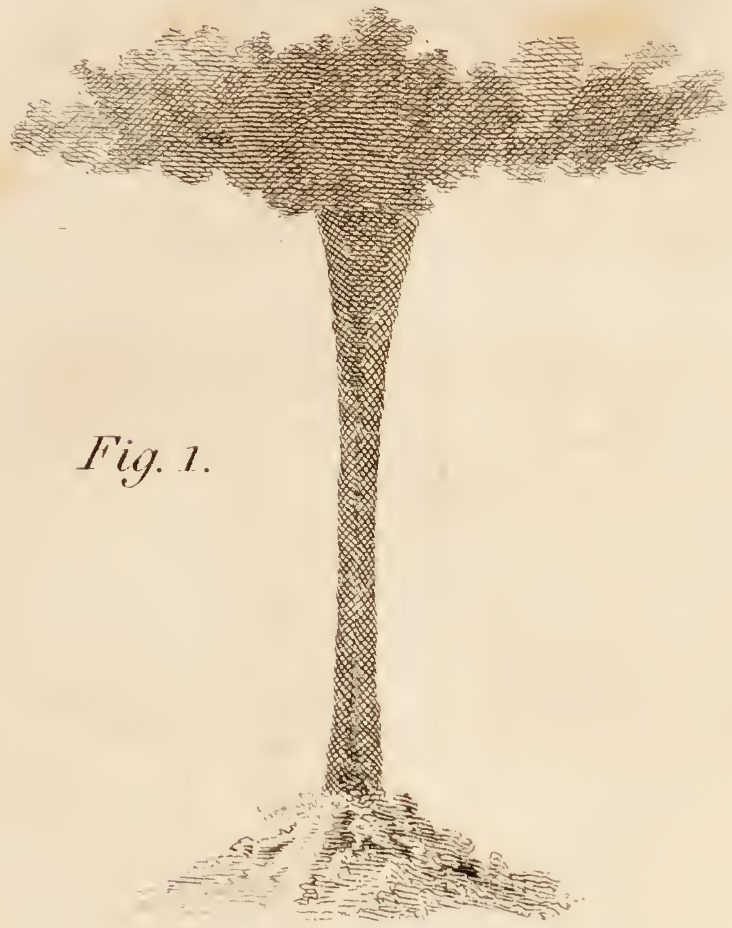
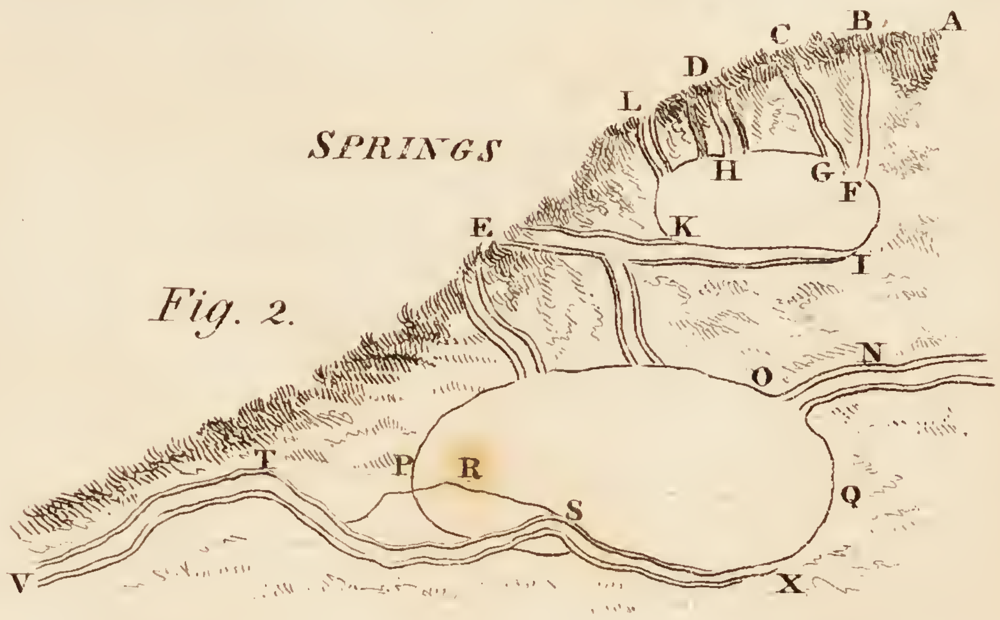


Fig. 1.

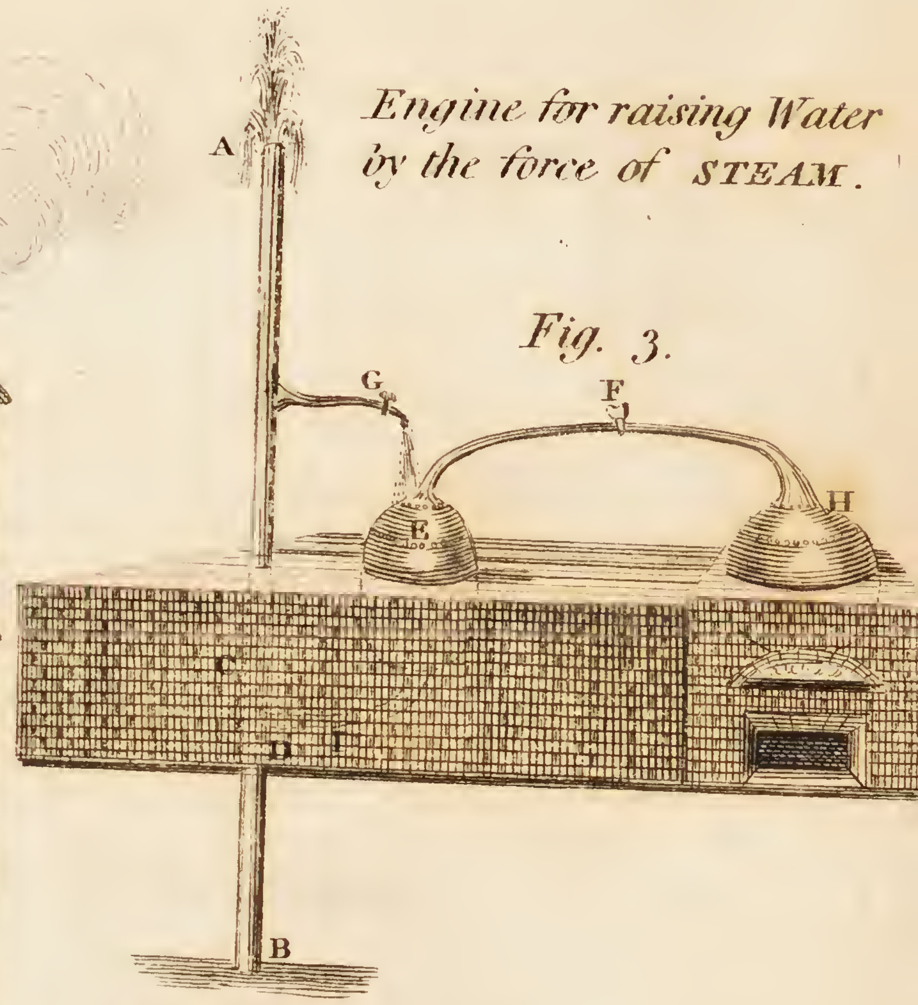
SPRINGS

Fig. 2.



Engine for raising Water by the force of STEAM.

Fig. 3.



STEAM Engine.

Fig. 4.

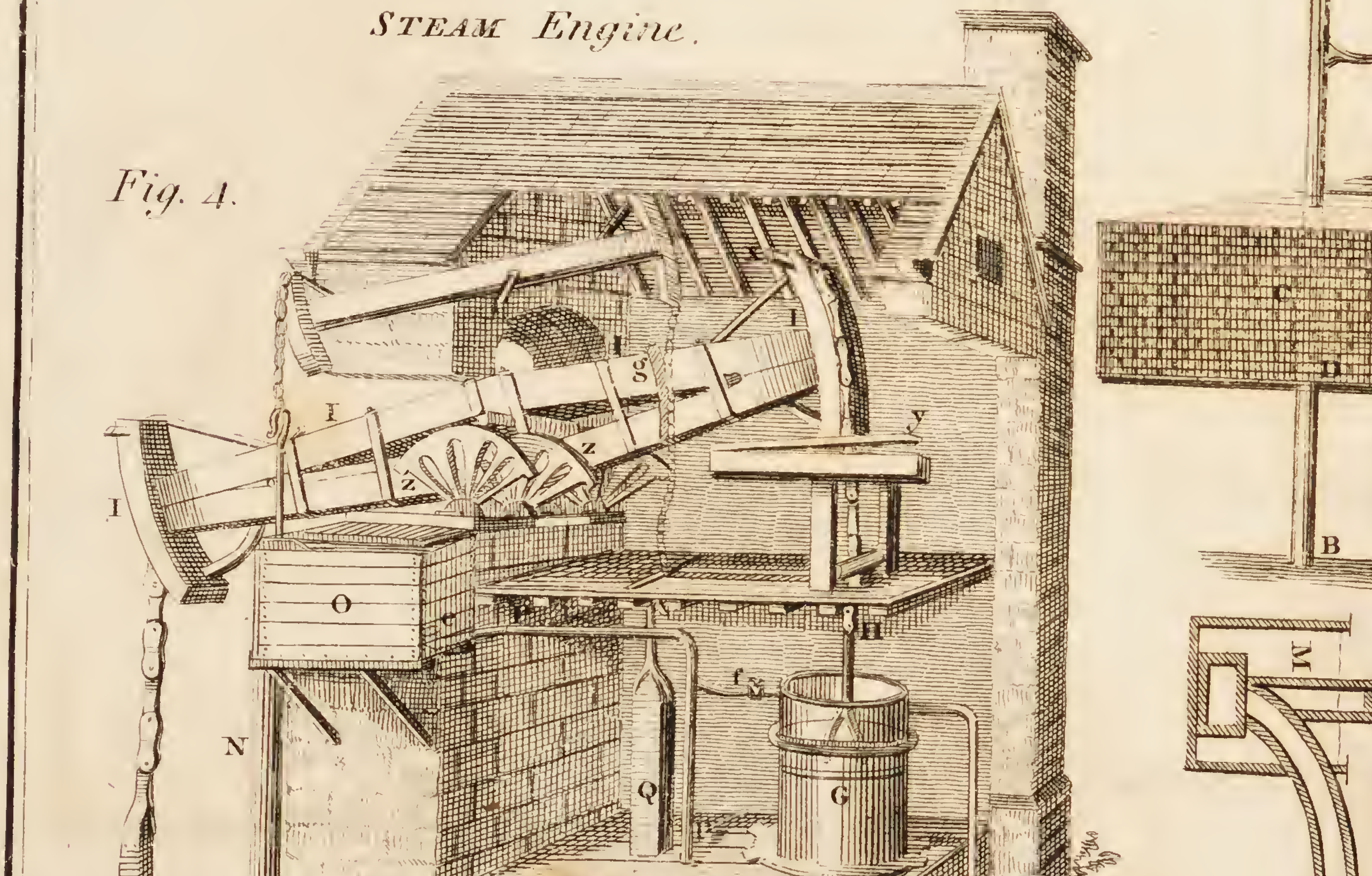
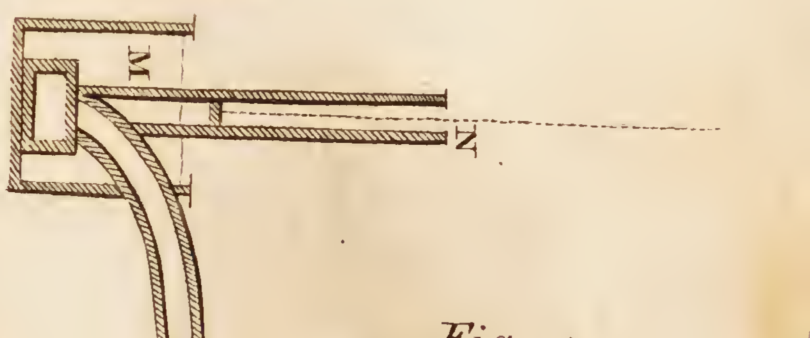


Fig. 5.



PLAN of Primrose Farm.

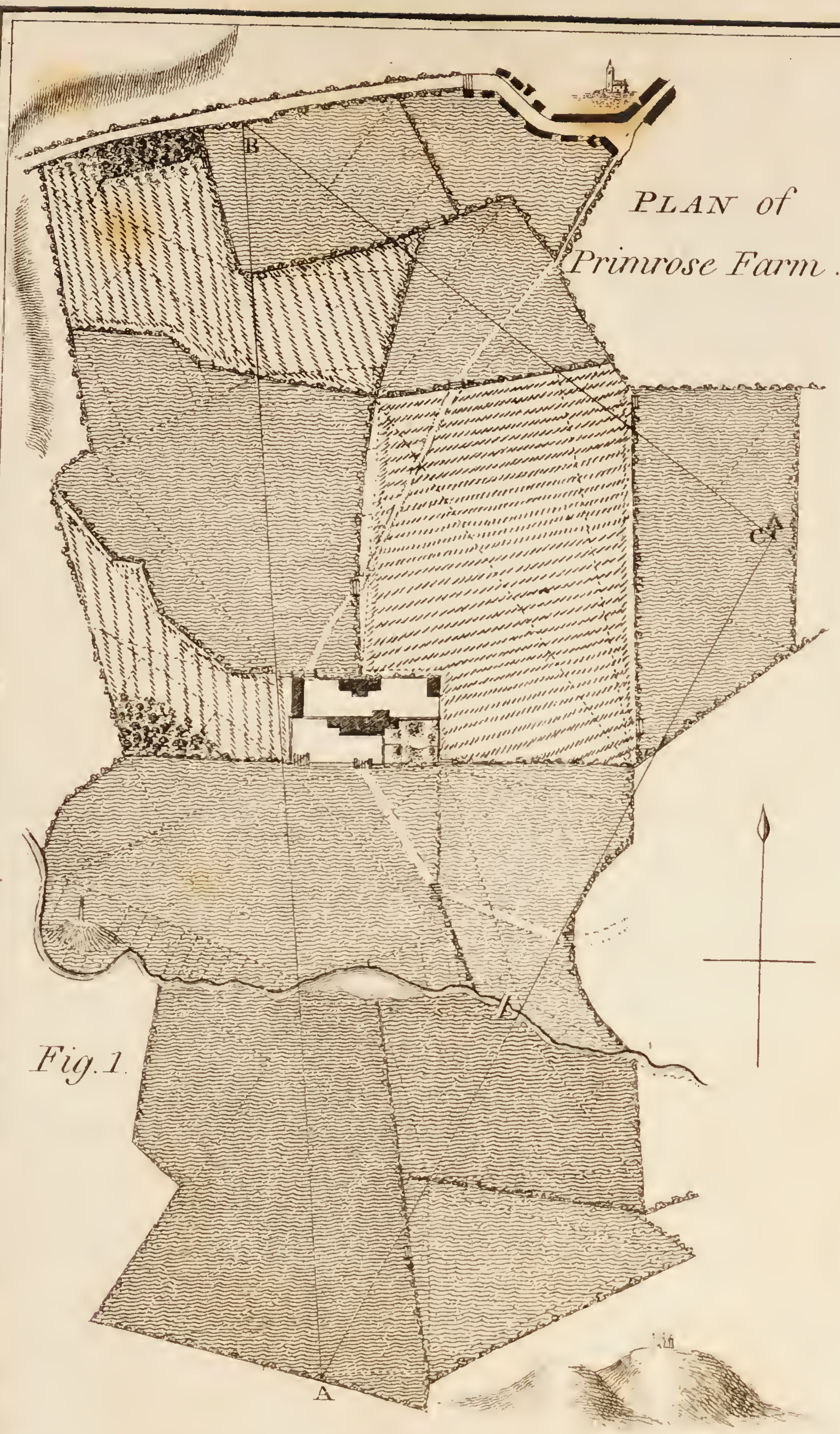
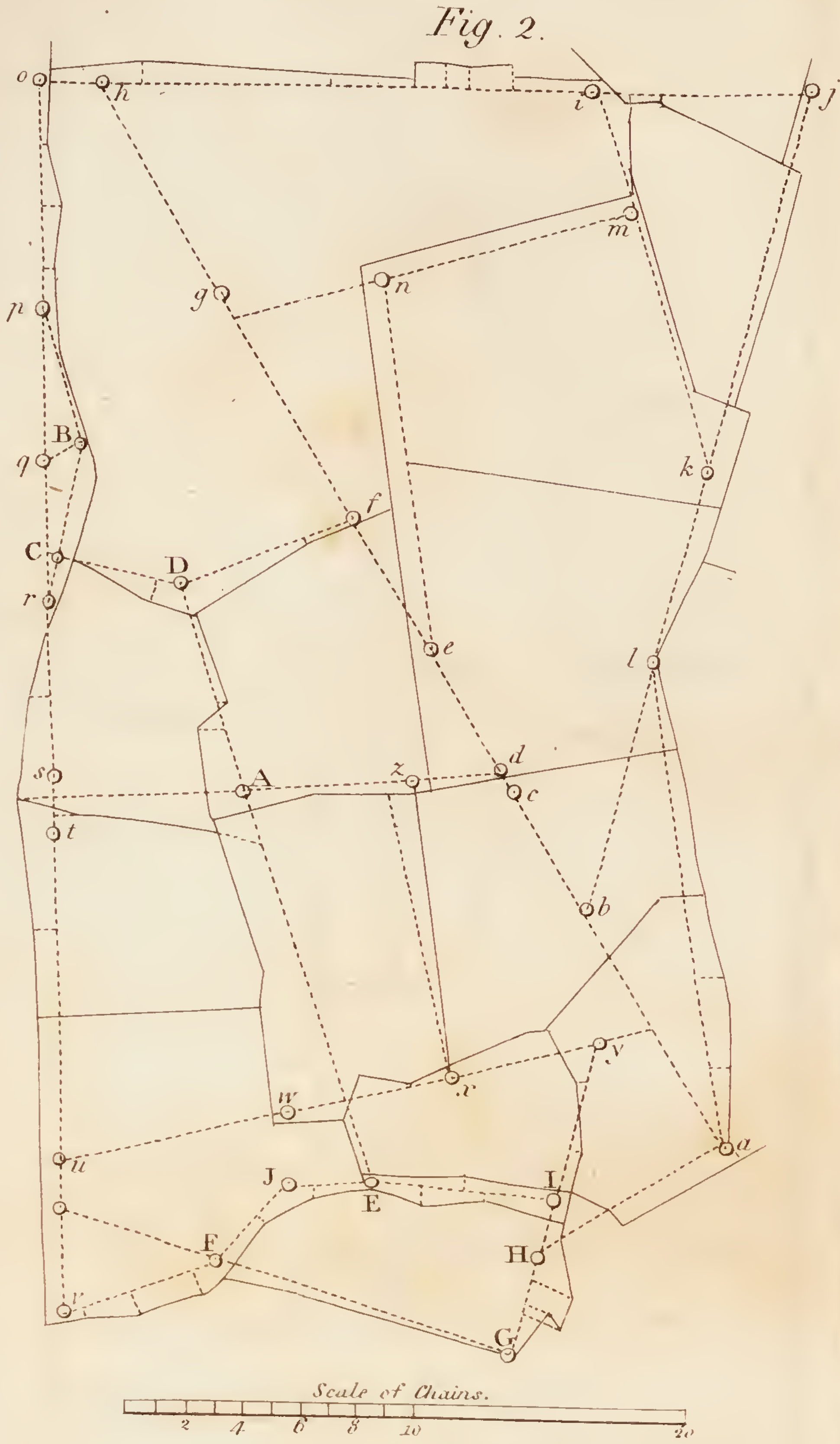


Fig. 1.

Fig. 2.



The French TELEGRAPH.
Fig. 3.

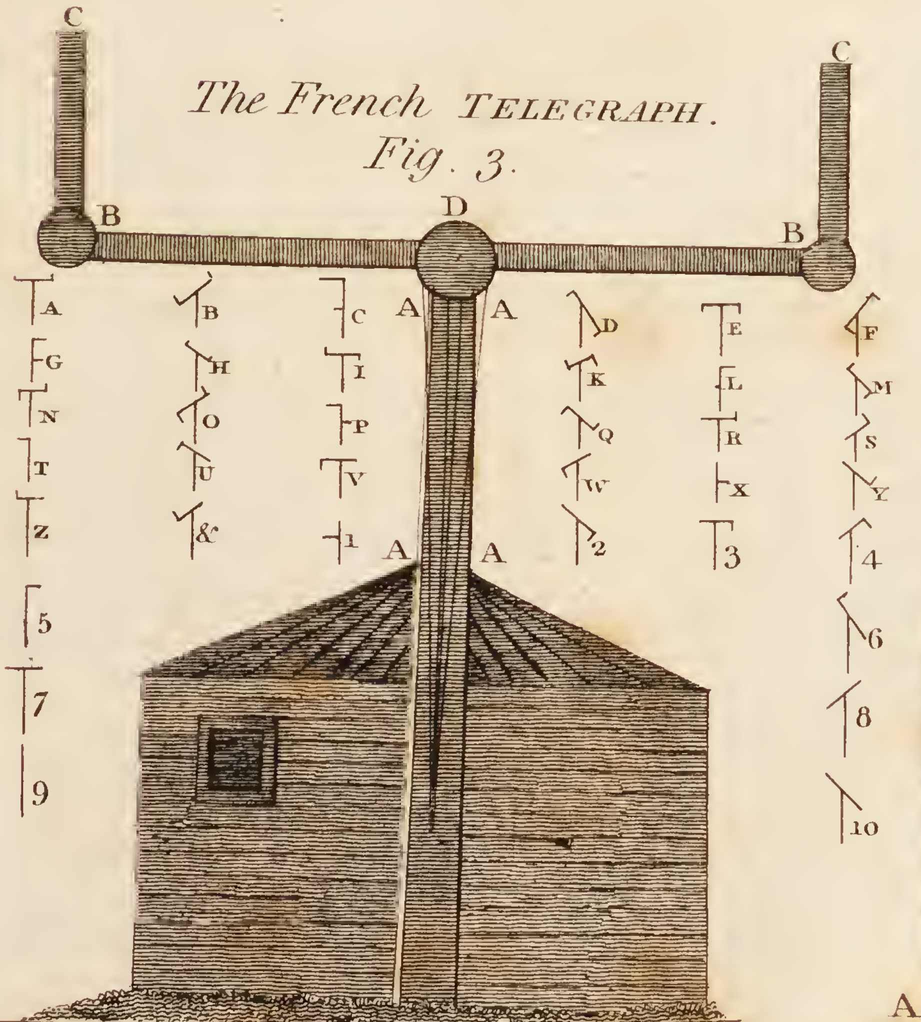
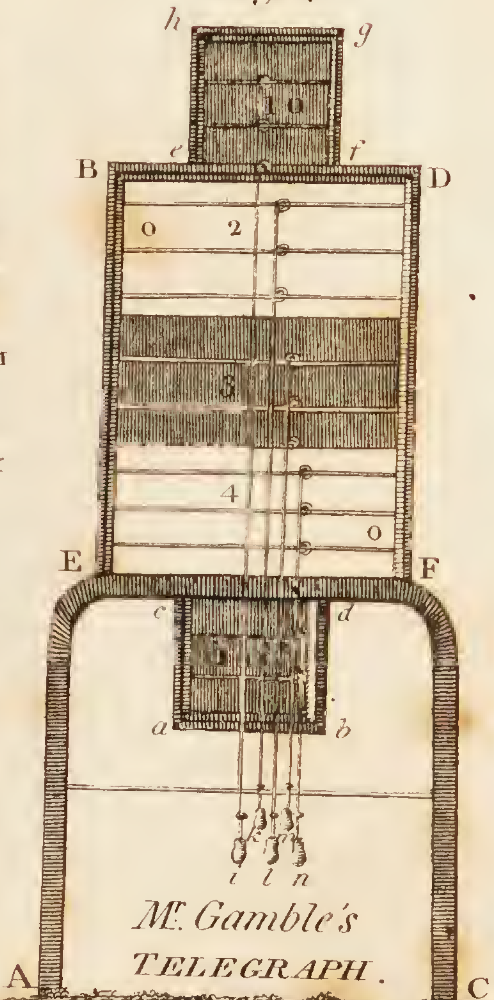
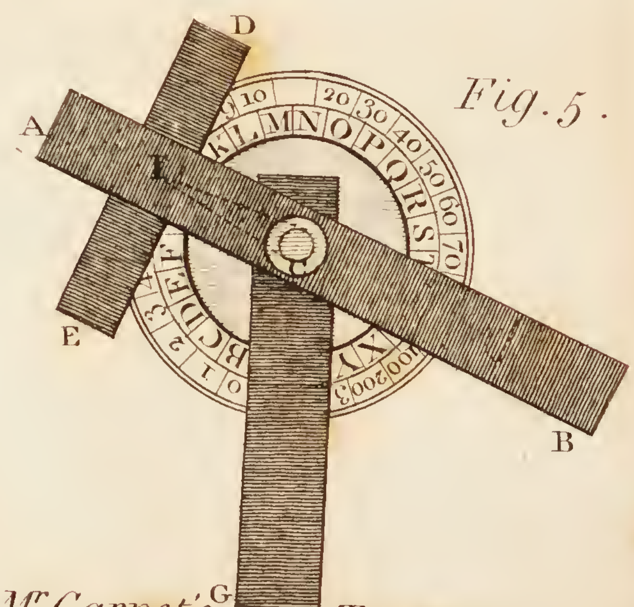


Fig. 4.



Mr. Gamble's TELEGRAPH.

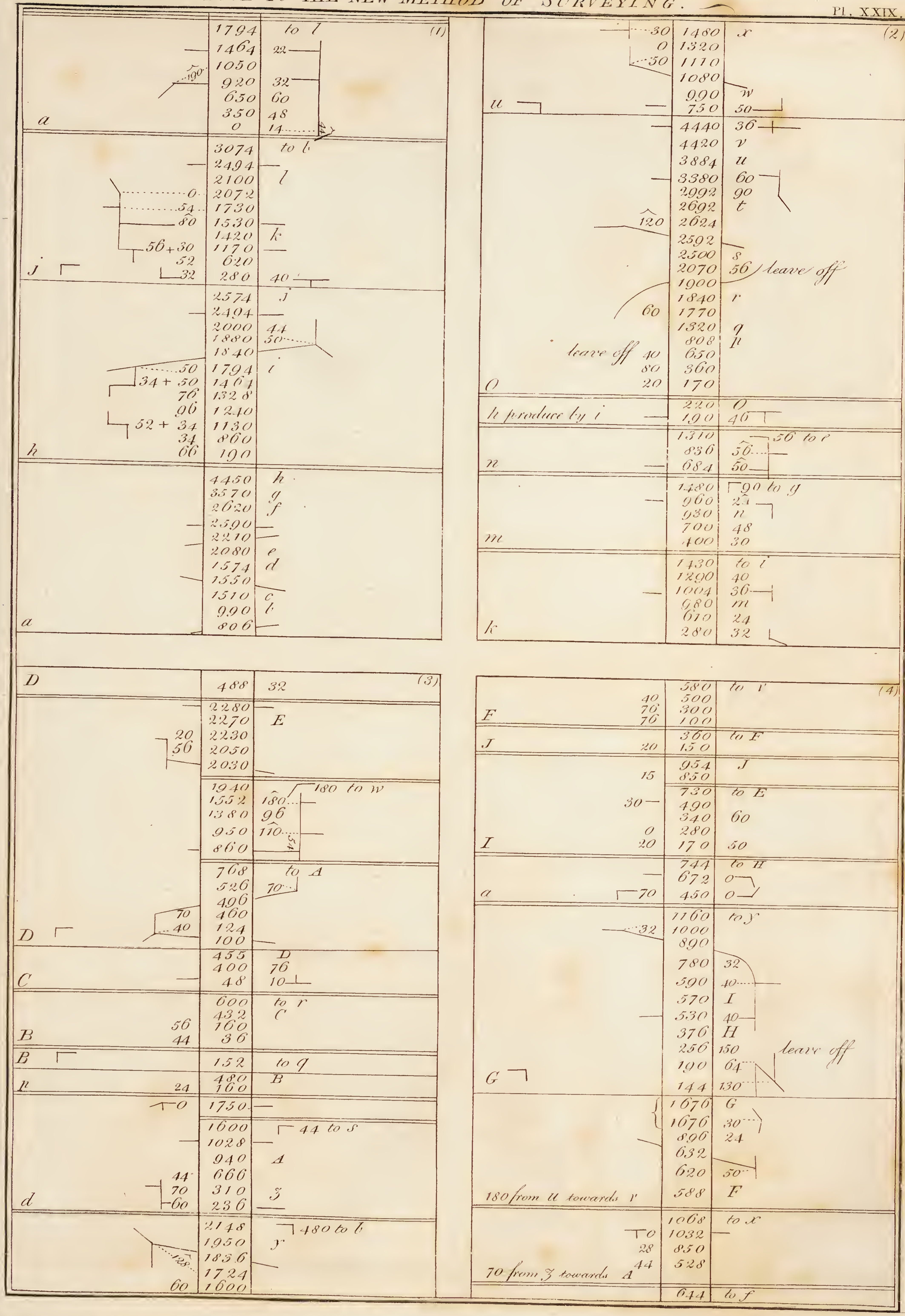
Fig. 5.



Mr. Garnet's TELEGRAPH.

Fig. 6.

FIELD-BOOK TO THE NEW METHOD OF SURVEYING.



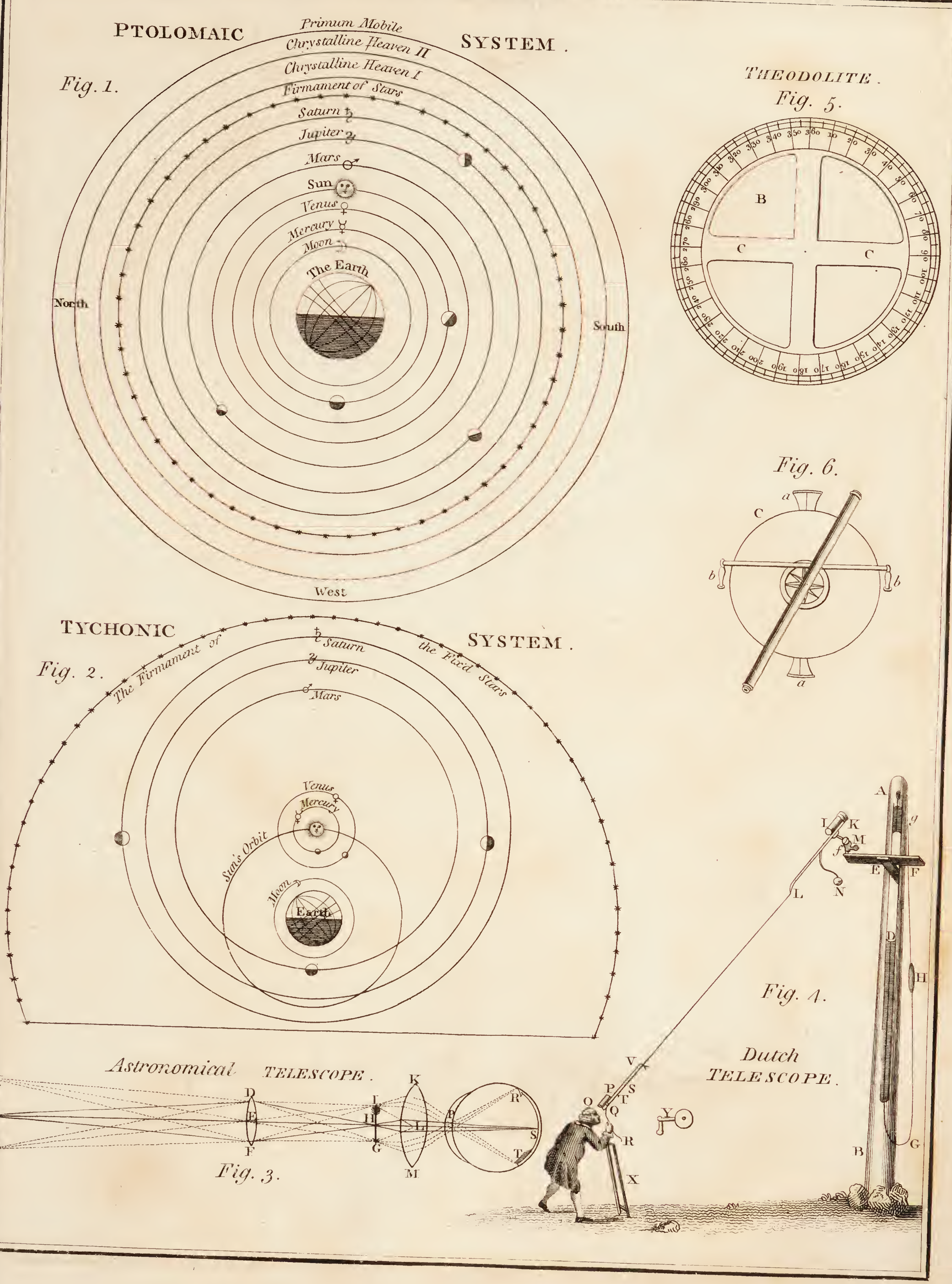

PTOLOMAIC SYSTEM.
Fig. 1.
Primum Mobile
Chrystalline Heaven II
Chrystalline Heaven I
Firmament of Stars
Saturn
Jupiter
Mars
Sun
Venus
Mercury
Moon
The Earth
North
South
West
THEODOLITE.
Fig. 5.
B
C
C
Fig. 6.
a
C
b
b
a
TYCHONIC SYSTEM.
Fig. 2.
The Firmament of the Fixed Stars
Saturn
Jupiter
Mars
Venus
Mercury
Sun's Orbit
Moon
Earth
Fig. 4.
Dutch TELESCOPE.
A
g
I
K
M
f
E
F
N
L
D
H
B
G
V
P
S
T
O
Q
R
X
Y
Astronomical TELESCOPE.
A
B
C
D
E
F
I
H
G
K
L
M
P
R
S
T
Fig. 3.

Plate XXXI.

The COPERNICAN or SOLAR SYSTEM.

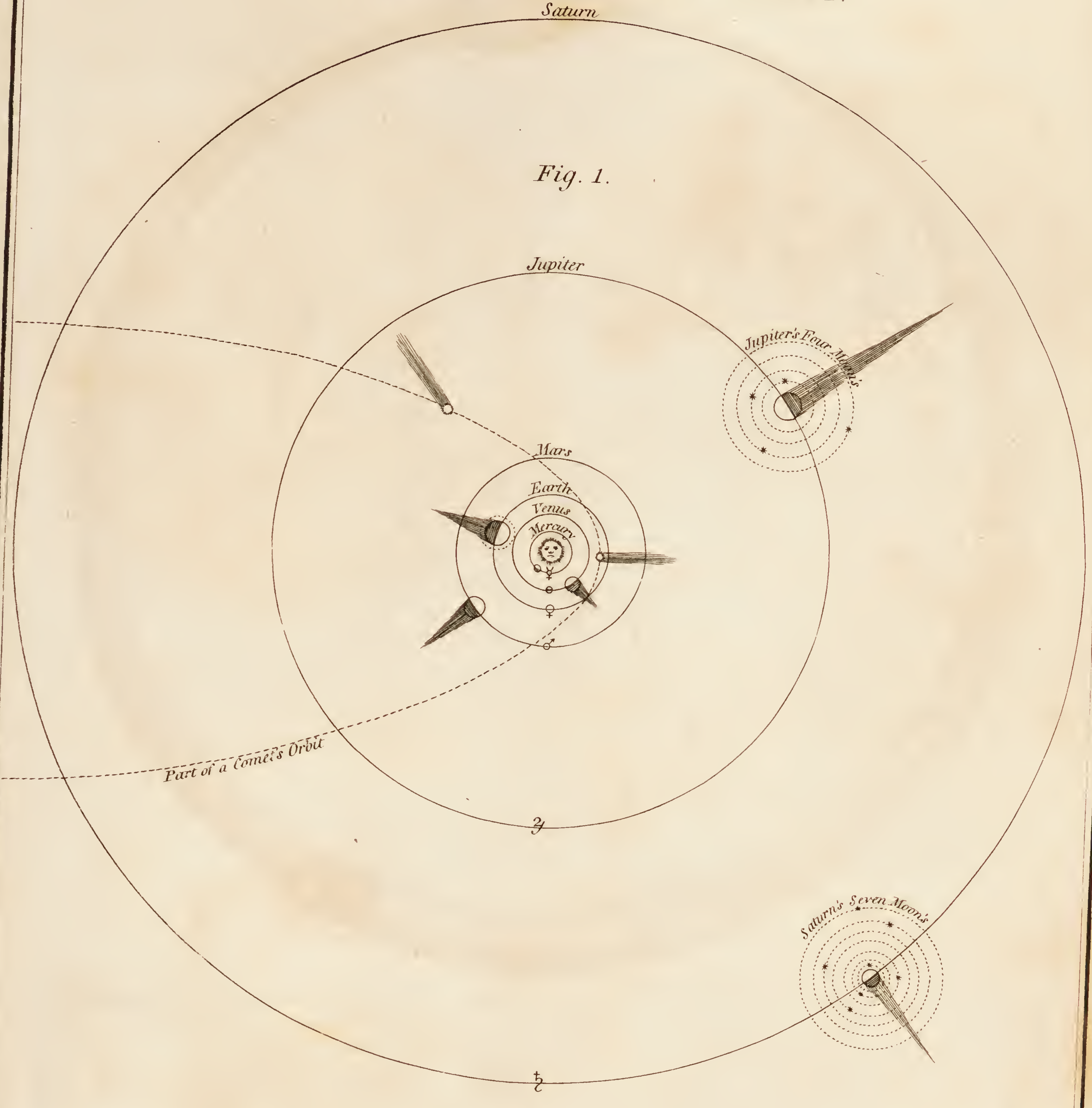


Fig. 2.

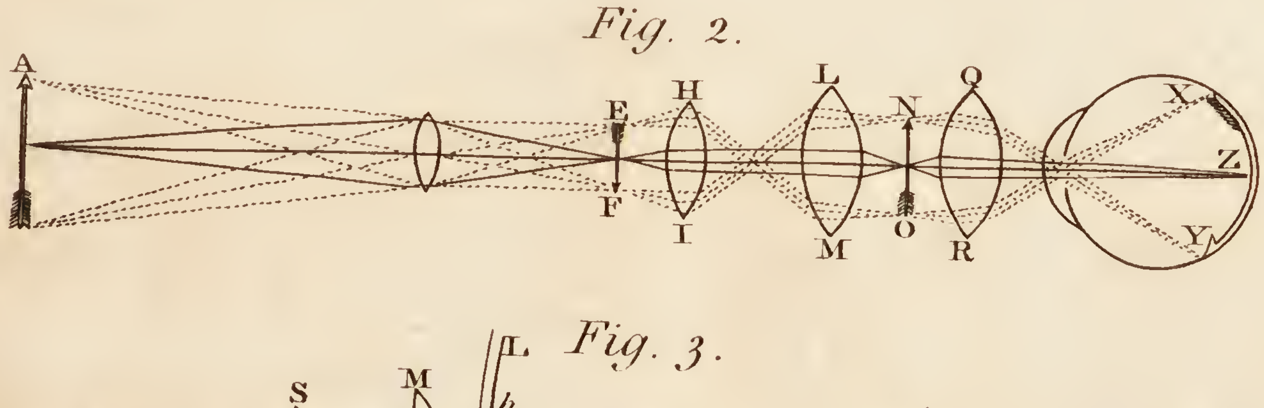
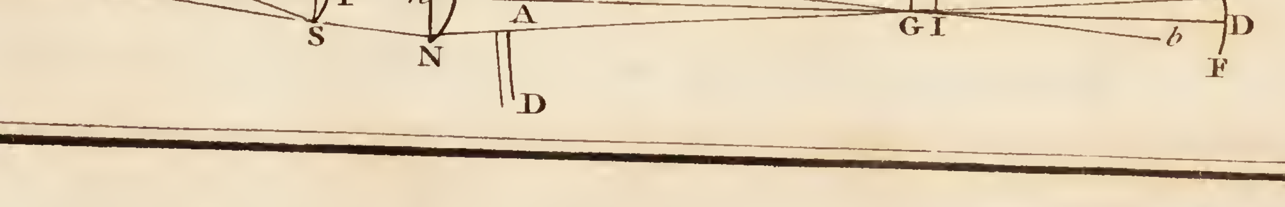


Fig. 3.



UNIVERSAL SOLAR SYSTEM.

Fig. 1.

Fig. 2.

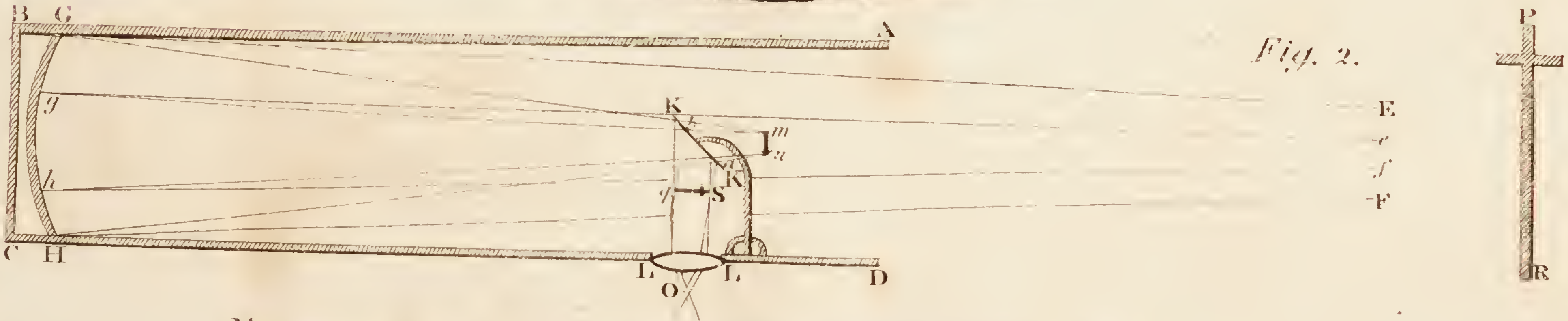
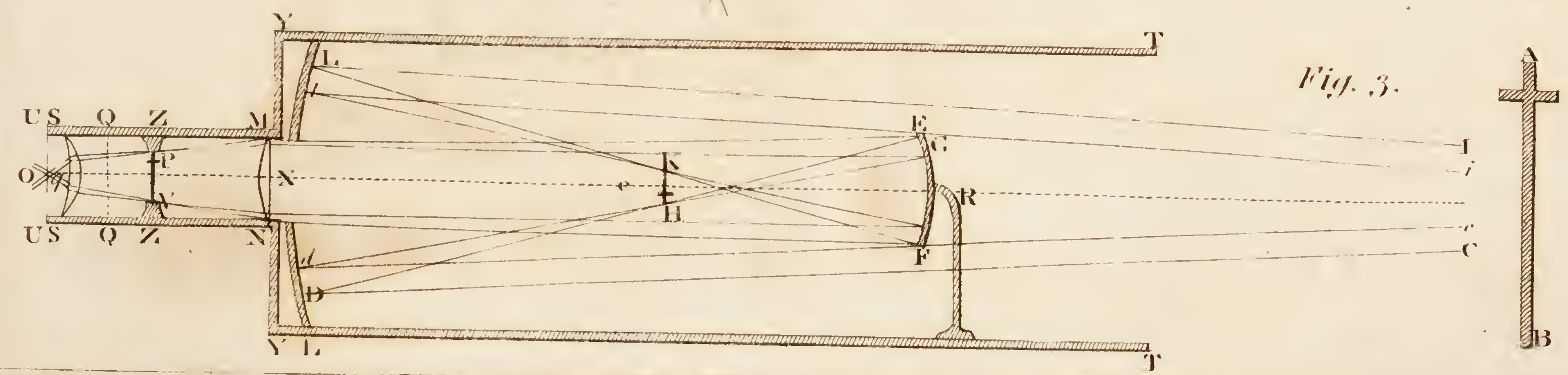


Fig. 3.



Plan & Elevation of a DOME, constructed without Centring.

Fig. 1.

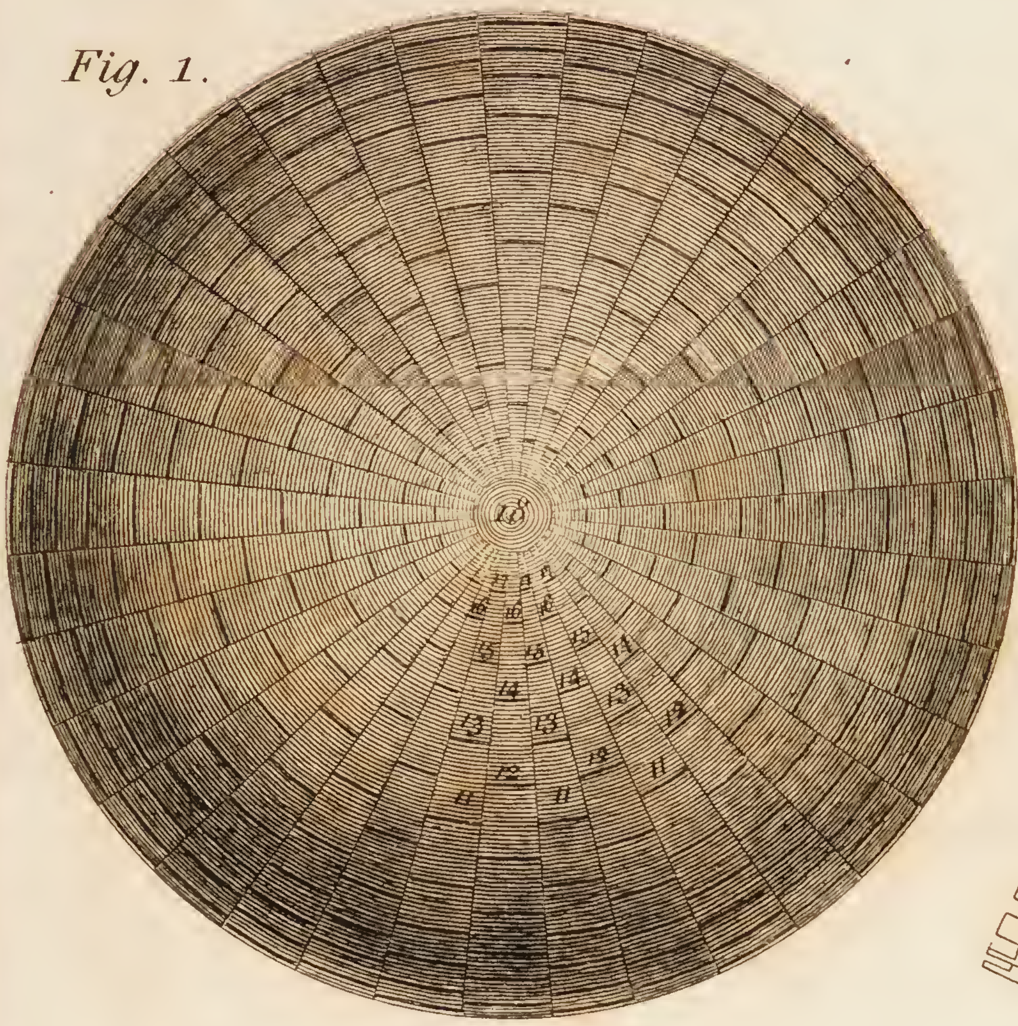


Fig. 2.

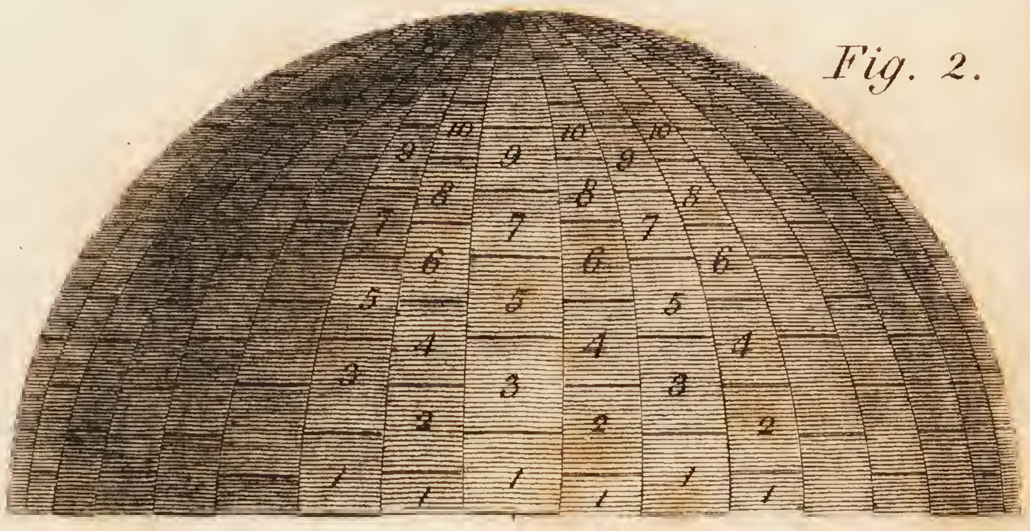


Fig. 3.

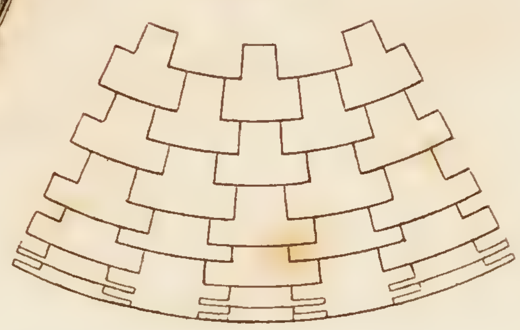
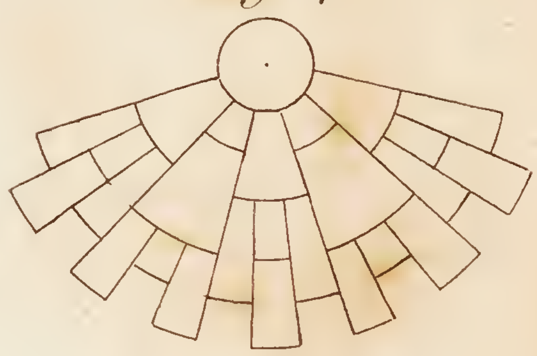
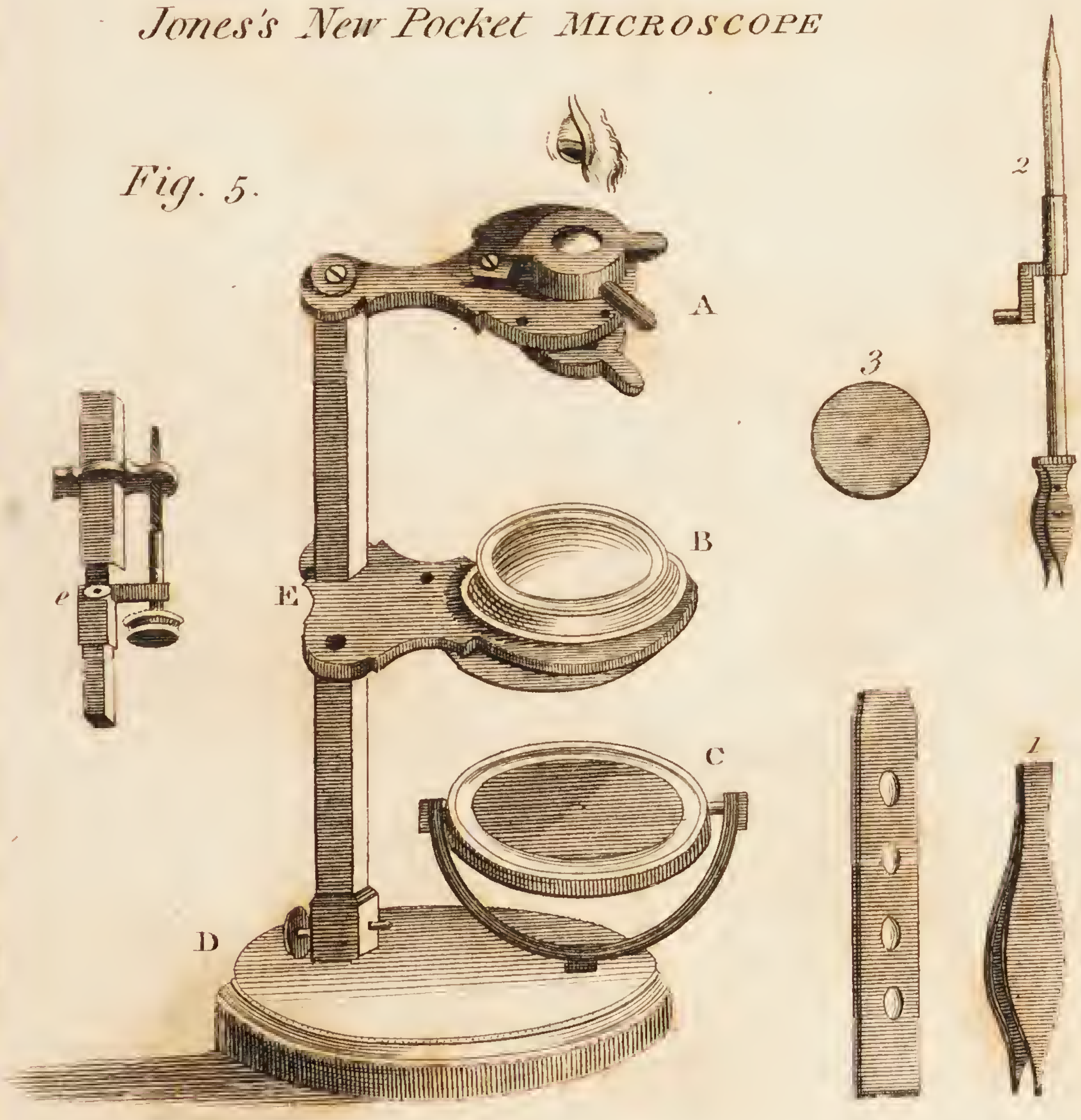


Fig. 4.



Jones's New Pocket MICROSCOPE

Fig. 5.

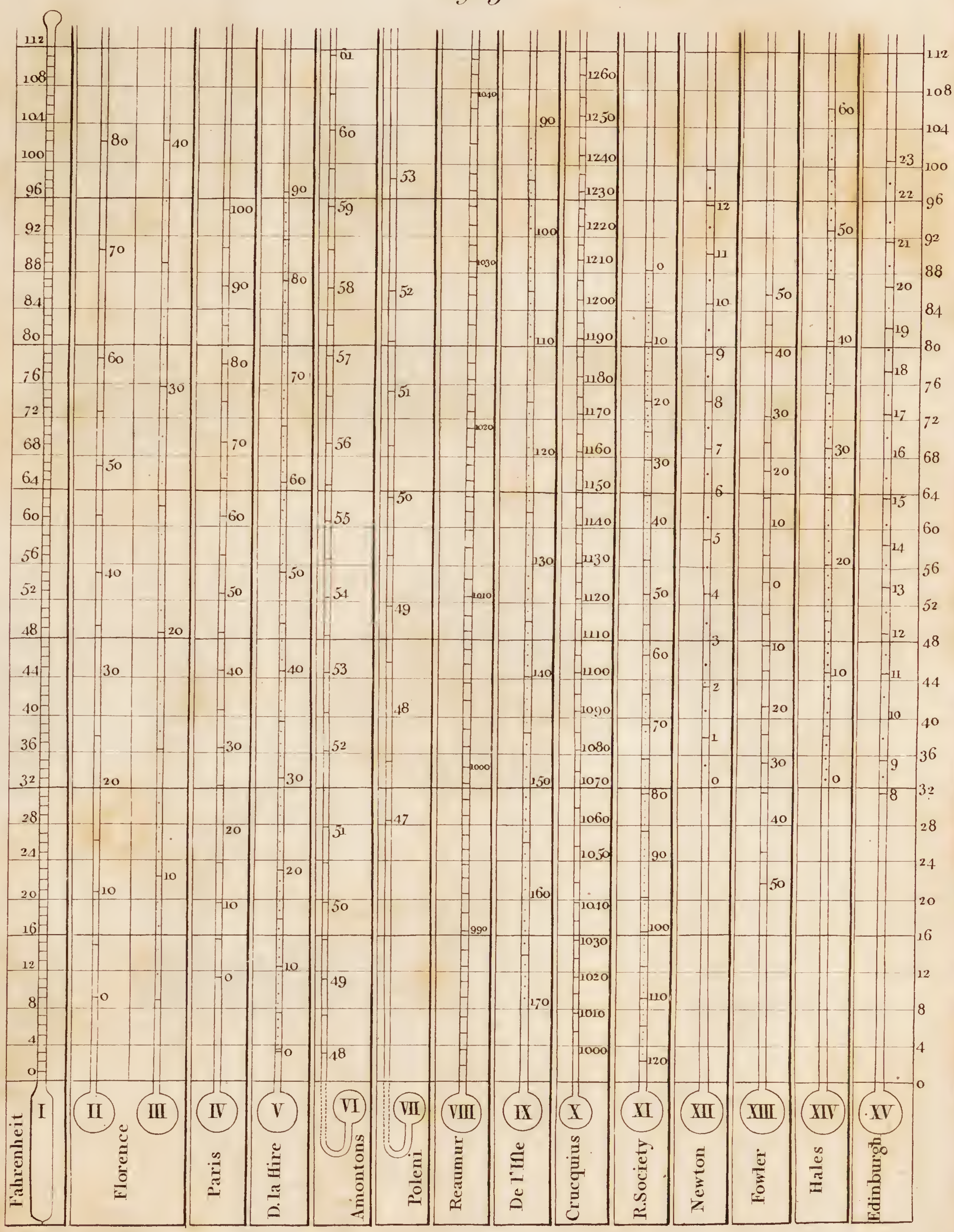


THERMOMETERS.

Fig. 1.

Fig. 2.

Fig. 3.



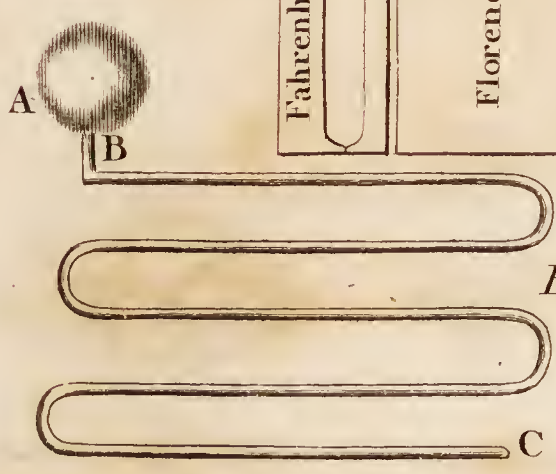
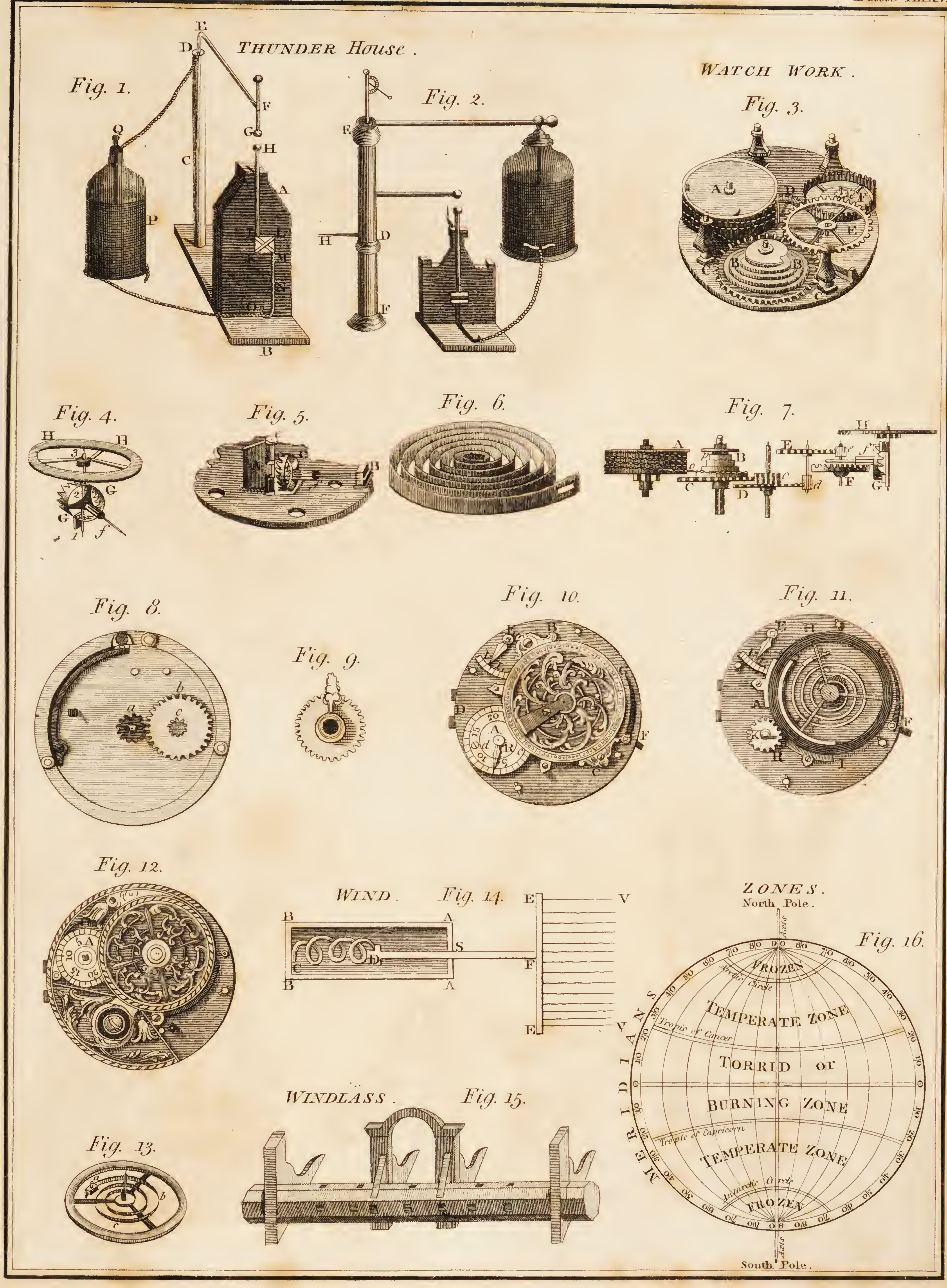


Fig. 4.


THUNDER House.
Fig. 1.
Fig. 2.
WATCH WORK.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
WIND.
Fig. 14.
ZONES.
North Pole.
Fig. 16.
FROZEN
Arctic Circle
TEMPERATE ZONE
Tropic of Cancer
TORRID or
BURNING ZONE
Tropic of Capricorn
TEMPERATE ZONE
Antarctic Circle
FROZEN
Axis
MERIDIANS
South Pole.
WINDLASS.
Fig. 15.
Fig. 13.

WHIRLING TABLE.

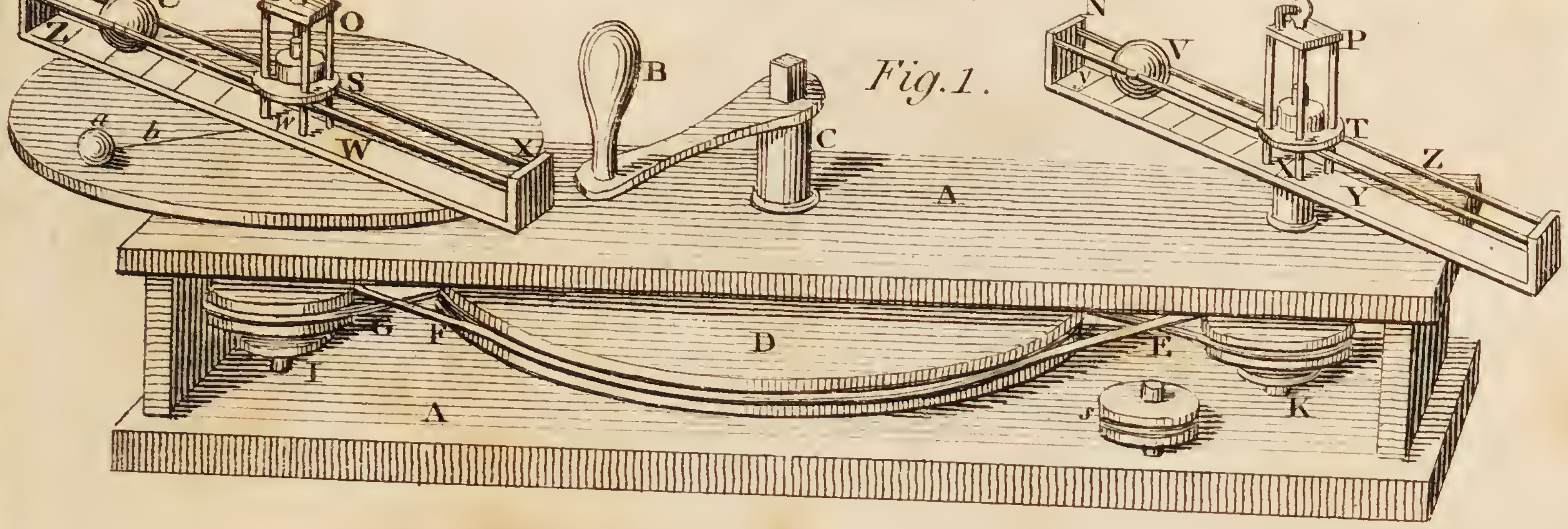


Fig. 1.

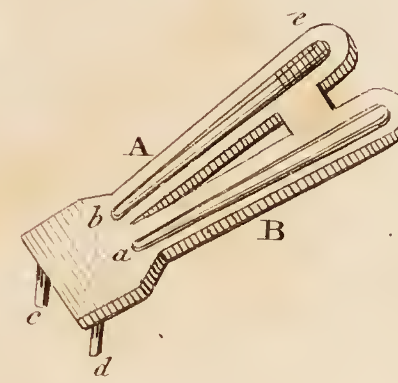


Fig. 2.

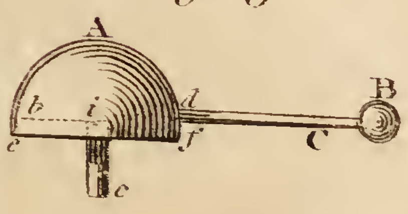


Fig. 3.

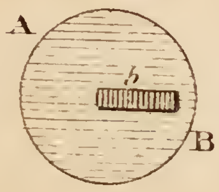


Fig. 4.

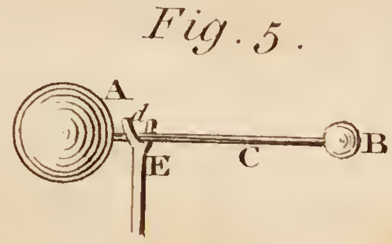


Fig. 5.

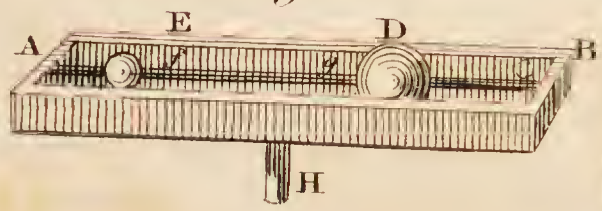


Fig. 6.

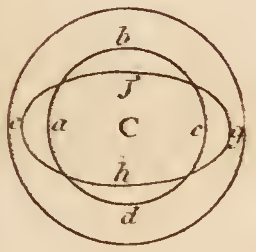


Fig. 7.

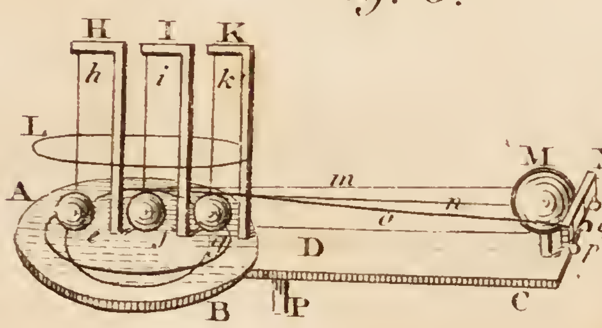


Fig. 8.

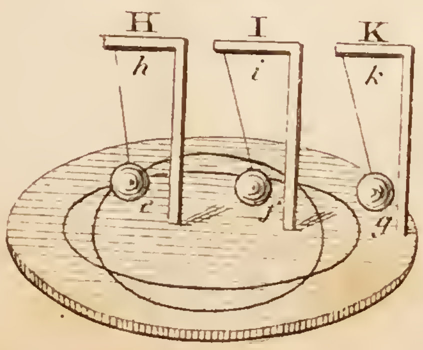


Fig. 9.

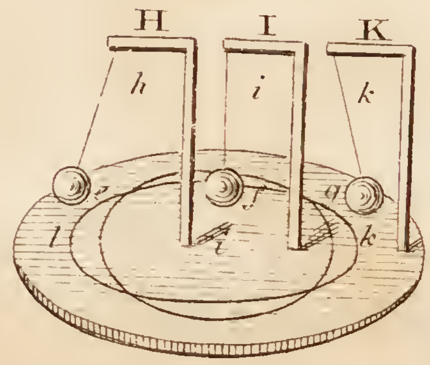


Fig. 10.

WIND - MILL

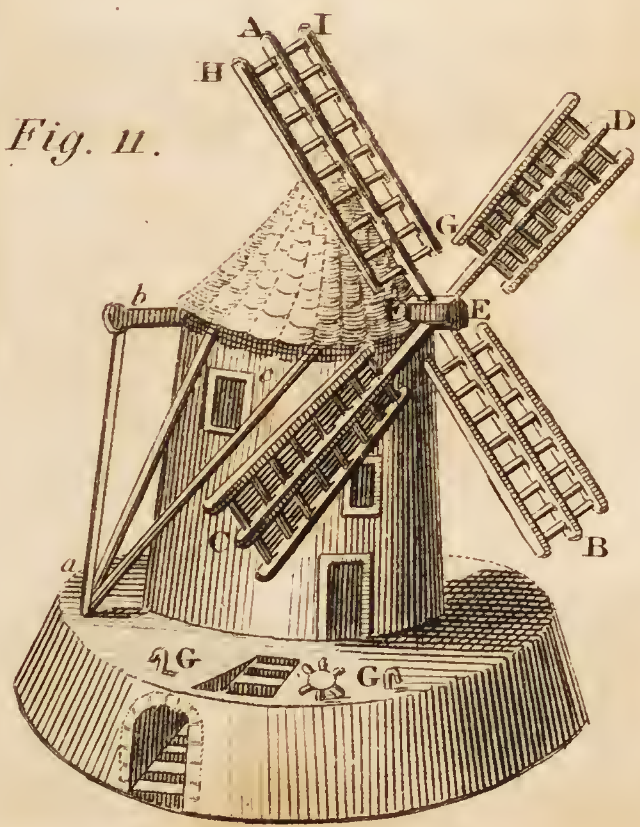


Fig. 11.

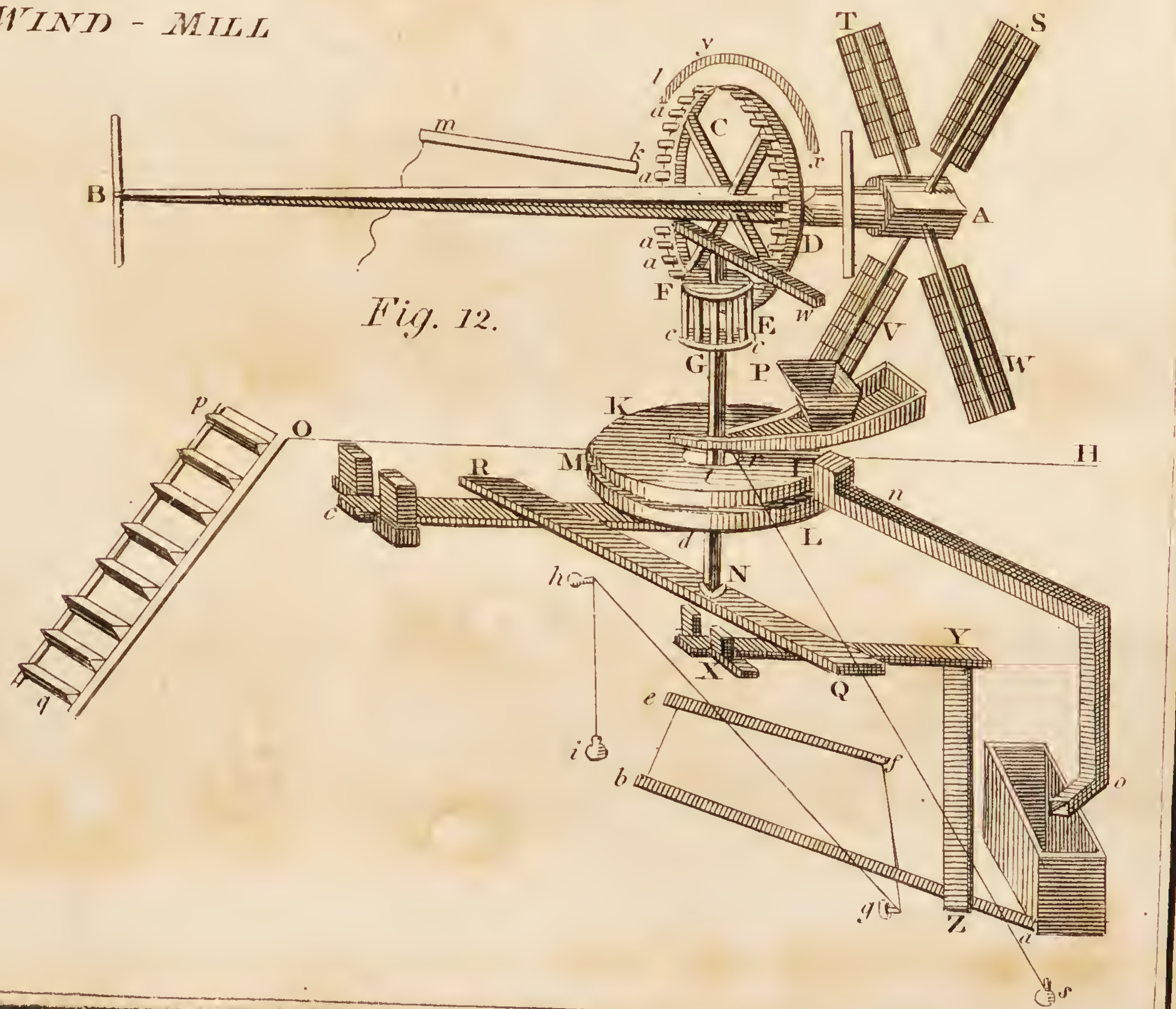
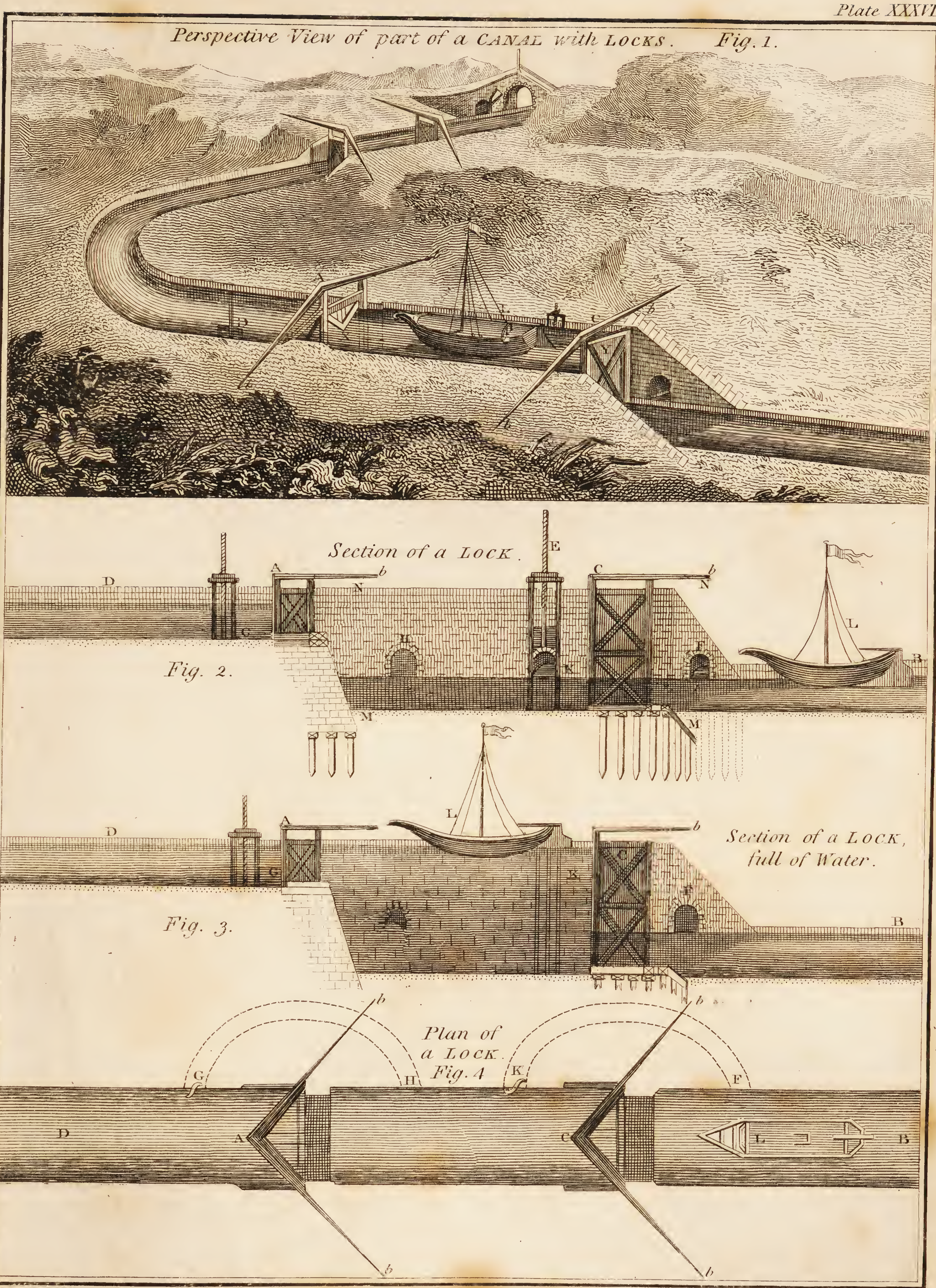


Fig. 12.


Perspective View of part of a CANAL with LOCKS. Fig. 1.
Section of a LOCK.
Fig. 2.
A
b
N
C
b
N
E
D
G
H
K
F
L
B
M
M
Section of a LOCK, full of Water.
Fig. 3.
Plan of a LOCK. Fig. 4
G
H
K
F
A
C
D
L
B
b
b
b
b